# Computer Algebra
# Independent Integration Tests

Summer 2023 edition

## 7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/188-7.1.5-Inverse-hyperbolic-sine-functions

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# CHAPTER 1

# INTRODUCTION


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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 371 ]. This is test number [ 188 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 99.73 ( 370 ) | 0.27 ( 1 ) |
| Mathematica | 99.73 ( 370 ) | 0.27 ( 1 ) |
| Maple | 67.12 ( 249 ) | 32.88 ( 122 ) |
| Fricas | 42.59 ( 158 ) | 57.41 ( 213 ) |
| Sympy | 27.22 ( 101 ) | 72.78 ( 270 ) |
| Maxima | 26.95 ( 100 ) | 73.05 ( 271 ) |
| Giac | 26.42 ( 98 ) | 73.58 ( 273 ) |
| Mupad | 21.02 ( 78 ) | 78.98 ( 293 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

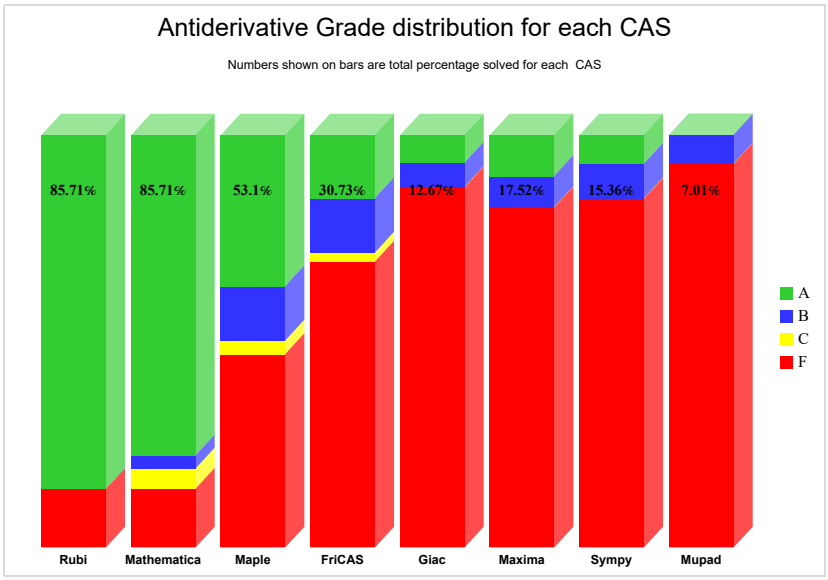
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

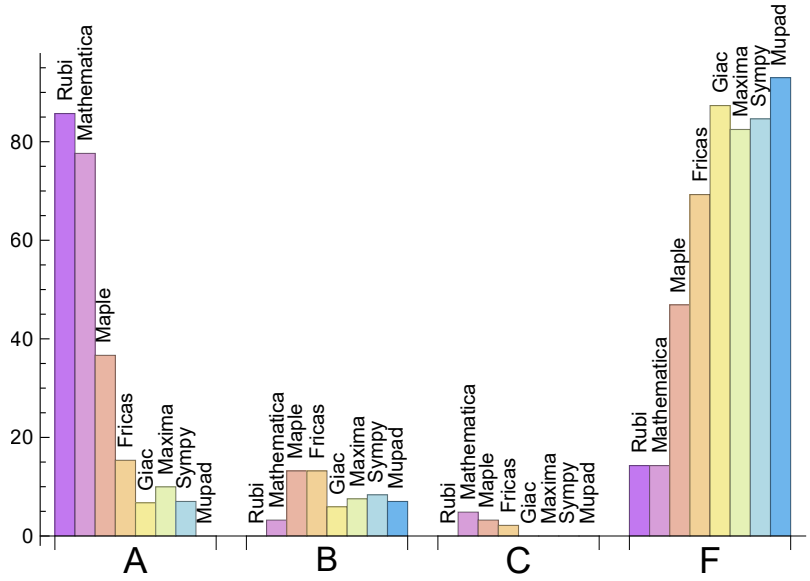
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 85.714 | 0.000 | 0.000 | 14.286 |
| Mathematica | 77.628 | 3.235 | 4.852 | 14.286 |
| Maple | 36.658 | 13.208 | 3.235 | 46.900 |
| Fricas | 15.364 | 13.208 | 2.156 | 69.272 |
| Maxima | 9.973 | 7.547 | 0.000 | 82.480 |
| Sympy | 7.008 | 8.356 | 0.000 | 84.636 |
| Giac | 6.739 | 5.930 | 0.000 | 87.332 |
| Mupad | 0.000 | 7.008 | 0.000 | 92.992 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of

error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 1 | 100.00 | 0.00 | 0.00 |
| Mathematica | 1 | 100.00 | 0.00 | 0.00 |
| Maple | 122 | 100.00 | 0.00 | 0.00 |
| Fricas | 213 | 59.62 | 0.00 | 40.38 |
| Maxima | 271 | 80.07 | 1.85 | 18.08 |
| Sympy | 270 | 83.33 | 6.30 | 10.37 |
| Giac | 273 | 82.42 | 0.37 | 17.22 |
| Mupad | 293 | 0.00 | 100.00 | 0.00 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Rubi | 0.23 |
| Fricas | 0.26 |
| Maple | 0.47 |
| Maxima | 1.34 |
| Mupad | 2.72 |
| Mathematica | 3.05 |
| Giac | 3.50 |
| Sympy | 7.12 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 33.54 | 1.08 | 25.00 | 1.00 |
| Giac | 105.78 | 1.62 | 34.50 | 1.11 |
| Fricas | 172.00 | 2.23 | 110.00 | 1.72 |
| Rubi | 181.34 | 1.00 | 132.50 | 1.00 |
| Mathematica | 201.22 | 1.07 | 115.00 | 0.99 |
| Sympy | 261.81 | 2.25 | 61.00 | 1.44 |
| Maple | 271.97 | 1.45 | 114.00 | 1.10 |
| Maxima | 294.87 | 10.92 | 92.00 | 1.35 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

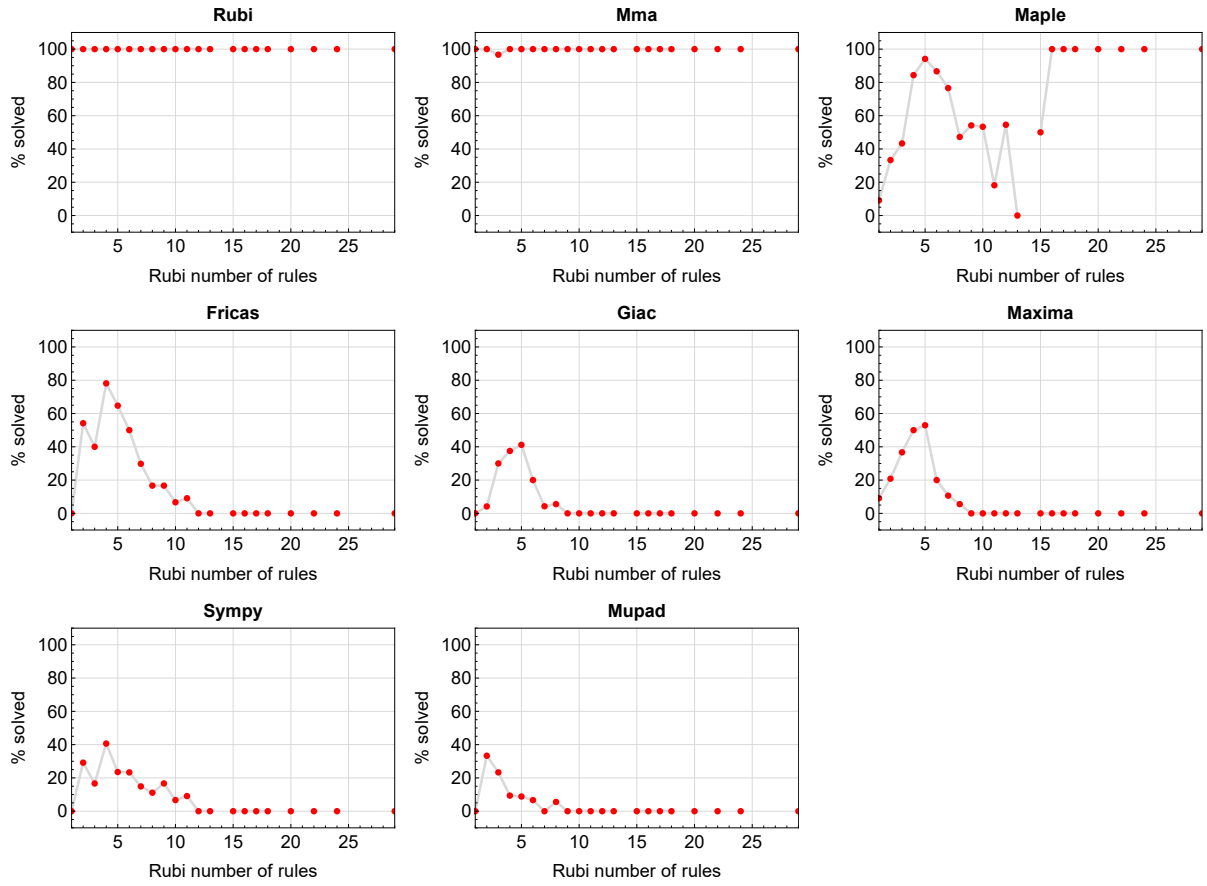



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

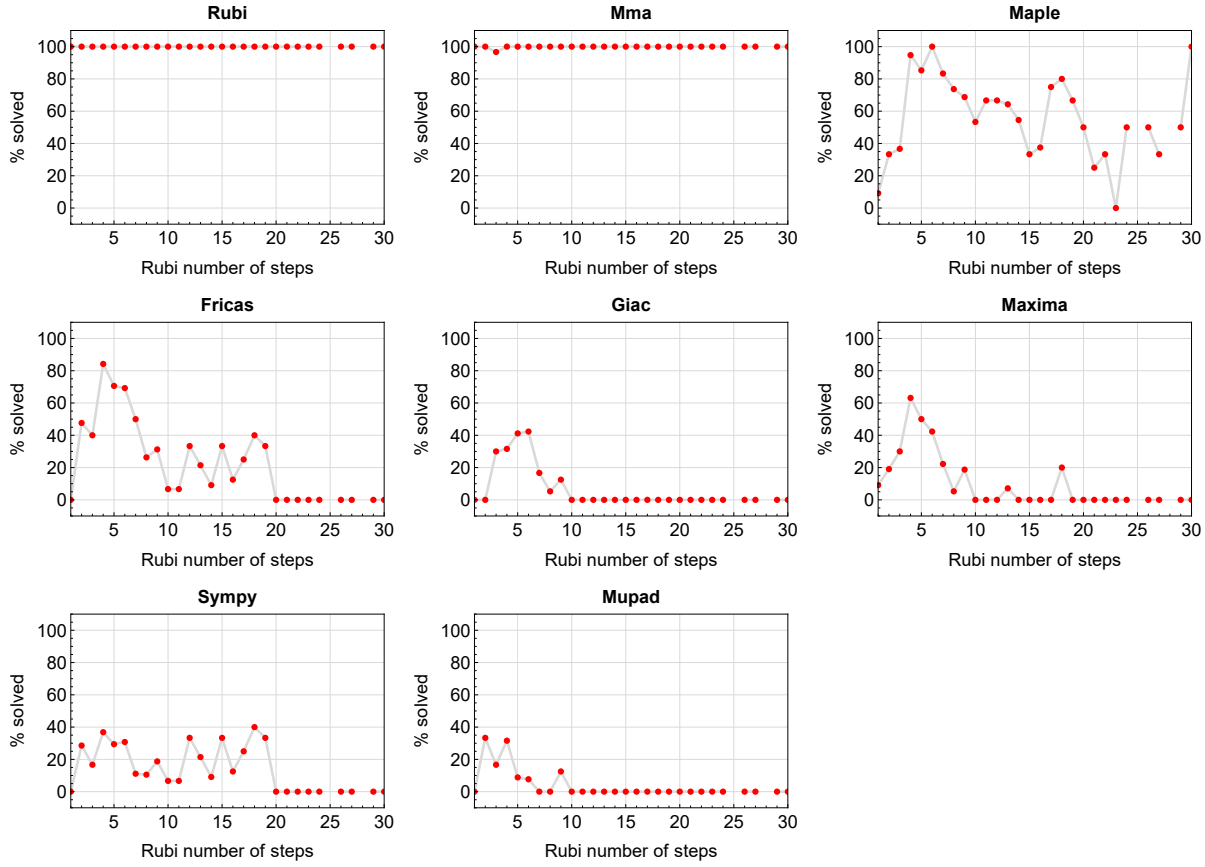



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

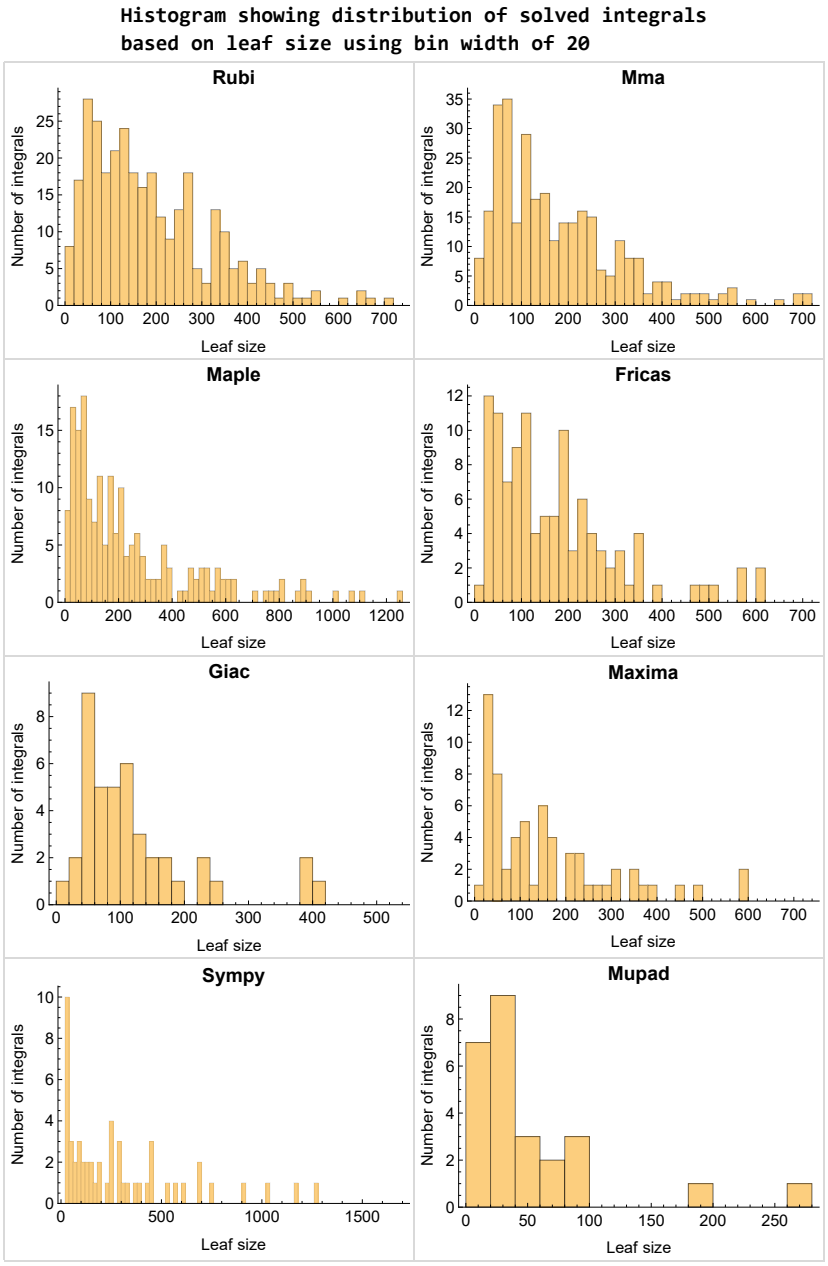



Figure 1.3: Solved integrals based on leaf size distribution

# 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

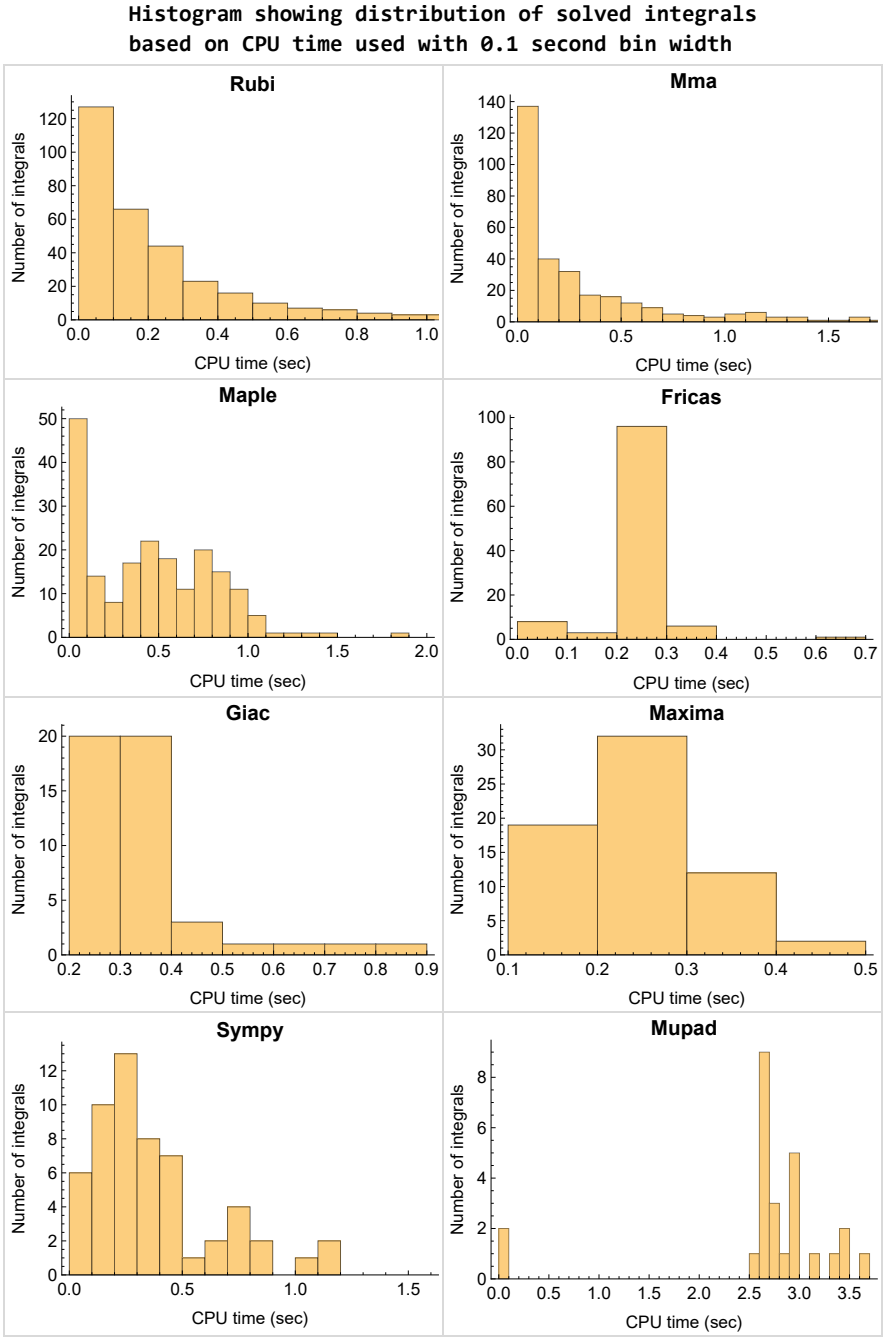



Figure 1.4: Solved integrals histogram based on CPU time used

# 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

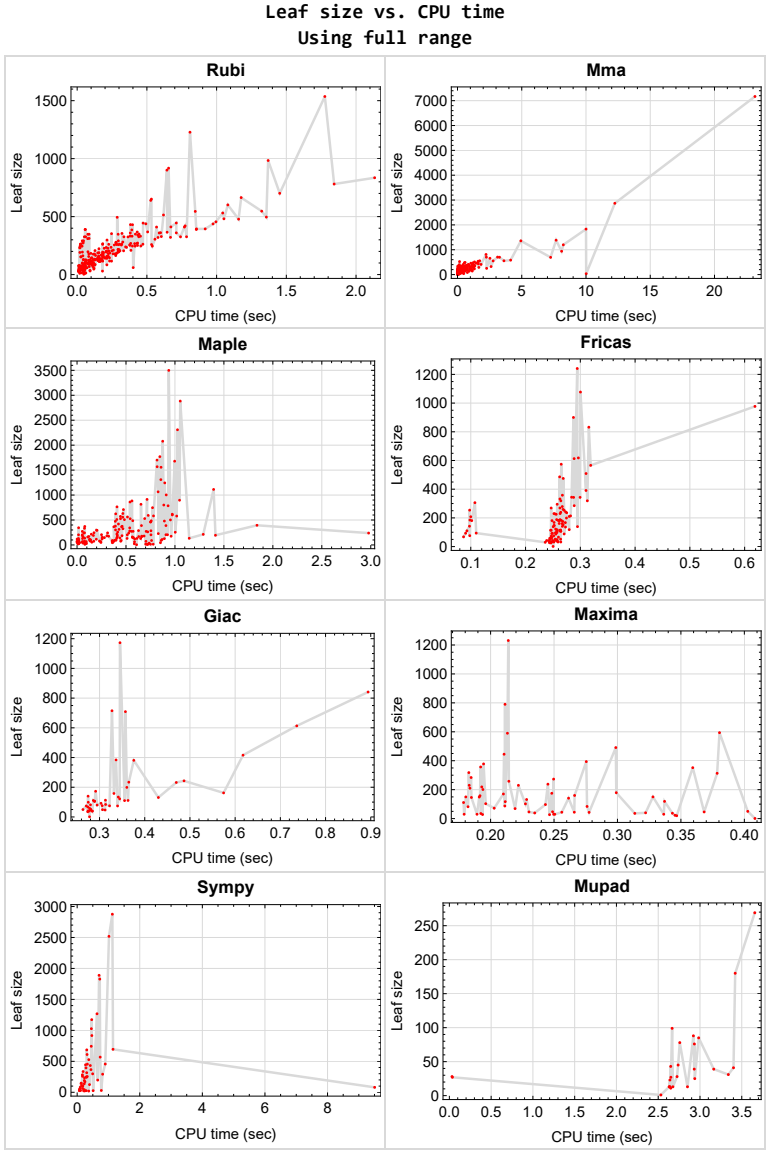



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {}

**Mathematica** {37, 38, 42, 46, 145, 154, 369}

**Maple** {}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

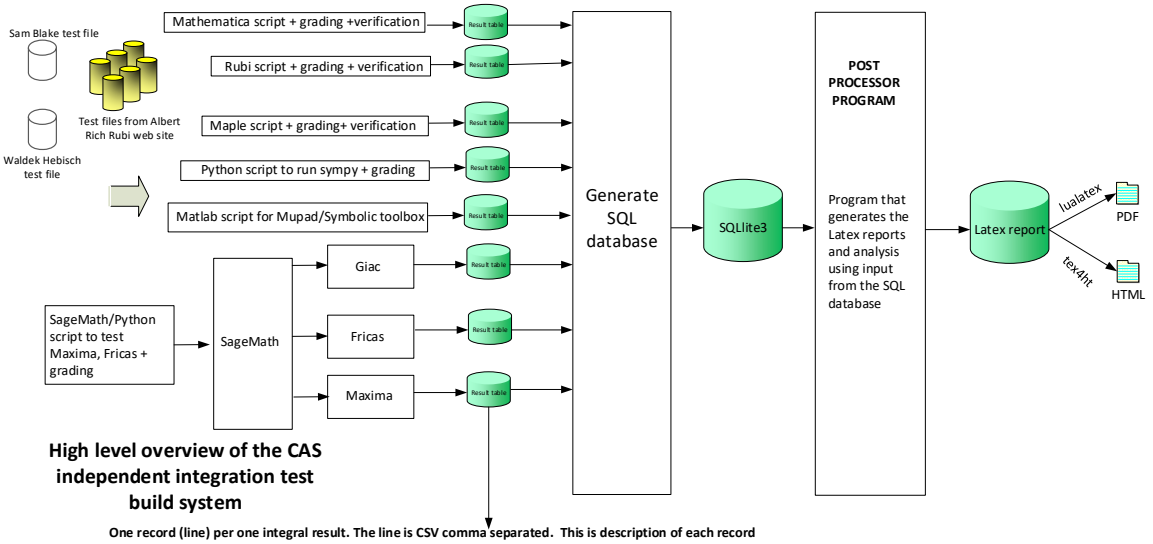
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.




**High level overview of the CAS independent integration test build system**

**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

# CHAPTER 2

# DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS . . . . . 22
2.2 Detailed conclusion table per each integral for all CAS systems . . . . . 27
2.3 Detailed conclusion table specific for Rubi results . . . . . 102

## 2.1 List of integrals sorted by grade for each CAS

Rubi . . . . . 22
Mma . . . . . 23
Maple . . . . . 23
Fricas . . . . . 24
Maxima . . . . . 24
Giac . . . . . 25
Mupad . . . . . 25
Sympy . . . . . 26

### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 366, 367, 368, 370, 371 }

**B grade** { }

**C grade** { }

**F normal fail** { 369 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 153, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 198, 199, 200, 201, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 285, 287, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 366, 367, 370, 371 }

**B grade** { 61, 103, 104, 119, 145, 152, 154, 196, 202, 302, 368, 369 }

**C grade** { 37, 38, 42, 46, 74, 125, 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

**F normal fail** { 31 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 17, 19, 20, 21, 22, 25, 26, 27, 37, 42, 50, 51, 58, 59, 60, 61, 63, 64, 65, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 81, 82, 83, 85, 86, 87, 89, 90, 91, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 137, 138, 139, 140, 141, 143, 144, 145, 147, 148, 154, 156, 157, 158, 159, 160, 164, 165, 166, 171, 172, 177, 178, 260, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 285, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 317, 324, 345, 348, 351, 353, 364, 365, 366, 368 }

**B grade** { 9, 10, 11, 18, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 52, 62, 66, 132, 142, 149, 150, 151, 152, 153, 162, 163, 168, 169, 170, 174, 175, 176, 266, 267, 268, 280, 343, 344, 349, 350, 352, 354, 355, 356, 357 }

**C grade** { 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

**F normal fail** { 2, 3, 16, 31, 54, 55, 56, 71, 78, 79, 80, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 287, 291, 307, 308, 309, 310, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 358, 359, 360, 361, 367, 369, 370, 371 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 4, 5, 6, 7, 12, 13, 14, 58, 59, 60, 61, 67, 68, 69, 75, 76, 77, 118, 119, 260, 261, 262, 266, 267, 268, 283, 284, 285, 286, 288, 292, 293, 294, 296, 297, 298, 299, 301, 304, 305, 306, 315, 316, 317, 322, 323, 324, 349, 350, 351, 353, 355, 356, 357, 366, 369, 371 }

**B grade** { 9, 10, 11, 15, 63, 64, 65, 66, 70, 115, 116, 117, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 272, 273, 274, 275, 276, 277, 280, 289, 300, 302, 314, 321, 352, 354, 367, 368, 370 }

**C grade** { 228, 229, 230, 231, 232, 233, 234, 235 }

**F normal fail** { 1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 114, 120, 126, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 269, 270, 271, 278, 279, 287, 290, 291, 295, 303, 318, 319, 320, 325, 326, 327, 343, 344, 345, 358, 359, 360, 361, 364, 365 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 307, 308, 309, 310, 311, 312, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 348 }

## Maxima

**A grade** { 4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 49, 50, 61, 64, 119, 285, 289, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 306, 317, 324, 350, 353, 354, 366, 367, 369 }

**B grade** { 58, 59, 60, 63, 65, 66, 115, 116, 117, 118, 122, 124, 134, 262, 268, 272, 273, 274, 276, 280, 283, 305, 349, 351, 352, 355, 356, 357 }

**C grade** { }

**F normal fail** { 1, 2, 3, 8, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 37, 38, 51, 52, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 123, 125, 126, 127, 128, 129, 130, 131, 132, 135, 137, 138, 139, 140, 141, 142, 144, 145, 147, 148, 149, 150, 151, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 260, 261, 263, 264, 265, 266, 267, 269, 270, 271, 275, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327,

328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

**F(-1) timedout fail** { 174, 175, 176, 177, 179 }

**F(-2) exception fail** { 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 121, 133, 143, 152, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259 }

## Giac

**A grade** { 6, 7, 58, 59, 60, 280, 283, 285, 292, 293, 294, 296, 297, 298, 299, 300, 301, 304, 305, 306, 349, 350, 351, 353, 369 }

**B grade** { 9, 15, 61, 63, 64, 65, 66, 115, 116, 117, 118, 119, 121, 289, 302, 352, 354, 355, 356, 357, 366, 367 }

**C grade** { }

**F normal fail** { 1, 2, 3, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 47, 48, 49, 50, 51, 52, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

**F(-1) timedout fail** { 93 }

**F(-2) exception fail** { 4, 5, 12, 13, 14, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 54, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341 }

## Mupad

**A grade** { }

**B grade** { 6, 7, 61, 119, 272, 273, 274, 275, 276, 277, 283, 285, 294, 301, 302, 304, 305, 317, 324, 353, 354, 366, 367, 368, 369, 371 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89,

90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 364, 365, 370 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 4, 5, 6, 7, 13, 14, 15, 60, 61, 67, 68, 69, 70, 75, 76, 77, 119, 283, 285, 294, 304, 349, 350, 351, 352, 366 }

**B grade** { 12, 58, 59, 115, 116, 117, 118, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 147, 148, 149, 150, 266, 267, 268, 272, 273, 274, 275, 276, 277, 367 }

**C grade** { }

**F normal fail** { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 193, 194, 195, 196, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 343, 344, 345, 348, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 369, 370, 371 }

**F(-1) timedout fail** { 43, 53, 192, 198, 199, 200, 201, 202, 203, 228, 236, 244, 245, 252, 253, 342, 347 }

**F(-2) exception fail** { 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 168 | 272 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.177 | 0.010 | 0.706 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 260 | 240 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.284 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 348 | 322 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.294 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 166 | 242 | 230 | 214 | 316 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.38 | 1.31 | 1.22 | 1.80 | 0.00 | 0.00 |
| time (sec) | N/A | 0.116 | 0.111 | 0.312 | 0.183 | 0.282 | 0.304 | 0.000 | 0.000 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 121 | 174 | 150 | 147 | 190 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 1.40 | 1.21 | 1.19 | 1.53 | 0.00 | 0.00 |
| time (sec) | N/A | 0.081 | 0.079 | 0.323 | 0.180 | 0.252 | 0.229 | 0.000 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 91 | 84 | 82 | 87 | 99 | 122 | 78 |
| N.S. | 1 | 1.00 | 0.94 | 0.87 | 0.85 | 0.90 | 1.02 | 1.26 | 0.80 |
| time (sec) | N/A | 0.036 | 0.032 | 0.016 | 0.182 | 0.255 | 0.173 | 0.345 | 2.758 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 31 | 30 | 43 | 26 | 41 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 1.00 | 1.43 | 0.87 | 1.37 | 0.93 |
| time (sec) | N/A | 0.010 | 0.007 | 0.013 | 0.189 | 0.250 | 0.070 | 0.275 | 0.032 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 175 | 279 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.185 | 0.043 | 0.538 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 79 | 174 | 103 | 253 | 0 | 232 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 2.12 | 1.26 | 3.09 | 0.00 | 2.83 | 0.00 |
| time (sec) | N/A | 0.041 | 0.071 | 0.961 | 0.196 | 0.270 | 0.000 | 0.470 | 0.000 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 166 | 273 | 158 | 566 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 2.13 | 1.23 | 4.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.062 | 0.247 | 0.573 | 0.192 | 0.319 | 0.000 | 0.000 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 205 | 510 | 0 | 977 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 2.79 | 0.00 | 5.34 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.098 | 0.300 | 0.406 | 0.000 | 0.619 | 0.000 | 0.000 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 368 | 368 | 354 | 526 | 590 | 475 | 743 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.43 | 1.60 | 1.29 | 2.02 | 0.00 | 0.00 |
| time (sec) | N/A | 0.505 | 0.357 | 0.444 | 0.213 | 0.269 | 0.450 | 0.000 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 239 | 248 | 364 | 378 | 319 | 454 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 1.52 | 1.58 | 1.33 | 1.90 | 0.00 | 0.00 |
| time (sec) | N/A | 0.347 | 0.225 | 0.455 | 0.194 | 0.264 | 0.326 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 142 | 183 | 219 | 183 | 233 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.31 | 1.56 | 1.31 | 1.66 | 0.00 | 0.00 |
| time (sec) | N/A | 0.213 | 1.148 | 0.306 | 0.193 | 0.262 | 0.234 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 74 | 72 | 72 | 96 | 82 | 111 | 0 |
| N.S. | 1 | 1.00 | 1.61 | 1.57 | 1.57 | 2.09 | 1.78 | 2.41 | 0.00 |
| time (sec) | N/A | 0.041 | 0.033 | 0.000 | 0.203 | 0.256 | 0.099 | 0.363 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 273 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.295 | 0.178 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 263 | 191 | 525 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.319 | 0.145 | 0.543 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 270 | 815 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 2.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.418 | 0.443 | 0.653 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 394 | 394 | 305 | 394 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.918 | 0.535 | 1.838 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 188 | 254 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.539 | 0.345 | 1.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 98 | 120 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.246 | 0.148 | 0.589 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 45 | 56 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.050 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 20 | 25 | 15 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.39 | 0.83 | 1.11 | 1.11 |
| time (sec) | N/A | 0.025 | 0.162 | 0.366 | 0.257 | 0.256 | 0.893 | 0.287 | 2.669 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 20 | 49 | 17 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 2.72 | 0.94 | 1.11 | 1.11 |
| time (sec) | N/A | 0.024 | 0.317 | 0.277 | 0.265 | 0.241 | 1.707 | 0.861 | 2.630 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 359 | 288 | 616 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.381 | 1.661 | 0.973 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 150 | 272 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.223 | 0.890 | 0.556 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 71 | 118 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.125 | 0.161 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 746 | 51 | 17 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 41.44 | 2.83 | 0.94 | 1.11 | 1.11 |
| time (sec) | N/A | 0.023 | 5.834 | 0.308 | 0.829 | 0.262 | 1.609 | 0.318 | 2.911 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 1050 | 92 | 19 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 58.33 | 5.11 | 1.06 | 1.11 | 1.11 |
| time (sec) | N/A | 0.022 | 4.551 | 0.353 | 1.221 | 0.265 | 5.726 | 1.299 | 2.675 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 271 | 32 | 17 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 15.06 | 1.78 | 0.94 | 1.11 | 1.11 |
| time (sec) | N/A | 0.207 | 5.324 | 1.562 | 0.852 | 0.281 | 5.170 | 0.383 | 2.810 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 20 | 20 | 15 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.11 | 0.83 | 1.11 | 1.11 |
| time (sec) | N/A | 0.022 | 0.302 | 0.948 | 0.239 | 0.251 | 0.901 | 0.281 | 2.899 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 607 | 34 | 17 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 33.72 | 1.89 | 0.94 | 1.11 | 1.11 |
| time (sec) | N/A | 0.022 | 0.669 | 0.928 | 0.961 | 0.277 | 14.414 | 0.297 | 2.650 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 640 | 640 | 410 | 1309 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.64 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.528 | 0.843 | 0.856 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 431 | 431 | 301 | 913 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.382 | 0.769 | 0.715 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 208 | 582 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.194 | 0.959 | 0.781 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 664 | 664 | 1358 | 747 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.05 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.176 | 4.933 | 0.771 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 781 | 781 | 1384 | 1701 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.77 | 2.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.843 | 7.674 | 0.817 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 918 | 918 | 536 | 2079 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.58 | 2.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.656 | 1.604 | 0.875 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 651 | 651 | 416 | 1568 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.64 | 2.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.533 | 1.163 | 0.818 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 289 | 1065 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 3.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.247 | 0.633 | 0.828 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 984 | 984 | 2869 | 1557 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.92 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.370 | 12.244 | 0.858 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 1228 | 1228 | 810 | 2884 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 2.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.809 | 2.221 | 1.054 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 901 | 901 | 555 | 2309 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.644 | 1.726 | 1.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 494 | 494 | 390 | 1679 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 3.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.287 | 0.887 | 0.997 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F(-2) | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 1536 | 1536 | 7168 | 3499 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 4.67 | 2.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.776 | 23.148 | 0.937 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 430 | 430 | 304 | 785 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 1.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.397 | 1.015 | 0.930 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 233 | 484 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.296 | 0.710 | 0.758 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 158 | 227 | 87 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 1.89 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.149 | 0.348 | 0.830 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 76 | 77 | 28 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 1.64 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.023 | 0.034 | 0.375 | 0.194 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 240 | 529 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.365 | 0.352 | 0.691 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 444 | 444 | 251 | 1770 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.57 | 3.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.473 | 0.407 | 0.846 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 32 | 34 | 34 | 0 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 0.94 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.130 | 0.103 | 2.496 | 0.923 | 0.263 | 0.000 | 73.328 | 2.651 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 438 | 438 | 397 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.493 | 0.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 332 | 304 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.380 | 0.154 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 206 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.206 | 0.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 36 | 32 | 34 | 51 | 31 | 34 | 34 |
| N.S. | 1 | 1.00 | 1.06 | 0.94 | 1.00 | 1.50 | 0.91 | 1.00 | 1.00 |
| time (sec) | N/A | 0.152 | 0.294 | 1.941 | 0.806 | 0.266 | 7.693 | 0.910 | 2.547 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 95 | 200 | 318 | 110 | 255 | 162 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.53 | 2.43 | 0.84 | 1.95 | 1.24 | 0.00 |
| time (sec) | N/A | 0.130 | 0.063 | 0.053 | 0.183 | 0.257 | 0.323 | 0.574 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 74 | 130 | 210 | 91 | 170 | 131 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.44 | 2.33 | 1.01 | 1.89 | 1.46 | 0.00 |
| time (sec) | N/A | 0.081 | 0.043 | 0.015 | 0.184 | 0.248 | 0.248 | 0.430 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 60 | 74 | 149 | 75 | 104 | 111 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.97 | 1.96 | 0.99 | 1.37 | 1.46 | 0.00 |
| time (sec) | N/A | 0.054 | 0.031 | 0.012 | 0.191 | 0.251 | 0.181 | 0.286 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 78 | 31 | 30 | 57 | 46 | 92 | 76 |
| N.S. | 1 | 1.00 | 2.29 | 0.91 | 0.88 | 1.68 | 1.35 | 2.71 | 2.24 |
| time (sec) | N/A | 0.011 | 0.158 | 0.013 | 0.179 | 0.246 | 0.078 | 0.303 | 2.933 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 153 | 388 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 2.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.179 | 0.011 | 0.658 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 68 | 111 | 167 | 0 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.19 | 1.95 | 2.93 | 0.00 | 1.93 | 0.00 |
| time (sec) | N/A | 0.067 | 0.030 | 0.242 | 0.179 | 0.273 | 0.000 | 0.355 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 110 | 103 | 146 | 236 | 0 | 199 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 1.12 | 1.59 | 2.57 | 0.00 | 2.16 | 0.00 |
| time (sec) | N/A | 0.079 | 0.163 | 0.017 | 0.185 | 0.275 | 0.000 | 0.360 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 149 | 203 | 284 | 285 | 0 | 381 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 1.57 | 2.20 | 2.21 | 0.00 | 2.95 | 0.00 |
| time (sec) | N/A | 0.115 | 0.189 | 0.015 | 0.185 | 0.289 | 0.000 | 0.376 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 179 | 345 | 357 | 343 | 0 | 709 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 2.07 | 2.14 | 2.05 | 0.00 | 4.25 | 0.00 |
| time (sec) | N/A | 0.182 | 0.181 | 0.016 | 0.192 | 0.300 | 0.000 | 0.357 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 331 | 331 | 145 | 293 | 0 | 182 | 366 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.44 | 0.89 | 0.00 | 0.55 | 1.11 | 0.00 | 0.00 |
| time (sec) | N/A | 0.394 | 0.501 | 0.204 | 0.000 | 0.265 | 0.441 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 211 | 107 | 190 | 0 | 146 | 243 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.51 | 0.90 | 0.00 | 0.69 | 1.15 | 0.00 | 0.00 |
| time (sec) | N/A | 0.260 | 0.411 | 0.167 | 0.000 | 0.252 | 0.300 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 79 | 113 | 0 | 114 | 138 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.90 | 0.00 | 0.90 | 1.10 | 0.00 | 0.00 |
| time (sec) | N/A | 0.166 | 0.226 | 0.127 | 0.000 | 0.246 | 0.226 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 47 | 46 | 0 | 88 | 63 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 1.02 | 0.00 | 1.96 | 1.40 | 0.00 | 0.00 |
| time (sec) | N/A | 0.035 | 0.019 | 0.081 | 0.000 | 0.270 | 0.100 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 251 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.248 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 178 | 206 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.279 | 0.089 | 0.250 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 279 | 374 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.354 | 0.089 | 0.461 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 478 | 478 | 1830 | 764 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.83 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.158 | 10.001 | 0.408 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 355 | 175 | 297 | 0 | 225 | 432 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.49 | 0.84 | 0.00 | 0.63 | 1.22 | 0.00 | 0.00 |
| time (sec) | N/A | 0.321 | 0.510 | 0.199 | 0.000 | 0.256 | 0.420 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 129 | 169 | 0 | 180 | 248 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.64 | 0.83 | 0.00 | 0.89 | 1.22 | 0.00 | 0.00 |
| time (sec) | N/A | 0.210 | 0.281 | 0.120 | 0.000 | 0.261 | 0.292 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 70 | 67 | 0 | 139 | 109 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.86 | 0.00 | 1.78 | 1.40 | 0.00 | 0.00 |
| time (sec) | N/A | 0.050 | 0.022 | 0.093 | 0.000 | 0.295 | 0.142 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 346 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.279 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 268 | 259 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.429 | 0.097 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 514 | 514 | 524 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.619 | 0.160 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 44 | 49 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.402 | 0.233 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 27 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.182 | 0.127 | 0.096 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.018 | 0.011 | 0.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 14 | 12 | 14 | 14 | 10 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.17 | 1.00 | 1.17 | 1.17 | 0.83 | 1.17 | 1.17 |
| time (sec) | N/A | 0.040 | 0.233 | 0.983 | 0.246 | 0.242 | 0.316 | 0.345 | 2.726 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 83 | 146 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.54 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.157 | 0.770 | 0.279 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 62 | 73 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.106 | 0.186 | 0.171 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 35 | 34 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.069 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 14 | 12 | 527 | 14 | 12 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.17 | 1.00 | 43.92 | 1.17 | 1.00 | 1.17 | 1.17 |
| time (sec) | N/A | 0.032 | 2.129 | 0.846 | 0.948 | 0.244 | 0.390 | 0.340 | 2.707 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 110 | 215 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.43 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.377 | 0.330 | 0.146 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 117 | 107 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.199 | 0.135 | 0.131 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 53 | 51 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.058 | 0.033 | 0.091 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 14 | 12 | 3273 | 14 | 12 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.17 | 1.00 | 272.75 | 1.17 | 1.00 | 1.17 | 1.17 |
| time (sec) | N/A | 0.040 | 1.587 | 0.835 | 15.191 | 0.246 | 0.472 | 0.330 | 2.835 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 15 | 0 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.94 | 0.00 | 1.12 |
| time (sec) | N/A | 0.042 | 0.344 | 0.322 | 0.575 | 0.272 | 12.848 | 0.000 | 2.820 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 545 | 545 | 345 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.847 | 0.664 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 228 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.412 | 0.135 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 109 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.093 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 14 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.88 | 1.12 | 1.12 |
| time (sec) | N/A | 0.043 | 0.212 | 0.310 | 0.595 | 0.257 | 0.573 | 1.398 | 2.662 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 496 | 496 | 656 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.357 | 2.525 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 250 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.534 | 2.267 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 111 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.166 | 0.133 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 582 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.741 | 4.136 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 272 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.189 | 0.400 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 389 | 939 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.854 | 8.103 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 458 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.272 | 1.447 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 411 | 411 | 498 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.672 | 1.599 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 217 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.307 | 1.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 111 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.099 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 269 | 269 | 322 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.384 | 2.613 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 155 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.175 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 365 | 555 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.645 | 3.614 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 207 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.214 | 0.233 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 445 | 445 | 703 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.711 | 3.117 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 238 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.294 | 0.192 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 79 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.059 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 71 | 93 | 1231 | 279 | 527 | 841 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.93 | 12.31 | 2.79 | 5.27 | 8.41 | 0.00 |
| time (sec) | N/A | 0.067 | 0.065 | 0.385 | 0.214 | 0.267 | 0.371 | 0.894 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 83 | 86 | 790 | 228 | 394 | 613 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.82 | 7.52 | 2.17 | 3.75 | 5.84 | 0.00 |
| time (sec) | N/A | 0.056 | 0.050 | 0.360 | 0.211 | 0.254 | 0.282 | 0.736 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 64 | 73 | 445 | 168 | 258 | 416 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.96 | 5.86 | 2.21 | 3.39 | 5.47 | 0.00 |
| time (sec) | N/A | 0.050 | 0.033 | 0.359 | 0.211 | 0.252 | 0.170 | 0.617 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 57 | 62 | 201 | 108 | 148 | 243 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.91 | 2.96 | 1.59 | 2.18 | 3.57 | 0.00 |
| time (sec) | N/A | 0.030 | 0.044 | 0.013 | 0.194 | 0.258 | 0.132 | 0.487 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 83 | 36 | 35 | 65 | 51 | 99 | 85 |
| N.S. | 1 | 1.00 | 2.13 | 0.92 | 0.90 | 1.67 | 1.31 | 2.54 | 2.18 |
| time (sec) | N/A | 0.017 | 0.158 | 0.061 | 0.192 | 0.247 | 0.082 | 0.275 | 2.984 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 70 | 129 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.019 | 0.650 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 43 | 54 | 0 | 175 | 0 | 134 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.10 | 0.00 | 3.57 | 0.00 | 2.73 | 0.00 |
| time (sec) | N/A | 0.040 | 0.030 | 0.365 | 0.000 | 0.268 | 0.000 | 0.342 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 57 | 60 | 117 | 118 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.02 | 1.98 | 2.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.040 | 0.039 | 0.368 | 0.212 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 69 | 74 | 0 | 343 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.88 | 0.00 | 4.08 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.050 | 0.083 | 0.446 | 0.000 | 0.288 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 61 | 80 | 258 | 210 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.68 | 0.89 | 2.87 | 2.33 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.048 | 0.049 | 0.362 | 0.214 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 61 | 94 | 0 | 509 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.53 | 0.82 | 0.00 | 4.43 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.062 | 0.057 | 0.398 | 0.000 | 0.310 | 0.000 | 0.000 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 155 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.148 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 192 | 218 | 0 | 618 | 1268 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.11 | 0.00 | 3.14 | 6.44 | 0.00 | 0.00 |
| time (sec) | N/A | 0.210 | 0.266 | 0.407 | 0.000 | 0.296 | 0.635 | 0.000 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 170 | 194 | 0 | 486 | 916 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 1.13 | 0.00 | 2.83 | 5.33 | 0.00 | 0.00 |
| time (sec) | N/A | 0.186 | 0.230 | 0.412 | 0.000 | 0.262 | 0.469 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 147 | 163 | 0 | 358 | 610 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 1.20 | 0.00 | 2.63 | 4.49 | 0.00 | 0.00 |
| time (sec) | N/A | 0.143 | 0.213 | 0.411 | 0.000 | 0.267 | 0.316 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 120 | 135 | 0 | 230 | 335 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 1.31 | 0.00 | 2.23 | 3.25 | 0.00 | 0.00 |
| time (sec) | N/A | 0.116 | 0.219 | 0.078 | 0.000 | 0.263 | 0.189 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 87 | 88 | 0 | 141 | 143 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.53 | 1.54 | 0.00 | 2.47 | 2.51 | 0.00 | 0.00 |
| time (sec) | N/A | 0.057 | 0.093 | 0.131 | 0.000 | 0.267 | 0.125 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 100 | 312 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.183 | 0.034 | 0.429 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 164 | 185 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.64 | 1.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.116 | 0.547 | 0.459 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 120 | 150 | 230 | 319 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.41 | 1.76 | 2.71 | 3.75 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.108 | 0.289 | 0.566 | 0.222 | 0.313 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 229 | 236 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.36 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.173 | 1.313 | 0.528 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 716 | 55 | 20 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 31.13 | 2.39 | 0.87 | 1.09 | 1.09 |
| time (sec) | N/A | 0.129 | 1.499 | 1.065 | 3.980 | 0.280 | 17.299 | 0.820 | 2.767 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 355 | 420 | 0 | 1077 | 2518 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 1.29 | 0.00 | 3.30 | 7.72 | 0.00 | 0.00 |
| time (sec) | N/A | 0.335 | 0.431 | 0.423 | 0.000 | 0.300 | 1.017 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 279 | 303 | 365 | 0 | 832 | 1828 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 1.31 | 0.00 | 2.98 | 6.55 | 0.00 | 0.00 |
| time (sec) | N/A | 0.278 | 0.364 | 0.376 | 0.000 | 0.316 | 0.722 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 258 | 302 | 0 | 613 | 1173 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 1.33 | 0.00 | 2.70 | 5.17 | 0.00 | 0.00 |
| time (sec) | N/A | 0.221 | 0.323 | 0.393 | 0.000 | 0.289 | 0.465 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 200 | 243 | 0 | 391 | 685 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 1.51 | 0.00 | 2.43 | 4.25 | 0.00 | 0.00 |
| time (sec) | N/A | 0.154 | 0.267 | 0.076 | 0.000 | 0.310 | 0.307 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 147 | 160 | 0 | 239 | 282 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 1.60 | 0.00 | 2.39 | 2.82 | 0.00 | 0.00 |
| time (sec) | N/A | 0.077 | 0.145 | 0.109 | 0.000 | 0.264 | 0.167 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 128 | 556 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 3.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.217 | 0.032 | 0.467 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 315 | 377 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.90 | 2.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.179 | 0.651 | 0.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 229 | 325 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.46 | 2.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.217 | 0.718 | 0.469 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 261 | 694 | 486 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.66 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.281 | 7.245 | 0.569 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 940 | 71 | 20 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 40.87 | 3.09 | 0.87 | 1.09 | 1.09 |
| time (sec) | N/A | 0.122 | 0.902 | 1.092 | 5.240 | 0.273 | 35.373 | 1.189 | 2.759 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 475 | 573 | 0 | 1241 | 2876 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.36 | 1.64 | 0.00 | 3.56 | 8.24 | 0.00 | 0.00 |
| time (sec) | N/A | 0.444 | 0.538 | 0.443 | 0.000 | 0.294 | 1.124 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 412 | 473 | 0 | 900 | 1889 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 1.68 | 0.00 | 3.20 | 6.72 | 0.00 | 0.00 |
| time (sec) | N/A | 0.327 | 0.454 | 0.391 | 0.000 | 0.287 | 0.700 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 300 | 371 | 0 | 574 | 1027 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.54 | 1.90 | 0.00 | 2.94 | 5.27 | 0.00 | 0.00 |
| time (sec) | N/A | 0.209 | 0.354 | 0.079 | 0.000 | 0.265 | 0.454 | 0.000 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 226 | 245 | 0 | 344 | 444 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.97 | 2.13 | 0.00 | 2.99 | 3.86 | 0.00 | 0.00 |
| time (sec) | N/A | 0.107 | 0.227 | 0.173 | 0.000 | 0.284 | 0.268 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 157 | 861 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 4.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.234 | 0.044 | 0.541 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 510 | 630 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.18 | 2.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.208 | 1.324 | 0.468 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 349 | 561 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.88 | 3.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.285 | 1.347 | 0.515 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 385 | 385 | 1198 | 883 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.11 | 2.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.392 | 8.235 | 0.561 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 25 | 25 | 19 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.83 | 1.09 | 1.09 |
| time (sec) | N/A | 0.046 | 0.453 | 0.782 | 0.281 | 0.245 | 0.901 | 0.333 | 2.789 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 151 | 194 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.302 | 0.389 | 1.414 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 109 | 134 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.232 | 0.312 | 1.146 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 102 | 130 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.200 | 0.301 | 0.511 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 61 | 66 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.112 | 0.099 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 49 | 60 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.062 | 0.051 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 25 | 31 | 34 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.35 | 1.48 | 1.09 | 1.09 |
| time (sec) | N/A | 0.050 | 0.723 | 0.205 | 0.300 | 0.243 | 1.009 | 0.346 | 2.627 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 256 | 281 | 602 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 2.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.266 | 1.251 | 0.973 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 193 | 388 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 2.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.189 | 1.099 | 0.908 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 138 | 342 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.183 | 0.911 | 0.538 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 97 | 160 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.095 | 0.439 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 77 | 128 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.120 | 0.182 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 1086 | 61 | 73 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 47.22 | 2.65 | 3.17 | 1.09 | 1.09 |
| time (sec) | N/A | 0.045 | 1.366 | 0.173 | 1.634 | 0.269 | 1.887 | 0.348 | 2.681 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 316 | 896 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 2.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.580 | 1.021 | 1.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 179 | 579 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 2.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.457 | 0.625 | 1.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 246 | 216 | 507 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 2.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.396 | 0.672 | 0.484 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 120 | 239 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.224 | 0.322 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 100 | 190 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.238 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 6716 | 91 | 112 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 292.00 | 3.96 | 4.87 | 1.09 | 1.09 |
| time (sec) | N/A | 0.046 | 0.904 | 0.152 | 68.046 | 0.281 | 3.600 | 0.366 | 2.701 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 410 | 410 | 1244 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 3.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.569 | 1.835 | 0.896 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 340 | 318 | 800 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 2.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.435 | 1.172 | 0.886 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 331 | 331 | 258 | 709 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.454 | 0.927 | 0.474 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 181 | 333 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.227 | 0.780 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 130 | 272 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.187 | 0.410 | 0.054 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 0 | 121 | 151 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 0.00 | 5.26 | 6.57 | 1.09 | 1.09 |
| time (sec) | N/A | 0.045 | 4.671 | 0.161 | 0.000 | 0.333 | 7.476 | 0.386 | 2.650 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 361 | 342 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.598 | 0.516 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 272 | 223 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.433 | 0.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 238 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.416 | 0.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 122 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.276 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 111 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.166 | 0.177 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 20 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.80 | 1.00 | 1.00 |
| time (sec) | N/A | 0.060 | 2.561 | 0.210 | 0.659 | 0.000 | 0.397 | 1.508 | 2.944 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 601 | 601 | 343 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.081 | 0.414 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 360 | 360 | 225 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.711 | 0.235 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 328 | 238 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.604 | 0.231 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 125 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.333 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 272 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.182 | 0.412 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 51 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 2.04 | 1.00 | 1.00 |
| time (sec) | N/A | 0.070 | 0.226 | 0.203 | 0.781 | 0.000 | 2.379 | 2.312 | 2.629 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 701 | 701 | 324 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.453 | 0.367 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 455 | 455 | 209 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.995 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 394 | 394 | 219 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.856 | 0.234 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 262 | 126 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.471 | 0.065 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 458 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.257 | 1.037 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 88 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 3.52 | 1.00 | 1.00 |
| time (sec) | N/A | 0.071 | 0.257 | 0.189 | 1.079 | 0.000 | 22.455 | 3.763 | 2.539 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 835 | 835 | 324 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.133 | 0.497 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 547 | 547 | 208 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.323 | 0.220 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 481 | 481 | 220 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.053 | 0.280 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 125 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.558 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 698 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 3.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.292 | 3.269 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.076 | 0.272 | 0.206 | 2.533 | 0.000 | 0.000 | 5.196 | 2.559 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 320 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.442 | 0.297 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 217 | 205 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.296 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 214 | 217 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.296 | 0.179 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 119 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.158 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 111 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.091 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 36 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 1.44 | 1.00 | 1.00 |
| time (sec) | N/A | 0.066 | 0.123 | 0.214 | 0.798 | 0.000 | 0.695 | 1.846 | 2.601 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 367 | 367 | 490 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.430 | 0.589 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 262 | 253 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.304 | 0.338 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 327 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.317 | 0.252 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 147 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.149 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 155 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.168 | 0.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 88 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 3.52 | 1.00 | 1.00 |
| time (sec) | N/A | 0.079 | 0.143 | 0.219 | 0.750 | 0.000 | 1.899 | 0.510 | 2.633 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 437 | 437 | 551 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.974 | 2.772 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 390 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.785 | 1.280 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 389 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.669 | 1.147 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.386 | 0.522 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 207 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.215 | 0.434 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 155 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 6.20 | 1.00 | 1.00 |
| time (sec) | N/A | 0.079 | 0.139 | 0.208 | 0.709 | 0.000 | 8.739 | 0.587 | 2.705 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 531 | 531 | 701 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.044 | 2.245 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 420 | 429 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.774 | 1.622 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 410 | 474 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.769 | 1.201 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 235 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.378 | 0.647 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 238 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.297 | 0.139 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 25 | 0 | 221 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 0.00 | 8.84 | 1.00 | 1.00 |
| time (sec) | N/A | 0.090 | 0.138 | 0.246 | 0.798 | 0.000 | 105.850 | 0.675 | 2.593 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 113 | 238 | 0 | 306 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.38 | 0.80 | 0.00 | 1.03 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.214 | 0.159 | 2.978 | 0.000 | 0.108 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 113 | 212 | 0 | 254 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.64 | 1.20 | 0.00 | 1.44 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.127 | 1.289 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 261 | 87 | 205 | 0 | 185 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.33 | 0.79 | 0.00 | 0.71 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.183 | 0.036 | 0.908 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 87 | 179 | 0 | 142 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 1.26 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.101 | 0.026 | 0.685 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 61 | 161 | 0 | 93 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.27 | 0.72 | 0.00 | 0.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.156 | 0.031 | 0.565 | 0.000 | 0.091 | 0.000 | 0.000 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 56 | 140 | 0 | 110 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.53 | 1.32 | 0.00 | 1.04 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.089 | 0.020 | 0.437 | 0.000 | 0.093 | 0.000 | 0.000 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 266 | 58 | 202 | 0 | 181 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.22 | 0.76 | 0.00 | 0.68 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.183 | 0.028 | 0.513 | 0.000 | 0.102 | 0.000 | 0.000 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F(-2) | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 61 | 176 | 0 | 208 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.42 | 1.21 | 0.00 | 1.43 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.105 | 0.031 | 0.661 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.165 | 0.096 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.162 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.166 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.153 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.155 | 0.050 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 109 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.152 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 106 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.170 | 0.060 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 110 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.164 | 0.060 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 256 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 10.24 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.147 | 74.280 | 0.910 | 0.000 | 0.288 | 0.000 | 2.121 | 2.706 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 186 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 7.44 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.140 | 120.369 | 0.219 | 0.000 | 0.283 | 0.000 | 1.823 | 2.659 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 100 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 4.00 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.150 | 66.169 | 0.239 | 0.000 | 0.312 | 20.338 | 1.612 | 2.680 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 55 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.20 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.140 | 104.771 | 0.237 | 0.000 | 0.268 | 3.880 | 1.149 | 2.973 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 55 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.20 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.125 | 61.732 | 0.087 | 0.000 | 0.266 | 3.403 | 0.599 | 2.799 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 83 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 3.32 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.137 | 43.205 | 0.329 | 0.000 | 0.265 | 6.559 | 0.751 | 2.798 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 97 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 3.88 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.139 | 39.195 | 0.253 | 0.000 | 0.279 | 14.788 | 2.983 | 2.660 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 111 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 4.44 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.145 | 73.145 | 0.250 | 0.000 | 0.280 | 61.702 | 0.759 | 2.703 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 330 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 13.20 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.136 | 37.754 | 0.899 | 0.000 | 0.301 | 0.000 | 172.173 | 2.757 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 240 | 0 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 9.60 | 0.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.136 | 74.088 | 0.225 | 0.000 | 0.282 | 0.000 | 2.364 | 2.810 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 130 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 5.20 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.139 | 34.309 | 0.216 | 0.000 | 0.268 | 39.495 | 2.244 | 2.762 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 71 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.84 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.130 | 57.210 | 0.232 | 0.000 | 0.278 | 6.792 | 1.465 | 2.815 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 71 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.84 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.129 | 13.866 | 0.074 | 0.000 | 0.282 | 5.190 | 0.698 | 2.931 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 99 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 3.96 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.124 | 16.330 | 0.263 | 0.000 | 0.279 | 9.407 | 0.906 | 2.737 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 113 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 4.52 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.148 | 10.218 | 0.244 | 0.000 | 0.271 | 18.432 | 3.848 | 2.817 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 23 | 0 | 127 | 22 | 25 | 25 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 5.08 | 0.88 | 1.00 | 1.00 |
| time (sec) | N/A | 0.145 | 53.079 | 0.259 | 0.000 | 0.280 | 68.421 | 0.921 | 2.809 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 127 | 204 | 0 | 199 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.56 | 0.00 | 1.52 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.135 | 0.082 | 0.924 | 0.000 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 110 | 167 | 0 | 161 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.56 | 0.00 | 1.50 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.089 | 0.065 | 0.661 | 0.000 | 0.261 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 61 | 91 | 238 | 98 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.49 | 3.90 | 1.61 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.054 | 0.039 | 0.860 | 0.245 | 0.257 | 0.000 | 0.000 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 24 | 23 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.092 | 0.052 | 0.585 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 47 | 44 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.092 | 0.049 | 0.852 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 69 | 85 | 63 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.97 | 1.20 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.087 | 0.171 | 0.587 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 266 | 592 | 0 | 332 | 694 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.13 | 2.52 | 0.00 | 1.41 | 2.95 | 0.00 | 0.00 |
| time (sec) | N/A | 0.243 | 0.162 | 0.727 | 0.000 | 0.263 | 1.145 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 211 | 479 | 0 | 259 | 568 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 2.53 | 0.00 | 1.37 | 3.01 | 0.00 | 0.00 |
| time (sec) | N/A | 0.156 | 0.110 | 0.892 | 0.000 | 0.269 | 0.736 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 124 | 262 | 394 | 160 | 298 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 2.47 | 3.72 | 1.51 | 2.81 | 0.00 | 0.00 |
| time (sec) | N/A | 0.084 | 0.065 | 0.722 | 0.275 | 0.267 | 0.495 | 0.000 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 37 | 36 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.119 | 0.478 | 0.736 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 70 | 72 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.121 | 0.222 | 0.704 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 108 | 110 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.29 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.213 | 0.211 | 0.869 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 179 | 32 | 26 | 0 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 11.93 | 2.13 | 1.73 | 0.00 | 0.87 |
| time (sec) | N/A | 0.053 | 0.016 | 0.778 | 0.299 | 0.264 | 0.289 | 0.000 | 2.641 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 132 | 32 | 26 | 0 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 8.80 | 2.13 | 1.73 | 0.00 | 0.87 |
| time (sec) | N/A | 0.050 | 0.015 | 0.717 | 0.228 | 0.245 | 0.231 | 0.000 | 2.676 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 84 | 32 | 24 | 0 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 5.60 | 2.13 | 1.60 | 0.00 | 0.87 |
| time (sec) | N/A | 0.033 | 0.013 | 0.701 | 0.276 | 0.259 | 0.207 | 0.000 | 2.850 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 0 | 30 | 22 | 0 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 0.00 | 2.73 | 2.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.056 | 0.024 | 0.717 | 0.000 | 0.236 | 0.240 | 0.000 | 2.650 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 14 | 150 | 32 | 26 | 0 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 11.54 | 2.46 | 2.00 | 0.00 | 1.00 |
| time (sec) | N/A | 0.062 | 0.012 | 0.724 | 0.328 | 0.247 | 0.511 | 0.000 | 2.631 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 0 | 32 | 29 | 0 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.00 | 2.13 | 1.93 | 0.00 | 0.87 |
| time (sec) | N/A | 0.057 | 0.011 | 0.750 | 0.000 | 0.246 | 0.772 | 0.000 | 2.634 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 128 | 203 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.158 | 0.426 | 0.755 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 98 | 168 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.129 | 0.300 | 0.630 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 62 | 131 | 119 | 115 | 0 | 76 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 2.85 | 2.59 | 2.50 | 0.00 | 1.65 | 0.00 |
| time (sec) | N/A | 0.044 | 0.066 | 0.612 | 0.337 | 0.264 | 0.000 | 0.306 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 30 | 82 | 31 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 1.00 | 2.73 | 1.03 | 1.00 | 1.00 |
| time (sec) | N/A | 0.060 | 0.512 | 0.372 | 0.332 | 0.237 | 1.613 | 0.303 | 2.648 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 610 | 82 | 32 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 20.33 | 2.73 | 1.07 | 1.00 | 1.00 |
| time (sec) | N/A | 0.087 | 2.209 | 0.322 | 2.535 | 0.256 | 1.769 | 0.310 | 2.630 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 44 | 71 | 102 | 52 | 42 | 74 | 45 |
| N.S. | 1 | 1.00 | 0.88 | 1.42 | 2.04 | 1.04 | 0.84 | 1.48 | 0.90 |
| time (sec) | N/A | 0.028 | 0.014 | 0.073 | 0.227 | 0.264 | 0.229 | 0.271 | 2.738 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 75 | 89 | 0 | 68 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.88 | 0.00 | 0.67 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.037 | 0.086 | 0.451 | 0.000 | 0.087 | 0.000 | 0.000 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 29 | 30 | 42 | 27 | 40 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.88 | 1.24 | 0.79 | 1.18 | 0.82 |
| time (sec) | N/A | 0.020 | 0.009 | 0.017 | 0.251 | 0.239 | 0.086 | 0.284 | 2.725 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 35 | 77 | 0 | 94 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.22 | 0.48 | 0.00 | 0.58 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.041 | 10.014 | 0.135 | 0.000 | 0.110 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.049 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 42 | 66 | 0 | 76 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.56 | 0.88 | 0.00 | 1.01 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.019 | 0.028 | 0.079 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 28 | 46 | 106 | 0 | 58 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.39 | 3.21 | 0.00 | 1.76 | 0.00 |
| time (sec) | N/A | 0.022 | 0.006 | 0.014 | 0.230 | 0.265 | 0.000 | 0.278 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 88 | 101 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.45 | 0.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.077 | 0.105 | 0.139 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 43 | 47 | 46 | 40 | 0 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.60 | 0.65 | 0.64 | 0.56 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.018 | 0.017 | 0.058 | 0.368 | 0.257 | 0.000 | 0.263 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 37 | 37 | 36 | 35 | 0 | 48 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.66 | 0.64 | 0.62 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 0.013 | 0.014 | 0.012 | 0.314 | 0.250 | 0.000 | 0.280 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 46 | 24 | 23 | 28 | 29 | 40 | 31 |
| N.S. | 1 | 1.00 | 1.31 | 0.69 | 0.66 | 0.80 | 0.83 | 1.14 | 0.89 |
| time (sec) | N/A | 0.009 | 0.075 | 0.010 | 0.345 | 0.243 | 0.107 | 0.277 | 3.339 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 78 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.008 | 0.229 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 21 | 20 | 25 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.77 | 0.96 | 0.00 | 1.35 | 0.00 |
| time (sec) | N/A | 0.010 | 0.009 | 0.010 | 0.347 | 0.246 | 0.000 | 0.274 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 34 | 31 | 30 | 32 | 0 | 52 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.67 | 0.65 | 0.70 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 0.012 | 0.011 | 0.011 | 0.336 | 0.252 | 0.000 | 0.280 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 39 | 41 | 40 | 37 | 0 | 67 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.66 | 0.65 | 0.60 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 0.015 | 0.013 | 0.012 | 0.322 | 0.259 | 0.000 | 0.274 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 44 | 51 | 50 | 42 | 0 | 82 | 0 |
| N.S. | 1 | 1.00 | 0.56 | 0.65 | 0.64 | 0.54 | 0.00 | 1.05 | 0.00 |
| time (sec) | N/A | 0.018 | 0.015 | 0.012 | 0.403 | 0.242 | 0.000 | 0.280 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 57 | 54 | 69 | 122 | 0 | 74 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.96 | 1.23 | 2.18 | 0.00 | 1.32 | 0.00 |
| time (sec) | N/A | 0.029 | 0.030 | 0.151 | 0.219 | 0.255 | 0.000 | 0.340 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 29 | 38 | 27 | 45 | 0 | 47 | 27 |
| N.S. | 1 | 1.00 | 0.88 | 1.15 | 0.82 | 1.36 | 0.00 | 1.42 | 0.82 |
| time (sec) | N/A | 0.015 | 0.018 | 0.015 | 0.246 | 0.255 | 0.000 | 0.312 | 0.036 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 77 | 31 | 43 | 96 | 0 | 49 | 25 |
| N.S. | 1 | 1.00 | 3.08 | 1.24 | 1.72 | 3.84 | 0.00 | 1.96 | 1.00 |
| time (sec) | N/A | 0.013 | 0.061 | 0.014 | 0.278 | 0.255 | 0.000 | 0.307 | 2.938 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 52 | 114 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.043 | 0.008 | 0.197 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 31 | 30 | 49 | 20 | 39 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.03 | 1.69 | 0.69 | 1.34 | 0.93 |
| time (sec) | N/A | 0.020 | 0.014 | 0.011 | 0.250 | 0.257 | 0.369 | 0.284 | 2.652 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 44 | 46 | 97 | 58 | 0 | 84 | 43 |
| N.S. | 1 | 1.00 | 0.88 | 0.92 | 1.94 | 1.16 | 0.00 | 1.68 | 0.86 |
| time (sec) | N/A | 0.033 | 0.018 | 0.013 | 0.243 | 0.244 | 0.000 | 0.313 | 2.649 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 48 | 50 | 47 | 62 | 0 | 75 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.93 | 0.87 | 1.15 | 0.00 | 1.39 | 0.00 |
| time (sec) | N/A | 0.031 | 0.022 | 0.013 | 0.249 | 0.248 | 0.000 | 0.322 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.038 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 66 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.029 | 0.034 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 58 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.024 | 0.035 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.017 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 52 | 120 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.014 | 0.256 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 61 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.024 | 0.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 62 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.029 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 149 | 0 | 0 | 269 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 1.76 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.028 | 0.088 | 0.000 | 0.000 | 0.263 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 180 | 0 | 0 | 188 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.40 | 0.00 | 0.00 | 1.46 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.046 | 0.098 | 0.000 | 0.000 | 0.257 | 0.000 | 0.000 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 76 | 0 | 0 | 114 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.50 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.011 | 0.015 | 0.000 | 0.000 | 0.247 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | F(-2) | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 48 | 47 | 44 | 52 | 0 | 0 | 39 |
| N.S. | 1 | 1.00 | 0.96 | 0.94 | 0.88 | 1.04 | 0.00 | 0.00 | 0.78 |
| time (sec) | N/A | 0.031 | 0.020 | 0.024 | 0.266 | 0.245 | 0.000 | 0.000 | 3.165 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 150 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.037 | 0.617 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 197 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.035 | 1.104 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 229 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.043 | 0.402 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 149 | 0 | 0 | 269 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 1.76 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.026 | 0.088 | 0.000 | 0.000 | 0.247 | 0.000 | 0.000 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 180 | 0 | 0 | 188 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.40 | 0.00 | 0.00 | 1.46 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.044 | 0.095 | 0.000 | 0.000 | 0.263 | 0.000 | 0.000 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 76 | 0 | 0 | 114 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.50 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.012 | 0.015 | 0.000 | 0.000 | 0.263 | 0.000 | 0.000 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | F(-2) | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 48 | 48 | 44 | 52 | 0 | 0 | 39 |
| N.S. | 1 | 1.00 | 0.96 | 0.96 | 0.88 | 1.04 | 0.00 | 0.00 | 0.78 |
| time (sec) | N/A | 0.030 | 0.020 | 0.025 | 0.256 | 0.254 | 0.000 | 0.000 | 2.931 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 146 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.019 | 0.574 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 244 | 196 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.021 | 1.153 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 272 | 227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.041 | 0.485 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 348 | 337 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.086 | 0.205 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 312 | 312 | 258 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.086 | 0.165 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 263 | 259 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.023 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 180 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.018 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 291 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.046 | 0.284 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 308 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.587 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 389 | 365 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.057 | 0.690 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 348 | 337 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.077 | 0.209 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 310 | 255 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.076 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 262 | 259 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.025 | 0.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 180 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.021 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 291 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.036 | 0.254 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 308 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.061 | 0.591 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 389 | 370 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.058 | 0.692 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 42 | 36 | 39 | 38 | 0 | 38 | 39 |
| N.S. | 1 | 1.00 | 1.05 | 0.90 | 0.98 | 0.95 | 0.00 | 0.95 | 0.98 |
| time (sec) | N/A | 0.036 | 0.169 | 0.510 | 0.971 | 0.268 | 0.000 | 1.984 | 3.070 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 261 | 244 | 1113 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 4.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.227 | 0.045 | 1.395 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 187 | 624 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 3.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.182 | 0.040 | 0.395 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 127 | 258 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.124 | 0.020 | 0.415 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 42 | 36 | 39 | 48 | 61 | 38 | 39 |
| N.S. | 1 | 1.00 | 1.05 | 0.90 | 0.98 | 1.20 | 1.52 | 0.95 | 0.98 |
| time (sec) | N/A | 0.035 | 0.280 | 0.340 | 0.486 | 0.250 | 163.878 | 5.648 | 2.856 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 42 | 36 | 474 | 91 | 0 | 38 | 39 |
| N.S. | 1 | 1.00 | 1.05 | 0.90 | 11.85 | 2.28 | 0.00 | 0.95 | 0.98 |
| time (sec) | N/A | 0.030 | 1.132 | 0.312 | 0.594 | 0.270 | 0.000 | 20.277 | 3.629 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 68 | 153 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.060 | 0.447 | 0.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 119 | 464 | 491 | 138 | 456 | 173 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 2.81 | 2.98 | 0.84 | 2.76 | 1.05 | 0.00 |
| time (sec) | N/A | 0.132 | 0.061 | 0.753 | 0.299 | 0.251 | 0.897 | 0.291 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 102 | 288 | 273 | 117 | 292 | 140 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 2.50 | 2.37 | 1.02 | 2.54 | 1.22 | 0.00 |
| time (sec) | N/A | 0.099 | 0.075 | 0.628 | 0.250 | 0.258 | 0.814 | 0.275 | 0.000 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 73 | 145 | 175 | 93 | 197 | 106 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 2.16 | 2.61 | 1.39 | 2.94 | 1.58 | 0.00 |
| time (sec) | N/A | 0.056 | 0.068 | 0.576 | 0.248 | 0.249 | 0.656 | 0.289 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 46 | 89 | 141 | 73 | 124 | 80 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 2.87 | 4.55 | 2.35 | 4.00 | 2.58 | 0.00 |
| time (sec) | N/A | 0.016 | 0.027 | 0.761 | 0.261 | 0.247 | 0.394 | 0.295 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 99 | 126 | 160 | 136 | 0 | 158 | 180 |
| N.S. | 1 | 1.00 | 1.11 | 1.42 | 1.80 | 1.53 | 0.00 | 1.78 | 2.02 |
| time (sec) | N/A | 0.088 | 0.051 | 0.892 | 0.266 | 0.273 | 0.000 | 0.332 | 3.422 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 110 | 284 | 170 | 183 | 0 | 234 | 269 |
| N.S. | 1 | 1.00 | 1.11 | 2.87 | 1.72 | 1.85 | 0.00 | 2.36 | 2.72 |
| time (sec) | N/A | 0.083 | 0.111 | 0.755 | 0.210 | 0.259 | 0.000 | 0.365 | 3.656 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 129 | 459 | 313 | 181 | 0 | 384 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 3.96 | 2.70 | 1.56 | 0.00 | 3.31 | 0.00 |
| time (sec) | N/A | 0.063 | 0.143 | 0.750 | 0.379 | 0.248 | 0.000 | 0.337 | 0.000 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 162 | 502 | 352 | 230 | 0 | 715 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 3.22 | 2.26 | 1.47 | 0.00 | 4.58 | 0.00 |
| time (sec) | N/A | 0.083 | 0.077 | 0.954 | 0.359 | 0.252 | 0.000 | 0.328 | 0.000 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 192 | 1003 | 594 | 295 | 0 | 1173 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 4.85 | 2.87 | 1.43 | 0.00 | 5.67 | 0.00 |
| time (sec) | N/A | 0.128 | 0.591 | 0.911 | 0.380 | 0.258 | 0.000 | 0.345 | 0.000 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 359 | 198 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.588 | 0.249 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 138 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.433 | 0.125 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 76 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.232 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 44 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.041 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 13 | 15 | 15 | 12 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.14 | 0.93 | 1.07 | 1.07 | 0.86 | 1.07 | 1.07 |
| time (sec) | N/A | 0.029 | 0.147 | 0.007 | 0.624 | 0.257 | 0.360 | 0.317 | 2.566 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 13 | 15 | 15 | 14 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.14 | 0.93 | 1.07 | 1.07 | 1.00 | 1.07 | 1.07 |
| time (sec) | N/A | 0.028 | 0.342 | 0.006 | 0.450 | 0.258 | 0.386 | 0.333 | 2.567 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 52 | 134 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 2.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.072 | 0.015 | 0.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.048 | 0.077 | 0.608 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 41 | 38 | 37 | 66 | 61 | 105 | 88 |
| N.S. | 1 | 1.00 | 0.91 | 0.84 | 0.82 | 1.47 | 1.36 | 2.33 | 1.96 |
| time (sec) | N/A | 0.038 | 0.019 | 0.020 | 0.343 | 0.260 | 0.179 | 0.289 | 2.921 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | B | B | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 41 | 0 | 39 | 152 | 80 | 113 | 99 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.85 | 3.30 | 1.74 | 2.46 | 2.15 |
| time (sec) | N/A | 0.045 | 0.030 | 0.000 | 0.235 | 0.269 | 9.504 | 0.313 | 2.666 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | B | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 357 | 46 | 0 | 242 | 0 | 0 | 41 |
| N.S. | 1 | 1.00 | 7.29 | 0.94 | 0.00 | 4.94 | 0.00 | 0.00 | 0.84 |
| time (sec) | N/A | 0.028 | 0.895 | 0.360 | 0.000 | 0.273 | 0.000 | 0.000 | 3.401 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | B | F | A | A | F | A | B |
| verified | N/A | N/A | No | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 148 | 0 | 1 | 1 | 0 | 1 | 1 |
| N.S. | 1 | 0.00 | 5.48 | 0.00 | 0.04 | 0.04 | 0.00 | 0.04 | 0.04 |
| time (sec) | N/A | 0.000 | 0.216 | 0.000 | 0.408 | 0.250 | 0.000 | 0.278 | 2.532 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 37 | 0 | 0 | 108 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 2.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.050 | 0.036 | 0.000 | 0.000 | 0.263 | 0.000 | 0.000 | 0.000 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 24 | 0 | 0 | 33 | 0 | 0 | 23 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 1.14 | 0.00 | 0.00 | 0.79 |
| time (sec) | N/A | 0.044 | 0.018 | 0.000 | 0.000 | 0.245 | 0.000 | 0.000 | 2.640 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [1.33299999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 8 | 5 | 1.00 | 12 | 0.417 |
| 2 | A | 10 | 6 | 1.00 | 14 | 0.429 |
| 3 | A | 12 | 7 | 1.00 | 14 | 0.500 |
| 4 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 5 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 6 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 7 | A | 3 | 2 | 1.00 | 8 | 0.250 |
| 8 | A | 8 | 5 | 1.00 | 16 | 0.312 |
| 9 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 10 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 11 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 12 | A | 18 | 7 | 1.00 | 18 | 0.389 |
| 13 | A | 13 | 7 | 1.00 | 18 | 0.389 |
| 14 | A | 9 | 7 | 1.00 | 16 | 0.438 |
| 15 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 16 | A | 10 | 6 | 1.00 | 18 | 0.333 |
| 17 | A | 10 | 7 | 1.00 | 18 | 0.389 |
| 18 | A | 13 | 10 | 1.00 | 18 | 0.556 |
| 19 | A | 27 | 7 | 1.00 | 18 | 0.389 |
| 20 | A | 17 | 6 | 1.00 | 18 | 0.333 |
| 21 | A | 11 | 7 | 1.00 | 16 | 0.438 |
| 22 | A | 4 | 4 | 1.00 | 10 | 0.400 |
| 23 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 24 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 25 | A | 19 | 7 | 1.00 | 18 | 0.389 |
| 26 | A | 11 | 7 | 1.00 | 16 | 0.438 |
| 27 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 28 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 29 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 30 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 31 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 32 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 33 | N/A | 0 | 0 | 1.00 | 18 | 0.000 |
| 34 | A | 16 | 12 | 1.00 | 30 | 0.400 |
| 35 | A | 13 | 8 | 1.00 | 30 | 0.267 |
| 36 | A | 8 | 6 | 1.00 | 28 | 0.214 |
| 37 | A | 22 | 20 | 1.00 | 30 | 0.667 |
| 38 | A | 35 | 22 | 1.00 | 30 | 0.733 |
| 39 | A | 24 | 17 | 1.00 | 30 | 0.567 |
| 40 | A | 20 | 12 | 1.00 | 30 | 0.400 |
| 41 | A | 12 | 9 | 1.00 | 28 | 0.321 |
| 42 | A | 29 | 24 | 1.00 | 30 | 0.800 |
| 43 | A | 30 | 18 | 1.00 | 30 | 0.600 |
| 44 | A | 26 | 15 | 1.00 | 30 | 0.500 |
| 45 | A | 14 | 10 | 1.00 | 28 | 0.357 |
| 46 | A | 37 | 29 | 1.00 | 30 | 0.967 |
| 47 | A | 13 | 7 | 1.00 | 30 | 0.233 |
| 48 | A | 9 | 7 | 1.00 | 30 | 0.233 |
| 49 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 50 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 51 | A | 10 | 7 | 1.00 | 30 | 0.233 |
| 52 | A | 13 | 10 | 1.00 | 30 | 0.333 |
| 53 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |
| 54 | A | 13 | 9 | 1.00 | 34 | 0.265 |
| 55 | A | 11 | 8 | 1.00 | 32 | 0.250 |
| 56 | A | 9 | 7 | 1.00 | 24 | 0.292 |
| 57 | N/A | 0 | 0 | 1.00 | 34 | 0.000 |
| 58 | A | 6 | 6 | 1.00 | 10 | 0.600 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 59 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 60 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 61 | A | 3 | 3 | 1.00 | 6 | 0.500 |
| 62 | A | 9 | 6 | 1.00 | 10 | 0.600 |
| 63 | A | 4 | 4 | 1.00 | 10 | 0.400 |
| 64 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 65 | A | 6 | 6 | 1.00 | 10 | 0.600 |
| 66 | A | 7 | 7 | 1.00 | 10 | 0.700 |
| 67 | A | 19 | 8 | 1.00 | 12 | 0.667 |
| 68 | A | 14 | 8 | 1.00 | 12 | 0.667 |
| 69 | A | 10 | 8 | 1.00 | 10 | 0.800 |
| 70 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 71 | A | 11 | 7 | 1.00 | 12 | 0.583 |
| 72 | A | 11 | 8 | 1.00 | 12 | 0.667 |
| 73 | A | 14 | 11 | 1.00 | 12 | 0.917 |
| 74 | A | 40 | 16 | 1.00 | 12 | 1.333 |
| 75 | A | 18 | 11 | 1.00 | 12 | 0.917 |
| 76 | A | 12 | 10 | 1.00 | 10 | 1.000 |
| 77 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 78 | A | 13 | 8 | 1.00 | 12 | 0.667 |
| 79 | A | 13 | 9 | 1.00 | 12 | 0.750 |
| 80 | A | 21 | 13 | 1.00 | 12 | 1.083 |
| 81 | A | 14 | 8 | 1.00 | 12 | 0.667 |
| 82 | A | 10 | 8 | 1.00 | 10 | 0.800 |
| 83 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 84 | N/A | 0 | 0 | 1.00 | 12 | 0.000 |
| 85 | A | 12 | 7 | 1.00 | 12 | 0.583 |
| 86 | A | 8 | 7 | 1.00 | 10 | 0.700 |
| 87 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 88 | N/A | 0 | 0 | 1.00 | 12 | 0.000 |
| 89 | A | 24 | 12 | 1.00 | 12 | 1.000 |
| 90 | A | 14 | 12 | 1.00 | 10 | 1.200 |
| 91 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 92 | N/A | 0 | 0 | 1.00 | 12 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 93 | N/A | 0 | 0 | 1.00 | 16 | 0.000 |
| 94 | A | 22 | 9 | 1.00 | 16 | 0.562 |
| 95 | A | 14 | 9 | 1.00 | 14 | 0.643 |
| 96 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 97 | N/A | 0 | 0 | 1.00 | 16 | 0.000 |
| 98 | A | 23 | 12 | 1.00 | 18 | 0.667 |
| 99 | A | 14 | 10 | 1.00 | 16 | 0.625 |
| 100 | A | 8 | 7 | 1.00 | 14 | 0.500 |
| 101 | A | 16 | 10 | 1.00 | 16 | 0.625 |
| 102 | A | 9 | 8 | 1.00 | 14 | 0.571 |
| 103 | A | 18 | 10 | 1.00 | 16 | 0.625 |
| 104 | A | 10 | 8 | 1.00 | 14 | 0.571 |
| 105 | A | 20 | 9 | 1.00 | 18 | 0.500 |
| 106 | A | 12 | 8 | 1.00 | 16 | 0.500 |
| 107 | A | 7 | 6 | 1.00 | 14 | 0.429 |
| 108 | A | 16 | 10 | 1.00 | 16 | 0.625 |
| 109 | A | 8 | 7 | 1.00 | 14 | 0.500 |
| 110 | A | 22 | 15 | 1.00 | 16 | 0.938 |
| 111 | A | 9 | 8 | 1.00 | 14 | 0.571 |
| 112 | A | 21 | 13 | 1.00 | 16 | 0.812 |
| 113 | A | 10 | 8 | 1.00 | 14 | 0.571 |
| 114 | A | 3 | 3 | 1.00 | 21 | 0.143 |
| 115 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 116 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 117 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 118 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 119 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 120 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 121 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 122 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 123 | A | 7 | 7 | 1.00 | 21 | 0.333 |
| 124 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 125 | A | 8 | 7 | 1.00 | 21 | 0.333 |
| 126 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 127 | A | 9 | 7 | 1.00 | 23 | 0.304 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 128 | A | 8 | 6 | 1.00 | 23 | 0.261 |
| 129 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 130 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 131 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 132 | A | 8 | 8 | 1.00 | 23 | 0.348 |
| 133 | A | 9 | 7 | 1.00 | 23 | 0.304 |
| 134 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 135 | A | 11 | 9 | 1.00 | 23 | 0.391 |
| 136 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 137 | A | 17 | 9 | 1.00 | 23 | 0.391 |
| 138 | A | 13 | 7 | 1.00 | 23 | 0.304 |
| 139 | A | 12 | 9 | 1.00 | 23 | 0.391 |
| 140 | A | 8 | 7 | 1.00 | 21 | 0.333 |
| 141 | A | 6 | 4 | 1.00 | 12 | 0.333 |
| 142 | A | 9 | 9 | 1.00 | 23 | 0.391 |
| 143 | A | 11 | 8 | 1.00 | 23 | 0.348 |
| 144 | A | 9 | 9 | 1.00 | 23 | 0.391 |
| 145 | A | 16 | 12 | 1.00 | 23 | 0.522 |
| 146 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 147 | A | 16 | 6 | 1.00 | 23 | 0.261 |
| 148 | A | 13 | 8 | 1.00 | 23 | 0.348 |
| 149 | A | 9 | 6 | 1.00 | 21 | 0.286 |
| 150 | A | 6 | 4 | 1.00 | 12 | 0.333 |
| 151 | A | 10 | 9 | 1.00 | 23 | 0.391 |
| 152 | A | 13 | 9 | 1.00 | 23 | 0.391 |
| 153 | A | 10 | 10 | 1.00 | 23 | 0.435 |
| 154 | A | 21 | 12 | 1.00 | 23 | 0.522 |
| 155 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 156 | A | 14 | 7 | 1.00 | 23 | 0.304 |
| 157 | A | 11 | 7 | 1.00 | 23 | 0.304 |
| 158 | A | 11 | 7 | 1.00 | 23 | 0.304 |
| 159 | A | 8 | 7 | 1.00 | 21 | 0.333 |
| 160 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 161 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 162 | A | 13 | 6 | 1.00 | 23 | 0.261 |
| 163 | A | 10 | 6 | 1.00 | 23 | 0.261 |
| 164 | A | 10 | 6 | 1.00 | 23 | 0.261 |
| 165 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 166 | A | 6 | 6 | 1.00 | 12 | 0.500 |
| 167 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 168 | A | 26 | 9 | 1.00 | 23 | 0.391 |
| 169 | A | 20 | 9 | 1.00 | 23 | 0.391 |
| 170 | A | 18 | 10 | 1.00 | 23 | 0.435 |
| 171 | A | 11 | 10 | 1.00 | 21 | 0.476 |
| 172 | A | 7 | 7 | 1.00 | 12 | 0.583 |
| 173 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 174 | A | 24 | 8 | 1.00 | 23 | 0.348 |
| 175 | A | 17 | 8 | 1.00 | 23 | 0.348 |
| 176 | A | 18 | 10 | 1.00 | 23 | 0.435 |
| 177 | A | 9 | 9 | 1.00 | 21 | 0.429 |
| 178 | A | 8 | 7 | 1.00 | 12 | 0.583 |
| 179 | N/A | 0 | 0 | 1.00 | 23 | 0.000 |
| 180 | A | 21 | 9 | 1.00 | 25 | 0.360 |
| 181 | A | 16 | 9 | 1.00 | 25 | 0.360 |
| 182 | A | 16 | 9 | 1.00 | 25 | 0.360 |
| 183 | A | 11 | 9 | 1.00 | 23 | 0.391 |
| 184 | A | 8 | 7 | 1.00 | 14 | 0.500 |
| 185 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 186 | A | 43 | 12 | 1.00 | 25 | 0.480 |
| 187 | A | 27 | 11 | 1.00 | 25 | 0.440 |
| 188 | A | 24 | 12 | 1.00 | 25 | 0.480 |
| 189 | A | 13 | 11 | 1.00 | 23 | 0.478 |
| 190 | A | 9 | 8 | 1.00 | 14 | 0.571 |
| 191 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 192 | A | 46 | 12 | 1.00 | 25 | 0.480 |
| 193 | A | 29 | 11 | 1.00 | 25 | 0.440 |
| 194 | A | 26 | 12 | 1.00 | 25 | 0.480 |
| 195 | A | 14 | 11 | 1.00 | 23 | 0.478 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 196 | A | 10 | 8 | 1.00 | 14 | 0.571 |
| 197 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 198 | A | 77 | 13 | 1.00 | 25 | 0.520 |
| 199 | A | 42 | 11 | 1.00 | 25 | 0.440 |
| 200 | A | 35 | 13 | 1.00 | 25 | 0.520 |
| 201 | A | 16 | 11 | 1.00 | 23 | 0.478 |
| 202 | A | 11 | 8 | 1.00 | 14 | 0.571 |
| 203 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 204 | A | 20 | 8 | 1.00 | 25 | 0.320 |
| 205 | A | 15 | 8 | 1.00 | 25 | 0.320 |
| 206 | A | 15 | 8 | 1.00 | 25 | 0.320 |
| 207 | A | 10 | 8 | 1.00 | 23 | 0.348 |
| 208 | A | 7 | 6 | 1.00 | 14 | 0.429 |
| 209 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 210 | A | 19 | 7 | 1.00 | 25 | 0.280 |
| 211 | A | 14 | 7 | 1.00 | 25 | 0.280 |
| 212 | A | 14 | 7 | 1.00 | 25 | 0.280 |
| 213 | A | 8 | 7 | 1.00 | 23 | 0.304 |
| 214 | A | 8 | 7 | 1.00 | 14 | 0.500 |
| 215 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 216 | A | 36 | 10 | 1.00 | 25 | 0.400 |
| 217 | A | 26 | 10 | 1.00 | 25 | 0.400 |
| 218 | A | 24 | 11 | 1.00 | 25 | 0.440 |
| 219 | A | 13 | 11 | 1.00 | 23 | 0.478 |
| 220 | A | 9 | 8 | 1.00 | 14 | 0.571 |
| 221 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 222 | A | 34 | 9 | 1.00 | 25 | 0.360 |
| 223 | A | 23 | 9 | 1.00 | 25 | 0.360 |
| 224 | A | 24 | 11 | 1.00 | 25 | 0.440 |
| 225 | A | 11 | 10 | 1.00 | 23 | 0.435 |
| 226 | A | 10 | 8 | 1.00 | 14 | 0.571 |
| 227 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 228 | A | 8 | 7 | 1.00 | 23 | 0.304 |
| 229 | A | 6 | 5 | 1.00 | 23 | 0.217 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 230 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 231 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 232 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 233 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 234 | A | 7 | 7 | 1.00 | 23 | 0.304 |
| 235 | A | 5 | 5 | 1.00 | 23 | 0.217 |
| 236 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 237 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 238 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 239 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 240 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 241 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 242 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 243 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 244 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 245 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 246 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 247 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 248 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 249 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 250 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 251 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 252 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 253 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 254 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 255 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 256 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 257 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 258 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 259 | N/A | 0 | 0 | 1.00 | 25 | 0.000 |
| 260 | A | 7 | 6 | 1.00 | 30 | 0.200 |
| 261 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 262 | A | 4 | 4 | 1.00 | 28 | 0.143 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 263 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 264 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 265 | A | 4 | 4 | 0.97 | 30 | 0.133 |
| 266 | A | 15 | 9 | 1.00 | 30 | 0.300 |
| 267 | A | 11 | 9 | 1.00 | 30 | 0.300 |
| 268 | A | 7 | 6 | 1.00 | 28 | 0.214 |
| 269 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 270 | A | 7 | 5 | 1.00 | 30 | 0.167 |
| 271 | A | 11 | 8 | 1.00 | 30 | 0.267 |
| 272 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 273 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 274 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 275 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 276 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 277 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 278 | A | 8 | 8 | 1.00 | 30 | 0.267 |
| 279 | A | 7 | 7 | 1.00 | 30 | 0.233 |
| 280 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 281 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 282 | N/A | 0 | 0 | 1.00 | 30 | 0.000 |
| 283 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 284 | A | 4 | 4 | 1.00 | 10 | 0.400 |
| 285 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 286 | A | 5 | 5 | 1.00 | 6 | 0.833 |
| 287 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 288 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 289 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 290 | A | 6 | 6 | 1.00 | 10 | 0.600 |
| 291 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 292 | A | 7 | 5 | 1.00 | 10 | 0.500 |
| 293 | A | 6 | 5 | 1.00 | 8 | 0.625 |
| 294 | A | 6 | 6 | 1.00 | 6 | 1.000 |
| 295 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 296 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 297 | A | 4 | 4 | 1.00 | 10 | 0.400 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 298 | A | 5 | 4 | 1.00 | 10 | 0.400 |
| 299 | A | 6 | 4 | 1.00 | 10 | 0.400 |
| 300 | A | 6 | 6 | 1.00 | 10 | 0.600 |
| 301 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 302 | A | 5 | 5 | 1.00 | 6 | 0.833 |
| 303 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 304 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 305 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 306 | A | 5 | 4 | 1.00 | 10 | 0.400 |
| 307 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 308 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 309 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 310 | A | 3 | 3 | 1.00 | 6 | 0.500 |
| 311 | A | 5 | 5 | 1.00 | 10 | 0.500 |
| 312 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 313 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 314 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 315 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 316 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 317 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 318 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 319 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 320 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 321 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 322 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 323 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 324 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 325 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 326 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 327 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 328 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 329 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 330 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 331 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 332 | A | 1 | 1 | 1.00 | 22 | 0.045 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 333 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 334 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 335 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 336 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 337 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 338 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 339 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 340 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 341 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 342 | N/A | 0 | 0 | 1.00 | 40 | 0.000 |
| 343 | A | 8 | 8 | 1.00 | 40 | 0.200 |
| 344 | A | 7 | 7 | 1.00 | 40 | 0.175 |
| 345 | A | 6 | 7 | 1.00 | 38 | 0.184 |
| 346 | N/A | 0 | 0 | 1.00 | 40 | 0.000 |
| 347 | N/A | 0 | 0 | 1.00 | 40 | 0.000 |
| 348 | A | 6 | 6 | 1.00 | 10 | 0.600 |
| 349 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 350 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 351 | A | 5 | 4 | 1.00 | 10 | 0.400 |
| 352 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 353 | A | 9 | 8 | 1.00 | 12 | 0.667 |
| 354 | A | 9 | 8 | 1.00 | 12 | 0.667 |
| 355 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 356 | A | 7 | 6 | 1.00 | 12 | 0.500 |
| 357 | A | 8 | 7 | 1.00 | 12 | 0.583 |
| 358 | A | 37 | 8 | 1.00 | 14 | 0.571 |
| 359 | A | 27 | 8 | 1.00 | 14 | 0.571 |
| 360 | A | 17 | 8 | 1.00 | 12 | 0.667 |
| 361 | A | 7 | 4 | 1.00 | 10 | 0.400 |
| 362 | N/A | 0 | 0 | 1.00 | 14 | 0.000 |
| 363 | N/A | 0 | 0 | 1.00 | 14 | 0.000 |
| 364 | A | 7 | 7 | 1.00 | 19 | 0.368 |
| 365 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 366 | A | 4 | 4 | 1.00 | 12 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 367 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 368 | A | 6 | 6 | 1.00 | 10 | 0.600 |
| 369 | F | 0 | 0 | N/A | 0.000 | N/A |
| 370 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 371 | A | 2 | 2 | 1.00 | 26 | 0.077 |

# CHAPTER 3

# LISTING OF INTEGRALS


3.1 $\int \frac{\text{arcsinh}(cx)}{d+ex}\,dx$ . . . 126
3.2 $\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx$ . . . 132
3.3 $\int \frac{\text{arcsinh}(cx)^3}{d+ex}\,dx$ . . . 138
3.4 $\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$ . . . 146
3.5 $\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx$ . . . 152
3.6 $\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$ . . . 157
3.7 $\int (a+b\text{arcsinh}(cx))\,dx$ . . . 162
3.8 $\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx$ . . . 166
3.9 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}\,dx$ . . . 172
3.10 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx$ . . . 177
3.11 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}\,dx$ . . . 182
3.12 $\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$ . . . 188
3.13 $\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$ . . . 197
3.14 $\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$ . . . 205
3.15 $\int (a+b\text{arcsinh}(cx))^2\,dx$ . . . 212
3.16 $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx$ . . . 216
3.17 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$ . . . 223
3.18 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$ . . . 230
3.19 $\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$ . . . 238
3.20 $\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx$ . . . 246
3.21 $\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx$ . . . 253
3.22 $\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$ . . . 258

3.23 $\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))}\,dx$ . . . 262
3.24 $\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx$ . . . 265
3.25 $\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 269
3.26 $\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 277
3.27 $\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 283
3.28 $\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 288
3.29 $\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 292
3.30 $\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))^2\,dx$ . . . 296
3.31 $\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 299
3.32 $\int \frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}\,dx$ . . . 303
3.33 $\int \frac{(d+ex)^m}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$ . . . 306
3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 310
3.35 $\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 319
3.36 $\int (f+gx)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 327
3.37 $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx$ . . . 333
3.38 $\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx$ . . . 348
3.39 $\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 366
3.40 $\int (f+gx)^2\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 380
3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 390
3.42 $\int \frac{\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx$ . . . 397
3.43 $\int (f+gx)^3\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 419
3.44 $\int (f+gx)^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 437
3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$ . . . 453
3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx$ . . . 462
3.47 $\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$ . . . 488
3.48 $\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$ . . . 495
3.49 $\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$ . . . 501
3.50 $\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$ . . . 506
3.51 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$ . . . 509
3.52 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$ . . . 516
3.53 $\int \frac{(a+b\operatorname{arcsinh}(cx))^n\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$ . . . 525
3.54 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$ . . . 529
3.55 $\int \frac{(a+b\operatorname{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$ . . . 539
3.56 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$ . . . 546

3.57 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$ . . . 552
3.58 $\int x^3\text{arcsinh}(a+bx)\,dx$ . . . 556
3.59 $\int x^2\text{arcsinh}(a+bx)\,dx$ . . . 563
3.60 $\int x\text{arcsinh}(a+bx)\,dx$ . . . 569
3.61 $\int \text{arcsinh}(a+bx)\,dx$ . . . 574
3.62 $\int \frac{\text{arcsinh}(a+bx)}{x}\,dx$ . . . 578
3.63 $\int \frac{\text{arcsinh}(a+bx)}{x^2}\,dx$ . . . 584
3.64 $\int \frac{\text{arcsinh}(a+bx)}{x^3}\,dx$ . . . 589
3.65 $\int \frac{\text{arcsinh}(a+bx)}{x^4}\,dx$ . . . 594
3.66 $\int \frac{\text{arcsinh}(a+bx)}{x^5}\,dx$ . . . 600
3.67 $\int x^3\text{arcsinh}(a+bx)^2\,dx$ . . . 608
3.68 $\int x^2\text{arcsinh}(a+bx)^2\,dx$ . . . 616
3.69 $\int x\text{arcsinh}(a+bx)^2\,dx$ . . . 623
3.70 $\int \text{arcsinh}(a+bx)^2\,dx$ . . . 629
3.71 $\int \frac{\text{arcsinh}(a+bx)^2}{x}\,dx$ . . . 633
3.72 $\int \frac{\text{arcsinh}(a+bx)^2}{x^2}\,dx$ . . . 640
3.73 $\int \frac{\text{arcsinh}(a+bx)^2}{x^3}\,dx$ . . . 647
3.74 $\int \frac{\text{arcsinh}(a+bx)^2}{x^4}\,dx$ . . . 655
3.75 $\int x^2\text{arcsinh}(a+bx)^3\,dx$ . . . 672
3.76 $\int x\text{arcsinh}(a+bx)^3\,dx$ . . . 681
3.77 $\int \text{arcsinh}(a+bx)^3\,dx$ . . . 688
3.78 $\int \frac{\text{arcsinh}(a+bx)^3}{x}\,dx$ . . . 693
3.79 $\int \frac{\text{arcsinh}(a+bx)^3}{x^2}\,dx$ . . . 701
3.80 $\int \frac{\text{arcsinh}(a+bx)^3}{x^3}\,dx$ . . . 709
3.81 $\int \frac{x^2}{\text{arcsinh}(a+bx)}\,dx$ . . . 722
3.82 $\int \frac{x}{\text{arcsinh}(a+bx)}\,dx$ . . . 727
3.83 $\int \frac{1}{\text{arcsinh}(a+bx)}\,dx$ . . . 732
3.84 $\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx$ . . . 736
3.85 $\int \frac{x^2}{\text{arcsinh}(a+bx)^2}\,dx$ . . . 739
3.86 $\int \frac{x}{\text{arcsinh}(a+bx)^2}\,dx$ . . . 745
3.87 $\int \frac{1}{\text{arcsinh}(a+bx)^2}\,dx$ . . . 750
3.88 $\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx$ . . . 754
3.89 $\int \frac{x^2}{\text{arcsinh}(a+bx)^3}\,dx$ . . . 757
3.90 $\int \frac{x}{\text{arcsinh}(a+bx)^3}\,dx$ . . . 767
3.91 $\int \frac{1}{\text{arcsinh}(a+bx)^3}\,dx$ . . . 775
3.92 $\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx$ . . . 781
3.93 $\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx$ . . . 786

3.94 $\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx$ . . . 789
3.95 $\int x(a+b\text{arcsinh}(c+dx))^n\,dx$ . . . 797
3.96 $\int (a+b\text{arcsinh}(c+dx))^n\,dx$ . . . 804
3.97 $\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx$ . . . 808
3.98 $\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 811
3.99 $\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 822
3.100 $\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 829
3.101 $\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$ . . . 834
3.102 $\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx$ . . . 841
3.103 $\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$ . . . 847
3.104 $\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$ . . . 856
3.105 $\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 862
3.106 $\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 870
3.107 $\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 876
3.108 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 881
3.109 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 888
3.110 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 893
3.111 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 903
3.112 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 909
3.113 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 919
3.114 $\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx$ . . . 925
3.115 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx$ . . . 929
3.116 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx$ . . . 936
3.117 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx$ . . . 943
3.118 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx$ . . . 949
3.119 $\int (a+b\text{arcsinh}(c+dx))\,dx$ . . . 954
3.120 $\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx$ . . . 958
3.121 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx$ . . . 963
3.122 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx$ . . . 968
3.123 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx$ . . . 973
3.124 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$ . . . 979
3.125 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx$ . . . 984
3.126 $\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 990
3.127 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 995
3.128 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 1003
3.129 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 1010
3.130 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 1017
3.131 $\int (a+b\text{arcsinh}(c+dx))^2\,dx$ . . . 1022

3.132 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx$ . . . 1027
3.133 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx$ . . . 1033
3.134 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx$ . . . 1038
3.135 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$ . . . 1043
3.136 $\int (ce+dex)^m (a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1050
3.137 $\int (ce+dex)^4 (a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1054
3.138 $\int (ce+dex)^3 (a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1066
3.139 $\int (ce+dex)^2 (a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1075
3.140 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1084
3.141 $\int (a+b\text{arcsinh}(c+dx))^3\,dx$ . . . 1091
3.142 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx$ . . . 1096
3.143 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$ . . . 1103
3.144 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$ . . . 1110
3.145 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$ . . . 1117
3.146 $\int (ce+dex)^m (a+b\text{arcsinh}(c+dx))^4\,dx$ . . . 1127
3.147 $\int (ce+dex)^3 (a+b\text{arcsinh}(c+dx))^4\,dx$ . . . 1131
3.148 $\int (ce+dex)^2 (a+b\text{arcsinh}(c+dx))^4\,dx$ . . . 1142
3.149 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx$ . . . 1151
3.150 $\int (a+b\text{arcsinh}(c+dx))^4\,dx$ . . . 1159
3.151 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$ . . . 1165
3.152 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$ . . . 1173
3.153 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$ . . . 1182
3.154 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$ . . . 1190
3.155 $\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1202
3.156 $\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1205
3.157 $\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1211
3.158 $\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1217
3.159 $\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1223
3.160 $\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx$ . . . 1228
3.161 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx$ . . . 1232
3.162 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx$ . . . 1235
3.163 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx$ . . . 1243
3.164 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx$ . . . 1250
3.165 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}\,dx$ . . . 1257
3.166 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}\,dx$ . . . 1263

3.167 $\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$ . . . 1268
3.168 $\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1272
3.169 $\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1286
3.170 $\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1298
3.171 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1309
3.172 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1319
3.173 $\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$ . . . 1327
3.174 $\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1334
3.175 $\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1344
3.176 $\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1353
3.177 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1362
3.178 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1369
3.179 $\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$ . . . 1385
3.180 $\int (ce+dex)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$ . . . 1389
3.181 $\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$ . . . 1397
3.182 $\int (ce+dex)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$ . . . 1404
3.183 $\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$ . . . 1411
3.184 $\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$ . . . 1417
3.185 $\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx$ . . . 1422
3.186 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$ . . . 1425
3.187 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$ . . . 1440
3.188 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$ . . . 1450
3.189 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$ . . . 1459
3.190 $\int (a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$ . . . 1466
3.191 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$ . . . 1472
3.192 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$ . . . 1475
3.193 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$ . . . 1490
3.194 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$ . . . 1502
3.195 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$ . . . 1513
3.196 $\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$ . . . 1521
3.197 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$ . . . 1527
3.198 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$ . . . 1530
3.199 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$ . . . 1543
3.200 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$ . . . 1559
3.201 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$ . . . 1574
3.202 $\int (a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$ . . . 1583
3.203 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$ . . . 1590

3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1593
3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1600
3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1607
3.207 $\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1614
3.208 $\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1619
3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$ . . . 1624
3.210 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1627
3.211 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1634
3.212 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1641
3.213 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1647
3.214 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1652
3.215 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$ . . . 1657
3.216 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1660
3.217 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1670
3.218 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1679
3.219 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1688
3.220 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1695
3.221 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$ . . . 1701
3.222 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1704
3.223 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1715
3.224 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1724
3.225 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1734
3.226 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1741
3.227 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$ . . . 1747
3.228 $\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx$ . . . 1750
3.229 $\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx$ . . . 1757
3.230 $\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx$ . . . 1763
3.231 $\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx$ . . . 1769
3.232 $\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$ . . . 1775
3.233 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx$ . . . 1781
3.234 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx$ . . . 1786
3.235 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx$ . . . 1793

3.236 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$ . . . 1799
3.237 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$ . . . 1803
3.238 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$ . . . 1807
3.239 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$ . . . 1811
3.240 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$ . . . 1815
3.241 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$ . . . 1819
3.242 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx$ . . . 1823
3.243 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx$ . . . 1827
3.244 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$ . . . 1831
3.245 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$ . . . 1834
3.246 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$ . . . 1837
3.247 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$ . . . 1841
3.248 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$ . . . 1844
3.249 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$ . . . 1848
3.250 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$ . . . 1852
3.251 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$ . . . 1856
3.252 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$ . . . 1860
3.253 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$ . . . 1863
3.254 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$ . . . 1866
3.255 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$ . . . 1870
3.256 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$ . . . 1873
3.257 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$ . . . 1877
3.258 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$ . . . 1881
3.259 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$ . . . 1885
3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx$ . . . 1889
3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx$ . . . 1894
3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx$ . . . 1899
3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx$ . . . 1904
3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx$ . . . 1908
3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx$ . . . 1913
3.266 $\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx$ . . . 1919
3.267 $\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2\,dx$ . . . 1927
3.268 $\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)\,dx$ . . . 1934
3.269 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx$ . . . 1940
3.270 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx$ . . . 1944

3.271 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$ . . . 1949
3.272 $\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$ . . . 1956
3.273 $\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$ . . . 1960
3.274 $\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$ . . . 1964
3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx$ . . . 1968
3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx$ . . . 1971
3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx$ . . . 1975
3.278 $\int \frac{\text{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$ . . . 1980
3.279 $\int \frac{\text{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$ . . . 1986
3.280 $\int \frac{\text{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$ . . . 1991
3.281 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\text{arcsinh}(a+bx)}\,dx$ . . . 1995
3.282 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\text{arcsinh}(a+bx)^2}\,dx$ . . . 1999
3.283 $\int x^3\text{arcsinh}(ax^2)\,dx$ . . . 2003
3.284 $\int x^2\text{arcsinh}(ax^2)\,dx$ . . . 2008
3.285 $\int x\text{arcsinh}(ax^2)\,dx$ . . . 2013
3.286 $\int \text{arcsinh}(ax^2)\,dx$ . . . 2017
3.287 $\int \frac{\text{arcsinh}(ax^2)}{x}\,dx$ . . . 2022
3.288 $\int \frac{\text{arcsinh}(ax^2)}{x^2}\,dx$ . . . 2026
3.289 $\int \frac{\text{arcsinh}(ax^2)}{x^3}\,dx$ . . . 2030
3.290 $\int \frac{\text{arcsinh}(ax^2)}{x^4}\,dx$ . . . 2035
3.291 $\int \frac{\text{arcsinh}(ax^5)}{x}\,dx$ . . . 2041
3.292 $\int x^2\text{arcsinh}\left(\sqrt{x}\right)\,dx$ . . . 2045
3.293 $\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx$ . . . 2050
3.294 $\int \text{arcsinh}\left(\sqrt{x}\right)\,dx$ . . . 2055
3.295 $\int \frac{\text{arcsinh}(\sqrt{x})}{x}\,dx$ . . . 2059
3.296 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^2}\,dx$ . . . 2063
3.297 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^3}\,dx$ . . . 2067
3.298 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^4}\,dx$ . . . 2071
3.299 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^5}\,dx$ . . . 2076
3.300 $\int x^2\text{arcsinh}\left(\frac{a}{x}\right)\,dx$ . . . 2081
3.301 $\int x\text{arcsinh}\left(\frac{a}{x}\right)\,dx$ . . . 2086
3.302 $\int \text{arcsinh}\left(\frac{a}{x}\right)\,dx$ . . . 2090
3.303 $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx$ . . . 2095
3.304 $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx$ . . . 2099
3.305 $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx$ . . . 2103

3.306 $\int \frac{\operatorname{arcsinh}(\frac{a}{x})}{x^4}\,dx$ . . . 2108
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3.310 $\int \operatorname{arcsinh}(ax^n)\,dx$ . . . 2124
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3.312 $\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx$ . . . 2132
3.313 $\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx$ . . . 2136
3.314 $\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^4\,dx$ . . . 2140
3.315 $\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^3\,dx$ . . . 2145
3.316 $\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^2\,dx$ . . . 2150
3.317 $\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)\,dx$ . . . 2154
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3.329 $\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{3/2}\,dx$ . . . 2211
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3.343 $\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$ . . . 2275

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3.348 $\int \text{arcsinh}\left(ce^{a+bx}\right)\,dx$ . . . 2304

3.349 $\int e^{\text{arcsinh}(a+bx)}x^3\,dx$ . . . 2309

3.350 $\int e^{\text{arcsinh}(a+bx)}x^2\,dx$ . . . 2315

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3.354 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}\,dx$ . . . 2337

3.355 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx$ . . . 2343

3.356 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4}\,dx$ . . . 2349

3.357 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5}\,dx$ . . . 2356

3.358 $\int e^{\text{arcsinh}(a+bx)^2}x^3\,dx$ . . . 2365

3.359 $\int e^{\text{arcsinh}(a+bx)^2}x^2\,dx$ . . . 2373

3.360 $\int e^{\text{arcsinh}(a+bx)^2}x\,dx$ . . . 2380

3.361 $\int e^{\text{arcsinh}(a+bx)^2}\,dx$ . . . 2385

3.362 $\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx$ . . . 2389

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3.364 $\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx$ . . . 2395

3.365 $\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx$ . . . 2400

3.366 $\int x^3\text{arcsinh}(a+bx^4)\,dx$ . . . 2403

3.367 $\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx$ . . . 2407

3.368 $\int \text{arcsinh}\left(\frac{c}{a+bx}\right)\,dx$ . . . 2411

3.369 $\int \frac{x}{\text{arcsinh}(\sinh(x))}\,dx$ . . . 2416

3.370 $\int \frac{\text{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$ . . . 2419

3.371 $\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx$ . . . 2423

# 3.1 $\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx$

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Rubi [A] (verified) . . . 126
Mathematica [A] (verified) . . . 129
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Fricas [F] . . . 130
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## Optimal result

Integrand size = 12, antiderivative size = 170

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[Out] -1/2*arcsinh(c*x)^2/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2
*d^2+e^2)^(1/2)))/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d
^2+e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5827, 5680, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e} - \frac{\operatorname{arcsinh}(cx)^2}{2e}$$

```
[In] Int[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x\cosh(x)}{cd + e\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \operatorname{Subst}\left(\int \frac{e^x x}{cd - \sqrt{c^2d^2+e^2} + ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&\quad + \operatorname{Subst}\left(\int \frac{e^x x}{cd + \sqrt{c^2d^2+e^2} + ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&= -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{\operatorname{Subst}\left(\int \log\left(1+\frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{\operatorname{Subst}\left(\int \log\left(1+\frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ex}{cd-\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ex}{cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,\frac{ee^{\operatorname{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[In] Integrate[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

## Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{c\,\operatorname{arcsinh}(cx)^2}{2e}+\frac{c\,\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}}{c}$ |
| default | $\frac{-\frac{c\,\operatorname{arcsinh}(cx)^2}{2e}+\frac{c\,\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}}{c}$ |

```
[In] int(arcsinh(c*x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*c*arcsinh(c*x)^2/e+c/e*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(
1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+c/e*arcsinh(c*x)*ln(
(c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2
)))+c/e*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c
^2*d^2+e^2)^(1/2)))+c/e*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
[Out] integral(arcsinh(c*x)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(cx)}{d+ex}\,dx$$

```
[In] integrate(asinh(c*x)/(e*x+d),x)
[Out] Integral(asinh(c*x)/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)}{d+e\,x}\,dx$$

```
[In] int(asinh(c*x)/(d + e*x),x)
[Out] int(asinh(c*x)/(d + e*x), x)
```

# 3.2 $\int \frac{\mathbf{arcsinh}(cx)^2}{d+ex}\,dx$

Optimal result . . . 132
Rubi [A] (verified) . . . 133
Mathematica [A] (verified) . . . 136
Maple [F] . . . 136
Fricas [F] . . . 136
Sympy [F] . . . 137
Maxima [F] . . . 137
Giac [F] . . . 137
Mupad [F(-1)] . . . 137

## Optimal result

Integrand size = 14, antiderivative size = 260

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = &-\frac{\operatorname{arcsinh}(cx)^3}{3e}+\frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\&-\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

```
[Out] -1/3*arcsinh(c*x)^3/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(
c*d-(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c
*d-(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2
*d^2+e^2)^(1/2)))/e
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5827, 5680, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = &\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{\operatorname{arcsinh}(cx)^2\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}\\ &+\frac{\operatorname{arcsinh}(cx)^2\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^3}{3e}\end{aligned}$$

```
[In] Int[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*ArcSinh[c*x]^3/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*
x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&= -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&\quad + \text{Subst}\left(\int \frac{e^x x^2}{cd+\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&= -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{2\text{Subst}\left(\int x\log\left(1+\frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{2\text{Subst}\left(\int x\log\left(1+\frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{2\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{ee^{x}}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{2\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{ee^{x}}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{2\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{ex}{-cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&\quad - \frac{2\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{ex}{cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx =$$

$$-\frac{\operatorname{arcsinh}(cx)^3 - 3\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) - 3\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) - 6\operatorname{arcsinh}(cx$$

```
[In] Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sq
rt[c^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
 + e^2]))])/e
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{ex+d}dx$$

```
[In] int(arcsinh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^2/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^2/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{asinh}^2(cx)}{d+ex}\,dx$$

```
[In] integrate(asinh(c*x)**2/(e*x+d),x)
[Out] Integral(asinh(c*x)**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)^2}{d+e\,x}\,dx$$

```
[In] int(asinh(c*x)^2/(d + e*x),x)
[Out] int(asinh(c*x)^2/(d + e*x), x)
```

# 3.3 $\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$

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## Optimal result

Integrand size = 14, antiderivative size = 348

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = &-\frac{\operatorname{arcsinh}(cx)^4}{4e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\&-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\&-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\&+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

```
[Out] -1/4*arcsinh(c*x)^4/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))
/(c*d-(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*
x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*
x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = &\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}\\ &+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^4}{4e}\end{aligned}$$

```
[In] Int[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] -1/4*ArcSinh[c*x]^4/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^A
rcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*PolyLog[3,
-((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[
4, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^3 \cosh(x)}{cd + e\sinh(x)}\, dx, x, \text{arcsinh}(cx)\right)$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \operatorname{Subst}\left(\int \frac{e^x x^3}{cd - \sqrt{c^2d^2+e^2} + ee^x}\, dx, x, \operatorname{arcsinh}(cx)\right) \\
&\quad + \operatorname{Subst}\left(\int \frac{e^x x^3}{cd + \sqrt{c^2d^2+e^2} + ee^x}\, dx, x, \operatorname{arcsinh}(cx)\right) \\
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1 + \frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1 + \frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{3\operatorname{Subst}\left(\int x^2 \log\left(1 + \frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\, dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{3\operatorname{Subst}\left(\int x^2 \log\left(1 + \frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\, dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1 + \frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1 + \frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -\frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\, dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{6\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -\frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\, dx, x, \operatorname{arcsinh}(cx)\right)}{e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{6\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad + \frac{6\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,\frac{ex}{-cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&\quad + \frac{6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,-\frac{ex}{cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(cx)^4}{4e} + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{3\operatorname{arcsinh}(cx)^2 \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{6\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{6 \operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{6 \operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.93

$$
\begin{aligned}
&\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx \\
&= \frac{-\operatorname{arcsinh}(cx)^4 + 4\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) + 4\operatorname{arcsinh}(cx)^3 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) + 12\operatorname{arcsinh}(}{}
\end{aligned}
$$

```
[In] Integrate[ArcSinh[c*x]^3/(d + e*x),x]

[Out] (-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[
c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[
c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
+ Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSinh[c*
x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSinh[c*x])
/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x])/(c*d
+ Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{ex+d}dx$$

```
[In] int(arcsinh(c*x)^3/(e*x+d),x)
[Out] int(arcsinh(c*x)^3/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
[Out] integral(arcsinh(c*x)^3/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}^3(cx)}{d+ex}\,dx$$

```
[In] integrate(asinh(c*x)**3/(e*x+d),x)
[Out] Integral(asinh(c*x)**3/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}\left(cx\right)^3}{ex+d}\,dx$$

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)^3}{d+e\,x}\,dx$$

```
[In] int(asinh(c*x)^3/(d + e*x),x)
[Out] int(asinh(c*x)^3/(d + e*x), x)
```

# 3.4 $\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 176

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b(4d(19c^2d^2-16e^2)+e(26c^2d^2-9e^2)\,x)\sqrt{1+c^2x^2}}{96c^3} - \frac{b(8c^4d^4-24c^2d^2e^2+3e^4)\operatorname{arcsinh}(cx)}{32c^4e} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e}$$

```
[Out] -1/32*b*(8*c^4*d^4-24*c^2*d^2*e^2+3*e^4)*arcsinh(c*x)/c^4/e+1/4*(e*x+d)^4*(
a+b*arcsinh(c*x))/e-7/48*b*d*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/16*b*(e*x+d)^3
*(c^2*x^2+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2-16*e^2)+e*(26*c^2*d^2-9*e^2)*x
)*(c^2*x^2+1)^(1/2)/c^3
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5828, 757, 847, 794, 221}

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx = \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e} - \frac{b\operatorname{arcsinh}(cx)\,(8c^4d^4-24c^2d^2e^2+3e^4)}{32c^4e} - \frac{b\sqrt{c^2x^2+1}(d+ex)^3}{16c} - \frac{7bd\sqrt{c^2x^2+1}(d+ex)^2}{48c} - \frac{b\sqrt{c^2x^2+1}(ex(26c^2d^2-9e^2)+4d(19c^2d^2-16e^2))}{96c^3}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-7*b*d*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(48*c) - (b*(d + e*x)^3*Sqrt[1 + c^2
*x^2])/(16*c) - (b*(4*d*(19*c^2*d^2 - 16*e^2) + e*(26*c^2*d^2 - 9*e^2)*x)*S
qrt[1 + c^2*x^2])/(96*c^3) - (b*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSin
h[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{1+c^2x^2}}\,dx}{4e} \\
&= -\frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e} - \frac{b\int \frac{(d+ex)^2\left(4c^2d^2-3e^2+7c^2dex\right)}{\sqrt{1+c^2x^2}}\,dx}{16ce} \\
&= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} \\
&\quad + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e} - \frac{b\int \frac{(d+ex)\left(c^2d\left(12c^2d^2-23e^2\right)+c^2e\left(26c^2d^2-9e^2\right)x\right)}{\sqrt{1+c^2x^2}}\,dx}{48c^3e} \\
&= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} \\
&\quad - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)\sqrt{1+c^2x^2}}{96c^3} \\
&\quad + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e} - \frac{\left(b\left(8c^4d^4-24c^2d^2e^2+3e^4\right)\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{32c^3e} \\
&= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} \\
&\quad - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)\sqrt{1+c^2x^2}}{96c^3} \\
&\quad - \frac{b\left(8c^4d^4-24c^2d^2e^2+3e^4\right)\operatorname{arcsinh}(cx)}{32c^4e} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{4e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{24ac^4x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right) - bc\sqrt{1+c^2x^2}\left(-e^2(64d+9ex)+c^2\left(96d^3+72d^2ex+32de^2x^2+6e^{3}\right.\right.}{96c^4}
\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2*
x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e
^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2
*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| parts | $\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\,\mathrm{arcsinh}(cx)x^4}{4}+c\,e^2\,\mathrm{arcsinh}(cx)x^3d+\frac{3c\,\mathrm{arcsinh}(cx)d^2e\,x^2}{2}+\mathrm{arcsinh}(cx)cx\,d^3+\frac{c\,\mathrm{arcsinh}(cx)d^4}{4e}-\frac{c^4d^4\,a}{4e}\right.}{}$ |
| derivativedivides | $\frac{a(ecx+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arcsinh}(cx)c^4d^4}{4e}+\mathrm{arcsinh}(cx)c^4d^3x+\frac{3e\,\mathrm{arcsinh}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arcsinh}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arcsinh}(cx)c^4x^4}{4}-\frac{c^4d^4\,\mathrm{arcs}}{}\right.}{}$ |
| default | $\frac{a(ecx+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arcsinh}(cx)c^4d^4}{4e}+\mathrm{arcsinh}(cx)c^4d^3x+\frac{3e\,\mathrm{arcsinh}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arcsinh}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arcsinh}(cx)c^4x^4}{4}-\frac{c^4d^4\,\mathrm{arcs}}{}\right.}{}$ |

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arcsinh(c*x)*x^4+c*e^2*arcsinh(c*x)*x^3*d+
3/2*c*arcsinh(c*x)*d^2*e*x^2+arcsinh(c*x)*c*x*d^3+1/4*c/e*arcsinh(c*x)*d^4-
1/4/c^3/e*(c^4*d^4*arcsinh(c*x)+e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c*x*
(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*d^3*c^3*e*(c^2*x^2+1)^(1/2)+6*d^2*c^2
*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+4*d*c*e^3*(1/3*c^2*x^2*(c
^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int (d+ex)^3(a+b\mathrm{arcsinh}(cx))\,dx$$

$$=\frac{24\,ac^4e^3x^4+96\,ac^4de^2x^3+144\,ac^4d^2ex^2+96\,ac^4d^3x+3\,(8\,bc^4e^3x^4+32\,bc^4de^2x^3+48\,bc^4d^2ex^2+32\,bc^4d}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c^
4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 + 32
*b*c^4*d^3*x + 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*
b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 - 64*b*c*d*e^2 + 9*(8*b*c
^3*d^2*e - b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.80

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{asinh}(cx) + \frac{3bd^2ex^2\operatorname{asinh}(cx)}{2} + bde^2x^3\operatorname{asinh}(cx) + \frac{be^3x^4\operatorname{asinh}(cx)}{4} - \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c*x
) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2*b*d*
e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(32*c**3
) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2
+ d*e**2*x**3 + e**3*x**4/4), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.31

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2$$
$$+ \frac{3}{4}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^2e$$
$$+ \frac{1}{3}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde^2$$
$$+ \frac{1}{32}\left(8x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)be^3$$
$$+ ad^3x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2*e + 1/3*(3*x^3*arcsinh
```

```
(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e^2 +
1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 +
1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*x) - sqr
t(c^2*x^2 + 1))*b*d^3/c
```

## Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^3, x)
```

# 3.5 $\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\operatorname{arcsinh}(cx)}{6e} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e}$$

```
[Out] -1/6*b*d*(2*d^2-3*e^2/c^2)*arcsinh(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))/
e-1/9*b*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*d^2-4*e^2)
*(c^2*x^2+1)^(1/2)/c^3
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5828, 757, 794, 221}

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e} - \frac{bd\operatorname{arcsinh}(cx)\left(2d^2-\frac{3e^2}{c^2}\right)}{6e} - \frac{b\sqrt{c^2x^2+1}(d+ex)^2}{9c} - \frac{b\sqrt{c^2x^2+1}(4(4c^2d^2-e^2)+5c^2dex)}{18c^3}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/9*(b*(d + e*x)^2*Sqrt[1 + c^2*x^2])/c - (b*(4*(4*c^2*d^2 - e^2) + 5*c^2*
d*e*x)*Sqrt[1 + c^2*x^2])/(18*c^3) - (b*d*(2*d^2 - (3*e^2)/c^2)*ArcSinh[c*x
])/(6*e) + ((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e} - \frac{b\int \frac{(d+ex)(3c^2d^2-2e^2+5c^2dex)}{\sqrt{1+c^2x^2}}\,dx}{9ce}$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3}$$
$$+ \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e} - \frac{1}{6}\left(bd\left(\frac{2cd^2}{e} - \frac{3e}{c}\right)\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\operatorname{arcsinh}(cx)}{6e} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{6ac^3x(3d^2+3dex+e^2x^2) - b\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2)) + 3bc(6c^2d^2x+2c^2e^2x^3+3d(e}{18c^3}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*
(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| parts | $\frac{a(ex+d)^3}{3e} + \frac{b\left(\frac{c\,e^2\operatorname{arcsinh}(cx)x^3}{3} + c\operatorname{arcsinh}(cx)de\,x^2 + \operatorname{arcsinh}(cx)cx\,d^2 + \frac{c\operatorname{arcsinh}(cx)d^3}{3e} - \frac{c^3d^3\operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c^2x^2+}}{3}\right.}{c}$ |
| derivativedivides | $\frac{a(ecx+cd)^3}{3c^2e} + \frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e} + \operatorname{arcsinh}(cx)c^3d^2x + e\operatorname{arcsinh}(cx)c^3dx^2 + \frac{e^2\operatorname{arcsinh}(cx)c^3x^3}{3} - \frac{c^3d^3\operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c^2x^2+}}{3}\right.}{c^2}}{c}$ |
| default | $\frac{a(ecx+cd)^3}{3c^2e} + \frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e} + \operatorname{arcsinh}(cx)c^3d^2x + e\operatorname{arcsinh}(cx)c^3dx^2 + \frac{e^2\operatorname{arcsinh}(cx)c^3x^3}{3} - \frac{c^3d^3\operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c^2x^2+}}{3}\right.}{c^2}}{c}$ |

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arcsinh(c*x)*x^3+c*arcsinh(c*x)*d*e*x^2+ar
csinh(c*x)*c*x*d^2+1/3*c/e*arcsinh(c*x)*d^3-1/3/c^2/e*(c^3*d^3*arcsinh(c*x)
+e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d^2*c^2*e*(c^2
*x^2+1)^(1/2)+3*d*c*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{6\,ac^3e^2x^3 + 18\,ac^3dex^2 + 18\,ac^3d^2x + 3\,(2\,bc^3e^2x^3 + 6\,bc^3dex^2 + 6\,bc^3d^2x + 3\,bcde)\log\left(cx + \sqrt{c^2x^2+1}\right) -}{18\,c^3}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2*
x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x + 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2 +
 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 - 4*b*e^2)*sqrt(c^2*
x^2 + 1))/c^3
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asinh}(cx) + bdex^2\operatorname{asinh}(cx) + \frac{be^2x^3\operatorname{asinh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{bdex\sqrt{c^2x^2+1}}{2c} - \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b*
d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 + 1)
/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)/(9
*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3), Ne
(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\begin{aligned}
&\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{1}{3}ae^2x^3 + adex^2 + \frac{1}{2}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bde \\
&\quad + \frac{1}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be^2 \\
&\quad + ad^2x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*
x/c^2 - arcsinh(c*x)/c^3))*b*d*e + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2 + a*d^2*x + (c*x*arcsinh(c
*x) - sqrt(c^2*x^2 + 1))*b*d^2/c
```

## Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\,\mathrm{asinh}(c\,x))\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^2, x)
```

# 3.6 $\int (d+ex)(a+b\textbf{arcsinh}(cx))\,dx$

Optimal result . . . 157
Rubi [A] (verified) . . . 157
Mathematica [A] (verified) . . . 159
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Maxima [A] (verification not implemented) . . . 160
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Mupad [B] (verification not implemented) . . . 161

## Optimal result

Integrand size = 14, antiderivative size = 97

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} - \frac{b\left(2d^2-\frac{e^2}{c^2}\right)\text{arcsinh}(cx)}{4e} + \frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}$$

```
[Out] -1/4*b*(2*d^2-e^2/c^2)*arcsinh(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))/e-3/
4*b*d*(c^2*x^2+1)^(1/2)/c-1/4*b*(e*x+d)*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5828, 757, 655, 221}

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = \frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e} - \frac{b\text{arcsinh}(cx)\left(2d^2-\frac{e^2}{c^2}\right)}{4e} - \frac{b\sqrt{c^2x^2+1}(d+ex)}{4c} - \frac{3bd\sqrt{c^2x^2+1}}{4c}$$

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2])/(4*c) - (b*(d + e*x)*Sqrt[1 + c^2*x^2])/(4*c) -
(b*(2*d^2 - e^2/c^2)*ArcSinh[c*x])/(4*e) + ((d + e*x)^2*(a + b*ArcSinh[c*x]
))/(2*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{\sqrt{1+c^2x^2}}\,dx}{2e} \\
&= -\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{2e} - \frac{b\int \frac{2c^2d^2-e^2+3c^2dex}{\sqrt{1+c^2x^2}}\,dx}{4ce} \\
&= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} \\
&\quad + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{2e} - \frac{1}{4}\left(b\left(\frac{2cd^2}{e} - \frac{e}{c}\right)\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} \\
&\quad - \frac{b\left(2d^2 - \frac{e^2}{c^2}\right)\operatorname{arcsinh}(cx)}{4e} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{2e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx = adx + \frac{1}{2}aex^2 - \frac{bd\sqrt{1+c^2x^2}}{c} - \frac{bex\sqrt{1+c^2x^2}}{4c} + \frac{be\operatorname{arcsinh}(cx)}{4c^2} + bdx\operatorname{arcsinh}(cx) + \frac{1}{2}bex^2\operatorname{arcsinh}(cx)$$

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2])
/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSinh
[c*x])/2
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| parts | $a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(cx)x^2e}{2}+\operatorname{arcsinh}(cx)dcx-\frac{e\left(\frac{cx\sqrt{c^2x^2+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)+2dc\sqrt{c^2x^2+1}}{2c}\right)}{c}$ | 84 |
| derivativedivides | $\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^2x+\frac{\operatorname{arcsinh}(cx)e\,c^2x^2}{2}-\frac{e\left(\frac{cx\sqrt{c^2x^2+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}-dc\sqrt{c^2x^2+1}\right)}{c}}{c}$ | 96 |
| default | $\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^2x+\frac{\operatorname{arcsinh}(cx)e\,c^2x^2}{2}-\frac{e\left(\frac{cx\sqrt{c^2x^2+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}-dc\sqrt{c^2x^2+1}\right)}{c}}{c}$ | 96 |

```
[In] int((e*x+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsinh(c*x)*x^2*e+arcsinh(c*x)*d*c*x-1/2/c*(e
*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+2*d*c*(c^2*x^2+1)^(1/2)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{2\,ac^2ex^2+4\,ac^2dx+(2\,bc^2ex^2+4\,bc^2dx+be)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcex+4\,bcd)\sqrt{c^2x^2+1}}{4\,c^2}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x + b*e)*log(
c*x + sqrt(c^2*x^2 + 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

## Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} adx+\frac{aex^2}{2}+bdx\operatorname{asinh}(cx)+\frac{bex^2\operatorname{asinh}(cx)}{2}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex\sqrt{c^2x^2+1}}{4c}+\frac{be\operatorname{asinh}(cx)}{4c^2} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c*x
)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{1}{2}\,aex^2+\frac{1}{4}\left(2\,x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)be$$
$$+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c*x)/c^3))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d/c
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{1}{2}aex^2 + \left(x\log\left(cx+\sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd \\
&\quad + \frac{1}{4}\left(2x^2\log\left(cx+\sqrt{c^2x^2+1}\right) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} + \frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)}{c^2|c|}\right)\right)be \\
&\quad + adx
\end{aligned}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*e*x^2 + (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d +
1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + log(
-x*abs(c) + sqrt(c^2*x^2 + 1))/(c^2*abs(c))))*b*e + a*d*x
```

## Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\begin{aligned}
\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx = {}& \frac{a\,x\,(2\,d+e\,x)}{2} - \frac{b\,d\left(\sqrt{c^2\,x^2+1} - c\,x\,\operatorname{asinh}(c\,x)\right)}{c} \\
& - \frac{b\,e\,x\,\sqrt{c^2\,x^2+1}}{4\,c} + b\,e\,x\,\operatorname{asinh}(c\,x)\left(\frac{x}{2} + \frac{1}{4\,c^2\,x}\right)
\end{aligned}$$

```
[In] int((a + b*asinh(c*x))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 - (b*d*((c^2*x^2 + 1)^(1/2) - c*x*asinh(c*x)))/c - (b*e
*x*(c^2*x^2 + 1)^(1/2))/(4*c) + b*e*x*asinh(c*x)*(x/2 + 1/(4*c^2*x))
```

# 3.7 $\int (a + b\mathbf{arcsinh}(cx))\, dx$

Optimal result . . . 162
Rubi [A] (verified) . . . 162
Mathematica [A] (verified) . . . 163
Maple [A] (verified) . . . 163
Fricas [A] (verification not implemented) . . . 164
Sympy [A] (verification not implemented) . . . 164
Maxima [A] (verification not implemented) . . . 164
Giac [A] (verification not implemented) . . . 165
Mupad [B] (verification not implemented) . . . 165

## Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a + b\text{arcsinh}(cx))\, dx = ax - \frac{b\sqrt{1 + c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

```
[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5772, 267}

$$\int (a + b\text{arcsinh}(cx))\, dx = ax + bx\text{arcsinh}(cx) - \frac{b\sqrt{c^2x^2 + 1}}{c}$$

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \operatorname{arcsinh}(cx)\,dx \\ &= ax + bx\operatorname{arcsinh}(cx) - (bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx \\ &= ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\operatorname{arcsinh}(cx)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\operatorname{arcsinh}(cx)$$

```
[In] Integrate[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $ax + \frac{b\left(\operatorname{arcsinh}(cx)cx - \sqrt{c^2x^2+1}\right)}{c}$ | 31 |
| parts | $ax + \frac{b\left(\operatorname{arcsinh}(cx)cx - \sqrt{c^2x^2+1}\right)}{c}$ | 31 |
| derivativedivides | $\frac{cxa + b\left(\operatorname{arcsinh}(cx)cx - \sqrt{c^2x^2+1}\right)}{c}$ | 33 |

```
[In] int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a + b\text{arcsinh}(cx))\, dx = \frac{bcx \log\left(cx + \sqrt{c^2x^2+1}\right) + acx - \sqrt{c^2x^2+1}b}{c}$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a + b\text{arcsinh}(cx))\, dx = ax + b\left(\begin{cases} x\,\text{asinh}\,(cx) - \frac{\sqrt{c^2x^2+1}}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\, dx = ax + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)b}{c}$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

## Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = \left(x \log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right) b + ax$$

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = a\,x - \frac{b\sqrt{c^2\,x^2+1}}{c} + b\,x\,\operatorname{asinh}(c\,x)$$

```
[In] int(a + b*asinh(c*x),x)
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

# 3.8 $\int \frac{a+b\mathbf{arcsinh}(cx)}{d+ex}\,dx$

Optimal result . . . 166
Rubi [A] (verified) . . . 166
Mathematica [A] (verified) . . . 169
Maple [A] (verified) . . . 169
Fricas [F] . . . 170
Sympy [F] . . . 170
Maxima [F] . . . 170
Giac [F] . . . 170
Mupad [F(-1)] . . . 171

## Optimal result

Integrand size = 16, antiderivative size = 187

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2)
)/(c*d-(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5827, 5680, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx = \frac{(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] -1/2*(a + b*ArcSinh[c*x])^2/(b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 + (
e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^ArcSinh
[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{cd+e\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \operatorname{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right) + \operatorname{Subst}\left(\int \frac{e^x(a+bx)}{cd+\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right)$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{ee^x}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{ee^x}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e}$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ex}{cd-\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ex}{cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e}$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2be} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{d+ex}\,dx$$
$$= \frac{-\left((a+b\operatorname{arcsinh}(cx))\left(a+b\operatorname{arcsinh}(cx)-2b\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-2b\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)\right)+2b^2\,\mathrm{P}}{2be}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x]
)/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt
[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/(2*b*e)
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| parts | $\frac{a\ln(ex+d)}{e}-\frac{b\operatorname{arcsinh}(cx)^2}{2e}+\frac{b\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right.}{c}\right.$ |
| derivativedivides | $\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(-\frac{\operatorname{arcsinh}(cx)^2}{2e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)}{cd+\sqrt{c^2d^2}}\right.\right.}{e}\right.}{c}$ |
| default | $\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(-\frac{\operatorname{arcsinh}(cx)^2}{2e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)}{cd+\sqrt{c^2d^2}}\right.\right.}{e}\right.}{c}$ |

```
[In] int((a+b*arcsinh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e-1/2*b*arcsinh(c*x)^2/e+b/e*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*
x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*arcsinh(
c*x)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e
^2)^(1/2)))+b/e*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/
(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d
^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))
```

## Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{d+ex}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x+d),x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x) + a*log(e*x + d)/e
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{d + ex}\,dx = \int \frac{a + b\,\operatorname{asinh}(c\,x)}{d + e\,x}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x),x)
[Out] int((a + b*asinh(c*x))/(d + e*x), x)
```

# 3.9 $\int \frac{a+b\mathbf{arcsinh}(cx)}{(d+ex)^2}\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} - \frac{bc\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/e/(e*x+d)-b*c*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)/
(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5828, 739, 212}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} - \frac{bc\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^
2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} + \frac{(bc)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e} \\ &= -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} - \frac{(bc)\operatorname{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx, x, \frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{e} \\ &= -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} - \frac{bc\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{\frac{a+b\operatorname{arcsinh}(cx)}{d+ex} + \frac{bc\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^
2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(78) = 156.

Time = 0.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| parts | $-\frac{a}{(ex+d)e}-\frac{bc\,\operatorname{arcsinh}(cx)}{(ecx+cd)e}-\frac{bc\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}$ |
| derivativedivides | $\frac{-\frac{a\,c^2}{(ecx+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsinh}(cx)}{(ecx+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$ |
| default | $\frac{-\frac{a\,c^2}{(ecx+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsinh}(cx)}{(ecx+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$ |

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e-b*c/(c*e*x+c*d)/e*arcsinh(c*x)-b*c/e^2/((c^2*d^2+e^2)/e^2)^(1/
2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*
((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(78) = 156.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.09

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -\frac{ac^2d^3+ade^2-(bc^2d^2e+be^3)x\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcdex+bcd^2)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x-cde+\sqrt{c^2d^2+}}{c^2d^4e+d^2e^3+(c^2}\right.}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*e^2 - (b*c^2*d^2*e + b*e^3)*x*log(c*x + sqrt(c^2*x^2 + 1)
) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqr
```

```
t(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*
sqrt(c^2*x^2 + 1))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d
*e^4)*x)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(cx\right)}{\left(d+ex\right)^{2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -b\left(\frac{\operatorname{arsinh}\left(cx\right)}{e^{2}x+de} - \frac{c\operatorname{arsinh}\left(\frac{cd\sqrt{e^{4}}x}{e\left|e^{2}x+de\right|} - \frac{\sqrt{e^{4}}}{c\left|e^{2}x+de\right|}\right)}{\sqrt{\frac{c^{2}d^{2}}{e^{2}}+1e^{2}}}\right) - \frac{a}{e^{2}x+de}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c*d*sqrt(e^4)*x/(e*abs(e^2*x + d
*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a/(e^
2*x + d*e)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(78) = 156.

Time = 0.47 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.83

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx$$

$$= \left(\frac{c\log\left(-c^{2}de+\sqrt{c^{2}d^{2}+e^{2}}|c||e|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{c^{2}d^{2}+e^{2}}|e|} - \frac{\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{(ex+d)e} - \frac{c\log\left(-c^{2}de+\sqrt{c^{2}d^{2}+e}\right.}{\sqrt{c}}\right.$$

$$- \frac{a}{(ex+d)e}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*abs(c)*abs(e))*sgn(1/(e*x + d))*sgn(e
)/(sqrt(c^2*d^2 + e^2)*abs(e)) - log(c*x + sqrt(c^2*x^2 + 1))/((e*x + d)*e)
 - c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*d/(e*x + d) + c^2
*d^2/(e*x + d)^2 + e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 + e^4)/((e*x + d)*e)
)*abs(e))/(sqrt(c^2*d^2 + e^2)*abs(e)*sgn(1/(e*x + d))*sgn(e)))*b - a/((e*x
 + d)*e)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^2, x)
```

# 3.10 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} - \frac{bc^3d\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/e/(e*x+d)^2-1/2*b*c^3*d*arctanh((-c^2*d*x+e)/(c^2*d
^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(3/2)-1/2*b*c*(c^2*x^2+1)^
(1/2)/(c^2*d^2+e^2)/(e*x+d)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5828, 745, 739, 212}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} - \frac{bc^3d\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}} - \frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)) - (a + b*ArcSinh[c
*x])/(2*e*(d + e*x)^2) - (b*c^3*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 + e^2
]*Sqrt[1 + c^2*x^2])])/(2*e*(c^2*d^2 + e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} + \frac{(bc)\int \frac{1}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{2e} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} + \frac{\left(bc^3d\right)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{2e\left(c^2d^2+e^2\right)} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} - \frac{\left(bc^3d\right)\operatorname{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx, x, \frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)} \\
&= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{2e(d+ex)^2} - \frac{bc^3d\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bc\sqrt{1+c^2x^2}}{(c^2d^2+e^2)(d+ex)} - \frac{b\operatorname{arcsinh}(cx)}{e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{e(c^2d^2+e^2)^{3/2}} - \frac{bc^3d\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e(c^2d^2+e^2)^{3/2}}\right)$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x))
 - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2 +
e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x
^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(117) = 234$.

Time = 0.57 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.13

| method | result |
|---|---|
| parts | $-\frac{a}{2(ex+d)^2e} - \frac{bc^2\operatorname{arcsinh}(cx)}{2(ecx+cd)^2e} - \frac{bc^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)} - \frac{bc^3d\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}\right.}{2e^2(}$ |
| derivativedivides | $-\frac{ac^3}{2(ecx+cd)^2e}+bc^3\left(-\frac{\operatorname{arcsinh}(cx)}{2(ecx+cd)^2e}+\frac{-\frac{e^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d}{}}\right.}{2e^3}\right.$ |
| default | $-\frac{ac^3}{2(ecx+cd)^2e}+bc^3\left(-\frac{\operatorname{arcsinh}(cx)}{2(ecx+cd)^2e}+\frac{-\frac{e^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d}{}}\right.}{2e^3}\right.$ |

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e-1/2*b*c^2/(c*e*x+c*d)^2/e*arcsinh(c*x)-1/2*b*c^2/e/(c^2*
d^2+e^2)/(c*x+d*c/e)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^
(1/2)-1/2*b*c^3/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^
2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)^2-2
*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(116) = 232$.

Time $= 0.32$ (sec) , antiderivative size $= 566$, normalized size of antiderivative $= 4.42$

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx =$$

$$-\frac{(a+b)c^4d^6 + (2\,a+b)c^2d^4e^2 + ad^2e^4 + (bc^4d^4e^2 + bc^2d^2e^4)x^2 - (bc^3d^3e^2x^2 + 2\,bc^3d^4ex + bc^3d^5)\sqrt{c^2d^2 +}}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*c^4*d^6 + (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
+ b*c^2*d^2*e^4)*x^2 - (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*sq
rt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqrt(c^2*d^2 + e^2)*(c^2*d*x -
e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x^2 + 1))/(e*x + d)
) + 2*(b*c^4*d^5*e + b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 +
 b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2
+ 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (b*c^3*d^5*e + b*c*d^3*e^3 + (b*c^3*d^
4*e^2 + b*c*d^2*e^4)*x)*sqrt(c^2*x^2 + 1))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4
*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2
*d^5*e^4 + d^3*e^6)*x)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(cx\right)}{\left(d+ex\right)^3}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.23

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{1}{2}\left(c\left(\frac{\sqrt{c^2x^2+1}}{c^2d^2ex+c^2d^3+e^3x+de^2}-\frac{c^2d\operatorname{arsinh}\left(\frac{cdx}{e\left|x+\frac{d}{e}\right|}-\frac{1}{c\left|x+\frac{d}{e}\right|}\right)}{\left(\frac{c^2d^2}{e^2}+1\right)^{\frac{3}{2}}e^4}\right)+\frac{\operatorname{arsinh}(cx)}{e^3x^2+2\,de^2x+d^2e}\right)b - \frac{a}{2\left(e^3x^2+2\,de^2x+d^2e\right)}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*
arcsinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/
2)*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^2 +
 2*d*e^2*x + d^2*e)
```

## Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{b\operatorname{arsinh}(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^3, x)
```

# 3.11 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 183

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3} - \frac{bc^3(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/e/(e*x+d)^3-1/6*b*c^3*(2*c^2*d^2-e^2)*arctanh((-c^2
*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(5/2)-1/6*b*
c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)^2-1/2*b*c^3*d*(c^2*x^2+1)^(1/2)/(
c^2*d^2+e^2)^2/(e*x+d)
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5828, 759, 821, 739, 212}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3} - \frac{bc^3(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}} - \frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}$$

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)^2) - (b*c^3*d*Sqrt[
1 + c^2*x^2])/(2*(c^2*d^2 + e^2)^2*(d + e*x)) - (a + b*ArcSinh[c*x])/(3*e*(
```

```
d + e*x)^3) - (b*c^3*(2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2
+ e^2]*Sqrt[1 + c^2*x^2])])/(6*e*(c^2*d^2 + e^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a + b\text{arcsinh}(cx)}{3e(d+ex)^3} + \frac{(bc)\int \frac{1}{(d+ex)^3\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3} - \frac{(bc^3)\int \frac{-2d+ex}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{6e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}$$
$$- \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3} + \frac{\left(bc^3(2c^2d^2-e^2)\right)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{6e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3}$$
$$- \frac{\left(bc^3(2c^2d^2-e^2)\right)\operatorname{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx, x, \frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{6e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}$$
$$- \frac{a+b\operatorname{arcsinh}(cx)}{3e(d+ex)^3} - \frac{bc^3(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = \frac{1}{6}\left(-\frac{2a}{e(d+ex)^3} - \frac{bc\sqrt{1+c^2x^2}(e^2+c^2d(4d+3ex))}{\left(c^2d^2+e^2\right)^2(d+ex)^2}\right.$$
$$- \frac{2b\operatorname{arcsinh}(cx)}{e(d+ex)^3} - \frac{bc^3(-2c^2d^2+e^2)\log(d+ex)}{e\left(c^2d^2+e^2\right)^{5/2}}$$
$$\left. + \frac{bc^3(-2c^2d^2+e^2)\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e\left(c^2d^2+e^2\right)^{5/2}}\right)$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x)
))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) - (
b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^3*(
-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^2]])
/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(168) = 336$.

Time $= 0.41$ (sec) , antiderivative size $= 510$, normalized size of antiderivative $= 2.79$

| method | result |
|---|---|
| parts | $-\frac{a}{3(ex+d)^3e} - \frac{b\,c^3\,\mathrm{arcsinh}(cx)}{3(ecx+cd)^3e} - \frac{b\,c^3\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)^2} - \frac{b\,c^4d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+$ |
| derivativedivides | $-\frac{a\,c^4}{3(ecx+cd)^3e} - \frac{b\,c^4\,\mathrm{arcsinh}(cx)}{3(ecx+cd)^3e} - \frac{b\,c^4\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)^2} - \frac{b\,c^5d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(cx+\frac{dc}{e}\right)} - \frac{b\,c^{}}{}$ |
| default | $-\frac{a\,c^4}{3(ecx+cd)^3e} - \frac{b\,c^4\,\mathrm{arcsinh}(cx)}{3(ecx+cd)^3e} - \frac{b\,c^4\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)^2} - \frac{b\,c^5d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(cx+\frac{dc}{e}\right)} - \frac{b\,c^{}}{}$ |

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/(e*x+d)^3/e-1/3*b*c^3/(c*e*x+c*d)^3/e*arcsinh(c*x)-1/6*b*c^3/e^2/(c^
2*d^2+e^2)/(c*x+d*c/e)^2*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e
^2)^(1/2)-1/2*b*c^4/e*d/(c^2*d^2+e^2)^2/(c*x+d*c/e)*((c*x+d*c/e)^2-2*d*c/e*
(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*b*c^5/e^2*d^2/(c^2*d^2+e^2)^2/((c^
2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d
^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1
/2))/(c*x+d*c/e))+1/6*b*c^3/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln(
(2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+
d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(167) = 334$.

Time $= 0.62$ (sec) , antiderivative size $= 977$, normalized size of antiderivative $= 5.34$

$$\int \frac{a+b\mathrm{arcsinh}(cx)}{(d+ex)^4}\,dx =$$

$$-\frac{(2\,a+3\,b)c^6d^9+3\,(2\,a+b)c^4d^7e^2+6\,ac^2d^5e^4+2\,ad^3e^6+3\,(bc^6d^6e^3+bc^4d^4e^5)x^3+9\,(bc^6d^7e^2+bc^4d^5e}{}$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*((2*a + 3*b)*c^6*d^9 + 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 + 2*a
*d^3*e^6 + 3*(b*c^6*d^6*e^3 + b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 + b*c^4
*d^5*e^4)*x^2 + (2*b*c^5*d^8 - b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 - b*c^3*d^3
*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 - b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e - b*
c^3*d^5*e^3)*x)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e - sqrt(c^2*d^2
+ e^2)*(c^2*d*x - e) + (c^2*d^2 - sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x
^2 + 1))/(e*x + d)) + 9*(b*c^6*d^8*e + b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^3
 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b*
c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6
*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(b*
c^6*d^9 + 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 + b*d^3*e^6 + (b*c^6*d^6*e^3 +
3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b*c^4
*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6*e^
3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (4*b*c^
5*d^8*e + 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 + b*c^3*d^4*e^5)
*x^2 + (7*b*c^5*d^7*e^2 + 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 +
1))/(c^6*d^12*e + 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 + d^6*e^7 + (c^6*d^9*e^4 +
 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 + d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 + 3*c^4*d^
8*e^5 + 3*c^2*d^6*e^7 + d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 + 3*c^4*d^9*e^4 + 3*
c^2*d^7*e^6 + d^5*e^8)*x)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(cx\right)}{\left(d+ex\right)^4}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

## Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx = \int \frac{b\,\operatorname{arsinh}\left(cx\right) + a}{\left(ex+d\right)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 + 3*c*d^2*e^2*x^2 + c
*d^3*e*x + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c^2
*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)*x^
3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*c^4*
d*e^2)*log(e*x + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7) + (3*
```

```
c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2 + (5*
c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^2 + (c^
6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2 + 3*(c^6
*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3*c^4*d^7*e
^3 + 3*c^2*d^5*e^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8
 + e^10)*x^3 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2
+ 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x) - I*(3*c^6*d
^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 + 3*c^4*d^4*e^2
+ 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x +
 d^3*e)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(ex+d)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d+e\,x)^4}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + e*x)^4,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^4, x)
```

# 3.12 $\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 368

$$\begin{aligned}\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx &= 2b^2d^3x-\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2-\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3\\&+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c}\\&+\frac{4bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3}\\&-\frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c}\\&+\frac{3be^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{16c^3}\\&-\frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c}\\&-\frac{be^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{8c}\\&-\frac{d^4(a+b\operatorname{arcsinh}(cx))^2}{4e}+\frac{3d^2e(a+b\operatorname{arcsinh}(cx))^2}{4c^2}\\&-\frac{3e^3(a+b\operatorname{arcsinh}(cx))^2}{32c^4}\\&+\frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e}\end{aligned}$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2-3/32*b^2*e^3*x^2/c^2+2/9*
b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arcsinh(c*x))^2/e+3/4*d^2*e*(a+
b*arcsinh(c*x))^2/c^2-3/32*e^3*(a+b*arcsinh(c*x))^2/c^4+1/4*(e*x+d)^4*(a+b*
arcsinh(c*x))^2/e-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/3*b*d*e^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-3/2*b*d^2*e*x*(a+b*arcsinh(c*x))
*(c^2*x^2+1)^(1/2)/c+3/16*b*e^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-
```

```
2/3*b*d*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/8*b*e^3*x^3*(a+b*a
rcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = &-\frac{3e^3(a+b\operatorname{arcsinh}(cx))^2}{32c^4}\\ &-\frac{2bd^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c}\\ &-\frac{3bd^2ex\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{2c}\\ &+\frac{3d^2e(a+b\operatorname{arcsinh}(cx))^2}{4c^2}\\ &-\frac{2bde^2x^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{3c}\\ &-\frac{be^3x^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{8c}\\ &+\frac{4bde^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{3c^3}\\ &+\frac{3be^3x\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{16c^3}\\ &-\frac{d^4(a+b\operatorname{arcsinh}(cx))^2}{4e}\\ &+\frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e}-\frac{4b^2de^2x}{3c^2}-\frac{3b^2e^3x^2}{32c^2}\\ &+2b^2d^3x+\frac{3}{4}b^2d^2ex^2+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4\end{aligned}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 - (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*
c) + (3*b*e^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^3) - (2*b*d*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) - (d^4*(a + b*ArcSinh[c*x])^2)/(4*e) +
 (3*d^2*e*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcSinh[c*x])^2)/
(32*c^4) + ((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{2e} \\
&= \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e} \\
&\quad - \frac{(bc)\int \left(\frac{d^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{4d^3ex(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{6d^2e^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{4de^3x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right.}{2e} \\
&= \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e} - \left(2bcd^3\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad - \frac{(bcd^4)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2e} - \left(3bcd^2e\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad - \left(2bcde^2\right)\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{2}\left(bce^3\right)\int \frac{x^4(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} \\
&\quad - \frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} - \frac{be^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{8c} \\
&\quad - \frac{d^4(a+b\operatorname{arcsinh}(cx))^2}{4e} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e} + \left(2b^2d^3\right)\int 1\,dx \\
&\quad + \frac{1}{2}\left(3b^2d^2e\right)\int x\,dx + \frac{(3bd^2e)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c} + \frac{1}{3}\left(2b^2de^2\right)\int x^2\,dx \\
&\quad + \frac{(4bde^2)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3c} + \frac{1}{8}\left(b^2e^3\right)\int x^3\,dx + \frac{(3be^3)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{8c}
\end{aligned}$$

$$
\begin{aligned}
&= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\
&\quad + \frac{4bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} - \frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} \\
&\quad + \frac{3be^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{16c^3} - \frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} \\
&\quad - \frac{be^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{8c} - \frac{d^4(a+b\operatorname{arcsinh}(cx))^2}{4e} \\
&\quad + \frac{3d^2e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e} \\
&\quad - \frac{(4b^2de^2)\int 1\,dx}{3c^2} - \frac{(3be^3)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c^3} - \frac{(3b^2e^3)\int x\,dx}{16c^2} \\
&= 2b^2d^3x - \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 - \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 \\
&\quad - \frac{2bd^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4bde^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c^3} \\
&\quad - \frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} + \frac{3be^3x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{16c^3} \\
&\quad - \frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{3c} - \frac{be^3x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{8c} \\
&\quad - \frac{d^4(a+b\operatorname{arcsinh}(cx))^2}{4e} + \frac{3d^2e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(cx))^2}{32c^4} + \frac{(d+ex)^4(a+b\operatorname{arcsinh}(cx))^2}{4e}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96

$$
\begin{aligned}
&\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx \\
&= \frac{c\big(72a^2c^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3) - 6ab\sqrt{1+c^2x^2}(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2 +
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[1
 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^
2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*
e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8*c^4
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2]*(-(
e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))
)*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x +
 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{a^2(ecx+cd)^4}{4c^3e}+\frac{b^2\left(\frac{e^3\left(8\operatorname{arcsinh}(cx)^2x^4c^4-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4+6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-3\operatorname{arcsinh}(cx)^2-3c^2x^2-\right.}{32}}{}$ |
| default | $\frac{a^2(ecx+cd)^4}{4c^3e}+\frac{b^2\left(\frac{e^3\left(8\operatorname{arcsinh}(cx)^2x^4c^4-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4+6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-3\operatorname{arcsinh}(cx)^2-3c^2x^2-\right.}{32}}{}$ |
| parts | $\frac{a^2(ex+d)^4}{4e}+\frac{b^2\left(72\operatorname{arcsinh}(cx)^2c^4x^4e^3+288\operatorname{arcsinh}(cx)^2c^4x^3de^2+432\operatorname{arcsinh}(cx)^2c^4x^2d^2e+288\operatorname{arcsinh}(cx)^2c^4xd^3\right.}{}$ |

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*a^2/c^3*(c*e*x+c*d)^4/e+b^2/c^3*(1/32*e^3*(8*arcsinh(c*x)^2*x^4*c^
4-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+c^4*x^4+6*arcsinh(c*x)*c*x*(c^2*
x^2+1)^(1/2)-3*arcsinh(c*x)^2-3*c^2*x^2-3)+1/9*d*c*e^2*(9*arcsinh(c*x)^2*x^
3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)-12*c*x)+3/4*d^2*c^2*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcsinh(c
*x)*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)+d^3*c^3*(arcsinh(c*x)^2
*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^3*(1/4/e*arcsinh(c*x)
*c^4*d^4+arcsinh(c*x)*c^4*d^3*x+3/2*e*arcsinh(c*x)*c^4*d^2*x^2+e^2*arcsinh(
c*x)*c^4*d*x^3+1/4*e^3*arcsinh(c*x)*c^4*x^4-1/4/e*(c^4*d^4*arcsinh(c*x)+e^4
*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c*x*(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))
+4*d^3*c^3*e*(c^2*x^2+1)^(1/2)+6*d^2*c^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2
*arcsinh(c*x))+4*d*c*e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/
2)))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.29

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$=\frac{9\left(8\,a^2+b^2\right)c^4e^3x^4+32\left(9\,a^2+2\,b^2\right)c^4de^2x^3+27\left(8\left(2\,a^2+b^2\right)c^4d^2e-b^2c^2e^3\right)x^2+9\left(8\,b^2c^4e^3x^4+32\,b^2c^4\right.}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27*
(8*(2*a^2 + b^2)*c^4*d^2*e - b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32*b
```

```
^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x + 24*b^2*c^2*d^2
*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 96*(3*(a^2 + 2*b^2)*c^4*d^
3 - 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3 + 144
*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x + 72*a*b*c^2*d^2*e - 9*a*b*e^3 - (6*b
^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 - 64*b^2*c*d*e^2 + 9
*(8*b^2*c^3*d^2*e - b^2*c*e^3)*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*d^3 - 64*
a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e - a*b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(364) = 728$.

Time $= 0.45$ (sec) , antiderivative size $= 743$, normalized size of antiderivative $= 2.02$

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2d^3x + \frac{3a^2d^2ex^2}{2} + a^2de^2x^3 + \frac{a^2e^3x^4}{4} + 2abd^3x\,\operatorname{asinh}(cx) + 3abd^2ex^2\,\operatorname{asinh}(cx) + 2abde^2x^3\,\operatorname{asinh}(cx) + \frac{ab} \\ a^2\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b*d
*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3*a*b*
d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*
a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16*c**4)
+ b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*asinh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d**2*e*asinh(c
*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2/(32*c**2) + 4
*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**2*e**3*x*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)**2/(32*c**4), N
e(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), Tru
e))
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.60

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{1}{4}b^2e^3x^4\operatorname{arsinh}(cx)^2 + b^2de^2x^3\operatorname{arsinh}(cx)^2$$
$$+\frac{1}{4}a^2e^3x^4 + \frac{3}{2}b^2d^2ex^2\operatorname{arsinh}(cx)^2 + a^2de^2x^3 + b^2d^3x\operatorname{arsinh}(cx)^2$$
$$+\frac{3}{2}a^2d^2ex^2 + \frac{3}{2}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abd^2e$$
$$+\frac{3}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2d^2e$$
$$+\frac{2}{3}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abde^2$$
$$-\frac{2}{9}\left(3c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6x}{c^2}\right)b^2de^2$$
$$+\frac{1}{16}\left(8x^4\operatorname{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)abe^3$$
$$+\frac{1}{32}\left(\left(\left(\frac{x^4}{c^2} - \frac{3x^2}{c^4} + \frac{3\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}x}{c^4} + \frac{3\operatorname{arsinh}(cx)}{c^5}\right.\right.\right.$$
$$+2b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2d^3x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^3}{c}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e^3*x^4*arcsinh(c*x)^2 + b^2*d*e^2*x^3*arcsinh(c*x)^2 + 1/4*a^2*e^3
*x^4 + 3/2*b^2*d^2*e*x^2*arcsinh(c*x)^2 + a^2*d*e^2*x^3 + b^2*d^3*x*arcsinh
(c*x)^2 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2*e + 3/4*(c^2*(x^2/c^2 - log(c*x + sqrt
(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*ar
csinh(c*x))*b^2*d^2*e + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/
c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2
/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e
^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x
^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*e^3 + 1/32*((x^4/c^2 - 3*x^2/c^4
 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c
^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*e^
3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*a
rcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

## Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^3, x)
```

# 3.13 $\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx &= 2b^2d^2x - \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 \\ &\quad - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\ &\quad + \frac{4be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} \\ &\quad - \frac{bdex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\ &\quad - \frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} \\ &\quad - \frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e} + \frac{de(a+b\operatorname{arcsinh}(cx))^2}{2c^2} \\ &\quad + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}\end{aligned}$$

```
[Out] 2*b^2*d^2*x-4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+b
*arcsinh(c*x))^2/e+1/2*d*e*(a+b*arcsinh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcs
inh(c*x))^2/e-2*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-b*d*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c-2/9*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx = &-\frac{2bd^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c}\\ &-\frac{bdex\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c}\\ &+\frac{de(a+b\operatorname{arcsinh}(cx))^2}{2c^2}\\ &-\frac{2be^2x^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{9c}\\ &+\frac{4be^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{9c^3}\\ &-\frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e}\\ &+\frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}\\ &-\frac{4b^2e^2x}{9c^2}+2b^2d^2x+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3\end{aligned}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e^2*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/c - (2*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c)
 - (d^3*(a + b*ArcSinh[c*x])^2)/(3*e) + (d*e*(a + b*ArcSinh[c*x])^2)/(2*c^2
) + ((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e} - \frac{(2bc)\int \frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e} - \frac{(2bc)\int \left(\frac{d^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3d^2ex(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3de^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{e^3x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right) dx}{3e}$$

$$= \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e} - \left(2bcd^2\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{(2bcd^3)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{3e} - (2bcde)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx - \frac{1}{3}\left(2bce^2\right)\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bdex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} - \frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e} + \left(2b^2d^2\right)\int 1\,dx + \left(b^2de\right)\int x\,dx + \frac{(bde)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{c} + \frac{1}{9}\left(2b^2e^2\right)\int x^2\,dx + \frac{(4be^2)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{9c}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} - \frac{bdex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} - \frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e} + \frac{de(a+b\operatorname{arcsinh}(cx))^2}{2c^2} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e} - \frac{(4b^2e^2)\int 1\,dx}{9c^2}$$

$$= 2b^2d^2x - \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + \frac{4be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} - \frac{bdex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} - \frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e} + \frac{de(a+b\operatorname{arcsinh}(cx))^2}{2c^2} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{18a^2c^3x(3d^2+3dex+e^2x^2)-6ab\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2))+b^2cx(-24e^2+c^2(108d^2$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^2
 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2 +
27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^
2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))
)*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2*e*x^
2))*ArcSinh[c*x]^2)/(54*c^3)
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| derivativedivides | $\frac{a^2(ecx+cd)^3}{3c^2e}+\frac{b^2\left(\frac{e^2\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27}+\frac{dce\left(2\operatorname{arcsinh}(c$ |
| default | $\frac{a^2(ecx+cd)^3}{3c^2e}+\frac{b^2\left(\frac{e^2\left(9\operatorname{arcsinh}(cx)^2x^3c^3-6\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27}+\frac{dce\left(2\operatorname{arcsinh}(c$ |
| parts | $\frac{a^2(ex+d)^3}{3e}+\frac{b^2\left(18\operatorname{arcsinh}(cx)^2c^3x^3e^2+54\operatorname{arcsinh}(cx)^2c^3x^2de+54\operatorname{arcsinh}(cx)^2c^3x\,d^2-12\ \operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^2x$ |

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*a^2/c^2*(c*e*x+c*d)^3/e+b^2/c^2*(1/27*e^2*(9*arcsinh(c*x)^2*x^3*c^
3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)-12*c*x)+1/2*d*c*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcsinh(c*x)*c*x*
(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)+d^2*c^2*(arcsinh(c*x)^2*x*c-2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3/e*arcsinh(c*x)*c^3*d^3
+arcsinh(c*x)*c^3*d^2*x+e*arcsinh(c*x)*c^3*d*x^2+1/3*e^2*arcsinh(c*x)*c^3*x
^3-1/3/e*(c^3*d^3*arcsinh(c*x)+e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*
x^2+1)^(1/2))+3*d^2*c^2*e*(c^2*x^2+1)^(1/2)+3*d*c*e^2*(1/2*c*x*(c^2*x^2+1)^
(1/2)-1/2*arcsinh(c*x))))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.33

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \frac{2\,(9\,a^2+2\,b^2)c^3e^2x^3+27\,(2\,a^2+b^2)c^3dex^2+9\,(2\,b^2c^3e^2x^3+6\,b^2c^3dex^2+6\,b^2c^3d^2x+3\,b^2cde)\log\left(cx+\sqrt{c}\right.}{}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*b
^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x + 3*b^2*c*d*e)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 - 4*b^2*c*e^2)*x + 6*(
6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x + 9*a*b*c*d*e - (
2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 - 4*b^2*e^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b*c^2*d
*e*x + 18*a*b*c^2*d^2 - 4*a*b*e^2)*sqrt(c^2*x^2 + 1))/c^3
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.90

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{asinh}(cx) + 2abdex^2\operatorname{asinh}(cx) + \frac{2abe^2x^3\operatorname{asinh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{abdex}{} \\ a^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*as
inh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*sqrt(
c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 +
1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**2*x/(9*c*
*2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c, 0)), (a**2
*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.58

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{1}{3}\,b^2e^2x^3\operatorname{arsinh}(cx)^2 + b^2dex^2\operatorname{arsinh}(cx)^2 + \frac{1}{3}\,a^2e^2x^3 + b^2d^2x\operatorname{arsinh}(cx)^2 + a^2dex^2 + \left(2\,x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abde + \frac{1}{2}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2de + \frac{2}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abe^2 - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx) - \frac{c^2x^3-6\,x}{c^2}\right)b^2e^2 + 2\,b^2d^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2d^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd^2}{c}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^2*x^3*arcsinh(c*x)^2 + b^2*d*e*x^2*arcsinh(c*x)^2 + 1/3*a^2*e^2*x
^3 + b^2*d^2*x*arcsinh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arcsinh(c*x) - c*(sqrt
(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d*e + 1/2*(c^2*(x^2/c^2 - log(
c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*
x)/c^3)*arcsinh(c*x))*b^2*d*e + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*
b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*
(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

## Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^2,x)
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^2, x)
```

# 3.14 $\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 140

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} - \frac{d^2(a+b\operatorname{arcsinh}(cx))^2}{2e} + \frac{e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e}$$

```
[Out] 2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arcsinh(c*x))^2/e+1/4*e*(a+b*arcsinh(c
*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))^2/e-2*b*d*(a+b*arcsinh(c*x))*(c
^2*x^2+1)^(1/2)/c-1/2*b*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = -\frac{2bd\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bex\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{2c} + \frac{e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} - \frac{d^2(a+b\operatorname{arcsinh}(cx))^2}{2e} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e} + 2b^2dx + \frac{1}{4}b^2ex^2$$

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c - (b*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c) - (d^2*(a + b*ArcS
inh[c*x])^2)/(2*e) + (e*(a + b*ArcSinh[c*x])^2)/(4*c^2) + ((d + e*x)^2*(a +
 b*ArcSinh[c*x])^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{e} \\
&= \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e} \\
&\quad - \frac{(bc)\int \left(\frac{d^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{2dex(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{e^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)\,dx}{e} \\
&= \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e} - (2bcd)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&\quad - \frac{(bcd^2)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{e} - (bce)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{2bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} \\
&\quad - \frac{d^2(a+b\operatorname{arcsinh}(cx))^2}{2e} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e} \\
&\quad + \left(2b^2d\right)\int 1\,dx + \frac{1}{2}\left(b^2e\right)\int x\,dx + \frac{(be)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c} \\
&= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} \\
&\quad - \frac{d^2(a+b\operatorname{arcsinh}(cx))^2}{2e} + \frac{e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \frac{c\left(2a^2cx(2d+ex)+b^2cx(8d+ex)-2ab(4d+ex)\sqrt{1+c^2x^2}\right)+2b\left(-bc(4d+ex)\sqrt{1+c^2x^2}+a(e+4c^2dx\right.}{4c^2}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
+ c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*x
+ 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh[c*
x]^2)/(4*c^2)
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b^2\left(\frac{e\left(2\operatorname{arcsinh}(cx)^2x^2c^2-2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{4c}+d\left(\operatorname{arcsinh}(cx)^2xc-2\ \text{arc}\right.}{c}$ |
| derivativedivides | $\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\frac{e\left(2\operatorname{arcsinh}(cx)^2x^2c^2-2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+dc\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(c\right.}{c}}{c}$ |
| default | $\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\frac{e\left(2\operatorname{arcsinh}(cx)^2x^2c^2-2\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+dc\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(c\right.}{c}}{c}$ |

```
[In] int((e*x+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*e*x^2+d*x)+b^2/c*(1/4*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcsinh(c*x)*c
*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)/c+d*(arcsinh(c*x)^2*x*c-2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c*(1/2*c*arcsinh(c*x)*x^2*e+arcs
inh(c*x)*d*c*x-1/2/c*(e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+2*d*c*
(c^2*x^2+1)^(1/2)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \frac{(2\,a^2+b^2)c^2ex^2+4\,(a^2+2\,b^2)c^2dx+(2\,b^2c^2ex^2+4\,b^2c^2dx+b^2e)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(2\,abc^2ex^2+\right.}{4\,c^2}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2 +
 4*b^2*c^2*d*x + b^2*e)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(2*a*b*c^2*e*x^2
 + 4*a*b*c^2*d*x + a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2dx+\frac{a^2ex^2}{2}+2abdx\operatorname{asinh}(cx)+abex^2\operatorname{asinh}(cx)-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{abex\sqrt{c^2x^2+1}}{2c}+\frac{abe\operatorname{asinh}(cx)}{2c^2}+b^2dx\operatorname{asin} \\ a^2\left(dx+\frac{ex^2}{2}\right) \end{cases}$$

```
[In] integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*asi
nh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*c)
 + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(c*x)*
*2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{1}{2}\,b^2ex^2\operatorname{arsinh}(cx)^2 + b^2dx\operatorname{arsinh}(cx)^2 + \frac{1}{2}\,a^2ex^2 + \frac{1}{2}\left(2\,x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abe + \frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2e + 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2dx + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abd}{c}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*x^2*arcsinh(c*x)^2 + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*e*x^2 + 1/2
*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*
e + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2
*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*e + 2*b^2*d*(x - sqrt
(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*a*b*d/c
```

## Giac [F(-2)]

Exception generated.

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x),x)
[Out] int((a + b*asinh(c*x))^2*(d + e*x), x)
```

# 3.15 $\int (a + b\mathbf{arcsinh}(cx))^2\,dx$

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## Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a + b\text{arcsinh}(cx))^2\,dx = 2b^2x - \frac{2b\sqrt{1+c^2x^2}(a + b\text{arcsinh}(cx))}{c} + x(a + b\text{arcsinh}(cx))^2$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5772, 5798, 8}

$$\int (a + b\text{arcsinh}(cx))^2\,dx = -\frac{2b\sqrt{c^2x^2+1}(a + b\text{arcsinh}(cx))}{c} + x(a + b\text{arcsinh}(cx))^2 + 2b^2x$$

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x(a+b\operatorname{arcsinh}(cx))^2 - (2bc)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx \\ &= -\frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + x(a+b\operatorname{arcsinh}(cx))^2 + \left(2b^2\right)\int 1\,dx \\ &= 2b^2x - \frac{2b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} + x(a+b\operatorname{arcsinh}(cx))^2\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\begin{aligned}\int (a+b\operatorname{arcsinh}(cx))^2\,dx = \left(a^2+2b^2\right)x - \frac{2ab\sqrt{1+c^2x^2}}{c} \\ + \frac{2b\left(acx - b\sqrt{1+c^2x^2}\right)\operatorname{arcsinh}(cx)}{c} + b^2x\operatorname{arcsinh}(cx)^2\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

## Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2xc-2\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 72 |
| default | $\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2xc-2\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 72 |
| parts | $a^2x+\frac{b^2\left(\operatorname{arcsinh}(cx)^2xc-2\,\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)}{c}+\frac{2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$ | 73 |

```
[In] int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time $= 0.26$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 2.09$

$$\int (a + b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \frac{b^2 cx \log\left(cx + \sqrt{c^2x^2+1}\right)^2 + (a^2 + 2\,b^2)cx - 2\sqrt{c^2x^2+1}ab + 2\left(abcx - \sqrt{c^2x^2+1}b^2\right)\log\left(cx + \sqrt{c^2x^2+1}\right.}{c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

## Sympy [A] (verification not implemented)

Time $= 0.10$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 1.78$

$$\int (a + b\operatorname{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a + b\operatorname{arcsinh}(cx))^2\,dx = b^2 x \operatorname{arsinh}(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\begin{aligned}&\int (a + b\operatorname{arcsinh}(cx))^2\,dx \\ &= 2\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab \\ &\quad + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2\,c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2 + a^2x\end{aligned}$$

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(cx))^2\,dx = \int (a + b\,\operatorname{asinh}(c\,x))^2\,dx$$

```
[In] int((a + b*asinh(c*x))^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2, x)
```

# 3.16 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{d+ex}\,dx$

Optimal result . . . 216
Rubi [A] (verified) . . . 217
Mathematica [A] (verified) . . . 220
Maple [F] . . . 220
Fricas [F] . . . 221
Sympy [F] . . . 221
Maxima [F] . . . 221
Giac [F] . . . 221
Mupad [F(-1)] . . . 222

## Optimal result

Integrand size = 18, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx &= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*x))*polylog(2,
-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*
x))*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*b^2
*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*b^2*po
lylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5827, 5680, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = &\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}\\ &+\frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}\\ &-\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be}-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] -1/3*(a + b*ArcSinh[c*x])^3/(b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log[1
 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (2*b*(a + b*ArcSinh
[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (
2*b*(a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^
2 + e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)\\ &= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right)\\ &\quad + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd+\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right)\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be} + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{(2b)\operatorname{Subst}\left(\int (a+bx)\log\left(1+\frac{ee^{x}}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{(2b)\operatorname{Subst}\left(\int (a+bx)\log\left(1+\frac{ee^{x}}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be} + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{(2b^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{ee^{x}}{cd-\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&\quad - \frac{(2b^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{ee^{x}}{cd+\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be} + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad + \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\
&\quad - \frac{(2b^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{ex}{-cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e} \\
&\quad - \frac{(2b^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{ex}{cd+\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e}
\end{aligned}
$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^3}{3be} + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}$$
$$+ \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$
$$+ \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}$$
$$+ \frac{2b(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$
$$- \frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{2b^2\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx$$
$$= \frac{-\frac{(a+b\operatorname{arcsinh}(cx))^3}{b} + 3(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) + 3(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*Pol
yLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e)
```

## Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))^2}{ex+d}\,dx$$

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{d+ex}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d),x)
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
[Out] a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x + d)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c\,x))^2}{d+e\,x}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x),x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x), x)
```

# 3.17 $\int \frac{(a+b\mathbf{arcsinh}(cx))^2}{(d+ex)^2}\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx &= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} \\ &+ \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &- \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &+ \frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} \\ &- \frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

```
[Out] -(a+b*arcsinh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b*c*(a+b*ar
csinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^
2*d^2+e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2
+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(
1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5843, 3403, 2296, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx = &\frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}\\ &-\frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e\sqrt{c^2d^2+e^2}}\\ &-\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)}+\frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\\ &-\frac{2b^2c\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcSinh[c*x])*Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2])
 - (2*b*c*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d
^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2 + e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2
+ e^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{(2bc)\operatorname{Subst}\left(\int \frac{a+bx}{cd+e\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{(4bc)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{-e+2cde^x+ee^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{(4bc)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2+e^2}+2ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2d^2+e^2}} \\
&\quad - \frac{(4bc)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cd+2\sqrt{c^2d^2+e^2}+2ee^x}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2d^2+e^2}}
\end{aligned}$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$- \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$- \frac{(2b^2c)\operatorname{Subst}\left(\int \log\left(1+\frac{2ee^x}{2cd-2\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e\sqrt{c^2d^2+e^2}}$$
$$+ \frac{(2b^2c)\operatorname{Subst}\left(\int \log\left(1+\frac{2ee^x}{2cd+2\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e\sqrt{c^2d^2+e^2}}$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$- \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$- \frac{(2b^2c)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ex}{2cd-2\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$+ \frac{(2b^2c)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ex}{2cd+2\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e\sqrt{c^2d^2+e^2}}$$

$$= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{e(d+ex)} + \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$- \frac{2bc(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$
$$+ \frac{2b^2c\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2b^2c\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx$$
$$= \frac{-\frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex} + \frac{2bc\left((a+b\operatorname{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)+b\operatorname{PolyLog}\left(2,\frac{ee^{\operatorname{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-b\operatorname{Poly}}{\sqrt{c^2d^2+e^2}}}{e}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
+ (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*
x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.00

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2c^2}{(ecx+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x}\right.\right.}{cd+\sqrt{}}\right.$ |
| default | $-\frac{a^2c^2}{(ecx+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x}\right.\right.}{cd+\sqrt{}}\right.$ |
| parts | $-\frac{a^2}{(ex+d)e}+b^2\left(-\frac{c^2\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\right.\right.}{e\sqrt{c^2d^2}}\right.$ |

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a^2*c^2/(c*e*x+c*d)/e+b^2*c^2*(-arcsinh(c*x)^2/e/(c*e*x+c*d)+2/e*arcs
inh(c*x)/(c^2*d^2+e^2)^(1/2)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^
2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e*arcsinh(c*x)/(c^2*d^2+e^2)^(1/2)*
ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(
1/2)))+2/e/(c^2*d^2+e^2)^(1/2)*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d
^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e/(c^2*d^2+e^2)^(1/2)*dilog((c
*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))
))+2*a*b*c^2*(-1/(c*e*x+c*d)/e*arcsinh(c*x)-1/e^2/((c^2*d^2+e^2)/e^2)^(1/2)
*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((
c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e))))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(log(c*x + sqrt(c^2*x^2 + 1))^2/(e^2*x + d*e) - integrate(2*(c^3*x^2 +
 sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^2*x^4 + c
^3*d*e*x^3 + c*e^2*x^2 + c*d*e*x + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e
)*sqrt(c^2*x^2 + 1)), x)) - 2*a*b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c
*d*sqrt(e^4)*x/(e*abs(e^2*x + d*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/(sqrt
(c^2*d^2/e^2 + 1)*e^2)) - a^2/(e^2*x + d*e)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^2,x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^2, x)
```

# 3.18 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 349

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx &= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\ &+ \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ &- \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ &+ \frac{b^2c^2\log(d+ex)}{e(c^2d^2+e^2)} + \frac{b^2c^3d\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} \\ &- \frac{b^2c^3d\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}}\end{aligned}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2+e^2)+b*c
^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^
2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)+b^2*c^3*d*poly
log(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)
^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1
/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(c^2*d
^2+e^2)/(e*x+d)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5828, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx &= -\frac{bc\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} \\ &\quad + \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\ &\quad - \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\ &\quad - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} + \frac{b^2c^3d\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\ &\quad - \frac{b^2c^3d\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*x)))
 - (a + b*ArcSinh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d*(a + b*ArcSinh[c*x])
*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2
)^(3/2)) - (b*c^3*d*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2)^(3/2)) + (b^2*c^2*Log[d + e*x])/(
e*(c^2*d^2 + e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2)) - (b^2*c^3*d*PolyLog[2, -((e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} + \frac{(bc)\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{e} \\
&= -\frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} + \frac{(bc^2)\operatorname{Subst}\left(\int \frac{a+bx}{(cd+e\sinh(x))^2}\,dx, x, \operatorname{arcsinh}(cx)\right)}{e} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{(b^2c^2)\operatorname{Subst}\left(\int \frac{\cosh(x)}{cd+e\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^2d^2+e^2} \\
&\quad + \frac{(bc^3d)\operatorname{Subst}\left(\int \frac{a+bx}{cd+e\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{e(c^2d^2+e^2)} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{(b^2c^2)\operatorname{Subst}\left(\int \frac{1}{cd+x}\,dx, x, cex\right)}{e(c^2d^2+e^2)} \\
&\quad + \frac{(2bc^3d)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{-e+2cde^x+ee^{2x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{e(c^2d^2+e^2)} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} + \frac{b^2c^2\log(d+ex)}{e(c^2d^2+e^2)} \\
&\quad + \frac{(2bc^3d)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2+e^2+2ee^x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{(c^2d^2+e^2)^{3/2}} \\
&\quad - \frac{(2bc^3d)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cd+2\sqrt{c^2d^2+e^2+2ee^x}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{(c^2d^2+e^2)^{3/2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&\quad - \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)} \\
&\quad - \frac{(b^2c^3d)\operatorname{Subst}\left(\int \log\left(1+\frac{2ee^x}{2cd-2\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&\quad + \frac{(b^2c^3d)\operatorname{Subst}\left(\int \log\left(1+\frac{2ee^x}{2cd+2\sqrt{c^2d^2+e^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&\quad - \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)} \\
&\quad - \frac{(b^2c^3d)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ex}{2cd-2\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&\quad + \frac{(b^2c^3d)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2ex}{2cd+2\sqrt{c^2d^2+e^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&= -\frac{bc\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{2e(d+ex)^2} \\
&\quad + \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} \\
&\quad - \frac{bc^3d(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)} \\
&\quad + \frac{b^2c^3d\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}} - \frac{b^2c^3d\operatorname{PolyLog}\left(2, -\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx$$

$$= \frac{-\frac{2bce\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2} + \frac{2b^2c^2\log(d+ex)}{c^2d^2+e^2} + \frac{2bc^3d\left((a+b\operatorname{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\right.\right.}{2e}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] ((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*
x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*d^
2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/(c*
d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])]
 - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/(c^2*d
^2 + e^2)^(3/2))/(2*e)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(365) = 730.

Time = 0.65 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2c^3}{2(ecx+cd)^2e}+b^2c^3\left(-\frac{\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4dc^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(cx)\right.}{2e(ecx+cd)^2\left(c^2d^2+e^2\right)}\right.$ |
| default | $-\frac{a^2c^3}{2(ecx+cd)^2e}+b^2c^3\left(-\frac{\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4dc^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(cx)\right.}{2e(ecx+cd)^2\left(c^2d^2+e^2\right)}\right.$ |
| parts | $-\frac{a^2}{2(ex+d)^2e}+b^2\left(-\frac{c^3\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4dc^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(\right.}{2e\left(c^2d^2+e^2\right)(ecx+cd)^2}\right.$ |

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e+b^2*c^3*(-1/2*arcsinh(c*x)*(2*d*c*e*(c^2*
x^2+1)^(1/2)-4*d*c^2*e*x+e^2*arcsinh(c*x)+2*(c^2*x^2+1)^(1/2)*e^2*c*x-2*c^2
*d^2-2*e^2*c^2*x^2+c^2*d^2*arcsinh(c*x))/e/(c*e*x+c*d)^2/(c^2*d^2+e^2)-2/e/
(c^2*d^2+e^2)*ln(c*x+(c^2*x^2+1)^(1/2))+1/e/(c^2*d^2+e^2)*ln(2*d*(c*x+(c^2*
x^2+1)^(1/2))*c+e*(c*x+(c^2*x^2+1)^(1/2))^2-e)+1/e/(c^2*d^2+e^2)^(3/2)*d*c*
arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+
(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*arcsinh(c*x)*ln((c*d+e*(c
*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))+1/e/(
c^2*d^2+e^2)^(3/2)*d*c*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^
(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*dilog((c*d+e
*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))))-a
*b*c^3/(c*e*x+c*d)^2/e*arcsinh(c*x)-a*b*c^3/e/(c^2*d^2+e^2)/(c*x+d*c/e)*((c
*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-a*b*c^4/e^2*d/(c^2
*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*
c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^
2+e^2)/e^2)^(1/2))/(c*x+d*c/e)))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\,\operatorname{arsinh}(cx)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*arcs
inh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/2)*e
^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*a*b - 1/2*b^2*(log(c*x +
 sqrt(c^2*x^2 + 1))^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 2*integrate((c^3*x^2
+ sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^3*x^5 +
2*c^3*d*e^2*x^4 + 2*c*d*e^2*x^2 + c*d^2*e*x + (c^3*d^2*e + c*e^3)*x^3 + (c^
2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sq
rt(c^2*x^2 + 1)), x)) - 1/2*a^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\,\operatorname{arsinh}(cx)+a)^2}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c\,x))^2}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^3, x)
```

# 3.19 $\int \frac{(d+ex)^3}{a+b\mathbf{arcsinh}(cx)}\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 394

$$\begin{aligned}\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = {}& \frac{d^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &-\frac{3de^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{3d^2e\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}\\ &+\frac{e^3\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{4bc^4}\\ &-\frac{e^3\operatorname{Chi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{8bc^4}\\ &-\frac{d^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &+\frac{3de^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{3d^2e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\\ &-\frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{4bc^4}\\ &-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right)}{8bc^4}\end{aligned}$$

```
[Out] d^3*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arcsinh(c*x))*cos
h(a/b)/b/c^3+3/4*d*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+3/2*d^2*
e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-1/4*e^3*cosh(2*a/b)*Shi(2*a/b
+2*arcsinh(c*x))/b/c^4+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arcsinh(c*x))/b/c^4-
d^3*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c+3/4*d*e^2*Shi(a/b+arcsinh(c*x))*sin
h(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2+1/4*e^3*
Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Shi(3*a/b+3*arcsinh(c
*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arcsinh(c*x))*sinh(4*a/b)/b/c^4
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556, 12}

$$\begin{aligned}\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = &\frac{e^3 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{4bc^4}\\ &-\frac{e^3 \sinh\left(\frac{4a}{b}\right) \operatorname{Chi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right)}{8bc^4}\\ &-\frac{e^3 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{4bc^4}\\ &+\frac{e^3 \cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right)}{8bc^4}\\ &-\frac{3de^2 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{3de^2 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{3de^2 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{3de^2 \sinh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{3d^2e \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\\ &+\frac{3d^2e \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\\ &+\frac{d^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &-\frac{d^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\end{aligned}$$

```
[In] Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (3*d*e^2*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*Cosh[(3*a)/b]*CoshI
```

```
ntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*a)/
b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (e^3*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (e^3*CoshIntegral[(4*a)/b + 4*Ar
cSinh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) - (d^3*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(b*c) + (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(
4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(
2*b*c^2) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b*
c^4) - (3*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*
c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^3}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\cosh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh(x)\sinh^2(x)}{a+bx} + \frac{e^3\cosh(x)\sinh^3(x)}{a+bx}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^4} \\
&= \frac{d^3\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^2} \\
&\quad + \frac{(3de^2)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^3} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^4} \\
&= \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^2} \\
&\quad + \frac{(3de^2)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^3} \\
&\quad + \frac{e^3\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4x)}{8(a+bx)}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c^4} \\
&\quad + \frac{\left(d^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad - \frac{\left(d^3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{arcsinh}(cx)\right)}{bc} - \frac{d^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{arcsinh}(cx)\right)}{bc} \\
&\quad + \frac{(3d^2 e)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2c^2} \\
&\quad - \frac{(3de^2)\operatorname{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{(3de^2)\operatorname{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{e^3 \operatorname{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{8c^4} - \frac{e^3 \operatorname{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^4} \\
&= \frac{d^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{arcsinh}(cx)\right)}{bc} - \frac{d^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{arcsinh}(cx)\right)}{bc} \\
&\quad - \frac{\left(3de^2 \cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{\left(3d^2 e \cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2c^2} \\
&\quad - \frac{\left(e^3 \cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^4} \\
&\quad + \frac{\left(3de^2 \cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{\left(e^3 \cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{8c^4} \\
&\quad + \frac{\left(3de^2 \sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad - \frac{\left(3d^2 e \sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2c^2} \\
&\quad + \frac{\left(e^3 \sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^4} \\
&\quad - \frac{\left(3de^2 \sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{4c^3} \\
&\quad - \frac{\left(e^3 \sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \operatorname{arcsinh}(cx)\right)}{8c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc} - \frac{3de^2 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3} \\
&\quad + \frac{3de^2 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3} - \frac{3d^2 e \operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{2bc^2} \\
&\quad + \frac{e^3 \operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{4bc^4} - \frac{e^3 \operatorname{Chi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right) \sinh\left(\frac{4a}{b}\right)}{8bc^4} \\
&\quad - \frac{d^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc} + \frac{3de^2 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3} \\
&\quad + \frac{3d^2 e \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2} - \frac{e^3 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{4bc^4} \\
&\quad - \frac{3de^2 \sinh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3} + \frac{e^3 \cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4a}{b}+4\operatorname{arcsinh}(cx)\right)}{8bc^4}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.77

$$
\begin{aligned}
&\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx \\
&= \frac{d^3\left(\cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right) - \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)}{bc} \\
&\quad + \frac{3de^2\left(-\cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right) + \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) + \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}\right.\right.}{4bc^3} \\
&\quad + \frac{e^3\left(2\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) \sinh\left(\frac{2a}{b}\right) - \operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) \sinh\left(\frac{4a}{b}\right) - 2\cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(2\left(\frac{a}{b}+a\right.\right.}{8bc^4} \\
&\quad - \frac{3d^2 e\left(\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right) - \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\right)}{2bc^2}
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSi
nh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*
SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[
(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]
]))/(2*b*c^2)
```

## Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $\frac{e^3 e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b} - \frac{e^3 e^{-\frac{4a}{b}} \operatorname{Ei}_1\left(-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b} + \frac{3e e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right) d^2}{4cb} - \frac{e^3 e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\right.}{8c^3b}$ |
| default | $\frac{e^3 e^{\frac{4a}{b}} \operatorname{Ei}_1\left(4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b} - \frac{e^3 e^{-\frac{4a}{b}} \operatorname{Ei}_1\left(-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b} + \frac{3e e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right) d^2}{4cb} - \frac{e^3 e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\right.}{8c^3b}$ |

```
[In] int((e*x+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/16/c^3*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/16/c^3*e^3/b*ex
p(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh(c
*x)+2*a/b)*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-3/4/c*e/
b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)*Ei(
1,-2*arcsinh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3
*a/b)-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)*d^3+3/8/c^2*d/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-
1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2*d/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)*e^2)
```

## Fricas [F]

$$\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^3}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arcsinh(c*x) + a), x)
```

## Sympy [F]

$$\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^3}{a+b\,\operatorname{asinh}(cx)}\,dx$$

```
[In] integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

## Maxima [F]

$$\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^3}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

## Giac [F]

$$\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^3}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d+e\,x)^3}{a+b\,\operatorname{asinh}(c\,x)}\,dx$$

```
[In] int((d + e*x)^3/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^3/(a + b*asinh(c*x)), x)
```

# 3.20 $\int \frac{(d+ex)^2}{a+b\mathbf{arcsinh}(cx)}\,dx$

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Fricas [F] . . . 251
Sympy [F] . . . 251
Maxima [F] . . . 251
Giac [F] . . . 251
Mupad [F(-1)] . . . 252

## Optimal result

Integrand size = 18, antiderivative size = 245

$$\begin{aligned}\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = &\frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &-\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{de\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2}\\ &-\frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &+\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{de\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{bc^2}\\ &-\frac{e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{arcsinh}(cx)\right)}{4bc^3}\end{aligned}$$

```
[Out] d^2*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-1/4*e^2*Chi(a/b+arcsinh(c*x))*cosh(
a/b)/b/c^3+1/4*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+d*e*cosh(2*a
/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-d^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c
+1/4*e^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arcsinh(c*x)
)*sinh(2*a/b)/b/c^2-1/4*e^2*Shi(3*a/b+3*arcsinh(c*x))*sinh(3*a/b)/b/c^3
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556}

$$\begin{aligned}\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = &-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3}\\ &+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3}\\ &-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{bc^2}\\ &+\frac{de\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{bc^2}\\ &+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}\\ &-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}\end{aligned}$$

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e^2*Cosh[a/b]*Cos
hIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e^2*Cosh[(3*a)/b]*CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]])/(b*c) + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) +
 (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b*c^2) - (e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^2}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\cosh(x)}{a+bx} + \frac{e^2\cosh(x)\sinh^2(x)}{a+bx} + \frac{cde\sinh(2x)}{a+bx}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^3} \\
&= \frac{d^2\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c} + \frac{(de)\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&\quad + \frac{e^2\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^2 \text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\, dx, x, \text{arcsinh}(cx)\right)}{c^3} \\
&\quad + \frac{\left(d^2 \cosh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{\left(de \cosh\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&\quad - \frac{\left(d^2 \sinh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{c} \\
&\quad - \frac{\left(de \sinh\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&= \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} - \frac{de \text{Chi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{bc^2} \\
&\quad - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} + \frac{de \cosh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right)}{bc^2} \\
&\quad - \frac{e^2 \text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} + \frac{e^2 \text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} \\
&= \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} - \frac{de \text{Chi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{bc^2} \\
&\quad - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} + \frac{de \cosh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right)}{bc^2} \\
&\quad - \frac{\left(e^2 \cosh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{\left(e^2 \cosh\left(\frac{3a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} \\
&\quad + \frac{\left(e^2 \sinh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} \\
&\quad - \frac{\left(e^2 \sinh\left(\frac{3a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\, dx, x, \text{arcsinh}(cx)\right)}{4c^3} \\
&= \frac{d^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} - \frac{e^2 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{4bc^3} \\
&\quad + \frac{e^2 \cosh\left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3a}{b} + 3\text{arcsinh}(cx)\right)}{4bc^3} - \frac{de \text{Chi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{bc^2} \\
&\quad - \frac{d^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{bc} + \frac{e^2 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b} + \text{arcsinh}(cx)\right)}{4bc^3} \\
&\quad + \frac{de \cosh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2a}{b} + 2\text{arcsinh}(cx)\right)}{bc^2} - \frac{e^2 \sinh\left(\frac{3a}{b}\right) \text{Shi}\left(\frac{3a}{b} + 3\text{arcsinh}(cx)\right)}{4bc^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx$$
$$= \frac{(4c^2d^2-e^2)\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)-4cde\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcs}\right.\right.}{}$$

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcS
inh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Cosh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

## Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| derivativedivides | $-\frac{e^2 e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^2e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e^2}{8c^2b}-\frac{e}{}$ |
| default | $-\frac{e^2 e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^2e^{-\frac{3a}{b}}\operatorname{Ei}_1\left(-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e^2}{8c^2b}-\frac{e}{}$ |

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8/c^2*e^2/b*exp
(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2+1/2/c*d*
e/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/2/c*d*e/b*exp(-2*a/b)*Ei(1,-2*a
rcsinh(c*x)-2*a/b))
```

## Fricas [F]

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arcsinh(c*x) + a), x)
```

## Sympy [F]

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^2}{a+b\,\operatorname{asinh}(cx)}\,dx$$

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

## Maxima [F]

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

## Giac [F]

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d+e\,x)^2}{a+b\,\operatorname{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((d + e*x)^2/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^2/(a + b*asinh(c*x)), x)
```

# 3.21 $\int \frac{d+ex}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result . . . 253
Rubi [A] (verified) . . . 253
Mathematica [A] (verified) . . . 256
Maple [A] (verified) . . . 256
Fricas [F] . . . 256
Sympy [F] . . . 257
Maxima [F] . . . 257
Giac [F] . . . 257
Mupad [F(-1)] . . . 257

## Optimal result

Integrand size = 16, antiderivative size = 116

$$\begin{aligned}\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx = &\frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &-\frac{e\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}\\ &-\frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &+\frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\end{aligned}$$

```
[Out] d*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh
(c*x))/b/c^2-d*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arcsin
h(c*x))*sinh(2*a/b)/b/c^2
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556, 12}

$$\begin{aligned}\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx = &-\frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\\ &+\frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{arcsinh}(cx)\right)}{2bc^2}\\ &+\frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\\ &-\frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{bc}\end{aligned}$$

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]

[Out] (d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (d*Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcS
inh[c*x]])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \left(\frac{cd\cosh(x)}{a+bx} + \frac{e\cosh(x)\sinh(x)}{a+bx}\right)\,dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&= \frac{d\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c} + \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \text{arcsinh}(cx)\right)}{c^2} \\
&\quad + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c} \\
&\quad - \frac{\left(d\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{c} \\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{2c^2} \\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\
&\quad + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{2c^2} \\
&\quad - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{arcsinh}(cx)\right)}{2c^2} \\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} - \frac{e\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} \\
&\quad - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2bc^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx$$
$$= \frac{2cd\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right) - e\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - 2cd\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(c\right.}{2bc^2}$$

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| derivativedivides | $-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b} - \frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)d}{2b} + \frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb} - \frac{e\,e^{-\frac{2a}{b}}\operatorname{Ei}_1\left(-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}}{c}$ |
| default | $-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b} - \frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)d}{2b} + \frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb} - \frac{e\,e^{-\frac{2a}{b}}\operatorname{Ei}_1\left(-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}}{c}$ |

```
[In] int((e*x+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*d+1/4*e/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4*e/c/b*exp(
-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

## Fricas [F]

$$\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b*arcsinh(c*x) + a), x)
```

## Sympy [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{d+ex}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((e*x+d)/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)/(a + b*asinh(c*x)), x)
```

## Maxima [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

## Giac [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{d+e\,x}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)/(a + b*asinh(c*x)), x)
```

# 3.22 $\int \frac{1}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result . . . 258
Rubi [A] (verified) . . . 258
Mathematica [A] (verified) . . . 259
Maple [A] (verified) . . . 260
Fricas [F] . . . 260
Sympy [F] . . . 260
Maxima [F] . . . 260
Giac [F] . . . 261
Mupad [F(-1)] . . . 261

## Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5774, 3384, 3379, 3382}

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc} \\ &= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc} \\ &\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc} \\ &= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

## Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$ | 56 |
| default | $\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$ | 56 |

```
[In] int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

## Fricas [F]

$$\int \frac{1}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\operatorname{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

## Sympy [F]

$$\int \frac{1}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{a+b\,\operatorname{asinh}\left(cx\right)}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\operatorname{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

## Giac [F]

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int(1/(a + b*asinh(c*x)),x)
[Out] int(1/(a + b*asinh(c*x)), x)
```

# 3.23 $\int \frac{1}{(d+ex)(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result . . . 262
Rubi [N/A] . . . 262
Mathematica [N/A] . . . 263
Maple [N/A] (verified) . . . 263
Fricas [N/A] . . . 263
Sympy [N/A] . . . 263
Maxima [N/A] . . . 264
Giac [N/A] . . . 264
Mupad [N/A] . . . 264

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arcsinh(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\left(cx\right))\left(d+ex\right)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

# 3.24 $\int \frac{1}{(d+ex)^2(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result . . . 265
Rubi [N/A] . . . 265
Mathematica [N/A] . . . 266
Maple [N/A] (verified) . . . 266
Fricas [N/A] . . . 266
Sympy [N/A] . . . 267
Maxima [N/A] . . . 267
Giac [N/A] . . . 267
Mupad [N/A] . . . 268

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)}dx$$

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2)
*arcsinh(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex)^2}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\left(d+e\,x\right)^2}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)^2), x)
```

# 3.25 $\int \frac{(d+ex)^2}{(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result . . . 269
Rubi [A] (verified) . . . 270
Mathematica [A] (verified) . . . 273
Maple [A] (verified) . . . 274
Fricas [F] . . . 275
Sympy [F] . . . 275
Maxima [F] . . . 275
Giac [F] . . . 276
Mupad [F(-1)] . . . 276

## Optimal result

Integrand size = 18, antiderivative size = 359

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = &-\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\ &- \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{2de\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\ &- \frac{d^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} \\ &+ \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} \\ &- \frac{3e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} \\ &+ \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} \\ &- \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\ &- \frac{2de\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\ &+ \frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c^3}\end{aligned}$$

```
[Out] 2*d*e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d^2*cosh(a/b)*Shi((a+
b*arcsinh(c*x))/b)/b^2/c-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^
```

```
3+3/4*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d^2*Chi((a+b*arcs
inh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^3-2*d*e*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Chi(3*(
a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))-2*d*e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^2*(c^2*x^
2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5829, 5773, 5819, 3384, 3379, 3382, 5778}

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = {} & \frac{e^2 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\ & - \frac{3e^2 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c^3} \\ & - \frac{e^2 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\ & + \frac{3e^2 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c^3} \\ & + \frac{2de \cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\ & - \frac{2de \sinh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\ & - \frac{d^2 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} \\ & + \frac{d^2 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))} \\ & - \frac{2dex\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^2\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}\end{aligned}$$

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 + c
^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a +
 b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^2) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b
^2*c) + (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b^2*c^3) -
(3*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b^2*c^3)
```

```
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e^2*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/
b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left( \frac{d^2}{(a+b\operatorname{arcsinh}(cx))^2} + \frac{2dex}{(a+b\operatorname{arcsinh}(cx))^2} + \frac{e^2x^2}{(a+b\operatorname{arcsinh}(cx))^2} \right) dx \\
&= d^2 \int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\, dx + (2de) \int \frac{x}{(a+b\operatorname{arcsinh}(cx))^2}\, dx + e^2 \int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^2}\, dx \\
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{(cd^2)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\, dx}{b} \\
&\quad + \frac{(2de)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2} \\
&\quad + \frac{e^2\operatorname{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right) dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^3} \\
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad - \frac{d^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad + \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad + \frac{\left(2de\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2} \\
&\quad - \frac{\left(2de\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad + \frac{2de\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{2de\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\
&\quad + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad - \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad + \frac{\left(3e^2\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad - \frac{\left(d^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad + \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad - \frac{\left(3e^2\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{4b^2c^3} \\
&= -\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad + \frac{2de\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{d^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} \\
&\quad + \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3} - \frac{3e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3} \\
&\quad + \frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{4b^2c^3} \\
&\quad - \frac{2de\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} + \frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)}{4b^2c^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.80

$$
\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx =
$$

$$
-\frac{\frac{4bc^2d^2\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} + \frac{8bc^2dex\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} + \frac{4bc^2e^2x^2\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} - 8cde\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) + (4c^2d^2 -}{}
$$

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*x
*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*x^
2])/(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcS
inh[c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] +
 3*e^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] -
3*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)
```

## Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.72

| method | result |
|---|---|
| derivativedivides | $\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e^2}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3e^2e^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\,\mathrm{arcsinh}(cx))}-\frac{3e^2}{}$ |
| default | $\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e^2}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3e^2e^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\,\mathrm{arcsinh}(cx))}-\frac{3e^2}{}$ |

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^
2/c^2/b/(a+b*arcsinh(c*x))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3
*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1
)^(1/2))/(a+b*arcsinh(c*x))-3/8/b^2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x
)-3*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh(c*x))+1/2*d^2/b^2*
exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^2/b/(a+b
*arcsinh(c*x))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c
*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsin
h(c*x)-a/b)+1/8/c^2/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^
2/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/2*(-2*c*x*(c^2*x^2+1)^(1/2)+2
*c^2*x^2+1)*d*e/c/b/(a+b*arcsinh(c*x))-e*d/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(
c*x)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(
c*x))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

## Fricas [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x)
 + a^2), x)
```

## Sympy [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^2}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

## Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^5 + 2*c^3*d*e*x^4 + 2*c*d*e*x^2 + c*d^2*x + (c^3*d^2 + c*e^2)*x
^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 + 2*d*e*x + (c^2*d^2 + e^2)*x^2 + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*
c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))
) + integrate((3*c^5*e^2*x^6 + 4*c^5*d*e*x^5 + 8*c^3*d*e*x^3 + (c^5*d^2 + 6
*c^3*e^2)*x^4 + 4*c*d*e*x + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^3*e^2*
x^4 + 4*c^3*d*e*x^3 - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) + (6*c^4
*e^2*x^5 + 8*c^4*d*e*x^4 + 8*c^2*d*e*x^2 + (2*c^4*d^2 + 7*c^2*e^2)*x^3 + 2*
d*e + (c^2*d^2 + 2*e^2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

## Giac [F]

$$\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^2}{(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((d + e*x)^2/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x)^2/(a + b*asinh(c*x))^2, x)
```

# 3.26 $\int \frac{d+ex}{(a+b\mathbf{arcsinh}(cx))^2}\,dx$

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## Optimal result

Integrand size = 16, antiderivative size = 180

$$\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{ex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{d\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}$$

```
[Out] e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d*cosh(a/b)*Shi((a+b*arcs
inh(c*x))/b)/b^2/c-d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a+b
*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(
c*x))-e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5829, 5773, 5819, 3384, 3379, 3382, 5778}

$$\begin{aligned}\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = &\frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}\\ &-\frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}\\ &-\frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}\\ &+\frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}\\ &-\frac{d\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{ex\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}\end{aligned}$$

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^2
])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c^2) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/
b])/(b^2*c) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) -
(e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left( \frac{d}{(a+b\operatorname{arcsinh}(cx))^2} + \frac{ex}{(a+b\operatorname{arcsinh}(cx))^2} \right) dx \\
&= d \int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\, dx + e \int \frac{x}{(a+b\operatorname{arcsinh}(cx))^2}\, dx \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{ex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\, dx}{b} \\
&\quad + \frac{e\operatorname{Subst}\left( \int \frac{\cosh\left(\frac{2a}{b} - \frac{2x}{b}\right)}{x}\, dx, x, a+b\operatorname{arcsinh}(cx) \right)}{b^2c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{ex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad - \frac{d\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2} \\
&\quad - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2} \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{ex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} \\
&\quad + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad - \frac{\left(d\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{ex\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} \\
&\quad + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2} - \frac{d\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} \\
&\quad + \frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

$$
\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = \\
-\frac{\frac{bcd\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} + \frac{bcex\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} - e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right) + cd\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right.}{b^2c^2}
$$

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*x
^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c
*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c*x])])/(b^2*c^2))
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{c^2x^2+1}\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)d}{2b^2}+\frac{\left(-2cx\sqrt{c^2x^2+1}+2c^2x^2+\right.}{4cb(a+b\,\mathrm{arcsinh}(cx))}$ |
| default | $\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{c^2x^2+1}\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)d}{2b^2}+\frac{\left(-2cx\sqrt{c^2x^2+1}+2c^2x^2+\right.}{4cb(a+b\,\mathrm{arcsinh}(cx))}$ |

```
[In] int((e*x+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*E
i(1,arcsinh(c*x)+a/b)*d-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*d-
1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d+1/4*(-2*c*x*(c^2*x^2+1)^(1/2)+2
*c^2*x^2+1)*e/c/b/(a+b*arcsinh(c*x))-1/2*e/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(
c*x)+2*a/b)-1/4*e/c/b*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*
x))-1/2*e/c/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

## Fricas [F]

$$\int \frac{d+ex}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{ex+d}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

## Sympy [F]

$$\int \frac{d+ex}{(a+b\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{d+ex}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

## Maxima [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^4 + c^3*d*x^3 + c*e*x^2 + c*d*x + (c^2*e*x^3 + c^2*d*x^2 + e*x +
d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
+ 1))) + integrate((2*c^5*e*x^5 + c^5*d*x^4 + 4*c^3*e*x^3 + 2*c^3*d*x^2 + 2
*c*e*x + (2*c^3*e*x^3 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*e*x^4
 + 2*c^4*d*x^3 + 4*c^2*e*x^2 + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4
 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*
x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*
x)*sqrt(c^2*x^2 + 1)), x)
```

## Giac [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{d+e\,x}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((d + e*x)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)/(a + b*asinh(c*x))^2, x)
```

# 3.27 $\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$

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## Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5773, 5819, 3384, 3379, 3382}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c} - \frac{\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{c\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx}{b} \\
&= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c} \\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c}
\end{aligned}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc(a+b\operatorname{arcsinh}(cx))} - \frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$
$$= \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)} - \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)}{b^2c}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

## Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\ \operatorname{arcsinh}(cx))} + \frac{e^{\frac{a}{b}}\ \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2} - \frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\ \operatorname{arcsinh}(cx))} - \frac{e^{-\frac{a}{b}}\ \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$ | 118 |
| default | $\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\ \operatorname{arcsinh}(cx))} + \frac{e^{\frac{a}{b}}\ \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2} - \frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\ \operatorname{arcsinh}(cx))} - \frac{e^{-\frac{a}{b}}\ \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$ | 118 |

```
[In] int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^
2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

## Fricas [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^2,x)
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

# 3.28 $\int \frac{1}{(d+ex)(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result . . . . . 288
Rubi [N/A] . . . . . 288
Mathematica [N/A] . . . . . 289
Maple [N/A] (verified) . . . . . 289
Fricas [N/A] . . . . . 289
Sympy [N/A] . . . . . 290
Maxima [N/A] . . . . . 290
Giac [N/A] . . . . . 291
Mupad [N/A] . . . . . 291

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 5.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\left(cx\right)+a)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arcsinh(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 746, normalized size of antiderivative = 41.44

$$\int \frac{1}{(d+ex)(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^3 + a*b*c^3*d*x^2 + a*b
*c*e*x + a*b*c*d + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (
b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d*x^4
 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*e*x - c*d)*(c^2*x^2 + 1) + c*d + (2*c^4*d
*x^3 - 2*c^2*e*x^2 + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^6 + 2*a
*b*c^5*d*e*x^5 + 4*a*b*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*a*b*x^4 + 2*a*b*
c*d*e*x + a*b*c*d^2 + (2*c^3*d^2 + c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^4 + 2*a*
b*c^3*d*e*x^3 + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^6 + 2*b^2*c
^5*d*e*x^5 + 4*b^2*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*b^2*x^4 + 2*b^2*c*d*
e*x + b^2*c*d^2 + (2*c^3*d^2 + c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4 + 2*b^2*c^
3*d*e*x^3 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^2*x^5 + 2*b^2*c^4
*d*e*x^4 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*b^2*x^3)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^2*x^5 + 2*a
*b*c^4*d*e*x^4 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*a*
b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)), x)
```

# 3.29 $\int \frac{1}{(d+ex)^2(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result . . . 292
Rubi [N/A] . . . 292
Mathematica [N/A] . . . 293
Maple [N/A] (verified) . . . 293
Fricas [N/A] . . . 293
Sympy [N/A] . . . 294
Maxima [N/A] . . . 294
Giac [N/A] . . . 295
Mupad [N/A] . . . 295

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)^2\left(a+b\,\text{arcsinh}\left(cx\right)\right)^2}dx$$

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 5.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\left(cx\right)+a)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*
x + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arcsi
nh(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 5.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2\left(d+ex\right)^2}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 1050, normalized size of antiderivative = 58.33

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(ex+d\right)^2\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^4 + 2*a*b*c^3*d*e*x^3
 + 2*a*b*c*d*e*x + a*b*c*d^2 + (c^3*d^2 + c*e^2)*a*b*x^2 + (b^2*c^3*e^2*x^4
 + 2*b^2*c^3*d*e*x^3 + 2*b^2*c*d*e*x + b^2*c*d^2 + (c^3*d^2 + c*e^2)*b^2*x^
2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2 + a*b
*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^5 - c^5*d*x^4 + 2*c^3*e
*x^3 - 2*c^3*d*x^2 + c*e*x + (c^3*e*x^3 - c^3*d*x^2 + 3*c*e*x + c*d)*(c^2*x
^2 + 1) - c*d + (2*c^4*e*x^4 - 2*c^4*d*x^3 + 5*c^2*e*x^2 - c^2*d*x + 2*e)*s
qrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*d^2*e + 2
*c^3*e^3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*a*b*x^4 + a*b
*c*d^3 + (6*c^3*d^2*e + c*e^3)*a*b*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*a*b*x^2 +
(a*b*c^3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 + a*b*c^3*d^3*
x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^5*d*e^2*x^6 + (3*c^5*d^2*e
+ 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*b^2*x^4 +
b^2*c*d^3 + (6*c^3*d^2*e + c*e^3)*b^2*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*b^2*x^2
 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^2*c^3*d^2*e*x^3 + b^2*c^3*d
^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^3*x^6 + 3*b^2*c^4*d*e^2*x^5 + 3*b^2*c^
2*d^2*e*x^2 + b^2*c^2*d^3*x + (3*c^4*d^2*e + c^2*e^3)*b^2*x^4 + (c^4*d^3 +
3*c^2*d*e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(
a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 + 3*a*b*c^2*d^2*e*x^2 + a*b*c^2*d^3*x
 + (3*c^4*d^2*e + c^2*e^3)*a*b*x^4 + (c^4*d^3 + 3*c^2*d*e^2)*a*b*x^3)*sqrt(
c^2*x^2 + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(d+e\,x)^2}\,dx$$

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2),x)
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2), x)
```

# 3.30 $\int (d+ex)^m (a+b\mathbf{arcsinh}(cx))^2\,dx$

Optimal result . . . 296
Rubi [N/A] . . . 296
Mathematica [N/A] . . . 297
Maple [N/A] (verified) . . . 297
Fricas [N/A] . . . 297
Sympy [N/A] . . . 297
Maxima [N/A] . . . 298
Giac [N/A] . . . 298
Mupad [N/A] . . . 298

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (d+ex)^m (a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))^2}{e(1+m)} - \frac{2bc\operatorname{Int}\left(\frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}},x\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)/e/(1+m)
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex)^m (a+b\operatorname{arcsinh}(cx))^2\,dx = \int (d+ex)^m (a+b\operatorname{arcsinh}(cx))^2\,dx$$

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2], x])/(e*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))^2}{e(1+m)} - \frac{(2bc)\int \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{e(1+m)}$$

## Mathematica [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx$$

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 1.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex+d)^m \left(a+b\,\text{arcsinh}\left(cx\right)\right)^2\,dx$$

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int \left(b\,\text{arsinh}\left(cx\right)+a\right)^2 (ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(e*x + d)^m, x)
```

## Sympy [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int \left(a+b\,\text{asinh}\left(cx\right)\right)^2 (d+ex)^m\,dx$$

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
[Out] Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

## Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 271, normalized size of antiderivative = 15.06

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int (b\,\text{arsinh}\,(cx)+a)^2 (ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (b^2*e*x + b^2*d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*(m + 1)) +
(e*x + d)^(m + 1)*a^2/(e*(m + 1)) + integrate(-2*((b^2*c^2*d*x - a*b*e*(m +
 1) - (a*b*c^2*e*(m + 1) - b^2*c^2*e)*x^2)*sqrt(c^2*x^2 + 1)*(e*x + d)^m +
(b^2*c^3*d*x^2 + b^2*c*d - (a*b*c^3*e*(m + 1) - b^2*c^3*e)*x^3 - (a*b*c*e*(
m + 1) - b^2*c*e)*x)*(e*x + d)^m)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e*(m +
1)*x^3 + c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1))
, x)
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int (b\,\text{arsinh}\,(cx)+a)^2 (ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)^m\,dx$$

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^m,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^m, x)
```

# 3.31 $\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx$

Optimal result . . . 299
Rubi [A] (verified) . . . 299
Mathematica [F] . . . 301
Maple [F] . . . 301
Fricas [F] . . . 301
Sympy [F] . . . 301
Maxima [F] . . . 302
Giac [F] . . . 302
Mupad [F(-1)] . . . 302

## Optimal result

Integrand size = 16, antiderivative size = 179

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx =$$
$$-\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\operatorname{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}}$$
$$+\frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))/e/(1+m)-b*c*(e*x+d)^(2+m)*AppellF1(2+m,1/2
,1/2,3+m,(e*x+d)/(d-e/(-c^2)^(1/2)),(e*x+d)/(d+e/(-c^2)^(1/2)))*(1+(-e*x-d)
/(d-e/(-c^2)^(1/2)))^(1/2)*(1+(-e*x-d)/(d+e/(-c^2)^(1/2)))^(1/2)/e^2/(1+m)/
(2+m)/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5828, 774, 138}

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx = \frac{(d+ex)^{m+1}(a+b\operatorname{arcsinh}(cx))}{e(m+1)}$$
$$-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}\operatorname{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (d
 + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d -
 e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt[1
+ c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{1+c^2x^2}}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{e(1+m)} - \frac{\left(bc\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-c^2}e}{c^2}}}\right)\operatorname{Subst}\left(\int \frac{x^{1+m}}{\sqrt{1-\frac{x}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{x}{d+\frac{e}{\sqrt{-c^2}}}}}\,dx, x, d+ex\right)}{e^2(1+m)\sqrt{1+c^2x^2}}$$

$$= -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\operatorname{AppellF1}\left(2+m, \frac{1}{2}, \frac{1}{2}, 3+m, \frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}, \frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}} + \frac{(d+ex)^{1+m}(a+b\operatorname{arcsinh}(cx))}{e(1+m)}$$

## Mathematica [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$$

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

## Maple [F]

$$\int (ex+d)^m\,(a+b\,\text{arcsinh}\,(cx))\,dx$$

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

## Fricas [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

## Sympy [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}\,(cx))\,(d+ex)^m\,dx$$

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
[Out] Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

## Maxima [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] b*((e*x + d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))/(e*(m + 1)) - integra
te((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + e*(m + 1)), x) -
integrate((c*e*x + c*d)*(e*x + d)^m/(c^3*e*(m + 1)*x^3 + c*e*(m + 1)*x + (c
^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1)), x)) + (e*x + d)^(m + 1)*a
/(e*(m + 1))
```

## Giac [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+e\,x\right)^m\,dx$$

```
[In] int((a + b*asinh(c*x))*(d + e*x)^m,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^m, x)
```

# 3.32 $\int \frac{(d+ex)^m}{a+b\mathbf{arcsinh}(cx)}\,dx$

Optimal result . . . . . 303
Rubi [N/A] . . . . . 303
Mathematica [N/A] . . . . . 304
Maple [N/A] (verified) . . . . . 304
Fricas [N/A] . . . . . 304
Sympy [N/A] . . . . . 304
Maxima [N/A] . . . . . 305
Giac [N/A] . . . . . 305
Mupad [N/A] . . . . . 305

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}\,dx = \operatorname{Int}\left(\frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}, x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}\,dx$$

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex)^m}{a+b\operatorname{arcsinh}(cx)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx$$

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{a+b\,\text{arcsinh}\,(cx)}dx$$

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\,\text{asinh}\,(cx)}\,dx$$

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+e\,x)^m}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

```
[In] int((d + e*x)^m/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^m/(a + b*asinh(c*x)), x)
```

# 3.33 $\int \frac{(d+ex)^m}{(a+b\mathbf{arcsinh}(cx))^2}\,dx$

Optimal result . . . 306
Rubi [N/A] . . . 306
Mathematica [N/A] . . . 307
Maple [N/A] (verified) . . . 307
Fricas [N/A] . . . 307
Sympy [N/A] . . . 307
Maxima [N/A] . . . 308
Giac [N/A] . . . 308
Mupad [N/A] . . . 309

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}, x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx$$

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{(a+b\,\text{arcsinh}\,(cx))^2}dx$$

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

## Sympy [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 607, normalized size of antiderivative = 33.72

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*(e*x + d)^m + (c^3*x^3 + c*x)*(e*x + d)^m)/(a*b*c^3*x
^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*e*(m +
1)*x^3 + c^3*d*x^2 + c*e*(m - 1)*x - c*d)*(c^2*x^2 + 1)*(e*x + d)^m + (2*c^
4*e*(m + 1)*x^4 + 2*c^4*d*x^3 + c^2*e*(3*m + 1)*x^2 + c^2*d*x + e*m)*sqrt(c
^2*x^2 + 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 + 2*c^3*e*(m + 1)*
x^3 + 2*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5 + a*b*
c^5*d*x^4 + 2*a*b*c^3*e*x^3 + 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d + (a*b*
c^3*e*x^3 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*x^5 + b^2*c^5*d*x^4 +
 2*b^2*c^3*e*x^3 + 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (b^2*c^3*e*x^3 +
 b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^4 + b^2*c^4*d*x^3 + b^2*c^2*
e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a
*b*c^4*e*x^4 + a*b*c^4*d*x^3 + a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 +
1)), x)
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^m}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

```
[In] int((d + e*x)^m/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)^m/(a + b*asinh(c*x))^2, x)
```

# 3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))\,dx$

Optimal result . . . 310
Rubi [A] (verified) . . . 311
Mathematica [A] (verified) . . . 316
Maple [B] (verified) . . . 316
Fricas [F] . . . 317
Sympy [F] . . . 317
Maxima [F(-2)] . . . 318
Giac [F(-2)] . . . 318
Mupad [F(-1)] . . . 318

## Optimal result

Integrand size = 30, antiderivative size = 640

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}} + \frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{3bfg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}$$
$$- \frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bg^3x^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{3bcfg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bcg^3x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}$$
$$+ \frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2}$$
$$+ \frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{f^2g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^2}$$
$$- \frac{g^3(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^4}$$
$$+ \frac{g^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4}$$
$$+ \frac{f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} - \frac{3fg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*f*g^2*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)+f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*g^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(c^2*d*x^2+d)^(1/2)
/c/(c^2*x^2+1)^(1/2)+2/15*b*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)
-1/4*b*c*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*f*g^2*x^2*(c^
2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/3*b*c*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/
```

```
16*b*c*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/25*b*c*g^3*x^5*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/16*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5845, 5838, 5785, 5783, 30, 5798, 5806, 5812, 272, 45, 5804, 12}

$$
\begin{aligned}
&\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{1}{2}f^3x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) + \frac{f^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}} \\
&\quad + \frac{f^2g(c^2x^2+1)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{c^2} + \frac{3fg^2x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{8c^2} \\
&\quad + \frac{3}{4}fg^2x^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) + \frac{g^3(c^2x^2+1)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad - \frac{g^3(c^2x^2+1)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3c^4} - \frac{3fg^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{c^2x^2+1}} \\
&\quad - \frac{bcf^3x^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}} - \frac{bf^2gx\sqrt{c^2dx^2+d}}{c\sqrt{c^2x^2+1}} - \frac{bcf^2gx^3\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}} - \frac{3bfg^2x^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}} \\
&\quad - \frac{3bcfg^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}} - \frac{bcg^3x^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}} - \frac{bg^3x^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}} + \frac{2bg^3x\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}
\end{aligned}
$$

```
[In] Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2])) + (2*b*g^3*x*Sqrt[
d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*c*f^3*x^2*Sqrt[d + c^2*d*x^
2])/(4*Sqrt[1 + c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[
1 + c^2*x^2]) - (b*c*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d + c^2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (3*b*c*f*g^2*x^
4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c*g^3*x^5*Sqrt[d + c^2*d
*x^2])/(25*Sqrt[1 + c^2*x^2]) + (f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/2 + (3*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (3
*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^2*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (f^3*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2]) - (3*f*g^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+c^2dx^2}\int (f+gx)^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\sqrt{d+c^2dx^2}\int \left(f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))+3f^2gx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))+3fg^2x^2\sqrt{1+\right.}}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(f^3\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(3f^2g\sqrt{d+c^2dx^2}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(3fg^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(g^3\sqrt{d+c^2dx^2}\right)\int x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{f^2g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^2} \\
&\quad - \frac{g^3(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^4} \\
&\quad + \frac{g^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{\left(f^3\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(bcf^3\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bf^2g\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)\,dx}{c\sqrt{1+c^2x^2}} + \frac{\left(3fg^2\sqrt{d+c^2dx^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3bcfg^2\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(bcg^3\sqrt{d+c^2dx^2}\right)\int \frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx}{\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}} - \frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bcfg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{f^2g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^2} \\
&\quad - \frac{g^3(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^4} \\
&\quad + \frac{g^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} - \frac{\left(3fg^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8c^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3bfg^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{8c\sqrt{1+c^2x^2}} - \frac{\left(bg^3\sqrt{d+c^2dx^2}\right)\int \left(-2+c^2x^2+3c^4x^4\right)\,dx}{15c^3\sqrt{1+c^2x^2}} \\
&= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}} + \frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bfg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} - \frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}} - \frac{bg^3x^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bcfg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bcg^3x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + \frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{f^2g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{c^2} \\
&\quad - \frac{g^3(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^4} \\
&\quad + \frac{g^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} - \frac{3fg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.64

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$$

$$= \frac{240ad(1+c^2x^2)^{3/2}\left(-16g^3+c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g^3x^3)\right)-9}{}$$

```
[In] Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*d*(1 + c^2*x^2)^(3/2)*(-16*g^3 + c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*x
^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x^3)) - 9600*b*c^
2*d*f^2*g*(3*c*x + 4*c^3*x^3 + c^5*x^5 - 3*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]
) + 128*b*d*g^3*(1 + c^2*x^2)*(30*c*x - 5*c^3*x^3 - 9*c^5*x^5 + 15*Sqrt[1 +
 c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3600*a*c*Sqrt[d]*f*(4*
c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*
Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*d*f^3*(1 + c^2*x^2)*(Cosh[2*ArcSinh[c*x]]
 - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675*b*c*d*f*g^2*
(1 + c^2*x^2)*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sin
h[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. 2(560) = 1120.

Time = 0.86 (sec) , antiderivative size = 1309, normalized size of antiderivative = 2.05

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1309 |
| parts | Expression too large to display | 1309 |

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^3*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2))+g^3*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4
*(c^2*d*x^2+d)^(3/2))+3*f*g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(1/2
*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/
(c^2*d)^(1/2)))+f^2*g/c^2/d*(c^2*d*x^2+d)^(3/2))+b*(1/16*(d*(c^2*x^2+1))^(1
/2)*f*arcsinh(c*x)^2*(4*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/c^3+1/800*(d*(c^2*
x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^
3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+5*arcsinh
(c*x))/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^
2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/
```

```
2))*f*g^2*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*
g*(36*arcsinh(c*x)*c^2*f^2-12*c^2*f^2-3*arcsinh(c*x)*g^2+g^2)/c^4/(c^2*x^2+
1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+
(c^2*x^2+1)^(1/2))*f^3*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1
))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(6*arcsinh(c*x)*c^2*f^2-6*c^2*
f^2-arcsinh(c*x)*g^2+g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(6*arcsinh(c*x)*c^2*f^2+6*c^2*f^2-arcsinh(c*x
)*g^2-g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*
(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^3*(1+2*arcsinh(c*x))/c/(c^2*x^
2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(36*arcsinh(c*x)*c^2*f^2+12*c^2*f^2-3*a
rcsinh(c*x)*g^2-g^2)/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5
-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(
c^2*x^2+1)^(1/2))*f*g^2*(1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/800*(d*(c^2*x^
2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*
(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(1+5*arcsinh(c*
x))/c^4/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(gx+f)^3(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\,(f+gx)^3\,dx$$

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.35 $\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 431

$$\begin{aligned}
&\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} - \frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2fg(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} - \frac{g^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] 1/2*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*g^2*x*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)+2/3*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*f*g
*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f^2*x^2*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-2/9*b*c*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c*g^2*x^4*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2
+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)
^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5845, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$\begin{aligned}
&\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{1}{2}f^2x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) + \frac{f^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}} \\
&\quad + \frac{2fg(c^2x^2+1)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{g^2x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{1}{4}g^2x^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) \\
&\quad - \frac{g^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{c^2x^2+1}} - \frac{bcf^2x^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}} - \frac{2bfgx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}} \\
&\quad - \frac{2bcfgx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}} - \frac{bg^2x^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}} - \frac{bcg^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}
\end{aligned}$$

```
[In] Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*f*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f^2*x^2*Sqr
t[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*g^2*x^2*Sqrt[d + c^2*d*x^2])/(
16*c*Sqrt[1 + c^2*x^2]) - (2*b*c*f*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt[1 + c
^2*x^2]) - (b*c*g^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (f^2*
x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/4 + (2*f*g*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(3*c^2) + (f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2]) - (g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt
[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+c^2dx^2}\int (f+gx)^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\sqrt{d+c^2dx^2}\int \left(f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) + 2fgx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) + g^2x^2\sqrt{1+c^2}\right.}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(f^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2fg\sqrt{d+c^2dx^2}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(g^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2fg(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{\left(f^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcf^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(2bfg\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)\,dx}{3c\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(g^2\sqrt{d+c^2dx^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(bcg^2\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{4\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2fg(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(g^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8c^2\sqrt{1+c^2x^2}} - \frac{\left(bg^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{8c\sqrt{1+c^2x^2}} \\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} - \frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8c^2} + \frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2fg(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}} - \frac{g^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.70

$$
\begin{aligned}
&\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx \\
&= \frac{48ac\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(12c^2f^2x+3g^2x(1+2c^2x^2)+16f(g+c^2gx^2)) - 256bcfg\sqrt{d+c^2dx^2}\left(3cx+c\right.}{}
\end{aligned}
$$

```
[In] Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]

[Out] (48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x
 + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f - g
)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] -
144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(A
rcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSi
nh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(1
152*c^3*Sqrt[1 + c^2*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(373) = 746.

Time = 0.72 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x\left(c^2dx^2+d\right)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\frac{2f$ |
| parts | $a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x\left(c^2dx^2+d\right)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\frac{2f$ |

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^2*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2))+g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(1/2
*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/
(c^2*d)^(1/2)))+2/3*f*g/c^2/d*(c^2*d*x^2+d)^(3/2))+b*(1/16*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)^2*(4*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3+1/256*(d*(c^2*x^
2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^
2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+4*arcsinh(c*x))/c^3/(c^2*x^
2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^
2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/
16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*
x^2+1)^(1/2))*f^2*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/4*(d*(c^2*x^2+1))^(1/
2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+
1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(arcsinh(c*
x)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*
x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^2*(1+2*arcsinh(c*x))/c/(c^2*x^2+1)+
1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2
-3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(3*arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/256*(d
*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2
*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+4*arcsinh(c*x))/c^3/
(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

## Sympy [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.36 $\int (f+gx)\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 28, antiderivative size = 227

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} + \frac{f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/3*g*(c^2*x^2+1)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1
)^(1/2)-1/4*b*c*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

$= \{5845, 5838, 5785, 5783, 30, 5798\}$

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{2}fx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx)) + \frac{f\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{4bc\sqrt{c^2x^2+1}} + \frac{g(c^2x^2+1)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3c^2} - \frac{bcfx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}} - \frac{bgx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}} - \frac{bcgx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

```
[In] Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*f*x^2*Sqrt[d
+ c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*c*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sq
rt[1 + c^2*x^2]) + (f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1
 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+c^2dx^2}\int (f+gx)\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\sqrt{d+c^2dx^2}\int \left(f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx)) + gx\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\right)\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(f\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(g\sqrt{d+c^2dx^2}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{2}fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3c^2} \\
&\quad + \frac{\left(f\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcf\sqrt{d+c^2dx^2}\right)\int x\,dx}{2\sqrt{1+c^2x^2}} - \frac{\left(bg\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)\,dx}{3c\sqrt{1+c^2x^2}}
\end{aligned}$$

$$= -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2} + \frac{f\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

## Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \frac{1}{6}a\sqrt{d+c^2dx^2}\left(\frac{2g}{c^2}+x(3f+2gx)\right) - \frac{bg\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9c^2\sqrt{1+c^2x^2}} + \frac{a\sqrt{d}f\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{2c} + \frac{bf\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}}$$

```
[In] Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^2
*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sqrt
[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(ArcS
inh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(195) = 390.

Time = 0.78 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.56

| method | result |
|---|---|
| default | $\frac{afx\sqrt{c^2dx^2+d}}{2} + \frac{afd\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}} + \frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{d(c^2x^2+1)}f\,\text{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c} + \frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c\right.\right.$ |
| parts | $\frac{afx\sqrt{c^2dx^2+d}}{2} + \frac{afd\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}} + \frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{d(c^2x^2+1)}f\,\text{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c} + \frac{\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c\right.\right.$ |

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+1/3*a*g/c^2/d*(c^2*d*x^2+d)^(3/2)+b*(1/4*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2+1/72*(d*(c^2*x^2+1))^(1/2
)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+
1)*g*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*
x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(-1+2*arcsinh(c*
x))/c/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+
1)*g*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c
*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2
+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*
f*(1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(3*arcsinh
(c*x)+1)/c^2/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{c^2dx^2+d}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x)
```

## Sympy [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int \sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\,(f+gx)\;dx$$

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)\,(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

# 3.37 $\int \frac{\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))}{f+gx}\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 664

$$
\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

```
[Out] a*(c^2*d*x^2+d)^(1/2)/g+b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g-b*c*x*(c^2*d*x
^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
```

```
(1/2)/b/g/(c^2*x^2+1)^(1/2)-1/2*(1+c^2*f^2/g^2)*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/c/(g*x+f)/(c^2*x^2+1)^(1/2)-a*arctanh((-c^2*f*x+g)/(c^2*f^2
+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/
(c^2*x^2+1)^(1/2)+b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f
^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/
2)-b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))
*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(
c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2
/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)
^(1/2)/b/c/(g*x+f)
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5845, 5839, 697, 5835, 6874, 267, 739, 212, 5856, 1668, 12, 5855, 5798, 8, 5843, 3403, 2296, 2221, 2317, 2438}

$$
\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx \\
&= -\frac{\sqrt{c^2dx^2+d}\left(\frac{c^2f^2}{g^2}+1\right)(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2x^2+1}(f+gx)} \\
&\quad + \frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} - \frac{cx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{c^2x^2+1}} \\
&\quad - \frac{a\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}} + \frac{a\sqrt{c^2dx^2+d}}{g} \\
&\quad + \frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}} \\
&\quad - \frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}} \\
&\quad + \frac{b\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^2\sqrt{c^2x^2+1}} \\
&\quad - \frac{b\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{g^2\sqrt{c^2x^2+1}} \\
&\quad + \frac{b\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{g} - \frac{bcx\sqrt{c^2dx^2+d}}{g\sqrt{c^2x^2+1}}
\end{aligned}
$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2])/g - (b*c*x*Sqrt[d + c^2*d*x^2])/(g*Sqrt[1 + c^2*x^2
]) + (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g - (c*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(2*b*g*Sqrt[1 + c^2*x^2]) - ((1 + (c^2*f^2)/g^2)*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 + c^2*x^2]) +
 (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f +
 g*x)) - (a*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(
Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[
c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g
^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d
+ c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*PolyL
og[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^2*Sqrt[1 + c^2
*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
```

```
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)} - \frac{\sqrt{d+c^2dx^2}\int \frac{(-g+2c^2fx+c^2gx^2)(a+b\text{arcsinh}(cx))^2}{(f+gx)^2}\,dx}{2bc\sqrt{1+c^2x^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)} + \frac{\sqrt{d+c^2dx^2}\int \frac{\left(\frac{c^2x}{g}+\frac{1+\frac{c^2f^2}{g^2}}{f+gx}\right)(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)} + \frac{\sqrt{d+c^2dx^2}\int \left(\frac{a(c^2f^2+g^2+c^2fgx+c^2g^2x^2)}{g^2(f+gx)\sqrt{1+c^2x^2}} + \frac{b(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\text{arcsinh}(cx)}{g^2(f+gx)\sqrt{1+c^2x^2}}\right)dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(a\sqrt{d+c^2dx^2}\right)\int \frac{c^2f^2+g^2+c^2fgx+c^2g^2x^2}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \frac{(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} + \frac{\left(a\sqrt{d+c^2dx^2}\right)\int \frac{c^2g^2(c^2f^2+g^2)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{c^2g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \left(\frac{c^2gx\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \frac{(c^2f^2+g^2)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\right)dx}{g^2\sqrt{1+c^2x^2}} \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} + \frac{\left(bc^2\sqrt{d+c^2dx^2}\right)\int \frac{x\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{g\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(a(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\int \frac{1}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\sqrt{d+c^2dx^2}}{g} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} - \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int 1\,dx}{g\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(a(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{1}{c^2f^2+g^2-x^2}\,dx, x, \frac{g-c^2fx}{\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2b(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2ce^x f-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^x g-2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^x g+2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g} \\
&\quad - \frac{cx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.93 (sec) , antiderivative size = 1358, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx$$

$$2ag\sqrt{d+c^2dx^2} + 2a\sqrt{d}\sqrt{c^2f^2+g^2}\log(f+gx) - 2ac\sqrt{d}f\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) - 2a\sqrt{d}\sqrt{c^2f^2+g}$$

$$=$$

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]

[Out] (2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x] -
 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*Sqr
t[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d +
```

```
 c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*Ar
cSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*((-I)*c*f + g)*(I*
c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g
^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Co
t[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSi
nh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*
f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*
g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I
/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*
x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)
*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcS
inh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (Ar
cCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x
])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2
*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*S
qrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c
*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[
-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(
I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^
2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2
*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh
[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSin
h[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g
+ Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g +
 I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))/Sqrt[-(c^2*
f^2) - g^2]))/Sqrt[1 + c^2*x^2]))/(2*g^2)
```

## Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.12

| method | result |
| --- | --- |
| default | $a\left(\frac{\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}-\frac{c^2 df\ln\left(\frac{-\frac{c^2df}{g}+c^2d\left(x+\frac{f}{g}\right)}{\sqrt{c^2d}}+\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}\right)}{g\sqrt{c^2d}}-\frac{d\left(c^2f^2+g^2\right.}{g}\right.$ |
| parts | $a\left(\frac{\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}-\frac{c^2 df\ln\left(\frac{-\frac{c^2df}{g}+c^2d\left(x+\frac{f}{g}\right)}{\sqrt{c^2d}}+\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}\right)}{g\sqrt{c^2d}}-\frac{d\left(c^2f^2+g^2\right.}{g}\right.$ |

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] a/g*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d*
f/g*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/
g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/(d*
(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d
*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g
^2)/g^2)^(1/2))/(x+f/g)))+b*(-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f
*arcsinh(c*x)^2*c/g^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1
/2)+1)*(-1+arcsinh(c*x))/(c^2*x^2+1)/g+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c
*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/(c^2*x^2+1)/g+(d*(c^2*x^2+1))^(1/2
)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)
^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-arcsinh(c*x)
*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^
(1/2)))+dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c
^2*f^2+g^2)^(1/2)))-dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2
))/(c*f+(c^2*f^2+g^2)^(1/2))))/g^2)
```

## Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] -(c*sqrt(d)*f*arcsinh(c*x)/g^2 - sqrt(c^2*d*f^2/g^2 + d)*arcsinh(c*f*x/abs(
g*x + f) - g/(c*abs(g*x + f)))/g - sqrt(c^2*d*x^2 + d)/g)*a + b*integrate(s
qrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(g*x + f), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{f+g\,x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x),x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x), x)
```

# 3.38 $\int \frac{\sqrt{d+c^2dx^2}(a+b\mathbf{arcsinh}(cx))}{(f+gx)^2}\,dx$

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Mathematica [C] (warning: unable to verify) . . . . . 361
Maple [B] (verified) . . . . . 362
Fricas [F] . . . . . 363
Sympy [F] . . . . . 364
Maxima [F] . . . . . 364
Giac [F(-2)] . . . . . 364
Mupad [F(-1)] . . . . . 365

## Optimal result

Integrand size = 30, antiderivative size = 781

$$\begin{aligned}\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx &= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} \\ &+ \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\ &+ \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\ &- \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\ &+ \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\ &+ \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\ &- \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\ &+ \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\ &+ \frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} \\ &- \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\ &+ \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\end{aligned}$$

```
[Out] -a*(c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g/(g*x+
f)+a*c^3*f^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(c^2*x^2+1)
^(1/2)+1/2*b*c^3*f^2*arcsinh(c*x)^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(
c^2*x^2+1)^(1/2)-1/2*(-c^2*f*x+g)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)/b/c/(c^2*f^2+g^2)/(g*x+f)^2/(c^2*x^2+1)^(1/2)+b*c*ln(g*x+f)*(c^2*d*x^2+d)
^(1/2)/g^2/(c^2*x^2+1)^(1/2)+a*c^2*f*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/
2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+
1)^(1/2)-b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+
b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2
)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*f*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d
```

```
)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+(
c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2
)*(c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2
```

## Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {5845, 5839, 37, 5834, 12, 1665, 858, 221, 739, 212, 5856, 5855, 5783, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\begin{aligned}
\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx = &-\frac{\sqrt{c^2dx^2+d}(g-c^2fx)^2(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2x^2+1}\,(c^2f^2+g^2)(f+gx)^2}\\
&+\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2}\\
&+\frac{ac^3f^2\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{g^2\sqrt{c^2x^2+1}\,(c^2f^2+g^2)}\\
&+\frac{ac^2f\sqrt{c^2dx^2+d}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\\
&-\frac{a\sqrt{c^2dx^2+d}}{g(f+gx)}\\
&-\frac{bc^2f\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\\
&+\frac{bc^2f\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\\
&-\frac{bc^2f\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\\
&+\frac{bc^2f\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\\
&-\frac{b\operatorname{arcsinh}(cx)\sqrt{c^2dx^2+d}}{g(f+gx)}\\
&+\frac{bc^3f^2\operatorname{arcsinh}(cx)^2\sqrt{c^2dx^2+d}}{2g^2\sqrt{c^2x^2+1}\,(c^2f^2+g^2)}\\
&+\frac{bc\sqrt{c^2dx^2+d}\log(f+gx)}{g^2\sqrt{c^2x^2+1}}
\end{aligned}$$

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d + c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c
*x])/(g*(f + g*x)) + (a*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(g^2*(c^2
*f^2 + g^2)*Sqrt[1 + c^2*x^2]) + (b*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x
]^2)/(2*g^2*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]) - ((g - c^2*f*x)^2*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(c^2*f^2 + g^2)*(f + g*x)^2*Sqrt[
1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(f + g*x)^2) + (a*c^2*f*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x
)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1
 + c^2*x^2]) - (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh
[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^
2*x^2]) + (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]
*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 + c^2*x^2]) - (b*c^
2*f*Sqrt[d + c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
 + g^2]))])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (b*c^2*f*Sqrt[d +
 c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/
(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5834

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && L
tQ[m + p + 1, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
```

```
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2} - \frac{\sqrt{d+c^2dx^2}\int \frac{(-2g+2c^2fx)(a+b\text{arcsinh}(cx))^2}{(f+gx)^3}\,dx}{2bc\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{\sqrt{d+c^2dx^2}\int \frac{\left(g-c^2fx\right)^2(a+b\operatorname{arcsinh}(cx))}{\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{\sqrt{d+c^2dx^2}\int \frac{\left(g-c^2fx\right)^2(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{\sqrt{d+c^2dx^2}\int \left(\frac{a\left(-g+c^2fx\right)^2}{(f+gx)^2\sqrt{1+c^2x^2}} + \frac{b\left(-g+c^2fx\right)^2\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\right)\,dx}{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{\left(a\sqrt{d+c^2dx^2}\right)\int \frac{\left(-g+c^2fx\right)^2}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \frac{\left(-g+c^2fx\right)^2\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad - \frac{\left(a\sqrt{d+c^2dx^2}\right)\int \frac{c^2f\left(c^2f^2+g^2\right)-c^4f^2\left(\frac{c^2f^2}{g}+g\right)x}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b\sqrt{d+c^2dx^2}\right)\int \left(\frac{c^4f^2\operatorname{arcsinh}(cx)}{g^2\sqrt{1+c^2x^2}} + \frac{\left(c^2f^2+g^2\right)^2\operatorname{arcsinh}(cx)}{g^2(f+gx)^2\sqrt{1+c^2x^2}} - \frac{2c^2f\left(c^2f^2+g^2\right)\operatorname{arcsinh}(cx)}{g^2(f+gx)\sqrt{1+c^2x^2}}\right)\,dx}{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{(g-c^2fx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad - \frac{\left(ac^2f\sqrt{d+c^2dx^2}\right)\int \frac{1}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} - \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(ac^4f^2\left(\frac{c^2f^2}{g}+g\right)\sqrt{d+c^2dx^2}\right)\int \frac{1}{\sqrt{1+c^2x^2}}\,dx}{g\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc^4f^2\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(b(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad - \frac{(g-c^2fx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{\left(ac^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{1}{c^2f^2+g^2-x^2}\,dx, x, \frac{g-c^2fx}{\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{(cf+g\sinh(x))^2}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(4bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2ce^xf-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(x)}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{1+c^2x^2}} \\
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{1}{cf+x}\,dx, x, cgx\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2ce^xf-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(4bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(4bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{2bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{e^xx}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{g\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{e^xx}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx,x,\operatorname{arcsinh}(cx)\right)}{g\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx,x,e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} - \frac{2bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{2bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc^2f\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
&\quad + \frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} - \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.67 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx$$

$$
\begin{aligned}
&-\frac{2ag\sqrt{d+c^2dx^2}}{f+gx} - \frac{2ac^2\sqrt{d}f\log(f+gx)}{\sqrt{c^2f^2+g^2}} + 2ac\sqrt{d}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right) + \frac{2ac^2\sqrt{d}f\log\left(d(g-c^2fx)+\sqrt{d}\sqrt{c^2f^2+g^2}\sqrt{d+}\right.}{\sqrt{c^2f^2+g^2}} \\
&= \text{---------------------

```
c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f -
 I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[(
(-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSi
nh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^
2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*
f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[
-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f
^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*
I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) -
 g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot
[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Co
t[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[(
(c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[
((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcS
inh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g
 - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f +
g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[
2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2
]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))))/(Sqrt[-(c^2*f^2) - g^2]*Sqrt[1 + c
^2*x^2]))/(2*g^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1700 vs. $2(745) = 1490$.

Time = 0.82 (sec) , antiderivative size = 1701, normalized size of antiderivative = 2.18

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1701 |
| parts | Expression too large to display | 1701 |

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] a/g^2*(-1/d/(c^2*f^2+g^2)*g^2/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+
d*(c^2*f^2+g^2)/g^2)^(3/2)-c^2*f*g/(c^2*f^2+g^2)*(((x+f/g)^2*c^2*d-2*c^2*d*
f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+c^2*d*(x+f/
g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)
^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2
*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+
```

```
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g)))+2*c^
2/(c^2*f^2+g^2)*g^2*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2
*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2*f^2+g
^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2
)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^
(1/2)))+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c/g^2+
b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x^3*c^4*f-b*(d
*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^2*x^2+1))^(1/2
)*arcsinh(c*x)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)*arcsin
h(c*x)/(c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)
/(c^2*x^2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*
x^2+1)^(1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+
1)/g/(g*x+f)-b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2
)^(1/2)*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2)
)/(-c*f+(c^2*f^2+g^2)^(1/2)))*f+b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)^(1/
2)/g^2/(c^2*f^2+g^2)^(1/2)*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(
c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*f+b*(d*(c^2*x^2+1))^(1/2)*c^
3/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*
(c*x+(c^2*x^2+1)^(1/2))-g)*f^2-2*b*(d*(c^2*x^2+1))^(1/2)*c^3/(c^2*x^2+1)^(1
/2)/g^2/(c^2*f^2+g^2)*ln(c*x+(c^2*x^2+1)^(1/2))*f^2-b*(d*(c^2*x^2+1))^(1/2)
*c^2/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*dilog((-(c*x+(c^2*x^2+1)^(1/
2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*f+b*(d*(c^2*x^2+
1))^(1/2)*c^2/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*dilog(((c*x+(c^2*x^
2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*f+b*(d*(c
^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)-2*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^
2+1)^(1/2)/(c^2*f^2+g^2)*ln(c*x+(c^2*x^2+1)^(1/2))
```

## Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\mathrm{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)}{(gx+f)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

## Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\left(cx\right))}{\left(f+gx\right)^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\left(cx\right)+a)}{\left(gx+f\right)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] -(c^2*d*f*arcsinh(c*f*x/(g*abs(x + f/g)) - 1/(c*abs(x + f/g)))/(sqrt(c^2*d*
f^2/g^2 + d)*g^3) - c*sqrt(d)*arcsinh(c*x)/g^2 + sqrt(c^2*d*x^2 + d)/(g^2*x
 + f*g))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(
g^2*x^2 + 2*f*g*x + f^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{(f+g\,x)^2}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

# 3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\mathbf{arcsinh}(cx))\,dx$


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## Optimal result

Integrand size = 30, antiderivative size = 918

$$
\begin{aligned}
&\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \\
&-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} \\
&-\frac{3bdfg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{2bcdf^2gx^3\sqrt{d+c^2dx^2}}{5\sqrt{1+c^2x^2}}-\frac{bdg^3x^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}} \\
&-\frac{bc^3df^3x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{7bcdfg^2x^4\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}}-\frac{3bc^3df^2gx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&-\frac{8bcdg^3x^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dfg^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x^2}}-\frac{bc^3dg^3x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&+\frac{3}{8}df^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))+\frac{3dfg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} \\
&+\frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{3df^2g(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&-\frac{dg^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&+\frac{dg^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&+\frac{3df^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}-\frac{3dfg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

```
[Out] 3/8*d*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/16*d*f*g^2*x*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)+1/4*d*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)+1/2*d*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/5*d*
f^2*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/5*d*g^3*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*(c^2*x^2+1
)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/35*b*d*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-5/16*b*c*d*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/32*b*d
*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/5*b*c*d*f^2*g*x^3*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(
c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-7/32*b*c*d*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/25*b*c^3*d*f^
2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-8/175*b*c*d*g^3*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/12*b*c^3*d*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3
/16*d*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/
32*d*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200, 5808, 5806, 5812, 272, 45, 5804,

12, 380}

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx =$$
$$-\frac{bc^3dg^3\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{bc^3dfg^2\sqrt{c^2dx^2+d}x^6}{12\sqrt{c^2x^2+1}}-\frac{8bcdg^3\sqrt{c^2dx^2+d}x^5}{175\sqrt{c^2x^2+1}}$$
$$-\frac{3bc^3df^2g\sqrt{c^2dx^2+d}x^5}{25\sqrt{c^2x^2+1}}-\frac{bc^3df^3\sqrt{c^2dx^2+d}x^4}{16\sqrt{c^2x^2+1}}$$
$$-\frac{7bcdfg^2\sqrt{c^2dx^2+d}x^4}{32\sqrt{c^2x^2+1}}+\frac{3}{8}dfg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x^3$$
$$+\frac{1}{2}dfg^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x^3-\frac{bdg^3\sqrt{c^2dx^2+d}x^3}{105c\sqrt{c^2x^2+1}}$$
$$-\frac{2bcdf^2g\sqrt{c^2dx^2+d}x^3}{5\sqrt{c^2x^2+1}}-\frac{5bcdf^3\sqrt{c^2dx^2+d}x^2}{16\sqrt{c^2x^2+1}}-\frac{3bdfg^2\sqrt{c^2dx^2+d}x^2}{32c\sqrt{c^2x^2+1}}$$
$$+\frac{3}{8}df^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x+\frac{3dfg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x}{16c^2}$$
$$+\frac{1}{4}df^3\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x+\frac{2bdg^3\sqrt{c^2dx^2+d}x}{35c^3\sqrt{c^2x^2+1}}$$
$$-\frac{3bdf^2g\sqrt{c^2dx^2+d}x}{5c\sqrt{c^2x^2+1}}+\frac{3df^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{c^2x^2+1}}$$
$$-\frac{3dfg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{32bc^3\sqrt{c^2x^2+1}}$$
$$+\frac{dg^3\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{7c^4}$$
$$-\frac{dg^3\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^4}$$
$$+\frac{3df^2g\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{5c^2}$$

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) + (2*b*d*g^3*x
*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^3*x^2*Sqrt[d
+ c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d + c^2*d*x^2]
)/(32*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(5*Sqr
t[1 + c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (7*b*c*
d*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(32*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (8*b*c*d*g^3*x^5*Sqrt[d +
 c^2*d*x^2])/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d + c^2*d*x^
2])/(12*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt
[1 + c^2*x^2]) + (3*d*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (
```

```
3*d*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*d*f*g^2
*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f^3*x*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*f*g^2*x^3*(1 + c^2*x^2)*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (3*d*f^2*g*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (d*g^3*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (d*g^3*(1 + c^2*x^2)^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (3*d*f^3*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2]) - (3*d*f*g^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^3\left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f^3(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)+3f^2gx(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)+3f\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(df^3\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(3df^2g\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(3dfg^2\sqrt{d+c^2dx^2}\right)\int x^2(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(dg^3\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)^{3/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\text{arcsinh}(cx)\right)$$
$$+ \frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\text{arcsinh}(cx)\right)$$
$$+ \frac{3df^2g\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\text{arcsinh}(cx)\right)}{5c^2}$$
$$- \frac{dg^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\text{arcsinh}(cx)\right)}{5c^4}$$
$$+ \frac{dg^3\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}\left(a+b\text{arcsinh}(cx)\right)}{7c^4}$$
$$+ \frac{\left(3df^3\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\text{arcsinh}(cx)\right)dx}{4\sqrt{1+c^2x^2}}$$
$$- \frac{\left(bcdf^3\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(3bdf^2g\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^2\,dx}{5c\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(3dfg^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}\left(a+b\text{arcsinh}(cx)\right)dx}{2\sqrt{1+c^2x^2}}$$
$$- \frac{\left(bcdfg^2\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)\,dx}{2\sqrt{1+c^2x^2}}$$
$$- \frac{\left(bcdg^3\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)}{35c^4}\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{3}{8} df^3 x \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) + \frac{3}{8} df g^2 x^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4} df^3 x \left(1 + c^2 x^2\right) \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{2} df g^2 x^3 \left(1 + c^2 x^2\right) \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \\
&\quad + \frac{3 df^2 g \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{5c^2} \\
&\quad - \frac{dg^3 \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{dg^3 \left(1 + c^2 x^2\right)^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{7c^4} \\
&\quad + \frac{\left(3df^3 \sqrt{d + c^2 d x^2}\right) \int \frac{a + b \operatorname{arcsinh}(cx)}{\sqrt{1 + c^2 x^2}} \, dx}{8\sqrt{1 + c^2 x^2}} - \frac{\left(bcdf^3 \sqrt{d + c^2 d x^2}\right) \int \left(x + c^2 x^3\right) \, dx}{4\sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(3bcdf^3 \sqrt{d + c^2 d x^2}\right) \int x \, dx}{8\sqrt{1 + c^2 x^2}} - \frac{\left(3bdf^2 g \sqrt{d + c^2 d x^2}\right) \int \left(1 + 2c^2 x^2 + c^4 x^4\right) \, dx}{5c\sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(3df g^2 \sqrt{d + c^2 d x^2}\right) \int \frac{x^2 (a + b \operatorname{arcsinh}(cx))}{\sqrt{1 + c^2 x^2}} \, dx}{8\sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(3bcdf g^2 \sqrt{d + c^2 d x^2}\right) \int x^3 \, dx}{8\sqrt{1 + c^2 x^2}} - \frac{\left(bcdf g^2 \sqrt{d + c^2 d x^2}\right) \int \left(x^3 + c^2 x^5\right) \, dx}{2\sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(bdg^3 \sqrt{d + c^2 d x^2}\right) \int \left(1 + c^2 x^2\right)^2 \left(-2 + 5c^2 x^2\right) \, dx}{35c^3 \sqrt{1 + c^2 x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdf^2gx^3\sqrt{d+c^2dx^2}}{5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3df^3x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdfg^2x^4\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}} - \frac{3bc^3df^2gx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dfg^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x^2}} + \frac{3}{8}df^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3dfg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3df^2g(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad - \frac{dg^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{dg^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&\quad + \frac{3df^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3dfg^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c^2\sqrt{1+c^2x^2}} - \frac{\left(3bdfg^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{16c\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bdg^3\sqrt{d+c^2dx^2}\right)\int \left(-2+c^2x^2+8c^4x^4+5c^6x^6\right)\,dx}{35c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} + \frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bdfg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{2bcdf^2gx^3\sqrt{d+c^2dx^2}}{5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bdg^3x^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}} - \frac{bc^3df^3x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdfg^2x^4\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bc^3df^2gx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{8bcdg^3x^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}} - \frac{bc^3dfg^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dg^3x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} + \frac{3}{8}df^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3dfg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3df^2g(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad - \frac{dg^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} \\
&\quad + \frac{dg^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&\quad + \frac{3df^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} - \frac{3dfg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.58

$$
\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}(a
$$

$$
+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}(-32g^3+c^2g(336f^2+105fgx+16g^2x^2)+4c^6x^3(35
$$

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(d^2*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(-32*g^3 + c^2*g*(336*f^2 +
 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2 +
20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^3))
 + b*(-35*c*g^2*(245*f + 1536*g*x) + 70*c^3*(1785*f^3 + 8064*f^2*g*x + 1260
*f*g^2*x^2 + 128*g^3*x^3) + 168*c^5*x^2*(1750*f^3 + 2240*f^2*g*x + 1225*f*g
^2*x^2 + 256*g^3*x^3) + 16*c^7*x^4*(3675*f^3 + 7056*f^2*g*x + 4900*f*g^2*x^
2 + 1200*g^3*x^3)))) + 88200*b*c*d^2*f*(2*c^2*f^2 - g^2)*(1 + c^2*x^2)*ArcS
```

```
inh[c*x]^2 + 176400*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Sqrt[
d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*d^2*(1 + c^
2*x^2)*ArcSinh[c*x]*(35*c*f*(16*c^2*f^2 - 3*g^2)*Sinh[2*ArcSinh[c*x]] + 35*
c*f*(2*c^2*f^2 + 3*g^2)*Sinh[4*ArcSinh[c*x]] + g*(64*(1 + c^2*x^2)^(5/2)*(-
2*g^2 + c^2*(21*f^2 + 5*g^2*x^2)) + 35*c*f*g*Sinh[6*ArcSinh[c*x]])))/(94080
0*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(802) = 1604$.

Time = 0.88 (sec) , antiderivative size = 2079, normalized size of antiderivative = 2.26

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2079 |
| parts | Expression too large to display | 2079 |

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^3*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln
(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^3*(1/7*x^2*(c
^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+3*f*g^2*(1/6*x*(c^2
*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*
d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^
(1/2))))+3/5*f^2*g/c^2/d*(c^2*d*x^2+d)^(5/2))+b*(3/32*(d*(c^2*x^2+1))^(1/2)
*f*arcsinh(c*x)^2*(2*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/6272*(d*(c^2*x^
2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^
5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c
*x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/768*(d*
(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c
^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2
*x^2+1)^(1/2))*f*g^2*(-1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/3200*(d*(c^2*x
^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3
*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(60*arcsinh(c*x)
*c^2*f^2-12*c^2*f^2+5*arcsinh(c*x)*g^2-g^2)*d/c^4/(c^2*x^2+1)+1/512*(d*(c^2
*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*
(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c^2*f^2-2*c^2*
f^2+12*arcsinh(c*x)*g^2-3*g^2)*d/c^3/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2
)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+
1)*g*(36*arcsinh(c*x)*c^2*f^2-12*c^2*f^2-3*arcsinh(c*x)*g^2+g^2)*d/c^4/(c^2
*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+
2*c*x+(c^2*x^2+1)^(1/2))*f*(32*arcsinh(c*x)*c^2*f^2-16*c^2*f^2-6*arcsinh(c*
x)*g^2+3*g^2)*d/c^3/(c^2*x^2+1)+3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c
^2*x^2+1)^(1/2)+1)*g*(8*arcsinh(c*x)*c^2*f^2-8*c^2*f^2-arcsinh(c*x)*g^2+g^2
```

```
)*d/c^4/(c^2*x^2+1)+3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1
/2)+1)*g*(8*arcsinh(c*x)*c^2*f^2+8*c^2*f^2-arcsinh(c*x)*g^2-g^2)*d/c^4/(c^2
*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+
2*c*x-(c^2*x^2+1)^(1/2))*f*(32*arcsinh(c*x)*c^2*f^2+16*c^2*f^2-6*arcsinh(c*
x)*g^2-3*g^2)*d/c^3/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^
3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(36*arcsinh(
c*x)*c^2*f^2+12*c^2*f^2-3*arcsinh(c*x)*g^2-g^2)*d/c^4/(c^2*x^2+1)+1/512*(d*
(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*
x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c^2*f^2+2*
c^2*f^2+12*arcsinh(c*x)*g^2+3*g^2)*d/c^3/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))
^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*
x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(60*arcsinh(c*x)*c^2*f
^2+12*c^2*f^2+5*arcsinh(c*x)*g^2+g^2)*d/c^4/(c^2*x^2+1)+1/768*(d*(c^2*x^2+1
))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^
2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1
/2))*f*g^2*(1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2
)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2
+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^
2+1)^(1/2)+1)*g^3*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)^3\,(d + c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^{\frac{3}{2}}(gx+f)^3(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3 +
 (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c^2
*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*f^2
*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2
*d*x^2 + d), x)
```

## Sympy [F]

$$\int (f+gx)^3 \left(d + c^2 dx^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \int \left(d(c^2x^2+1)\right)^{\frac{3}{2}} (a + b\,\text{asinh}\,(cx)) (f+gx)^3\, dx$$

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d + c^2 dx^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d + c^2 dx^2\right)^{3/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\,(d + c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\textbf{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 651

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{bc^3df^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdg^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc^3dfgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{bc^3dg^2x^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{dg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} + \frac{3df^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} - \frac{dg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/16*d*g^2*x*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/4*d*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/
6*d*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/5*d*f*g*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*x*(c^2*d
*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/
```

```
15*b*c*d*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^2*x^4
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/96*b*c*d*g^2*x^4*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-1/36*b*c^3*d*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*f^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/32*d*g^2*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200, 5808, 5806, 5812}

$$
\begin{aligned}
&\int (f+gx)^2 \left(d \right. \\
&\left. + c^2 dx^2\right)^{3/2} (a + b\operatorname{arcsinh}(cx))\, dx = \frac{3}{8} df^2 x \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{4} df^2 x \left(c^2 x^2 + 1\right) \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx)) \\
&+ \frac{3df^2 \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{c^2 x^2 + 1}} \\
&+ \frac{2dfg\left(c^2 x^2 + 1\right)^2 \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx))}{5c^2} \\
&+ \frac{dg^2 x \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8} dg^2 x^3 \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{6} dg^2 x^3 \left(c^2 x^2 + 1\right) \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx)) \\
&- \frac{dg^2 \sqrt{c^2 dx^2 + d}(a + b\operatorname{arcsinh}(cx))^2}{32bc^3 \sqrt{c^2 x^2 + 1}} - \frac{5bcdf^2 x^2 \sqrt{c^2 dx^2 + d}}{16\sqrt{c^2 x^2 + 1}} \\
&- \frac{2bdfgx\sqrt{c^2 dx^2 + d}}{5c\sqrt{c^2 x^2 + 1}} - \frac{4bcdfgx^3 \sqrt{c^2 dx^2 + d}}{15\sqrt{c^2 x^2 + 1}} - \frac{bdg^2 x^2 \sqrt{c^2 dx^2 + d}}{32c\sqrt{c^2 x^2 + 1}} \\
&- \frac{7bcdg^2 x^4 \sqrt{c^2 dx^2 + d}}{96\sqrt{c^2 x^2 + 1}} - \frac{bc^3 df^2 x^4 \sqrt{c^2 dx^2 + d}}{16\sqrt{c^2 x^2 + 1}} \\
&- \frac{2bc^3 dfgx^5 \sqrt{c^2 dx^2 + d}}{25\sqrt{c^2 x^2 + 1}} - \frac{bc^3 dg^2 x^6 \sqrt{c^2 dx^2 + d}}{36\sqrt{c^2 x^2 + 1}}
\end{aligned}
$$

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^2*x
^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d + c^2*
d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (4*b*c*d*f*g*x^3*Sqrt[d + c^2*d*x^2])/(1
5*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c
^2*x^2]) - (7*b*c*d*g^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^
```

```
2*x^6*Sqrt[d + c^2*d*x^2])/(36*Sqrt[1 + c^2*x^2]) + (3*d*f^2*x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8
 + (d*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*
g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (2*d*f*
g*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*
f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
- (d*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2
*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
```

```
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + 2fgx(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx)) + g^2x\right.}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(df^2\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2dfg\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(dg^2\sqrt{d+c^2dx^2}\right)\int x^2(1+c^2x^2)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad + \frac{\left(3df^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcdf^2\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(2bdfg\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^2\,dx}{5c\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(dg^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcdg^2\sqrt{d+c^2dx^2}\right)\int x^3\left(1+c^2x^2\right)\,dx}{6\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad + \frac{\left(3df^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcdf^2\sqrt{d+c^2dx^2}\right)\int \left(x+c^2x^3\right)\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3bcdf^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(2bdfg\sqrt{d+c^2dx^2}\right)\int \left(1+2c^2x^2+c^4x^4\right)\,dx}{5c\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(dg^2\sqrt{d+c^2dx^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcdg^2\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcdg^2\sqrt{d+c^2dx^2}\right)\int \left(x^3+c^2x^5\right)\,dx}{6\sqrt{1+c^2x^2}} \\
&= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3df^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdg^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc^3dfgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dg^2x^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{dg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad + \frac{3df^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(dg^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16c^2\sqrt{1+c^2x^2}} - \frac{\left(bdg^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{16c\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} \\
&\quad - \frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{bc^3df^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} \\
&\quad - \frac{7bcdg^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc^3dfgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dg^2x^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{dg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} \\
&\quad + \frac{3df^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} - \frac{dg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.64

$$
\int (f+gx)^2\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(b(-175g^2+90c^2(85f^2+256fgx+20g^2x^2)+120c^4x^2(150f^2+128fgx
$$

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]

[Out] (-(d^2*(1 + c^2*x^2)*(b*(-175*g^2 + 90*c^2*(85*f^2 + 256*f*g*x + 20*g^2*x^2
) + 120*c^4*x^2*(150*f^2 + 128*f*g*x + 35*g^2*x^2) + 16*c^6*x^4*(225*f^2 +
288*f*g*x + 100*g^2*x^2)) - 240*a*c*Sqrt[1 + c^2*x^2]*(96*f*g*(1 + c^2*x^2)
^2 + 30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4))))
 + 1800*b*d^2*(6*c^2*f^2 - g^2)*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 3600*a*d^(3/
2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(15*(16*c^2*
f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 15*(2*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]]
 + g*(384*c*f*(1 + c^2*x^2)^(5/2) + 5*g*Sinh[6*ArcSinh[c*x]])))/(57600*c^3*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. 2(567) = 1134.

Time = 0.82 (sec) , antiderivative size = 1568, normalized size of antiderivative = 2.41

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1568 |
| parts | Expression too large to display | 1568 |

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln
(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^2*(1/6*x*(c^2
*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*
d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^
(1/2))))+2/5*f*g/c^2/d*(c^2*d*x^2+d)^(5/2))+b*(1/32*(d*(c^2*x^2+1))^(1/2)*a
rcsinh(c*x)^2*(6*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/2304*(d*(c^2*x^2+1)
)^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2
*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/
2))*g^2*(-1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/400*(d*(c^2*x^2+1))^(1/2)*(
16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(
1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+5*arcsinh(c*x))*d/c^2/(c
^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2
)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(8*arcsin
h(c*x)*c^2*f^2-2*c^2*f^2+4*arcsinh(c*x)*g^2-g^2)*d/c^3/(c^2*x^2+1)+1/48*(d*
(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(
c^2*x^2+1)^(1/2)+1)*f*g*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2
*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/
2))*(32*arcsinh(c*x)*c^2*f^2-16*c^2*f^2-2*arcsinh(c*x)*g^2+g^2)*d/c^3/(c^2*
x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+
arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2
*x^2+1)^(1/2)+1)*f*g*(arcsinh(c*x)+1)*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(3
2*arcsinh(c*x)*c^2*f^2+16*c^2*f^2-2*arcsinh(c*x)*g^2-g^2)*d/c^3/(c^2*x^2+1)
+1/48*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2-3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(3*arcsinh(c*x)+1)*d/c^2/(c^2*x^2+1)+1/512
*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*
c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(8*arcsinh(c*x)*c^2*f^2+
2*c^2*f^2+4*arcsinh(c*x)*g^2+g^2)*d/c^3/(c^2*x^2+1)+1/400*(d*(c^2*x^2+1))^(
1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^
2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(1+5*arcsinh(c*x))*d/c
^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2
+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^
2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+6*arcsinh(c*x))*d/c^3/(c^2*x
```

```
^2+1))
```

## Fricas [F]

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\, dx = \int \left(c^2 dx^2+d\right)^{\frac{3}{2}} (gx+f)^2 (b\,\text{arsinh}\,(cx)+a)\, dx$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a*c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f*g*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\, dx = \int \left(d(c^2x^2+1)\right)^{\frac{3}{2}} (a+b\,\text{asinh}\,(cx)) (f+gx)^2\, dx$$

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d + c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^2 \left(a+b\,\text{asinh}(c\,x)\right) \left(d\,c^2\,x^2+d\right)^{3/2} dx$$

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\mathbf{arcsinh}(cx))\,dx$

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## Optimal result

Integrand size = 28, antiderivative size = 353

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{bc^3dfx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^3dgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + \frac{3}{8}dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{dg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} + \frac{3df\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(c^2*x^2+1)*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/5*d*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-5/16*b*c*d*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/15*b*c*d*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
+3/16*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{3}{8}dfx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))+\frac{3df\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{c^2x^2+1}}$$
$$+\frac{dg\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{5c^2}-\frac{5bcdfx^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$
$$-\frac{bdgx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}-\frac{2bcdgx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bc^3dfx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{bc^3dgx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}$$

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*d*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f*x^2*S
qrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (2*b*c*d*g*x^3*Sqrt[d + c^2*d*
x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt
[1 + c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2])
 + (3*d*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f*x*(1 + c^2*x
^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*g*(1 + c^2*x^2)^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*f*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f(1+c^2x^2)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)+gx(1+c^2x^2)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)\right)dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(df\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(dg\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}} \\
&= \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right) \\
&\quad + \frac{dg(1+c^2x^2)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5c^2} \\
&\quad + \frac{\left(3df\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\operatorname{arcsinh}(cx)\right)dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcdf\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\,dx}{4\sqrt{1+c^2x^2}} - \frac{\left(bdg\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^2\,dx}{5c\sqrt{1+c^2x^2}} \\
&= \frac{3}{8}dfx\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right) + \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right) \\
&\quad + \frac{dg(1+c^2x^2)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5c^2} \\
&\quad + \frac{\left(3df\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bcdf\sqrt{d+c^2dx^2}\right)\int \left(x+c^2x^3\right)\,dx}{4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(3bcdf\sqrt{d+c^2dx^2}\right)\int x\,dx}{8\sqrt{1+c^2x^2}} - \frac{\left(bdg\sqrt{d+c^2dx^2}\right)\int \left(1+2c^2x^2+c^4x^4\right)\,dx}{5c\sqrt{1+c^2x^2}} \\
&= -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dfx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^3dgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} + \frac{3}{8}dfx\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right) \\
&\quad + \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right) \\
&\quad + \frac{dg(1+c^2x^2)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5c^2} \\
&\quad + \frac{3df\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{16bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.82

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-240a\sqrt{1+c^2x^2}\left(8g(1+c^2x^2)^2+5c^2fx(5+2c^2x^2)\right)+bc(128gx(15+$$

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]

[Out] (-(d^2*(1 + c^2*x^2)*(-240*a*Sqrt[1 + c^2*x^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^2
*f*x*(5 + 2*c^2*x^2)) + b*c*(128*g*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 75*f*(
17 + 40*c^2*x^2 + 8*c^4*x^4)))) + 1800*b*c*d^2*f*(1 + c^2*x^2)*ArcSinh[c*x]
^2 + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(32*g*(1
+ c^2*x^2)^(5/2) + 40*c*f*Sinh[2*ArcSinh[c*x]] + 5*c*f*Sinh[4*ArcSinh[c*x]]
))/(9600*c^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(305) = 610$.

Time = 0.83 (sec) , antiderivative size = 1065, normalized size of antiderivative = 3.02

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1065 |
| parts | Expression too large to display | 1065 |

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*l
n(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/5*a*g/c^2/d*(c
^2*d*x^2+d)^(5/2)+b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsi
nh(c*x)^2*d+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^
(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^
(1/2)+1)*g*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2
)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(
1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/96*(d
*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*
(c^2*x^2+1)^(1/2)+1)*g*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2)
)*f*(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2
+c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)*d/c^2/
```

```
(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/
2)+2*c*x-(c^2*x^2+1)^(1/2))*f*(1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/96*(d*(c
^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^
2*x^2+1)^(1/2)+1)*g*(3*arcsinh(c*x)+1)*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+
1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*
x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+
1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4
*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(
1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
+ b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int (f+gx)\left(d + c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))\,(f+gx)\,dx$$

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\,(d + c^2dx^2\big)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}dx$$

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

# 3.42 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\mathbf{arcsinh}(cx))}{f+gx}\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 984

$$\begin{aligned}
\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\operatorname{arcsinh}(cx)\right)}{f+gx}\,dx &= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} \\
&- \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&- \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} \\
&- \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&+ \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&- \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&- \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&- \frac{d(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)\sqrt{1+c^2x^2}} \\
&+ \frac{d(c^2f^2+g^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^2(f+gx)} \\
&- \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&+ \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&- \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&+ \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&- \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] a*d*(c^2*f^2+g^2)*(c^2*d*x^2+d)^(1/2)/g^3+b*d*(c^2*f^2+g^2)*arcsinh(c*x)*(c
^2*d*x^2+d)^(1/2)/g^3-1/2*c^2*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
g^2+1/3*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g-1/3*b*c*d*x*
(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-b*c*d*(c^2*f^2+g^2)*x*(c^2*d*x^2+d)
^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
```

```
*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/4*c
*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)-1/2*c
*d*(c^2*f^2+g^2)*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^3/(c^2*x^2+
1)^(1/2)-1/2*d*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c
/g^4/(g*x+f)/(c^2*x^2+1)^(1/2)-a*d*(c^2*f^2+g^2)^(3/2)*arctanh((-c^2*f*x+g)
/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)
^(1/2)+b*d*(c^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c
^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2
+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)^(
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d
*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c^2*f^2+g^2)^(3/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c
^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2+g^2)*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*
(c^2*d*x^2+d)^(1/2)/b/c/g^2/(g*x+f)
```

## Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 984, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5845, 5840, 5785, 5783, 30, 5798, 5839, 697, 5835, 6874, 267, 739, 212, 5856, 1668,

12, 5855, 8, 5843, 3403, 2296, 2221, 2317, 2438}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = -\frac{bdx^3\sqrt{c^2dx^2+d}c^3}{9g\sqrt{c^2x^2+1}}$$
$$+\frac{bdfx^2\sqrt{c^2dx^2+d}c^3}{4g^2\sqrt{c^2x^2+1}} - \frac{dfx\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))c^2}{2g^2}$$
$$-\frac{d(c^2f^2+g^2)\,x\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2c}{2bg^3\sqrt{c^2x^2+1}}$$
$$-\frac{df\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2c}{4bg^2\sqrt{c^2x^2+1}} - \frac{bd(c^2f^2+g^2)\,x\sqrt{c^2dx^2+d}c}{g^3\sqrt{c^2x^2+1}}$$
$$-\frac{bdx\sqrt{c^2dx^2+d}c}{3g\sqrt{c^2x^2+1}} + \frac{bd(c^2f^2+g^2)\sqrt{c^2dx^2+d}\text{arcsinh}(cx)}{g^3}$$
$$+\frac{d(c^2x^2+1)\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{3g}$$
$$-\frac{ad(c^2f^2+g^2)^{3/2}\sqrt{c^2dx^2+d}\text{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{c^2x^2+1}}\right)}{g^4\sqrt{c^2x^2+1}}$$
$$+\frac{bd(c^2f^2+g^2)^{3/2}\sqrt{c^2dx^2+d}\text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^4\sqrt{c^2x^2+1}}$$
$$-\frac{bd(c^2f^2+g^2)^{3/2}\sqrt{c^2dx^2+d}\text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)}{g^4\sqrt{c^2x^2+1}}$$
$$+\frac{bd(c^2f^2+g^2)^{3/2}\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{c^2x^2+1}}$$
$$-\frac{bd(c^2f^2+g^2)^{3/2}\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{c^2x^2+1}}$$
$$+\frac{ad(c^2f^2+g^2)\sqrt{c^2dx^2+d}}{g^3}$$
$$+\frac{d(c^2f^2+g^2)\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2bg^2(f+gx)c}$$
$$-\frac{d(c^2f^2+g^2)^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{2bg^4(f+gx)\sqrt{c^2x^2+1}c}$$

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])/g^3 - (b*c*d*x*Sqrt[d + c^2*d*x^2
])/(3*g*Sqrt[1 + c^2*x^2]) - (b*c*d*(c^2*f^2 + g^2)*x*Sqrt[d + c^2*d*x^2])/
(g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d + c^2*d*x^2])/(4*g^2*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d + c^2*d*x^2])/(9*g*Sqrt[1 + c^2*x^2]) +
(b*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g^3 - (c^2*d*f*x*Sqr
```

```
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + (d*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) - (c*d*f*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2)/(4*b*g^2*Sqrt[1 + c^2*x^2]) - (c*d*(c^2*f^2 + g^2)*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^3*Sqrt[1 + c^2*x^2]) - (d*(c^
2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f +
g*x)*Sqrt[1 + c^2*x^2]) + (d*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*d*(c^2*f^2 + g^2
)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt
[1 + c^2*x^2])])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[
d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/(g^4*Sqrt[1 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1
 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]*PolyLog[2, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
```

```
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5840

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(-\frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{g^2}+\frac{c^2x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{g}+\frac{\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}(a+b\operatorname{ar}}{g^2(f+gx)}\right.}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(c^2df\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(c^2d\sqrt{d+c^2dx^2}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g\sqrt{1+c^2x^2}} \\
&= -\frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2}+\frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{\left(-g+2c^2fx+c^2gx^2\right)(a+b\operatorname{arcsinh}(cx))^2}{(f+gx)^2}\,dx}{2bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(c^2df\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bc^3df\sqrt{d+c^2dx^2}\right)\int x\,dx}{2g^2\sqrt{1+c^2x^2}} - \frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)\,dx}{3g\sqrt{1+c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{\left(\frac{c^2x}{g}+\frac{1+\frac{c^2f^2}{g^2}}{f+gx}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} \\
&= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \left(\frac{a(c^2f^2+g^2+c^2fgx+c^2g^2x^2)}{g^2(f+gx)\sqrt{1+c^2x^2}} + \frac{b(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\operatorname{arcsinh}(cx)}{g^2(f+gx)\sqrt{1+c^2x^2}}\right)dx}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(ad\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{c^2f^2+g^2+c^2fgx+c^2g^2x^2}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(ad\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{c^2g^2(c^2f^2+g^2)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{c^2g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \left(\frac{c^2gx\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \frac{(c^2f^2+g^2)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\right)\,dx}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad + \frac{\left(bc^2d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{x\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{g\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(ad\left(1+\frac{c^2f^2}{g^2}\right)(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\int \frac{1}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} \\
&\quad - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{\left(bcd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int 1\,dx}{g\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(ad\left(1+\frac{c^2f^2}{g^2}\right)(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{1}{c^2f^2+g^2-x^2}\,dx, x, \frac{g-c^2fx}{\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bd\left(1+\frac{c^2f^2}{g^2}\right)(c^2f^2+g^2)\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2ce^xf-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} \\
&- \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} \\
&+ \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&+ \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&- \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&- \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&+ \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&- \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&+ \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&- \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&- \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}} \\
&+ \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} \\
&\quad + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} \\
&\quad - \frac{d\left(1+\frac{c^2f^2}{g^2}\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.24 (sec) , antiderivative size = 2869, normalized size of antiderivative = 2.92

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]

[Out] (a*d*Sqrt[d + c^2*d*x^2]*(8*g^2 + c^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)))/(6*g^
3) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*c*d^(3/2)*f*(2
```

```
*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*g^4) - (a*d^
(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^
2]*Sqrt[d + c^2*d*x^2]])/g^4 + (b*d*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1
+ c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2
*((-I)*c*f + g)*(I*c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2
])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcT
anh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]
+ (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c
*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)
*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g
^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I
)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I
*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c
*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[(
(1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c
*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi
+ (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c
*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/
(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))
] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-
(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + S
qrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, (
(I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[
(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot
[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g
^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4
]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/
4]))]))/Sqrt[-(c^2*f^2) - g^2]))/Sqrt[1 + c^2*x^2]))/(2*g^2) + (b*d*Sqrt[d
+ c^2*d*x^2]*((-9*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])]) +
PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - PolyLog[2,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/Sqrt[c^2*f^2 + g^2] +
(-18*c*g*(4*c^2*f^2 + g^2)*x + 18*g*(4*c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x] - 18*c*f*(2*c^2*f^2 + g^2)*ArcSinh[c*x]^2 + 9*c*f*g^2*Cosh[2*ArcS
inh[c*x]] + 6*g^3*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 9*(8*c^4*f^4 + 8*c^2*
f^2*g^2 + g^4)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f
^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I
*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)
*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[
-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot
[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*
f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1
/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(
```

```
f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan
[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E
^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])]
 - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt
[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g
 + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[(
(-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g
^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2
) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt
[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*A
rcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*
ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f)
+ I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f
 + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))/Sqrt[
-(c^2*f^2) - g^2]) - 18*c*f*g^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 2*g^3*S
inh[3*ArcSinh[c*x]])/g^4))/(72*Sqrt[1 + c^2*x^2])
```

## Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 1557, normalized size of antiderivative = 1.58

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1557 |
| parts | Expression too large to display | 1557 |

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] a/g*(1/3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)-c^
2*d*f/g*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2*d-2*c^2*d*f
/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2*f^2+g^2)/g^2-4*c^
4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2
*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2))+d*(c^
2*f^2+g^2)/g^2*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(
1/2)-c^2*d*f/g*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d
-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g
^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(
x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d
*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))))+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f
^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c
^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)/(
```

```
c^2*x^2+1)^(1/2)*f^3*arcsinh(c*x)^2*c^3*d/g^4-3/4*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c*d/g^2+1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^
2*x^2+1)/g*arcsinh(c*x)*x^4*c^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(
1/2)/g*x^3*c^3+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)*x^2
*c^2-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g*c*x+1/8*b*(d*(c^2*x^
2+1))^(1/2)*f*c*d/(c^2*x^2+1)^(1/2)/g^2+b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+
1)/g^3*arcsinh(c*x)*c^2*f^2+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arc
sinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g^3*x*c^3*f^2+b*(c^2*
f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*l
n((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^
(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4
*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(
c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^4*d/(c^2*x^2+1)/g^2*ar
csinh(c*x)*x^3+1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c^3*d/(c^2*x^2+1)^(1/2)/g^2*x^
2-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2*d/(c^2*x^2+1)/g^2*arcsinh(c*x)*x+b*(d*(
c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^2*c^4*f^2
```

## Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^{3/2}}{f+g\,x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x), x)
```

# 3.43 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\mathbf{arcsinh}(cx))\, dx$

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Maple [B] (verified) . . . . . 432
Fricas [F] . . . . . 434
Sympy [F(-1)] . . . . . 435
Maxima [F(-2)] . . . . . 435
Giac [F(-2)] . . . . . 435
Mupad [F(-1)] . . . . . 436

## Optimal result

Integrand size = 30, antiderivative size = 1228

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}$$
$$+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{25bcd^2f^3x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{15bd^2fg^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}$$
$$-\frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{bd^2g^3x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} - \frac{5bc^3d^2f^3x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}$$
$$-\frac{59bcd^2fg^2x^4\sqrt{d+c^2dx^2}}{256\sqrt{1+c^2x^2}} - \frac{9bc^3d^2f^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{bcd^2g^3x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}$$
$$-\frac{17bc^3d^2fg^2x^6\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{3bc^5d^2f^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{19bc^3d^2g^3x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}}$$
$$-\frac{3bc^5d^2fg^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} - \frac{bc^5d^2g^3x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}} - \frac{bd^2f^3(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}$$
$$+\frac{5}{16}d^2f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{15d^2fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2}$$
$$+\frac{15}{64}d^2fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{3d^2f^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2}$$
$$-\frac{d^2g^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4}$$
$$+\frac{d^2g^3(1+c^2x^2)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4}$$
$$+\frac{5d^2f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} - \frac{15d^2fg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}$$

```
[Out] -1/36*b*d^2*f^3*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+15/64*d^2*f*g^2*x^3
*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f^3*x*(c^2*x^2+1)*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)-1/7*d^2*g^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*
x^2+d)^(1/2)/c^4+1/9*d^2*g^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)/c^4+5/16*d^2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-3/7*b*d^2*
f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-15/256*b*d^2*f*g^2*x^2*(c^2
*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/7*b*c*d^2*f^2*g*x^3*(c^2*d*x^2+d)^(1/
```

```
2)/(c^2*x^2+1)^(1/2)-59/256*b*c*d^2*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+
1)^(1/2)-9/35*b*c^3*d^2*f^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-17/
96*b*c^3*d^2*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/49*b*c^5*d^2
*f^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/64*b*c^5*d^2*f*g^2*x^8*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-15/256*d^2*f*g^2*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)+15/128*d^2*f*g^2*x*(a+b*arcsinh
(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)+3/7*d^2*f^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)/c^2+2/63*b*d^2*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2
)-25/96*b*c*d^2*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/189*b*d^2*g
^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^3*x^4*(c^2*
d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/21*b*c*d^2*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-1/81*b*c^5*d^2*g^3*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*
f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 1228, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200, 5808, 5806, 5812, 272, 45,

5804, 12, 380}

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx =$$
$$-\frac{bc^5d^2g^3\sqrt{c^2dx^2+d}x^9}{81\sqrt{c^2x^2+1}} - \frac{3bc^5d^2fg^2\sqrt{c^2dx^2+d}x^8}{64\sqrt{c^2x^2+1}}$$
$$-\frac{19bc^3d^2g^3\sqrt{c^2dx^2+d}x^7}{441\sqrt{c^2x^2+1}} - \frac{3bc^5d^2f^2g\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}$$
$$-\frac{17bc^3d^2fg^2\sqrt{c^2dx^2+d}x^6}{96\sqrt{c^2x^2+1}} - \frac{bcd^2g^3\sqrt{c^2dx^2+d}x^5}{21\sqrt{c^2x^2+1}}$$
$$-\frac{9bc^3d^2f^2g\sqrt{c^2dx^2+d}x^5}{35\sqrt{c^2x^2+1}} - \frac{5bc^3d^2f^3\sqrt{c^2dx^2+d}x^4}{96\sqrt{c^2x^2+1}}$$
$$-\frac{59bcd^2fg^2\sqrt{c^2dx^2+d}x^4}{256\sqrt{c^2x^2+1}} + \frac{15}{64}d^2fg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x^3$$
$$+\frac{3}{8}d^2fg^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x^3$$
$$+\frac{5}{16}d^2fg^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x^3$$
$$-\frac{bd^2g^3\sqrt{c^2dx^2+d}x^3}{189c\sqrt{c^2x^2+1}} - \frac{3bcd^2f^2g\sqrt{c^2dx^2+d}x^3}{7\sqrt{c^2x^2+1}}$$
$$-\frac{25bcd^2f^3\sqrt{c^2dx^2+d}x^2}{96\sqrt{c^2x^2+1}} - \frac{15bd^2fg^2\sqrt{c^2dx^2+d}x^2}{256c\sqrt{c^2x^2+1}}$$
$$+\frac{5}{16}d^2f^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x$$
$$+\frac{15d^2fg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x}{128c^2}$$
$$+\frac{1}{6}d^2f^3\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x$$
$$+\frac{5}{24}d^2f^3\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))x + \frac{2bd^2g^3\sqrt{c^2dx^2+d}x}{63c^3\sqrt{c^2x^2+1}}$$
$$-\frac{3bd^2f^2g\sqrt{c^2dx^2+d}x}{7c\sqrt{c^2x^2+1}} + \frac{5d^2f^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{32bc\sqrt{c^2x^2+1}}$$
$$-\frac{15d^2fg^2\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))^2}{256bc^3\sqrt{c^2x^2+1}}$$
$$+\frac{d^2g^3(c^2x^2+1)^4\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{9c^4}$$
$$-\frac{d^2g^3(c^2x^2+1)^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{7c^4}$$
$$+\frac{3d^2f^2g(c^2x^2+1)^3\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{7c^2}$$
$$-\frac{bd^2f^3(c^2x^2+1)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g
^3*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f^3*x^2*
Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d +
c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d + c^2*d
*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(189*c*S
qrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 +
c^2*x^2]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(256*Sqrt[1 + c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^2*x^2]) -
(b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x
^2])/(81*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (15*d^2*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(128*c^2) + (15*
d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*d^2*f^3*x*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (5*d^2*f*g^2*x^
3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (d^2*f^3*x*(
1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (3*d^2*f*g^2*x
^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*d^2*f^2
*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) - (d^2
*g^3*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (d
^2*g^3*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*c^4) +
(5*d^2*f^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2
*x^2]) - (15*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c
^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^3\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + 3f^2gx(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + 3\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2f^3\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(3d^2f^2g\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(3d^2fg^2\sqrt{d+c^2dx^2}\right)\int x^2(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(d^2g^3\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= \frac{1}{6} d^2 f^3 x \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \\
&\quad + \frac{3}{8} d^2 f g^2 x^3 \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx)) \\
&\quad + \frac{3 d^2 f^2 g (1 + c^2 x^2)^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{7 c^2} \\
&\quad - \frac{d^2 g^3 (1 + c^2 x^2)^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{7 c^4} \\
&\quad + \frac{d^2 g^3 (1 + c^2 x^2)^4 \sqrt{d + c^2 d x^2} (a + b \operatorname{arcsinh}(cx))}{9 c^4} \\
&\quad + \frac{\left(5 d^2 f^3 \sqrt{d + c^2 d x^2}\right) \int (1 + c^2 x^2)^{3/2} (a + b \operatorname{arcsinh}(cx)) \, dx}{6 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(b c d^2 f^3 \sqrt{d + c^2 d x^2}\right) \int x (1 + c^2 x^2)^2 \, dx}{6 \sqrt{1 + c^2 x^2}} - \frac{\left(3 b d^2 f^2 g \sqrt{d + c^2 d x^2}\right) \int (1 + c^2 x^2)^3 \, dx}{7 c \sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(15 d^2 f g^2 \sqrt{d + c^2 d x^2}\right) \int x^2 (1 + c^2 x^2)^{3/2} (a + b \operatorname{arcsinh}(cx)) \, dx}{8 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(3 b c d^2 f g^2 \sqrt{d + c^2 d x^2}\right) \int x^3 (1 + c^2 x^2)^2 \, dx}{8 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(b c d^2 g^3 \sqrt{d + c^2 d x^2}\right) \int \frac{(1 + c^2 x^2)^3 (-2 + 7 c^2 x^2)}{63 c^4} \, dx}{\sqrt{1 + c^2 x^2}}
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{bd^2f^3(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{3d^2f^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2}\\
&-\frac{d^2g^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4}\\
&+\frac{d^2g^3(1+c^2x^2)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4}\\
&+\frac{\left(5d^2f^3\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{8\sqrt{1+c^2x^2}}\\
&-\frac{\left(5bcd^2f^3\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\,dx}{24\sqrt{1+c^2x^2}}\\
&-\frac{\left(3bd^2f^2g\sqrt{d+c^2dx^2}\right)\int(1+3c^2x^2+3c^4x^4+c^6x^6)\,dx}{7c\sqrt{1+c^2x^2}}\\
&+\frac{\left(15d^2fg^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{16\sqrt{1+c^2x^2}}\\
&-\frac{\left(3bcd^2fg^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x(1+c^2x)^2\,dx,x,x^2\right)}{16\sqrt{1+c^2x^2}}\\
&-\frac{\left(5bcd^2fg^2\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)\,dx}{16\sqrt{1+c^2x^2}}\\
&-\frac{\left(bd^2g^3\sqrt{d+c^2dx^2}\right)\int(1+c^2x^2)^3(-2+7c^2x^2)\,dx}{63c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{9bc^3d^2f^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bc^5d^2f^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{bd^2f^3(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{15}{64}d^2fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\qquad + \frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\qquad + \frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\qquad + \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\qquad + \frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\qquad + \frac{3d^2f^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\qquad - \frac{d^2g^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&\qquad + \frac{d^2g^3(1+c^2x^2)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4} \\
&\quad + \frac{\left(5d^2f^3\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2f^3\sqrt{d+c^2dx^2}\right)\int \left(x+c^2x^3\right)\,dx}{24\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2f^3\sqrt{d+c^2dx^2}\right)\int x\,dx}{16\sqrt{1+c^2x^2}} + \frac{\left(15d^2fg^2\sqrt{d+c^2dx^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}} \\
&\qquad - \frac{\left(3bcd^2fg^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \left(x+2c^2x^2+c^4x^3\right)\,dx, x, x^2\right)}{16\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(15bcd^2fg^2\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{64\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2fg^2\sqrt{d+c^2dx^2}\right)\int \left(x^3+c^2x^5\right)\,dx}{16\sqrt{1+c^2x^2}} \\
&\qquad - \frac{\left(bd^2g^3\sqrt{d+c^2dx^2}\right)\int \left(-2+c^2x^2+15c^4x^4+19c^6x^6+7c^8x^8\right)\,dx}{63c^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} + \frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{25bcd^2f^3x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} \\
&\quad - \frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{bd^2g^3x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} - \frac{5bc^3d^2f^3x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} \\
&\quad - \frac{59bcd^2fg^2x^4\sqrt{d+c^2dx^2}}{256\sqrt{1+c^2x^2}} - \frac{9bc^3d^2f^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{bcd^2g^3x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}} \\
&\quad - \frac{17bc^3d^2fg^2x^6\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{3bc^5d^2f^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{19bc^3d^2g^3x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}} - \frac{3bc^5d^2fg^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2g^3x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}} - \frac{bd^2f^3\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{15d^2fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2} \\
&\quad + \frac{15}{64}d^2fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3d^2f^2g\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad - \frac{d^2g^3\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&\quad + \frac{d^2g^3\left(1+c^2x^2\right)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4} \\
&\quad + \frac{5d^2f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(15d^2fg^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{128c^2\sqrt{1+c^2x^2}} - \frac{\left(15bd^2fg^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{128c\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} + \frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{25bcd^2f^3x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} \\
&\quad - \frac{15bd^2fg^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}} - \frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{bd^2g^3x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} \\
&\quad - \frac{5bc^3d^2f^3x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{59bcd^2fg^2x^4\sqrt{d+c^2dx^2}}{256\sqrt{1+c^2x^2}} - \frac{9bc^3d^2f^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2g^3x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}} - \frac{17bc^3d^2fg^2x^6\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{3bc^5d^2f^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{19bc^3d^2g^3x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}} - \frac{3bc^5d^2fg^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2g^3x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}} - \frac{bd^2f^3(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{15d^2fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2} \\
&\quad + \frac{15}{64}d^2fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{3d^2f^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad - \frac{d^2g^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} \\
&\quad + \frac{d^2g^3(1+c^2x^2)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4} \\
&\quad + \frac{5d^2f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} - \frac{15d^2fg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 810, normalized size of antiderivative = 0.66

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \frac{-d^3(1+c^2x^2)\left(-20160a\sqrt{1+c^2x^2}(-256g^3+c^2g(3456f^2+945fgx+128g^2x^2)+16c^8x$$

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(d^3*(1 + c^2*x^2)*(-20160*a*Sqrt[1 + c^2*x^2]*(-256*g^3 + c^2*g*(3456*f^
2 + 945*f*g*x + 128*g^2*x^2) + 16*c^8*x^5*(84*f^3 + 216*f^2*g*x + 189*f*g^2
*x^2 + 56*g^3*x^3) + 8*c^6*x^3*(546*f^3 + 1296*f^2*g*x + 1071*f*g^2*x^2 + 3
04*g^3*x^3) + 6*c^4*x*(924*f^3 + 1728*f^2*g*x + 1239*f*g^2*x^2 + 320*g^3*x^
3)) + b*(-315*c*g^2*(7539*f + 16384*g*x) + 30240*c^5*x^2*(1848*f^3 + 2304*f
^2*g*x + 1239*f*g^2*x^2 + 256*g^3*x^3) + 3360*c^3*(6279*f^3 + 20736*f^2*g*x
 + 2835*f*g^2*x^2 + 256*g^3*x^3) + 2304*c^7*x^4*(9555*f^3 + 18144*f^2*g*x +
 12495*f*g^2*x^2 + 3040*g^3*x^3) + 640*c^9*x^6*(7056*f^3 + 15552*f^2*g*x +
11907*f*g^2*x^2 + 3136*g^3*x^3)))) + 3175200*b*c*d^3*f*(8*c^2*f^2 - 3*g^2)*
(1 + c^2*x^2)*ArcSinh[c*x]^2 + 6350400*a*c*d^(5/2)*f*(8*c^2*f^2 - 3*g^2)*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + 2520*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(27648*c^2*f^2*g*Sqrt[1 + c^2*x^2
] - 2048*g^3*Sqrt[1 + c^2*x^2] + 82944*c^4*f^2*g*x^2*Sqrt[1 + c^2*x^2] + 10
24*c^2*g^3*x^2*Sqrt[1 + c^2*x^2] + 82944*c^6*f^2*g*x^4*Sqrt[1 + c^2*x^2] +
15360*c^4*g^3*x^4*Sqrt[1 + c^2*x^2] + 27648*c^8*f^2*g*x^6*Sqrt[1 + c^2*x^2]
 + 19456*c^6*g^3*x^6*Sqrt[1 + c^2*x^2] + 7168*c^8*g^3*x^8*Sqrt[1 + c^2*x^2]
 + 3024*c*f*(5*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 1512*c*f*(2*c^2*f^2 +
g^2)*Sinh[4*ArcSinh[c*x]] + 336*c^3*f^3*Sinh[6*ArcSinh[c*x]] + 1008*c*f*g^2
*Sinh[6*ArcSinh[c*x]] + 189*c*f*g^2*Sinh[8*ArcSinh[c*x]]))/(162570240*c^4*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2883 vs. 2(1080) = 2160.

Time = 1.05 (sec) , antiderivative size = 2884, normalized size of antiderivative = 2.35

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2884 |
| parts | Expression too large to display | 2884 |

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^3*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1
/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2)
)/(c^2*d)^(1/2))))+g^3*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d
*x^2+d)^(7/2))+3*f*g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*(c^2
*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)
^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2))))
)+3/7*f^2*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*f*arc
sinh(c*x)^2*(8*c^2*f^2-3*g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/41472*(d*(c^2*x^2
+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8+576*c^7
*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^
4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(
-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^(1/2)*(128*c
^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+256*c^6*x^6*(c^2*x^2+1)^(1
/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^3+32*c^2*x^2*(c^2*x^
2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*f*g^2*(-1+8*arcsinh(c*x))*d^2/c^3/(c^2*
x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/
2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^
2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*g*(28*arcsinh(c*x)*c^2*f^2
-4*c^2*f^2+7*arcsinh(c*x)*g^2-g^2)*d^2/c^4/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1
))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^
2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1
/2))*f*(6*arcsinh(c*x)*c^2*f^2-c^2*f^2+18*arcsinh(c*x)*g^2-3*g^2)*d^2/c^3/(
c^2*x^2+1)+3/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(
1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(
1/2)+1)*f^2*g*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1)
)^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^
2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c^2*f^2-2*c^2*f^2+4*a
rcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^
4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(8
1*arcsinh(c*x)*c^2*f^2-27*c^2*f^2-6*arcsinh(c*x)*g^2+2*g^2)*d^2/c^4/(c^2*x^
2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c
*x+(c^2*x^2+1)^(1/2))*f*(10*arcsinh(c*x)*c^2*f^2-5*c^2*f^2-2*arcsinh(c*x)*g
^2+g^2)*d^2/c^3/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x
^2+1)^(1/2)+1)*g*(10*arcsinh(c*x)*c^2*f^2-10*c^2*f^2-arcsinh(c*x)*g^2+g^2)*
d^2/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1
/2)+1)*g*(10*arcsinh(c*x)*c^2*f^2+10*c^2*f^2-arcsinh(c*x)*g^2-g^2)*d^2/c^4/
(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1
/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*(10*arcsinh(c*x)*c^2*f^2+5*c^2*f^2-2*arcsinh
(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-
4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(81*arcs
inh(c*x)*c^2*f^2+27*c^2*f^2-6*arcsinh(c*x)*g^2-2*g^2)*d^2/c^4/(c^2*x^2+1)+3
/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x
^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c
^2*f^2+2*c^2*f^2+4*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+3/640*(d*(c^2*
x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^
```

```
3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*f^2*g*(1+5*arcsin
h(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6
*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-1
8*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*f*(6*arcsinh(c*x)*c^2*
f^2+c^2*f^2+18*arcsinh(c*x)*g^2+3*g^2)*d^2/c^3/(c^2*x^2+1)+3/25088*(d*(c^2*
x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*
x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7
*c*x*(c^2*x^2+1)^(1/2)+1)*g*(28*arcsinh(c*x)*c^2*f^2+4*c^2*f^2+7*arcsinh(c*
x)*g^2+g^2)*d^2/c^4/(c^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-
128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272
*c^5*x^5-160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1
/2)+8*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1
/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c^9*x^9*(c^2*x^2+1)^(1/2)+7
04*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6-432*c^5*x^5*(c^2*x^2+1
)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(c^2*x^2
+1)^(1/2)+1)*g^3*(1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)^3\,(d + c^2dx^2)^{5/2}\,(a+b\mathrm{arcsinh}(cx))\,dx = \int (c^2dx^2+d)^{\frac{5}{2}}(gx+f)^3(b\,\mathrm{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a*c
^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f^3
+ 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3 + (2*
b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\,(d + c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.44 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\mathbf{arcsinh}(cx))\, dx$

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## Optimal result

Integrand size = 30, antiderivative size = 901

$$\begin{aligned}
\int (f+gx)^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx &= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} \\
&- \frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{5bd^2g^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}} - \frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&- \frac{5bc^3d^2f^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{59bcd^2g^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}} \\
&- \frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{17bc^3d^2g^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}} \\
&- \frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{bc^5d^2g^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} \\
&- \frac{bd^2f^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{5d^2g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2} \\
&+ \frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{2d^2fg(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&+ \frac{5d^2f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} \\
&- \frac{5d^2g^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] -1/36*b*d^2*f^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/128*d^2*g^2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+
5/24*d^2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/48*d^2*
g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(c^2*x^2
+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/7*d^2*f*g*(c^2*x^2+1)^3*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(c^2*d*x^2+d)^(1/2)/
```

```
c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-5/256*b*d^2*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^
2*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^2*x^4*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*g^2*x^4*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-17/288*b*c^3*d^2*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/49*b
*c^5*d^2*f*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*g^2*x
^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^2*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-5/256*d^2*g^2*(a+b*arcsinh(c*x))^
2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200, 5808, 5806, 5812, 272, 45}

$$\int (f+gx)^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{bc^5d^2g^2\sqrt{c^2dx^2+d}x^8}{64\sqrt{c^2x^2+1}}$$
$$-\frac{2bc^5d^2fg\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}} - \frac{17bc^3d^2g^2\sqrt{c^2dx^2+d}x^6}{288\sqrt{c^2x^2+1}}$$
$$-\frac{6bc^3d^2fg\sqrt{c^2dx^2+d}x^5}{35\sqrt{c^2x^2+1}} - \frac{5bc^3d^2f^2\sqrt{c^2dx^2+d}x^4}{96\sqrt{c^2x^2+1}}$$
$$-\frac{59bcd^2g^2\sqrt{c^2dx^2+d}x^4}{768\sqrt{c^2x^2+1}} + \frac{5}{64}d^2g^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x^3$$
$$+\frac{1}{8}d^2g^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x^3$$
$$+\frac{5}{48}d^2g^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x^3$$
$$-\frac{2bcd^2fg\sqrt{c^2dx^2+d}x^3}{7\sqrt{c^2x^2+1}} - \frac{25bcd^2f^2\sqrt{c^2dx^2+d}x^2}{96\sqrt{c^2x^2+1}}$$
$$-\frac{5bd^2g^2\sqrt{c^2dx^2+d}x^2}{256c\sqrt{c^2x^2+1}} + \frac{5}{16}d^2f^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x$$
$$+\frac{5d^2g^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x}{128c^2}$$
$$+\frac{1}{6}d^2f^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x$$
$$+\frac{5}{24}d^2f^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))x$$
$$-\frac{2bd^2fg\sqrt{c^2dx^2+d}x}{7c\sqrt{c^2x^2+1}} + \frac{5d^2f^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{c^2x^2+1}}$$
$$-\frac{5d^2g^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{c^2x^2+1}}$$
$$+\frac{2d^2fg\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{7c^2}$$
$$-\frac{bd^2f^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}$$

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*
f^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*f*g*x^3*Sqrt[d + c^
2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d + c^2*d*x^2])
/(96*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*g^2*x^4*Sqrt[d + c^2*d*x^2])/(768*Sqr
t[1 + c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (b*c
```

```
^5*d^2*g^2*x^8*Sqrt[d + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*d^2*f^2*(1
+ c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*d^2*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/24 + (5*d^2*g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/48 + (d^2*f^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/6 + (d^2*g^2*x^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/8 + (2*d^2*f*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*d^2*f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c
*Sqrt[1 + c^2*x^2]) - (5*d^2*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2
)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
```

```
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \Big(f^2(1+c^2x^2)^{5/2}\left(a+b\text{arcsinh}(cx)\right)+2fgx(1+c^2x^2)^{5/2}\left(a+b\text{arcsinh}(cx)\right)+g$$

$$
\begin{aligned}
&= \frac{\left(d^2 f^2 \sqrt{d + c^2 d x^2}\right) \int \left(1 + c^2 x^2\right)^{5/2} \left(a + b \operatorname{arcsinh}(c x)\right) dx}{\sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(2 d^2 f g \sqrt{d + c^2 d x^2}\right) \int x \left(1 + c^2 x^2\right)^{5/2} \left(a + b \operatorname{arcsinh}(c x)\right) dx}{\sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(d^2 g^2 \sqrt{d + c^2 d x^2}\right) \int x^2 \left(1 + c^2 x^2\right)^{5/2} \left(a + b \operatorname{arcsinh}(c x)\right) dx}{\sqrt{1 + c^2 x^2}} \\
&= \frac{1}{6} d^2 f^2 x \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} \left(a + b \operatorname{arcsinh}(c x)\right) \\
&\quad + \frac{1}{8} d^2 g^2 x^3 \left(1 + c^2 x^2\right)^2 \sqrt{d + c^2 d x^2} \left(a + b \operatorname{arcsinh}(c x)\right) \\
&\quad + \frac{2 d^2 f g \left(1 + c^2 x^2\right)^3 \sqrt{d + c^2 d x^2} \left(a + b \operatorname{arcsinh}(c x)\right)}{7 c^2} \\
&\quad + \frac{\left(5 d^2 f^2 \sqrt{d + c^2 d x^2}\right) \int \left(1 + c^2 x^2\right)^{3/2} \left(a + b \operatorname{arcsinh}(c x)\right) dx}{6 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(b c d^2 f^2 \sqrt{d + c^2 d x^2}\right) \int x \left(1 + c^2 x^2\right)^2 \, dx}{6 \sqrt{1 + c^2 x^2}} - \frac{\left(2 b d^2 f g \sqrt{d + c^2 d x^2}\right) \int \left(1 + c^2 x^2\right)^3 \, dx}{7 c \sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(5 d^2 g^2 \sqrt{d + c^2 d x^2}\right) \int x^2 \left(1 + c^2 x^2\right)^{3/2} \left(a + b \operatorname{arcsinh}(c x)\right) dx}{8 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(b c d^2 g^2 \sqrt{d + c^2 d x^2}\right) \int x^3 \left(1 + c^2 x^2\right)^2 \, dx}{8 \sqrt{1 + c^2 x^2}}
\end{aligned}
$$

$$
\begin{aligned}
= -\frac{bd^2f^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} &+ \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+ \frac{2d^2fg(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&+ \frac{\left(5d^2f^2\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{8\sqrt{1+c^2x^2}} \\
&- \frac{\left(5bcd^2f^2\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\,dx}{24\sqrt{1+c^2x^2}} \\
&- \frac{\left(2bd^2fg\sqrt{d+c^2dx^2}\right)\int(1+3c^2x^2+3c^4x^4+c^6x^6)\,dx}{7c\sqrt{1+c^2x^2}} \\
&+ \frac{\left(5d^2g^2\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{16\sqrt{1+c^2x^2}} \\
&- \frac{\left(bcd^2g^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int x(1+c^2x)^2\,dx,x,x^2\right)}{16\sqrt{1+c^2x^2}} \\
&- \frac{\left(5bcd^2g^2\sqrt{d+c^2dx^2}\right)\int x^3(1+c^2x^2)\,dx}{48\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{bd^2f^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2d^2fg(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{\left(5d^2f^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2f^2\sqrt{d+c^2dx^2}\right)\int \left(x+c^2x^3\right)\,dx}{24\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2f^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{16\sqrt{1+c^2x^2}} + \frac{\left(5d^2g^2\sqrt{d+c^2dx^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{64\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcd^2g^2\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \left(x+2c^2x^2+c^4x^3\right)\,dx, x, x^2\right)}{16\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2g^2\sqrt{d+c^2dx^2}\right)\int x^3\,dx}{64\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2g^2\sqrt{d+c^2dx^2}\right)\int \left(x^3+c^2x^5\right)\,dx}{48\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&\quad - \frac{5bc^3d^2f^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{59bcd^2g^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}} - \frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} \\
&\quad - \frac{17bc^3d^2g^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}} - \frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{bc^5d^2g^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2f^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5d^2g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2} + \frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2d^2fg(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{5d^2f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} - \frac{\left(5d^2g^2\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{128c^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bd^2g^2\sqrt{d+c^2dx^2}\right)\int x\,dx}{128c\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{5bd^2g^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}} \\
&\quad - \frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{5bc^3d^2f^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{59bcd^2g^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}} \\
&\quad - \frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{17bc^3d^2g^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}} - \frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2g^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}} - \frac{bd^2f^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{5d^2g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2} \\
&\quad + \frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{2d^2fg\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{5d^2f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}} - \frac{5d^2g^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.62

$$
\int (f+gx)^2\left(d+c^2dx^2\right)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(b(-87955g^2+1120c^2(2093f^2+4608fgx+315g^2x^2)+3360c^4x^2(1848f^2
$$

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]

[Out] (-(d^3*(1 + c^2*x^2)*(b*(-87955*g^2 + 1120*c^2*(2093*f^2 + 4608*f*g*x + 315
*g^2*x^2) + 3360*c^4*x^2*(1848*f^2 + 1536*f*g*x + 413*g^2*x^2) + 640*c^8*x^
6*(784*f^2 + 1152*f*g*x + 441*g^2*x^2) + 1792*c^6*x^4*(1365*f^2 + 1728*f*g*
x + 595*g^2*x^2)) - 6720*a*c*Sqrt[1 + c^2*x^2]*(768*f*g*(1 + c^2*x^2)^3 + 5
6*c^2*f^2*x*(33 + 26*c^2*x^2 + 8*c^4*x^4) + 7*g^2*x*(15 + 118*c^2*x^2 + 136
*c^4*x^4 + 48*c^6*x^6)))) + 352800*b*d^3*(8*c^2*f^2 - g^2)*(1 + c^2*x^2)*Ar
cSinh[c*x]^2 + 705600*a*d^(5/2)*(8*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d
```

```
+ c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 840*b*d^3*(1 + c^2*
x^2)*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c^2*x^2] + 18432*c^3*f*g*x^2*Sqrt[1
+ c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 + c^2*x^2] + 6144*c^7*f*g*x^6*Sqrt[1
+ c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 168*(6*c^2*f^2 +
 g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2*Sinh[6*ArcSinh[c*x]] + 112*g^2*Sin
h[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSinh[c*x]]))/(18063360*c^3*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. 2(791) = 1582.

Time = 1.03 (sec) , antiderivative size = 2309, normalized size of antiderivative = 2.56

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2309 |
| parts | Expression too large to display | 2309 |

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^2*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1
/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2)
)/(c^2*d)^(1/2))))+g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*(c^2
*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)
^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2))))
)+2/7*f*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*arcsinh
(c*x)^2*(8*c^2*f^2-g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/16384*(d*(c^2*x^2+1))^(
1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+256*c^6*x^6*(c^
2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^3+32*c^2*
x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+8*arcsinh(c*x))*d^2/
c^3/(c^2*x^2+1)+1/3136*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^
2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3
*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+7*arcsinh(
c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x
^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*
c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(6*arcsinh(c*x)*c^2*f^2-
c^2*f^2+6*arcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/320*(d*(c^2*x^2+1))^(
1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^
2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+5*arcsinh(c*x))*d^
2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^
2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*
(24*arcsinh(c*x)*c^2*f^2-6*c^2*f^2+4*arcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2
+1)+1/64*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2
*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)
```

```
+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(
c^2*x^2+1)^(1/2))*(30*arcsinh(c*x)*c^2*f^2-15*c^2*f^2-2*arcsinh(c*x)*g^2+g^
2)*d^2/c^3/(c^2*x^2+1)+5/64*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^
(1/2)+1)*f*g*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/64*(d*(c^2*x^2+1))^(1/
2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+
1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x
-(c^2*x^2+1)^(1/2))*(30*arcsinh(c*x)*c^2*f^2+15*c^2*f^2-2*arcsinh(c*x)*g^2-
g^2)*d^2/c^3/(c^2*x^2+1)+1/64*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c
^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(3*arcsinh(c*x)+1)
*d^2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2
*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2
))*(24*arcsinh(c*x)*c^2*f^2+6*c^2*f^2+4*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*
x^2+1)+1/320*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)
+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)
+1)*f*g*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)
*(32*c^7*x^7-32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)
^(1/2)+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(6*
arcsinh(c*x)*c^2*f^2+c^2*f^2+6*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+1/
3136*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6
*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)
+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*
x^2+1)+1/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*c^8*x^8*(c^2*x^2+1)^(
1/2)+320*c^7*x^7-256*c^6*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5-160*c^4*x^4*(c^2
*x^2+1)^(1/2)+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/
2))*g^2*(1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)^2\left(d + c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c^2
*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*d^2*
g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
+ d), x)
```

## Sympy [F]

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}} (a + b\,\mathrm{asinh}\,(cx)) \left(f+gx\right)^2\, dx$$

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d + c^2 dx^2\right)^{5/2} (a + b\operatorname{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\,(d + c^2dx^2)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\mathbf{arcsinh}(cx))\,dx$

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Rubi [A] (verified) . . . 454
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Mupad [F(-1)] . . . 461

## Optimal result

Integrand size = 28, antiderivative size = 494

$$\begin{aligned}
&\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \\
&-\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&-\frac{5bc^3d^2fx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5d^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&-\frac{bd^2f\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{d^2g\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&+\frac{5d^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}
\end{aligned}$$

```
[Out] -1/36*b*d^2*f*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/7*d^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2-1/7*b*d^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f*x^2
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*g*x^3*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-3/35*b*c^3*d^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d
^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200}

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{5}{24}d^2fx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{5d^2f\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{c^2x^2+1}}$$
$$+\frac{d^2g\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{7c^2}-\frac{25bcd^2fx^2\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}$$
$$-\frac{bd^2f\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}-\frac{bd^2gx\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}-\frac{bcd^2gx^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}}$$
$$-\frac{bc^5d^2gx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{5bc^3d^2fx^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{3bc^3d^2gx^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}$$

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*d^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f*
x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c*d^2*g*x^3*Sqrt[d + c
^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d + c^2*d*x^2])/
(96*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1
 + c^2*x^2]) - (b*c^5*d^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2])
 - (b*d^2*f*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*f*x*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (d^2*f*x*(1 + c^2*x^2)^2*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (d^2*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx)) + gx(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2f\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(d^2g\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^{5/2}(a+b\text{arcsinh}(cx))\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) + \frac{d^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{7c^2} + \frac{\left(5d^2f\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^{3/2}(a+b\text{arcsinh}(cx))\,dx}{6\sqrt{1+c^2x^2}} - \frac{\left(bcd^2f\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)^2\,dx}{6\sqrt{1+c^2x^2}} - \frac{\left(bd^2g\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^3\,dx}{7c\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{bd^2 f (1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{24}d^2 fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2 fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{d^2 g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{\left(5d^2 f\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{8\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2 f\sqrt{d+c^2dx^2}\right)\int x(1+c^2x^2)\,dx}{24\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bd^2 g\sqrt{d+c^2dx^2}\right)\int (1+3c^2x^2+3c^4x^4+c^6x^6)\,dx}{7c\sqrt{1+c^2x^2}} \\
&= -\frac{bd^2 gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2 gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3 d^2 gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5 d^2 gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{bd^2 f(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} \\
&\quad + \frac{5}{16}d^2 fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2 fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2 fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{d^2 g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{\left(5d^2 f\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{16\sqrt{1+c^2x^2}} - \frac{\left(5bcd^2 f\sqrt{d+c^2dx^2}\right)\int (x+c^2x^3)\,dx}{24\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(5bcd^2 f\sqrt{d+c^2dx^2}\right)\int x\,dx}{16\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&\quad - \frac{5bc^3d^2fx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{3bc^3d^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - \frac{bc^5d^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2f\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{16}d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&\quad + \frac{d^2g\left(1+c^2x^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&\quad + \frac{5d^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.79

$$
\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a + b\operatorname{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}\left(48g(1+c^2x^2)^3+7c^2fx(33+26c^2x^2+8c^4x^4)\right)+bc(\right.
$$

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(d^3*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(48*g*(1 + c^2*x^2)^3 + 7*c
^2*f*x*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + b*c*(2304*g*x*(35 + 35*c^2*x^2 + 21
*c^4*x^4 + 5*c^6*x^6) + 245*f*(299 + 792*c^2*x^2 + 312*c^4*x^4 + 64*c^6*x^6
)))) + 88200*b*c*d^3*f*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 176400*a*c*d^(5/2)*f*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] + 420*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(192*g*Sqrt[1 + c^2*x^2] + 576*c
^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[1 + c^2*x^2] + 192*c^6*g*x^
6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x]] + 63*c*f*Sinh[4*ArcSinh[
c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]]))/(564480*c^2*Sqrt[1 + c^2*x^2]*Sqrt[d +
 c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. 2(430) = 860.

Time = 1.00 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.40

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1679 |
| parts | Expression too large to display | 1679 |

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2
*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+1/7*a*g*(c^2*d*x^2+d)^(7/2)/c^2/d+b*(5/32*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2*d^2+1/6272*(d*(c^2*x^2+1))^(
1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*
x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2
*x^2+1)^(1/2)+1)*g*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x
^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4
*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1
)^(1/2))*f*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2
)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1
)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+5*arcsinh(c*x))*d^2/c^2
/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(
1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(-1+
4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+3*arcs
inh(c*x))*d^2/c^2/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2
*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(-1+2*arcsinh(c*x))*d^2/c
/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
g*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+15/256*(d
*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1
)^(1/2))*f*(1+2*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)
*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1
)*g*(3*arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c
^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4
*c*x-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/640*(d*(c^
2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*
x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(1+5*arcsinh(
c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*c^6*x
^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5-48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3-18*
c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*f*(1+6*arcsinh(c*x))*d^2
/c/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2
+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*
```

```
(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*g*(1+7*arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1))
```

## Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b*c
^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sqrt(
c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int (f+gx)\,(d + c^2dx^2\big)^{5/2}\,(a+b\text{arcsinh}(cx))\,dx = \int \left(d\big(c^2x^2+1\big)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))\,(f+gx)\,\,dx$$

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (f+gx)\,(d + c^2dx^2\big)^{5/2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}dx$$

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

# 3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\mathbf{arcsinh}(cx))}{f+gx}\,dx$

Optimal result . . . . . 463
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Mathematica [C] (warning: unable to verify) . . . . . 484
Maple [B] (verified) . . . . . 484
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Sympy [F] . . . . . 486
Maxima [F(-2)] . . . . . 486
Giac [F(-2)] . . . . . 487
Mupad [F(-1)] . . . . . 487

## Optimal result

Integrand size = 30, antiderivative size = 1536

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\operatorname{arcsinh}(cx)\right)}{f+gx}\,dx = \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}}$$
$$- \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}}$$
$$+ \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}}$$
$$+ \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5}$$
$$- \frac{c^2d^2fx\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{8g^2} - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{2g^4}$$
$$- \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{4g^2} - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g}$$
$$+ \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g^3}$$
$$+ \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5g} + \frac{cd^2f\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{16bg^2\sqrt{1+c^2x^2}}$$
$$- \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{4bg^4\sqrt{1+c^2x^2}}$$
$$- \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bg^5\sqrt{1+c^2x^2}}$$
$$- \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}}$$
$$+ \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^4(f+gx)}$$
$$- \frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$+ \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$- \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$+ \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$- \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$

```
[Out] 1/4*b*c^3*d^2*f*(c^2*f^2+2*g^2)*x^2*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/
2)-1/4*c*d^2*f*(c^2*f^2+2*g^2)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g
^4/(c^2*x^2+1)^(1/2)-1/2*c*d^2*(c^2*f^2+g^2)^2*x*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/g^5/(c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2+g^2)^3*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)/(c^2*x^2+1)^(1/2)-1/3*d^2*(c^2
*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g+1/5*d^2*(c^2*x^2+1)^2*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g+1/3*d^2*(c^2*f^2+2*g^2)*(c^2*x^2+1)*(a
+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^3+b*d^2*(c^2*f^2+g^2)^2*arcsinh(c*x)
*(c^2*d*x^2+d)^(1/2)/g^5+a*d^2*(c^2*f^2+g^2)^2*(c^2*d*x^2+d)^(1/2)/g^5-1/8*
c^2*d^2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*x^3*(a
+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2+2/15*b*c*d^2*x*(c^2*d*x^2+d)^(1/2)
/g/(c^2*x^2+1)^(1/2)-1/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(
1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)+1/2*d^2*(c^
2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c
/g^4/(g*x+f)-a*d^2*(c^2*f^2+g^2)^(5/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(
1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)+b*d^2*(c^
2*f^2+g^2)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1
/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d
)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-b*c*d^2*(c^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)
/g^5/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x
^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1
/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)
-1/2*c^2*d^2*f*(c^2*f^2+2*g^2)*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^4
-1/3*b*c*d^2*(c^2*f^2+2*g^2)*x*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/
16*b*c^3*d^2*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d^2*
(c^2*f^2+2*g^2)*x^3*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/16*b*c^5*d^
2*f*x^4*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+1/16*c*d^2*f*(a+b*arcsinh
(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 1536, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.967$, Rules used = {5845, 5840, 5785, 5783, 30, 5798, 5806, 5812, 272, 45, 5804, 12, 5839, 697, 5835,

6874, 267, 739, 212, 5856, 1668, 5855, 8, 5843, 3403, 2296, 2221, 2317, 2438}

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\operatorname{arcsinh}(cx)\right)}{f+gx}\,dx = -\frac{bd^2x^5\sqrt{c^2dx^2+d}c^5}{25g\sqrt{c^2x^2+1}}$$
$$+\frac{bd^2fx^4\sqrt{c^2dx^2+d}c^5}{16g^2\sqrt{c^2x^2+1}} - \frac{d^2fx^3\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))c^4}{4g^2}$$
$$-\frac{bd^2\left(c^2f^2+2g^2\right)x^3\sqrt{c^2dx^2+d}c^3}{9g^3\sqrt{c^2x^2+1}} - \frac{bd^2x^3\sqrt{c^2dx^2+d}c^3}{45g\sqrt{c^2x^2+1}}$$
$$+\frac{bd^2f\left(c^2f^2+2g^2\right)x^2\sqrt{c^2dx^2+d}c^3}{4g^4\sqrt{c^2x^2+1}} + \frac{bd^2fx^2\sqrt{c^2dx^2+d}c^3}{16g^2\sqrt{c^2x^2+1}}$$
$$-\frac{d^2f\left(c^2f^2+2g^2\right)x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))c^2}{2g^4}$$
$$-\frac{d^2fx\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))c^2}{8g^2}$$
$$-\frac{d^2f\left(c^2f^2+2g^2\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2c}{4bg^4\sqrt{c^2x^2+1}}$$
$$-\frac{d^2\left(c^2f^2+g^2\right)^2x\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2c}{2bg^5\sqrt{c^2x^2+1}}$$
$$+\frac{d^2f\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))^2c}{16bg^2\sqrt{c^2x^2+1}}$$
$$-\frac{bd^2\left(c^2f^2+g^2\right)^2x\sqrt{c^2dx^2+d}c}{g^5\sqrt{c^2x^2+1}} - \frac{bd^2\left(c^2f^2+2g^2\right)x\sqrt{c^2dx^2+d}c}{3g^3\sqrt{c^2x^2+1}}$$
$$+\frac{2bd^2x\sqrt{c^2dx^2+d}c}{15g\sqrt{c^2x^2+1}} + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{c^2dx^2+d}\operatorname{arcsinh}(cx)}{g^5}$$
$$+\frac{d^2\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{5g}$$
$$+\frac{d^2\left(c^2f^2+2g^2\right)\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3g^3}$$
$$-\frac{d^2\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}(a+b\operatorname{arcsinh}(cx))}{3g}$$
$$-\frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{c^2dx^2+d}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{c^2x^2+1}}\right)}{g^6\sqrt{c^2x^2+1}}$$
$$+\frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{c^2dx^2+d}\operatorname{arcsinh}(cx)\log\left(\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^6\sqrt{c^2x^2+1}}$$
$$-\frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{c^2dx^2+d}\operatorname{arcsinh}(cx)\log\left(\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)}{g^6\sqrt{c^2x^2+1}}$$
$$+\frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{c^2x^2+1}}$$
$$-\frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{c^2dx^2+d}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{c^2x^2+1}}$$
$$+\frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{c^2dx^2+d}}$$

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2])/g^5 + (2*b*c*d^2*x*Sqrt[d + c
^2*d*x^2])/(15*g*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d +
 c^2*d*x^2])/(g^5*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + 2*g^2)*x*Sqrt[d
+ c^2*d*x^2])/(3*g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d + c^2*d*x
^2])/(16*g^2*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*(c^2*f^2 + 2*g^2)*x^2*Sqrt[d
 + c^2*d*x^2])/(4*g^4*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d + c^2*d*x^
2])/(45*g*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*(c^2*f^2 + 2*g^2)*x^3*Sqrt[d + c^
2*d*x^2])/(9*g^3*Sqrt[1 + c^2*x^2]) + (b*c^5*d^2*f*x^4*Sqrt[d + c^2*d*x^2])
/(16*g^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c^2*d*x^2])/(25*g*Sqr
t[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
)/g^5 - (c^2*d^2*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*g^2) - (c
^2*d^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g
^4) - (c^4*d^2*f*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - (d
^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) + (d^2*(c^
2*f^2 + 2*g^2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g
^3) + (d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*g)
+ (c*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*g^2*Sqrt[1 + c
^2*x^2]) - (c*d^2*f*(c^2*f^2 + 2*g^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*g^4*Sqrt[1 + c^2*x^2]) - (c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^5*Sqrt[1 + c^2*x^2]) - (d^2*(c^2*f^
2 + g^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^6*(f + g*x)
*Sqrt[1 + c^2*x^2]) + (d^2*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 + g
^2)^(5/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sq
rt[1 + c^2*x^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*
f^2 + g^2])])/(g^6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
 g^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^
2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^
6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^2*d*x^2]*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^6*Sqrt[1 + c
^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
```

```
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5840

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(\frac{\left(-c^4f^3-2c^2fg^2\right)\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{g^4} + \frac{c^2\left(c^2f^2+2g^2\right)x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{g^3} - \frac{c^4f}{\sqrt{1+c^2}}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g^2\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(c^4d^2\sqrt{d+c^2dx^2}\right)\int x^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(d^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\right)\int \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx}{g^4\sqrt{1+c^2x^2}}$$
$$- \frac{\left(c^2d^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g^4\sqrt{1+c^2x^2}}$$
$$+ \frac{\left(c^2d^2\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}\right)\int x\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\,dx}{g^3\sqrt{1+c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{c^2 d^2 f (c^2 f^2 + 2g^2)\, x \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4 d^2 f x^3 \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2 (1 + c^2 x^2)\, \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2 (c^2 f^2 + 2g^2)\, (1 + c^2 x^2)\, \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2 (1 + c^2 x^2)^2\, \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{d^2 (c^2 f^2 + g^2)^2\, \sqrt{1 + c^2 x^2}\, \sqrt{d + c^2 d x^2}\,(a + b\operatorname{arcsinh}(cx))^2}{2bcg^4 (f + gx)} \\
&\quad - \frac{\left(c^4 d^2 f \sqrt{d + c^2 d x^2}\right) \int \frac{x^2 (a + b\operatorname{arcsinh}(cx))}{\sqrt{1 + c^2 x^2}}\, dx}{4g^2 \sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(bc^5 d^2 f \sqrt{d + c^2 d x^2}\right) \int x^3\, dx}{4g^2 \sqrt{1 + c^2 x^2}} - \frac{\left(bc^5 d^2 \sqrt{d + c^2 d x^2}\right) \int \frac{-2 + c^2 x^2 + 3c^4 x^4}{15c^4}\, dx}{g \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(d^2 (c^2 f^2 + g^2)^2\, \sqrt{d + c^2 d x^2}\right) \int \frac{(-g + 2c^2 f x + c^2 g x^2)(a + b\operatorname{arcsinh}(cx))^2}{(f + gx)^2}\, dx}{2bcg^4 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(c^2 d^2 f (c^2 f^2 + 2g^2)\, \sqrt{d + c^2 d x^2}\right) \int \frac{a + b\operatorname{arcsinh}(cx)}{\sqrt{1 + c^2 x^2}}\, dx}{2g^4 \sqrt{1 + c^2 x^2}} \\
&\quad + \frac{\left(bc^3 d^2 f (c^2 f^2 + 2g^2)\, \sqrt{d + c^2 d x^2}\right) \int x\, dx}{2g^4 \sqrt{1 + c^2 x^2}} \\
&\quad - \frac{\left(bcd^2 (c^2 f^2 + 2g^2)\, \sqrt{d + c^2 d x^2}\right) \int (1 + c^2 x^2)\, dx}{3g^3 \sqrt{1 + c^2 x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2f(c^2f^2+2g^2)\,x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2(c^2f^2+2g^2)\,x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2(c^2f^2+2g^2)(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad - \frac{cd^2f(c^2f^2+2g^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2(c^2f^2+g^2)^2\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2(c^2f^2+g^2)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad + \frac{\left(c^2d^2f\sqrt{d+c^2dx^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{8g^2\sqrt{1+c^2x^2}} + \frac{\left(bc^3d^2f\sqrt{d+c^2dx^2}\right)\int x\,dx}{8g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int \left(-2+c^2x^2+3c^4x^4\right)\,dx}{15g\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(d^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}\right)\int \frac{\left(\frac{c^2x}{g}+\frac{1+\frac{c^2f^2}{g^2}}{f+gx}\right)(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{g^4\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&+ \frac{bc^3d^2f(c^2f^2+2g^2)\,x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&- \frac{bc^3d^2(c^2f^2+2g^2)\,x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&- \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&- \frac{c^2d^2f(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&- \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&- \frac{d^2(1+c^2x^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&+ \frac{d^2(c^2f^2+2g^2)\,(1+c^2x^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&+ \frac{d^2(1+c^2x^2)^2\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&+ \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&- \frac{cd^2f(c^2f^2+2g^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&- \frac{cd^2(c^2f^2+g^2)^2\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&- \frac{d^2(c^2f^2+g^2)^3\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&+ \frac{d^2(c^2f^2+g^2)^2\,\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&+ \frac{\left(d^2(c^2f^2+g^2)^2\,\sqrt{d+c^2dx^2}\right)\int\left(\frac{a(c^2f^2+g^2+c^2fgx+c^2g^2x^2)}{g^2(f+gx)\sqrt{1+c^2x^2}} + \frac{b(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\operatorname{arcsinh}(cx)}{g^2(f+gx)\sqrt{1+c^2x^2}}\right)dx}{g^4\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f(c^2f^2+2g^2)\,x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2(c^2f^2+2g^2)\,x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2(1+c^2x^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2(c^2f^2+2g^2)\,(1+c^2x^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2(1+c^2x^2)^2\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f(c^2f^2+2g^2)\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2(c^2f^2+g^2)^2\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2(c^2f^2+g^2)^3\,\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2(c^2f^2+g^2)^2\,\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad + \frac{\left(ad^2(c^2f^2+g^2)^2\,\sqrt{d+c^2dx^2}\right)\int \frac{c^2f^2+g^2+c^2fgx+c^2g^2x^2}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd^2(c^2f^2+g^2)^2\,\sqrt{d+c^2dx^2}\right)\int \frac{(c^2f^2+g^2+c^2fgx+c^2g^2x^2)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^4(f+gx)} \\
&\quad + \frac{\left(ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\right)\int \frac{c^2g^2\left(c^2f^2+g^2\right)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{c^2g^8\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\right)\int \left(\frac{c^2gx\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}} + \frac{\left(c^2f^2+g^2\right)\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\right)\,dx}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\operatorname{arcsinh}(cx)\right)^2}{2bcg^4(f+gx)} \\
&\quad + \frac{\left(bc^2d^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\right)\int \frac{x\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{g^5\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(ad^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\right)\int \frac{1}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\right)\int \frac{\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

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\begin{aligned}
&= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} + \frac{bc^3d^2f(c^2f^2+2g^2)\,x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2(c^2f^2+2g^2)\,x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2(c^2f^2+2g^2)(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f(c^2f^2+2g^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2(c^2f^2+g^2)^2\,x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2(c^2f^2+g^2)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{\left(bcd^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}\right)\int 1\,dx}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(ad^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{1}{c^2f^2+g^2-x^2}\,dx, x, \frac{g-c^2fx}{\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{x}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bd^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2ce^x f-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

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\begin{aligned}
&= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+g^2)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2(c^2f^2+2g^2)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f(c^2f^2+2g^2)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2(c^2f^2+2g^2)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f(c^2f^2+2g^2)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2(c^2f^2+2g^2)(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f(c^2f^2+2g^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2(c^2f^2+g^2)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2(c^2f^2+g^2)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{ad^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(2bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(2bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{e^x x}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{g^5\sqrt{1+c^2x^2}}
\end{aligned}
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$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{\left(bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx,x,\operatorname{arcsinh}(cx)\right)}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{\left(bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} \\
&\quad + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}} \\
&\quad - \frac{bc^3d^2\left(c^2f^2+2g^2\right)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}} + \frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5} - \frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2} \\
&\quad - \frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4} \\
&\quad - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2} \\
&\quad - \frac{d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} \\
&\quad + \frac{d^2\left(c^2f^2+2g^2\right)\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3} \\
&\quad + \frac{d^2\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g} \\
&\quad + \frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2f\left(c^2f^2+2g^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}} \\
&\quad - \frac{cd^2\left(c^2f^2+g^2\right)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}} \\
&\quad - \frac{d^2\left(c^2f^2+g^2\right)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}} \\
&\quad + \frac{d^2\left(c^2f^2+g^2\right)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)} \\
&\quad - \frac{ad^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad + \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}} \\
&\quad - \frac{bd^2\left(c^2f^2+g^2\right)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}
\end{aligned}
$$

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 7168, normalized size of antiderivative = 4.67

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
[Out] Result too large to show
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3498 vs. 2(1420) = 2840.

Time = 0.94 (sec) , antiderivative size = 3499, normalized size of antiderivative = 2.28

| method | result | size |
|---|---|---|
| default | Expression too large to display | 3499 |
| parts | Expression too large to display | 3499 |

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
[Out] b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilog
((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(
1/2)))-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*x^5*c^5+33/128*
b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)/g^2+1/8*b*(d*(c^2*x^2+1))
^(1/2)*f^3*d^2*c^3/(c^2*x^2+1)^(1/2)/g^4-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/
(c^2*x^2+1)^(1/2)/g*x^3*c^3-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(
1/2)/g*c*x-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2
)/g^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f
^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^6/(c^2*x^2+1)/g^4*arc
sinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^6/(c^2*x^2+1)/g^2*arcsinh
(c*x)*x^5-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/
2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*
f^2+g^2)^(1/2)))*c^2*f^2-11/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^4/(c^2*x^2+1)
/g^2*arcsinh(c*x)*x^3-9/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^2/(c^2*x^2+1)/g^2
*arcsinh(c*x)*x-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^4/(c^2*x^2+1)/g^4*arc
sinh(c*x)*x+1/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^
4*c^6*f^2+8/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^2*
c^4*f^2+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g
^6*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^
2+g^2)^(1/2)))*c^4*f^4-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2
*x^2+1)^(1/2)/g^6*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))
```

```
/(c*f+(c^2*f^2+g^2)^(1/2)))*c^4*f^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+
g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2
*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2+b*(d*(c^2*x^2+1))^(1/2
)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x)*x^2*c^6*f^4-b*(d*(c^2*x^2+1))^(1/2)*d^2/
(c^2*x^2+1)^(1/2)/g^5*x*c^5*f^4-7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)
^(1/2)/g^3*x*c^3*f^2+1/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+1)^(1/
2)/g^2*x^4+9/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)/g^2*x^2
+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^5/(c^2*x^2+1)^(1/2)/g^4*x^2+a/g*(1/5
*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(5/2)-c^2*d*f/g*
(1/8*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/
g)+d*(c^2*f^2+g^2)/g^2)^(3/2)+3/16*(4*c^2*d^2*(c^2*f^2+g^2)/g^2-4*c^4*d^2*f
^2/g^2)/c^2/d*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2*d-2*c
^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2*f^2+g^2)/g^
2-4*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2))
)+d*(c^2*f^2+g^2)/g^2*(1/3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(3/2)-c^2*d*f/g*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)
^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2
*f^2+g^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d
)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c
^2*d)^(1/2))+d*(c^2*f^2+g^2)/g^2*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c
^2*f^2+g^2)/g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2
)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^
(1/2)-d*(c^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)
/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c
^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g)))))+23/15*b*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)-1/9*b*(d*(c^2*x^2+1))^(1/2)*d^2/
(c^2*x^2+1)^(1/2)/g^3*x^3*c^5*f^2-15/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*f*arcsinh(c*x)^2*d^2*c/g^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+
1)/g*arcsinh(c*x)*x^6*c^6+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*a
rcsinh(c*x)*c^2*f^2+b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x
)*c^4*f^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f^5*arcsinh(c*x)^2*
d^2*c^5/g^6+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^
4*c^4+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2+
b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*ln((-
(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2
)))*arcsinh(c*x)-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1
)^(1/2)/g^2*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^
2*f^2+g^2)^(1/2)))*arcsinh(c*x)-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*f^3*arcsinh(c*x)^2*d^2*c^3/g^4+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)
^(1/2)/(c^2*x^2+1)^(1/2)/g^6*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^
2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*c^4*f^4-b*d^2*(d*(c^2*x^
2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*ln(((c*x+(c^2*x^2+1)^
(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*c
^4*f^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/
```

```
g^4*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+
g^2)^(1/2)))*arcsinh(c*x)*c^2*f^2-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^
2)^(1/2)/(c^2*x^2+1)^(1/2)/g^4*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g
^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*c^2*f^2
```

## Fricas [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{gx+f}\,dx$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

## Sympy [F]

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \int \frac{\left(d(c^2x^2+1)\right)^{\frac{5}{2}}\left(a+b\,\text{asinh}\left(cx\right)\right)}{f+gx}\,dx$$

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\text{arcsinh}(cx)\right)}{f+gx}\,dx = \int \frac{\left(a+b\,\text{asinh}(c\,x)\right)\,\left(d\,c^2\,x^2+d\right)^{5/2}}{f+g\,x}\,dx$$

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x), x)
```

# 3.47 $\int \frac{(f+gx)^3(a+b\mathbf{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 430

$$\begin{aligned}\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = &-\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}\\ &-\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}\\ &+\frac{3f^2g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}}\\ &-\frac{2g^3(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c^4\sqrt{d+c^2dx^2}}\\ &+\frac{3fg^2x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}}\\ &+\frac{g^3x^2(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{3c^2\sqrt{d+c^2dx^2}}\\ &+\frac{f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}\\ &-\frac{3fg^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 3*f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2/3*g^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^4/(c^2*d*x^2+d)^(1/2)+3/2*f*g^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/3*g^3*x^2*(c^2*x^2+1)*(a+b*arc
sinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x
^2+d)^(1/2)+2/3*b*g^3*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-3/4*b*f*g
^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx &= \frac{f^3\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}} \\ &+ \frac{3f^2g(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{c^2dx^2+d}} \\ &+ \frac{3fg^2x(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{c^2dx^2+d}} \\ &+ \frac{g^3x^2(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3c^2\sqrt{c^2dx^2+d}} \\ &- \frac{2g^3(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{3c^4\sqrt{c^2dx^2+d}} \\ &- \frac{3fg^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}} \\ &- \frac{3bf^2gx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}} - \frac{3bfg^2x^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}} \\ &- \frac{bg^3x^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}} + \frac{2bg^3x\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}\end{aligned}$$

```
[In] Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b*g^3*x*Sqrt[
1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 + c^2*x^2
])/(4*c*Sqrt[d + c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 + c^2*x^2])/(9*c*Sqrt[d +
c^2*d*x^2]) + (3*f^2*g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^
2*d*x^2]) - (2*g^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^4*Sqrt[d + c^2*
d*x^2]) + (3*f*g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*c^2*Sqrt[d + c^
2*d*x^2]) + (g^3*x^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^2*Sqrt[d + c^
2*d*x^2]) + (f^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d +
c^2*d*x^2]) - (3*f*g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*S
qrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3f^2gx(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{3fg^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{g^3x^3(a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(3f^2g\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$+ \frac{\left(3fg^2\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(g^3\sqrt{1+c^2x^2}\right)\int \frac{x^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{3f^2g(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{3fg^2x(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{g^3x^2(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} - \frac{\left(3bf^2g\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(3fg^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^2\sqrt{d+c^2dx^2}} - \frac{\left(3bfg^2\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{d+c^2dx^2}}$$
$$- \frac{\left(2g^3\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{3c^2\sqrt{d+c^2dx^2}} - \frac{\left(bg^3\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3c\sqrt{d+c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}$$
$$- \frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} + \frac{3f^2g(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}$$
$$- \frac{2g^3(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c^4\sqrt{d+c^2dx^2}} + \frac{3fg^2x(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{g^3x^2(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c^2\sqrt{d+c^2dx^2}} + \frac{f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$
$$- \frac{3fg^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}} + \frac{\left(2bg^3\sqrt{1+c^2x^2}\right)\int 1\,dx}{3c^3\sqrt{d+c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} - \frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}$$
$$+ \frac{3f^2g(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} - \frac{2g^3(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c^4\sqrt{d+c^2dx^2}}$$
$$+ \frac{3fg^2x(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}} + \frac{g^3x^2(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{3c^2\sqrt{d+c^2dx^2}}$$
$$+ \frac{f^3\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} - \frac{3fg^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}$$

## Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.71

$$\int \frac{(f+gx)^3(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{4\sqrt{d}g\left(-2bcx\sqrt{1+c^2x^2}(-6g^2+c^2(27f^2+g^2x^2))+3a(1+c^2x^2)\left(-4g^2+c^2(18f^2+9fgx+2g^2x^2)\right)\right)+12}{}$$

```
[In] Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]

[Out] (4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
+ 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b*S
qrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*ArcSin
h[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36*a*c*
f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(382) = 764.

Time = 0.93 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| default | $a\left(\frac{f^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+g^3\left(\frac{x^2\sqrt{c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}}{3dc^4}\right)+3fg^2\left(\frac{x\sqrt{c^2dx^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}}\right)\right.$ |
| parts | $a\left(\frac{f^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+g^3\left(\frac{x^2\sqrt{c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}}{3dc^4}\right)+3fg^2\left(\frac{x\sqrt{c^2dx^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}}\right)\right.$ |

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)

[Out] a*(f^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^3*(1/3
*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+3*f*g^2*(1/2*
x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2))+3*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2*x^2
+1))^(1/2)*f*arcsinh(c*x)^2*(2*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/c^3/d+1/72*
(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*
x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)+3/16*(d*(c
^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(
1/2))*f*g^2*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+1))^(1/2)
```

```
*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(4*arcsinh(c*x)*c^2*f^2-4*c^2*f^2-arcs
inh(c*x)*g^2+g^2)/c^4/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*
(c^2*x^2+1)^(1/2)+1)*g*(4*arcsinh(c*x)*c^2*f^2+4*c^2*f^2-arcsinh(c*x)*g^2-g
^2)/c^4/d/(c^2*x^2+1)+3/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*
x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(1+2*arcsinh(c*x))/c^3/d/(c^2*x
^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(3*arcsinh(c*x)+1)/c^4/d/(c^2*x^2+1))
```

## Fricas [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^3(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))\,(f+gx)^3}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^3(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.48 $\int \frac{(f+gx)^2(a+b\mathbf{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$


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Maxima [F(-2)] . . . 500
Giac [F] . . . 500
Mupad [F(-1)] . . . 500


## Optimal result

Integrand size = 30, antiderivative size = 258

$$\begin{aligned}\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx &= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} \\ &+ \frac{2fg(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} \\ &+ \frac{g^2x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}} \\ &+ \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} \\ &- \frac{g^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] 2*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/2*g^2*x*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(
1/2)+1/2*f^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
-1/4*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = &\frac{f^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}\\ &+\frac{2fg(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{c^2dx^2+d}}\\ &+\frac{g^2x(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{c^2dx^2+d}}\\ &-\frac{g^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{c^2dx^2+d}}\\ &-\frac{2bfgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}-\frac{bg^2x^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}\end{aligned}$$

```
[In] Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
+ c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (2*f*g*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
]))/(2*c^2*Sqrt[d + c^2*d*x^2]) + (f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/(2*b*c*Sqrt[d + c^2*d*x^2]) - (g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{2fgx(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{g^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(2fg\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(g^2\sqrt{1+c^2x^2}\right)\int \frac{x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2fg(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{g^2x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} - \frac{\left(2bfg\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(g^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^2\sqrt{d+c^2dx^2}} - \frac{\left(bg^2\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{d+c^2dx^2}} \\
&= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} \\
&\quad + \frac{2fg(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{g^2x(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}} \\
&\quad + \frac{f^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} - \frac{g^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$
\begin{aligned}
&\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx \\
&= \frac{4c\sqrt{d}g\left(-4bcfx\sqrt{1+c^2x^2}+a(4f+gx)\,(1+c^2x^2)\right)+4bc\sqrt{d}g(4f+gx)\,(1+c^2x^2)\operatorname{arcsinh}(cx)+2b\sqrt{d}(2c^2}{}
\end{aligned}
$$

```
[In] Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]

[Out] (4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2))
 + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(2*
c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 + c^
2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(230) = 460$.

Time = 0.76 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $a\left(\frac{f^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+g^2\left(\frac{x\sqrt{c^2dx^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}}\right)+\frac{2fg\sqrt{c^2dx^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,a}{4\sqrt{}}\right.$ |
| parts | $a\left(\frac{f^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+g^2\left(\frac{x\sqrt{c^2dx^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2c^2\sqrt{c^2d}}\right)+\frac{2fg\sqrt{c^2dx^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\,a}{4\sqrt{}}\right.$ |

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] a*(f^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^2*(1/2
*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)
^(1/2))/(c^2*d)^(1/2))+2*f*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2*x^2+
1))^(1/2)*arcsinh(c*x)^2*(2*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3/d+1/16*(d*(c
^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(
1/2))*g^2*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)*(c^2*
x^2+c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(arcsinh(c*x)+1)/c^2/
d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(
1/2)+2*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1))
```

## Fricas [F]

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\,\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*a
rcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(cx))\,(f+gx)^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

## Giac [F]

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)^2\,(a+b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.49 $\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result . . . 501
Rubi [A] (verified) . . . 501
Mathematica [A] (verified) . . . 503
Maple [B] (verified) . . . 503
Fricas [F] . . . 504
Sympy [F] . . . 504
Maxima [A] (verification not implemented) . . . 504
Giac [F] . . . 505
Mupad [F(-1)] . . . 505

## Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-b*g*x*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5845, 5838, 5783, 5798, 8}

$$\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{f\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}} + \frac{g(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{c^2dx^2+d}} - \frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[In] Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (g*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (f*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}} + \frac{gx(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\right)\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(g\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\operatorname{arcsinh}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} - \frac{\left(bg\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{2\sqrt{d}g\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\right)+2b\sqrt{d}g(1+c^2x^2)\operatorname{arcsinh}(cx)+bc\sqrt{d}f\sqrt{1+c^2x^2}\operatorname{arcsinh}(cx)^2+2ac}{2c^2\sqrt{d}\sqrt{d+c^2dx^2}}$$

```
[In] Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1 +
 c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + 2
*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*c^2
*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.

Time = 0.83 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.89

| method | result |
|---|---|
| default | $\frac{af\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2dx^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)g(-1+a}{2c^2d(c^2x^2+1)}\right.$ |
| parts | $\frac{af\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2dx^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)g(-1+a}{2c^2d(c^2x^2+1)}\right.$ |

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*f*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+a*g/c^2/d*(
c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*arc
sinh(c*x)^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(
-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(
c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)/c^2/d/(c^2*x^2+1))
```

## Fricas [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))\,(f+gx)}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{bf\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}} - \frac{bgx}{c\sqrt{d}} + \frac{af\,\text{arsinh}\,(cx)}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2+d}bg\,\text{arsinh}\,(cx)}{c^2d} + \frac{\sqrt{c^2dx^2+d}ag}{c^2d}$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*f*arcsinh(c*x)^2/(c*sqrt(d)) - b*g*x/(c*sqrt(d)) + a*f*arcsinh(c*x)/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*g*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a*g/(c^2*d)
```

## Giac [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.50 $\int \frac{a+b\mathbf{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal result . . . 506
Rubi [A] (verified) . . . 506
Mathematica [A] (verified) . . . 507
Maple [A] (verified) . . . 507
Fricas [F] . . . 507
Sympy [F] . . . 508
Maxima [A] (verification not implemented) . . . 508
Giac [F] . . . 508
Mupad [F(-1)] . . . 508

## Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5783}

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d\,(1+c^2x^2)}} + \frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcTa
nh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

| method | result | size |
|---|---|---|
| default | $\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}} + \frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,dc}$ | 77 |
| parts | $\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{\sqrt{c^2d}} + \frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,dc}$ | 77 |

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(c*x)^2
```

## Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(cx)}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{b\,\operatorname{arsinh}(cx)^2}{2\,c\sqrt{d}} + \frac{a\,\operatorname{arsinh}(cx)}{c\sqrt{d}}$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

## Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

# 3.51 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 325

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}$$

```
[Out] (a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2))
)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*
x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(
1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)+b*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(
c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2
)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5843, 3403, 2296, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}} \\ &- \frac{\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}} \\ &+ \frac{b\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}} \\ &- \frac{b\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - S
qrt[c^2*f^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f
^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^
2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2
*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^Arc
Sinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c
^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}\operatorname{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&= \frac{\left(2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf-g+e^{2x}g}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(2g\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(2g\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.74

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{d}\left(g-c^2fx\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\operatorname{arcsinh}(cx)\left(\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)+\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcs}}}{-cf+}\right.\right.}{\sqrt{d+c^2dx^2}}}{\sqrt{c^2f^2+g^2}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-((a*ArcTanh[(Sqrt[d]*(g - c^2*f*x))/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x
^2])])/Sqrt[d]) + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[c*
x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 +
 g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/S
qrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $-\frac{a\ln\left(\frac{\frac{2d\left(c^2f^2+g^2\right)}{g^2}-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\sqrt{c^2f^2+g^2}\,\sqrt{}\right.$ |
| parts | $-\frac{a\ln\left(\frac{\frac{2d\left(c^2f^2+g^2\right)}{g^2}-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,\sqrt{c^2f^2+g^2}\,\sqrt{}\right.$ |

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f
/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c
^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*((d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/
2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*(
ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)
^(1/2)))-ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f
^2+g^2)^(1/2))))+(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2
```

```
)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*(dilog((-(c*x+(c^2*x^2+1)^(1/2))*
g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-dilog(((c*x+(c^2*x^2
+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))))
```

## Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x^
2 + d*g*x + d*f), x)
```

## Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}\,(f+gx)}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(f+g\,x)\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.52 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

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## Optimal result

Integrand size = 30, antiderivative size = 444

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx &= -\frac{g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{(c^2f^2+g^2)\,(f+gx)\sqrt{d+c^2dx^2}}\\ &+\frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\ &-\frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\ &+\frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}}\\ &+\frac{bc^2f\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\ &-\frac{bc^2f\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\end{aligned}$$

```
[Out] -g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(c^2*f^2+g^2)/(g*x+f)/(c^2*d*x^2+d)^(1/2)
+b*c*ln(g*x+f)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(c^2*d*x^2+d)^(1/2)+c^2*f*(a
+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*
(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+
1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)+b*c^2*f*polylog(2,-(c*x+(c
^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^
2)^(3/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(
c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+
d)^(1/2)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5845, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx &= -\frac{g(c^2x^2+1)\,(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)\,(f+gx)} \\ &+ \frac{c^2f\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}} \\ &- \frac{c^2f\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}} \\ &+ \frac{bc^2f\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}} \\ &- \frac{bc^2f\sqrt{c^2x^2+1}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}} \\ &+ \frac{bc\sqrt{c^2x^2+1}\log(f+gx)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((c^2*f^2 + g^2)*(f + g*x)*Sqrt[d
+ c^2*d*x^2])) + (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d
+ c^2*d*x^2]) - (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d +
 c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[f + g*x])/((c^2*f^2 + g^2)*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
 - (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\left(c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\sinh(x))^2}\,dx, x, \text{arcsinh}(cx)\right)}{\sqrt{d+c^2dx^2}} \\
&= -\frac{g(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(c^2f\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(bcg\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)}{cf+g\sinh(x)}\,dx, x, \text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&= -\frac{g(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{cf+x}\,dx, x, cgx\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2c^2f\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf-g+e^{2x}g}\,dx, x, \text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&= -\frac{g(1+c^2x^2)\left(a+b\text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(2c^2fg\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(2c^2fg\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg+2\sqrt{c^2f^2+g^2}}\,dx, x, \text{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{g(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(bc^2f\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf-2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(bc^2f\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^xg}{2cf+2\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&= -\frac{g(1+c^2x^2)\left(a+b\operatorname{arcsinh}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} \\
&\quad - \frac{\left(bc^2f\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf-2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{\left(bc^2f\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2gx}{2cf+2\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{g(1+c^2x^2)\,(a+b\operatorname{arcsinh}(cx))}{(c^2f^2+g^2)\,(f+gx)\sqrt{d+c^2dx^2}} \\
&\quad + \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{c^2f\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}} + \frac{bc^2f\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}} \\
&\quad - \frac{bc^2f\sqrt{1+c^2x^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.57

$$
\begin{aligned}
&\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx \\
&= \frac{c\sqrt{1+c^2x^2}\left(-\frac{g\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{cf+cgx}+b\log(f+gx)+\frac{cf\left((a+b\operatorname{arcsinh}(cx))\left(\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\operatorname{arcs}}}{cf+\sqrt{}}\right.\right.\right.}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]

[Out] (c*Sqrt[1 + c^2*x^2]*(-((g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c*f + c
*g*x)) + b*Log[f + g*x] + (c*f*((a + b*ArcSinh[c*x])*(Log[1 + (E^ArcSinh[c*
x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])]) + b*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2
 + g^2])] - b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]
))/Sqrt[c^2*f^2 + g^2]))/((c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. 2(442) = 884.

Time = 0.85 (sec) , antiderivative size = 1770, normalized size of antiderivative = 3.99

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1770 |
| parts | Expression too large to display | 1770 |

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d
*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*c^4*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f-b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^2*c^2
*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(
g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^
2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)
/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+
1)/(c^2*f^2+g^2)/(g*x+f)*g+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6
*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^3*ln((c
*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*f^2-2*b*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^
2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^
2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*
g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+
g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*(c^2*f^2+g^2)^(1/2)*f-
b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+
c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*
f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*(c^2*f^2+g^2
)^(1/2)*f+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*dilog((-(c*x+(c^2*x^2+
1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g
^2)^(1/2)*f-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*
f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*dilog(((c*x+(c^2*x^2
+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g
^2)^(1/2)*f+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*
f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln((c*x+(c^2*x^2+1)^(1
/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*g^2-2*b*(d*(c^2*x^2+1))^(1/2)*(c^
2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f
^2*g^2+g^4)*c*ln(c*x+(c^2*x^2+1)^(1/2))*g^2
```

## Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d*f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

## Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}\,(f+gx)^2}\,dx$$

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

## Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c\,x)}{(f+g\,x)^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)), x)
```

# 3.53 $\int \frac{(a+b\mathbf{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result . . . 525
Rubi [N/A] . . . 525
Mathematica [N/A] . . . 526
Maple [N/A] (verified) . . . 526
Fricas [N/A] . . . 526
Sympy [F(-1)] . . . 527
Maxima [N/A] . . . 527
Giac [N/A] . . . 527
Mupad [N/A] . . . 528

## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}, x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

```
[In] Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] Defer[Int][((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

```
[In] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
[Out] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x]
```

## Maple [N/A] (verified)

Not integrable

Time = 2.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^n \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

```
[In] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^n \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
[Out] integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^n \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Giac [N/A]

Not integrable

Time = 73.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^n \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\left(a+b\,\text{asinh}(c\,x)\right)^n}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2),x)
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2), x)
```

# 3.54 $\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result . . . 529
Rubi [A] (verified) . . . 530
Mathematica [A] (verified) . . . 536
Maple [F] . . . 537
Fricas [F] . . . 537
Sympy [F] . . . 537
Maxima [F] . . . 537
Giac [F(-2)] . . . 538
Mupad [F(-1)] . . . 538

## Optimal result

Integrand size = 34, antiderivative size = 438

$$
\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{2b^2m \operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{2b^2m \operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}
$$

```
[Out] 1/12*m*(a+b*arcsinh(c*x))^4/b^2/c+1/3*(a+b*arcsinh(c*x))^3*ln(h*(g*x+f)^m)/
b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g
/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))^2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))^2*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+2*b*m*(a+b*arcs
```

```
inh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c
+2*b*m*(a+b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^
2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+
g^2)^(1/2)))/c
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {5783, 5846, 5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$
\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{3bc} + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{2b^2m \operatorname{PolyLog}\left(4, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{2b^2m \operatorname{PolyLog}\left(4, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}
$$

```
[In] Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^4)/(12*b^2*c) - (m*(a + b*ArcSinh[c*x])^3*Log[1 + (
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(3*b*c) - (m*(a + b*ArcSinh
[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(3*b*c) +
 ((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (m*(a + b*ArcSinh[c*
x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m
*(a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4,
```

```
 -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4
, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5846

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[g*(m/(b*c*S
qrt[d]*(n + 1))), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\int \frac{(a+b\operatorname{arcsinh}(cx))^3}{f+gx}\,dx}{3bc} \\
&= \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\operatorname{Subst}\left(\int \frac{(a+bx)^3\cosh(x)}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3bc} \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{(gm)\operatorname{Subst}\left(\int \frac{e^x(a+bx)^3}{cf+e^xg-\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3bc} \\
&\quad - \frac{(gm)\operatorname{Subst}\left(\int \frac{e^x(a+bx)^3}{cf+e^xg+\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{3bc}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad + \frac{m\operatorname{Subst}\left(\int (a+bx)^2 \log\left(1+\frac{e^x g}{cf-\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int (a+bx)^2 \log\left(1+\frac{e^x g}{cf+\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{(2bm)\operatorname{Subst}\left(\int (a+bx) \operatorname{PolyLog}\left(2, -\frac{e^x g}{cf-\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{(2bm)\operatorname{Subst}\left(\int (a+bx) \operatorname{PolyLog}\left(2, -\frac{e^x g}{cf+\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{(2b^2m)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{e^xg}{cf-\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad - \frac{(2b^2m)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{e^xg}{cf+\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{(2b^2m)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,\frac{gx}{-cf+\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c} \\
&\quad - \frac{(2b^2m)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,-\frac{gx}{cf+\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c}
\end{aligned}
$$

$$= \frac{m(a+b\operatorname{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc}$$
$$+ \frac{(a+b\operatorname{arcsinh}(cx))^3 \log\left(h(f+gx)^m\right)}{3bc}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))^2 \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$+ \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$+ \frac{2bm(a+b\operatorname{arcsinh}(cx)) \operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$- \frac{2b^2m \operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{2b^2m \operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx =$$
$$-\frac{-\frac{m(a+b\operatorname{arcsinh}(cx))^4}{4b} + m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right) + m(a+b\operatorname{arcsinh}(cx))^3 \log\left(1+\frac{e^{\operatorname{arc}}}{cf+}\right.}{}$$

```
[In] Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]

[Out] -1/3*(-1/4*(m*(a + b*ArcSinh[c*x])^4)/b + m*(a + b*ArcSinh[c*x])^3*Log[1 +
(E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^3*
Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh[c*
x])^3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^ArcS
inh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])*Poly
Log[3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*PolyLog[4
, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a + b*ArcSi
nh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] -
2*b*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g
^2]))]))/(b*c)
```

## Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2 \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

```
[In] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2 \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h)
/sqrt(c^2*x^2 + 1), x)
```

## Sympy [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}\,(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

## Maxima [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2 \log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{720,[0,4,1,1,1,1,4,0]%%%}+%%%{-1260,[0,4,1,1,1,1,3,1]%%%}+%%%
{360,[
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,(a+b\,\operatorname{asinh}(c\,x))^2}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2), x)
```

# 3.55 $\int \frac{(a+b\operatorname{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal result . . . 539
Rubi [A] (verified) . . . 540
Mathematica [A] (verified) . . . 544
Maple [F] . . . 544
Fricas [F] . . . 544
Sympy [F] . . . 545
Maxima [F] . . . 545
Giac [F] . . . 545
Mupad [F(-1)] . . . 545

## Optimal result

Integrand size = 32, antiderivative size = 332

$$\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} + \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}$$

```
[Out] 1/6*m*(a+b*arcsinh(c*x))^3/b^2/c+1/2*(a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/b
/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))/b/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+(c^2*
x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5783, 5846, 5827, 5680, 2221, 2611, 2320, 6724}

$$\int \frac{(a+b\operatorname{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

$$= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))^2\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{2bc}$$
$$- \frac{m(a+b\operatorname{arcsinh}(cx))^2\log\left(\frac{ge^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{2bc} + \frac{(a+b\operatorname{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{2bc}$$
$$+ \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

```
[In] Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^3)/(6*b^2*c) - (m*(a + b*ArcSinh[c*x])^2*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(2*b*c) - (m*(a + b*ArcSinh[
c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(2*b*c) +
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (m*(a + b*ArcSinh[c*x
])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*(a
 + b*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])
)])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])
/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5846

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[g*(m/(b*c*S
qrt[d]*(n + 1))), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{f+gx}\,dx}{2bc} \\
&= \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\operatorname{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{cf+g\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2bc} \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{(gm)\operatorname{Subst}\left(\int \frac{e^x(a+bx)^2}{cf+e^xg-\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2bc} \\
&\quad - \frac{(gm)\operatorname{Subst}\left(\int \frac{e^x(a+bx)^2}{cf+e^xg+\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right)}{2bc} \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} \\
&\quad + \frac{m\operatorname{Subst}\left(\int (a+bx)\log\left(1+\frac{e^xg}{cf-\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int (a+bx)\log\left(1+\frac{e^xg}{cf+\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{(bm)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -\frac{e^xg}{cf-\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{(bm)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -\frac{e^xg}{cf+\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{(bm)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{gx}{-cf+\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c} \\
&\quad + \frac{(bm)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{gx}{cf+\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c} \\
&= \frac{m(a+b\operatorname{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))^2 \log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(cx))^2 \log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\operatorname{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$
$$= \frac{\frac{m(a+b\operatorname{arcsinh}(cx))^3}{3b} - m(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right) - m(a+b\operatorname{arcsinh}(cx))^2\log\left(1+\frac{e^{\operatorname{arcsinh}(c}}{cf+\sqrt{c^2f^2}}\right.}{}$$

```
[In] Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] ((m*(a + b*ArcSinh[c*x])^3)/(3*b) - m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^Arc
Sinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - m*(a + b*ArcSinh[c*x])^2*Log[1
+ (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] + (a + b*ArcSinh[c*x])^2*
Log[h*(f + g*x)^m] + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSinh[c*
x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + b*PolyLog[3, (E^ArcSinh[c*x]*g)/(-
(c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, -
((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[3, -((E^ArcS
inh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

## Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(cx))\ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

```
[In] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,(a+b\,\text{asinh}(c\,x))}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2),x)
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2), x)
```

# 3.56 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

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Rubi [A] (verified) . . . 546
Mathematica [A] (verified) . . . 549
Maple [F] . . . 550
Fricas [F] . . . 550
Sympy [F] . . . 550
Maxima [F] . . . 550
Giac [F] . . . 551
Mupad [F(-1)] . . . 551

## Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - \frac{m\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

```
[Out] 1/2*m*arcsinh(c*x)^2/c+arcsinh(c*x)*ln(h*(g*x+f)^m)/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^2*x^2+1
)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{221, 2451, 5827, 5680, 2221, 2317, 2438\}$

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = -\frac{m\,\mathrm{PolyLog}\left(2, -\frac{e^{\mathrm{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\mathrm{PolyLog}\left(2, -\frac{e^{\mathrm{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\mathrm{arcsinh}(cx)\log\left(\frac{ge^{\mathrm{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c} - \frac{m\mathrm{arcsinh}(cx)\log\left(\frac{ge^{\mathrm{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{c} + \frac{\mathrm{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} + \frac{m\mathrm{arcsinh}(cx)^2}{2c}$$

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f
- Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (m*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*PolyLog[
2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\operatorname{arcsinh}(cx)}{cf+cgx}\,dx \\
&= \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\operatorname{Subst}\left(\int \frac{x\cosh(x)}{c^2 f + cg\sinh(x)}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&= \frac{m\operatorname{arcsinh}(cx)^2}{2c} + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} \\
&\quad - (gm)\operatorname{Subst}\left(\int \frac{e^x x}{c^2 f + ce^x g - c\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&\quad - (gm)\operatorname{Subst}\left(\int \frac{e^x x}{c^2 f + ce^x g + c\sqrt{c^2f^2+g^2}}\,dx, x, \operatorname{arcsinh}(cx)\right) \\
&= \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int \log\left(1+\frac{ce^x g}{c^2f-c\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int \log\left(1+\frac{ce^x g}{c^2f+c\sqrt{c^2f^2+g^2}}\right)\,dx, x, \operatorname{arcsinh}(cx)\right)}{c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{cgx}{c^2f-c\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c} \\
&\quad + \frac{m\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{cgx}{c^2f+c\sqrt{c^2f^2+g^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(cx)}\right)}{c} \\
&= \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} \\
&\quad - \frac{m\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx &= \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{ce^{\operatorname{arcsinh}(cx)}g}{c^2f-c\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{ce^{\operatorname{arcsinh}(cx)}g}{c^2f+c\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{\operatorname{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - \frac{m\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}
$$

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c^
2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m]
)/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c -
 (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

## Maple [F]

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

```
[In] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

## Fricas [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Sympy [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

## Maxima [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2), x)
```

# 3.57 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\mathbf{arcsinh}(cx))}\,dx$

Optimal result . . . 552
Rubi [N/A] . . . 552
Mathematica [N/A] . . . 553
Maple [N/A] (verified) . . . 553
Fricas [N/A] . . . 553
Sympy [N/A] . . . 554
Maxima [N/A] . . . 554
Giac [N/A] . . . 554
Mupad [N/A] . . . 555

## Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \operatorname{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}, x\right)$$

```
[Out] Unintegrable(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\ln\left(h(gx+f)^m\right)}{(a+b\,\text{arcsinh}\left(cx\right))\sqrt{c^2x^2+1}}dx$$

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
[Out] integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)*
arcsinh(c*x) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 7.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{(a+b\,\text{asinh}\left(cx\right))\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
[Out] Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\ln\left(h\left(f+g\,x\right)^m\right)}{\left(a+b\,\text{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
[Out] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

# 3.58 $\int x^3 \mathbf{arcsinh}(a+bx)\,dx$

Optimal result . . . 556
Rubi [A] (verified) . . . 556
Mathematica [A] (verified) . . . 559
Maple [A] (verified) . . . 559
Fricas [A] (verification not implemented) . . . 560
Sympy [B] (verification not implemented) . . . 561
Maxima [B] (verification not implemented) . . . 561
Giac [A] (verification not implemented) . . . 562
Mupad [F(-1)] . . . 562

## Optimal result

Integrand size = 10, antiderivative size = 131

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx = \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{(4a(16-19a^2)-(9-26a^2)(a+bx))\sqrt{1+(a+bx)^2}}{96b^4} - \frac{(3-24a^2+8a^4)\operatorname{arcsinh}(a+bx)}{32b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)$$

```
[Out] -1/32*(8*a^4-24*a^2+3)*arcsinh(b*x+a)/b^4+1/4*x^4*arcsinh(b*x+a)+7/48*a*x^2
*(1+(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1+(b*x+a)^2)^(1/2)/b-1/96*(4*a*(-19*a^2+
16)-(-26*a^2+9)*(b*x+a))*(1+(b*x+a)^2)^(1/2)/b^4
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5859, 5828, 757, 847, 794, 221}

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx = -\frac{(4a(16-19a^2)-(9-26a^2)(a+bx))\sqrt{(a+bx)^2+1}}{96b^4} - \frac{(8a^4-24a^2+3)\operatorname{arcsinh}(a+bx)}{32b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx) + \frac{7ax^2\sqrt{(a+bx)^2+1}}{48b^2} - \frac{x^3\sqrt{(a+bx)^2+1}}{16b}$$

```
[In] Int[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[1 + (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 + (a + b*x)^2])/(16*
b) - ((4*a*(16 - 19*a^2) - (9 - 26*a^2)*(a + b*x))*Sqrt[1 + (a + b*x)^2])/(
```

```
96*b^4) - ((3 - 24*a^2 + 8*a^4)*ArcSinh[a + b*x])/(32*b^4) + (x^4*ArcSinh[a
 + b*x])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{4}x^4\operatorname{arcsinh}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx) \\
&\quad - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3-4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx) \\
&\quad - \frac{1}{48}\text{Subst}\left(\int \frac{\left(\frac{a(23-12a^2)}{b^3}-\frac{(9-26a^2)x}{b^3}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} \\
&\quad - \frac{\left(4a(16-19a^2)-(9-26a^2)(a+bx)\right)\sqrt{1+(a+bx)^2}}{96b^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx) - \frac{(3-24a^2+8a^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{32b^4} \\
&= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} \\
&\quad - \frac{\left(4a(16-19a^2)-(9-26a^2)(a+bx)\right)\sqrt{1+(a+bx)^2}}{96b^4} \\
&\quad - \frac{(3-24a^2+8a^4)\operatorname{arcsinh}(a+bx)}{32b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int x^3 \text{arcsinh}(a+bx)\,dx$$

$$= \frac{\sqrt{1+a^2+2abx+b^2x^2}(50a^3+9bx-26a^2bx-6b^3x^3+a(-55+14b^2x^2))-3(3-24a^2+8a^4-8b^4x^4)\,a}{96b^4}$$

```
[In] Integrate[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*x^3
 + a*(-55 + 14*b^2*x^2)) - 3*(3 - 24*a^2 + 8*a^4 - 8*b^4*x^4)*ArcSinh[a + b
*x])/(96*b^4)
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| derivativedivides | $-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}-\frac{(bx+a)^3\sqrt{1+(bx+a}$ |
| default | $-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}-\frac{(bx+a)^3\sqrt{1+(bx+a}$ |
| parts | $\frac{x^4\operatorname{arcsinh}(bx+a)}{4}-\frac{b\left(\frac{x^3\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}-\frac{7a\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a}}{b^2}\right.\right.\right.\right.\right.$ |

```
[In] int(x^3*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(-arcsinh(b*x+a)*a^3*(b*x+a)+3/2*arcsinh(b*x+a)*a^2*(b*x+a)^2-arcsinh
(b*x+a)*a*(b*x+a)^3+1/4*arcsinh(b*x+a)*(b*x+a)^4-1/16*(b*x+a)^3*(1+(b*x+a)^
2)^(1/2)+3/32*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/32*arcsinh(b*x+a)+a^3*(1+(b*x+a
)^2)^(1/2)-3/2*a^2*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))+a*(
1/3*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-2/3*(1+(b*x+a)^2)^(1/2)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int x^3\operatorname{arcsinh}(a+bx)\,dx$$

$$=\frac{3\left(8\,b^4x^4-8\,a^4+24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2-\right.\right.}{96\,b^4}$$

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b*x + 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.

Time $= 0.32$ (sec) , antiderivative size $= 255$, normalized size of antiderivative $= 1.95$

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx$$

$$= \begin{cases} -\frac{a^4 \operatorname{asinh}(a+bx)}{4b^4} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}}{48b^4} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}}{48b^3} + \frac{3a^2 \operatorname{asinh}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}}{48b^2} - \frac{55a\sqrt{a} \\ \frac{x^4 \operatorname{asinh}(a)}{4} \end{cases}$$

```
[In] integrate(x**3*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(48
*b**3) + 3*a**2*asinh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**2) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(96*b**4
) + x**4*asinh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(16*b
) + 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(32*b**3) - 3*asinh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(114) = 228$.

Time $= 0.18$ (sec) , antiderivative size $= 318$, normalized size of antiderivative $= 2.43$

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx = \frac{1}{4}\,x^4 \operatorname{arsinh}(bx+a)$$

$$-\frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2\,abx+a^2+1}x^3}{b^2} - \frac{14\sqrt{b^2x^2+2\,abx+a^2+1}ax^2}{b^3} + \frac{105\,a^4 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^5}\right.$$

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b^
2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a^2*x/b^4 - 90*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*
```

```
a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3
/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x/b^4 + 9*(a^2 + 1)^2*
arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 55*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a/b^5)*b
```

## Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.24

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx = \frac{1}{4}x^4 \log\left(bx + a + \sqrt{(bx+a)^2+1}\right)$$
$$-\frac{1}{96}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x\left(\frac{3\,x}{b^2}-\frac{7\,a}{b^3}\right)+\frac{26\,a^2b^3-9\,b^3}{b^7}\right)x-\frac{5\,(10\,a^3b^2-11\,ab^2)}{b^7}\right)-\frac{3\,(8\,a^4-}{}\right.$$

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 - 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 - 11*a*b^2)/b^7) - 3*(8*a^4 - 24*a^2 + 3)*log(-a*b - (x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^4*abs(b)))*b
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx = \int x^3 \operatorname{asinh}(a+b\,x)\,dx$$

```
[In] int(x^3*asinh(a + b*x),x)
```

```
[Out] int(x^3*asinh(a + b*x), x)
```

# 3.59 $\int x^2 \mathbf{arcsinh}(a+bx)\,dx$

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## Optimal result

Integrand size = 10, antiderivative size = 90

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx = -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3} - \frac{a(3-2a^2)\operatorname{arcsinh}(a+bx)}{6b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)$$

```
[Out] -1/6*a*(-2*a^2+3)*arcsinh(b*x+a)/b^3+1/3*x^3*arcsinh(b*x+a)-1/9*x^2*(1+(b*x
+a)^2)^(1/2)/b+1/18*(5*a*b*x-11*a^2+4)*(1+(b*x+a)^2)^(1/2)/b^3
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5828, 757, 794, 221}

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx = -\frac{a(3-2a^2)\operatorname{arcsinh}(a+bx)}{6b^3} + \frac{(-11a^2+5abx+4)\sqrt{(a+bx)^2+1}}{18b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx) - \frac{x^2\sqrt{(a+bx)^2+1}}{9b}$$

```
[In] Int[x^2*ArcSinh[a + b*x],x]
```

```
[Out] -1/9*(x^2*Sqrt[1 + (a + b*x)^2])/b + ((4 - 11*a^2 + 5*a*b*x)*Sqrt[1 + (a +
b*x)^2])/(18*b^3) - (a*(3 - 2*a^2)*ArcSinh[a + b*x])/(6*b^3) + (x^3*ArcSinh
[a + b*x])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \text{arcsinh}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\text{arcsinh}(a+bx) - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$
\begin{aligned}
&= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx) \\
&\quad - \frac{1}{9}\operatorname{Subst}\left(\int \frac{\left(-\frac{2-3a^2}{b^2} - \frac{5ax}{b^2}\right)\left(-\frac{a}{b} + \frac{x}{b}\right)}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3} \\
&\quad + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx) - \frac{(a(3-2a^2))\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6b^3} \\
&= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3} \\
&\quad - \frac{a(3-2a^2)\operatorname{arcsinh}(a+bx)}{6b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int x^2\operatorname{arcsinh}(a+bx)\,dx \\
&= \frac{(4-11a^2+5abx-2b^2x^2)\sqrt{1+a^2+2abx+b^2x^2} + (-9a+6a^3+6b^3x^3)\operatorname{arcsinh}(a+bx)}{18b^3}
\end{aligned}
$$

```
[In] Integrate[x^2*ArcSinh[a + b*x],x]
```

```
[Out] ((4 - 11*a^2 + 5*a*b*x - 2*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-9
*a + 6*a^3 + 6*b^3*x^3)*ArcSinh[a + b*x])/(18*b^3)
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-\frac{(bx+a)^2\sqrt{1+(bx+a)^2}}{9}+\frac{2\sqrt{1+(bx+a)^2}}{9}-a^2\sqrt{1+(}$ $\over b^3$ |
| default | $\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-\frac{(bx+a)^2\sqrt{1+(bx+a)^2}}{9}+\frac{2\sqrt{1+(bx+a)^2}}{9}-a^2\sqrt{1+(}$ $\over b^3$ |
| parts | $\frac{x^3\operatorname{arcsinh}(bx+a)}{3}-\frac{b\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+}\right.}{b\sqrt{b^2}}\right.}{2b}\right.}{3b}\right.}{3}$ |

```
[In] int(x^2*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(arcsinh(b*x+a)*a^2*(b*x+a)-arcsinh(b*x+a)*a*(b*x+a)^2+1/3*arcsinh(b*
x+a)*(b*x+a)^3-1/9*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(1+(b*x+a)^2)^(1/2)-a^
2*(1+(b*x+a)^2)^(1/2)+a*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a)
))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^2\operatorname{arcsinh}(a+bx)\,dx$$
$$=\frac{3\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+2\,abx}}{18\,b^3}$$

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))/b^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(83) = 166$.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx$$

$$= \begin{cases} \frac{a^3 \operatorname{asinh}(a+bx)}{3b^3} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}}{18b^2} - \frac{a \operatorname{asinh}(a+bx)}{2b^3} + \frac{x^3 \operatorname{asinh}(a+bx)}{3} - \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{9b} \\ \frac{x^3 \operatorname{asinh}(a)}{3} \end{cases}$$

```
[In] integrate(x**2*asinh(b*x+a),x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(18*b**2
) - a*asinh(a + b*x)/(2*b**3) + x**3*asinh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)/(9*b) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(9*b*
*3), Ne(b, 0)), (x**3*asinh(a)/3, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(78) = 156$.

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.33

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx = \frac{1}{3}x^3 \operatorname{arsinh}(bx+a)$$
$$-\frac{1}{18}b\left(\frac{2\sqrt{b^2x^2+2abx+a^2+1}x^2}{b^2} - \frac{15a^3 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^4} - \frac{5\sqrt{b^2x^2+2abx+a^2+1}ax}{b^3} +$$

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/
b^2 - 15*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3 + 9*(a^2 + 1)*a*arcsinh(2*(
b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 15*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)/b^
4)
```

## Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx = \frac{1}{3}x^3 \log\left(bx+a+\sqrt{(bx+a)^2+1}\right)$$
$$-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b-4\,b}{b^5}\right)+\frac{3\,(2\,a^3-3\,a)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2}\right.\right.\right.}{b^3|b|}$$

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b - 4*b)/b^5) + 3*(2*a^3 - 3*a
)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*ab
s(b)))*b
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}(a+bx)\,dx = \int x^2 \operatorname{asinh}(a+b\,x)\,dx$$

```
[In] int(x^2*asinh(a + b*x),x)
```

```
[Out] int(x^2*asinh(a + b*x), x)
```

# 3.60 $\int x\mathbf{arcsinh}(a+bx)\,dx$

Optimal result . . . 569
Rubi [A] (verified) . . . 569
Mathematica [A] (verified) . . . 571
Maple [A] (verified) . . . 571
Fricas [A] (verification not implemented) . . . 572
Sympy [A] (verification not implemented) . . . 572
Maxima [B] (verification not implemented) . . . 572
Giac [A] (verification not implemented) . . . 573
Mupad [F(-1)] . . . 573

## Optimal result

Integrand size = 8, antiderivative size = 76

$$\int x\operatorname{arcsinh}(a+bx)\,dx = \frac{3a\sqrt{1+(a+bx)^2}}{4b^2} - \frac{x\sqrt{1+(a+bx)^2}}{4b} + \frac{(1-2a^2)\operatorname{arcsinh}(a+bx)}{4b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)$$

```
[Out] 1/4*(-2*a^2+1)*arcsinh(b*x+a)/b^2+1/2*x^2*arcsinh(b*x+a)+3/4*a*(1+(b*x+a)^2
)^(1/2)/b^2-1/4*x*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5828, 757, 655, 221}

$$\int x\operatorname{arcsinh}(a+bx)\,dx = \frac{(1-2a^2)\operatorname{arcsinh}(a+bx)}{4b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx) + \frac{3a\sqrt{(a+bx)^2+1}}{4b^2} - \frac{x\sqrt{(a+bx)^2+1}}{4b}$$

```
[In] Int[x*ArcSinh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 + (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 + (a + b*x)^2])/(4*b) + ((1
 - 2*a^2)*ArcSinh[a + b*x])/(4*b^2) + (x^2*ArcSinh[a + b*x])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{x\sqrt{1+(a+bx)^2}}{4b} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-\frac{1-2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2} - \frac{x\sqrt{1+(a+bx)^2}}{4b} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx) \\
&\quad + \frac{(1-2a^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{4b^2}
\end{aligned}
$$

$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2} - \frac{x\sqrt{1+(a+bx)^2}}{4b} + \frac{(1-2a^2)\operatorname{arcsinh}(a+bx)}{4b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int x\operatorname{arcsinh}(a+bx)\,dx = \frac{(3a-bx)\sqrt{1+a^2+2abx+b^2x^2} + (1-2a^2+2b^2x^2)\operatorname{arcsinh}(a+bx)}{4b^2}$$

```
[In] Integrate[x*ArcSinh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 - 2*a^2 + 2*b^2*x^2)*Ar
cSinh[a + b*x])/(4*b^2)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2} - \operatorname{arcsinh}(bx+a)a(bx+a) - \frac{(bx+a)\sqrt{1+(bx+a)^2}}{4} + \frac{\operatorname{arcsinh}(bx+a)}{4} + a\sqrt{1+(bx+a)^2}}{b^2}$ |
| default | $\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2} - \operatorname{arcsinh}(bx+a)a(bx+a) - \frac{(bx+a)\sqrt{1+(bx+a)^2}}{4} + \frac{\operatorname{arcsinh}(bx+a)}{4} + a\sqrt{1+(bx+a)^2}}{b^2}$ |
| parts | $\frac{x^2\operatorname{arcsinh}(bx+a)}{2} - \frac{b\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2} - \frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2} - \frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b} - \frac{\left(a^2+1\right)\ln$ |

```
[In] int(x*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)*(b*x+a)^2-arcsinh(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(1
+(b*x+a)^2)^(1/2)+1/4*arcsinh(b*x+a)+a*(1+(b*x+a)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int x\operatorname{arcsinh}(a+bx)\,dx$$
$$=\frac{(2\,b^2x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{4\,b^2}$$

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x\operatorname{arcsinh}(a+bx)\,dx$$
$$=\begin{cases}-\frac{a^2\operatorname{asinh}(a+bx)}{2b^2}+\frac{3a\sqrt{a^2+2abx+b^2x^2+1}}{4b^2}+\frac{x^2\operatorname{asinh}(a+bx)}{2}-\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{4b}+\frac{\operatorname{asinh}(a+bx)}{4b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{asinh}(a)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(4*b**2) + x**2*asinh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x**
*2 + 1)/(4*b) + asinh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*asinh(a)/2, True)
)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(64) = 128.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.96

$$\int x\operatorname{arcsinh}(a+bx)\,dx=\frac{1}{2}\,x^2\operatorname{arsinh}\left(bx+a\right)$$
$$-\frac{1}{4}\,b\left(\frac{3\,a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}x}{b^2}-\frac{(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^3}\right.$$

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(b*x + a) - 1/4*b*(3*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2))/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - (a
^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)
```

## Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int x\operatorname{arcsinh}(a+bx)\,dx = \frac{1}{2}x^2\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)$$
$$-\frac{1}{4}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b^2|b|}\right)$$

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(x/b^2 - 3*a/b^3) - (2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b)))*b
```

## Mupad [F(-1)]

Timed out.

$$\int x\operatorname{arcsinh}(a+bx)\,dx = \int x\,\operatorname{asinh}(a+b\,x)\;dx$$

```
[In] int(x*asinh(a + b*x),x)
```

```
[Out] int(x*asinh(a + b*x), x)
```

# 3.61 $\int \operatorname{arcsinh}(a+bx)\,dx$

Optimal result . . . 574
Rubi [A] (verified) . . . 574
Mathematica [B] (verified) . . . 575
Maple [A] (verified) . . . 575
Fricas [A] (verification not implemented) . . . 576
Sympy [A] (verification not implemented) . . . 576
Maxima [A] (verification not implemented) . . . 577
Giac [B] (verification not implemented) . . . 577
Mupad [B] (verification not implemented) . . . 577

## Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \operatorname{arcsinh}(a+bx)\,dx = -\frac{\sqrt{1+(a+bx)^2}}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b}$$

```
[Out] (b*x+a)*arcsinh(b*x+a)/b-(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 267}

$$\int \operatorname{arcsinh}(a+bx)\,dx = \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{\sqrt{(a+bx)^2+1}}{b}$$

```
[In] Int[ArcSinh[a + b*x],x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/b) + ((a + b*x)*ArcSinh[a + b*x])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}(\int \operatorname{arcsinh}(x)\,dx, x, a+bx)}{b} \\ &= \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\ &= -\frac{\sqrt{1+(a+bx)^2}}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b}\end{aligned}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \operatorname{arcsinh}(a+bx)\,dx &= x\operatorname{arcsinh}(a+bx) \\ &\quad - \frac{\sqrt{1+a^2+2abx+b^2x^2} + 2a\operatorname{arctanh}\left(\frac{bx}{\sqrt{1+a^2}-\sqrt{1+a^2+2abx+b^2x^2}}\right)}{b}\end{aligned}$$

```
[In] Integrate[ArcSinh[a + b*x],x]
```

```
[Out] x*ArcSinh[a + b*x] - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + 2*a*ArcTanh[(b*x)
/(Sqrt[1 + a^2] - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/b
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(bx+a)\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$ | 31 |
| default | $\frac{(bx+a)\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$ | 31 |
| parts | $x\operatorname{arcsinh}(bx+a)-b\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)$ | 84 |

```
[In] int(arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*((b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \operatorname{arcsinh}(a+bx)\,dx = \frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

```
[In] integrate(arcsinh(b*x+a),x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))/b
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \operatorname{arcsinh}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{asinh}(a+bx)}{b}+x\operatorname{asinh}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for } b\neq 0 \\ x\operatorname{asinh}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(b*x+a),x)
[Out] Piecewise((a*asinh(a + b*x)/b + x*asinh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \operatorname{arcsinh}(a+bx)\,dx = \frac{(bx+a)\operatorname{arsinh}(bx+a) - \sqrt{(bx+a)^2+1}}{b}$$

```
[In] integrate(arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arcsinh(b*x + a) - sqrt((b*x + a)^2 + 1))/b
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\begin{aligned}&\int \operatorname{arcsinh}(a+bx)\,dx \\ &= -b\left(\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\right) \\ &\quad + x\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)\end{aligned}$$

```
[In] integrate(arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b*
abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2) + x*log(b*x + a + sqrt((b*
x + a)^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \operatorname{arcsinh}(a+bx)\,dx = x\,\operatorname{asinh}(a+b\,x) - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{b} \\ + \frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}\end{aligned}$$

```
[In] int(asinh(a + b*x),x)
```

```
[Out] x*asinh(a + b*x) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/b + (a*log((a^2 + b^
2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

# 3.62 $\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx$

Optimal result . . . 578
Rubi [A] (verified) . . . 578
Mathematica [A] (verified) . . . 581
Maple [B] (verified) . . . 581
Fricas [F] . . . 582
Sympy [F] . . . 582
Maxima [F] . . . 582
Giac [F] . . . 583
Mupad [F(-1)] . . . 583

## Optimal result

Integrand size = 10, antiderivative size = 131

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

```
[Out] -1/2*arcsinh(b*x+a)^2+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a
^2+1)^(1/2)))+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))+polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5859, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right) + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right) - \frac{1}{2}\operatorname{arcsinh}(a+bx)^2$$

```
[In] Int[ArcSinh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 +
 a^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \frac{\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - \operatorname{Subst}\left(\int \log\left(1+\frac{e^x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad - \operatorname{Subst}\left(\int \log\left(1+\frac{e^x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right) \\
&\quad - \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)
\end{aligned}$$

$$= -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)$$
$$+ \operatorname{arcsinh}(a+bx)\log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$
$$+ \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right)b}\right)$$
$$+ \operatorname{arcsinh}(a+bx)\log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right)b}\right)$$
$$+ \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{-a+\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

```
[In] Integrate[ArcSinh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(a
/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((
-(a/b) + Sqrt[1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcSinh[a + b*x]/(-a + Sqrt[
1 + a^2]))] + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(153) = 306.

Time = 0.66 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.96

| method | result |
|---|---|
| derivativedivides | $-\frac{\operatorname{arcsinh}(bx+a)^2}{2} + \frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx-}{a+\sqrt{a^2+1}}\right.\right.}{a^2+1}$ |
| default | $-\frac{\operatorname{arcsinh}(bx+a)^2}{2} + \frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx-}{a+\sqrt{a^2+1}}\right.\right.}{a^2+1}$ |

```
[In] int(arcsinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsinh(b*x+a)^2+(a^2+1+(a^2+1)^(1/2)*a)/(a^2+1)*arcsinh(b*x+a)*(2*ln(
((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a^2+ln(((a^2+1)^
(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*ln(((a^2+1)^(1/2)-b*x-(
1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*(a^2+1)^(1/2)*a+ln(((a^2+1)^(1/2)+b*
x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))+dilog(((a^2+1)^(1/2)+b*x+(1+(b*
x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(
1/2))/(a+(a^2+1)^(1/2)))+a*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b
*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-a*arcsinh(b*x+a)/(a^2+1)^(1/2)*l
n(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{asinh}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(asinh(b*x+a)/x,x)
[Out] Integral(asinh(a + b*x)/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(b*x + a)/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x}\,dx$$

```
[In] int(asinh(a + b*x)/x,x)
[Out] int(asinh(a + b*x)/x, x)
```

# 3.63 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx$

Optimal result . . . 584
Rubi [A] (verified) . . . 584
Mathematica [A] (verified) . . . 585
Maple [A] (verified) . . . 586
Fricas [B] (verification not implemented) . . . 586
Sympy [F] . . . 587
Maxima [B] (verification not implemented) . . . 587
Giac [B] (verification not implemented) . . . 587
Mupad [F(-1)] . . . 588

## Optimal result

Integrand size = 10, antiderivative size = 57

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = -\frac{\operatorname{arcsinh}(a+bx)}{x} - \frac{b\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

```
[Out] -arcsinh(b*x+a)/x-b*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b*x+a)^2)^(1/2)
)/(a^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5859, 5828, 739, 212}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = -\frac{b\operatorname{arctanh}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\operatorname{arcsinh}(a+bx)}{x}$$

```
[In] Int[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\text{arcsinh}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right) \\
&= -\frac{\text{arcsinh}(a+bx)}{x} - \frac{b\text{arctanh}\left(\frac{b\left(\frac{1}{b}+\frac{a(a+bx)}{b}\right)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)}{x^2}\,dx = -\frac{\text{arcsinh}(a+bx)}{x} - \frac{b\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

```
[In] Integrate[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| parts | $-\frac{\operatorname{arcsinh}(bx+a)}{x}-\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}$ | 68 |
| derivativedivides | $b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$ | 75 |
| default | $b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$ | 75 |

```
[In] int(arcsinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(b*x+a)/x-b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.93

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx$$

$$= \frac{\sqrt{a^2+1}bx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+(a^2+1)x\log\left(-bx-a+\sqrt{b}\right.}{(a^2+1)x}$$

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + (a^
2 + 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (a^2 - (a^2 +
1)*x + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/((a^2 + 1)*x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^2}\,dx$$

```
[In] integrate(asinh(b*x+a)/x**2,x)
[Out] Integral(asinh(a + b*x)/x**2, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.95

$$\begin{aligned}&\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx \\ &= -\frac{b\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}\right)}{\sqrt{a^2+1}} \\ &\quad -\frac{\operatorname{arsinh}(bx+a)}{x}\end{aligned}$$

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="maxima")
[Out] -b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(b*x + a)/x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx &= \frac{b\log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} \\ &\quad -\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{x}\end{aligned}$$

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="giac")
[Out] b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 +
1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1)
))/sqrt(a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 + 1))/x
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^2}\,dx$$

```
[In] int(asinh(a + b*x)/x^2,x)
[Out] int(asinh(a + b*x)/x^2, x)
```

# 3.64 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx$

Optimal result . . . 589
Rubi [A] (verified) . . . 589
Mathematica [A] (verified) . . . 591
Maple [A] (verified) . . . 591
Fricas [B] (verification not implemented) . . . 592
Sympy [F] . . . 592
Maxima [A] (verification not implemented) . . . 592
Giac [B] (verification not implemented) . . . 593
Mupad [F(-1)] . . . 593

## Optimal result

Integrand size = 10, antiderivative size = 92

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x} - \frac{\operatorname{arcsinh}(a+bx)}{2x^2} + \frac{ab^2\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)^{3/2}}$$

```
[Out] -1/2*arcsinh(b*x+a)/x^2+1/2*a*b^2*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b
*x+a)^2)^(1/2))/(a^2+1)^(3/2)-1/2*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5828, 745, 739, 212}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \frac{ab^2\operatorname{arctanh}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2\left(a^2+1\right)^{3/2}} - \frac{b\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)x} - \frac{\operatorname{arcsinh}(a+bx)}{2x^2}$$

```
[In] Int[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x) - ArcSinh[a + b*x]/(2*x^2) + (
a*b^2*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(2*
(1 + a^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x} - \frac{\text{arcsinh}(a+bx)}{2x^2} - \frac{(ab)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2\left(1+a^2\right)} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x} - \frac{\text{arcsinh}(a+bx)}{2x^2} + \frac{(ab)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x} - \frac{\text{arcsinh}(a+bx)}{2x^2} + \frac{ab^2\text{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)^{3/2}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = -\frac{\operatorname{arcsinh}(a+bx)+\frac{bx\left(\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}+abx\log(x)-abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)\right)}{(1+a^2)^{3/2}}}{2x^2}$$

```
[In] Integrate[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*(ArcSinh[a + b*x] + (b*x*(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x
^2] + a*b*x*Log[x] - a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2
 + 2*a*b*x + b^2*x^2]]))/(1 + a^2)^(3/2))/x^2
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| parts | $-\frac{\operatorname{arcsinh}(bx+a)}{2x^2}+\frac{b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\left(a^2+1\right)^{\frac{3}{2}}}\right)}{2}$ | 103 |
| derivativedivides | $b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{2(a^2+1)^{\frac{3}{2}}}\right)$ | 112 |
| default | $b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{2(a^2+1)^{\frac{3}{2}}}\right)$ | 112 |

```
[In] int(arcsinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsinh(b*x+a)/x^2+1/2*b*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a
*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(78) = 156.

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.57

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx$$

$$= \frac{\sqrt{a^2+1}ab^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)b^2x^2+(a^4+2\,a^2$$

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*a*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a
)/x) - (a^2 + 1)*b^2*x^2 + (a^4 + 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*b*x -
 (a^4 - (a^4 + 2*a^2 + 1)*x^2 + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)))/((a^4 + 2*a^2 + 1)*x^2)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^3}\,dx$$

```
[In] integrate(asinh(b*x+a)/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)/x**3, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx$$

$$= \frac{1}{2}\left(\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{(a^2+1)x}\right.$$

$$\left.-\frac{\operatorname{arsinh}(bx+a)}{2\,x^2}\right)$$

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^
2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((a
^2 + 1)*x))*b - 1/2*arcsinh(b*x + a)/x^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(78) = 156$.

Time $= 0.36$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 2.16$

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = -\frac{1}{2}\left(\frac{ab\log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{(a^2+1)^{\frac{3}{2}}} - \frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)(a^2+1)}\right) - \frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqr
t(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(
a^2 + 1)))/(a^2 + 1)^(3/2) - 2*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))*a*b + a^2*abs(b) + abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^2 - a^2 - 1)*(a^2 + 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 + 1))/
x^2
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^3}\,dx$$

```
[In] int(asinh(a + b*x)/x^3,x)
```

```
[Out] int(asinh(a + b*x)/x^3, x)
```

# 3.65 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx$

Optimal result . . . 594
Rubi [A] (verified) . . . 594
Mathematica [A] (verified) . . . 596
Maple [A] (verified) . . . 597
Fricas [B] (verification not implemented) . . . 597
Sympy [F] . . . 598
Maxima [B] (verification not implemented) . . . 598
Giac [B] (verification not implemented) . . . 599
Mupad [F(-1)] . . . 599

## Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)}{3x^3} + \frac{\left(1-2a^2\right)b^3 \operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{6\left(1+a^2\right)^{5/2}}$$

```
[Out] -1/3*arcsinh(b*x+a)/x^3+1/6*(-2*a^2+1)*b^3*arctanh((1+a*(b*x+a))/(a^2+1)^(1
/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(5/2)-1/6*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^
2+1/2*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5859, 5828, 759, 821, 739, 212}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = \frac{\left(1-2a^2\right)b^3 \operatorname{arctanh}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6\left(a^2+1\right)^{5/2}} + \frac{ab^2\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)^2 x} - \frac{b\sqrt{(a+bx)^2+1}}{6\left(a^2+1\right)x^2} - \frac{\operatorname{arcsinh}(a+bx)}{3x^3}$$

```
[In] Int[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -1/6*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^
2])/(2*(1 + a^2)^2*x) - ArcSinh[a + b*x]/(3*x^3) + ((1 - 2*a^2)*b^3*ArcTanh
```

```
[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(6*(1 + a^2)^(5/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} - \frac{\text{arcsinh}(a+bx)}{3x^3} - \frac{b^2\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6\left(1+a^2\right)} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\text{arcsinh}(a+bx)}{3x^3} \\
&\quad - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6\left(1+a^2\right)^2} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\text{arcsinh}(a+bx)}{3x^3} \\
&\quad + \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)}{6\left(1+a^2\right)^2} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\text{arcsinh}(a+bx)}{3x^3} \\
&\quad + \frac{\left(1-2a^2\right)b^3\text{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{6\left(1+a^2\right)^{5/2}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{\text{arcsinh}(a+bx)}{x^4}\,dx \\
&= \frac{-\sqrt{1+a^2}bx(1+a^2-3abx)\sqrt{1+a^2+2abx+b^2x^2} - 2(1+a^2)^{5/2}\,\text{arcsinh}(a+bx) + (-1+2a^2)\,b^3x^3\log(x}{6\left(1+a^2\right)^{5/2}x^3}
\end{aligned}$$

```
[In] Integrate[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] (-(Sqrt[1 + a^2]*b*x*(1 + a^2 - 3*a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
 - 2*(1 + a^2)^(5/2)*ArcSinh[a + b*x] + (-1 + 2*a^2)*b^3*x^3*Log[x] + (1 -
2*a^2)*b^3*x^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x +
 b^2*x^2]])/(6*(1 + a^2)^(5/2)*x^3)
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| parts | $-\frac{\operatorname{arcsinh}(bx+a)}{3x^3}+\frac{b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2}}{x}\right.}{\left(a^2+1\right)^{\frac{3}{2}}}\right.}{2\left(a^2+1\right)}\right.}{3}$ |
| derivativedivides | $b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+}\right.}{bx}\right.}{\left(a^2+1\right)^{\frac{3}{2}}}\right.}{2(a^2+1)}\right.$ |
| default | $b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+}\right.}{bx}\right.}{\left(a^2+1\right)^{\frac{3}{2}}}\right.}{2(a^2+1)}\right.$ |

```
[In] int(arcsinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(b*x+a)/x^3+1/3*b*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-3/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b/(a^2+1)^
(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
)+1/2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(111) = 222$.

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx$$

$$= \frac{(2\,a^2-1)\sqrt{a^2+1}b^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+3\,(a^3+a)b^3x^3+}$$

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 - 1)*sqrt(a^2 + 1)*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^
2 + 1) + a)/x) + 3*(a^3 + a)*b^3*x^3 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*x^3*log(
```

```
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(a^6 + 3*a^4 - (a^6 + 3*a
^4 + 3*a^2 + 1)*x^3 + 3*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)) + (3*(a^3 + a)*b^2*x^2 - (a^4 + 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))/((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^4}\,dx$$

```
[In] integrate(asinh(b*x+a)/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)/x**4, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(111) = 222$.

Time $= 0.18$ (sec) , antiderivative size $= 284$, normalized size of antiderivative $= 2.20$

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx =$$

$$-\frac{1}{6}\left(\frac{3\,a^2b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}\right)}{(a^2+1)^{\frac{5}{2}}} - \frac{b^2\operatorname{arsinh}\left(\frac{}{\sqrt{-4\,a^2b}}\right.}{}\right.$$

$$-\frac{\operatorname{arsinh}(bx+a)}{3\,x^3}$$

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - b^2*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2)
 - 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x) + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/((a^2 + 1)*x^2))*b - 1/3*arcsinh(b*x + a)/x^3
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.

Time = 0.38 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx$$

$$= \frac{1}{6}\,b\left(\frac{(2\,a^2b^2-b^2)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^4+2\,a^2+1)\sqrt{a^2+1}} - \frac{2\left(2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^2b^2-\right.}{}\right.$$

$$\left. - \frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{3\,x^3}\right)$$

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*((2*a^2*b^2 - b^2)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1) + 2*sqrt(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 2*(2*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*a^4*b^2 - 4*a^5*b*abs(b) - (x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^3*b^2 - 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))*a^2*b^2 - 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*b^2 - 4*a*b*abs(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2 - a^2 - 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 + 1)
)/x^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^4}\,dx$$

```
[In] int(asinh(a + b*x)/x^4,x)
```

```
[Out] int(asinh(a + b*x)/x^4, x)
```

# 3.66 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$

Optimal result . . . 600
Rubi [A] (verified) . . . 600
Mathematica [A] (verified) . . . 603
Maple [B] (verified) . . . 604
Fricas [B] (verification not implemented) . . . 605
Sympy [F] . . . 605
Maxima [B] (verification not implemented) . . . 605
Giac [B] (verification not implemented) . . . 606
Mupad [F(-1)] . . . 607

## Optimal result

Integrand size = 10, antiderivative size = 167

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} - \frac{\operatorname{arcsinh}(a+bx)}{4x^4} - \frac{a\left(3-2a^2\right)b^4\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{8\left(1+a^2\right)^{7/2}}$$

```
[Out] -1/4*arcsinh(b*x+a)/x^4-1/8*a*(-2*a^2+3)*b^4*arctanh((1+a*(b*x+a))/(a^2+1)^
(1/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(7/2)-1/12*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)
/x^3+5/24*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x^2+1/24*(-11*a^2+4)*b^3*(1+(
b*x+a)^2)^(1/2)/(a^2+1)^3/x
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5859, 5828, 759, 849, 821, 739, 212}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = -\frac{a\left(3-2a^2\right)b^4\operatorname{arctanh}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8\left(a^2+1\right)^{7/2}} + \frac{\left(4-11a^2\right)b^3\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^3x} + \frac{5ab^2\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^2x^2} - \frac{b\sqrt{(a+bx)^2+1}}{12\left(a^2+1\right)x^3} - \frac{\operatorname{arcsinh}(a+bx)}{4x^4}$$

```
[In] Int[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -1/12*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x^3) + (5*a*b^2*Sqrt[1 + (a + b*
x)^2])/(24*(1 + a^2)^2*x^2) + ((4 - 11*a^2)*b^3*Sqrt[1 + (a + b*x)^2])/(24*
(1 + a^2)^3*x) - ArcSinh[a + b*x]/(4*x^4) - (a*(3 - 2*a^2)*b^4*ArcTanh[(1 +
 a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(8*(1 + a^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^5}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} - \frac{\text{arcsinh}(a+bx)}{4x^4} - \frac{b^2\text{Subst}\left(\int \frac{\frac{3a}{b}+\frac{2x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)}{12\left(1+a^2\right)} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} - \frac{\text{arcsinh}(a+bx)}{4x^4} \\
&\quad + \frac{b^4\text{Subst}\left(\int \frac{-\frac{2\left(2-3a^2\right)}{b^2}+\frac{5ax}{b^2}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)}{24\left(1+a^2\right)^2} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} \\
&\quad - \frac{\text{arcsinh}(a+bx)}{4x^4} + \frac{\left(a(3-2a^2)\,b^3\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8\left(1+a^2\right)^3} \\
&= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} \\
&\quad - \frac{\text{arcsinh}(a+bx)}{4x^4} - \frac{\left(a(3-2a^2)\,b^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)}{8\left(1+a^2\right)^3}
\end{aligned}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x}$$
$$- \frac{\operatorname{arcsinh}(a+bx)}{4x^4} - \frac{a(3-2a^2)\,b^4\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{8\left(1+a^2\right)^{7/2}}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$$
$$= \frac{1}{8}\left(-\frac{b\sqrt{1+a^2+2abx+b^2x^2}(2+2a^4-5abx-5a^3bx-4b^2x^2+a^2(4+11b^2x^2))}{3\left(1+a^2\right)^3x^3}\right.$$
$$- \frac{2\operatorname{arcsinh}(a+bx)}{x^4} - \frac{a(-3+2a^2)\,b^4\log(x)}{\left(1+a^2\right)^{7/2}}$$
$$\left. + \frac{a(-3+2a^2)\,b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\left(1+a^2\right)^{7/2}}\right)$$

```
[In] Integrate[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] (-1/3*(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 - 5*a*b*x - 5*a^3*b*x
 - 4*b^2*x^2 + a^2*(4 + 11*b^2*x^2)))/((1 + a^2)^3*x^3) - (2*ArcSinh[a + b*
x])/x^4 - (a*(-3 + 2*a^2)*b^4*Log[x])/(1 + a^2)^(7/2) + (a*(-3 + 2*a^2)*b^4
*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1
 + a^2)^(7/2))/8
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(145) = 290.

Time = 0.02 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.07

| method | result |
| --- | --- |
| parts | $-\frac{\operatorname{arcsinh}(bx+a)}{4x^4}+b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{3\left(a^2+1\right)x^3}-\frac{5ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+}{}\right.}{2\left(a^2+1\right)}\right.}{}\right.}{}\right.$ |
| derivativedivides | $b^4\left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3}-\frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)b^2x^2}-\frac{3a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+2a}{}\right.}{2\left(a^2+1\right.}\right.}{}\right.}{}\right.$ |
| default | $b^4\left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3}-\frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)b^2x^2}-\frac{3a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+2a}{}\right.}{2\left(a^2+1\right)}\right.}{}\right.}{}\right.$ |

```
[In] int(arcsinh(b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsinh(b*x+a)/x^4+1/4*b*(-1/3/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-5/3*a*b/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a*b/
(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b/(a^2+1)^(3/2)*ln((2
*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+1/2*b^2/(
a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x))-2/3*b^2/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b/(a
^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2))/x)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(145) = 290.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$$

$$= \frac{3\,(2\,a^3-3\,a)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(11\,a^4+7\,a^2$$

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(2*a^3 - 3*a)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sq
rt(a^2 + 1) + a)/x) - (11*a^4 + 7*a^2 - 4)*b^4*x^4 + 6*(a^8 + 4*a^6 + 6*a^4
 + 4*a^2 + 1)*x^4*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*(a^
8 + 4*a^6 - (a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4 + 6*a^4 + 4*a^2 + 1)*log(
b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - ((11*a^4 + 7*a^2 - 4)*b^3*x^
3 - 5*(a^5 + 2*a^3 + a)*b^2*x^2 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*b*x)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^5}\,dx$$

```
[In] integrate(asinh(b*x+a)/x**5,x)
```

```
[Out] Integral(asinh(a + b*x)/x**5, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(145) = 290.

Time = 0.19 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$$

$$= \frac{1}{24}\left(\frac{15\,a^3b^3\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2|x|}}\right)}{(a^2+1)^{\frac{7}{2}}}-\frac{9\,ab^3\operatorname{arsinh}\left(\frac{}{\sqrt{-}}\right.}{}\right.$$

$$\left.-\frac{\operatorname{arsinh}(bx+a)}{4\,x^4}\right.$$

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(15*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 9*a*b^3*arcsinh(2*a*b*x/(sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^
(5/2) - 15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^3*x) + 4*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/((a^2 + 1)^2*x) + 5*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x^2) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
((a^2 + 1)*x^3))*b - 1/4*arcsinh(b*x + a)/x^4
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(145) = 290$.

Time = 0.36 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = -\frac{1}{24}\,b\left(\frac{3\,(2\,a^3b^3-3\,ab^3)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^6+3\,a^4+3\,a^2+1)\sqrt{a^2+1}} - \frac{2\left(6\,\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^5\right.}{}$$

$$-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{4\,x^4}$$

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/24*b*(3*(2*a^3*b^3 - 3*a*b^3)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1))
 - 2*(6*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^3*b^3 - 16*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 42*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^2*a^6*b^2*abs(b) + 20*a^8*b^2*abs(b) - 9*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 8*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^3*a^3*b^3 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^
5*b^3 + 36*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b)
+ 56*a^6*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a
*b^3 + 60*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 36*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 48*a^4*b^2*abs
(b) + 9*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 12*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*b^2*abs(b) + 8*a^2*b^2*abs(b) - 4*b
^2*abs(b))/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^2 - a^2 - 1)^3)) - 1/4*log(b*x + a + sqrt((b*x + a)^2 + 1))/x^4
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^5}\,dx$$

```
[In] int(asinh(a + b*x)/x^5,x)
[Out] int(asinh(a + b*x)/x^5, x)
```

# 3.67 $\int x^3 \mathbf{arcsinh}(a+bx)^2\,dx$

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## Optimal result

Integrand size = 12, antiderivative size = 331

$$\begin{aligned}\int x^3 \operatorname{arcsinh}(a+bx)^2\,dx = {} & \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} \\ & + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\ & + \frac{2a^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^4} \\ & + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{16b^4} \\ & - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^4} \\ & + \frac{2a(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\ & - \frac{(a+bx)^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b^4} \\ & - \frac{3\operatorname{arcsinh}(a+bx)^2}{32b^4} + \frac{3a^2\operatorname{arcsinh}(a+bx)^2}{4b^4} \\ & - \frac{a^4\operatorname{arcsinh}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2\end{aligned}$$

```
[Out] 4/3*a*x/b^3-2*a^3*x/b^3-3/32*(b*x+a)^2/b^4+3/4*a^2*(b*x+a)^2/b^4-2/9*a*(b*x
+a)^3/b^4+1/32*(b*x+a)^4/b^4-3/32*arcsinh(b*x+a)^2/b^4+3/4*a^2*arcsinh(b*x+
a)^2/b^4-1/4*a^4*arcsinh(b*x+a)^2/b^4+1/4*x^4*arcsinh(b*x+a)^2-4/3*a*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2*a^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
^4+3/16*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4-3/2*a^2*(b*x+a)*arcs
inh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2/3*a*(b*x+a)^2*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)/b^4-1/8*(b*x+a)^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int x^3 \operatorname{arcsinh}(a+bx)^2\,dx = &-\frac{a^4 \operatorname{arcsinh}(a+bx)^2}{4b^4} + \frac{2a^3\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{b^4} \\ &- \frac{2a^3 x}{b^3} + \frac{3a^2 \operatorname{arcsinh}(a+bx)^2}{4b^4} \\ &- \frac{3a^2(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{2b^4} \\ &+ \frac{3a^2(a+bx)^2}{4b^4} + \frac{2a(a+bx)^2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{3b^4} \\ &- \frac{4a\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{3b^4} - \frac{3\operatorname{arcsinh}(a+bx)^2}{32b^4} \\ &- \frac{(a+bx)^3\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{8b^4} \\ &+ \frac{3(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{16b^4} \\ &+ \frac{1}{4}x^4 \operatorname{arcsinh}(a+bx)^2 - \frac{2a(a+bx)^3}{9b^4} \\ &+ \frac{(a+bx)^4}{32b^4} - \frac{3(a+bx)^2}{32b^4} + \frac{4ax}{3b^3}\end{aligned}$$

```
[In] Int[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (4*a*x)/(3*b^3) - (2*a^3*x)/b^3 - (3*(a + b*x)^2)/(32*b^4) + (3*a^2*(a + b*
x)^2)/(4*b^4) - (2*a*(a + b*x)^3)/(9*b^4) + (a + b*x)^4/(32*b^4) - (4*a*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) + (2*a^3*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x])/b^4 + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(16*b^4) - (3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b
^4) + (2*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) - ((
a + b*x)^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(8*b^4) - (3*ArcSinh[a +
 b*x]^2)/(32*b^4) + (3*a^2*ArcSinh[a + b*x]^2)/(4*b^4) - (a^4*ArcSinh[a + b
*x]^2)/(4*b^4) + (x^4*ArcSinh[a + b*x]^2)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

`rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]`

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4 \operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4\operatorname{arcsinh}(x)}{b^4\sqrt{1+x^2}} - \frac{4a^3x\operatorname{arcsinh}(x)}{b^4\sqrt{1+x^2}}\right.\right. \\
&\qquad \left.\left. + \frac{6a^2x^2\operatorname{arcsinh}(x)}{b^4\sqrt{1+x^2}} - \frac{4ax^3\operatorname{arcsinh}(x)}{b^4\sqrt{1+x^2}} + \frac{x^4\operatorname{arcsinh}(x)}{b^4\sqrt{1+x^2}}\right)\,dx, x, a+bx\right) \\
&= \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{x^4\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} \\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{x^3\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^4} \\
&\quad - \frac{(3a^2)\,\text{Subst}\left(\int \frac{x^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^4} \\
&\quad + \frac{(2a^3)\,\text{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^4} - \frac{a^4\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} \\
&= \frac{2a^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^4} - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^4} \\
&\quad + \frac{2a(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\
&\quad - \frac{(a+bx)^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b^4} - \frac{a^4\operatorname{arcsinh}(a+bx)^2}{4b^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2 + \frac{\text{Subst}\left(\int x^3\,dx, x, a+bx\right)}{8b^4} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{x^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b^4} - \frac{(2a)\text{Subst}\left(\int x^2\,dx, x, a+bx\right)}{3b^4} \\
&\quad - \frac{(4a)\text{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{3b^4} + \frac{(3a^2)\,\text{Subst}\left(\int x\,dx, x, a+bx\right)}{2b^4} \\
&\quad + \frac{(3a^2)\,\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} - \frac{(2a^3)\,\text{Subst}\left(\int 1\,dx, x, a+bx\right)}{b^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2a^3x}{b^3} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} \\
&\quad - \frac{4a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} + \frac{2a^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^4} \\
&\quad + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{16b^4} \\
&\quad - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^4} \\
&\quad + \frac{2a(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\
&\quad - \frac{(a+bx)^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b^4} + \frac{3a^2\operatorname{arcsinh}(a+bx)^2}{4b^4} \\
&\quad - \frac{a^4\operatorname{arcsinh}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2 - \frac{3\operatorname{Subst}(\int x\,dx, x, a+bx)}{16b^4} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{16b^4} + \frac{(4a)\operatorname{Subst}(\int 1\,dx, x, a+bx)}{3b^4} \\
&= \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} \\
&\quad + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\
&\quad + \frac{2a^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^4} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{16b^4} \\
&\quad - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^4} \\
&\quad + \frac{2a(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\
&\quad - \frac{(a+bx)^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b^4} - \frac{3\operatorname{arcsinh}(a+bx)^2}{32b^4} \\
&\quad + \frac{3a^2\operatorname{arcsinh}(a+bx)^2}{4b^4} - \frac{a^4\operatorname{arcsinh}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.44

$$
\begin{aligned}
&\int x^3\operatorname{arcsinh}(a+bx)^2\,dx \\
&= \frac{bx(-300a^3 + 78a^2bx + a(330 - 28b^2x^2) + 9bx(-3+b^2x^2)) + 6\sqrt{1+a^2+2abx+b^2x^2}(50a^3 + 9bx - 26a^2b}{288b^4}
\end{aligned}
$$

```
[In] Integrate[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-300*a^3 + 78*a^2*b*x + a*(330 - 28*b^2*x^2) + 9*b*x*(-3 + b^2*x^2))
+ 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*
x^3 + a*(-55 + 14*b^2*x^2))*ArcSinh[a + b*x] - 9*(3 - 24*a^2 + 8*a^4 - 8*b^
4*x^4)*ArcSinh[a + b*x]^2)/(288*b^4)
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)^4}{4}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^3}{8}+\frac{3\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{16}-\frac{3\operatorname{arcsinh}(bx+a)^2}{32}+\frac{(bx+a)}{32}$ |
| default | $\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)^4}{4}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^3}{8}+\frac{3\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{16}-\frac{3\operatorname{arcsinh}(bx+a)^2}{32}+\frac{(bx+a)}{32}$ |

```
[In] int(x^3*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arcsinh(b*x+a)^2*(b*x+a)^4-1/8*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2
)*(b*x+a)^3+3/16*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)-3/32*arcsinh(b*
x+a)^2+1/32*(b*x+a)^4-3/32*(b*x+a)^2-3/32-1/9*a*(9*(b*x+a)^3*arcsinh(b*x+a)
^2-6*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2+2*(b*x+a)^3+12*arcsinh(b*
x+a)*(1+(b*x+a)^2)^(1/2)-12*b*x-12*a)+3/4*a^2*(2*arcsinh(b*x+a)^2*(b*x+a)^2
-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)
-a^3*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2
*a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.55

$$\int x^3 \operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \frac{9\,b^4x^4 - 28\,ab^3x^3 + 3\,(26\,a^2-9)b^2x^2 - 30\,(10\,a^3-11\,a)bx + 9\,(8\,b^4x^4 - 8\,a^4 + 24\,a^2 - 3)\log\left(bx+a+\sqrt{}\right.}{}$$

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*b^4*x^4 - 28*a*b^3*x^3 + 3*(26*a^2 - 9)*b^2*x^2 - 30*(10*a^3 - 11*
a)*b*x + 9*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2 - 6*(6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b
*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)))/b^4
```

## Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.11

$$\int x^3 \operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \begin{cases} -\frac{a^4 \operatorname{asinh}^2(a+bx)}{4b^4} - \frac{25a^3x}{24b^3} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^4} + \frac{13a^2x^2}{48b^2} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^3} + \frac{3a^2\operatorname{asinh}}{4} \\ \frac{x^4 \operatorname{asinh}^2(a)}{4} \end{cases}$$

```
[In] integrate(x**3*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)**2/(4*b**4) - 25*a**3*x/(24*b**3) + 25*a**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**4) + 13*a**2*x*
*2/(48*b**2) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)/(24*b**3) + 3*a**2*asinh(a + b*x)**2/(4*b**4) - 7*a*x**3/(72*b) + 7*a*x**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**2) + 55*a*x/(4
8*b**3) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(48*b**4
) + x**4*asinh(a + b*x)**2/4 + x**4/32 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*asinh(a + b*x)/(8*b) - 3*x**2/(32*b**2) + 3*x*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)*asinh(a + b*x)/(16*b**3) - 3*asinh(a + b*x)**2/(32*b**4), N
e(b, 0)), (x**4*asinh(a)**2/4, True))
```

## Maxima [F]

$$\int x^3 \operatorname{arcsinh}(a+bx)^2\,dx = \int x^3 \operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(1/2*
(b^3*x^6 + 2*a*b^2*x^5 + (a^2*b + b)*x^4 + (b^2*x^5 + a*b*x^4)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

## Giac [F]

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx = \int x^3\,\text{arsinh}\,(bx+a)^2\,\,dx$$

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx = \int x^3\,\text{asinh}(a+b\,x)^2\,dx$$

```
[In] int(x^3*asinh(a + b*x)^2,x)
[Out] int(x^3*asinh(a + b*x)^2, x)
```

# 3.68 $\int x^2 \mathbf{arcsinh}(a+bx)^2\,dx$

Optimal result . . . 616
Rubi [A] (verified) . . . 617
Mathematica [A] (verified) . . . 620
Maple [A] (verified) . . . 620
Fricas [A] (verification not implemented) . . . 620
Sympy [A] (verification not implemented) . . . 621
Maxima [F] . . . 621
Giac [F] . . . 622
Mupad [F(-1)] . . . 622

## Optimal result

Integrand size = 12, antiderivative size = 211

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx = &-\frac{4x}{9b^2}+\frac{2a^2x}{b^2}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}\\ &+\frac{4\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3}\\ &-\frac{2a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3}\\ &+\frac{a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3}\\ &-\frac{2(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3}\\ &-\frac{a\operatorname{arcsinh}(a+bx)^2}{2b^3}+\frac{a^3\operatorname{arcsinh}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2\end{aligned}$$

```
[Out] -4/9*x/b^2+2*a^2*x/b^2-1/2*a*(b*x+a)^2/b^3+2/27*(b*x+a)^3/b^3-1/2*a*arcsinh
(b*x+a)^2/b^3+1/3*a^3*arcsinh(b*x+a)^2/b^3+1/3*x^3*arcsinh(b*x+a)^2+4/9*arc
sinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2*a^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2
)/b^3+a*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2/9*(b*x+a)^2*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx = {} & \frac{a^3 \operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{2a^2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{b^3} + \frac{2a^2 x}{b^2} \\ & - \frac{a \operatorname{arcsinh}(a+bx)^2}{2b^3} + \frac{a(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{b^3} \\ & - \frac{2(a+bx)^2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{9b^3} \\ & + \frac{4\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{9b^3} \\ & + \frac{1}{3}x^3 \operatorname{arcsinh}(a+bx)^2 - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} - \frac{4x}{9b^2}\end{aligned}$$

```
[In] Int[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-4*x)/(9*b^2) + (2*a^2*x)/b^2 - (a*(a + b*x)^2)/(2*b^3) + (2*(a + b*x)^3)/
(27*b^3) + (4*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(9*b^3) - (2*a^2*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b^3 + (a*(a + b*x)*Sqrt[1 + (a + b*x)^2
]*ArcSinh[a + b*x])/b^3 - (2*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
b*x])/(9*b^3) - (a*ArcSinh[a + b*x]^2)/(2*b^3) + (a^3*ArcSinh[a + b*x]^2)/(
3*b^3) + (x^3*ArcSinh[a + b*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \operatorname{arcsinh}(x)^2\, dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\, dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2 - \frac{2}{3}\operatorname{Subst}\left(\int \left(-\frac{a^3\operatorname{arcsinh}(x)}{b^3\sqrt{1+x^2}} + \frac{3a^2x\operatorname{arcsinh}(x)}{b^3\sqrt{1+x^2}} - \frac{3ax^2\operatorname{arcsinh}(x)}{b^3\sqrt{1+x^2}} + \frac{x^3\operatorname{arcsinh}(x)}{b^3\sqrt{1+x^2}}\right)\,dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2 - \frac{2\operatorname{Subst}\left(\int \frac{x^3\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{3b^3} + \frac{(2a)\operatorname{Subst}\left(\int \frac{x^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3} - \frac{(2a^2)\operatorname{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3} + \frac{(2a^3)\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{3b^3}$$

$$= -\frac{2a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} - \frac{2(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2 + \frac{2\operatorname{Subst}\left(\int x^2\,dx, x, a+bx\right)}{9b^3} + \frac{4\operatorname{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{9b^3} - \frac{a\operatorname{Subst}\left(\int x\,dx, x, a+bx\right)}{b^3} - \frac{a\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3} + \frac{(2a^2)\operatorname{Subst}\left(\int 1\,dx, x, a+bx\right)}{b^3}$$

$$= \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} - \frac{2(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3} - \frac{a\operatorname{arcsinh}(a+bx)^2}{2b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2 - \frac{4\operatorname{Subst}\left(\int 1\,dx, x, a+bx\right)}{9b^3}$$

$$= -\frac{4x}{9b^2} + \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^3} - \frac{2(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{9b^3} - \frac{a\operatorname{arcsinh}(a+bx)^2}{2b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^2$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx$$
$$= \frac{bx(-24+66a^2-15abx+4b^2x^2)-6\sqrt{1+a^2+2abx+b^2x^2}(-4+11a^2-5abx+2b^2x^2)\operatorname{arcsinh}(a+bx)+}{54b^3}$$

```
[In] Integrate[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-24 + 66*a^2 - 15*a*b*x + 4*b^2*x^2) - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2
*x^2]*(-4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x] + 9*(-3*a + 2*a^
3 + 2*b^3*x^3)*ArcSinh[a + b*x]^2)/(54*b^3)
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| derivativedivides | $\frac{(bx+a)^3\operatorname{arcsinh}(bx+a)^2}{3}+\frac{4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{9}-\frac{2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^2}{9}-\frac{4bx}{9}-\frac{4a}{9}+\frac{2(bx+a)^3}{27}-\frac{a\left(2\operatorname{arcsinh}\right.}{}$ |
| default | $\frac{(bx+a)^3\operatorname{arcsinh}(bx+a)^2}{3}+\frac{4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{9}-\frac{2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^2}{9}-\frac{4bx}{9}-\frac{4a}{9}+\frac{2(bx+a)^3}{27}-\frac{a\left(2\operatorname{arcsinh}\right.}{}$ |

```
[In] int(x^2*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^3*arcsinh(b*x+a)^2+4/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2
)-2/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2-4/9*b*x-4/9*a+2/27*(b*x+
a)^3-1/2*a*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/
2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)+a^2*(arcsinh(b*x+a)^2*(b*x+a)-2*ar
csinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx$$
$$= \frac{4\,b^3x^3-15\,ab^2x^2+6\,(11\,a^2-4)bx+9\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-6\,(2}{54\,b^3}$$

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*b^3*x^3 - 15*a*b^2*x^2 + 6*(11*a^2 - 4)*b*x + 9*(2*b^3*x^3 + 2*a^3
- 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^2*x^2 -
5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^3
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \begin{cases} \frac{a^3 \operatorname{asinh}^2(a+bx)}{3b^3} + \frac{11a^2x}{9b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^3} - \frac{5ax^2}{18b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^2} - \frac{a\operatorname{asinh}^2(a+bx)}{2b^3} \\ \frac{x^3 \operatorname{asinh}^2(a)}{3} \end{cases}$$

```
[In] integrate(x**2*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**2/(3*b**3) + 11*a**2*x/(9*b**2) - 11*a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3) - 5*a*x**2/(18*
b) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**2) - a
*asinh(a + b*x)**2/(2*b**3) + x**3*asinh(a + b*x)**2/3 + 2*x**3/27 - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b) - 4*x/(9*b**2) +
 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3), Ne(b, 0)),
 (x**3*asinh(a)**2/3, True))
```

## Maxima [F]

$$\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx = \int x^2 \operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2/3*
(b^3*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

## Giac [F]

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx = \int x^2\,\text{arsinh}\,(bx+a)^2\,\,dx$$

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx = \int x^2\,\text{asinh}(a+b\,x)^2\,dx$$

```
[In] int(x^2*asinh(a + b*x)^2,x)
[Out] int(x^2*asinh(a + b*x)^2, x)
```

# 3.69 $\int x\text{arcsinh}(a+bx)^2\,dx$

Optimal result . . . 623
Rubi [A] (verified) . . . 623
Mathematica [A] (verified) . . . 626
Maple [A] (verified) . . . 626
Fricas [A] (verification not implemented) . . . 626
Sympy [A] (verification not implemented) . . . 627
Maxima [F] . . . 627
Giac [F] . . . 627
Mupad [F(-1)] . . . 628

## Optimal result

Integrand size = 10, antiderivative size = 126

$$\int x\text{arcsinh}(a+bx)^2\,dx = -\frac{2ax}{b} + \frac{(a+bx)^2}{4b^2} + \frac{2a\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{2b^2} + \frac{\text{arcsinh}(a+bx)^2}{4b^2} - \frac{a^2\text{arcsinh}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\text{arcsinh}(a+bx)^2$$

```
[Out] -2*a*x/b+1/4*(b*x+a)^2/b^2+1/4*arcsinh(b*x+a)^2/b^2-1/2*a^2*arcsinh(b*x+a)^
2/b^2+1/2*x^2*arcsinh(b*x+a)^2+2*a*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2-1
/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$\int x\text{arcsinh}(a+bx)^2\,dx = -\frac{a^2\text{arcsinh}(a+bx)^2}{2b^2} - \frac{\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)}{2b^2} + \frac{\text{arcsinh}(a+bx)^2}{4b^2} + \frac{2a\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{b^2} + \frac{1}{2}x^2\text{arcsinh}(a+bx)^2 + \frac{(a+bx)^2}{4b^2} - \frac{2ax}{b}$$

```
[In] Int[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*a*x)/b + (a + b*x)^2/(4*b^2) + (2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x])/b^2 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b^2) + Arc
```

```
Sinh[a + b*x]^2/(4*b^2) - (a^2*ArcSinh[a + b*x]^2)/(2*b^2) + (x^2*ArcSinh[a
 + b*x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^2 - \text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^2 \\
&\quad - \text{Subst}\left(\int \left(\frac{a^2\operatorname{arcsinh}(x)}{b^2\sqrt{1+x^2}} - \frac{2ax\operatorname{arcsinh}(x)}{b^2\sqrt{1+x^2}} + \frac{x^2\operatorname{arcsinh}(x)}{b^2\sqrt{1+x^2}}\right)\,dx, x, a+bx\right) \\
&= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{x^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^2} \\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^2} - \frac{a^2\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^2} \\
&= \frac{2a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^2} \\
&\quad - \frac{a^2\operatorname{arcsinh}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^2 + \frac{\text{Subst}(\int x\,dx, x, a+bx)}{2b^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^2} - \frac{(2a)\text{Subst}(\int 1\,dx, x, a+bx)}{b^2} \\
&= -\frac{2ax}{b} + \frac{(a+bx)^2}{4b^2} + \frac{2a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^2} \\
&\quad - \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^2} + \frac{\operatorname{arcsinh}(a+bx)^2}{4b^2} \\
&\quad - \frac{a^2\operatorname{arcsinh}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^2
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx$$
$$=\frac{bx(-6a+bx)+2(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)+\left(1-2a^2+2b^2x^2\right)\operatorname{arcsinh}(a+bx)^2}{4b^2}$$

```
[In] Integrate[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-6*a + b*x) + 2*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + (1 - 2*a^2 + 2*b^2*x^2)*ArcSinh[a + b*x]^2)/(4*b^2)
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\operatorname{arcsinh}(bx+a)^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a)^2(bx+a)-2\right.}{b^2}$ |
| default | $\frac{\frac{\operatorname{arcsinh}(bx+a)^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a)^2(bx+a)-2\right.}{b^2}$ |

```
[In] int(x*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)-1/2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^2+1/4*(b*x+a)^2+1/4-a*(arcsinh(b*x+a)^2*(b
*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx$$
$$=\frac{b^2x^2-6\,abx+\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx-}{4\,b^2}$$

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2 - 6*a*b*x + (2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^2 - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*
log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \begin{cases} -\frac{a^2\operatorname{asinh}^2(a+bx)}{2b^2} - \frac{3ax}{2b} + \frac{3a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b^2} + \frac{x^2\operatorname{asinh}^2(a+bx)}{2} + \frac{x^2}{4} - \frac{x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b} + \\ \frac{x^2\operatorname{asinh}^2(a)}{2} \end{cases}$$

```
[In] integrate(x*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**2/(2*b**2) - 3*a*x/(2*b) + 3*a*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b**2) + x**2*asinh(a + b*x)**2/
2 + x**2/4 - x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b) +
asinh(a + b*x)**2/(4*b**2), Ne(b, 0)), (x**2*asinh(a)**2/2, True))
```

## Maxima [F]

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx = \int x\operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate((b^3
*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^
3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + a), x)
```

## Giac [F]

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx = \int x\operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x \operatorname{arcsinh}(a+bx)^2\,dx = \int x\,\operatorname{asinh}(a+b\,x)^2\,dx$$

```
[In] int(x*asinh(a + b*x)^2,x)
[Out] int(x*asinh(a + b*x)^2, x)
```

# 3.70 $\int \operatorname{arcsinh}(a+bx)^2\,dx$

Optimal result . . . 629
Rubi [A] (verified) . . . 629
Mathematica [A] (verified) . . . 630
Maple [A] (verified) . . . 631
Fricas [B] (verification not implemented) . . . 631
Sympy [A] (verification not implemented) . . . 631
Maxima [F] . . . 632
Giac [F] . . . 632
Mupad [F(-1)] . . . 632

## Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = 2x - \frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^2}{b}$$

```
[Out] 2*x+(b*x+a)*arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5798, 8}

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \frac{(a+bx)\operatorname{arcsinh}(a+bx)^2}{b} - \frac{2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{b} + 2x$$

```
[In] Int[ArcSinh[a + b*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b + ((a + b*x)*ArcSinh[a +
 b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \text{arcsinh}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{x\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= -\frac{2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b} + \frac{2\text{Subst}(\int 1\,dx, x, a+bx)}{b} \\
&= 2x - \frac{2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \text{arcsinh}(a+bx)^2\,dx \\
&= \frac{2(a+bx) - 2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx) + (a+bx)\text{arcsinh}(a+bx)^2}{b}
\end{aligned}$$

```
[In] Integrate[ArcSinh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) - 2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (a + b*x)*ArcSinh
[a + b*x]^2)/b
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$ | 46 |
| default | $\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$ | 46 |

```
[In] int(arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*
a)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,bx-2\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2-}\right.}{b}$$

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*b*x - 2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)))/b
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(a+bx)^2\,dx$$

$$= \begin{cases} \frac{a\operatorname{asinh}^2(a+bx)}{b} + x\operatorname{asinh}^2(a+bx) + 2x - \frac{2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{b} & \text{for } b \neq 0 \\ x\operatorname{asinh}^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**2/b + x*asinh(a + b*x)**2 + 2*x - 2*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/b, Ne(b, 0)), (x*asinh(a)**2, Tru
e))
```

## Maxima [F]

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^3 + 3*a*
b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a),
 x)
```

## Giac [F]

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\,dx$$

```
[In] int(asinh(a + b*x)^2,x)
```

```
[Out] int(asinh(a + b*x)^2, x)
```

# 3.71 $\int \frac{\mathbf{arcsinh}(a+bx)^2}{x}\,dx$

Optimal result . . . 633
Rubi [A] (verified) . . . 634
Mathematica [A] (verified) . . . 637
Maple [F] . . . 638
Fricas [F] . . . 638
Sympy [F] . . . 638
Maxima [F] . . . 638
Giac [F] . . . 639
Mupad [F(-1)] . . . 639

## Optimal result

Integrand size = 12, antiderivative size = 205

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = &-\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ &+ \operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ &+ 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ &+ 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ &- 2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) - 2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\end{aligned}$$

```
[Out] -1/3*arcsinh(b*x+a)^3+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)
^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^
(1/2)))-2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))-2*polylo
g(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5859, 5827, 5680, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx &= 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\\ &\quad + 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\\ &\quad - 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) - 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\\ &\quad + \operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\\ &\quad + \operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right) - \frac{1}{3}\operatorname{arcsinh}(a+bx)^3\end{aligned}$$

```
[In] Int[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a
- Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt
[1 + a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
+ a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3,
 E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$= -\frac{1}{3}\text{arcsinh}(a+bx)^3 + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$
\begin{aligned}
&= -\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 2\operatorname{Subst}\left(\int x \log\left(1 + \frac{e^x}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right)\, dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad - 2\operatorname{Subst}\left(\int x \log\left(1 + \frac{e^x}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right)\, dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= -\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 2\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -\frac{e^x}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right)\, dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad - 2\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -\frac{e^x}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right)\, dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= -\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 2\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{x}{a-\sqrt{1+a^2}}\right)}{x}\, dx, x, e^{\operatorname{arcsinh}(a+bx)}\right) \\
&\quad - 2\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{x}{a+\sqrt{1+a^2}}\right)}{x}\, dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^2 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) - 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.22

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = &-\frac{1}{3}\operatorname{arcsinh}(a+bx)^3 + \operatorname{arcsinh}(a+bx)^2 \log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right)b}\right) \\
&+ \operatorname{arcsinh}(a+bx)^2 \log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right)b}\right) \\
&+ 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right)b}\right) \\
&+ 2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right)b}\right) \\
&- 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) - 2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 2*ArcSinh[a + b*x]*PolyLog[2, -(E^ArcSi
nh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 2*ArcSinh[a + b*x]*PolyLog[2
, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 2*PolyLog[3, E^Ar
cSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a +
Sqrt[1 + a^2])]
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^2}{x}dx$$

```
[In] int(arcsinh(b*x+a)^2/x,x)
[Out] int(arcsinh(b*x+a)^2/x,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^2/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asinh}^2(a+bx)}{x}\,dx$$

```
[In] integrate(asinh(b*x+a)**2/x,x)
[Out] Integral(asinh(a + b*x)**2/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x}\,dx$$

```
[In] int(asinh(a + b*x)^2/x,x)
[Out] int(asinh(a + b*x)^2/x, x)
```

# 3.72 $\int \frac{\mathbf{arcsinh}(a+bx)^2}{x^2}\,dx$

Optimal result . . . 640
Rubi [A] (verified) . . . 640
Mathematica [A] (verified) . . . 644
Maple [A] (verified) . . . 644
Fricas [F] . . . 645
Sympy [F] . . . 645
Maxima [F] . . . 645
Giac [F] . . . 646
Mupad [F(-1)] . . . 646

## Optimal result

Integrand size = 12, antiderivative size = 178

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = -\frac{\operatorname{arcsinh}(a+bx)^2}{x} - \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{2b\,\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\,\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$

```
[Out] -arcsinh(b*x+a)^2/x-2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used

$= \{5859, 5828, 5843, 3403, 2296, 2221, 2317, 2438\}$

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = -\frac{2b\,\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} + \frac{2b\,\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} - \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} + \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}} - \frac{\operatorname{arcsinh}(a+bx)^2}{x}$$

```
[In] Int[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^2/x) - (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/
(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (2*b*PolyLog[2, E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*PolyLog[2, E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\text{arcsinh}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right) \\
&= -\frac{\text{arcsinh}(a+bx)^2}{x} + 4\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)
\end{aligned}$$

$$= -\frac{\operatorname{arcsinh}(a+bx)^2}{x} + \frac{4\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}$$
$$- \frac{4\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}$$

$$= -\frac{\operatorname{arcsinh}(a+bx)^2}{x} - \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$- \frac{(2b)\text{Subst}\left(\int \log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}$$
$$+ \frac{(2b)\text{Subst}\left(\int \log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}$$

$$= -\frac{\operatorname{arcsinh}(a+bx)^2}{x} - \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$- \frac{(2b)\text{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{(2b)\text{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{\sqrt{1+a^2}}$$

$$= -\frac{\operatorname{arcsinh}(a+bx)^2}{x} - \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{2b\,\mathrm{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\,\mathrm{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \frac{-\operatorname{arcsinh}(a+bx)\left(\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)+2bx\left(-\log\left(\frac{a+\sqrt{1+a^2}-e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)+\log\left(\frac{-a+\sqrt{1+a^2}+e^{\operatorname{arcsinh}(a+}\right.\right.\right.}{\sqrt{1+a^2}x}$$

```
[In] Integrate[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] (-(ArcSinh[a + b*x]*(Sqrt[1 + a^2]*ArcSinh[a + b*x] + 2*b*x*(-Log[(a + Sqrt
[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + Log[(-a + Sqrt[1 + a
^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])]))) - 2*b*x*PolyLog[2, E^Arc
Sinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*b*x*PolyLog[2, E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x)
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)\right)}{\sqrt{a^2+1}}+\frac{2\,\mathrm{dilog}}{}\right.$ |
| default | $b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)\right)}{\sqrt{a^2+1}}+\frac{2\,\mathrm{dilog}}{}\right.$ |

```
[In] int(arcsinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arcsinh(b*x+a)^2/b/x+2*arcsinh(b*x+a)*(ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))-ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-
```

```
a+(a^2+1)^(1/2))))/(a^2+1)^(1/2)+2/(a^2+1)^(1/2)*dilog(((a^2+1)^(1/2)-b*x-(
1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/(a^2+1)^(1/2)*dilog(((a^2+1)^(1/2)
+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{asinh}^2\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(asinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**2, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x + integrate(2*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^4 + 3*a*b^2*x^3 +
 (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x^2}\,dx$$

```
[In] int(asinh(a + b*x)^2/x^2,x)
[Out] int(asinh(a + b*x)^2/x^2, x)
```

# 3.73 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx$

Optimal result . . . 647
Rubi [A] (verified) . . . 648
Mathematica [A] (verified) . . . 652
Maple [A] (verified) . . . 653
Fricas [F] . . . 653
Sympy [F] . . . 654
Maxima [F] . . . 654
Giac [F] . . . 654
Mupad [F(-1)] . . . 654

## Optimal result

Integrand size = 12, antiderivative size = 235

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = &-\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2} \\ &+ \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} \\ &- \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{b^2\log(x)}{1+a^2} \\ &+ \frac{ab^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} - \frac{ab^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}\end{aligned}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/x^2+b^2*ln(x)/(a^2+1)+a*b^2*arcsinh(b*x+a)*ln(1-(b*x+
a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2*arcsinh(b*x+a
)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a*b^2*p
olylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2
*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-b*a
rcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5859, 5828, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = &\frac{ab^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}} - \frac{ab^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}} \\ &+ \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}} \\ &- \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{(a^2+1)^{3/2}} \\ &- \frac{b\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{(a^2+1)\,x} + \frac{b^2\log(x)}{a^2+1} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2}\end{aligned}$$

```
[In] Int[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x)) - ArcSinh[a + b
*x]^2/(2*x^2) + (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqr
t[1 + a^2])])/(1 + a^2)^(3/2) - (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (b^2*Log[x])/(1 + a^2) + (a
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) -
(a*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \text{arcsinh}(a+bx)\right) \\
&= -\frac{b\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\text{arcsinh}(a+bx)^2}{2x^2} \\
&\quad + \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad - \frac{(ab)\,\text{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&= -\frac{b\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\text{arcsinh}(a+bx)^2}{2x^2} \\
&\quad - \frac{(2ab)\,\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+x}\,dx, x, \frac{a}{b}+x\right)}{1+a^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{1+a^2} \\
&\quad - \frac{(2ab)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{(1+a^2)^{3/2}} \\
&\quad + \frac{(2ab)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{(1+a^2)^{3/2}} \\
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2} \\
&\quad + \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} \\
&\quad - \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{b^2\log(x)}{1+a^2} \\
&\quad + \frac{(ab^2)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{(1+a^2)^{3/2}} \\
&\quad - \frac{(ab^2)\operatorname{Subst}\left(\int \log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{(1+a^2)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2} \\
&\quad + \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} \\
&\quad - \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{b^2\log(x)}{1+a^2} \\
&\quad + \frac{(ab^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{(1+a^2)^{3/2}} \\
&\quad - \frac{(ab^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{(1+a^2)^{3/2}} \\
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x} - \frac{\operatorname{arcsinh}(a+bx)^2}{2x^2} \\
&\quad + \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} \\
&\quad - \frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{b^2\log(x)}{1+a^2} \\
&\quad + \frac{ab^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} - \frac{ab^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19

$$
\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx =
$$

$$
-\frac{2\sqrt{1+a^2}bx\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx) + \sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^2 + a^2\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^2 +}{}
$$

```
[In] Integrate[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -1/2*(2*Sqrt[1 + a^2]*b*x*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + Sqrt[1 +
 a^2]*ArcSinh[a + b*x]^2 + a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^2 + 2*a*b^2*x
```

```
^2*ArcSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[
1 + a^2])] - 2*a*b^2*x^2*ArcSinh[a + b*x]*Log[(-a + Sqrt[1 + a^2] + E^ArcSi
nh[a + b*x])/(-a + Sqrt[1 + a^2])] - 2*Sqrt[1 + a^2]*b^2*x^2*Log[x] - 2*a*b
^2*x^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*a*b^2*x^2*Pol
yLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/((1 + a^2)^(3/2)*x^2)
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| derivativedivides | $b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(-2(bx+a)^2+4a(bx+a)+\operatorname{arcsinh}(bx+a)+a^2\operatorname{arcsinh}(bx+a)-2a\sqrt{1+(bx+a)^2}+2(bx+a)\sqrt{1+(b\right.}{2b^2x^2(a^2+1)}$ |
| default | $b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(-2(bx+a)^2+4a(bx+a)+\operatorname{arcsinh}(bx+a)+a^2\operatorname{arcsinh}(bx+a)-2a\sqrt{1+(bx+a)^2}+2(bx+a)\sqrt{1+(b\right.}{2b^2x^2(a^2+1)}$ |

```
[In] int(arcsinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arcsinh(b*x+a)*(-2*(b*x+a)^2+4*a*(b*x+a)+arcsinh(b*x+a)+a^2*arcsi
nh(b*x+a)-2*a*(1+(b*x+a)^2)^(1/2)+2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-2*a^2)/b^2/
x^2/(a^2+1)-1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+
a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1
)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/(a^2+1)^(3/2)*a*dilo
g(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/(a^2+1)^(3/2
)*a*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-2/(a^
2+1)*ln(b*x+a+(1+(b*x+a)^2)^(1/2))+1/(a^2+1)*ln(2*a*(b*x+a+(1+(b*x+a)^2)^(1
/2))-(b*x+a+(1+(b*x+a)^2)^(1/2))^2+1))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}^2(a+bx)}{x^3}\,dx$$

```
[In] integrate(asinh(b*x+a)**2/x**3,x)
[Out] Integral(asinh(a + b*x)**2/x**3, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^2}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="maxima")
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^2 + integrate((b^
3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b)
 + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^5 + 3*a*b^2*x
^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^2}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x^3}\,dx$$

```
[In] int(asinh(a + b*x)^2/x^3,x)
[Out] int(asinh(a + b*x)^2/x^3, x)
```

# 3.74 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx$

Optimal result . . . 655
Rubi [A] (verified) . . . 656
Mathematica [C] (verified) . . . 667
Maple [A] (verified) . . . 669
Fricas [F] . . . 670
Sympy [F] . . . 670
Maxima [F] . . . 671
Giac [F] . . . 671
Mupad [F(-1)] . . . 671

## Optimal result

Integrand size = 12, antiderivative size = 478

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = &-\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\ &+ \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\ &- \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\ &+ \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\ &+ \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\ &- \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} - \frac{ab^3\log(x)}{\left(1+a^2\right)^2} \\ &- \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} + \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\ &+ \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} - \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}}\end{aligned}$$

```
[Out] -1/3*b^2/(a^2+1)/x-1/3*arcsinh(b*x+a)^2/x^3-a*b^3*ln(x)/(a^2+1)^2-a^2*b^3*a
rcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(
5/2)+1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/
2)))/(a^2+1)^(3/2)+a^2*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/
```

```
(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a^2*b^3*polylog(2,(b*x+a+(1+
(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(5/2)+1/3*b^3*polylog(2,(b*x+a
+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a^2*b^3*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*polylog(
2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-1/3*b*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^2+a*b^2*arcsinh(b*x+a)*(1+(b*x+a)^2)
^(1/2)/(a^2+1)^2/x
```

## Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {5859, 5828, 5843, 3406, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31, 6873, 12, 6874, 32}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = &\frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}} - \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}\\ &- \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}} + \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}\\ &+ \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}}\\ &- \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}\\ &- \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{3\left(a^2+1\right)^{3/2}}\\ &+ \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{5/2}}\\ &+ \frac{ab^2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{\left(a^2+1\right)^2x}\\ &- \frac{b\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{3\left(a^2+1\right)x^2}\\ &- \frac{ab^3\log(x)}{\left(a^2+1\right)^2} - \frac{b^2}{3\left(a^2+1\right)x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3}\end{aligned}$$

```
[In] Int[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] -1/3*b^2/((1 + a^2)*x) - (b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*(1 +
 a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)^2*x)
```

```
 - ArcSinh[a + b*x]^2/(3*x^3) - (a^2*b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*ArcSinh[a + b*x]*Log
[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^
3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^
2)^(5/2) - (b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])])/(3*(1 + a^2)^(3/2)) - (a*b^3*Log[x])/(1 + a^2)^2 - (a^2*b^3*PolyLog[
2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*PolyLog[
2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*
PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(5/2) - (b^3*
PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\text{arcsinh}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\text{arcsinh}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3}\,dx, x, \text{arcsinh}(a+bx)\right)$$

$$
\begin{aligned}
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\cosh(x)}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{x\sinh(x)}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&\quad - \frac{(2ab)\operatorname{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&= -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)^2 x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} - \frac{b\operatorname{Subst}\left(\int \frac{b^2 x\sinh(x)}{(a-\sinh(x))^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&\quad - \frac{(2ab^2)\operatorname{Subst}\left(\int \frac{\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad + \frac{(2a^2b^2)\operatorname{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+x\right)^2}\,dx, x, \frac{a}{b}+x\right)}{3\left(1+a^2\right)} \\
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)^2 x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} + \frac{(4a^2b^2)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{(2ab^3)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+x}\,dx, x, \frac{a}{b}+x\right)}{3\left(1+a^2\right)^2} - \frac{b^3\operatorname{Subst}\left(\int \frac{x\sinh(x)}{(a-\sinh(x))^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{2ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} - \frac{2ab^3\log(x)}{3\left(1+a^2\right)^2} \\
&\quad + \frac{\left(4a^2b^2\right)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b} - \frac{2\sqrt{1+a^2}}{b} + \frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{\left(4a^2b^2\right)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b} + \frac{2\sqrt{1+a^2}}{b} + \frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \left(\frac{ax}{(a-\sinh(x))^2} - \frac{x}{a-\sinh(x)}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{2ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} - \frac{2ab^3\log(x)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{\left(2a^2b^3\right)\operatorname{Subst}\left(\int \log\left(1 + \frac{2e^x}{\left(-\frac{2a}{b} - \frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{\left(2a^2b^3\right)\operatorname{Subst}\left(\int \log\left(1 + \frac{2e^x}{\left(-\frac{2a}{b} + \frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{b^3\operatorname{Subst}\left(\int \frac{x}{a-\sinh(x)}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)} \\
&\quad - \frac{\left(ab^3\right)\operatorname{Subst}\left(\int \frac{x}{(a-\sinh(x))^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} - \frac{2ab^3\log(x)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{(2a^2b^3)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{(2a^2b^3)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{(ab^3)\operatorname{Subst}\left(\int \frac{\cosh(x)}{a-\sinh(x)}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{(a^2b^3)\operatorname{Subst}\left(\int \frac{x}{a-\sinh(x)}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad + \frac{(2b^3)\operatorname{Subst}\left(\int \frac{e^x x}{1+2ae^x-e^{2x}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{2ab^3\log(x)}{3\left(1+a^2\right)^2} - \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} - \frac{(ab^3)\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, -a-bx\right)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{(2a^2b^3)\operatorname{Subst}\left(\int \frac{e^x x}{1+2ae^x-e^{2x}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^2} \\
&\quad - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{e^x x}{2a-2\sqrt{1+a^2}-2e^x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{(2b^3)\operatorname{Subst}\left(\int \frac{e^x x}{2a+2\sqrt{1+a^2}-2e^x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{2a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} - \frac{ab^3\log(x)}{\left(1+a^2\right)^2} \\
&\quad - \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} + \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{\left(2a^2b^3\right)\operatorname{Subst}\left(\int \frac{e^x x}{2a-2\sqrt{1+a^2}-2e^x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{\left(2a^2b^3\right)\operatorname{Subst}\left(\int \frac{e^x x}{2a+2\sqrt{1+a^2}-2e^x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \log\left(1-\frac{2e^x}{2a-2\sqrt{1+a^2}}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{b^3\operatorname{Subst}\left(\int \log\left(1-\frac{2e^x}{2a+2\sqrt{1+a^2}}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} - \frac{ab^3\log(x)}{\left(1+a^2\right)^2} \\
&\quad - \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} + \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{\left(a^2b^3\right)\operatorname{Subst}\left(\int \log\left(1-\frac{2e^x}{2a-2\sqrt{1+a^2}}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{\left(a^2b^3\right)\operatorname{Subst}\left(\int \log\left(1-\frac{2e^x}{2a+2\sqrt{1+a^2}}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{2x}{2a-2\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{b^3\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{2x}{2a+2\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} - \frac{ab^3\log(x)}{\left(1+a^2\right)^2} \\
&\quad - \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} + \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{2a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{5/2}} - \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{\left(a^2b^3\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{2x}{2a-2\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{\left(a^2b^3\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{2x}{2a+2\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{3\left(1+a^2\right)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3\left(1+a^2\right)x^2} \\
&\quad + \frac{ab^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{\left(1+a^2\right)^2 x} - \frac{\operatorname{arcsinh}(a+bx)^2}{3x^3} \\
&\quad - \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad + \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{a^2b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} \\
&\quad - \frac{b^3\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} - \frac{ab^3\log(x)}{\left(1+a^2\right)^2} \\
&\quad - \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} + \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{a^2b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} - \frac{b^3\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 1830, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \frac{1}{3}\Bigg(-\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x^2} - \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}$$

$$- \frac{b^2\left(1+a^2-3a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)\right)}{\left(1+a^2\right)^2 x}$$

$$+ \frac{ib^3\pi\operatorname{arctanh}\left(\frac{-1-a\tanh\left(\frac{1}{2}\operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}} - \frac{2ia^2b^3\pi\operatorname{arctanh}\left(\frac{-1-a\tanh\left(\frac{1}{2}\operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{5/2}}$$

$$- \frac{3ab^3\log\left(-\frac{bx}{a}\right)}{\left(1+a^2\right)^2}$$

$$+ \frac{b^3\left(-2\arccos(ia)\operatorname{arctanh}\left(\frac{(-i+a)\cot\left(\frac{1}{4}(\pi+2i\operatorname{arcsinh}(a+bx))\right)}{\sqrt{-1-a^2}}\right) - (\pi-2i\operatorname{arcsinh}(a+bx))\operatorname{arctanh}\left(\frac{(i+a)\tan\left(\frac{1}{4}\right.\right.\right.}{}$$

$$- \frac{2a^2b^3\left(-2\arccos(ia)\operatorname{arctanh}\left(\frac{(-i+a)\cot\left(\frac{1}{4}(\pi+2i\operatorname{arcsinh}(a+bx))\right)}{\sqrt{-1-a^2}}\right) - (\pi-2i\operatorname{arcsinh}(a+bx))\operatorname{arctanh}\left(\frac{(i+a)\,\mathrm{ta}}{}\right.\right.$$

```
[In] Integrate[ArcSinh[a + b*x]^2/x^4,x]

[Out] (-((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x^2)) - ArcSinh[a
+ b*x]^2/x^3 - (b^2*(1 + a^2 - 3*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]))
/((1 + a^2)^2*x) + (I*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSinh[a + b*x]/2])/Sqrt
[1 + a^2]])/(1 + a^2)^(5/2) - ((2*I)*a^2*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSin
h[a + b*x]/2])/Sqrt[1 + a^2]])/(1 + a^2)^(5/2) - (3*a*b^3*Log[-((b*x)/a)])/
(1 + a^2)^2 + (b^3*(-2*ArcCos[I*a]*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSin
h[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)*ArcSinh[a + b*x])*ArcTanh[((I
 + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (ArcCos[I*a]
+ (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a
^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1
 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcSinh[a + b*x]/2)*Sqrt[b*x])] +
(ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4
])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])
/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(ArcSinh[a + b*x]/2))/(Sqrt[2]*S
qrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a + I*(-1 + Sqrt[-1 - a^2]))*
(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(I + a - Sqrt[-1 - a^2]*Cot[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a] - (2*I)*ArcTanh[((-I + a)*Co
```

```
t[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a - I*(1
 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a +
Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] + I*(PolyLog[2, -((((
-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a +
 b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))
] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2
*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])])))/(-1 - a^2)^(5/2) - (2*a^2*b^3*(-2*ArcCos[I*a]*ArcTanh[((
-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)
*ArcSinh[a + b*x])*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/S
qrt[-1 - a^2]] + (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)
*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcS
inh[a + b*x]/2)*Sqrt[b*x])] + (ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(
Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(A
rcSinh[a + b*x]/2))/(Sqrt[2]*Sqrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-
I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*
(a + I*(-1 + Sqrt[-1 - a^2]))*(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(
I + a - Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a
] - (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 -
 a^2]])*Log[((I + a)*(a - I*(1 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*
x])/4])] + I*(PolyLog[2, -((((-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^
2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi
 + (2*I)*ArcSinh[a + b*x])/4]))] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a
 + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1
 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])])))/(-1 - a^2)^(5/2))/3
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| derivativedivides | $b^3\left(-\frac{a^4\operatorname{arcsinh}(bx+a)^2-4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^3+7\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^2(bx+a)-3\operatorname{arcsinh}(bx-\right.$ |
| default | $b^3\left(-\frac{a^4\operatorname{arcsinh}(bx+a)^2-4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^3+7\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^2(bx+a)-3\operatorname{arcsinh}(bx-\right.$ |

```
[In] int(arcsinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*(a^4*arcsinh(b*x+a)^2-4*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^3+7*
arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^2*(b*x+a)-3*arcsinh(b*x+a)*(1+(b*x+a)^
2)^(1/2)*a*(b*x+a)^2-3*arcsinh(b*x+a)*a^4+9*arcsinh(b*x+a)*a^3*(b*x+a)-9*ar
csinh(b*x+a)*a^2*(b*x+a)^2+3*arcsinh(b*x+a)*a*(b*x+a)^3+2*a^2*arcsinh(b*x+a
)^2+a^4-2*a^3*(b*x+a)+a^2*(b*x+a)^2-arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a+ar
csinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+a^2-2*a*(b*x+a)+(
b*x+a)^2)/(a^2+1)^2/b^3/x^3+2/(a^2+1)^2*a*ln(b*x+a+(1+(b*x+a)^2)^(1/2))-1/(
a^2+1)^2*a*ln(2*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-(b*x+a+(1+(b*x+a)^2)^(1/2))^2
+1)-1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1
/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+
b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/3/(a^2+1)^(5/2)*dilog(((a^2+
1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1)^(5/2)*dilo
g(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3/(a^2+1)^(
5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+
1)^(1/2)))-2/3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b
*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)^(1
/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3/(a^2+1)^(5/2)*a^2*dilog
(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}^2\left(a+bx\right)}{x^4}\,dx$$

```
[In] integrate(asinh(b*x+a)**2/x**4,x)
[Out] Integral(asinh(a + b*x)**2/x**4, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^3 + integrate(2/3
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^6 + 3*a*b
^2*x^5 + (3*a^2*b + b)*x^4 + (a^3 + a)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^2 +
1)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x^4}\,dx$$

```
[In] int(asinh(a + b*x)^2/x^4,x)
```

```
[Out] int(asinh(a + b*x)^2/x^4, x)
```

# 3.75 $\int x^2 \mathbf{arcsinh}(a+bx)^3\,dx$

Optimal result . . . 672
Rubi [A] (verified) . . . 673
Mathematica [A] (verified) . . . 678
Maple [A] (verified) . . . 678
Fricas [A] (verification not implemented) . . . 679
Sympy [A] (verification not implemented) . . . 679
Maxima [F] . . . 680
Giac [F] . . . 680
Mupad [F(-1)] . . . 680

## Optimal result

Integrand size = 12, antiderivative size = 355

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = {} & \frac{14\sqrt{1+(a+bx)^2}}{9b^3} - \frac{6a^2\sqrt{1+(a+bx)^2}}{b^3} \\ & + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} - \frac{2\left(1+(a+bx)^2\right)^{3/2}}{27b^3} \\ & - \frac{3a\operatorname{arcsinh}(a+bx)}{4b^3} - \frac{4(a+bx)\operatorname{arcsinh}(a+bx)}{3b^3} \\ & + \frac{6a^2(a+bx)\operatorname{arcsinh}(a+bx)}{b^3} - \frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} \\ & + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} + \frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} \\ & - \frac{3a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^3} \\ & + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b^3} \\ & - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} \\ & - \frac{a\operatorname{arcsinh}(a+bx)^3}{2b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3\end{aligned}$$

```
[Out] -2/27*(1+(b*x+a)^2)^(3/2)/b^3-3/4*a*arcsinh(b*x+a)/b^3-4/3*(b*x+a)*arcsinh(
b*x+a)/b^3+6*a^2*(b*x+a)*arcsinh(b*x+a)/b^3-3/2*a*(b*x+a)^2*arcsinh(b*x+a)/
b^3+2/9*(b*x+a)^3*arcsinh(b*x+a)/b^3-1/2*a*arcsinh(b*x+a)^3/b^3+1/3*a^3*arc
sinh(b*x+a)^3/b^3+1/3*x^3*arcsinh(b*x+a)^3+14/9*(1+(b*x+a)^2)^(1/2)/b^3-6*a
^2*(1+(b*x+a)^2)^(1/2)/b^3+3/4*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3+2/3*arcsin
h(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-3*a^2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/
2)/b^3+3/2*a*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a)^2
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5859, 5828, 5843, 3398, 3377, 2718, 3392, 30, 2715, 8, 2713}

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = {} & \frac{a^3 \operatorname{arcsinh}(a+bx)^3}{3b^3} + \frac{6a^2(a+bx)\operatorname{arcsinh}(a+bx)}{b^3} \\ & - \frac{3a^2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{b^3} \\ & - \frac{6a^2\sqrt{(a+bx)^2+1}}{b^3} + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} \\ & - \frac{\sqrt{(a+bx)^2+1}(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{3b^3} \\ & - \frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} \\ & + \frac{3a\sqrt{(a+bx)^2+1}(a+bx)\operatorname{arcsinh}(a+bx)^2}{2b^3} \\ & - \frac{4(a+bx)\operatorname{arcsinh}(a+bx)}{3b^3} - \frac{a\operatorname{arcsinh}(a+bx)^3}{2b^3} \\ & + \frac{2\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{3a\operatorname{arcsinh}(a+bx)}{4b^3} \\ & + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 + \frac{3a\sqrt{(a+bx)^2+1}(a+bx)}{4b^3} \\ & - \frac{2\left((a+bx)^2+1\right)^{3/2}}{27b^3} + \frac{14\sqrt{(a+bx)^2+1}}{9b^3}\end{aligned}$$

```
[In] Int[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] (14*Sqrt[1 + (a + b*x)^2])/(9*b^3) - (6*a^2*Sqrt[1 + (a + b*x)^2])/b^3 + (3
*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b^3) - (2*(1 + (a + b*x)^2)^(3/2))/(
27*b^3) - (3*a*ArcSinh[a + b*x])/(4*b^3) - (4*(a + b*x)*ArcSinh[a + b*x])/(
3*b^3) + (6*a^2*(a + b*x)*ArcSinh[a + b*x])/b^3 - (3*a*(a + b*x)^2*ArcSinh[
a + b*x])/(2*b^3) + (2*(a + b*x)^3*ArcSinh[a + b*x])/(9*b^3) + (2*Sqrt[1 +
(a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^3) - (3*a^2*Sqrt[1 + (a + b*x)^2]*Arc
Sinh[a + b*x]^2)/b^3 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
]^2)/(2*b^3) - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^
3) - (a*ArcSinh[a + b*x]^3)/(2*b^3) + (a^3*ArcSinh[a + b*x]^3)/(3*b^3) + (x
^3*ArcSinh[a + b*x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
```

```
- Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
- 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \operatorname{arcsinh}(x)^3\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 - \text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \operatorname{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 - \text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 - \text{Subst}\left(\int \left(-\frac{a^3x^2}{b^3}+\frac{3a^2x^2\sinh(x)}{b^3}-\frac{3ax^2\sinh^2(x)}{b^3}\right.\right. \\
&\qquad \left.\left. +\frac{x^2\sinh^3(x)}{b^3}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 \\
&\quad - \frac{\text{Subst}\left(\int x^2\sinh^3(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{(3a)\text{Subst}\left(\int x^2\sinh^2(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(3a^2)\,\text{Subst}\left(\int x^2\sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} \\
&\quad - \frac{3a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^3} \\
&\quad + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b^3} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} \\
&\quad + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 - \frac{2\operatorname{Subst}\left(\int \sinh^3(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{9b^3} \\
&\quad + \frac{2\operatorname{Subst}\left(\int x^2\sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{3b^3} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int x^2\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^3} \\
&\quad + \frac{(3a)\operatorname{Subst}\left(\int \sinh^2(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^3} \\
&\quad + \frac{(6a^2)\operatorname{Subst}(\int x\cosh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{b^3} \\
&= \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} + \frac{6a^2(a+bx)\operatorname{arcsinh}(a+bx)}{b^3} \\
&\quad - \frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} \\
&\quad + \frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{3a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^3} \\
&\quad + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b^3} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{a\operatorname{arcsinh}(a+bx)^3}{2b^3} \\
&\quad + \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 \\
&\quad + \frac{2\operatorname{Subst}\left(\int (1-x^2)\ dx, x, \sqrt{1+(a+bx)^2}\right)}{9b^3} \\
&\quad - \frac{4\operatorname{Subst}(\int x\cosh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{3b^3} \\
&\quad - \frac{(3a)\operatorname{Subst}(\int 1\,dx, x, \operatorname{arcsinh}(a+bx))}{4b^3} \\
&\quad - \frac{(6a^2)\operatorname{Subst}(\int \sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{1+(a+bx)^2}}{9b^3} - \frac{6a^2\sqrt{1+(a+bx)^2}}{b^3} + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} \\
&\quad - \frac{2(1+(a+bx)^2)^{3/2}}{27b^3} - \frac{3a\operatorname{arcsinh}(a+bx)}{4b^3} \\
&\quad - \frac{4(a+bx)\operatorname{arcsinh}(a+bx)}{3b^3} + \frac{6a^2(a+bx)\operatorname{arcsinh}(a+bx)}{b^3} \\
&\quad - \frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} \\
&\quad + \frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{3a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^3} \\
&\quad + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b^3} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} \\
&\quad - \frac{a\operatorname{arcsinh}(a+bx)^3}{2b^3} + \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} \\
&\quad + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3 + \frac{4\operatorname{Subst}(\int \sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{3b^3} \\
&= \frac{14\sqrt{1+(a+bx)^2}}{9b^3} - \frac{6a^2\sqrt{1+(a+bx)^2}}{b^3} + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} \\
&\quad - \frac{2(1+(a+bx)^2)^{3/2}}{27b^3} - \frac{3a\operatorname{arcsinh}(a+bx)}{4b^3} \\
&\quad - \frac{4(a+bx)\operatorname{arcsinh}(a+bx)}{3b^3} + \frac{6a^2(a+bx)\operatorname{arcsinh}(a+bx)}{b^3} \\
&\quad - \frac{3a(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b^3} + \frac{2(a+bx)^3\operatorname{arcsinh}(a+bx)}{9b^3} \\
&\quad + \frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{3a^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^3} \\
&\quad + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b^3} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{3b^3} - \frac{a\operatorname{arcsinh}(a+bx)^3}{2b^3} \\
&\quad + \frac{a^3\operatorname{arcsinh}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\operatorname{arcsinh}(a+bx)^3
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.49

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx$$
$$= \frac{(160-575a^2+65abx-8b^2x^2)\sqrt{1+a^2+2abx+b^2x^2}+3(170a^3+132a^2bx+8bx(-6+b^2x^2)-15a(5+$$

```
[In] Integrate[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] ((160 - 575*a^2 + 65*a*b*x - 8*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] +
 3*(170*a^3 + 132*a^2*b*x + 8*b*x*(-6 + b^2*x^2) - 15*a*(5 + 2*b^2*x^2))*Ar
cSinh[a + b*x] - 18*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4 + 11*a^2 - 5*a*b*
x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 18*(-3*a + 2*a^3 + 2*b^3*x^3)*ArcSinh[a
 + b*x]^3)/(108*b^3)
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)^3}{3}+\frac{2\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}}{3}-\frac{\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)^2}{3}-\frac{4(bx+a)\operatorname{arcsinh}(bx+a)}{3}+\frac{40\sqrt{1+(b}}{27}$ |
| default | $\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)^3}{3}+\frac{2\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}}{3}-\frac{\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)^2}{3}-\frac{4(bx+a)\operatorname{arcsinh}(bx+a)}{3}+\frac{40\sqrt{1+(b}}{27}$ |

```
[In] int(x^2*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*arcsinh(b*x+a)^3*(b*x+a)^3+2/3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1
/2)-1/3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2-4/3*(b*x+a)*arcsinh(
b*x+a)+40/27*(1+(b*x+a)^2)^(1/2)+2/9*arcsinh(b*x+a)*(b*x+a)^3-2/27*(b*x+a)^
2*(1+(b*x+a)^2)^(1/2)-1/4*a*(4*arcsinh(b*x+a)^3*(b*x+a)^2-6*arcsinh(b*x+a)^
2*(1+(b*x+a)^2)^(1/2)*(b*x+a)+2*arcsinh(b*x+a)^3+6*arcsinh(b*x+a)*(b*x+a)^2
-3*(b*x+a)*(1+(b*x+a)^2)^(1/2)+3*arcsinh(b*x+a))+a^2*(arcsinh(b*x+a)^3*(b*x
+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b
*x+a)^2)^(1/2)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.63

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \frac{18\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 - 18\,(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+}}{}$$

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(18*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))^3 - 18*(2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(8*b^3*x^
3 - 30*a*b^2*x^2 + 170*a^3 + 12*(11*a^2 - 4)*b*x - 75*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (8*b^2*x^2 - 65*a*b*x + 575*a^2 - 160)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

## Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.22

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \begin{cases} \frac{a^3 \operatorname{asinh}^3(a+bx)}{3b^3} + \frac{85a^3 \operatorname{asinh}(a+bx)}{18b^3} + \frac{11a^2 x \operatorname{asinh}(a+bx)}{3b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{6b^3} - \frac{575a^2\sqrt{a^2+2abx+b^2x^2+1}}{108b^3} - \\ \frac{x^3 \operatorname{asinh}^3(a)}{3} \end{cases}$$

```
[In] integrate(x**2*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**3/(3*b**3) + 85*a**3*asinh(a + b*x)/(18*b**
3) + 11*a**2*x*asinh(a + b*x)/(3*b**2) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)*asinh(a + b*x)**2/(6*b**3) - 575*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(108*b**3) - 5*a*x**2*asinh(a + b*x)/(6*b) + 5*a*x*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(6*b**2) + 65*a*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)/(108*b**2) - a*asinh(a + b*x)**3/(2*b**3) - 25*a*asin
h(a + b*x)/(12*b**3) + x**3*asinh(a + b*x)**3/3 + 2*x**3*asinh(a + b*x)/9 -
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b) - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b) - 4*x*asinh(a + b*x)/(3*b**2)
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b**3) + 40*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b**3), Ne(b, 0)), (x**3*asinh(a)**3/
3, True))
```

## Maxima [F]

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = \int x^2\,\operatorname{arsinh}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate((b^3
*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*
x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(
3/2) + a), x)
```

## Giac [F]

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = \int x^2\,\operatorname{arsinh}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = \int x^2\,\operatorname{asinh}\left(a+b\,x\right)^3\,dx$$

```
[In] int(x^2*asinh(a + b*x)^3,x)
```

```
[Out] int(x^2*asinh(a + b*x)^3, x)
```

# 3.76 $\int x\operatorname{arcsinh}(a+bx)^3\,dx$

Optimal result . . . 681
Rubi [A] (verified) . . . 681
Mathematica [A] (verified) . . . 685
Maple [A] (verified) . . . 685
Fricas [A] (verification not implemented) . . . 686
Sympy [A] (verification not implemented) . . . 686
Maxima [F] . . . 687
Giac [F] . . . 687
Mupad [F(-1)] . . . 687

## Optimal result

Integrand size = 10, antiderivative size = 203

$$\begin{aligned}\int x\operatorname{arcsinh}(a+bx)^3\,dx = {}& \frac{6a\sqrt{1+(a+bx)^2}}{b^2} - \frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2} + \frac{3\operatorname{arcsinh}(a+bx)}{8b^2} \\ & - \frac{6a(a+bx)\operatorname{arcsinh}(a+bx)}{b^2} + \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{4b^2} \\ & + \frac{3a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^2} \\ & - \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{4b^2} + \frac{\operatorname{arcsinh}(a+bx)^3}{4b^2} \\ & - \frac{a^2\operatorname{arcsinh}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3\end{aligned}$$

```
[Out] 3/8*arcsinh(b*x+a)/b^2-6*a*(b*x+a)*arcsinh(b*x+a)/b^2+3/4*(b*x+a)^2*arcsinh
(b*x+a)/b^2+1/4*arcsinh(b*x+a)^3/b^2-1/2*a^2*arcsinh(b*x+a)^3/b^2+1/2*x^2*a
rcsinh(b*x+a)^3+6*a*(1+(b*x+a)^2)^(1/2)/b^2-3/8*(b*x+a)*(1+(b*x+a)^2)^(1/2)
/b^2+3*a*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^2-3/4*(b*x+a)*arcsinh(b*x+a
)^2*(1+(b*x+a)^2)^(1/2)/b^2
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules

used = {5859, 5828, 5843, 3398, 3377, 2718, 3392, 30, 2715, 8}

$$\begin{aligned}\int x\operatorname{arcsinh}(a+bx)^3\,dx = &-\frac{a^2\operatorname{arcsinh}(a+bx)^3}{2b^2}+\frac{\operatorname{arcsinh}(a+bx)^3}{4b^2}\\ &-\frac{3(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{4b^2}\\ &+\frac{3a\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{b^2}\\ &+\frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{4b^2}-\frac{6a(a+bx)\operatorname{arcsinh}(a+bx)}{b^2}\\ &+\frac{3\operatorname{arcsinh}(a+bx)}{8b^2}+\frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3\\ &-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2}+\frac{6a\sqrt{(a+bx)^2+1}}{b^2}\end{aligned}$$

```
[In] Int[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (6*a*Sqrt[1 + (a + b*x)^2])/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(8*b^
2) + (3*ArcSinh[a + b*x])/(8*b^2) - (6*a*(a + b*x)*ArcSinh[a + b*x])/b^2 +
(3*(a + b*x)^2*ArcSinh[a + b*x])/(4*b^2) + (3*a*Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)
/(4*b^2) + ArcSinh[a + b*x]^3/(4*b^2) - (a^2*ArcSinh[a + b*x]^3)/(2*b^2) +
(x^2*ArcSinh[a + b*x]^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\operatorname{arcsinh}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 - \frac{3}{2}\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2 \operatorname{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 - \frac{3}{2}\operatorname{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2\,dx, x, \operatorname{arcsinh}(a+bx)\right)$$

$$\begin{aligned}= {} & \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 \\ & - \frac{3}{2}\operatorname{Subst}\left(\int \left(\frac{a^2x^2}{b^2} - \frac{2ax^2\sinh(x)}{b^2} + \frac{x^2\sinh^2(x)}{b^2}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)\end{aligned}$$

$$\begin{aligned}= {} & -\frac{a^2\operatorname{arcsinh}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 \\ & - \frac{3\operatorname{Subst}\left(\int x^2\sinh^2(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^2} \\ & + \frac{(3a)\operatorname{Subst}\left(\int x^2\sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^2}\end{aligned}$$

$$\begin{aligned}= {} & \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{4b^2} + \frac{3a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^2} \\ & - \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{4b^2} - \frac{a^2\operatorname{arcsinh}(a+bx)^3}{2b^2} \\ & + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 + \frac{3\operatorname{Subst}\left(\int x^2\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^2} \\ & - \frac{3\operatorname{Subst}\left(\int \sinh^2(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^2} \\ & - \frac{(6a)\operatorname{Subst}(\int x\cosh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{b^2}\end{aligned}$$

$$\begin{aligned}= {} & -\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2} - \frac{6a(a+bx)\operatorname{arcsinh}(a+bx)}{b^2} \\ & + \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{4b^2} + \frac{3a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b^2} \\ & - \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{4b^2} + \frac{\operatorname{arcsinh}(a+bx)^3}{4b^2} \\ & - \frac{a^2\operatorname{arcsinh}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\operatorname{arcsinh}(a+bx)^3 + \frac{3\operatorname{Subst}(\int 1\,dx, x, \operatorname{arcsinh}(a+bx))}{8b^2} \\ & + \frac{(6a)\operatorname{Subst}(\int \sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx))}{b^2}\end{aligned}$$

$$= \frac{6a\sqrt{1+(a+bx)^2}}{b^2} - \frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2} + \frac{3\text{arcsinh}(a+bx)}{8b^2}$$
$$- \frac{6a(a+bx)\text{arcsinh}(a+bx)}{b^2} + \frac{3(a+bx)^2\text{arcsinh}(a+bx)}{4b^2}$$
$$+ \frac{3a\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{b^2} - \frac{3(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{4b^2}$$
$$+ \frac{\text{arcsinh}(a+bx)^3}{4b^2} - \frac{a^2\text{arcsinh}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\text{arcsinh}(a+bx)^3$$

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

$$\int x\text{arcsinh}(a+bx)^3\,dx$$
$$= \frac{3(15a-bx)\sqrt{1+a^2+2abx+b^2x^2} + (3-42a^2-36abx+6b^2x^2)\,\text{arcsinh}(a+bx) + 6(3a-bx)\sqrt{1+a^2-}}{8b^2}$$

```
[In] Integrate[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (3*(15*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3 - 42*a^2 - 36*a*b*x
+ 6*b^2*x^2)*ArcSinh[a + b*x] + 6*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*
x^2]*ArcSinh[a + b*x]^2 + (2 - 4*a^2 + 4*b^2*x^2)*ArcSinh[a + b*x]^3)/(8*b^
2)
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| derivativedivides | $\frac{\text{arcsinh}(bx+a)^3\left(1+(bx+a)^2\right)}{2} - \frac{3\,\text{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4} - \frac{\text{arcsinh}(bx+a)^3}{4} + \frac{3\,\text{arcsinh}(bx+a)\left(1+(bx+a)^2\right)}{4} - \frac{3(bx+a)}{}$ |
| default | $\frac{\text{arcsinh}(bx+a)^3\left(1+(bx+a)^2\right)}{2} - \frac{3\,\text{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4} - \frac{\text{arcsinh}(bx+a)^3}{4} + \frac{3\,\text{arcsinh}(bx+a)\left(1+(bx+a)^2\right)}{4} - \frac{3(bx+a)}{}$ |

```
[In] int(x*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^3*(1+(b*x+a)^2)-3/4*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^3+3/4*arcsinh(b*x+a)*(1+(b*x+a)^2)-3/8*(
b*x+a)*(1+(b*x+a)^2)^(1/2)-3/8*arcsinh(b*x+a)-a*(arcsinh(b*x+a)^3*(b*x+a)-3
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)
^2)^(1/2)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int x\operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \frac{2\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-6\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)\log\left(bx-\right.}{}$$

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^3 - 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(2*b^2*x^2 - 12*a*b*x - 14*a^2 + 1)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(b*x - 15*a))/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int x\operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \begin{cases} -\frac{a^2\operatorname{asinh}^3(a+bx)}{2b^2}-\frac{21a^2\operatorname{asinh}(a+bx)}{4b^2}-\frac{9ax\operatorname{asinh}(a+bx)}{2b}+\frac{9a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b^2}+\frac{45a\sqrt{a^2+2abx+b^2x^2+1}}{8b^2}+\frac{x^2a}{} \\ \frac{x^2\operatorname{asinh}^3(a)}{2} \end{cases}$$

```
[In] integrate(x*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**3/(2*b**2) - 21*a**2*asinh(a + b*x)/(4*b**
2) - 9*a*x*asinh(a + b*x)/(2*b) + 9*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
asinh(a + b*x)**2/(4*b**2) + 45*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(8*b
**2) + x**2*asinh(a + b*x)**3/2 + 3*x**2*asinh(a + b*x)/4 - 3*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) - 3*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(8*b) + asinh(a + b*x)**3/(4*b**2) + 3*asinh(a + b*x)/(8*
b**2), Ne(b, 0)), (x**2*asinh(a)**3/2, True))
```

## Maxima [F]

$$\int x\text{arcsinh}(a+bx)^3\,dx = \int x\,\text{arsinh}\,(bx+a)^3\,\,dx$$

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3/2*
(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + a), x)
```

## Giac [F]

$$\int x\text{arcsinh}(a+bx)^3\,dx = \int x\,\text{arsinh}\,(bx+a)^3\,\,dx$$

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(a+bx)^3\,dx = \int x\,\text{asinh}(a+b\,x)^3\,dx$$

```
[In] int(x*asinh(a + b*x)^3,x)
```

```
[Out] int(x*asinh(a + b*x)^3, x)
```

# 3.77 $\int \operatorname{arcsinh}(a+bx)^3\,dx$

Optimal result . . . 688
Rubi [A] (verified) . . . 688
Mathematica [A] (verified) . . . 690
Maple [A] (verified) . . . 690
Fricas [A] (verification not implemented) . . . 690
Sympy [A] (verification not implemented) . . . 691
Maxima [F] . . . 691
Giac [F] . . . 691
Mupad [F(-1)] . . . 692

## Optimal result

Integrand size = 8, antiderivative size = 78

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b}$$

```
[Out] 6*(b*x+a)*arcsinh(b*x+a)/b+(b*x+a)*arcsinh(b*x+a)^3/b-6*(1+(b*x+a)^2)^(1/2)
/b-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5798, 267}

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b} - \frac{3\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{b} + \frac{6(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{6\sqrt{(a+bx)^2+1}}{b}$$

```
[In] Int[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2])/b + (6*(a + b*x)*ArcSinh[a + b*x])/b - (3*Sqrt[1
 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/b + ((a + b*x)*ArcSinh[a + b*x]^3)/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{arcsinh}(x)^3\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b} - \frac{3\text{Subst}\left(\int \frac{x\operatorname{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= -\frac{3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b} \\
&\quad + \frac{6\text{Subst}(\int \operatorname{arcsinh}(x)\,dx, x, a+bx)}{b} \\
&= \frac{6(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b} \\
&\quad + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b} - \frac{6\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\operatorname{arcsinh}(a+bx)}{b} \\
&\quad - \frac{3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \operatorname{arcsinh}(a+bx)^3\,dx$$
$$= \frac{-6\sqrt{1+(a+bx)^2}+6(a+bx)\operatorname{arcsinh}(a+bx)-3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2+(a+bx)\operatorname{arcsinh}(a+}{b}$$

```
[In] Integrate[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2] + 6*(a + b*x)*ArcSinh[a + b*x] - 3*Sqrt[1 + (a +
b*x)^2]*ArcSinh[a + b*x]^2 + (a + b*x)*ArcSinh[a + b*x]^3)/b
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6(bx+a)\operatorname{arcsinh}(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$ | 67 |
| default | $\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6(bx+a)\operatorname{arcsinh}(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$ | 67 |

```
[In] int(arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x
+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int \operatorname{arcsinh}(a+bx)^3\,dx$$
$$= \frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,abx+}\right.}{b}$$

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 3*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 6*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))/b
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \begin{cases} \frac{a\,\operatorname{asinh}^3(a+bx)}{b} + \frac{6a\,\operatorname{asinh}(a+bx)}{b} + x\,\operatorname{asinh}^3(a+bx) + 6x\,\operatorname{asinh}(a+bx) - \frac{3\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)}{b} - \frac{6\sqrt{a^2} \\ x\,\operatorname{asinh}^3(a) \end{cases}$$

```
[In] integrate(asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**3/b + 6*a*asinh(a + b*x)/b + x*asinh(a + b*x)*
*3 + 6*x*asinh(a + b*x) - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/b - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a
)**3, True))
```

## Maxima [F]

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{arsinh}(bx+a)^3\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^3 + 3*
a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a
), x)
```

## Giac [F]

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{arsinh}(bx+a)^3\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{asinh}(a+b\,x)^3\,dx$$

```
[In] int(asinh(a + b*x)^3,x)
[Out] int(asinh(a + b*x)^3, x)
```

# 3.78 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx$

Optimal result . . . 693
Rubi [A] (verified) . . . 694
Mathematica [A] (verified) . . . 698
Maple [F] . . . 699
Fricas [F] . . . 699
Sympy [F] . . . 699
Maxima [F] . . . 700
Giac [F] . . . 700
Mupad [F(-1)] . . . 700

## Optimal result

Integrand size = 12, antiderivative size = 275

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = &-\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ &+ \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ &+ 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ &+ 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ &- 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ &- 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ &+ 6\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + 6\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\end{aligned}$$

```
[Out] -1/4*arcsinh(b*x+a)^4+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+6*polylog(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+6*poly
log(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx &= 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) \\ &\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right) \\ &\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) \\ &\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right) \\ &\quad + 6 \operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) + 6 \operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right) \\ &\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) \\ &\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right) - \frac{1}{4}\operatorname{arcsinh}(a+bx)^4\end{aligned}$$

```
[In] Int[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a
- Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt
[1 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[
1 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 +
a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)^3}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^3\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \frac{\operatorname{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 3\operatorname{Subst}\left(\int x^2 \log\left(1+\frac{e^x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad - 3\operatorname{Subst}\left(\int x^2 \log\left(1+\frac{e^x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -\frac{e^x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad - 6\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -\frac{e^x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 6\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -\frac{e^x}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right) dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&\quad + 6\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -\frac{e^x}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right) dx, x, \operatorname{arcsinh}(a+bx)\right) \\
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{x}{a-\sqrt{1+a^2}}\right)}{x} dx, x, e^{\operatorname{arcsinh}(a+bx)}\right) \\
&\quad + 6\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{x}{a+\sqrt{1+a^2}}\right)}{x} dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 - \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\
&\quad + 6\operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + 6\operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.26

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx &= -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3 \log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad + \operatorname{arcsinh}(a+bx)^3 \log\left(1 + \frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad + 3\operatorname{arcsinh}(a+bx)^2 \operatorname{PolyLog}\left(2, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} - \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad - 6\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(3, -\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b} + \frac{\sqrt{1+a^2}}{b}\right) b}\right) \\
&\quad + 6\operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + 6\operatorname{PolyLog}\left(4, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 3*ArcSinh[a + b*x]^2*PolyLog[2, -(E^Arc
Sinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 3*ArcSinh[a + b*x]^2*PolyL
og[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 6*ArcSinh[a +
 b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] - 6*
ArcSinh[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b
)*b))] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4
, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(bx+a\right)^3}{x}dx$$

```
[In] int(arcsinh(b*x+a)^3/x,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asinh}^3\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(asinh(b*x+a)**3/x,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="maxima")
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x}\,dx$$

```
[In] int(asinh(a + b*x)^3/x,x)
[Out] int(asinh(a + b*x)^3/x, x)
```

# 3.79 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx$

Optimal result . . . 701
Rubi [A] (verified) . . . 702
Mathematica [A] (verified) . . . 706
Maple [F] . . . 706
Fricas [F] . . . 707
Sympy [F] . . . 707
Maxima [F] . . . 707
Giac [F] . . . 707
Mupad [F(-1)] . . . 708

## Optimal result

Integrand size = 12, antiderivative size = 268

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = &-\frac{\operatorname{arcsinh}(a+bx)^3}{x} - \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\\ &+\frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\\ &-\frac{6b\operatorname{arcsinh}(a+bx)\,\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\\ &+\frac{6b\operatorname{arcsinh}(a+bx)\,\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\\ &+\frac{6b\,\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{6b\,\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\end{aligned}$$

```
[Out] -arcsinh(b*x+a)^3/x-3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6*b*arcsinh(b*x+a)*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6*b*arcsinh(b*x+
a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5859, 5828, 5843, 3403, 2296, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = &-\frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} \\ &+ \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} \\ &+ \frac{6b\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} - \frac{6b\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} \\ &- \frac{3b\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}} \\ &+ \frac{3b\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}} - \frac{\operatorname{arcsinh}(a+bx)^3}{x}\end{aligned}$$

```
[In] Int[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^3/x) - (3*b*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x
]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (3*b*ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (6*b*ArcSinh[a + b*x]
*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (6*b*A
rcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1
 + a^2] + (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 +
 a^2] - (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^
2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)^3}{x} + 3\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\text{arcsinh}(a+bx)^3}{x} + 3\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right) \\
&= -\frac{\text{arcsinh}(a+bx)^3}{x} + 6\text{Subst}\left(\int \frac{e^x x^2}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \text{arcsinh}(a+bx)\right) \\
&= -\frac{\text{arcsinh}(a+bx)^3}{x} + \frac{6\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{6\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}} \\
&= -\frac{\text{arcsinh}(a+bx)^3}{x} - \frac{3b\text{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{3b\text{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{(6b)\text{Subst}\left(\int x\log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{(6b)\text{Subst}\left(\int x\log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(a+bx)^3}{x} - \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{(6b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx,x,\operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{(6b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx,x,\operatorname{arcsinh}(a+bx)\right)}{\sqrt{1+a^2}} \\
&= -\frac{\operatorname{arcsinh}(a+bx)^3}{x} - \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\
&\quad + \frac{(6b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{x}{a-\sqrt{1+a^2}}\right)}{x}\,dx,x,e^{\operatorname{arcsinh}(a+bx)}\right)}{\sqrt{1+a^2}} \\
&\quad - \frac{(6b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{x}{a+\sqrt{1+a^2}}\right)}{x}\,dx,x,e^{\operatorname{arcsinh}(a+bx)}\right)}{\sqrt{1+a^2}}
\end{aligned}
$$

$$= -\frac{\operatorname{arcsinh}(a+bx)^3}{x} - \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$- \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{6b\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$
$$+ \frac{6b\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{6b\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx =$$
$$-\frac{\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^3 - 3bx\operatorname{arcsinh}(a+bx)^2 \log\left(\frac{a+\sqrt{1+a^2}-e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) + 3bx\operatorname{arcsinh}(a+bx)^2 \log\left(\right.}{}$$

```
[In] Integrate[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -((Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 - 3*b*x*ArcSinh[a + b*x]^2*Log[(a + Sqr
t[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + 3*b*x*ArcSinh[a + b
*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] +
 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]
- 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
 - 6*b*x*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b*x*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x))
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x^2}\,dx$$

```
[In] int(arcsinh(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^2,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}^3\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(asinh(b*x+a)**3/x**2,x)
[Out] Integral(asinh(a + b*x)**3/x**2, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x + integrate(3*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^4 + 3*a*b^2*x^3
 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^3/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x^2}\,dx$$

```
[In] int(asinh(a + b*x)^3/x^2,x)
```

```
[Out] int(asinh(a + b*x)^3/x^2, x)
```

# 3.80 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx$

Optimal result . . . 709
Rubi [A] (verified) . . . 710
Mathematica [A] (verified) . . . 719
Maple [F] . . . 720
Fricas [F] . . . 720
Sympy [F] . . . 720
Maxima [F] . . . 720
Giac [F] . . . 721
Mupad [F(-1)] . . . 721

## Optimal result

Integrand size = 12, antiderivative size = 514

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx &= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\ &\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\ &\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\ &\quad + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\ &\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\ &\quad + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\ &\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} \\ &\quad + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\ &\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} \\ &\quad - \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}}\end{aligned}$$

```
[Out] -3/2*b^2*arcsinh(b*x+a)^2/(a^2+1)-1/2*arcsinh(b*x+a)^3/x^2+3*b^2*arcsinh(b*
x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)+3/2*a*b^2*
arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1
)^(3/2)+3*b^2*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))/(a^2+1)-3/2*a*b^2*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))
/(a-(a^2+1)^(1/2)))/(a^2+1)+3*a*b^2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)-3*a*b^2*arcsinh(b*x+a)*polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3*a*b^2*polyl
og(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*a*b^2*p
olylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3/2*b
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules

used = {5859, 5828, 5843, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{a^2+1} + \frac{3b^2\operatorname{PolyLog}\left(2, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{a^2+1} - \frac{3ab^2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}} + \frac{3ab^2\operatorname{PolyLog}\left(3, \frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(a^2+1\right)} + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{2\left(a^2+1\right)^{3/2}} - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\left(a^2+1\right)^{3/2}} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{a^2+1} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{a^2+1} - \frac{3b\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{2\left(a^2+1\right)x} - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2}$$

```
[In] Int[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSinh[a + b*x]^2)/(2*(1 + a^2)) - (3*b*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2)/(2*(1 + a^2)*x) - ArcSinh[a + b*x]^3/(2*x^2) + (3*b^2*ArcSi
nh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3
*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
(2*(1 + a^2)^(3/2)) + (3*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)) + (3*b^2*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3*a*b^2*ArcSinh[a + b*
x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (3
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b
^2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1
 + a^2)^(3/2) - (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])
```

```
/(1 + a^2)^(3/2) + (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{arcsinh}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\text{arcsinh}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \text{arcsinh}(a+bx)\right) \\
&= -\frac{3b\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\text{arcsinh}(a+bx)^3}{2x^2} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad - \frac{(3ab)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{2\left(1+a^2\right)} \\
&= -\frac{3b^2\text{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\text{arcsinh}(a+bx)^3}{2x^2} + \frac{(3b)\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad - \frac{(3ab)\text{Subst}\left(\int \frac{e^x x^2}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \text{arcsinh}(a+bx)\right)}{1+a^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{(3ab)\operatorname{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{(3ab)\operatorname{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{(3b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{e^x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{1+a^2} \\
&\quad - \frac{(3b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{e^x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{1+a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{(3ab^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{(3ab^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{(3b^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{x}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{1+a^2} \\
&\quad - \frac{(3b^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{x}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{1+a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{(3ab^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2e^x}{\left(-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}\right)b}\right.\right.}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{\left.\left.(3ab^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2e^x}{\left(-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}\right)b}\right)\,dx,x,\operatorname{arcsinh}(a+bx)\right)\right)}{\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{(3ab^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{x}{a-\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{(3ab^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{x}{a+\sqrt{1+a^2}}\right)}{x}\,dx, x, e^{\operatorname{arcsinh}(a+bx)}\right)}{\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2\left(1+a^2\right)x} \\
&\quad - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} \\
&\quad + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad + \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} \\
&\quad - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} \\
&\quad - \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}} + \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02

$$
\begin{aligned}
&\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx \\
&= \frac{-3\sqrt{1+a^2}b^2x^2\operatorname{arcsinh}(a+bx)^2 - 3\sqrt{1+a^2}bx\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2 - \sqrt{1+a^2}\operatorname{arcsinh}
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]^2 - 3*Sqrt[1 + a^2]*b*x*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2 - Sqrt[1 + a^2]*ArcSinh[a + b*
x]^3 - a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSi
nh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2]
)] - 3*a*b^2*x^2*ArcSinh[a + b*x]^2*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a +
b*x])/(a + Sqrt[1 + a^2])] + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]*Log[(
-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] + 3*a*b^2*x^
2*ArcSinh[a + b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sq
rt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] + a*ArcSinh[a + b*x])*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] - a*ArcSi
```

```
nh[a + b*x])*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])] - 6*a*b^2*x
^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*a*b^2*x^2*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)*x^2)
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x^3}\,dx$$

```
[In] int(arcsinh(b*x+a)^3/x^3,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^3,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^3}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}^3(a+bx)}{x^3}\,dx$$

```
[In] integrate(asinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**3, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^3}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x^2 + integrate(3/2
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^5 + 3*a
*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2
+ 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^3}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^3/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x^3}\,dx$$

```
[In] int(asinh(a + b*x)^3/x^3,x)
[Out] int(asinh(a + b*x)^3/x^3, x)
```

# 3.81 $\int \frac{x^2}{\text{arcsinh}(a+bx)}\,dx$

Optimal result . . . 722
Rubi [A] (verified) . . . 722
Mathematica [A] (verified) . . . 725
Maple [A] (verified) . . . 725
Fricas [F] . . . 725
Sympy [F] . . . 726
Maxima [F] . . . 726
Giac [F] . . . 726
Mupad [F(-1)] . . . 726

## Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{x^2}{\text{arcsinh}(a+bx)}\,dx = -\frac{\text{Chi}(\text{arcsinh}(a+bx))}{4b^3} + \frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{b^3} + \frac{\text{Chi}(3\text{arcsinh}(a+bx))}{4b^3} - \frac{a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3}$$

```
[Out] -1/4*Chi(arcsinh(b*x+a))/b^3+a^2*Chi(arcsinh(b*x+a))/b^3+1/4*Chi(3*arcsinh(
b*x+a))/b^3-a*Shi(2*arcsinh(b*x+a))/b^3
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5830, 6873, 12, 6874, 3382, 5556, 3379}

$$\int \frac{x^2}{\text{arcsinh}(a+bx)}\,dx = \frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{b^3} - \frac{\text{Chi}(\text{arcsinh}(a+bx))}{4b^3} + \frac{\text{Chi}(3\text{arcsinh}(a+bx))}{4b^3} - \frac{a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3}$$

```
[In] Int[x^2/ArcSinh[a + b*x],x]
```

```
[Out] -1/4*CoshIntegral[ArcSinh[a + b*x]]/b^3 + (a^2*CoshIntegral[ArcSinh[a + b*x
]])/b^3 + CoshIntegral[3*ArcSinh[a + b*x]]/(4*b^3) - (a*SinhIntegral[2*ArcS
inh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{b^2 x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2\cosh(x)}{x} - \frac{2a\cosh(x)\sinh(x)}{x} + \frac{\cosh(x)\sinh^2(x)}{x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{b^3} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{b^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^3} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^3} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= -\frac{\text{Chi}(\text{arcsinh}(a+bx))}{4b^3} + \frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{b^3} \\
&\quad + \frac{\text{Chi}(3\text{arcsinh}(a+bx))}{4b^3} - \frac{a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \frac{(-1+4a^2)\operatorname{Chi}(\operatorname{arcsinh}(a+bx)) + \operatorname{Chi}(3\operatorname{arcsinh}(a+bx)) - 4a\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{4b^3}$$

```
[In] Integrate[x^2/ArcSinh[a + b*x],x]
```

```
[Out] ((-1 + 4*a^2)*CoshIntegral[ArcSinh[a + b*x]] + CoshIntegral[3*ArcSinh[a + b
*x]] - 4*a*SinhIntegral[2*ArcSinh[a + b*x]])/(4*b^3)
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(bx+a))}{4}-a\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))+a^2\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{b^3}$ | 49 |
| default | $\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(bx+a))}{4}-a\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))+a^2\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{b^3}$ | 49 |

```
[In] int(x^2/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/4*Chi(arcsinh(b*x+a))+1/4*Chi(3*arcsinh(b*x+a))-a*Shi(2*arcsinh(b
*x+a))+a^2*Chi(arcsinh(b*x+a)))
```

## Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a), x)
```

## Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{asinh}(a+bx)}\,dx$$

```
[In] integrate(x**2/asinh(b*x+a),x)
[Out] Integral(x**2/asinh(a + b*x), x)
```

## Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

## Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{asinh}(a+b\,x)}\,dx$$

```
[In] int(x^2/asinh(a + b*x),x)
[Out] int(x^2/asinh(a + b*x), x)
```

# 3.82 $\int \frac{x}{\text{arcsinh}(a+bx)}\,dx$

Optimal result . . . 727
Rubi [A] (verified) . . . 727
Mathematica [A] (verified) . . . 729
Maple [A] (verified) . . . 729
Fricas [F] . . . 730
Sympy [F] . . . 730
Maxima [F] . . . 730
Giac [F] . . . 730
Mupad [F(-1)] . . . 731

## Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{x}{\text{arcsinh}(a+bx)}\,dx = -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2}$$

```
[Out] -a*Chi(arcsinh(b*x+a))/b^2+1/2*Shi(2*arcsinh(b*x+a))/b^2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5859, 5830, 6873, 12, 6874, 3382, 5556, 3379}

$$\int \frac{x}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2} - \frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2}$$

```
[In] Int[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{bx}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a\cosh(x)}{x} + \frac{\cosh(x)\sinh(x)}{x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2} \\
&= -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$
\int \frac{x}{\text{arcsinh}(a+bx)}\,dx = -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2}
$$

```
[In] Integrate[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
](/(2*b^2)
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{Shi}(2\,\text{arcsinh}(bx+a))}{2} - a\,\text{Chi}(\text{arcsinh}(bx+a))}{b^2}$ | 27 |
| default | $\frac{\text{Shi}(2\,\text{arcsinh}(bx+a))}{2} - a\,\text{Chi}(\text{arcsinh}(bx+a))}{b^2}$ | 27 |

```
[In] int(x/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*Shi(2*arcsinh(b*x+a))-a*Chi(arcsinh(b*x+a)))
```

## Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(x/arcsinh(b*x + a), x)
```

## Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

```
[In] integrate(x/asinh(b*x+a),x)
[Out] Integral(x/asinh(a + b*x), x)
```

## Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x/arcsinh(b*x + a), x)
```

## Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(x/arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

```
[In] int(x/asinh(a + b*x),x)
[Out] int(x/asinh(a + b*x), x)
```

# 3.83 $\int \frac{1}{\mathbf{arcsinh}(a+bx)}\,dx$

Optimal result . . . 732
Rubi [A] (verified) . . . 732
Mathematica [A] (verified) . . . 733
Maple [A] (verified) . . . 733
Fricas [F] . . . 734
Sympy [F] . . . 734
Maxima [F] . . . 734
Giac [F] . . . 734
Mupad [F(-1)] . . . 735

## Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{b}$$

```
[Out] Chi(arcsinh(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5858, 5774, 3382}

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{b}$$

```
[In] Int[ArcSinh[a + b*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\ &= \frac{\text{Chi}(\text{arcsinh}(a+bx))}{b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(\text{arcsinh}(a+bx))}{b}$$

```
[In] Integrate[ArcSinh[a + b*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\text{Chi}(\text{arcsinh}(bx+a))}{b}$ | 12 |
| default | $\frac{\text{Chi}(\text{arcsinh}(bx+a))}{b}$ | 12 |

```
[In] int(1/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] Chi(arcsinh(b*x+a))/b
```

## Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(1/arcsinh(b*x + a), x)
```

## Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

```
[In] integrate(1/asinh(b*x+a),x)
[Out] Integral(1/asinh(a + b*x), x)
```

## Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(1/arcsinh(b*x + a), x)
```

## Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(1/arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

```
[In] int(1/asinh(a + b*x),x)
[Out] int(1/asinh(a + b*x), x)
```

# 3.84 $\int \frac{1}{x\mathbf{arcsinh}(a+bx)}\,dx$

Optimal result . . . 736
Rubi [N/A] . . . 736
Mathematica [N/A] . . . 737
Maple [N/A] (verified) . . . 737
Fricas [N/A] . . . 737
Sympy [N/A] . . . 737
Maxima [N/A] . . . 738
Giac [N/A] . . . 738
Mupad [N/A] . . . 738

## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)}\,dx = \operatorname{Int}\left(\frac{1}{x\operatorname{arcsinh}(a+bx)}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\operatorname{arcsinh}(a+bx)}\,dx$$

```
[In] Int[1/(x*ArcSinh[a + b*x]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{(-\frac{a}{b}+\frac{x}{b})\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{b}$$

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\text{arcsinh}(a+bx)}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a + b*x]),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ \text{arcsinh}\left(bx+a\right)}dx$$

```
[In] int(1/x/arcsinh(b*x+a),x)
[Out] int(1/x/arcsinh(b*x+a),x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\text{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(b*x + a)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\text{asinh}\left(a+bx\right)}\,dx$$

```
[In] integrate(1/x/asinh(b*x+a),x)
[Out] Integral(1/(x*asinh(a + b*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\text{arsinh}\,(bx+a)}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\text{arsinh}\,(bx+a)}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\text{asinh}\,(a+b\,x)}\,dx$$

```
[In] int(1/(x*asinh(a + b*x)),x)
[Out] int(1/(x*asinh(a + b*x)), x)
```

# 3.85 $\int \frac{x^2}{\text{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 739
Rubi [A] (verified) . . . 739
Mathematica [A] (verified) . . . 742
Maple [A] (verified) . . . 742
Fricas [F] . . . 743
Sympy [F] . . . 743
Maxima [F] . . . 743
Giac [F] . . . 744
Mupad [F(-1)] . . . 744

## Optimal result

Integrand size = 12, antiderivative size = 154

$$\begin{aligned}\int \frac{x^2}{\text{arcsinh}(a+bx)^2}\,dx = &-\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)}+\frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)}\\&-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)}\\&-\frac{2a\text{Chi}(2\text{arcsinh}(a+bx))}{b^3}-\frac{\text{Shi}(\text{arcsinh}(a+bx))}{4b^3}\\&+\frac{a^2\text{Shi}(\text{arcsinh}(a+bx))}{b^3}+\frac{3\text{Shi}(3\text{arcsinh}(a+bx))}{4b^3}\end{aligned}$$

```
[Out] -2*a*Chi(2*arcsinh(b*x+a))/b^3-1/4*Shi(arcsinh(b*x+a))/b^3+a^2*Shi(arcsinh(
b*x+a))/b^3+3/4*Shi(3*arcsinh(b*x+a))/b^3-a^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsi
nh(b*x+a)+2*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)-(b*x+a)^2*(1+(
b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used

$= \{5859, 5829, 5773, 5819, 3379, 5778, 3382\}$

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{a^2\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{b^3} - \frac{a^2\sqrt{(a+bx)^2+1}}{b^3\operatorname{arcsinh}(a+bx)} - \frac{2a\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{b^3} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{4b^3} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(a+bx))}{4b^3} + \frac{2a(a+bx)\sqrt{(a+bx)^2+1}}{b^3\operatorname{arcsinh}(a+bx)} - \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{b^3\operatorname{arcsinh}(a+bx)}$$

```
[In] Int[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x])) + (2*a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]) - ((a + b*x)^2*Sqrt[1 + (a + b*x)
^2])/(b^3*ArcSinh[a + b*x]) - (2*a*CoshIntegral[2*ArcSinh[a + b*x]])/b^3 -
SinhIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*SinhIntegral[ArcSinh[a + b*x]
])/b^3 + (3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\text{arcsinh}(x)^2} - \frac{2ax}{b^2\text{arcsinh}(x)^2} + \frac{x^2}{b^2\text{arcsinh}(x)^2}\right)\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)} \\
&\quad + \frac{\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4x} + \frac{3\sinh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} \\
&\quad - \frac{2a\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{b^3} - \frac{\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^3} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^3} \\
&\quad + \frac{a^2\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)} - \frac{2a\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{b^3} \\
&\quad - \frac{\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{4b^3} + \frac{a^2\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{b^3} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(a+bx))}{4b^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$
\begin{aligned}
&\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx \\
&= \frac{-\frac{4b^2x^2\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)} - 8a\operatorname{Chi}(2\operatorname{arcsinh}(a+bx)) + (-1+4a^2)\operatorname{Shi}(\operatorname{arcsinh}(a+bx)) + 3\operatorname{Shi}(3\operatorname{arcsinh}(a+
}{4b^3}
\end{aligned}
$$

```
[In] Integrate[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] ((-4*b^2*x^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/ArcSinh[a + b*x] - 8*a*Cosh
Integral[2*ArcSinh[a + b*x]] + (-1 + 4*a^2)*SinhIntegral[ArcSinh[a + b*x]]
+ 3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\sqrt{1+(bx+a)^2}}{4\operatorname{arcsinh}(bx+a)} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{4} - \frac{\cosh(3\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(bx+a))}{4} - \frac{a(2\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-s}{\operatorname{arcsinh}(bx+a)}}{b^3}$ |
| default | $\frac{\frac{\sqrt{1+(bx+a)^2}}{4\operatorname{arcsinh}(bx+a)} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{4} - \frac{\cosh(3\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(bx+a))}{4} - \frac{a(2\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-s}{\operatorname{arcsinh}(bx+a)}}{b^3}$ |

```
[In] int(x^2/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/4/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/4*Shi(arcsinh(b*x+a))-1/4/a
rcsinh(b*x+a)*cosh(3*arcsinh(b*x+a))+3/4*Shi(3*arcsinh(b*x+a))-a*(2*Chi(2*a
rcsinh(b*x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)+a^2*(S
hi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

## Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^2/arcsinh(b*x + a)^2, x)
```

## Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

```
[In] integrate(x**2/asinh(b*x+a)**2,x)
[Out] Integral(x**2/asinh(a + b*x)**2, x)
```

## Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="maxima")
[Out] -(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*
a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a
*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b
*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((3*b^5*x^6 + 14*a*
b^4*x^5 + 2*(13*a^2*b^3 + 3*b^3)*x^4 + 8*(3*a^3*b^2 + 2*a*b^2)*x^3 + (11*a^
4*b + 14*a^2*b + 3*b)*x^2 + (3*b^3*x^4 + 8*a*b^2*x^3 + (7*a^2*b + b)*x^2 +
2*(a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^5 + 2*a^3 + a)*x + (6*b
^4*x^5 + 22*a*b^3*x^4 + (30*a^2*b^2 + 7*b^2)*x^3 + (18*a^3*b + 13*a*b)*x^2
+ 2*(2*a^4 + 3*a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^5*x^4 + 4
*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2
*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*
x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2/arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^2/asinh(a + b*x)^2,x)
[Out] int(x^2/asinh(a + b*x)^2, x)
```

# 3.86 $\int \frac{x}{\text{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 745
Rubi [A] (verified) . . . 745
Mathematica [A] (verified) . . . 747
Maple [A] (verified) . . . 748
Fricas [F] . . . 748
Sympy [F] . . . 748
Maxima [F] . . . 748
Giac [F] . . . 749
Mupad [F(-1)] . . . 749

## Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{x}{\text{arcsinh}(a+bx)^2}\,dx = \frac{a\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{a\text{Shi}(\text{arcsinh}(a+bx))}{b^2}$$

```
[Out] Chi(2*arcsinh(b*x+a))/b^2-a*Shi(arcsinh(b*x+a))/b^2+a*(1+(b*x+a)^2)^(1/2)/b
^2/arcsinh(b*x+a)-(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5859, 5829, 5773, 5819, 3379, 5778, 3382}

$$\int \frac{x}{\text{arcsinh}(a+bx)^2}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{a\text{Shi}(\text{arcsinh}(a+bx))}{b^2} + \frac{a\sqrt{(a+bx)^2+1}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\text{arcsinh}(a+bx)}$$

```
[In] Int[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(b^2*ArcSinh[a + b*x]) - ((a + b*x)*Sqrt[1 + (a +
 b*x)^2])/(b^2*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b^2 - (
a*SinhIntegral[ArcSinh[a + b*x]])/b^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\text{arcsinh}(x)^2}+\frac{x}{b\text{arcsinh}(x)^2}\right)\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^2} \\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b^2} \\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} \\
&\quad + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{a\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} \\
&\quad + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{a\text{Shi}(\text{arcsinh}(a+bx))}{b^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{x}{\text{arcsinh}(a+bx)^2}\,dx = -\frac{bx\sqrt{1+(a+bx)^2} - \text{arcsinh}(a+bx)\text{Chi}(2\text{arcsinh}(a+bx)) + a\text{arcsinh}(a+bx)\text{Shi}(\text{arcsinh}(a+bx))}{b^2\text{arcsinh}(a+bx)}$$

```
[In] Integrate[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((b*x*Sqrt[1 + (a + b*x)^2] - ArcSinh[a + b*x]*CoshIntegral[2*ArcSinh[a +
b*x]] + a*ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/(b^2*ArcSinh[a +
 b*x]))
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\sinh(2\ \operatorname{arcsinh}(bx+a))}{2\ \operatorname{arcsinh}(bx+a)}+\operatorname{Chi}(2\ \operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Shi}(\operatorname{arcsinh}(bx+a))\ \operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)}{\operatorname{arcsinh}(bx+a)}}{b^2}$ | 73 |
| default | $\frac{-\frac{\sinh(2\ \operatorname{arcsinh}(bx+a))}{2\ \operatorname{arcsinh}(bx+a)}+\operatorname{Chi}(2\ \operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Shi}(\operatorname{arcsinh}(bx+a))\ \operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)}{\operatorname{arcsinh}(bx+a)}}{b^2}$ | 73 |

```
[In] int(x/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2/arcsinh(b*x+a)*sinh(2*arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a))-a*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

## Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^2, x)
```

## Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

```
[In] integrate(x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x/asinh(a + b*x)**2, x)
```

## Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*
b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2
```

```
*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((2*b^5*x^5 + 9*a*b^4*x
^4 + a^5 + 4*(4*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2 + 5*a*b^2)*x^2 +
(2*b^3*x^3 + 5*a*b^2*x^2 + 4*a^2*b*x + a^3 + a)*(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*(3*a^4*b + 4*a^2*b + b)*x + (4*b^4*x^4 + 14*a*b^3*x^3 + 2*a^4 + 2*(9
*a^2*b^2 + 2*b^2)*x^2 + 3*a^2 + (10*a^3*b + 7*a*b)*x + 1)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) + a)/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*
b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2
*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x/asinh(a + b*x)^2,x)
```

```
[Out] int(x/asinh(a + b*x)^2, x)
```

# 3.87 $\int \frac{1}{\text{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 750
Rubi [A] (verified) . . . 750
Mathematica [A] (verified) . . . 751
Maple [A] (verified) . . . 752
Fricas [F] . . . 752
Sympy [F] . . . 752
Maxima [F] . . . 752
Giac [F] . . . 753
Mupad [F(-1)] . . . 753

## Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{1}{\text{arcsinh}(a+bx)^2}\,dx = -\frac{\sqrt{1+(a+bx)^2}}{b\,\text{arcsinh}(a+bx)} + \frac{\text{Shi}(\text{arcsinh}(a+bx))}{b}$$

```
[Out] Shi(arcsinh(b*x+a))/b-(1+(b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3379}

$$\int \frac{1}{\text{arcsinh}(a+bx)^2}\,dx = \frac{\text{Shi}(\text{arcsinh}(a+bx))}{b} - \frac{\sqrt{(a+bx)^2+1}}{b\,\text{arcsinh}(a+bx)}$$

```
[In] Int[ArcSinh[a + b*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x])) + SinhIntegral[ArcSinh[a + b*
x]]/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
```

```
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Shi}(\text{arcsinh}(a+bx))}{b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{\text{arcsinh}(a+bx)^2}\,dx = \frac{-\frac{\sqrt{1+(a+bx)^2}}{\text{arcsinh}(a+bx)} + \text{Shi}(\text{arcsinh}(a+bx))}{b}$$

```
[In] Integrate[ArcSinh[a + b*x]^(-2),x]
```

```
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]) + SinhIntegral[ArcSinh[a + b*x]]
)/b
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{b}$ | 34 |
| default | $\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{b}$ | 34 |

```
[In] int(1/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+Shi(arcsinh(b*x+a)))
```

## Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^(-2), x)
```

## Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

```
[In] integrate(1/asinh(b*x+a)**2,x)
[Out] Integral(asinh(a + b*x)**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="maxima")
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

```
 + integrate((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*
x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 - 1) + 2*a^2 + 4*(a^3*b +
 a*b)*x + (2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2
 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^
2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x +
 a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^(-2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/asinh(a + b*x)^2,x)
```

```
[Out] int(1/asinh(a + b*x)^2, x)
```

# 3.88 $\int \frac{1}{x\mathbf{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 754
Rubi [N/A] . . . 754
Mathematica [N/A] . . . 755
Maple [N/A] (verified) . . . 755
Fricas [N/A] . . . 755
Sympy [N/A] . . . 755
Maxima [N/A] . . . 756
Giac [N/A] . . . 756
Mupad [N/A] . . . 756

## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx = \operatorname{Int}\left(\frac{1}{x\operatorname{arcsinh}(a+bx)^2}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx$$

```
[In] Int[1/(x*ArcSinh[a + b*x]^2),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^2), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{(-\frac{a}{b}+\frac{x}{b})\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b}$$

## Mathematica [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^2),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,\text{arcsinh}\left(bx+a\right)^2}dx$$

```
[In] int(1/x/arcsinh(b*x+a)^2,x)
[Out] int(1/x/arcsinh(b*x+a)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(b*x + a)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{asinh}^2\left(a+bx\right)}\,dx$$

```
[In] integrate(1/x/asinh(b*x+a)**2,x)
[Out] Integral(1/(x*asinh(a + b*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 527, normalized size of antiderivative = 43.92

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(b^2*x^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))) - integrate((a*b^4*x^4 + 4*a^2*b^3*x^3 + a^5 + 2*a^3 + 2*(3*a^3*
b^2 + a*b^2)*x^2 + (a*b^2*x^2 + a^3 + 2*(a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 4*(a^4*b + a^2*b)*x + (2*a*b^3*x^3 + 2*a^4 + 2*(3*a^2*b^2 +
 b^2)*x^2 + 3*a^2 + (6*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1) + a)/((b^5*x^6 + 4*a*b^4*x^5 + 2*(3*a^2*b^3 + b^3)*x^4 + 4*(a^3*b^2 + a
*b^2)*x^3 + (a^4*b + 2*a^2*b + b)*x^2 + (b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(b^4*x^5 + 3*a*b^3*x^4 + (3*a^2*b^2 + b^
2)*x^3 + (a^3*b + a*b)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a
+ sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x*asinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*asinh(a + b*x)^2), x)
```

# 3.89 $\int \frac{x^2}{\text{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 757
Rubi [A] (verified) . . . 758
Mathematica [A] (verified) . . . 763
Maple [A] (verified) . . . 763
Fricas [F] . . . 764
Sympy [F] . . . 764
Maxima [F] . . . 764
Giac [F] . . . 766
Mupad [F(-1)] . . . 766

## Optimal result

Integrand size = 12, antiderivative size = 257

$$\begin{aligned}\int \frac{x^2}{\text{arcsinh}(a+bx)^3}\,dx = &-\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\text{arcsinh}(a+bx)^2}+\frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\text{arcsinh}(a+bx)^2}\\&-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\text{arcsinh}(a+bx)^2}+\frac{a}{b^3\text{arcsinh}(a+bx)}\\&-\frac{a+bx}{b^3\text{arcsinh}(a+bx)}-\frac{a^2(a+bx)}{2b^3\text{arcsinh}(a+bx)}\\&+\frac{2a(a+bx)^2}{b^3\text{arcsinh}(a+bx)}-\frac{3(a+bx)^3}{2b^3\text{arcsinh}(a+bx)}\\&-\frac{\text{Chi}(\text{arcsinh}(a+bx))}{8b^3}+\frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{2b^3}\\&+\frac{9\text{Chi}(3\text{arcsinh}(a+bx))}{8b^3}-\frac{2a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3}\end{aligned}$$

```
[Out] a/b^3/arcsinh(b*x+a)+(-b*x-a)/b^3/arcsinh(b*x+a)-1/2*a^2*(b*x+a)/b^3/arcsin
h(b*x+a)+2*a*(b*x+a)^2/b^3/arcsinh(b*x+a)-3/2*(b*x+a)^3/b^3/arcsinh(b*x+a)-
1/8*Chi(arcsinh(b*x+a))/b^3+1/2*a^2*Chi(arcsinh(b*x+a))/b^3+9/8*Chi(3*arcsi
nh(b*x+a))/b^3-2*a*Shi(2*arcsinh(b*x+a))/b^3-1/2*a^2*(1+(b*x+a)^2)^(1/2)/b^
3/arcsinh(b*x+a)^2+a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2-1/2*(
b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5859, 5829, 5773, 5818, 5774, 3382, 5779, 5780, 5556, 12, 3379, 5783}

$$\begin{aligned}\int \frac{x^2}{\text{arcsinh}(a+bx)^3}\,dx = &\frac{a^2\text{Chi}(\text{arcsinh}(a+bx))}{2b^3} - \frac{a^2(a+bx)}{2b^3\text{arcsinh}(a+bx)} \\ &- \frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\text{arcsinh}(a+bx)^2} - \frac{\text{Chi}(\text{arcsinh}(a+bx))}{8b^3} \\ &+ \frac{9\text{Chi}(3\text{arcsinh}(a+bx))}{8b^3} - \frac{2a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3} \\ &- \frac{3(a+bx)^3}{2b^3\text{arcsinh}(a+bx)} + \frac{2a(a+bx)^2}{b^3\text{arcsinh}(a+bx)} \\ &- \frac{\sqrt{(a+bx)^2+1}(a+bx)^2}{2b^3\text{arcsinh}(a+bx)^2} - \frac{a+bx}{b^3\text{arcsinh}(a+bx)} \\ &+ \frac{a\sqrt{(a+bx)^2+1}(a+bx)}{b^3\text{arcsinh}(a+bx)^2} + \frac{a}{b^3\text{arcsinh}(a+bx)}\end{aligned}$$

```
[In] Int[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) + (a*(a + b*x)*Sq
rt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) - ((a + b*x)^2*Sqrt[1 + (a +
b*x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + a/(b^3*ArcSinh[a + b*x]) - (a + b*x)/
(b^3*ArcSinh[a + b*x]) - (a^2*(a + b*x))/(2*b^3*ArcSinh[a + b*x]) + (2*a*(a
 + b*x)^2)/(b^3*ArcSinh[a + b*x]) - (3*(a + b*x)^3)/(2*b^3*ArcSinh[a + b*x]
) - CoshIntegral[ArcSinh[a + b*x]]/(8*b^3) + (a^2*CoshIntegral[ArcSinh[a +
b*x]])/(2*b^3) + (9*CoshIntegral[3*ArcSinh[a + b*x]])/(8*b^3) - (2*a*SinhIn
tegral[2*ArcSinh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\text{arcsinh}(x)^3} - \frac{2ax}{b^2\text{arcsinh}(x)^3} + \frac{x^2}{b^2\text{arcsinh}(x)^3}\right)\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{2b^3} \\
&\quad - \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} \\
&\quad - \frac{(2a)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^3} \\
&\quad + \frac{a^2\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{2b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)} \\
&\quad - \frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)} - \frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{b^3} + \frac{9\operatorname{Subst}\left(\int \frac{x^2}{\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{2b^3} \\
&\quad - \frac{(4a)\operatorname{Subst}\left(\int \frac{x}{\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{2b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad + \frac{a}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)} \\
&\quad - \frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{9\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^3} \\
&\quad - \frac{(4a)\operatorname{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{a^2\operatorname{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad + \frac{a}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)} \\
&\quad - \frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{b^3} + \frac{a^2\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b^3} \\
&\quad + \frac{9\operatorname{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^3} \\
&\quad - \frac{(4a)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad + \frac{a}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)} \\
&\quad + \frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)} - \frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{b^3} \\
&\quad + \frac{a^2\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b^3} - \frac{9\operatorname{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^3} \\
&\quad + \frac{9\operatorname{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^3} \\
&\quad - \frac{(2a)\operatorname{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2} \\
&\quad + \frac{a}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)} - \frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)} \\
&\quad + \frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)} - \frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)} - \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{8b^3} \\
&\quad + \frac{a^2\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b^3} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(a+bx))}{8b^3} - \frac{2a\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx$$
$$= \frac{-\frac{4bx\left(bx\sqrt{1+a^2+2abx+b^2x^2}+(2+2a^2+5abx+3b^2x^2)\operatorname{arcsinh}(a+bx)\right)}{\operatorname{arcsinh}(a+bx)^2}+(-1+4a^2)\operatorname{Chi}(\operatorname{arcsinh}(a+bx))+9\operatorname{Chi}(3\operatorname{arcsin}}{8b^3}$$

```
[In] Integrate[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] ((-4*b*x*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (2 + 2*a^2 + 5*a*b*x + 3*
b^2*x^2)*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2 + (-1 + 4*a^2)*CoshIntegral[
ArcSinh[a + b*x]] + 9*CoshIntegral[3*ArcSinh[a + b*x]] - 16*a*SinhIntegral[
2*ArcSinh[a + b*x]])/(8*b^3)
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| derivativedivides | $\frac{\sqrt{1+(bx+a)^2}}{8\operatorname{arcsinh}(bx+a)^2}+\frac{bx+a}{8\operatorname{arcsinh}(bx+a)}-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{8}-\frac{\cosh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)^2}-\frac{3\sinh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)}+\frac{9\operatorname{Chi}(3\operatorname{arcsinh}(bx+}{8}$ |
| default | $\frac{\sqrt{1+(bx+a)^2}}{8\operatorname{arcsinh}(bx+a)^2}+\frac{bx+a}{8\operatorname{arcsinh}(bx+a)}-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{8}-\frac{\cosh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)^2}-\frac{3\sinh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)}+\frac{9\operatorname{Chi}(3\operatorname{arcsinh}(bx+}{8}$ |

```
[In] int(x^2/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/8/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+1/8/arcsinh(b*x+a)*(b*x+a)-
1/8*Chi(arcsinh(b*x+a))-1/8/arcsinh(b*x+a)^2*cosh(3*arcsinh(b*x+a))-3/8/arc
sinh(b*x+a)*sinh(3*arcsinh(b*x+a))+9/8*Chi(3*arcsinh(b*x+a))-1/2*a*(4*Shi(2
*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*cosh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-s
inh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)^2+1/2*a^2*(Chi(arcsinh(b*x+a))*arcsin
h(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

## Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^2/arcsinh(b*x + a)^3, x)
```

## Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

```
[In] integrate(x**2/asinh(b*x+a)**3,x)
[Out] Integral(x**2/asinh(a + b*x)**3, x)
```

## Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(b^8*x^9 + 7*a*b^7*x^8 + 3*(7*a^2*b^6 + b^6)*x^7 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^6 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^5 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^4 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^3 + (a^7*b
+ 3*a^5*b + 3*a^3*b + a*b)*x^2 + (b^5*x^6 + 4*a*b^4*x^5 + (6*a^2*b^3 + b^3)
*x^4 + 2*(2*a^3*b^2 + a*b^2)*x^3 + (a^4*b + a^2*b)*x^2)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + (3*b^6*x^7 + 15*a*b^5*x^6 + 5*(6*a^2*b^4 + b^4)*x^5 + 15
*(2*a^3*b^3 + a*b^3)*x^4 + (15*a^4*b^2 + 15*a^2*b^2 + 2*b^2)*x^3 + (3*a^5*b
 + 5*a^3*b + 2*a*b)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (3*b^8*x^9 + 23*a*
b^7*x^8 + (77*a^2*b^6 + 9*b^6)*x^7 + 3*(49*a^3*b^5 + 17*a*b^5)*x^6 + (175*a
^4*b^4 + 120*a^2*b^4 + 9*b^4)*x^5 + (133*a^5*b^3 + 150*a^3*b^3 + 33*a*b^3)*
x^4 + 3*(21*a^6*b^2 + 35*a^4*b^2 + 15*a^2*b^2 + b^2)*x^3 + (17*a^7*b + 39*a
^5*b + 27*a^3*b + 5*a*b)*x^2 + (3*b^5*x^6 + 14*a*b^4*x^5 + 2*(13*a^2*b^3 +
2*b^3)*x^4 + 12*(2*a^3*b^2 + a*b^2)*x^3 + (11*a^4*b + 12*a^2*b + b)*x^2 + 2
*(a^5 + 2*a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (9*b^6*x^7 + 51
*a*b^5*x^6 + (120*a^2*b^4 + 17*b^4)*x^5 + 5*(30*a^3*b^3 + 13*a*b^3)*x^4 + (
105*a^4*b^2 + 93*a^2*b^2 + 10*b^2)*x^3 + (39*a^5*b + 59*a^3*b + 20*a*b)*x^2
```

```
 + 2*(3*a^6 + 7*a^4 + 5*a^2 + 1)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^8
+ 3*a^6 + 3*a^4 + a^2)*x + (9*b^7*x^8 + 60*a*b^6*x^7 + (171*a^2*b^5 + 22*b^
5)*x^6 + 2*(135*a^3*b^4 + 52*a*b^4)*x^5 + (255*a^4*b^3 + 196*a^2*b^3 + 18*b
^3)*x^4 + 2*(72*a^5*b^2 + 92*a^3*b^2 + 25*a*b^2)*x^3 + (45*a^6*b + 86*a^4*b
 + 46*a^2*b + 5*b)*x^2 + 2*(3*a^7 + 8*a^5 + 7*a^3 + 2*a)*x)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^7*x^8 + 18*a*b^6*x^7 + (45*a^2*b^5 + 7*b^5)*x^6 + 4*(15*a^3*b^4 + 7*a*b^4)
*x^5 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^4 + 2*(9*a^5*b^2 + 14*a^3*b^2 +
5*a*b^2)*x^3 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(5*a^2*b^6
+ b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b^4 + 6*a^2
*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 + a^2*b^2 +
 (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2
+ 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a*b^6*x^4 +
 a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3*b^4 + 3*a
*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integrate(1/2*(9
*b^10*x^10 + 82*a*b^9*x^9 + 2*a^10 + 2*(167*a^2*b^8 + 18*b^8)*x^8 + 8*a^8 +
 32*(25*a^3*b^7 + 8*a*b^7)*x^7 + 2*(623*a^4*b^6 + 394*a^2*b^6 + 27*b^6)*x^6
 + 12*a^6 + 4*(329*a^5*b^5 + 342*a^3*b^5 + 69*a*b^5)*x^5 + 4*(238*a^6*b^4 +
 365*a^4*b^4 + 144*a^2*b^4 + 9*b^4)*x^4 + 8*a^4 + 16*(29*a^7*b^3 + 61*a^5*b
^3 + 39*a^3*b^3 + 7*a*b^3)*x^3 + (9*b^6*x^6 + 46*a*b^5*x^5 + 2*a^6 + 4*(24*
a^2*b^4 + b^4)*x^4 + 4*a^4 + 8*(13*a^3*b^3 + 2*a*b^3)*x^3 + (61*a^4*b^2 + 2
4*a^2*b^2 - b^2)*x^2 + 2*a^2 + 2*(9*a^5*b + 8*a^3*b - a*b)*x)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + (145*a^8*b^2 + 396*a^6*b^2 + 366*a^4*b^2 + 124*a^2*b^2
 + 9*b^2)*x^2 + (36*b^7*x^7 + 220*a*b^6*x^6 + 8*a^7 + 8*(71*a^2*b^5 + 6*b^5
)*x^5 + 20*a^5 + 16*(50*a^3*b^4 + 13*a*b^4)*x^4 + (660*a^4*b^3 + 356*a^2*b^
3 + 13*b^3)*x^3 + 16*a^3 + (316*a^5*b^2 + 300*a^3*b^2 + 39*a*b^2)*x^2 + 2*(
40*a^6*b + 62*a^4*b + 21*a^2*b - b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^
(3/2) + (54*b^8*x^8 + 384*a*b^7*x^7 + 12*a^8 + 6*(197*a^2*b^6 + 20*b^6)*x^6
 + 36*a^6 + 12*(171*a^3*b^5 + 52*a*b^5)*x^5 + (2190*a^4*b^4 + 1332*a^2*b^4
+ 83*b^4)*x^4 + 38*a^4 + 4*(366*a^5*b^3 + 372*a^3*b^3 + 71*a*b^3)*x^3 + (59
4*a^6*b^2 + 912*a^4*b^2 + 357*a^2*b^2 + 19*b^2)*x^2 + 16*a^2 + 2*(66*a^7*b
+ 144*a^5*b + 97*a^3*b + 19*a*b)*x + 2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*a
^2 + 2*(13*a^9*b + 44*a^7*b + 54*a^5*b + 28*a^3*b + 5*a*b)*x + (36*b^9*x^9
+ 292*a*b^8*x^8 + 8*a^9 + 4*(261*a^2*b^7 + 28*b^7)*x^7 + 28*a^7 + 4*(539*a^
3*b^6 + 172*a*b^6)*x^6 + (2828*a^4*b^5 + 1788*a^2*b^5 + 123*b^5)*x^5 + 36*a
^5 + (2436*a^5*b^4 + 2540*a^3*b^4 + 519*a*b^4)*x^4 + (1372*a^6*b^3 + 2120*a
^4*b^3 + 855*a^2*b^3 + 57*b^3)*x^3 + 20*a^3 + (484*a^7*b^2 + 1032*a^5*b^2 +
 681*a^3*b^2 + 133*a*b^2)*x^2 + 2*(48*a^8*b + 134*a^6*b + 129*a^4*b + 48*a^
2*b + 5*b)*x + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^8 + 8*a*b^9
*x^7 + a^8*b^2 + 4*a^6*b^2 + 4*(7*a^2*b^8 + b^8)*x^6 + 6*a^4*b^2 + 8*(7*a^3
*b^7 + 3*a*b^7)*x^5 + 2*(35*a^4*b^6 + 30*a^2*b^6 + 3*b^6)*x^4 + 4*a^2*b^2 +
 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^3 + (b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*
```

```
b^4*x^2 + 4*a^3*b^3*x + a^4*b^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6
*b^4 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^2 + 4*(b^7*x^5 + 5*a*b^6*x^4 + a^5*b
^2 + a^3*b^2 + (10*a^2*b^5 + b^5)*x^3 + (10*a^3*b^4 + 3*a*b^4)*x^2 + (5*a^4
*b^3 + 3*a^2*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^6 + 6*a
*b^7*x^5 + a^6*b^2 + 2*a^4*b^2 + (15*a^2*b^6 + 2*b^6)*x^4 + a^2*b^2 + 4*(5*
a^3*b^5 + 2*a*b^5)*x^3 + (15*a^4*b^4 + 12*a^2*b^4 + b^4)*x^2 + 2*(3*a^5*b^3
 + 4*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + b^2 + 8*(a^7*b^3 +
 3*a^5*b^3 + 3*a^3*b^3 + a*b^3)*x + 4*(b^9*x^7 + 7*a*b^8*x^6 + a^7*b^2 + 3*
a^5*b^2 + 3*(7*a^2*b^7 + b^7)*x^5 + 3*a^3*b^2 + 5*(7*a^3*b^6 + 3*a*b^6)*x^4
 + (35*a^4*b^5 + 30*a^2*b^5 + 3*b^5)*x^3 + a*b^2 + 3*(7*a^5*b^4 + 10*a^3*b^
4 + 3*a*b^4)*x^2 + (7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))),
 x)
```

## Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^2/asinh(a + b*x)^3,x)
```

```
[Out] int(x^2/asinh(a + b*x)^3, x)
```

# 3.90 $\int \frac{x}{\text{arcsinh}(a+bx)^3}\,dx$

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## Optimal result

Integrand size = 10, antiderivative size = 147

$$\int \frac{x}{\text{arcsinh}(a+bx)^3}\,dx = \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)} - \frac{a\text{Chi}(\text{arcsinh}(a+bx))}{2b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{b^2}$$

```
[Out] -1/2/b^2/arcsinh(b*x+a)+1/2*a*(b*x+a)/b^2/arcsinh(b*x+a)-(b*x+a)^2/b^2/arcs
inh(b*x+a)-1/2*a*Chi(arcsinh(b*x+a))/b^2+Shi(2*arcsinh(b*x+a))/b^2+1/2*a*(1
+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)^2-1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/
arcsinh(b*x+a)^2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5859, 5829, 5773, 5818, 5774, 3382, 5779, 5780, 5556, 12, 3379, 5783}

$$\int \frac{x}{\text{arcsinh}(a+bx)^3}\,dx = -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{2b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)} - \frac{\sqrt{(a+bx)^2+1}(a+bx)}{2b^2\text{arcsinh}(a+bx)^2} - \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a\sqrt{(a+bx)^2+1}}{2b^2\text{arcsinh}(a+bx)^2}$$

```
[In] Int[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 +
(a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - 1/(2*b^2*ArcSinh[a + b*x]) + (a*
(a + b*x))/(2*b^2*ArcSinh[a + b*x]) - (a + b*x)^2/(b^2*ArcSinh[a + b*x]) -
(a*CoshIntegral[ArcSinh[a + b*x]])/(2*b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
```

```
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\text{arcsinh}(x)^3} + \frac{x}{b\text{arcsinh}(x)^3}\right)\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{2b^2}$$
$$+ \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{2b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2}$$
$$- \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)}$$
$$+ \frac{2\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{2b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2}$$
$$- \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)}$$
$$+ \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2}$$
$$- \frac{a\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)}$$
$$- \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)} - \frac{a\text{Chi}(\text{arcsinh}(a+bx))}{2b^2} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)}$$
$$- \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)} - \frac{a\text{Chi}(\text{arcsinh}(a+bx))}{2b^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\text{arcsinh}(a+bx)^2}$$
$$- \frac{1}{2b^2\text{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)^2}{b^2\text{arcsinh}(a+bx)}$$
$$- \frac{a\text{Chi}(\text{arcsinh}(a+bx))}{2b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{b^2}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx =$$
$$-\frac{bx\sqrt{1+a^2+2abx+b^2x^2}+\operatorname{arcsinh}(a+bx)+a^2\operatorname{arcsinh}(a+bx)+3abx\operatorname{arcsinh}(a+bx)+2b^2x^2\operatorname{arcsin}}{2b^2\operatorname{arcsinh}(a+}$$

```
[In] Integrate[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ArcSinh[a + b*x] + a^2*ArcSin
h[a + b*x] + 3*a*b*x*ArcSinh[a + b*x] + 2*b^2*x^2*ArcSinh[a + b*x] + a*ArcS
inh[a + b*x]^2*CoshIntegral[ArcSinh[a + b*x]] - 2*ArcSinh[a + b*x]^2*SinhIn
tegral[2*ArcSinh[a + b*x]])/(b^2*ArcSinh[a + b*x]^2)
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)^2}-\frac{\cosh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Chi}(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-(bx+a)\operatorname{arcsinh}\right.}{2\operatorname{arcsinh}(bx+a)^2}}{b^2}$ |
| default | $\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)^2}-\frac{\cosh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Chi}(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-(bx+a)\operatorname{arcsinh}\right.}{2\operatorname{arcsinh}(bx+a)^2}}{b^2}$ |

```
[In] int(x/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/4/arcsinh(b*x+a)^2*sinh(2*arcsinh(b*x+a))-1/2/arcsinh(b*x+a)*cosh
(2*arcsinh(b*x+a))+Shi(2*arcsinh(b*x+a))-1/2*a*(Chi(arcsinh(b*x+a))*arcsinh
(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

## Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^3, x)
```

## Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{asinh}^3(a+bx)}\,dx$$

```
[In] integrate(x/asinh(b*x+a)**3,x)
```

```
[Out] Integral(x/asinh(a + b*x)**3, x)
```

## Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{arsinh}(bx+a)^3}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (2*b^8*x^8 + 15*a*b^7*x^
7 + a^8 + (49*a^2*b^6 + 6*b^6)*x^6 + 3*a^6 + (91*a^3*b^5 + 33*a*b^5)*x^5 +
3*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 3*a^4 + (77*a^5*b^3 + 90*a^3*b^3
+ 21*a*b^3)*x^3 + (35*a^6*b^2 + 60*a^4*b^2 + 27*a^2*b^2 + 2*b^2)*x^2 + (2*b
^5*x^5 + 9*a*b^4*x^4 + a^5 + 2*(8*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2
 + 3*a*b^2)*x^2 + 6*(a^4*b + a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + (6*b^6*x^6 + 33*a*b^5*x^5 + 3*a^6 + 5*(15*a^2*b^4 + 2*b^4)*x^4 + 7*a^
4 + (90*a^3*b^3 + 37*a*b^3)*x^3 + (60*a^4*b^2 + 51*a^2*b^2 + 5*b^2)*x^2 + 5
*a^2 + (21*a^5*b + 31*a^3*b + 10*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ a^2 + 3*(3*a^7*b + 7*a^5*b + 5*a^3*b + a*b)*x + (6*b^7*x^7 + 39*a*b^6*x^6
 + 3*a^7 + 2*(54*a^2*b^5 + 7*b^5)*x^5 + 8*a^5 + (165*a^3*b^4 + 64*a*b^4)*x^
4 + (150*a^4*b^3 + 116*a^2*b^3 + 11*b^3)*x^3 + 7*a^3 + (81*a^5*b^2 + 104*a^
3*b^2 + 29*a*b^2)*x^2 + (24*a^6*b + 46*a^4*b + 25*a^2*b + 3*b)*x + 2*a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (3*b^7*x^7 + 18*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^
4 + 7*a*b^4)*x^4 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 1
4*a^3*b^2 + 5*a*b^2)*x^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(
5*a^2*b^6 + b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b
^4 + 6*a^2*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

```
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 +
 a^2*b^2 + (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*
b*x + a^2 + 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a
*b^6*x^4 + a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3
*b^4 + 3*a*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integr
ate(1/2*(4*b^9*x^9 + 35*a*b^8*x^8 + 3*a^9 + 8*(17*a^2*b^7 + 2*b^7)*x^7 + 12
*a^7 + 4*(77*a^3*b^6 + 27*a*b^6)*x^6 + 8*(56*a^4*b^5 + 39*a^2*b^5 + 3*b^5)*
x^5 + 18*a^5 + 2*(217*a^5*b^4 + 250*a^3*b^4 + 57*a*b^4)*x^4 + 8*(35*a^6*b^3
 + 60*a^4*b^3 + 27*a^2*b^3 + 2*b^3)*x^3 + (4*b^5*x^5 + 19*a*b^4*x^4 + 36*a^
2*b^3*x^3 + 34*a^3*b^2*x^2 + 16*a^4*b*x + 3*a^5 - 3*a)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1)^2 + 12*a^3 + 4*(29*a^7*b^2 + 69*a^5*b^2 + 51*a^3*b^2 + 11*a*b^2)*
x^2 + (16*b^6*x^6 + 92*a*b^5*x^5 + 12*a^6 + 4*(55*a^2*b^4 + 4*b^4)*x^4 + 12
*a^4 + 20*(14*a^3*b^3 + 3*a*b^3)*x^3 + 4*(50*a^4*b^2 + 21*a^2*b^2)*x^2 - 3*
a^2 + (76*a^5*b + 52*a^3*b - 3*a*b)*x - 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + 3*(8*b^7*x^7 + 54*a*b^6*x^6 + 6*a^7 + 4*(39*a^2*b^5 + 4*b^5)*x^5 + 12
*a^5 + 2*(125*a^3*b^4 + 38*a*b^4)*x^4 + 8*(30*a^4*b^3 + 18*a^2*b^3 + b^3)*x
^3 + 7*a^3 + (138*a^5*b^2 + 136*a^3*b^2 + 23*a*b^2)*x^2 + 2*(22*a^6*b + 32*
a^4*b + 11*a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(7*a^8*b + 22*a^
6*b + 24*a^4*b + 10*a^2*b + b)*x + (16*b^8*x^8 + 124*a*b^7*x^7 + 12*a^8 + 1
2*(35*a^2*b^6 + 4*b^6)*x^6 + 36*a^6 + 4*(203*a^3*b^5 + 69*a*b^5)*x^5 + 4*(2
45*a^4*b^4 + 165*a^2*b^4 + 12*b^4)*x^4 + 39*a^4 + 3*(252*a^5*b^3 + 280*a^3*
b^3 + 61*a*b^3)*x^3 + (364*a^6*b^2 + 600*a^4*b^2 + 261*a^2*b^2 + 19*b^2)*x^
2 + 18*a^2 + (100*a^7*b + 228*a^5*b + 165*a^3*b + 37*a*b)*x + 3)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1) + 3*a)/((b^9*x^8 + 8*a*b^8*x^7 + a^8*b + 4*a^6*b +
 4*(7*a^2*b^7 + b^7)*x^6 + 8*(7*a^3*b^6 + 3*a*b^6)*x^5 + 6*a^4*b + 2*(35*a^4
*b^5 + 30*a^2*b^5 + 3*b^5)*x^4 + 8*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^3 +
 (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 4*a^2*b + 4*(7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3
)*x^2 + 4*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + a^3*b + (10*a^2*b^4 + b^4)*x^3 +
 (10*a^3*b^3 + 3*a*b^3)*x^2 + (5*a^4*b^2 + 3*a^2*b^2)*x)*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2) + 6*(b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 2*a^4*b + (15*a^2*b^
5 + 2*b^5)*x^4 + 4*(5*a^3*b^4 + 2*a*b^4)*x^3 + a^2*b + (15*a^4*b^3 + 12*a^2
*b^3 + b^3)*x^2 + 2*(3*a^5*b^2 + 4*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1) + 8*(a^7*b^2 + 3*a^5*b^2 + 3*a^3*b^2 + a*b^2)*x + 4*(b^8*x^7 + 7*
a*b^7*x^6 + a^7*b + 3*a^5*b + 3*(7*a^2*b^6 + b^6)*x^5 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^4 + 3*a^3*b + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^3 + 3*(7*a^5*b^3 +
 10*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^
2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x/arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x/asinh(a + b*x)^3,x)
[Out] int(x/asinh(a + b*x)^3, x)
```

# 3.91 $\int \frac{1}{\mathbf{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 775
Rubi [A] (verified) . . . 775
Mathematica [A] (verified) . . . 777
Maple [A] (verified) . . . 777
Fricas [F] . . . 777
Sympy [F] . . . 778
Maxima [F] . . . 778
Giac [F] . . . 779
Mupad [F(-1)] . . . 780

## Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{\sqrt{1+(a+bx)^2}}{2b\operatorname{arcsinh}(a+bx)^2} - \frac{a+bx}{2b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b}$$

```
[Out] 1/2*(-b*x-a)/b/arcsinh(b*x+a)+1/2*Chi(arcsinh(b*x+a))/b-1/2*(1+(b*x+a)^2)^(
1/2)/b/arcsinh(b*x+a)^2
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5858, 5773, 5818, 5774, 3382}

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b} - \frac{a+bx}{2b\operatorname{arcsinh}(a+bx)} - \frac{\sqrt{(a+bx)^2+1}}{2b\operatorname{arcsinh}(a+bx)^2}$$

```
[In] Int[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] -1/2*Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x]^2) - (a + b*x)/(2*b*ArcSinh[
a + b*x]) + CoshIntegral[ArcSinh[a + b*x]]/(2*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\text{arcsinh}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{2b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\text{arcsinh}(a+bx)^2} - \frac{a+bx}{2b\text{arcsinh}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{2b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\text{arcsinh}(a+bx)^2} - \frac{a+bx}{2b\text{arcsinh}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\text{arcsinh}(a+bx)^2} - \frac{a+bx}{2b\text{arcsinh}(a+bx)} + \frac{\text{Chi}(\text{arcsinh}(a+bx))}{2b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \frac{-\frac{\sqrt{1+(a+bx)^2}}{\operatorname{arcsinh}(a+bx)^2} - \frac{a+bx}{\operatorname{arcsinh}(a+bx)} + \operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b}$$

```
[In] Integrate[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2) - (a + b*x)/ArcSinh[a + b*x] +
 CoshIntegral[ArcSinh[a + b*x]])/(2*b)
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\operatorname{arcsinh}(bx+a)^2} - \frac{bx+a}{2\operatorname{arcsinh}(bx+a)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{2}}{b}$ | 51 |
| default | $\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\operatorname{arcsinh}(bx+a)^2} - \frac{bx+a}{2\operatorname{arcsinh}(bx+a)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{2}}{b}$ | 51 |

```
[In] int(1/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-1/2/arcsinh(b*x+a)*(b*x+a)+1
/2*Chi(arcsinh(b*x+a)))
```

## Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^(-3), x)
```

## Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{asinh}^3(a+bx)}\,dx$$

```
[In] integrate(1/asinh(b*x+a)**3,x)
```

```
[Out] Integral(asinh(a + b*x)**(-3), x)
```

## Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{arsinh}(bx+a)^3}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^7*x^
7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*
a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 +
 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3
*b*x + a^4 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^5 + 5*a*b^4*
x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*
a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9
*a^2*b + b)*x + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + 3*(15*a^2*b^4 + 2*b^4)*
x^4 + 6*a^4 + 12*(5*a^3*b^3 + 2*a*b^3)*x^3 + (45*a^4*b^2 + 36*a^2*b^2 + 4*b
^2)*x^2 + 4*a^2 + 2*(9*a^5*b + 12*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1) + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/((
b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^
3*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4
*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(
2*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^
2 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 +
b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2
```

```
)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2) + integrate(1/2*(b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^
2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30
*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 +
(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4 + 3)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (4*
b^5*x^5 + 20*a*b^4*x^4 + 4*a^5 + 4*(10*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4*(10*a
^3*b^2 + 3*a*b^2)*x^2 + (20*a^4*b + 12*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + 3*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6 + 2*(15*a^2*b^4
 + 2*b^4)*x^4 + 4*a^4 + 8*(5*a^3*b^3 + 2*a*b^3)*x^3 + (30*a^4*b^2 + 24*a^2*
b^2 + b^2)*x^2 + a^2 + 2*(6*a^5*b + 8*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*
x + a^2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (4*b^7*x^7 +
 28*a*b^6*x^6 + 4*a^7 + 12*(7*a^2*b^5 + b^5)*x^5 + 12*a^5 + 20*(7*a^3*b^4 +
 3*a*b^4)*x^4 + (140*a^4*b^3 + 120*a^2*b^3 + 9*b^3)*x^3 + 9*a^3 + 3*(28*a^5
*b^2 + 40*a^3*b^2 + 9*a*b^2)*x^2 + (28*a^6*b + 60*a^4*b + 27*a^2*b + b)*x +
 a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^8*x^8 + 8*a*b^7*x^7 + a^8 +
4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b
^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)
*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 +
4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2
 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*a^2 + 8*(a^7*b
 + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b
^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*
b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^
6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^(-3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/asinh(a + b*x)^3,x)
[Out] int(1/asinh(a + b*x)^3, x)
```

# 3.92 $\int \frac{1}{x\mathbf{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 781
Rubi [N/A] . . . 781
Mathematica [N/A] . . . 782
Maple [N/A] (verified) . . . 782
Fricas [N/A] . . . 782
Sympy [N/A] . . . 782
Maxima [N/A] . . . 783
Giac [N/A] . . . 784
Mupad [N/A] . . . 785

## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(a+bx)^3}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^3,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx$$

```
[In] Int[1/(x*ArcSinh[a + b*x]^3),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(-\frac{a}{b}+\frac{x}{b})\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b}$$

## Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\operatorname{arcsinh}(a+bx)^3}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^3),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^3), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\operatorname{arcsinh}\left(bx+a\right)^3}dx$$

```
[In] int(1/x/arcsinh(b*x+a)^3,x)
[Out] int(1/x/arcsinh(b*x+a)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(b*x + a)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

```
[In] integrate(1/x/asinh(b*x+a)**3,x)
[Out] Integral(1/(x*asinh(a + b*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 15.19 (sec) , antiderivative size = 3273, normalized size of antiderivative = 272.75

$$\int \frac{1}{x\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x - (a*b^7*x^7 + 7*a^2*b^6*x
^6 + a^8 + 3*a^6 + 3*(7*a^3*b^5 + a*b^5)*x^5 + 5*(7*a^4*b^4 + 3*a^2*b^4)*x^
4 + 3*a^4 + (35*a^5*b^3 + 30*a^3*b^3 + 3*a*b^3)*x^3 + 3*(7*a^6*b^2 + 10*a^4
*b^2 + 3*a^2*b^2)*x^2 + (a*b^4*x^4 + a^5 + 2*(2*a^2*b^3 + b^3)*x^3 + 2*a^3
+ 6*(a^3*b^2 + a*b^2)*x^2 + 2*(2*a^4*b + 3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + (3*a*b^5*x^5 + 3*a^6 + (15*a^2*b^4 + 4*b^4)*x^4 + 7
*a^4 + (30*a^3*b^3 + 19*a*b^3)*x^3 + (30*a^4*b^2 + 33*a^2*b^2 + 5*b^2)*x^2
+ 5*a^2 + 5*(3*a^5*b + 5*a^3*b + 2*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + a^2 + (7*a^7*b + 15*a^5*b + 9*a^3*b + a*b)*x + (3*a*b^6*x^6 + 3*a^7 + 2
*(9*a^2*b^5 + b^5)*x^5 + 8*a^5 + (45*a^3*b^4 + 16*a*b^4)*x^4 + (60*a^4*b^3
+ 44*a^2*b^3 + 3*b^3)*x^3 + 7*a^3 + (45*a^5*b^2 + 56*a^3*b^2 + 13*a*b^2)*x^
2 + (18*a^6*b + 34*a^4*b + 17*a^2*b + b)*x + 2*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^7*x^7 + 1
8*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^4 + 7*a*b^4)*x^4 + (45
*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 14*a^3*b^2 + 5*a*b^2)*x
^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/((b^8*x^8 + 6*a*b^7*x^7 + 3*(5*a^2*b^6 + b^6)*x^6 + 4*(5*a^3*b^5 + 3*a*b^
5)*x^5 + 3*(5*a^4*b^4 + 6*a^2*b^4 + b^4)*x^4 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b
^3)*x^3 + (a^6*b^2 + 3*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + (b^5*x^5 + 3*a*b^4*
x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*
(b^6*x^6 + 4*a*b^5*x^5 + (6*a^2*b^4 + b^4)*x^4 + 2*(2*a^3*b^3 + a*b^3)*x^3
+ (a^4*b^2 + a^2*b^2)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 3*(b^7*x^7 + 5*a
*b^6*x^6 + 2*(5*a^2*b^5 + b^5)*x^5 + 2*(5*a^3*b^4 + 3*a*b^4)*x^4 + (5*a^4*b
^3 + 6*a^2*b^3 + b^3)*x^3 + (a^5*b^2 + 2*a^3*b^2 + a*b^2)*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*(a*b^9*x^9 + 10*a^2*b^8*x^8 + 2*a^10 + 8*a^8 + 4*(11*a^3*b^
7 + a*b^7)*x^7 + 16*(7*a^4*b^6 + 2*a^2*b^6)*x^6 + 12*a^6 + 2*(91*a^5*b^5 +
54*a^3*b^5 + 3*a*b^5)*x^5 + 4*(49*a^6*b^4 + 50*a^4*b^4 + 9*a^2*b^4)*x^4 + 8
```

```
*a^4 + 4*(35*a^7*b^3 + 55*a^5*b^3 + 21*a^3*b^3 + a*b^3)*x^3 + (a*b^5*x^5 +
2*a^6 + 2*(3*a^2*b^4 + 2*b^4)*x^4 + 4*a^4 + 2*(7*a^3*b^3 + 8*a*b^3)*x^3 + 8
*(2*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + 2*a^2 + (9*a^5*b + 16*a^3*b + 7*a*b)*x
)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 16*(4*a^8*b^2 + 9*a^6*b^2 + 6*a^4*b^2 +
 a^2*b^2)*x^2 + (4*a*b^6*x^6 + 8*a^7 + 4*(7*a^2*b^5 + 3*b^5)*x^5 + 20*a^5 +
 16*(5*a^3*b^4 + 4*a*b^4)*x^4 + 2*(60*a^4*b^3 + 70*a^2*b^3 + 11*b^3)*x^3 +
16*a^3 + (100*a^5*b^2 + 156*a^3*b^2 + 57*a*b^2)*x^2 + (44*a^6*b + 88*a^4*b
+ 51*a^2*b + 7*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*a*b^7*x
^7 + 12*a^8 + 12*(4*a^2*b^6 + b^6)*x^6 + 36*a^6 + 6*(27*a^3*b^5 + 14*a*b^5)
*x^5 + 4*(75*a^4*b^4 + 63*a^2*b^4 + 5*b^4)*x^4 + 38*a^4 + (330*a^5*b^3 + 40
8*a^3*b^3 + 95*a*b^3)*x^3 + 2*(108*a^6*b^2 + 186*a^4*b^2 + 84*a^2*b^2 + 5*b
^2)*x^2 + 16*a^2 + (78*a^7*b + 180*a^5*b + 131*a^3*b + 29*a*b)*x + 2)*(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 2*a^2 + (17*a^9*b + 52*a^7*b + 54*a^5*b + 20*a^3
*b + a*b)*x + (4*a*b^8*x^8 + 8*a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 28*a^7 + 20*
(7*a^3*b^6 + 2*a*b^6)*x^6 + 2*(154*a^4*b^5 + 84*a^2*b^5 + 3*b^5)*x^5 + 36*a
^5 + (420*a^5*b^4 + 380*a^3*b^4 + 51*a*b^4)*x^4 + (364*a^6*b^3 + 500*a^4*b^
3 + 153*a^2*b^3 + 3*b^3)*x^3 + 20*a^3 + (196*a^7*b^2 + 384*a^5*b^2 + 213*a^
3*b^2 + 25*a*b^2)*x^2 + (60*a^8*b + 160*a^6*b + 141*a^4*b + 42*a^2*b + b)*x
 + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^11 + 8*a*b^9*x^10 + 4*(
7*a^2*b^8 + b^8)*x^9 + 8*(7*a^3*b^7 + 3*a*b^7)*x^8 + 2*(35*a^4*b^6 + 30*a^2
*b^6 + 3*b^6)*x^7 + 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^6 + 4*(7*a^6*b^4
 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^5 + 8*(a^7*b^3 + 3*a^5*b^3 + 3*a^3*b^3 +
 a*b^3)*x^4 + (a^8*b^2 + 4*a^6*b^2 + 6*a^4*b^2 + 4*a^2*b^2 + b^2)*x^3 + (b^
6*x^7 + 4*a*b^5*x^6 + 6*a^2*b^4*x^5 + 4*a^3*b^3*x^4 + a^4*b^2*x^3)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^2 + 4*(b^7*x^8 + 5*a*b^6*x^7 + (10*a^2*b^5 + b^5)*x^6
 + (10*a^3*b^4 + 3*a*b^4)*x^5 + (5*a^4*b^3 + 3*a^2*b^3)*x^4 + (a^5*b^2 + a^
3*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^9 + 6*a*b^7*x^8
+ (15*a^2*b^6 + 2*b^6)*x^7 + 4*(5*a^3*b^5 + 2*a*b^5)*x^6 + (15*a^4*b^4 + 12
*a^2*b^4 + b^4)*x^5 + 2*(3*a^5*b^3 + 4*a^3*b^3 + a*b^3)*x^4 + (a^6*b^2 + 2*
a^4*b^2 + a^2*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(b^9*x^10 + 7*a*b
^8*x^9 + 3*(7*a^2*b^7 + b^7)*x^8 + 5*(7*a^3*b^6 + 3*a*b^6)*x^7 + (35*a^4*b^
5 + 30*a^2*b^5 + 3*b^5)*x^6 + 3*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^5 + (7
*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x^4 + (a^7*b^2 + 3*a^5*b^2 + 3*a^3
*b^2 + a*b^2)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\mathrm{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\mathrm{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/(x*asinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*asinh(a + b*x)^3), x)
```

# 3.93 $\int x^m(a+b\operatorname{arcsinh}(c+dx))^n\,dx$

Optimal result . . . 786
Rubi [N/A] . . . 786
Mathematica [N/A] . . . 787
Maple [N/A] (verified) . . . 787
Fricas [N/A] . . . 787
Sympy [N/A] . . . 787
Maxima [N/A] . . . 788
Giac [F(-1)] . . . 788
Mupad [N/A] . . . 788

## Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m(a+b\operatorname{arcsinh}(c+dx))^n\,dx = \operatorname{Int}(x^m(a+b\operatorname{arcsinh}(c+dx))^n, x)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m(a+b\operatorname{arcsinh}(c+dx))^n\,dx = \int x^m(a+b\operatorname{arcsinh}(c+dx))^n\,dx$$

```
[In] Int[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Defer[Subst][Defer[Int][(-(c/d) + x/d)^m*(a + b*ArcSinh[x])^n, x], x, c + d
*x]/d
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^m (a+b\operatorname{arcsinh}(x))^n\,dx, x, c+dx\right)}{d}$$

## Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \int x^m(a+b\text{arcsinh}(c+dx))^n\,dx$$

```
[In] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n,x]
[Out] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^m(a+b\,\text{arcsinh}\,(dx+c))^n\,dx$$

```
[In] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x^m\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

## Sympy [N/A]

Not integrable

Time = 12.85 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \int x^m(a+b\,\text{asinh}\,(c+dx))^n\,dx$$

```
[In] integrate(x**m*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x**m*(a + b*asinh(c + d*x))**n, x)
```

## Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x^m\,dx$$

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

## Giac [F(-1)]

Timed out.

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] Timed out
```

## Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx = \int x^m\,(a+b\,\text{asinh}(c+d\,x))^n\,dx$$

```
[In] int(x^m*(a + b*asinh(c + d*x))^n,x)
[Out] int(x^m*(a + b*asinh(c + d*x))^n, x)
```

# 3.94 $\int x^2(a+b\operatorname{arcsinh}(c+dx))^n\,dx$


Optimal result . . . 789
Rubi [A] (verified) . . . 790
Mathematica [A] (verified) . . . 794
Maple [F] . . . 795
Fricas [F] . . . 795
Sympy [F] . . . 795
Maxima [F] . . . 795
Giac [F] . . . 796
Mupad [F(-1)] . . . 796


## Optimal result

Integrand size = 16, antiderivative size = 545

$$
\begin{aligned}
&\int x^2(a+b\operatorname{arcsinh}(c+dx))^n\,dx \\
&= \frac{3^{-1-n}e^{-\frac{3a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3} \\
&\quad - \frac{2^{-2-n}ce^{-\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&\quad - \frac{e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&\quad + \frac{c^2e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad + \frac{e^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&\quad - \frac{c^2e^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad - \frac{2^{-2-n}ce^{\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&\quad - \frac{3^{-1-n}e^{\frac{3a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3}
\end{aligned}
$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-3*(a+b*arcsinh(d*x+c))/b)/d^
3/exp(3*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*(a+b*arcsinh(d*x+c))^
n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^3/exp(2*a/b)/(((-a-b*arcsinh(d*x+c
```

```
))/b)^n)-1/8*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/
exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*c^2*(a+b*arcsinh(d*x+c))^n*GAMMA
(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/
8*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^3/(((
a+b*arcsinh(d*x+c))/b)^n)-1/2*c^2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n
,(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*exp(2*
a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,2*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*
arcsinh(d*x+c))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,3*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5859, 5830, 6873, 12, 6874, 3388, 2212, 5556, 3389}

$$
\begin{aligned}
&\int x^2(a+b\operatorname{arcsinh}(c+dx))^n\,dx \\
&= \frac{c^2 e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad - \frac{c^2 e^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad + \frac{3^{-n-1}e^{-\frac{3a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3} \\
&\quad - \frac{c2^{-n-2}e^{-\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&\quad - \frac{e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&\quad + \frac{e^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&\quad - \frac{c2^{-n-2}e^{\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&\quad - \frac{3^{-n-1}e^{\frac{3a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3}
\end{aligned}
$$

```
[In] Int[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c +
d*x]))/b])/(8*d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
```

```
 n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(d^3*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*d^3*E^(a/b)*(-(
(a + b*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^3*E^(a/b)*(-((a + b*ArcSinh[c + d*
x])/b))^n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSin
h[c + d*x])/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a +
b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^3*((a
+ b*ArcSinh[c + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c +
d*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^3*((a + b*ArcSinh[
c + d*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[
1 + n, (3*(a + b*ArcSinh[c + d*x]))/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)
^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^2 (a+b\text{arcsinh}(x))^n\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2 dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^n \cosh(x)(c-\sinh(x))^2}{d^2}\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)(c-\sinh(x))^2\,dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&= \frac{\text{Subst}\left(\int \left(c^2(a+bx)^n\cosh(x) - 2c(a+bx)^n\cosh(x)\sinh(x) + (a+bx)^n\cosh(x)\sinh^2(x)\right)\,dx, x, \text{a}\right.}{d^3} \\
&= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\sinh^2(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&\quad - \frac{(2c)\text{Subst}\left(\int (a+bx)^n\cosh(x)\sinh(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int (a+bx)^n\cosh(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}(a+bx)^n \cosh(x) + \frac{1}{4}(a+bx)^n \cosh(3x)\right)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&\quad - \frac{(2c)\text{Subst}\left(\int \frac{1}{2}(a+bx)^n \sinh(2x)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int e^{-x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int e^{x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^3} \\
&= \frac{c^2 e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n \left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad - \frac{c^2 e^{a/b}(a+b\text{arcsinh}(c+dx))^n \left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad - \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\, dx, x, \text{arcsinh}(c+dx)\right)}{4d^3} \\
&\quad + \frac{\text{Subst}\left(\int (a+bx)^n \cosh(3x)\, dx, x, \text{arcsinh}(c+dx)\right)}{4d^3} \\
&\quad - \frac{c\text{Subst}\left(\int (a+bx)^n \sinh(2x)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^3} \\
&= \frac{c^2 e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n \left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad - \frac{c^2 e^{a/b}(a+b\text{arcsinh}(c+dx))^n \left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&\quad + \frac{\text{Subst}\left(\int e^{-3x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{8d^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{-x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{8d^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{8d^3} \\
&\quad + \frac{\text{Subst}\left(\int e^{3x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{8d^3} \\
&\quad + \frac{c\text{Subst}\left(\int e^{-2x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^3} \\
&\quad - \frac{c\text{Subst}\left(\int e^{2x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^3}
\end{aligned}
$$

$$= \frac{3^{-1-n} e^{-\frac{3a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3}$$
$$- \frac{2^{-2-n} c e^{-\frac{2a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3}$$
$$- \frac{e^{-\frac{a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3}$$
$$+ \frac{c^2 e^{-\frac{a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3}$$
$$+ \frac{e^{a/b} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8d^3}$$
$$- \frac{c^2 e^{a/b} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^3}$$
$$- \frac{2^{-2-n} c e^{\frac{2a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^3}$$
$$- \frac{3^{-1-n} e^{\frac{3a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8d^3}$$

## Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.63

$$\int x^2 (a + b\operatorname{arcsinh}(c+dx))^n \, dx$$
$$= \frac{2^{-3-n} 3^{-1-n} e^{-\frac{3a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{b^2}\right)^{-n} \left(-2^n 3^{1+n} (-1+4c^2)\, e^{\frac{4a}{b}} \left(-\frac{a+b\operatorname{arcsinh}(c\right.\right.}{b}$$

```
[In] Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*(-(2^n*3^(1 + n)*(-1 + 4*
c^2)*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSi
nh[c + d*x]]) + 2^n*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c + d*x]))/b] - 2*3^(1 + n)*c*E^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[
1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*3^(1 + n)*(-1 + 4*c^2)*E^((2*
a)/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)
] - E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*(2*3^(1 + n)*c*Gamma[1 +
n, (2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*E^(a/b)*Gamma[1 + n, (3*(a + b*Arc
Sinh[c + d*x]))/b])))/(d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^
n)
```

## Maple [F]

$$\int x^2(a+b\,\text{arcsinh}\,(dx+c))^n\,dx$$

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

## Fricas [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

## Sympy [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx = \int x^2(a+b\,\text{asinh}\,(c+dx))^n\,dx$$

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x**2*(a + b*asinh(c + d*x))**n, x)
```

## Maxima [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

## Giac [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx = \int x^2\,(a+b\,\text{asinh}(c+d\,x))^n\,dx$$

```
[In] int(x^2*(a + b*asinh(c + d*x))^n,x)
[Out] int(x^2*(a + b*asinh(c + d*x))^n, x)
```

# 3.95 $\int x(a+b\operatorname{arcsinh}(c+dx))^n\,dx$

Optimal result . . . 797
Rubi [A] (verified) . . . 798
Mathematica [A] (verified) . . . 801
Maple [F] . . . 801
Fricas [F] . . . 802
Sympy [F] . . . 802
Maxima [F] . . . 802
Giac [F] . . . 802
Mupad [F(-1)] . . . 803

## Optimal result

Integrand size = 14, antiderivative size = 267

$$\begin{aligned}
&\int x(a+b\operatorname{arcsinh}(c+dx))^n\,dx \\
&= \frac{2^{-3-n}e^{-\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2} \\
&\quad - \frac{ce^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad + \frac{ce^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad + \frac{2^{-3-n}e^{\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2}
\end{aligned}$$

```
[Out] 2^(-3-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^2/ex
p(2*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-1/2*c*(a+b*arcsinh(d*x+c))^n*GAMMA(1
+n,(-a-b*arcsinh(d*x+c))/b)/d^2/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*
c*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^2/(((
a+b*arcsinh(d*x+c))/b)^n)+2^(-3-n)*exp(2*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,2*(a+b*arcsinh(d*x+c))/b)/d^2/(((a+b*arcsinh(d*x+c))/b)^n)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5859, 5830, 6873, 12, 6874, 3388, 2212, 5556, 3389}

$$\int x(a+b\operatorname{arcsinh}(c+dx))^n\,dx$$

$$= \frac{2^{-n-3}e^{-\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2}$$
$$- \frac{ce^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2}$$
$$+ \frac{ce^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2}$$
$$+ \frac{2^{-n-3}e^{\frac{2a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2}$$

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c +
d*x]))/b])/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*(a + b*
ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^2*E^(
a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) + (c*E^(a/b)*(a + b*ArcSinh[c + d*x
])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^2*((a + b*ArcSinh[c + d
*x])/b)^n) + (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^2*((a + b*ArcSinh[c + d*x])/b)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)(a+b\text{arcsinh}(x))^n\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^n \cosh(x)(-c+\sinh(x))}{d}\, dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)(-c+\sinh(x))\, dx, x, \text{arcsinh}(c+dx)\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \left(-c(a+bx)^n \cosh(x) + (a+bx)^n \cosh(x)\sinh(x)\right)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\sinh(x)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^2} \\
&\quad - \frac{c\,\text{Subst}\left(\int (a+bx)^n \cosh(x)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{1}{2}(a+bx)^n \sinh(2x)\, dx, x, \text{arcsinh}(c+dx)\right)}{d^2} \\
&\quad - \frac{c\,\text{Subst}\left(\int e^{-x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^2} \\
&\quad - \frac{c\,\text{Subst}\left(\int e^{x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^2} \\
&= \\
&\quad - \frac{ce^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n \left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad + \frac{ce^{a/b}(a+b\text{arcsinh}(c+dx))^n \left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad + \frac{\text{Subst}\left(\int (a+bx)^n \sinh(2x)\, dx, x, \text{arcsinh}(c+dx)\right)}{2d^2} \\
&= \\
&\quad - \frac{ce^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n \left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad + \frac{ce^{a/b}(a+b\text{arcsinh}(c+dx))^n \left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad - \frac{\text{Subst}\left(\int e^{-2x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{4d^2} \\
&\quad + \frac{\text{Subst}\left(\int e^{2x}(a+bx)^n\, dx, x, \text{arcsinh}(c+dx)\right)}{4d^2}
\end{aligned}
$$

$$= \frac{2^{-3-n} e^{-\frac{2a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2}$$
$$- \frac{c e^{-\frac{a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2}$$
$$+ \frac{c e^{a/b} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d^2}$$
$$+ \frac{2^{-3-n} e^{\frac{2a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{d^2}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.85

$$\int x(a + b\operatorname{arcsinh}(c+dx))^n \, dx$$
$$= \frac{2^{-3-n} e^{-\frac{2a}{b}} (a + b\operatorname{arcsinh}(c+dx))^n \left(-\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{b^2}\right)^{-n} \left(2^{2+n} c e^{\frac{3a}{b}} \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^n \Gamma\left(1+n, \frac{a}{b}+\right.\right.$$

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*(2^(2 + n)*c*E^((3*a)/b)*(-((a + b*A
rcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSinh[c + d*x]] + (a/b + ArcSin
h[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] - 2^(2 + n)*c*E
^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b
)] + E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, (2*(a + b*A
rcSinh[c + d*x]))/b]))/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))
^n)
```

## Maple [F]

$$\int x(a + b \operatorname{arcsinh}(dx+c))^n \, dx$$

```
[In] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

## Fricas [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x, x)
```

## Sympy [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx = \int x(a+b\,\text{asinh}\,(c+dx))^n\,dx$$

```
[In] integrate(x*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x*(a + b*asinh(c + d*x))**n, x)
```

## Maxima [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

## Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^n x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx = \int x\,(a+b\,\text{asinh}(c+d\,x))^n\,dx$$

```
[In] int(x*(a + b*asinh(c + d*x))^n,x)
[Out] int(x*(a + b*asinh(c + d*x))^n, x)
```

# 3.96 $\int (a + b\operatorname{arcsinh}(c + dx))^n \, dx$

Optimal result . . . 804
Rubi [A] (verified) . . . 804
Mathematica [A] (verified) . . . 806
Maple [F] . . . 806
Fricas [F] . . . 806
Sympy [F] . . . 807
Maxima [F] . . . 807
Giac [F] . . . 807
Mupad [F(-1)] . . . 807

## Optimal result

Integrand size = 12, antiderivative size = 128

$$\int (a + b\operatorname{arcsinh}(c + dx))^n \, dx$$
$$= \frac{e^{-\frac{a}{b}} (a + b\operatorname{arcsinh}(c + dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$
$$- \frac{e^{a/b} (a + b\operatorname{arcsinh}(c + dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1 + n, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d/exp(a/b)/((
(-a-b*arcsinh(d*x+c))/b)^n)-1/2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(
a+b*arcsinh(d*x+c))/b)/d/(((a+b*arcsinh(d*x+c))/b)^n)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5774, 3388, 2212}

$$\int (a + b\operatorname{arcsinh}(c + dx))^n \, dx$$
$$= \frac{e^{-\frac{a}{b}} (a + b\operatorname{arcsinh}(c + dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(n + 1, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$
$$- \frac{e^{a/b} (a + b\operatorname{arcsinh}(c + dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(n + 1, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2
*d*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (E^(a/b)*(a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d*((a + b*ArcSinh[c +
 d*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\text{arcsinh}(x))^n\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int x^n \cosh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)} x^n\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&\quad + \frac{\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)} x^n\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd}
\end{aligned}$$

$$= \frac{e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n \left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$
$$- \frac{e^{a/b}(a+b\operatorname{arcsinh}(c+dx))^n \left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, \frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2d}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int (a+b\operatorname{arcsinh}(c+dx))^n \, dx$$
$$= \frac{e^{-\frac{a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n \left(-e^{\frac{2a}{b}} \left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)^{-n} \Gamma\left(1+n, \frac{a}{b}+\operatorname{arcsinh}(c+dx)\right) + \left(-\frac{a+b\operatorname{arcsinh}}{b}\right.\right.}{2d}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*(-((E^((2*a)/b)*Gamma[1 + n, a/b + ArcSinh[c +
d*x]])/(a/b + ArcSinh[c + d*x])^n) + Gamma[1 + n, -((a + b*ArcSinh[c + d*x]
)/b)]/(-((a + b*ArcSinh[c + d*x])/b))^n))/(2*d*E^(a/b))
```

## Maple [F]

$$\int (a+b\,\operatorname{arcsinh}(dx+c))^n \, dx$$

```
[In] int((a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n,x)
```

## Fricas [F]

$$\int (a+b\operatorname{arcsinh}(c+dx))^n \, dx = \int (b\operatorname{arsinh}(dx+c)+a)^n \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n, x)
```

## Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (a + b \, \text{asinh}\,(c + dx))^n \, dx$$

```
[In] integrate((a+b*asinh(d*x+c))**n,x)
[Out] Integral((a + b*asinh(c + d*x))**n, x)
```

## Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \, \text{arsinh}\,(dx + c) + a)^n \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

## Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \, \text{arsinh}\,(dx + c) + a)^n \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (a + b \, \text{asinh}(c + d \, x))^n \, dx$$

```
[In] int((a + b*asinh(c + d*x))^n,x)
[Out] int((a + b*asinh(c + d*x))^n, x)
```

# 3.97 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^n}{x}\,dx$

Optimal result . . . 808
Rubi [N/A] . . . 808
Mathematica [N/A] . . . 809
Maple [N/A] (verified) . . . 809
Fricas [N/A] . . . 809
Sympy [N/A] . . . 809
Maxima [N/A] . . . 810
Giac [N/A] . . . 810
Mupad [N/A] . . . 810

## Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^n}{x}\,dx = \operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^n}{x}, x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^n/x,x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^n}{x}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^n/x,x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^n/(-(c/d) + x/d), x], x, c + d*x
]/d
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^n}{-\frac{c}{d}+\frac{x}{d}}\,dx, x, c+dx\right)}{d}$$

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n/x,x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^n/x, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^n}{x}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^n/x,x)
[Out] int((a+b*arcsinh(d*x+c))^n/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n/x, x)
```

## Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^n}{x}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**n/x,x)
[Out] Integral((a + b*asinh(c + d*x))**n/x, x)
```

## Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

## Giac [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

## Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^n}{x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^n/x,x)
[Out] int((a + b*asinh(c + d*x))^n/x, x)
```

# 3.98 $\int x^2 \sqrt{a + b\mathbf{arcsinh}(c + dx)}\, dx$

Optimal result . . . . . 812
Rubi [A] (verified) . . . . . 813
Mathematica [A] (verified) . . . . . 819
Maple [F] . . . . . 820
Fricas [F(-2)] . . . . . 820
Sympy [F] . . . . . 820
Maxima [F] . . . . . 820
Giac [F] . . . . . 821
Mupad [F(-1)] . . . . . 821

## Optimal result

Integrand size = 18, antiderivative size = 496

$$\begin{aligned}\int x^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = {} & \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} \\ & + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\ & - \frac{c\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{2d^3} \\ & - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} \\ & + \frac{\sqrt{b}c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\ & + \frac{\sqrt{b}ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\ & + \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3} \\ & + \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} \\ & - \frac{\sqrt{b}c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\ & + \frac{\sqrt{b}ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\ & - \frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3}\end{aligned}$$

```
[Out] 1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/d^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/d^3/exp(3*a/b)+1/16*c*exp(2*a/b)*erf(2^(1/2)*(a+b*a
rcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+1/16*c*erfi(2^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3/exp(2*
a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)
/d^3+1/4*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1
/2)/d^3+1/16*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^3/
```

```
exp(a/b)-1/4*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/
d^3/exp(a/b)+c^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d^3+1/3*(d*x+c)^3*(a+b*
arcsinh(d*x+c))^(1/2)/d^3-1/2*c*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))
^(1/2)/d^3
```

## Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5830, 6873, 6874, 5433, 5406, 2236, 2235, 5432, 5407, 5480, 5408}

$$\begin{aligned}\int x^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = {} & \frac{\sqrt{\pi}\sqrt{b}c^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\ & - \frac{\sqrt{\pi}\sqrt{b}c^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\ & + \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} \\ & - \frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} \\ & + \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ce^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\ & + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3} \\ & + \frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} \\ & + \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ce^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\ & - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3} \\ & + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\ & - \frac{c\cosh(2\operatorname{arcsinh}(c+dx))\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d^3}\end{aligned}$$

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (c^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^3 + ((c + d*x)^3*Sqrt[a + b*
ArcSinh[c + d*x]])/(3*d^3) - (c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh
[c + d*x]])/(2*d^3) - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d^3) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(4*d^3) + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*d^3) + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3)
 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^3*E^
(a/b)) - (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(4*d^3*E^(a/b)) + (Sqrt[b]*c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(8*d^3*E^((2*a)/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3*E^((3*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5408

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
```

```
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^2 \sqrt{a+b\text{arcsinh}(x)}\, dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2 dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= \frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2 dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2 dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{2\text{Subst}\left(\int \left(c^2x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+cx^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)\right) dx, x, \sqrt{}\right.}{bd^3} \\
&= \frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{(2c)\text{Subst}\left(\int x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{(2c^2)\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{c^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{3d^3} \\
&\quad - \frac{c\sqrt{a+b\text{arcsinh}(c+dx)}\cosh(2\text{arcsinh}(c+dx))}{2d^3} \\
&\quad + \frac{\text{Subst}\left(\int \sinh^3\left(\frac{a}{b}-\frac{x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{3d^3} \\
&\quad + \frac{c\text{Subst}\left(\int \cosh\left(\frac{2a}{b}-\frac{2x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int \sinh\left(\frac{a}{b}-\frac{x^2}{b}\right) dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\
&\quad - \frac{c\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{2d^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int \left(\frac{1}{4}\sinh\left(\frac{3a}{b}-\frac{3x^2}{b}\right) - \frac{3}{4}\sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3d^3} \\
&\quad + \frac{c\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^3} \\
&\quad + \frac{c\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^3} \\
&\quad + \frac{c^2\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2d^3} \\
&\quad - \frac{c^2\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2d^3} \\
&= \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\
&\quad - \frac{c\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{2d^3} \\
&\quad + \frac{\sqrt{b}c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\
&\quad - \frac{\sqrt{b}c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int \sinh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{12d^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int \sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\
&\quad - \frac{c\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{2d^3} \\
&\quad + \frac{\sqrt{b}c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\
&\quad - \frac{\sqrt{b}c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^3} \\
&= \frac{c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d^3} \\
&\quad - \frac{c\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{2d^3} \\
&\quad - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} + \frac{\sqrt{b}c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\
&\quad + \frac{\sqrt{b}ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} + \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3} \\
&\quad + \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} - \frac{\sqrt{b}c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} \\
&\quad + \frac{\sqrt{b}ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} - \frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.32

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx$$

$$= \frac{-36(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)} + 144c^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)} - 72c\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{}$$

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-36*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] + 144*c^2*(c + d*x)*Sqrt[a + b*
ArcSinh[c + d*x]] - 72*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*
x]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]] + 9*Sqrt[b]*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]] - 36*Sqrt[b]*c^2*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 9*Sqrt[b]*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 36*Sqrt[b]*c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 9*Sqrt[b]*(-1 + 4*c^2
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]
) - 9*Sqrt[b]*c*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*Sinh[(2*a)/b] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[b]*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] + Sqrt[b]*
Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/
b] + 12*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[3*ArcSinh[c + d*x]])/(144*d^3)
```

## Maple [F]

$$\int x^2 \sqrt{a + b \operatorname{arcsinh}(dx + c)} dx$$

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(x**2*sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{b \operatorname{arsinh}(dx + c) + a} x^2\, dx$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

## Giac [F]

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{b \operatorname{arsinh}(dx + c) + a} x^2\, dx$$

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int x^2 \sqrt{a + b \operatorname{asinh}(c + d\, x)}\, dx$$

```
[In] int(x^2*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(x^2*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.99 $\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 822
Rubi [A] (verified) . . . 823
Mathematica [A] (verified) . . . 826
Maple [F] . . . 827
Fricas [F(-2)] . . . 827
Sympy [F] . . . 827
Maxima [F] . . . 827
Giac [F] . . . 828
Mupad [F(-1)] . . . 828

## Optimal result

Integrand size = 16, antiderivative size = 259

$$\begin{aligned}\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = &-\frac{c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^2}\\ &+\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2}\\ &-\frac{\sqrt{b}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2}\\ &-\frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\ &+\frac{\sqrt{b}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2}\\ &-\frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\end{aligned}$$

```
[Out] -1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^
(1/2)*Pi^(1/2)/d^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-1/4*c*exp(a/b)*erf((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2+1/4*c*erfi((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2/exp(a/b)-c*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/
2)/d^2+1/4*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5406, 2236, 2235, 5432, 5407}

$$\begin{aligned}\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = &-\frac{\sqrt{\pi}\sqrt{b}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2}\\ &-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\ &+\frac{\sqrt{\pi}\sqrt{b}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2}\\ &-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\ &-\frac{c(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d^2}\\ &+\frac{\cosh(2\text{arcsinh}(c+dx))\sqrt{a+b\text{arcsinh}(c+dx)}}{4d^2}\end{aligned}$$

```
[In] Int[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -((c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^2) + (Sqrt[a + b*ArcSinh[c +
 d*x]]*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2) - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d^2) + (Sqrt
[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2*E^(a/b))
- (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(16*d^2*E^((2*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)\sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= -\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int \left(cx^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{\text{Subst}\left(\int x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&\quad -\frac{(2c)\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{c(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d^2} \\
&\quad +\frac{\sqrt{a+b\text{arcsinh}(c+dx)}\cosh(2\text{arcsinh}(c+dx))}{4d^2} \\
&\quad -\frac{\text{Subst}\left(\int \cosh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4d^2} \\
&\quad -\frac{c\text{Subst}\left(\int \sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^2} \\
&\quad + \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^2} \\
&\quad - \frac{c\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2d^2} \\
&\quad + \frac{c\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2d^2} \\
&= -\frac{c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d^2} + \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{\sqrt{b}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2} - \frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} \\
&\quad + \frac{\sqrt{b}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2} - \frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

$$
\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx =
$$

$$
-\frac{-8\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx)) + 16ce^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}\right.}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c-}}\right.}{}
$$

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c + d*x]],x]

[Out] -1/32*(-8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + (16*c*Sqr
t[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*x]
])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)
]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/
b]) + Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]))/d^2
```

## Maple [F]

$$\int x\sqrt{a+b\,\text{arcsinh}\,(dx+c)}dx$$

```
[In] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int x\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$

```
[In] integrate(x*(a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(x*sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(dx+c)+ax}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

## Giac [F]

$$\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+ax}\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int x\,\sqrt{a+b\,\operatorname{asinh}\left(c+d\,x\right)}\,dx$$

```
[In] int(x*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(x*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.100 $\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 829
Rubi [A] (verified) . . . 829
Mathematica [A] (verified) . . . 831
Maple [F] . . . 832
Fricas [F(-2)] . . . 832
Sympy [F] . . . 832
Maxima [F] . . . 832
Giac [F] . . . 833
Mupad [F(-1)] . . . 833

## Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5819, 3389, 2211, 2236, 2235}

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} + \frac{(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d} \\
&\quad - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} \\
&\quad - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\begin{aligned}
&\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx \\
&= \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}
\end{aligned}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

## Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(dx + c)}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{a + b\operatorname{asinh}(c + dx)}\, dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{b\operatorname{arsinh}(dx + c) + a}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

# 3.101 $\int x(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$

Optimal result . . . 834
Rubi [A] (verified) . . . 835
Mathematica [A] (verified) . . . 838
Maple [F] . . . 839
Fricas [F(-2)] . . . 839
Sympy [F] . . . 840
Maxima [F] . . . 840
Giac [F] . . . 840
Mupad [F(-1)] . . . 840

## Optimal result

Integrand size = 16, antiderivative size = 326

$$\int x(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d^2} - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d^2} + \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} - \frac{3b^{3/2}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} - \frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} - \frac{3b^{3/2}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} + \frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} - \frac{3b\sqrt{a+b\operatorname{arcsinh}(c+dx)}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2}$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(3/2)*co
sh(2*arcsinh(d*x+c))/d^2-3/128*b^(3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2+3/128*b^(3/2)*erfi(2^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-3/8*b^(3/2)
*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2-3/8*b^(3/2
)*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)-3/16*b*s
inh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2+3/2*b*c*(1+(d*x+c)^2)^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5432, 5407, 2236, 2235, 5406}

$$\int x(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3\sqrt{\pi}b^{3/2}ce^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2}$$
$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} - \frac{3\sqrt{\pi}b^{3/2}ce^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2}$$
$$+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2}$$
$$+\frac{3bc\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d^2} - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d^2}$$
$$-\frac{3b\sinh(2\operatorname{arcsinh}(c+dx))\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{16d^2}$$
$$+\frac{\cosh(2\operatorname{arcsinh}(c+dx))(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d^2}$$

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (3*b*c*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d^2) - (c*(c
+ d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/d^2 + ((a + b*ArcSinh[c + d*x])^(3/2
)*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (3*b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2) - (3*b^(3/2)*E^((2*a)/b)*Sqrt[P
i/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d^2) - (3*b^(
3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2*E^(a/b))
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(64*d^2*E^((2*a)/b)) - (3*b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[2*ArcSinh
[c + d*x]])/(16*d^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)(a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int (a+bx)^{3/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= -\frac{2\text{Subst}\left(\int x^4\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int x^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int \left(cx^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{\text{Subst}\left(\int x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&\quad -\frac{(2c)\text{Subst}\left(\int x^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{c(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d^2} \\
&\quad +\frac{(a+b\text{arcsinh}(c+dx))^{3/2}\cosh(2\text{arcsinh}(c+dx))}{4d^2} \\
&\quad -\frac{3\text{Subst}\left(\int x^2\cosh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4d^2} \\
&\quad -\frac{(3c)\text{Subst}\left(\int x^2\sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{d^2} \\
&= \frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d^2} - \frac{c(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d^2} \\
&\quad +\frac{(a+b\text{arcsinh}(c+dx))^{3/2}\cosh(2\text{arcsinh}(c+dx))}{4d^2} \\
&\quad -\frac{3b\sqrt{a+b\text{arcsinh}(c+dx)}\sinh(2\text{arcsinh}(c+dx))}{16d^2} \\
&\quad -\frac{(3b)\text{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{16d^2} \\
&\quad -\frac{(3bc)\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d^2} - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{3b\sqrt{a+b\operatorname{arcsinh}(c+dx)}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d^2} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d^2} \\
&\quad - \frac{(3bc)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^2} \\
&\quad - \frac{(3bc)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^2} \\
&= \frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d^2} - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{3b^{3/2}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} - \frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} \\
&\quad - \frac{3b^{3/2}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} \\
&\quad + \frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} \\
&\quad - \frac{3b\sqrt{a+b\operatorname{arcsinh}(c+dx)}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.79

$$
\begin{aligned}
&\int x(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \\
&\qquad -64ace^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}}\right)-16\sqrt{}
\end{aligned}
$$

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] ((-64*a*c*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + Arc
Sinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh
[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) - 16*Sqrt[b]*c
*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c +
 d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 4*a*(8*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b])
- Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(C
osh[(2*a)/b] + Sinh[(2*a)/b])) + Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b])
+ (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*
(4*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] - 3*Sinh[2*ArcSinh[c + d*x]]))
)/(128*d^2)
```

## Maple [F]

$$\int x(a+b\,\mathrm{arcsinh}\left(dx+c\right))^{\frac{3}{2}}\,dx$$

```
[In] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(a+b\mathrm{arcsinh}(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int x(a+b\,\text{asinh}\,(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral(x*(a + b*asinh(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

## Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int x\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(x*(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.102 $\int (a + b\mathbf{arcsinh}(c + dx))^{3/2}\, dx$

Optimal result . . . 841
Rubi [A] (verified) . . . 841
Mathematica [A] (verified) . . . 844
Maple [F] . . . 844
Fricas [F(-2)] . . . 845
Sympy [F] . . . 845
Maxima [F] . . . 845
Giac [F] . . . 845
Mupad [F(-1)] . . . 846

## Optimal result

Integrand size = 14, antiderivative size = 150

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = -\frac{3b\sqrt{1 + (c + dx)^2}\sqrt{a + b\operatorname{arcsinh}(c + dx)}}{2d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{3/2}}{d} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = \frac{3\sqrt{\pi}b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} - \frac{3b\sqrt{(c + dx)^2 + 1}\sqrt{a + b\operatorname{arcsinh}(c + dx)}}{2d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{3/2}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} - \frac{(3b)\text{Subst}\left(\int \frac{x\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{4d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{(3b)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{(3b)\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8d} + \frac{(3b)\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{(3b)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4d} + \frac{(3b)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d}$$
$$+ \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$
$$+ \frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\text{arcsinh}(c+dx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b}\right.\right.}{8d}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

## Maple [F]

$$\int (a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \int (a + b\operatorname{asinh}(c + dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \int (b\operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \int (b\operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \int (a + b\,\operatorname{asinh}(c + d\,x))^{3/2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

# 3.103 $\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . 847
Rubi [A] (verified) . . . 848
Mathematica [B] (verified) . . . 853
Maple [F] . . . 854
Fricas [F(-2)] . . . 854
Sympy [F] . . . 854
Maxima [F] . . . 854
Giac [F] . . . 855
Mupad [F(-1)] . . . 855

## Optimal result

Integrand size = 16, antiderivative size = 389

$$\begin{aligned}\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx &= -\frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2}\\ &+\frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2}\\ &-\frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2}\\ &+\frac{15b^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{64d^2}\\ &+\frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2}\\ &-\frac{15b^{5/2}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\ &-\frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2}\\ &+\frac{15b^{5/2}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\ &-\frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2}\\ &-\frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2}\end{aligned}$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(5/2)*co
sh(2*arcsinh(d*x+c))/d^2-5/16*b*(a+b*arcsinh(d*x+c))^(3/2)*sinh(2*arcsinh(d
```

```
*x+c))/d^2-15/512*b^(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-15/16*b^(5/2)*c*exp(a/b
)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2+15/16*b^(5/2)*c*erfi
((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)+5/2*b*c*(a+b*arc
sinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d^2-15/4*b^2*c*(d*x+c)*(a+b*arcsinh(
d*x+c))^(1/2)/d^2+15/64*b^2*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/
2)/d^2
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5432, 5406, 2236, 2235, 5407}

$$\begin{aligned}
\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx &= -\frac{15\sqrt{\pi}b^{5/2}ce^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} \\
&- \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2} \\
&+ \frac{15\sqrt{\pi}b^{5/2}ce^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} \\
&- \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2} \\
&- \frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2} \\
&+ \frac{15b^2\cosh(2\operatorname{arcsinh}(c+dx))\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d^2} \\
&+ \frac{5bc\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2} \\
&- \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&- \frac{5b\sinh(2\operatorname{arcsinh}(c+dx))(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{16d^2} \\
&+ \frac{\cosh(2\operatorname{arcsinh}(c+dx))(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d^2}
\end{aligned}$$

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-15*b^2*c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d^2) + (5*b*c*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d^2) - (c*(c + d*x)*(a +
```

```
b*ArcSinh[c + d*x])^(5/2))/d^2 + (15*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[
2*ArcSinh[c + d*x]])/(64*d^2) + ((a + b*ArcSinh[c + d*x])^(5/2)*Cosh[2*ArcS
inh[c + d*x]])/(4*d^2) - (15*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]])/(16*d^2) - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d^2) + (15*b^(5/2)*c*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^2*E^(a/b)) - (15*
b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2
56*d^2*E^((2*a)/b)) - (5*b*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[2*ArcSinh[c
+ d*x]])/(16*d^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)(a+b\text{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int (a+bx)^{5/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= -\frac{2\text{Subst}\left(\int x^6\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int x^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int \left(cx^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{\text{Subst}\left(\int x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&\quad -\frac{(2c)\text{Subst}\left(\int x^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{5\operatorname{Subst}\left(\int x^4 \cosh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^2} \\
&\quad - \frac{(5c)\operatorname{Subst}\left(\int x^4 \sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{d^2} \\
&= \frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2} - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2} \\
&\quad - \frac{(15b)\operatorname{Subst}\left(\int x^2 \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d^2} \\
&\quad - \frac{(15bc)\operatorname{Subst}\left(\int x^2 \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2d^2} \\
&= -\frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2} \\
&\quad + \frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2} \\
&\quad - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&\quad + \frac{15b^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{64d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int \cosh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{64d^2} \\
&\quad - \frac{(15b^2c)\operatorname{Subst}\left(\int \sinh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2} \\
&\quad + \frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2} \\
&\quad - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&\quad + \frac{15b^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{64d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{128d^2} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{128d^2} \\
&\quad - \frac{(15b^2c)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^2} \\
&\quad + \frac{(15b^2c)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d^2} \\
&= -\frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2} \\
&\quad + \frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2} \\
&\quad - \frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2} \\
&\quad + \frac{15b^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{64d^2} \\
&\quad + \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2} \\
&\quad - \frac{15b^{5/2}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} - \frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2} \\
&\quad + \frac{15b^{5/2}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} \\
&\quad - \frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2} \\
&\quad - \frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 939 vs. $2(389) = 778$.

Time = 8.10 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.41

$$\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{-1920b^2c^2\sqrt{a+b\operatorname{arcsinh}(c+dx)} - 1920b^2cdx\sqrt{a+b\operatorname{arcsinh}(c+dx)} + 1280abc\sqrt{1+c^2+2}}{}$$

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-1920*b^2*c^2*Sqrt[a + b*ArcSinh[c + d*x]] - 1920*b^2*c*d*x*Sqrt[a + b*Arc
Sinh[c + d*x]] + 1280*a*b*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*Sqrt[a + b*Ar
cSinh[c + d*x]] - 1024*a*b*c^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]
] - 1024*a*b*c*d*x*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]] + 1280*b^2
*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c
 + d*x]] - 512*b^2*c^2*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] - 512
*b^2*c*d*x*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] + 128*a^2*Sqrt[a
 + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 120*b^2*Sqrt[a + b*ArcSin
h[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 256*a*b*ArcSinh[c + d*x]*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 128*b^2*ArcSinh[c + d*x]^2*Sqr
t[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - 128*a^2*Sqrt[b]*c*Sqrt
[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 480*b^(5/2)*c*S
qrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 15*b^(5/2)*S
qrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
] + (256*a^2*b*c*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]] + (256*a^2*b*c*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)])/(E^(a/b)*Sqr
t[a + b*ArcSinh[c + d*x]]) + 128*a^2*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 480*b^(5/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 32*Sqrt[b]*(4*a^2 - 15*b^2)*c*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 15*b
^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh
[(2*a)/b] - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 160*a*b*Sqrt[a + b*ArcSinh[c +
 d*x]]*Sinh[2*ArcSinh[c + d*x]] - 160*b^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSi
nh[c + d*x]]*Sinh[2*ArcSinh[c + d*x]])/(512*d^2)
```

## Maple [F]

$$\int x(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(a+b\mathrm{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x(a+b\mathrm{arcsinh}(c+dx))^{5/2}\,dx = \int x(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*(a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral(x*(a + b*asinh(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int x(a+b\mathrm{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

## Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int x\,(a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(x*(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.104 $\int (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx$

Optimal result . . . 856
Rubi [A] (verified) . . . 856
Mathematica [B] (verified) . . . 859
Maple [F] . . . 860
Fricas [F(-2)] . . . 860
Sympy [F] . . . 860
Maxima [F] . . . 861
Giac [F] . . . 861
Mupad [F(-1)] . . . 861

## Optimal result

Integrand size = 14, antiderivative size = 179

$$\int (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx = \frac{15b^2(c+dx)\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a + b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a + b\operatorname{arcsinh}(c+dx))^{5/2}}{d} + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5819, 3389, 2211, 2236, 2235}

$$\int (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx = \frac{15\sqrt{\pi}b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{15b^2(c+dx)\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{(c+dx)^2+1}(a + b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a + b\operatorname{arcsinh}(c+dx))^{5/2}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\text{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d} - \frac{(5b)\text{Subst}\left(\int \frac{x(a+b\text{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{5b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\,\text{Subst}\left(\int \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{4d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d} - \frac{(15b^3)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{8d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} \\
&\quad - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.

Time = 1.45 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.56

$$
\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}}\right) + 4a\sqrt{b}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

## Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}\, dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b\mathrm{arcsinh}(c + dx))^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a + b\mathrm{arcsinh}(c + dx))^{5/2}\, dx = \int (a + b\,\mathrm{asinh}\,(c + dx))^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

# 3.105 $\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 862
Rubi [A] (verified) . . . 863
Mathematica [A] (verified) . . . 867
Maple [F] . . . 868
Fricas [F(-2)] . . . 868
Sympy [F] . . . 868
Maxima [F] . . . 868
Giac [F] . . . 869
Mupad [F(-1)] . . . 869

## Optimal result

Integrand size = 18, antiderivative size = 411

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = &-\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}\\ &+\frac{c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}\\ &+\frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}\\ &+\frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}\\ &-\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}\\ &+\frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}\\ &-\frac{ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}\\ &+\frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}\end{aligned}$$

```
[Out] 1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/d^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/d^3/exp(3*a/b)/b^(1/2)+1/4*c*exp(2*a/b)*erf(2^(1/2)*(a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)-1/4*c*erfi(2^(1/2)*
(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/exp(2*a/b)/b^(1/2)
-1/8*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/2)+
1/2*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/
2)-1/8*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b^(1/
2)+1/2*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b
^(1/2)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5830, 6873, 6874, 5407, 2236, 2235, 5406, 5737}

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{\sqrt{\pi}c^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{\sqrt{\pi}c^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{\sqrt{\frac{\pi}{2}}ce^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} - \frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} - \frac{\sqrt{\frac{\pi}{2}}ce^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} - \frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}$$

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/8*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt[b]*
d^3) + (c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*
Sqrt[b]*d^3) + (c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^3) + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d^3) - (Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^3*E^(a/b)) + (c^2*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d^3*E^(a/b)) - (c*S
qrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*Sqrt[b]*
d^3*E^((2*a)/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(8*Sqrt[b]*d^3*E^((3*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
```

```
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(-\frac{c}{d}+\frac{x}{d}\right)^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2}{\sqrt{a+bx}}\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{2\text{Subst}\left(\int \left(c^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+c\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)\,dx, x, \sqrt{a+b}\right.}{bd^3} \\
&= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{(2c)\text{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{(2c^2)\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\text{Subst}\left(\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{c\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad - \frac{c\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&\quad + \frac{c^2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^3} \\
&= \frac{c^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} \\
&\quad + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} - \frac{ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} \\
&\quad + \frac{\text{Subst}\left(\int \cosh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd^3} \\
&\quad - \frac{\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd^3} \\
&= \frac{c^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} \\
&\quad + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} - \frac{ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} \\
&\quad + \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd^3} \\
&\quad + \frac{\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd^3}
\end{aligned}
$$

$$= -\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}$$
$$+ \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}$$
$$- \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}$$
$$- \frac{ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}$$

## Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$
$$= \frac{\sqrt{\frac{\pi}{6}}\left(\sqrt{2}\cosh\left(\frac{3a}{b}\right)\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right) - \sqrt{6}\cosh\left(\frac{a}{b}\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right) + 4\sqrt{6}c^2\cosh\left(\frac{a}{b}\right)\text{e}\right.$$

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[Pi/6]*(Sqrt[2]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]] - Sqrt[6]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]
+ 4*Sqrt[6]*c^2*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 4*Sq
rt[3]*c*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
+ Sqrt[2]*Cosh[(3*a)/b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
] + Sqrt[6]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 4*Sqrt[6
]*c^2*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + Sqrt[6]*(-1 +
4*c^2)*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) +
4*Sqrt[3]*c*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)
/b] + 4*Sqrt[3]*c*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh
[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[2]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]]*Sinh[(3*a)/b] - Sqrt[2]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]]*Sinh[(3*a)/b]))/(8*Sqrt[b]*d^3)
```

## Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,\operatorname{arcsinh}\left(dx+c\right)}}dx$$

```
[In] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\operatorname{asinh}\left(c+dx\right)}}\,dx$$

```
[In] integrate(x**2/(a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(x**2/sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\operatorname{asinh}\left(c+d\,x\right)}}\,dx$$

```
[In] int(x^2/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(x^2/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.106 $\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 870
Rubi [A] (verified) . . . 871
Mathematica [A] (verified) . . . 873
Maple [F] . . . 874
Fricas [F(-2)] . . . 874
Sympy [F] . . . 874
Maxima [F] . . . 874
Giac [F] . . . 875
Mupad [F(-1)] . . . 875

## Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = -\frac{ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}$$

```
[Out] -1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/d^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/d^2/exp(2*a/b)/b^(1/2)-1/2*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d^2/b^(1/2)-1/2*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d^2/exp(a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5830, 6873, 6874, 5407, 2236, 2235, 5406}

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} - \frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2} - \frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} + \frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}$$

```
[In] Int[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/2*(c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt[b
]*d^2) - (E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(4*Sqrt[b]*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/
Sqrt[b]])/(2*Sqrt[b]*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(4*Sqrt[b]*d^2*E^((2*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)}{\sqrt{a+bx}}\,dx, x, \text{arcsinh}(c+dx)\right)}{d} \\
&= -\frac{2\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\text{Subst}\left(\int \left(c\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&\quad -\frac{(2c)\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd^2} \\
&\quad +\frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd^2} \\
&\quad -\frac{c\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&\quad -\frac{c\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd^2} \\
&= -\frac{ce^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2} \\
&\quad -\frac{ce^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06

$$
\begin{aligned}
&\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx \\
&= \frac{e^{-\frac{a}{b}}\left(4ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)-4c\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{\sqrt{2\pi}\left(\text{erfi}\left(\frac{\sqrt{2}}{}\right.\right.}{8d^2}
\end{aligned}
$$

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((4*c*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c +
 d*x]] - 4*c*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSin
h[c + d*x])/b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) - (Sqrt[2*Pi]*(Erfi
[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 + Sinh[(2*a)/b])))/Sqrt[b])/(8*d^2)
```

## Maple [F]

$$\int \frac{x}{\sqrt{a+b\,\operatorname{arcsinh}\left(dx+c\right)}}dx$$

```
[In] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}}\,dx$$

```
[In] integrate(x/(a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(x/sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{x}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \int \frac{x}{\sqrt{b\,\mathrm{arsinh}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x}{\sqrt{b\,\operatorname{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int(x/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.107 $\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 876
Rubi [A] (verified) . . . 876
Mathematica [A] (verified) . . . 878
Maple [F] . . . 878
Fricas [F(-2)] . . . 879
Sympy [F] . . . 879
Maxima [F] . . . 879
Giac [F] . . . 879
Mupad [F(-1)] . . . 880

## Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5858, 5774, 3388, 2211, 2236, 2235}

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd} \\
&\quad + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd} \\
&= \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$
\begin{aligned}
&\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx \\
&= \frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\,\Gamma\left(\frac{1}{2}, \frac{a}{b}+\text{arcsinh}(c+dx)\right) + \sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\text{arcsinh}(c+dx)}}
\end{aligned}
$$

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$
\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\,(dx+c)}}dx
$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.108 $\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 881
Rubi [A] (verified) . . . 882
Mathematica [A] (verified) . . . 886
Maple [F] . . . 886
Fricas [F(-2)] . . . 886
Sympy [F] . . . 887
Maxima [F] . . . 887
Giac [F] . . . 887
Mupad [F(-1)] . . . 887

## Optimal result

Integrand size = 16, antiderivative size = 269

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}$$

```
[Out] 1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/d^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/d^2/exp(2*a/b)+c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*Pi^(1/2)/b^(3/2)/d^2-c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/b^(3/2)/d^2/exp(a/b)+2*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*
x+c))^(1/2)-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5829, 5773, 5819, 3389, 2211, 2236, 2235, 5778, 3388}

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} - \frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{2c\sqrt{(c+dx)^2+1}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{(c+dx)^2+1}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}}$$

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (c*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d^2) + (E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b
^(3/2)*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3
/2)*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(b^(3/2)*d^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{c}{d(a+b\text{arcsinh}(x))^{3/2}}+\frac{x}{d(a+b\text{arcsinh}(x))^{3/2}}\right)\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d^2} \\
&\quad - \frac{(2c)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{bd^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d^2} \\
&\quad + \frac{(2c)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{2\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d^2} \\
&\quad + \frac{2\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d^2} \\
&\quad + \frac{c\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^2d^2} \\
&\quad - \frac{c\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^2d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} \\
&\quad + \frac{(2c)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d^2} \\
&\quad - \frac{(2c)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} \\
&\quad - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.20

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\frac{4\sqrt{b}c\sqrt{1+(c+dx)^2}}{\sqrt{a+b\text{arcsinh}(c+dx)}} - 2c\sqrt{\pi}\cosh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right) + \sqrt{2\pi}\cosh($$

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] ((4*Sqrt[b]*c*Sqrt[1 + (c + d*x)^2])/Sqrt[a + b*ArcSinh[c + d*x]] - 2*c*Sqr
t[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + Sqrt[2*Pi]*Cos
h[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 2*c*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 2*c*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) - Sqrt[2*Pi]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + Sqrt[
2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] +
Sinh[(2*a)/b]) - (2*Sqrt[b]*Sinh[2*ArcSinh[c + d*x]])/Sqrt[a + b*ArcSinh[c
+ d*x]])/(2*b^(3/2)*d^2)
```

## Maple [F]

$$\int \frac{x}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(a+b\,\operatorname{asinh}(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral(x/(a + b*asinh(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(a+b\,\operatorname{asinh}(c+d\,x))^{3/2}}\,dx$$

```
[In] int(x/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.109 $\int \frac{1}{(a+b\mathbf{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 888
Rubi [A] (verified) . . . 888
Mathematica [A] (verified) . . . 890
Maple [F] . . . 891
Fricas [F(-2)] . . . 891
Sympy [F] . . . 891
Maxima [F] . . . 892
Giac [F] . . . 892
Mupad [F(-1)] . . . 892

## Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3389, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad + \frac{2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} \\
&\quad + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(c+dx)}\right) + e^{\frac{2a}{b}+\text{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\text{arcsinh}(}\right.}{bd}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{3/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.110 $\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 893
Rubi [A] (verified) . . . 894
Mathematica [A] (verified) . . . 900
Maple [F] . . . 901
Fricas [F(-2)] . . . 901
Sympy [F] . . . 901
Maxima [F] . . . 901
Giac [F] . . . 902
Mupad [F(-1)] . . . 902

## Optimal result

Integrand size = 16, antiderivative size = 365

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} + \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}$$

```
[Out] -2/3*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^
2-2/3*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^2/exp(a
/b)-2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/d^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2
^(1/2)*Pi^(1/2)/b^(5/2)/d^2/exp(2*a/b)+2/3*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b
*arcsinh(d*x+c))^(3/2)-2/3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d
*x+c))^(3/2)-4/3/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)+4/3*c*(d*x+c)/b^2/d^2/(
a+b*arcsinh(d*x+c))^(1/2)-8/3*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5859, 5829, 5773, 5818, 5774, 3388, 2211, 2236, 2235, 5779, 5780, 5556, 12, 3389, 5783}

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{\pi}ce^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2\sqrt{\pi}ce^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} + \frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2\sqrt{(c+dx)^2+1}(c+dx)}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{2c\sqrt{(c+dx)^2+1}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) -
4/(3*b^2*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*c*(c + d*x))/(3*b^2*d^2*Sqr
t[a + b*ArcSinh[c + d*x]]) - (8*(c + d*x)^2)/(3*b^2*d^2*Sqrt[a + b*ArcSinh[
c + d*x]]) - (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(3*b^(5/2)*d^2) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2) - (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d^2*E^(a/b)) + (2*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
```

```
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d(a+b\text{arcsinh}(x))^{5/2}} + \frac{x}{d(a+b\text{arcsinh}(x))^{5/2}}\right)\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd^2} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd^2} \\
&\quad - \frac{(2c)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{16\text{Subst}\left(\int \frac{x}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d^2} \\
&\quad - \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{16\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&\quad - \frac{(4c)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{16\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&\quad - \frac{(2c)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&\quad - \frac{(2c)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{8\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&\quad - \frac{(4c)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d^2} \\
&\quad - \frac{(4c)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} \\
&\quad - \frac{2ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} \\
&\quad - \frac{4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&\quad + \frac{4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} \\
&\quad - \frac{2ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} \\
&\quad - \frac{8\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d^2} \\
&\quad + \frac{8\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d^2}
\end{aligned}
$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$- \frac{4}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$
$$- \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}$$
$$- \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}$$
$$+ \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}$$

## Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.52

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx =$$

$$-\frac{-4a\sqrt{b}c(c+dx) - 2b^{3/2}c\sqrt{1+(c+dx)^2} - 4b^{3/2}c(c+dx)\operatorname{arcsinh}(c+dx) + 4a\sqrt{b}\cosh(2\operatorname{arcsinh}(c+dx)) +}{}$$

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(5/2),x]

[Out] -1/3*(-4*a*Sqrt[b]*c*(c + d*x) - 2*b^(3/2)*c*Sqrt[1 + (c + d*x)^2] - 4*b^(3
/2)*c*(c + d*x)*ArcSinh[c + d*x] + 4*a*Sqrt[b]*Cosh[2*ArcSinh[c + d*x]] + 4
*b^(3/2)*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b*Ar
cSinh[c + d*x])^(3/2)*Cosh[a/b]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] +
 2*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 2*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])
^(3/2)*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 2*Sqrt[2*Pi]*
(a + b*ArcSinh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]] + 2*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Erf[Sq
rt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*c*Sqrt[Pi]*(a + b*ArcSinh
[c + d*x])^(3/2)*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 2*S
qrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 2*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/
2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + b^(
3/2)*Sinh[2*ArcSinh[c + d*x]])/(b^(5/2)*d^2*(a + b*ArcSinh[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{x}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{5/2}}\,dx$$

```
[In] int(x/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.111 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 903
Rubi [A] (verified) . . . 903
Mathematica [A] (verified) . . . 906
Maple [F] . . . 907
Fricas [F(-2)] . . . 907
Sympy [F] . . . 907
Maxima [F] . . . 907
Giac [F] . . . 908
Mupad [F(-1)] . . . 908

## Optimal result

Integrand size = 14, antiderivative size = 158

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{5858, 5773, 5818, 5774, 3388, 2211, 2236, 2235\}$

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\sqrt{(c+dx)^2+1}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
```

```
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{4\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{4\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

$$
\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\operatorname{arcsinh}(c+dx)}\right)-2b\operatorname{arcsinh}(c+dx)+\right.\right.
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{5/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.112 $\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$


Optimal result . . . 909
Rubi [A] (verified) . . . 910
Mathematica [A] (verified) . . . 916
Maple [F] . . . 916
Fricas [F(-2)] . . . 917
Sympy [F] . . . 917
Maxima [F] . . . 917
Giac [F] . . . 917
Mupad [F(-1)] . . . 918


## Optimal result

Integrand size = 16, antiderivative size = 445

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} - \frac{4ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}$$

```
[Out] -4/15/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*(d*x+c)/b^2/d^2/(a+b*arcsin
h(d*x+c))^(3/2)-8/15*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*ex
p(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2-4/15*c*
erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2/exp(a/b)+8/15
*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2
)/b^(7/2)/d^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(7/2)/d^2/exp(2*a/b)+2/5*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsi
nh(d*x+c))^(5/2)-2/5*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))
^(5/2)+8/15*c*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)-32/15*
(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5859, 5829, 5773, 5818, 5819, 3389, 2211, 2236, 2235, 5779, 5778, 3388, 5783}

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{4\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} - \frac{4\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} - \frac{32\sqrt{(c+dx)^2+1}(c+dx)}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{8c\sqrt{(c+dx)^2+1}}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2\sqrt{(c+dx)^2+1}(c+dx)}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} + \frac{2c\sqrt{(c+dx)^2+1}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) -
4/(15*b^2*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*c*(c + d*x))/(15*b^2*d^2
*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*(c + d*x)^2)/(15*b^2*d^2*(a + b*ArcSi
nh[c + d*x])^(3/2)) + (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a + b*Ar
cSinh[c + d*x]]) - (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a
+ b*ArcSinh[c + d*x]]) + (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(15*b^(7/2)*d^2) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2) - (4*c*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d^2*E^(a/b)) + (8*Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2
*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d(a+b\text{arcsinh}(x))^{7/2}} + \frac{x}{d(a+b\text{arcsinh}(x))^{7/2}}\right)\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd^2} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd^2} \\
&\quad - \frac{(2c)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8(c+dx)^2}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{16\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d^2} \\
&\quad - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{4c(c+dx)}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{32\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{15b^4d^2} \\
&\quad - \frac{(8c)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{15b^3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{4c(c+dx)}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{16\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d^2} \\
&\quad + \frac{16\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d^2} \\
&\quad + \frac{(8c)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{4c(c+dx)}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{32\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d^2} \\
&\quad + \frac{32\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d^2} \\
&\quad + \frac{(4c)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d^2} \\
&\quad - \frac{(4c)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8(c+dx)^2}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} \\
&\quad + \frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} \\
&\quad + \frac{(8c)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d^2} \\
&\quad - \frac{(8c)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d^2} \\
&= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}} \\
&\quad - \frac{4}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8(c+dx)^2}{15b^2d^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}} + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} \\
&\quad + \frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} - \frac{4ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} \\
&\quad + \frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.58

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \frac{4ab^{3/2}c(c+dx)+8a^2\sqrt{b}c\sqrt{1+(c+dx)^2}+6b^{5/2}c\sqrt{1+(c+dx)^2}+4b^{5/2}c$$

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[1 + (c + d*x)^2] + 6*b^(5/2
)*c*Sqrt[1 + (c + d*x)^2] + 4*b^(5/2)*c*(c + d*x)*ArcSinh[c + d*x] + 16*a*b
^(3/2)*c*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x] + 8*b^(5/2)*c*Sqrt[1 + (c +
 d*x)^2]*ArcSinh[c + d*x]^2 - 4*a*b^(3/2)*Cosh[2*ArcSinh[c + d*x]] - 4*b^(5
/2)*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] + 4*c*Sqrt[Pi]*(a + b*ArcSinh
[c + d*x])^(5/2)*Cosh[a/b]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 8*Sq
rt[2*Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]] - 4*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])^(5/2
)*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a +
b*ArcSinh[c + d*x])^(5/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]] + 4*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*Sqrt[Pi]*(a + b*ArcSinh[c +
d*x])^(5/2)*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[2
*Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]]*Sinh[(2*a)/b] - 8*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 16*a^2*S
qrt[b]*Sinh[2*ArcSinh[c + d*x]] - 3*b^(5/2)*Sinh[2*ArcSinh[c + d*x]] - 32*a
*b^(3/2)*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] - 16*b^(5/2)*ArcSinh[c +
 d*x]^2*Sinh[2*ArcSinh[c + d*x]])/(15*b^(7/2)*d^2*(a + b*ArcSinh[c + d*x])^
(5/2))
```

## Maple [F]

$$\int \frac{x}{(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(7/2), x)
```

## Maxima [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{7/2}}\,dx$$

```
[In] int(x/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.113 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 919
Rubi [A] (verified) . . . 919
Mathematica [A] (verified) . . . 923
Maple [F] . . . 923
Fricas [F(-2)] . . . 923
Sympy [F] . . . 924
Maxima [F] . . . 924
Giac [F] . . . 924
Mupad [F(-1)] . . . 924

## Optimal result

Integrand size = 14, antiderivative size = 195

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{5858, 5773, 5818, 5819, 3389, 2211, 2236, 2235\}$

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{4\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} - \frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2\sqrt{(c+dx)^2+1}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
```

```
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{8\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{15b^3d}
\end{aligned}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$
$$-\frac{4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$
$$+\frac{4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d}$$
$$+\frac{8\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$+\frac{4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{-6b^2e^{\text{arcsinh}(c+dx)} - 2e^{-\text{arcsinh}(c+dx)}(4a^2+2ab(-1+4\text{arcsinh}(c+dx))+b$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c+d\,x))^{7/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.114 $\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 925
Rubi [A] (verified) . . . 925
Mathematica [A] (verified) . . . 926
Maple [F] . . . 927
Fricas [F] . . . 927
Sympy [F] . . . 927
Maxima [F] . . . 927
Giac [F] . . . 928
Mupad [F(-1)] . . . 928

## Optimal result

Integrand size = 21, antiderivative size = 91

$$\begin{aligned}&\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx\\&=\frac{(e(c+dx))^{1+m}(a+b\operatorname{arcsinh}(c+dx))}{de(1+m)}\\&\quad-\frac{b(e(c+dx))^{2+m}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-(c+dx)^2\right)}{de^2(1+m)(2+m)}\end{aligned}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*hyperg
eom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+m)/(2+m)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5859, 5776, 371}

$$\begin{aligned}&\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx\\&=\frac{(e(c+dx))^{m+1}(a+b\operatorname{arcsinh}(c+dx))}{de(m+1)}\\&\quad-\frac{b(e(c+dx))^{m+2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+dx)^2\right)}{de^2(m+1)(m+2)}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])/
(d*e^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (ex)^m (a + b\text{arcsinh}(x))\,dx, x, c + dx\right)}{d} \\ &= \frac{(e(c + dx))^{1+m}(a + b\text{arcsinh}(c + dx))}{de(1 + m)} - \frac{b\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{1+x^2}}\,dx, x, c + dx\right)}{de(1 + m)} \\ &= \frac{(e(c + dx))^{1+m}(a + b\text{arcsinh}(c + dx))}{de(1 + m)} \\ &\quad - \frac{b(e(c + dx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -(c + dx)^2\right)}{de^2(1 + m)(2 + m)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))\,dx = -\frac{(c + dx)(e(c + dx))^m \left(-((2 + m)(a + b\text{arcsinh}(c + dx))) + b(c + dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2},\right.\right.}{d(1 + m)(2 + m)}$$

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(e*(c + d*x))^m*(-((2 + m)*(a + b*ArcSinh[c + d*x])) + b*(c +
d*x)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2]))/(d*(1 + m
)*(2 + m)))
```

## Maple [F]

$$\int (dex + ce)^m \left(a + b \operatorname{arcsinh}\left(dx + c\right)\right) dx$$

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

## Fricas [F]

$$\int (ce + dex)^m (a + b\operatorname{arcsinh}(c + dx))\, dx = \int (b \operatorname{arsinh}\left(dx + c\right) + a)(dex + ce)^m\, dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

## Sympy [F]

$$\int (ce + dex)^m (a + b\operatorname{arcsinh}(c + dx))\, dx = \int \left(e(c + dx)\right)^m \left(a + b \operatorname{asinh}\left(c + dx\right)\right)\, dx$$

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int (ce + dex)^m (a + b\operatorname{arcsinh}(c + dx))\, dx = \int (b \operatorname{arsinh}\left(dx + c\right) + a)(dex + ce)^m\, dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] b*((d*e^m*x + c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d*(m + 1)) - integrate((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m)*(d*x +
 c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1), x) - integ
rate((d*e^m*x + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 +
 c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x
^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)), x)) + (d*e*x + c*e)^(m + 1)*a/(d*e*(m + 1))
```

## Giac [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)), x)
```

# 3.115 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 929
Rubi [A] (verified) . . . 929
Mathematica [A] (verified) . . . 931
Maple [A] (verified) . . . 931
Fricas [B] (verification not implemented) . . . 932
Sympy [B] (verification not implemented) . . . 932
Maxima [B] (verification not implemented) . . . 933
Giac [B] (verification not implemented) . . . 934
Mupad [F(-1)] . . . 935

## Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx = -\frac{be^4\sqrt{1+(c+dx)^2}}{5d} + \frac{2be^4(1+(c+dx)^2)^{3/2}}{15d} - \frac{be^4(1+(c+dx)^2)^{5/2}}{25d} + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{5d}$$

```
[Out] 2/15*b*e^4*(1+(d*x+c)^2)^(3/2)/d-1/25*b*e^4*(1+(d*x+c)^2)^(5/2)/d+1/5*e^4*(
d*x+c)^5*(a+b*arcsinh(d*x+c))/d-1/5*b*e^4*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 272, 45}

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{5d} - \frac{be^4((c+dx)^2+1)^{5/2}}{25d} + \frac{2be^4((c+dx)^2+1)^{3/2}}{15d} - \frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/5*(b*e^4*Sqrt[1 + (c + d*x)^2])/d + (2*b*e^4*(1 + (c + d*x)^2)^(3/2))/(1
5*d) - (b*e^4*(1 + (c + d*x)^2)^(5/2))/(25*d) + (e^4*(c + d*x)^5*(a + b*Arc
Sinh[c + d*x]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^4 x^4 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int x^4 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\
&= \frac{e^4 (c + dx)^5 (a + b\text{arcsinh}(c + dx))}{5d} - \frac{(be^4)\, \text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}}\, dx, x, c + dx\right)}{5d} \\
&= \frac{e^4 (c + dx)^5 (a + b\text{arcsinh}(c + dx))}{5d} - \frac{(be^4)\, \text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\, dx, x, (c + dx)^2\right)}{10d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{5d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}} - 2\sqrt{1+x} + (1+x)^{3/2}\right)\, dx, x, (c+dx)^2\right)}{10d} \\
&= -\frac{be^4\sqrt{1+(c+dx)^2}}{5d} + \frac{2be^4(1+(c+dx)^2)^{3/2}}{15d} \\
&\quad - \frac{be^4(1+(c+dx)^2)^{5/2}}{25d} + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{5d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$
\begin{aligned}
&\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx \\
&= \frac{e^4\left(-\frac{1}{75}b\sqrt{1+(c+dx)^2}\left(5-10(c+dx)^2+3(1+(c+dx)^2)^2\right)+\frac{1}{5}(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))\right)}{d}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(-1/75*(b*Sqrt[1 + (c + d*x)^2]*(5 - 10*(c + d*x)^2 + 3*(1 + (c + d*x)
^2)^2)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/d
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{e^4a(dx+c)^5}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$ | 93 |
| default | $\frac{\frac{e^4a(dx+c)^5}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$ | 93 |
| parts | $\frac{e^4a(dx+c)^5}{5d}+\frac{e^4b\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$ | 95 |

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*e^4*a*(d*x+c)^5+e^4*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4
*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^
(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(86) = 172$.

Time $= 0.27$ (sec) , antiderivative size $= 279$, normalized size of antiderivative $= 2.79$

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))\, dx$$

$$= \frac{15\, ad^5e^4x^5 + 75\, acd^4e^4x^4 + 150\, ac^2d^3e^4x^3 + 150\, ac^3d^2e^4x^2 + 75\, ac^4de^4x + 15\, (bd^5e^4x^5 + 5\, bcd^4e^4x^4 + 10\, bc$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*a
*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x^4
 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5*e^
4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (3*b*d^4*e^4*x^4 + 12
*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 - 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 - 2*b*c)*d*e^
4*x + (3*b*c^4 - 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(85) = 170$.

Time $= 0.37$ (sec) , antiderivative size $= 527$, normalized size of antiderivative $= 5.27$

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))\, dx$$

$$= \begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4 \operatorname{asinh}(c+dx)}{5d} + bc^4e^4x \operatorname{asinh}(c + dx) - \frac{bc^4e^4\sqrt{c} \\ c^4e^4x(a + b \operatorname{asinh}(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*asinh(c + d*x)/(5*d)
 + b*c**4*e**4*x*asinh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)/(25*d) + 2*b*c**3*d*e**4*x**2*asinh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 2*b*c**2*d**2*e**4*x**3*asinh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d) + b*c*d**3*e**4*x**4*asin
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + b*d**4*e**4*x**5*as
inh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(
c)), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(86) = 172$.

Time = 0.21 (sec) , antiderivative size = 1231, normalized size of antiderivative = 12.31

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))\, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2
*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c
^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*arcsinh
(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh
(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2
/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c^2*d^2*e^4 +
1/24*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^
2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2
*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3
/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*c*d^3*e^4 + 1/600*(120*x^
5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^4/d^2 - 54*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3*x/d^5 - 32*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^2 + 1)*c^3*arcsi
nh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 + 945*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^
2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c
^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d^6)*d)*b*d^4*e^4
 + a*c^4*e^4*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^4
*e^4/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(86) = 172$.

Time = 0.89 (sec) , antiderivative size = 841, normalized size of antiderivative = 8.41

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{5}\,ad^4e^4x^5 + acd^3e^4x^4 + 2\,ac^2d^2e^4x^3 + 2\,ac^3de^4x^2$$
$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\sqrt{}\right.\right.$$
$$+\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2-1)\log\left(-\right.\right.\right.\right.$$
$$+\frac{1}{3}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2d-4}{d^5}\right.\right.\right.$$
$$+\frac{1}{24}\left(24\,x^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\,x\left(\frac{3\,x}{d^2}-\frac{7\,c}{d^3}\right)+\frac{26\,c\right.\right.\right.\right.$$
$$+\frac{1}{600}\left(120\,x^5\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\left(3\,x\left(\frac{4\,x}{d^2}-\frac{9\,c}{d^3}\right)-\right.\right.\right.\right.$$
$$+\,ac^4e^4x$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)
)/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^4*e^4 + (2*x^2*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c
/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))*abs(d))/(d^2*abs(d)))*d)*b*c^3*d*e^4 + 1/3*(6*x^3*log(d*x + c + sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2
 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d)
- sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c^2*d^2*e^4
 + 1/24*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)
/d^7)*x - 5*(10*c^3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b
*c*d^3*e^4 + 1/600*(120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*(3*x*(4*x/d^2 - 9*c/d^3) + (47*c
^2*d^5 - 16*d^5)/d^9)*x - 7*(22*c^3*d^4 - 23*c*d^4)/d^9)*x + (274*c^4*d^3 -
 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 - 40*c^3 + 15*c)*log(-c*d - (x*abs(
d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^5*abs(d)))*d)*b*d^4*e^4
+ a*c^4*e^4*x
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)), x)
```

# 3.116 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 936
Rubi [A] (verified) . . . 936
Mathematica [A] (verified) . . . 938
Maple [A] (verified) . . . 938
Fricas [B] (verification not implemented) . . . 939
Sympy [B] (verification not implemented) . . . 939
Maxima [B] (verification not implemented) . . . 940
Giac [B] (verification not implemented) . . . 941
Mupad [F(-1)] . . . 942

## Optimal result

Integrand size = 21, antiderivative size = 105

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} - \frac{3be^3\operatorname{arcsinh}(c+dx)}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{4d}$$

```
[Out] -3/32*b*e^3*arcsinh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d+3/32*
b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2
)/d
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 327, 221}

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{4d} - \frac{3be^3\operatorname{arcsinh}(c+dx)}{32d} - \frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3}{16d} + \frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)}{32d}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(32*d) - (b*e^3*(c + d*x)^3*Sqrt[
1 + (c + d*x)^2])/(16*d) - (3*b*e^3*ArcSinh[c + d*x])/(32*d) + (e^3*(c + d*
x)^4*(a + b*ArcSinh[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\ &= \frac{e^3 \text{Subst}\left(\int x^3 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\ &= \frac{e^3 (c + dx)^4 (a + b\text{arcsinh}(c + dx))}{4d} - \frac{(be^3)\, \text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\, dx, x, c + dx\right)}{4d}\end{aligned}$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{4d} + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{16d}$$

$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{4d} - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d}$$

$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} - \frac{3be^3\operatorname{arcsinh}(c+dx)}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{4d}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \frac{e^3\left(3b(c+dx)\sqrt{1+(c+dx)^2} - 2b(c+dx)^3\sqrt{1+(c+dx)^2} - 3b\operatorname{arcsinh}(c+dx) + 8(c+dx)^4(a+b\operatorname{arcsin}\right.}{32d}$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (c + d
*x)^2] - 3*b*ArcSinh[c + d*x] + 8*(c + d*x)^4*(a + b*ArcSinh[c + d*x])))/(3
2*d)
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{e^3a(dx+c)^4}{4}+e^3b\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3\operatorname{arcsinh}(dx+c)}{32}\right)}{d}$ | 86 |
| default | $\frac{\frac{e^3a(dx+c)^4}{4}+e^3b\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3\operatorname{arcsinh}(dx+c)}{32}\right)}{d}$ | 86 |
| parts | $\frac{e^3a(dx+c)^4}{4d}+\frac{e^3b\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3\operatorname{arcsinh}(dx+c)}{32}\right)}{d}$ | 88 |

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3
*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(93) = 186$.

Time $= 0.25$ (sec) , antiderivative size $= 228$, normalized size of antiderivative $= 2.17$

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))\, dx$$

$$= \frac{8\,ad^4e^3x^4 + 32\,acd^3e^3x^3 + 48\,ac^2d^2e^3x^2 + 32\,ac^3de^3x + (8\,bd^4e^3x^4 + 32\,bcd^3e^3x^3 + 48\,bc^2d^2e^3x^2 + 32\,bc^3d}{}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c^
3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2 +
32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 - b)*d*e^
3*x + (2*b*c^3 - 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(94) = 188$.

Time $= 0.28$ (sec) , antiderivative size $= 394$, normalized size of antiderivative $= 3.75$

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))\, dx$$

$$= \begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{asinh}(c+dx)}{4d} + bc^3e^3x\operatorname{asinh}(c + dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2+1}}{16d} - \\ c^3e^3x(a + b\operatorname{asinh}(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*asinh(c + d*x)/(4*d) + b*c**3*e**3*x*asinh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*asinh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/16 + b*c*d**2*e**3*x**3*asinh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(32*d) + b*d**3*e**3*x**4*asinh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/32 - 3*b*e**3*asinh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*asinh(c)), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 790, normalized size of antiderivative = 7.52

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2$$
$$+\frac{3}{4}\left(2x^2\operatorname{arsinh}(dx+c) - d\left(\frac{3c^2\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2+2cdx+c^2+1}x}{d^2} - \frac{(c^2+1)\operatorname{arsi}\right.\right.$$
$$+\frac{1}{6}\left(6x^3\operatorname{arsinh}(dx+c) - d\left(\frac{2\sqrt{d^2x^2+2cdx+c^2+1}x^2}{d^2} - \frac{15c^3\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^4} - \frac{5\sqrt{d^2x^2}\right.\right.$$
$$+\frac{1}{96}\left(24x^4\operatorname{arsinh}(dx+c) - \left(\frac{6\sqrt{d^2x^2+2cdx+c^2+1}x^3}{d^2} - \frac{14\sqrt{d^2x^2+2cdx+c^2+1}cx^2}{d^3} + \frac{105c^4\operatorname{ars}\right.\right.$$
$$+ ac^3e^3x + \frac{\left((dx+c)\operatorname{arsinh}(dx+c) - \sqrt{(dx+c)^2+1}\right)bc^3e^3}{d}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arcs
inh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/96*(24*x^4*arcsi
nh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*
x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*e^3*x + ((d*x + c)*arcsinh(
d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^3*e^3/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(93) = 186$.

Time = 0.74 (sec) , antiderivative size = 613, normalized size of antiderivative = 5.84

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2$$
$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\right.\right.$$
$$+\frac{3}{4}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2-1)\,\mathrm{lo}\right.\right.$$
$$+\frac{1}{6}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2d-\right.\right.\right.$$
$$+\frac{1}{96}\left(24\,x^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\,x\left(\frac{3\,x}{d^2}-\frac{7\,c}{d^3}\right)+\frac{26\right.\right.\right.\right.$$
$$+ac^3e^3x$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 - (d*(c*log(-c*d
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)))*b*c^3*e^3 + 3/4*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 -
 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*
abs(d)))*d)*b*c^2*d*e^3 + 1/6*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (1
1*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c*d^2*e^3 + 1/96*(24*x^4*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)*x - 5*(10*c^
3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d - (x*abs(d) - sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b*d^3*e^3 + a*c^3*e
^3*x
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)), x)
```

# 3.117 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 943
Rubi [A] (verified) . . . 943
Mathematica [A] (verified) . . . 945
Maple [A] (verified) . . . 945
Fricas [B] (verification not implemented) . . . 945
Sympy [B] (verification not implemented) . . . 946
Maxima [B] (verification not implemented) . . . 946
Giac [B] (verification not implemented) . . . 947
Mupad [F(-1)] . . . 948

## Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{be^2\sqrt{1+(c+dx)^2}}{3d} - \frac{be^2(1+(c+dx)^2)^{3/2}}{9d} + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{3d}$$

```
[Out] -1/9*b*e^2*(1+(d*x+c)^2)^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1
/3*b*e^2*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 272, 45}

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{3d} - \frac{be^2((c+dx)^2+1)^{3/2}}{9d} + \frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2])/(3*d) - (b*e^2*(1 + (c + d*x)^2)^(3/2))/(9*d)
 + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2 x^2 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a + b\text{arcsinh}(x))\, dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b\text{arcsinh}(c + dx))}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}}\, dx, x, c + dx\right)}{3d} \\
&= \frac{e^2 (c + dx)^3 (a + b\text{arcsinh}(c + dx))}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\, dx, x, (c + dx)^2\right)}{6d} \\
&= \frac{e^2 (c + dx)^3 (a + b\text{arcsinh}(c + dx))}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right)\, dx, x, (c + dx)^2\right)}{6d} \\
&= \frac{be^2 \sqrt{1 + (c + dx)^2}}{3d} - \frac{be^2 \left(1 + (c + dx)^2\right)^{3/2}}{9d} + \frac{e^2 (c + dx)^3 (a + b\text{arcsinh}(c + dx))}{3d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \frac{e^2\left(-\frac{1}{9}b(-2+c^2+2cdx+d^2x^2)\sqrt{1+(c+dx)^2}+\frac{1}{3}(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))\right)}{d}$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-1/9*(b*(-2 + c^2 + 2*c*d*x + d^2*x^2)*Sqrt[1 + (c + d*x)^2]) + ((c +
 d*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{a\,e^2(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9}+\frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$ | 73 |
| default | $\frac{\frac{a\,e^2(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9}+\frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$ | 73 |
| parts | $\frac{a\,e^2(dx+c)^3}{3d}+\frac{e^2b\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9}+\frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$ | 75 |

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a*e^2*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*
(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(66) = 132.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.21

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \frac{3\,ad^3e^2x^3+9\,acd^2e^2x^2+9\,ac^2de^2x+3\left(bd^3e^2x^3+3\,bcd^2e^2x^2+3\,bc^2de^2x+bc^3e^2\right)\log\left(dx+c+\sqrt{d^2x^2+}\right.}{9\,d}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*x
^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 - 2*b
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(63) = 126$.

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.39

$$\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))\, dx$$

$$= \begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\,\text{asinh}\,(c+dx)}{3d} + bc^2e^2x\,\text{asinh}\,(c + dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + bcde^2x^2\,\text{asin} \\ c^2e^2x(a + b\,\text{asinh}\,(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*asinh(c + d*x)/(3*d) + b*c**2*e**2*x*asinh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + b*c*d*e**2*x**2*asinh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + b*d**2*e**2*x**3*asinh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*asinh(c)), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(66) = 132$.

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.86

$$\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))\, dx = \frac{1}{3}\, ad^2e^2x^3 + acde^2x^2$$

$$+ \frac{1}{2}\left(2\,x^2\,\text{arsinh}\,(dx + c) - d\left(\frac{3\,c^2\,\text{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}x}{d^2} - \frac{(c^2+1)\,\text{arsi}\right.\right.$$

$$+ \frac{1}{18}\left(6\,x^3\,\text{arsinh}\,(dx + c) - d\left(\frac{2\,\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}x^2}{d^2} - \frac{15\,c^3\,\text{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^4} - \frac{5\,\sqrt{d^2x}\right.\right.$$

$$+ ac^2e^2x + \frac{\left((dx + c)\,\text{arsinh}\,(dx + c) - \sqrt{(dx + c)^2 + 1}\right)bc^2e^2}{d}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*b*c*d*e^2 + 1/18*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2
 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)
*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)*(c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + ((d*x + c)*arcsinh(d*x + c) - s
qrt((d*x + c)^2 + 1))*b*c^2*e^2/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(66) = 132$.

Time $= 0.62$ (sec) , antiderivative size $= 416$, normalized size of antiderivative $= 5.47$

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))\, dx = \frac{1}{3} ad^2e^2x^3 + acde^2x^2$$
$$- \left( d \left( \frac{c \log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2} \right) - x \log\left(dx + c + \right.$$
$$+ \frac{1}{2} \left( 2\,x^2 \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \left( \sqrt{d^2x^2 + 2\,cdx + c^2 + 1} \left( \frac{x}{d^2} - \frac{3\,c}{d^3} \right) - \frac{(2\,c^2 - 1)\,\mathrm{lo}\right.\right.\right.$$
$$+ \frac{1}{18} \left( 6\,x^3 \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \left( \sqrt{d^2x^2 + 2\,cdx + c^2 + 1} \left( x \left( \frac{2\,x}{d^2} - \frac{5\,c}{d^3} \right) + \frac{11\,c^2d}{d^5}\right.\right.\right.\right.$$
$$+ ac^2e^2x$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^2*e^2 +
 1/2*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d)
 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c*d*e^2 +
1/18*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(
2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(
d))/(d^3*abs(d)))*d)*b*d^2*e^2 + a*c^2*e^2*x
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)), x)
```

# 3.118 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 949
Rubi [A] (verified) . . . 949
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Fricas [A] (verification not implemented) . . . 951
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## Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))\,dx = -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{be\operatorname{arcsinh}(c+dx)}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{2d}$$

```
[Out] 1/4*b*e*arcsinh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d-1/4*b*e*(d*
x+c)*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5859, 12, 5776, 327, 221}

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{2d} + \frac{be\operatorname{arcsinh}(c+dx)}{4d} - \frac{be\sqrt{(c+dx)^2+1}(c+dx)}{4d}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/4*(b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/d + (b*e*ArcSinh[c + d*x])/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}(\int ex(a+b\text{arcsinh}(x))\,dx, x, c+dx)}{d} \\
&= \frac{e\text{Subst}(\int x(a+b\text{arcsinh}(x))\,dx, x, c+dx)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))}{2d} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{be\text{arcsinh}(c+dx)}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))}{2d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \frac{e\Big(-b(c+dx)\sqrt{1+(c+dx)^2}+b\operatorname{arcsinh}(c+dx)+2(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))\Big)}{4d}$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e*(-(b*(c + d*x)*Sqrt[1 + (c + d*x)^2]) + b*ArcSinh[c + d*x] + 2*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])))/(4*d)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$ | 62 |
| default | $\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$ | 62 |
| parts | $ea\left(\frac{1}{2}dx^2+cx\right)+\frac{eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$ | 63 |

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a*(d*x+c)^2+e*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \frac{2\,ad^2ex^2+4\,acdex+(2\,bd^2ex^2+4\,bcdex+(2\,bc^2+b)e)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-(bdex+}{4\,d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
+ b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d*e*x + b*c*e
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx$$

$$= \begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\,\operatorname{asinh}(c+dx)}{2d} + bcex\,\operatorname{asinh}(c+dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{bdex^2\,\operatorname{asinh}(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+d^2} \\ cex(a + b\,\operatorname{asinh}(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*asinh(c + d*x)/(2*d) + b*c*e*x
*asinh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(4*d) + b*d*e*
x**2*asinh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + b*e*
asinh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c)), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(60) = 120$.

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.96

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx = \frac{1}{2}\,adex^2$$

$$+\frac{1}{4}\left(2\,x^2\,\operatorname{arsinh}(dx + c) - d\left(\frac{3\,c^2\,\operatorname{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}x}{d^2} - \frac{(c^2+1)\,\text{arsi}\right.\right.$$

$$+ acex + \frac{\left((dx + c)\,\operatorname{arsinh}(dx + c) - \sqrt{(dx + c)^2 + 1}\right)bce}{d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1
)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*d*e + a*c*e*x +
 ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c*e/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(60) = 120$.

Time = 0.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.57

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx = \frac{1}{2}adex^2$$
$$- \left( d\left( \frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2} \right) - x\log\left(dx + c + \right.$$
$$+ \frac{1}{4}\left( 2\,x^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \left( \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\left( \frac{x}{d^2} - \frac{3\,c}{d^3} \right) - \frac{(2\,c^2 - 1)\,\mathrm{lo}\right.\right.\right.$$
$$+ acex$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*e*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c*e + 1/4*(2*x^2*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*d*e + a*c*e*x
```

## Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx = \int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}(c + d\,x)\right)\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x)), x)
```

# 3.119 $\int (a + b\mathbf{arcsinh}(c + dx))\, dx$

Optimal result . . . 954
Rubi [A] (verified) . . . 954
Mathematica [B] (verified) . . . 955
Maple [A] (verified) . . . 956
Fricas [A] (verification not implemented) . . . 956
Sympy [A] (verification not implemented) . . . 956
Maxima [A] (verification not implemented) . . . 957
Giac [B] (verification not implemented) . . . 957
Mupad [B] (verification not implemented) . . . 957

## Optimal result

Integrand size = 10, antiderivative size = 39

$$\int (a + b\text{arcsinh}(c + dx))\, dx = ax - \frac{b\sqrt{1 + (c + dx)^2}}{d} + \frac{b(c + dx)\text{arcsinh}(c + dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arcsinh(d*x+c)/d-b*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5858, 5772, 267}

$$\int (a + b\text{arcsinh}(c + dx))\, dx = ax + \frac{b(c + dx)\text{arcsinh}(c + dx)}{d} - \frac{b\sqrt{(c + dx)^2 + 1}}{d}$$

```
[In] Int[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \operatorname{arcsinh}(c+dx)\,dx \\
&= ax + \frac{b\text{Subst}(\int \operatorname{arcsinh}(x)\,dx, x, c+dx)}{d} \\
&= ax + \frac{b(c+dx)\operatorname{arcsinh}(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= ax - \frac{b\sqrt{1+(c+dx)^2}}{d} + \frac{b(c+dx)\operatorname{arcsinh}(c+dx)}{d}
\end{aligned}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(39) = 78$.

Time $= 0.16$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 2.13$

$$\begin{aligned}
&\int (a + b\operatorname{arcsinh}(c+dx))\,dx \\
&= ax + bx\operatorname{arcsinh}(c+dx) \\
&\quad - \frac{b\left(\sqrt{1+c^2+2cdx+d^2x^2} + 2c\operatorname{arctanh}\left(\frac{dx}{\sqrt{1+c^2}-\sqrt{1+c^2+2cdx+d^2x^2}}\right)\right)}{d}
\end{aligned}$$

```
[In] Integrate[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcSinh[c + d*x] - (b*(Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] + 2*c*Ar
cTanh[(d*x)/(Sqrt[1 + c^2] - Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])]))/d
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $ax+\frac{b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$ | 36 |
| parts | $ax+\frac{b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$ | 36 |
| derivativedivides | $\frac{(dx+c)a+b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$ | 41 |

```
[In] int(a+b*arcsinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int (a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$=\frac{adx+(bdx+bc)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\sqrt{d^2x^2+2\,cdx+c^2+1}b}{d}$$

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b)/d
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int (a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$=ax+b\left(\begin{cases}\frac{c\operatorname{asinh}(c+dx)}{d}+x\operatorname{asinh}(c+dx)-\frac{\sqrt{c^2+2cdx+d^2x^2+1}}{d} & \text{for } d\neq 0\\ x\operatorname{asinh}(c) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*asinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*asinh(c + d*x)/d + x*asinh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*asinh(c), True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int (a + b\operatorname{arcsinh}(c + dx))\,dx = ax + \frac{\left((dx + c)\operatorname{arsinh}(dx + c) - \sqrt{(dx + c)^2 + 1}\right)b}{d}$$

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int (a + b\operatorname{arcsinh}(c + dx))\,dx = \\ -\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \right.\right. \\ \left.\left. + ax\right.\right.$$

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d
*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt((d
*x + c)^2 + 1)))*b + a*x
```

## Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int (a + b\operatorname{arcsinh}(c + dx))\,dx = a\,x + b\,x\,\operatorname{asinh}(c + d\,x) - \frac{b\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1}}{d} \\ + \frac{b\,c\,d^2\,\ln\left(\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1} + \frac{x\,d^2 + c\,d}{\sqrt{d^2}}\right)}{\left(d^2\right)^{3/2}}$$

```
[In] int(a + b*asinh(c + d*x),x)
```

```
[Out] a*x + b*x*asinh(c + d*x) - (b*(c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2))/d + (b*c
*d^2*log((c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2) + (c*d + d^2*x)/(d^2)^(1/2)))/
(d^2)^(3/2)
```

# 3.120 $\int \frac{a+b\mathbf{arcsinh}(c+dx)}{ce+dex}\,dx$

Optimal result . . . 958
Rubi [A] (verified) . . . 958
Mathematica [A] (verified) . . . 960
Maple [A] (verified) . . . 961
Fricas [F] . . . 961
Sympy [F] . . . 961
Maxima [F] . . . 962
Giac [F] . . . 962
Mupad [F(-1)] . . . 962

## Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx = \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{2bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{b\,\mathrm{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^2/b/d/e+(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x
+c)^2)^(1/2))^2)/d/e-1/2*b*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx = \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{2bde} + \frac{\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de} - \frac{b\,\mathrm{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcSinh[c + d*x])*Log[1 - E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*
d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x}\,dx, x, c+dx\right)}{de} \\
&= -\frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^2}{2bde} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^2}{2bde} + \frac{(a+b\text{arcsinh}(c+dx))\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{\text{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^2}{2bde} + \frac{(a+b\text{arcsinh}(c+dx))\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}\right)}{2de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^2}{2bde} + \frac{(a+b\text{arcsinh}(c+dx))\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}\right)}{2de}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx \\
&= \frac{-\left((a+b\text{arcsinh}(c+dx))\left(a+b\text{arcsinh}(c+dx)-2b\log\left(1-e^{2\text{arcsinh}(c+dx)}\right)\right)\right)+b^2\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(c+d}\right.}{2bde}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]

[Out] (-((a + b*ArcSinh[c + d*x])*(a + b*ArcSinh[c + d*x] - 2*b*Log[1 - E^(2*ArcS
inh[c + d*x])])) + b^2*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*b*d*e)
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+\operatorname{arcsinh}(dx+c)\ln$ |
| default | $\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+\operatorname{arcsinh}(dx+c)\ln$ |
| parts | $\frac{a\ln(dx+c)}{ed}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)-$ |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/e*ln(d*x+c)+b/e*(-1/2*arcsinh(d*x+c)^2+arcsinh(d*x+c)*ln(1+d*x+c+(1+
(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)*ln(1
-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx = \int \frac{b\,\operatorname{arsinh}\left(dx+c\right)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

## Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx = \frac{\int \frac{a}{c+dx}\,dx + \int \frac{b\,\operatorname{asinh}\left(c+dx\right)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*asinh(c + d*x)/(c + d*x), x))/e
```

## Maxima [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] b*integrate(log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x) + a*log(
d*e*x + c*e)/(d*e)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x), x)
```

# 3.121 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^2}\,dx$

Optimal result . . . 963
Rubi [A] (verified) . . . 963
Mathematica [A] (verified) . . . 965
Maple [A] (verified) . . . 965
Fricas [B] (verification not implemented) . . . 966
Sympy [F] . . . 966
Maxima [F(-2)] . . . 966
Giac [B] (verification not implemented) . . . 967
Mupad [F(-1)] . . . 967

## Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^2}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{de^2(c+dx)} - \frac{b\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}$$

```
[Out] (-a-b*arcsinh(d*x+c))/d/e^2/(d*x+c)-b*arctanh((1+(d*x+c)^2)^(1/2))/d/e^2
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5776, 272, 65, 213}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^2}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{de^2(c+dx)} - \frac{b\operatorname{arctanh}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])/(d*e^2*(c + d*x))) - (b*ArcTanh[Sqrt[1 + (c + d*
x)^2]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{2de^2} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{de^2}
\end{aligned}$$

$$= -\frac{a + b\text{arcsinh}(c + dx)}{de^2(c + dx)} - \frac{b\text{arctanh}\left(\sqrt{1 + (c + dx)^2}\right)}{de^2}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^2}\, dx = -\frac{\frac{a+b\text{arcsinh}(c+dx)}{c+dx} + b\text{arctanh}\left(\sqrt{1 + (c + dx)^2}\right)}{de^2}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c + d*x])/(c + d*x) + b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(
d*e^2))
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\text{arcsinh}(dx+c)}{dx+c}-\text{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2}}{d}$ | 54 |
| default | $\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\text{arcsinh}(dx+c)}{dx+c}-\text{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2}}{d}$ | 54 |
| parts | $-\frac{a}{e^2(dx+c)d}+\frac{b\left(-\frac{\text{arcsinh}(dx+c)}{dx+c}-\text{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2 d}$ | 56 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2
)^(1/2))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2}\, dx$$
$$= \frac{bdx \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - ac - (bcdx + bc^2)\log\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1} + 1\right.}{cd^2}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - a*c - (b*c*d*x +
b*c^2)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + (b*d*x + b*c
)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c*d*x + b*c^2)*log
(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1))/(c*d^2*e^2*x + c^2*d*e^
2)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2}\, dx = \frac{\int \frac{a}{c^2 + 2cdx + d^2x^2}\, dx + \int \frac{b\,\operatorname{asinh}(c + dx)}{c^2 + 2cdx + d^2x^2}\, dx}{e^2}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*asinh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2}\, dx$$
$$= -b\left(\frac{\log\left(dx + c + \sqrt{(dx + c)^2 + 1}\right)}{(dex + ce)de} + \frac{d\log\left(\sqrt{\frac{e^2}{(dex+ce)^2} + 1} + \frac{\sqrt{d^2e^4}}{(dex+ce)de}\right)}{e^2|d|^2\operatorname{sgn}\left(\frac{1}{dex+ce}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}\right)$$
$$- \frac{a}{(dex + ce)de}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -b*(log(d*x + c + sqrt((d*x + c)^2 + 1))/((d*e*x + c*e)*d*e) + d*log(sqrt(e
^2/(d*e*x + c*e)^2 + 1) + sqrt(d^2*e^4)/((d*e*x + c*e)*d*e))/(e^2*abs(d)^2*
sgn(1/(d*e*x + c*e))*sgn(d)*sgn(e))) - a/((d*e*x + c*e)*d*e)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2}\, dx = \int \frac{a + b\,\operatorname{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^2}\, dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2, x)
```

# 3.122 $\int \frac{a+b\mathbf{arcsinh}(c+dx)}{(ce+dex)^3}\,dx$

Optimal result . . . 968
Rubi [A] (verified) . . . 968
Mathematica [A] (verified) . . . 969
Maple [A] (verified) . . . 970
Fricas [B] (verification not implemented) . . . 970
Sympy [F] . . . 971
Maxima [B] (verification not implemented) . . . 971
Giac [F] . . . 971
Mupad [F(-1)] . . . 972

## Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)} - \frac{a+b\operatorname{arcsinh}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] 1/2*(-a-b*arcsinh(d*x+c))/d/e^3/(d*x+c)^2-1/2*b*(1+(d*x+c)^2)^(1/2)/d/e^3/(
d*x+c)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5859, 12, 5776, 270}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{2de^3(c+dx)^2} - \frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(b*Sqrt[1 + (c + d*x)^2])/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])
/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{e^3x^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^3}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^3} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)} - \frac{a+b\text{arcsinh}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = \frac{-\frac{b\sqrt{1+(c+dx)^2}}{2(c+dx)} + \frac{-a-b\text{arcsinh}(c+dx)}{2(c+dx)^2}}{de^3}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] (-1/2*(b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (-a - b*ArcSinh[c + d*x])/(2*(c
 + d*x)^2))/(d*e^3)
```

## Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$ | 60 |
| default | $\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$ | 60 |
| parts | $-\frac{a}{2e^3(dx+c)^2d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3d}$ | 62 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*
(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.00

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^3}\,dx$$
$$=\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-(bc^2dx+bc^3)\sqrt{d^2x^2+2\,cdx+c^2+1}}{2\left(c^2d^3e^3x^2+2\,c^3d^2e^3x+c^4de^3\right)}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*
e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

## Sympy [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = \frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time $= 0.21$ (sec) , antiderivative size $= 117$, normalized size of antiderivative $= 1.98$

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{1}{2}b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d}{d^3e^3x+cd^2e^3} + \frac{\text{arsinh}\,(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right) - \frac{a}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsi
nh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^3}\,dx = \int \frac{a + b\,\operatorname{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3,x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3, x)
```

# 3.123 $\int \frac{a+b\mathbf{arcsinh}(c+dx)}{(ce+dex)^4}\,dx$

Optimal result . . . 973
Rubi [A] (verified) . . . 973
Mathematica [A] (verified) . . . 975
Maple [A] (verified) . . . 976
Fricas [B] (verification not implemented) . . . 976
Sympy [F] . . . 977
Maxima [F] . . . 977
Giac [F] . . . 977
Mupad [F(-1)] . . . 978

## Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\operatorname{arcsinh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}$$

```
[Out] 1/3*(-a-b*arcsinh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctanh((1+(d*x+c)^2)^(1/2)
)/d/e^4-1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5776, 272, 44, 65, 213}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\operatorname{arctanh}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4} - \frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -1/6*(b*Sqrt[1 + (c + d*x)^2])/(d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x
])/(3*d*e^4*(c + d*x)^3) + (b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(6*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{e^4x^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^4}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{6de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{12de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{6de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3} + \frac{b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\begin{aligned}
&\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx \\
&= \frac{-\frac{2(a+b\text{arcsinh}(c+dx))}{(c+dx)^3} + b\left(-\frac{\sqrt{1+(c+dx)^2}}{(c+dx)^2} + \text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)\right)}{6de^4}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*(a + b*ArcSinh[c + d*x]))/(c + d*x)^3 + b*(-(Sqrt[1 + (c + d*x)^2]/(c
+ d*x)^2) + ArcTanh[Sqrt[1 + (c + d*x)^2]]))/(6*d*e^4)
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$ | 74 |
| default | $\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$ | 74 |
| parts | $-\frac{a}{3e^4(dx+c)^3d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4d}$ | 76 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.08

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx = -\frac{2\,ac^3-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\right.}{}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*
c^5*d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) - 2*
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)) + (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x +
 b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) + (b*c^3*d*x
+ b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^
4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

## Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\,\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**
**4), x) + Integral(b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

## Maxima [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx = \int \frac{b\,\operatorname{arsinh}(dx+c)+a}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(2*(d^2*x^2 + 2*c*d*x + c^2 + log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)))/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) -
 I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))/(d*e^4) - 6*i
ntegrate(1/3/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d
^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^
4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5
*c*d^4*e^4*x^4 + c^5*e^4 + c^3*e^4 + (10*c^2*d^3*e^4 + d^3*e^4)*x^3 + (10*c
^3*d^2*e^4 + 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 + 3*c^2*d*e^4)*x)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*
d^2*e^4*x + c^3*d*e^4)
```

## Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx = \int \frac{b\,\operatorname{arsinh}(dx+c)+a}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^4}\,dx = \int \frac{a + b\,\operatorname{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^4}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4, x)
```

# 3.124 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$

Optimal result . . . 979
Rubi [A] (verified) . . . 979
Mathematica [A] (verified) . . . 981
Maple [A] (verified) . . . 981
Fricas [B] (verification not implemented) . . . 981
Sympy [F] . . . 982
Maxima [B] (verification not implemented) . . . 982
Giac [F] . . . 983
Mupad [F(-1)] . . . 983

## Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3} + \frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)} - \frac{a+b\operatorname{arcsinh}(c+dx)}{4de^5(c+dx)^4}$$

```
[Out] 1/4*(-a-b*arcsinh(d*x+c))/d/e^5/(d*x+c)^4-1/12*b*(1+(d*x+c)^2)^(1/2)/d/e^5/
(d*x+c)^3+1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^5/(d*x+c)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 277, 270}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{4de^5(c+dx)^4} + \frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)} - \frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(b*Sqrt[1 + (c + d*x)^2])/(d*e^5*(c + d*x)^3) + (b*Sqrt[1 + (c + d*x)
^2])/(6*d*e^5*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(4*d*e^5*(c + d*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{e^5x^5}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^5}\,dx, x, c+dx\right)}{de^5} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4de^5} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3} - \frac{a+b\text{arcsinh}(c+dx)}{4de^5(c+dx)^4} - \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{6de^5} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3} + \frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)} - \frac{a+b\text{arcsinh}(c+dx)}{4de^5(c+dx)^4}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$$
$$= -\frac{b(c+dx)\left(1-2(c+dx)^2\right)\sqrt{1+(c+dx)^2}+3(a+b\operatorname{arcsinh}(c+dx))}{12de^5(c+dx)^4}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(b*(c + d*x)*(1 - 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] + 3*(a + b*Arc
Sinh[c + d*x]))/(d*e^5*(c + d*x)^4)
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$ | 80 |
| default | $\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$ | 80 |
| parts | $-\frac{a}{4e^5(dx+c)^4d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5d}$ | 82 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arcsinh(d*x+c)-1/12/(d*x+c)
^3*(1+(d*x+c)^2)^(1/2)+1/6/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.33

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$$
$$= \frac{3\,ad^4x^4+12\,acd^3x^3+18\,ac^2d^2x^2+12\,ac^3dx-3\,bc^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+(2\,bc^4d^3x^3+}{12\left(c^4d^5e^5x^4+4\,c^5d^4e^5x^3+6\,c^6d^3e^5x^2+4\,c^7d^2e^5\right.}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 - b*c^5 + (6*b*c^6 - b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*x^2
+ 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

## Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\, dx$$
$$= \frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\, dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\, dx}{e^5}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*asinh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(80) = 160$.

Time $= 0.21$ (sec) , antiderivative size $= 258$, normalized size of antiderivative $= 2.87$

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\, dx$$
$$= \frac{1}{12}\, b\left( \frac{(2\,d^4x^4 + 8\,cd^3x^3 + 2\,c^4 + (12\,c^2d^2 + d^2)x^2 + c^2 + 2\,(4\,c^3d + cd)x - 1)d}{(d^5e^5x^3 + 3\,cd^4e^5x^2 + 3\,c^2d^3e^5x + c^3d^2e^5)\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}} - \frac{3\ \text{arsinh}}{d^5e^5x^4 + 4\,cd^4e^5x^3 + 6\,c^2d} \right.$$
$$\left. - \frac{a}{4\,(d^5e^5x^4 + 4\,cd^4e^5x^3 + 6\,c^2d^3e^5x^2 + 4\,c^3d^2e^5x + c^4de^5)} \right.$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 + d^2)*x^2 + c^2 + 2
*(4*c^3*d + c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x
 + c^3*d^2*e^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*arcsinh(d*x + c)/(d^
5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e
^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2
*e^5*x + c^4*d*e^5)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^5}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^5}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5,x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5, x)
```

# 3.125 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^6}\,dx$

Optimal result . . . 984
Rubi [A] (verified) . . . 984
Mathematica [C] (verified) . . . 986
Maple [A] (verified) . . . 987
Fricas [B] (verification not implemented) . . . 987
Sympy [F] . . . 988
Maxima [F] . . . 988
Giac [F] . . . 989
Mupad [F(-1)] . . . 989

## Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\operatorname{arcsinh}(c+dx)}{5de^6(c+dx)^5} - \frac{3b\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{40de^6}$$

```
[Out] 1/5*(-a-b*arcsinh(d*x+c))/d/e^6/(d*x+c)^5-3/40*b*arctanh((1+(d*x+c)^2)^(1/2
))/d/e^6-1/20*b*(1+(d*x+c)^2)^(1/2)/d/e^6/(d*x+c)^4+3/40*b*(1+(d*x+c)^2)^(1
/2)/d/e^6/(d*x+c)^2
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5776, 272, 44, 65, 213}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = -\frac{a+b\operatorname{arcsinh}(c+dx)}{5de^6(c+dx)^5} - \frac{3b\operatorname{arctanh}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6} + \frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2} - \frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -1/20*(b*Sqrt[1 + (c + d*x)^2])/(d*e^6*(c + d*x)^4) + (3*b*Sqrt[1 + (c + d*
x)^2])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])/(5*d*e^6*(c + d*x)
^5) - (3*b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(40*d*e^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{e^6x^6}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^6}\,dx, x, c+dx\right)}{de^6} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} + \frac{b\text{Subst}\left(\int \frac{1}{x^5\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de^6} \\
&= -\frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} + \frac{b\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{10de^6} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} - \frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} - \frac{(3b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{40de^6} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} \\
&\quad - \frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{80de^6} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{40de^6} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} \\
&\quad - \frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5} - \frac{3b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{40de^6}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\begin{aligned}
&\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx \\
&= -\frac{\frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^5} + b\sqrt{1+(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, 3, \frac{3}{2}, 1+(c+dx)^2\right)}{5de^6}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -1/5*((a + b*ArcSinh[c + d*x])/(c + d*x)^5 + b*Sqrt[1 + (c + d*x)^2]*Hyperg
eometric2F1[1/2, 3, 3/2, 1 + (c + d*x)^2])/(d*e^6)
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6}}{d}$ | 94 |
| default | $\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6}}{d}$ | 94 |
| parts | $-\frac{a}{5e^6(dx+c)^5d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6d}$ | 96 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arcsinh(d*x+c)-1/20/(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)+3/40/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/40*arctanh(1/(1
+(d*x+c)^2)^(1/2))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(101) = 202$.

Time = 0.31 (sec) , antiderivative size = 509, normalized size of antiderivative = 4.43

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = -\frac{8\,ac^5-8\left(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+10\,bc^3d^2x^2+5\,bc^4dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3
*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 +
```

```
5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)
 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b
*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(b*
c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 + 5*b*c
^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) - (3
*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 - 2*b*c^6 + (9*b*c^7 - 2*b*c^5)*
d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^6*x^
4 + 10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d*e^6)
```

## Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^6}\, dx$$

$$= \frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\, dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\, dx}{e^6}$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*asinh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

## Maxima [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^6}\, dx = \int \frac{b\,\text{arsinh}\,(dx + c) + a}{(dex + ce)^6}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*(2*(3*d^4*x^4 + 12*c*d^3*x^3 + 3*c^4 + (18*c^2*d^2 - d^2)*x^2 - c^2
+ 2*(6*c^3*d - c*d)*x - 3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))
/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 +
 5*c^4*d^2*e^6*x + c^5*d*e^6) - 3*I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d
^2*x + c*d)/d + 1))/(d*e^6) + 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x
^7 + c^8*e^6 + c^6*e^6 + (28*c^2*d^6*e^6 + d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6
 + 3*c*d^5*e^6)*x^5 + 5*(14*c^4*d^4*e^6 + 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^
3*e^6 + 5*c^3*d^3*e^6)*x^3 + (28*c^6*d^2*e^6 + 15*c^4*d^2*e^6)*x^2 + 2*(4*c
^7*d*e^6 + 3*c^5*d*e^6)*x + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 + c^5*
e^6 + (21*c^2*d^5*e^6 + d^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 + c*d^4*e^6)*x^4 +
5*(7*c^4*d^3*e^6 + 2*c^2*d^3*e^6)*x^3 + (21*c^5*d^2*e^6 + 10*c^3*d^2*e^6)*x
^2 + (7*c^6*d*e^6 + 5*c^4*d*e^6)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))
 - 1/5*a/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e
^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^6)
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^6}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^6}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6,x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6, x)
```

# 3.126 $\int (ce+dex)^m (a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 990
Rubi [A] (verified) . . . 990
Mathematica [A] (verified) . . . 992
Maple [F] . . . 992
Fricas [F] . . . 993
Sympy [F] . . . 993
Maxima [F] . . . 993
Giac [F] . . . 994
Mupad [F(-1)] . . . 994

## Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}
&\int (ce+dex)^m (a+b\operatorname{arcsinh}(c+dx))^2\,dx \\
&= \frac{(e(c+dx))^{1+m}(a+b\operatorname{arcsinh}(c+dx))^2}{de(1+m)} \\
&\quad - \frac{2b(e(c+dx))^{2+m}(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-(c+dx)^2\right)}{de^2(1+m)(2+m)} \\
&\quad + \frac{2b^2(e(c+dx))^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};-(c+dx)^2\right)}{de^3(1+m)(2+m)(3+m)}
\end{aligned}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/d/e/(1+m)-2*b*(e*(d*x+c))^(2+m)*(a
+b*arcsinh(d*x+c))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+
m)/(2+m)+2*b^2*(e*(d*x+c))^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2
*m, 5/2+1/2*m],-(d*x+c)^2)/d/e^3/(3+m)/(m^2+3*m+2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

$= \{5859, 5776, 5817\}$

$$\int (ce+dex)^m (a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$= \frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1, \frac{m}{2}+\frac{3}{2}, \frac{m}{2}+\frac{3}{2}; \frac{m}{2}+2, \frac{m}{2}+\frac{5}{2}; -(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}$$
$$- \frac{2b(e(c+dx))^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, -(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2(m+1)(m+2)}$$
$$+ \frac{(e(c+dx))^{m+1}(a+b\operatorname{arcsinh}(c+dx))^2}{de(m+1)}$$

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, -(c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)) + (2*b^2*(e*(c + d*x))
^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 -(c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^m (a + b\text{arcsinh}(x))^2\, dx, x, c + dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}(a + b\text{arcsinh}(c+dx))^2}{de(1+m)} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{1+m}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\, dx, x, c+dx\right)}{de(1+m)}$$

$$= \frac{(e(c+dx))^{1+m}(a + b\text{arcsinh}(c+dx))^2}{de(1+m)}$$
$$- \frac{2b(e(c+dx))^{2+m}(a + b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -(c+dx)^2\right)}{de^2(1+m)(2+m)}$$
$$+ \frac{2b^2(e(c+dx))^{3+m}\,{}_3F_2\left(1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}; 2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}; -(c+dx)^2\right)}{de^3(1+m)(2+m)(3+m)}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int (ce + dex)^m (a + b\text{arcsinh}(c+dx))^2\, dx$$
$$= \frac{(c+dx)(e(c+dx))^m \left((a + b\text{arcsinh}(c+dx))^2 - \frac{2b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -(c+dx)^2\right)}{2+m}\right.}{d(1+m)}$$

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcSinh[c + d*x])^2 - (2*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*
x)^2])/(2 + m) + (2*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)^2])/((2 + m)*(3 + m))))/(d*(1 + m))
```

## Maple [F]

$$\int (dex + ce)^m (a + b\,\text{arcsinh}(dx + c))^2\, dx$$

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

## Fricas [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*(d*e*x + c
*e)^m, x)
```

## Sympy [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx = \int (e(c+dx))^m\,(a+b\,\text{asinh}\,(c+dx))^2\,dx$$

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**2, x)
```

## Maxima [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*d*e^m*x + b^2*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^2/(d*e*(m + 1)) + integ
rate(-2*((b^2*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b - (a*b*d^2*e^m*
(m + 1) - b^2*d^2*e^m)*x^2 - 2*(a*b*c*d*e^m*(m + 1) - b^2*c*d*e^m)*x)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((a*b*d^3*e^m*(m + 1) - b^2*d^3*
e^m)*x^3 + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a*b - (c^3*e^m + c*e^m)*b^2 +
3*(a*b*c*d^2*e^m*(m + 1) - b^2*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*(m + 1) + d*e
^m*(m + 1))*a*b - (3*c^2*d*e^m + d*e^m)*b^2)*x)*(d*x + c)^m)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 +
 c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x
^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)), x)
```

## Giac [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2, x)
```

# 3.127 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 995
Rubi [A] (verified) . . . 995
Mathematica [A] (verified) . . . 998
Maple [A] (verified) . . . 999
Fricas [B] (verification not implemented) . . . 999
Sympy [B] (verification not implemented) . . . 1000
Maxima [F] . . . 1001
Giac [F] . . . 1002
Mupad [F(-1)] . . . 1002

## Optimal result

Integrand size = 23, antiderivative size = 197

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$= \frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d}$$
$$+ \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d}$$
$$- \frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{25d}$$
$$+ \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^2}{5d}$$

```
[Out] 16/75*b^2*e^4*x-8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^4
*(d*x+c)^5*(a+b*arcsinh(d*x+c))^2/d-16/75*b*e^4*(a+b*arcsinh(d*x+c))*(1+(d*
x+c)^2)^(1/2)/d+8/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/
2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{5859, 12, 5776, 5812, 5798, 8, 30\}$

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$= \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^2}{5d}$$
$$- \frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{25d}$$
$$+ \frac{8be^4\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{75d}$$
$$- \frac{16be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{75d}$$
$$+ \frac{2b^2e^4(c+dx)^5}{125d} - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{16}{75}b^2e^4x$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (16*b^2*e^4*x)/75 - (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*
d) + (8*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(
75*d) - (2*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])
)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int e^4 x^4 (a + b\operatorname{arcsinh}(x))^2 \, dx, x, c + dx\right)}{d} \\
&= \frac{e^4 \operatorname{Subst}\left(\int x^4 (a + b\operatorname{arcsinh}(x))^2 \, dx, x, c + dx\right)}{d} \\
&= \frac{e^4 (c + dx)^5 (a + b\operatorname{arcsinh}(c + dx))^2}{5d} - \frac{(2be^4) \operatorname{Subst}\left(\int \frac{x^5 (a + b\operatorname{arcsinh}(x))}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{5d} \\
&= -\frac{2be^4 (c + dx)^4 \sqrt{1 + (c + dx)^2} (a + b\operatorname{arcsinh}(c + dx))}{25d} \\
&\quad + \frac{e^4 (c + dx)^5 (a + b\operatorname{arcsinh}(c + dx))^2}{5d} \\
&\quad + \frac{(8be^4) \operatorname{Subst}\left(\int \frac{x^3 (a + b\operatorname{arcsinh}(x))}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{25d} \\
&\quad + \frac{(2b^2 e^4) \operatorname{Subst}\left(\int x^4 \, dx, x, c + dx\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)^5}{125d} + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad - \frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^2}{5d} \\
&\quad - \frac{(16be^4)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{75d} \\
&\quad - \frac{(8b^2e^4)\operatorname{Subst}\left(\int x^2\,dx, x, c+dx\right)}{75d} \\
&= -\frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad - \frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^2}{5d} + \frac{(16b^2e^4)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{75d} \\
&= \frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} \\
&\quad - \frac{16be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad - \frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^2}{5d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$
\begin{aligned}
&\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx \\
&= \frac{e^4\Big(240b^2(c+dx) - 40b^2(c+dx)^3 + 9(25a^2+2b^2)(c+dx)^5 + 30ab\sqrt{1+(c+dx)^2}(-8+4(c+dx)^2-3(c
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]

[Out] (e^4*(240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[1 + (c + d*x)^2]*(-8 + 4*(c + d*x)^2 - 3*(c + d*x)^4) + 30
```

```
*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[1 + (c + d*x)^2] + 4*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2] - 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 2
25*b^2*(c + d*x)^5*ArcSinh[c + d*x]^2))/(1125*d)
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^2}{5}-\frac{16\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8\operatorname{arcsinh}$ |
| default | $\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^2}{5}-\frac{16\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8\operatorname{arcsinh}$ |
| parts | $\frac{e^4a^2(dx+c)^5}{5d}+\frac{e^4b^2\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^2}{5}-\frac{16\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8\operatorname{arcs}}{d}$ |

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*e^4*a^2*(d*x+c)^5+e^4*b^2*(1/5*(d*x+c)^5*arcsinh(d*x+c)^2-16/75*ar
csinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d*x+c)*(1+(d*x+c)^2
)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c
+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+2*e^4*a*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1
/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(
1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(177) = 354$.

Time = 0.30 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.14

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$

$$=\frac{9\,(25\,a^2+2\,b^2)d^5e^4x^5+45\,(25\,a^2+2\,b^2)cd^4e^4x^4+10\,(9\,(25\,a^2+2\,b^2)c^2-4\,b^2)d^3e^4x^3+30\,(3\,(25\,a^2+2\,b^2$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
+ 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2)*
c^3 - 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^
2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^3*e^
4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 30*(15*a*b*d^5*e^4*x^5 + 75*a*b
```

```
*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 + 75*a*b
*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e^4*x^3 +
 2*(9*b^2*c^2 - 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*e^4*x + (3*b
^2*c^4 - 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 30*(3*a*b*d^4*e^4*x^4 + 12*a*b*
c*d^3*e^4*x^3 + 2*(9*a*b*c^2 - 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^3 - 2*a*b*c)
*d*e^4*x + (3*a*b*c^4 - 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(184) = 368$.

Time $= 0.63$ (sec) , antiderivative size $= 1268$, normalized size of antiderivative $= 6.44$

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
**4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
sinh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*asinh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*asinh(c
 + d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*asinh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/25 + 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*asinh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/75 + 2*a*b*d**4*e**4*x**5*asinh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/(75*d) + b**2*c**5*e**4*asinh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*asinh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*asinh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*asinh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*c**2
*e**4*x/75 + 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
 + d*x)/(75*d) + b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)/25 - 8*b**2*c*d*e**4*x**2/75 + 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + b**2*d**4*e**4*x**5*asinh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*d**2*e**4*x**3/225 + 8*b**
```

```
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**2, True))
```

## Maxima [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*d^4*e^4*x^5 + a^2*c*d^3*e^4*x^4 + 2*a^2*c^2*d^2*e^4*x^3 + 2*a^2*c^3
*d*e^4*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^3*d*e^4 + 2/3*(6
*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15
*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*
c^2*d^2*e^4 + 1/12*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 +
9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d
^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*c*d^3*e^4
 + 1/300*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^
3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^
2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
+ 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/
sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d
^6)*d)*a*b*d^4*e^4 + a^2*c^4*e^4*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((
d*x + c)^2 + 1))*a*b*c^4*e^4/d + 1/5*(b^2*d^4*e^4*x^5 + 5*b^2*c*d^3*e^4*x^4
 + 10*b^2*c^2*d^2*e^4*x^3 + 10*b^2*c^3*d*e^4*x^2 + 5*b^2*c^4*e^4*x)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2/5*(b^2*d^7*e^4*x^
7 + 7*b^2*c*d^6*e^4*x^6 + (21*c^2*d^5*e^4 + d^5*e^4)*b^2*x^5 + 5*(7*c^3*d^4
```

```
*e^4 + c*d^4*e^4)*b^2*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*b^2*x^3 + 10*
(2*c^5*d^2*e^4 + c^3*d^2*e^4)*b^2*x^2 + 5*(c^6*d*e^4 + c^4*d*e^4)*b^2*x + (
b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 20*b^2*c^3
*d^3*e^4*x^3 + 15*b^2*c^4*d^2*e^4*x^2 + 5*b^2*c^5*d*e^4*x)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2, x)
```

# 3.128 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1003
Rubi [A] (verified) . . . 1003
Mathematica [A] (verified) . . . 1006
Maple [A] (verified) . . . 1006
Fricas [B] (verification not implemented) . . . 1007
Sympy [B] (verification not implemented) . . . 1007
Maxima [F] . . . 1008
Giac [F] . . . 1009
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## Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=-\frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}+\frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{16d}\\&\quad-\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{8d}\\&\quad-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^2}{32d}+\frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{4d}\end{aligned}$$

```
[Out] -3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arcsinh(d*
x+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d+3/16*b*e^3*(d*x+c)*(a+
b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*
x+c))*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$\begin{aligned}&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{8d}\\&\quad+\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)(a+b\operatorname{arcsinh}(c+dx))}{16d}\\&\quad-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}-\frac{3b^2e^3(c+dx)^2}{32d}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) + (3*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(16*d) - (b*e^3*(c
 + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(8*d) - (3*e^3*(a
 + b*ArcSinh[c + d*x])^2)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x]
)^2)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int e^3 x^3 (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3 \operatorname{Subst}\left(\int x^3 (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3 (c+dx)^4 (a+b\operatorname{arcsinh}(c+dx))^2}{4d} - \frac{(be^3)\operatorname{Subst}\left(\int \frac{x^4 (a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{be^3 (c+dx)^3 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{8d} + \frac{e^3 (c+dx)^4 (a+b\operatorname{arcsinh}(c+dx))^2}{4d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{x^2 (a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} + \frac{(b^2e^3)\operatorname{Subst}\left(\int x^3\,dx, x, c+dx\right)}{8d} \\
&= \frac{b^2 e^3 (c+dx)^4}{32d} + \frac{3be^3 (c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{16d} \\
&\quad - \frac{be^3 (c+dx)^3 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{8d} \\
&\quad + \frac{e^3 (c+dx)^4 (a+b\operatorname{arcsinh}(c+dx))^2}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{16d} - \frac{(3b^2e^3)\operatorname{Subst}(\int x\,dx, x, c+dx)}{16d} \\
&= -\frac{3b^2 e^3 (c+dx)^2}{32d} + \frac{b^2 e^3 (c+dx)^4}{32d} \\
&\quad + \frac{3be^3 (c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{16d} \\
&\quad - \frac{be^3 (c+dx)^3 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{8d} \\
&\quad - \frac{3e^3 (a+b\operatorname{arcsinh}(c+dx))^2}{32d} + \frac{e^3 (c+dx)^4 (a+b\operatorname{arcsinh}(c+dx))^2}{4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$

$$= \frac{e^3\Big(-3b^2(c+dx)^2+(8a^2+b^2)(c+dx)^4+2ab(c+dx)(3-2(c+dx)^2)\sqrt{1+(c+dx)^2}-6ab\operatorname{arcsinh}(c+$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^3*(-3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*(c + d*x)*(3 -
 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] - 6*a*b*ArcSinh[c + d*x] + 2*b*(c + d
*x)*(8*a*(c + d*x)^3 + 3*b*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*x]
^2))/(32*d)
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3\operatorname{arcsi}}{}\right.$ |
| default | $\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3\operatorname{arcsi}}{}\right.$ |
| parts | $\frac{e^3a^2(dx+c)^4}{4d}+\frac{e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3\,a}{}\right.}{d}$ |

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*x
+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*(1+(d*x+c)^2)^(1/2)*(d*x+c)*a
rcsinh(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+2*e
^3*a*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/3
2*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(156) = 312$.

Time $= 0.26$ (sec) , antiderivative size $= 486$, normalized size of antiderivative $= 2.83$

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^2 \, dx$$

$$= \frac{(8\,a^2 + b^2)d^4e^3x^4 + 4\,(8\,a^2 + b^2)cd^3e^3x^3 + 3\,(2\,(8\,a^2 + b^2)c^2 - b^2)d^2e^3x^2 + 2\,(2\,(8\,a^2 + b^2)c^3 - 3\,b^2c)de^3x -$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*a
^2 + b^2)*c^2 - b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*e^3*
x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 + 32
*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48*a*b*
c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b^2*d^3
*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 - b^2)*d*e^3*x + (2*b^2*c^3 -
 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x^2 + 3*(2
*a*b*c^2 - a*b)*d*e^3*x + (2*a*b*c^3 - 3*a*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(155) = 310$.

Time $= 0.47$ (sec) , antiderivative size $= 916$, normalized size of antiderivative $= 5.33$

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^2 \, dx$$

$$= \begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{asinh}(c+dx)}{2d} + 2abc^3e^3x\operatorname{asinh}(c+dx) - \frac{abc^3e^3\sqrt{c^2+2cdx+} \\ c^3e^3x(a + b\operatorname{asinh}(c))^2 \end{cases}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*asinh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*asinh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 2*a*b*c*d**2*e**3*x**3*asinh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + a*b*d**3*e**3*x**4*asinh(c +
```

```
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 - 3*a*b*e**3*asinh(c + d*x)/
(16*d) + b**2*c**4*e**3*asinh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*asinh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)*asinh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/8 - 3*b**2*c*e**3*x/16 + 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*asinh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*d*e**3*x**2/32 + 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/16 - 3*b**2*e**3*a
sinh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**2, True)
)
```

## Maxima [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^3(b\,\operatorname{arsinh}(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arcsinh(d*x + c
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/48*(24
*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2*c^3*e^3*x + 2*((d
*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/4*(b^2*
d^3*e^3*x^4 + 4*b^2*c*d^2*e^3*x^3 + 6*b^2*c^2*d*e^3*x^2 + 4*b^2*c^3*e^3*x)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(1/2*(b^2*d^6
```

```
*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 + d^4*e^3)*b^2*x^4 + 4*(5*
c^3*d^3*e^3 + c*d^3*e^3)*b^2*x^3 + 2*(7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^2*x^
2 + 4*(c^5*d*e^3 + c^3*d*e^3)*b^2*x + (b^2*d^5*e^3*x^5 + 5*b^2*c*d^4*e^3*x^
4 + 10*b^2*c^2*d^3*e^3*x^3 + 10*b^2*c^3*d^2*e^3*x^2 + 4*b^2*c^4*d*e^3*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x
+ c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2, x)
```

# 3.129 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1010
Rubi [A] (verified) . . . 1010
Mathematica [A] (verified) . . . 1013
Maple [A] (verified) . . . 1013
Fricas [B] (verification not implemented) . . . 1014
Sympy [B] (verification not implemented) . . . 1014
Maxima [F] . . . 1015
Giac [F] . . . 1015
Mupad [F(-1)] . . . 1016

## Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=-\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d}\\&\quad-\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d}\\&\quad+\frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{3d}\end{aligned}$$

```
[Out] -4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+
c))^2/d+4/9*b*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-2/9*b*e^2*(d*x
+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5812, 5798, 8, 30}

$$\begin{aligned}&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{3d}\\&\quad-\frac{2be^2\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{9d}\\&\quad+\frac{4be^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{9d}+\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4}{9}b^2e^2x\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) + (4*b*e^2*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) - (2*b*e^2*(c + d*x)^2*Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh
[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int e^2 x^2 (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2 \operatorname{Subst}\left(\int x^2 (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2 (c+dx)^3 (a+b\operatorname{arcsinh}(c+dx))^2}{3d} - \frac{(2be^2)\operatorname{Subst}\left(\int \frac{x^3 (a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3d} \\
&= -\frac{2be^2 (c+dx)^2 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{e^2 (c+dx)^3 (a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{(4be^2)\operatorname{Subst}\left(\int \frac{x (a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&\quad + \frac{(2b^2e^2)\operatorname{Subst}\left(\int x^2\,dx, x, c+dx\right)}{9d} \\
&= \frac{2b^2e^2(c+dx)^3}{27d} + \frac{4be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad - \frac{2be^2 (c+dx)^2 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{e^2 (c+dx)^3 (a+b\operatorname{arcsinh}(c+dx))^2}{3d} - \frac{(4b^2e^2)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{9d} \\
&= -\frac{4}{9}b^2e^2x + \frac{2b^2e^2(c+dx)^3}{27d} + \frac{4be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad - \frac{2be^2 (c+dx)^2 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{e^2 (c+dx)^3 (a+b\operatorname{arcsinh}(c+dx))^2}{3d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$=\frac{e^2\Big(-12b^2(c+dx)+\left(9a^2+2b^2\right)(c+dx)^3+6ab(2-(c+dx)^2)\sqrt{1+(c+dx)^2}+6b\Big(3a(c+dx)^3+2b\sqrt{}}{27d}$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*(-12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*(2 - (c + d*x
)^2)*Sqrt[1 + (c + d*x)^2] + 6*b*(3*a*(c + d*x)^3 + 2*b*Sqrt[1 + (c + d*x)^
2] - b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 9*b^2*(c + d*x
)^3*ArcSinh[c + d*x]^2))/(27*d)
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^2e^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}(dx+c)^2}{9}-\frac{4dx}{9}-\frac{4c}{9}\right.}{d}$ |
| default | $\frac{\frac{a^2e^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}(dx+c)^2}{9}-\frac{4dx}{9}-\frac{4c}{9}\right.}{d}$ |
| parts | $\frac{a^2e^2(dx+c)^3}{3d}+\frac{e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}(dx+c)^2}{9}-\frac{4dx}{9}-\right.}{d}$ |

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*e^2*(d*x+c)^3+e^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arcs
inh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+
c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+2*e^2*a*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-
1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(122) = 244$.

Time $= 0.27$ (sec) , antiderivative size $= 358$, normalized size of antiderivative $= 2.63$

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^2 \, dx$$

$$= \frac{(9\,a^2 + 2\,b^2) d^3 e^2 x^3 + 3\,(9\,a^2 + 2\,b^2) c d^2 e^2 x^2 + 3\,((9\,a^2 + 2\,b^2) c^2 - 4\,b^2) d e^2 x + 9\,(b^2 d^3 e^2 x^3 + 3\,b^2 c d^2 e^2 x^2 + 3$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((9
*a^2 + 2*b^2)*c^2 - 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x
^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*d*e^
2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 - 2*b^2
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 - 2*a*b)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(126) = 252$.

Time $= 0.32$ (sec) , antiderivative size $= 610$, normalized size of antiderivative $= 4.49$

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^2 \, dx$$

$$= \begin{cases} a^2 c^2 e^2 x + a^2 c d e^2 x^2 + \frac{a^2 d^2 e^2 x^3}{3} + \frac{2abc^3 e^2 \operatorname{asinh}(c+dx)}{3d} + 2abc^2 e^2 x \operatorname{asinh}(c + dx) - \frac{2abc^2 e^2 \sqrt{c^2+2cdx+d^2x^2+1}}{9d} + 2ab \\ c^2 e^2 x (a + b \operatorname{asinh}(c))^2 \end{cases}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*asinh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*asinh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a*b*c*d*e**2*
x**2*asinh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9
 + 2*a*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/9 + 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
(9*d) + b**2*c**3*e**2*asinh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*asinh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*asinh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
```

```
1)*asinh(c + d*x)/9 + b**2*d**2*e**2*x**3*asinh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/9 - 4*b**2*e**2*x/9 + 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**2,
True))
```

## Maxima [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^2(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*a*b*c*d*e^2 + 1/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1
)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*(c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 2*((d*x + c)*arcsinh(d*x +
 c) - sqrt((d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*e^2*x^3 + 3*b^2*c
*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^2 - integrate(2/3*(b^2*d^5*e^2*x^5 + 5*b^2*c*d^4*e^2*x^4 + (10*c^2*d^3*
e^2 + d^3*e^2)*b^2*x^3 + 3*(3*c^3*d^2*e^2 + c*d^2*e^2)*b^2*x^2 + 3*(c^4*d*e
^2 + c^2*d*e^2)*b^2*x + (b^2*d^4*e^2*x^4 + 4*b^2*c*d^3*e^2*x^3 + 6*b^2*c^2*
d^2*e^2*x^2 + 3*b^2*c^3*d*e^2*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3
*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^2(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2, x)
```

# 3.130 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1017
Rubi [A] (verified) . . . 1017
Mathematica [A] (verified) . . . 1019
Maple [A] (verified) . . . 1019
Fricas [B] (verification not implemented) . . . 1020
Sympy [B] (verification not implemented) . . . 1020
Maxima [F] . . . 1021
Giac [F] . . . 1021
Mupad [F(-1)] . . . 1021

## Optimal result

Integrand size = 21, antiderivative size = 103

$$\begin{aligned}&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{b^2e(c+dx)^2}{4d}-\frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{2d}\\&\quad+\frac{e(a+b\operatorname{arcsinh}(c+dx))^2}{4d}+\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{2d}\end{aligned}$$

```
[Out] 1/4*b^2*e*(d*x+c)^2/d+1/4*e*(a+b*arcsinh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*a
rcsinh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)
/d
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$\begin{aligned}&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}(c+dx)(a+b\operatorname{arcsinh}(c+dx))}{2d}\\&\quad+\frac{e(a+b\operatorname{arcsinh}(c+dx))^2}{4d}+\frac{b^2e(c+dx)^2}{4d}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x]))/(2*d) + (e*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int ex(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{e\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{2d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{2d} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} + \frac{(b^2e)\,\text{Subst}(\int x\,dx, x, c+dx)}{2d} \\
&= \frac{b^2e(c+dx)^2}{4d} - \frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{2d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^2}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

$$
\begin{aligned}
&\int (ce+dex)(a+b\text{arcsinh}(c+dx))^2\,dx \\
&= \frac{e\Big((2a^2+b^2)(c+dx)^2 - 2ab(c+dx)\sqrt{1+(c+dx)^2} + 2ab\text{arcsinh}(c+dx) + 2b(c+dx)\left(2a(c+dx) - b\right.}{4d}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*((2*a^2 + b^2)*(c + d*x)^2 - 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + 2*a
*b*ArcSinh[c + d*x] + 2*b*(c + d*x)*(2*a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2
])*ArcSinh[c + d*x] + b^2*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^2))/(4*d)
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{\text{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\,\text{arcsinh}(dx+c)}{2}-\frac{\text{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\frac{1}{4}\right)+2eab\left($ <br> $d$ |
| default | $\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{\text{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\,\text{arcsinh}(dx+c)}{2}-\frac{\text{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\frac{1}{4}\right)+2eab\left($ <br> $d$ |
| parts | $e\,a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^2\left(\frac{\text{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\,\text{arcsinh}(dx+c)}{2}-\frac{\text{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}\right.}{d}$ |

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+(
d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1
/4)+2*e*a*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1
/4*arcsinh(d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(93) = 186$.

Time $= 0.26$ (sec) , antiderivative size $= 230$, normalized size of antiderivative $= 2.23$

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx$$

$$= \frac{(2\,a^2 + b^2)d^2ex^2 + 2\,(2\,a^2 + b^2)cdex + (2\,b^2d^2ex^2 + 4\,b^2cdex + (2\,b^2c^2 + b^2)e)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx}\right.}{}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2 +
 4*b^2*c*d*e*x + (2*b^2*c^2 + b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 + a*b)*e -
(b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(88) = 176$.

Time $= 0.19$ (sec) , antiderivative size $= 335$, normalized size of antiderivative $= 3.25$

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx$$

$$= \begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\operatorname{asinh}(c+dx)}{d} + 2abcex\,\operatorname{asinh}(c + dx) - \frac{abce\sqrt{c^2+2cdx+d^2x^2+1}}{2d} + abdex^2\,\operatorname{asinh}(c + dx) - \\ cex(a + b\,\operatorname{asinh}(c))^2 \end{cases}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*asinh(c + d*x)/d + 2*a
*b*c*e*x*asinh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d
) + a*b*d*e*x**2*asinh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/2 + a*b*e*asinh(c + d*x)/(2*d) + b**2*c**2*e*asinh(c + d*x)**2/(2*d) +
b**2*c*e*x*asinh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
```

```
+ d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**2*d*e*x**2*asinh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/2 + b**2*e*asinh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))
**2, True))
```

## Maxima [F]

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \int (dex + ce)(b\,\mathrm{arsinh}\,(dx + c) + a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*d*e + a^2*c
*e*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c*e/d + 1
/2*(b^2*d*e*x^2 + 2*b^2*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2 - integrate((b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + (5*c^2*d^2*e + d^2*
e)*b^2*x^2 + 2*(c^3*d*e + c*d*e)*b^2*x + (b^2*d^3*e*x^3 + 3*b^2*c*d^2*e*x^2
 + 2*b^2*c^2*d*e*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x
 + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \int (dex + ce)(b\,\mathrm{arsinh}\,(dx + c) + a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \int (c\,e + d\,e\,x)\,(a + b\,\mathrm{asinh}(c + d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2, x)
```

# 3.131 $\int (a + b\operatorname{arcsinh}(c + dx))^2\,dx$

Optimal result . . . 1022
Rubi [A] (verified) . . . 1022
Mathematica [A] (verified) . . . 1023
Maple [A] (verified) . . . 1024
Fricas [B] (verification not implemented) . . . 1024
Sympy [B] (verification not implemented) . . . 1025
Maxima [F] . . . 1025
Giac [F] . . . 1025
Mupad [F(-1)] . . . 1026

## Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (a + b\operatorname{arcsinh}(c + dx))^2\,dx = 2b^2x - \frac{2b\sqrt{1 + (c + dx)^2}(a + b\operatorname{arcsinh}(c + dx))}{d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^2}{d}$$

```
[Out] 2*b^2*x+(d*x+c)*(a+b*arcsinh(d*x+c))^2/d-2*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c
)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5772, 5798, 8}

$$\int (a + b\operatorname{arcsinh}(c + dx))^2\,dx = -\frac{2b\sqrt{(c + dx)^2 + 1}(a + b\operatorname{arcsinh}(c + dx))}{d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^2}{d} + 2b^2x$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + ((c + d*
x)*(a + b*ArcSinh[c + d*x])^2)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d} - \frac{(2b)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d} + \frac{(2b^2)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{d} \\
&= 2b^2x - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int (a+b\operatorname{arcsinh}(c+dx))^2\,dx \\
&= \frac{\left(a^2+2b^2\right)(c+dx) - 2ab\sqrt{1+(c+dx)^2} + 2b\left(ac+adx-b\sqrt{1+(c+dx)^2}\right)\operatorname{arcsinh}(c+dx) + b^2(c+d
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2] + 2*b*(a*c + a*d*x -
 b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(c + d*x)*ArcSinh[c + d*x]
^2)/d
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| parts | $a^2x+\frac{b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)}{d}+\frac{2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(d$ |
| derivativedivides | $\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(d$ |
| default | $\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(d$ |

```
[In] int((a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2/d*((d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+
2*d*x+2*c)+2*a*b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.47

$$\int (a+b\operatorname{arcsinh}(c+dx))^2\,dx$$

$$=\frac{(a^2+2\,b^2)dx+(b^2dx+b^2c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2-2\sqrt{d^2x^2+2\,cdx+c^2+1}ab+2\left(ab$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(51) = 102$.

Time $= 0.12$ (sec) , antiderivative size $= 143$, normalized size of antiderivative $= 2.51$

$$\int (a + b\operatorname{arcsinh}(c + dx))^2 \, dx$$

$$= \begin{cases} a^2 x + \frac{2abc \operatorname{asinh}(c+dx)}{d} + 2abx \operatorname{asinh}(c + dx) - \frac{2ab\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{b^2 c \operatorname{asinh}^2(c+dx)}{d} + b^2 x \operatorname{asinh}^2(c + dx) + \\ x(a + b \operatorname{asinh}(c))^2 \end{cases}$$

```
[In] integrate((a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*asinh(c + d*x)/d + 2*a*b*x*asinh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + b**2*c*asinh(c + d*x)**2/d + b*
*2*x*asinh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**2, True))
```

## Maxima [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^2 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2*(d^3*x^
3 + 2*c*d^2*x^2 + (c^2*d + d)*x + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d^2*x^
2 + c*d*x))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c
*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c)
, x))*b^2 + a^2*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*
a*b/d
```

## Giac [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^2 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(c + dx))^2\,dx = \int (a + b\,\operatorname{asinh}(c + d\,x))^2\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2,x)
[Out] int((a + b*asinh(c + d*x))^2, x)
```

# 3.132 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx$

Optimal result . . . . . 1027
Rubi [A] (verified) . . . . . 1027
Mathematica [A] (verified) . . . . . 1030
Maple [B] (verified) . . . . . 1030
Fricas [F] . . . . . 1031
Sympy [F] . . . . . 1031
Maxima [F] . . . . . 1031
Giac [F] . . . . . 1032
Mupad [F(-1)] . . . . . 1032

## Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx = \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))^2 \log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{b(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{b^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}$$

```
[Out] 1/3*(a+b*arcsinh(d*x+c))^3/b/d/e+(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-b*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)
^2)^(1/2))^2)/d/e-1/2*b^2*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5775, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx = -\frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de} + \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3bde} + \frac{\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de} - \frac{b^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcSinh[c + d*x])^2*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (b*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b^2*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(
2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
```

```
b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x}\,dx, x, c+dx\right)}{de} \\
&= -\frac{\text{Subst}\left(\int x^2 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^3}{3bde} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\text{arcsinh}(c+dx))^2 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{2\text{Subst}\left(\int x \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\text{arcsinh}(c+dx))^2 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad + \frac{b\text{Subst}\left(\int \text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))^2 \log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b^2 \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de} \\
&= \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))^2 \log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{b^2 \operatorname{PolyLog}\left(3, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$
\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx \\
&= \frac{-2(a+b\operatorname{arcsinh}(c+dx))^2\left(a+b\operatorname{arcsinh}(c+dx)-3b\log\left(1-e^{2\operatorname{arcsinh}(c+dx)}\right)\right)+6b^2(a+b\operatorname{arcsinh}(c+dx))}{6bde}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2*(a + b*ArcSinh[c + d*x] - 3*b*Log[1 - E^(2*A
rcSinh[c + d*x])]) + 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSinh
[c + d*x])] - 3*b^3*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(6*b*d*e)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(136) = 272.

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.69

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)$ |
| default | $\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)$ |
| parts | $\frac{a^2\ln(dx+c)}{ed}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c\right.\right.$ |

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)+b^2/e*(-1/3*arcsinh(d*x+c)^3+arcsinh(d*x+c)^2*ln(1+d*x
+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/
2))-2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^2*ln(1-d*x-c-(1+
(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2*p
olylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+2*a*b/e*(-1/2*arcsinh(d*x+c)^2+arcsinh
(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2
))+arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1+(d*x+c
)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d*e*x + c
*e), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx = \frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*asinh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*asinh(c + d*x)/(c + d*x), x))/e
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(b^2*log(d*x + c + sqrt((d*x + c)^2 +
 1))^2/(d*e*x + c*e) + 2*a*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x +
c*e), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x), x)
```

# 3.133 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx$

Optimal result . . . 1033
Rubi [A] (verified) . . . 1033
Mathematica [A] (verified) . . . 1035
Maple [A] (verified) . . . 1036
Fricas [F] . . . 1036
Sympy [F] . . . 1037
Maxima [F(-2)] . . . 1037
Giac [F] . . . 1037
Mupad [F(-1)] . . . 1037

## Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{4b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} - \frac{2b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} + \frac{2b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$

```
[Out] -(a+b*arcsinh(d*x+c))^2/d/e^2/(d*x+c)-4*b*(a+b*arcsinh(d*x+c))*arctanh(d*x+
c+(1+(d*x+c)^2)^(1/2))/d/e^2-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/
e^2+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5816, 4267, 2317, 2438}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{4b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{2b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} + \frac{2b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^2/(d*e^2*(c + d*x))) - (4*b*(a + b*ArcSinh[c + d
*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (2*b^2*PolyLog[2, -E^ArcSinh[c
+ d*x]])/(d*e^2) + (2*b^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(c+dx))}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{4b(a+b\text{arcsinh}(c+dx))\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{4b(a+b\text{arcsinh}(c+dx))\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{arcsinh}(c+dx)}\right)}{de^2} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{4b(a+b\text{arcsinh}(c+dx))\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{2b^2\,\text{PolyLog}\left(2, -e^{\text{arcsinh}(c+dx)}\right)}{de^2} + \frac{2b^2\,\text{PolyLog}\left(2, e^{\text{arcsinh}(c+dx)}\right)}{de^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.64

$$\begin{aligned}
&\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx \\
&= \frac{-\frac{a^2}{c+dx} - 2ab\left(\frac{\text{arcsinh}(c+dx)}{c+dx} + \log\left(\frac{1}{2}(c+dx)\text{csch}\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right) - \log\left(\sinh\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)\right) +}{}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) - 2*a*b*(ArcSinh[c + d*x]/(c + d*x) + Log[((c + d*x)*Csch
[ArcSinh[c + d*x]/2])/2] - Log[Sinh[ArcSinh[c + d*x]/2]]) + b^2*(ArcSinh[c
+ d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] -
2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] -
2*PolyLog[2, E^(-ArcSinh[c + d*x])]))/(d*e^2)
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx\right.}{e^2}}{d}$ |
| default | $\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx\right.}{e^2}}{d}$ |
| parts | $-\frac{a^2}{e^2(dx+c)d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)\right.}{e^2d}$ |

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*
arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*polylog(2,d*x+c+(1+(d*x+c)
^2)^(1/2)))+2*a*b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1
/2))))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^2*e^2*x
^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\,\text{asinh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\,\text{asinh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*asinh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*asinh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

# 3.134 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx$

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Mathematica [A] (verified) . . . 1040
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Giac [F] . . . 1042
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## Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -1/2*(a+b*arcsinh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-b*(a+b*arcs
inh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 12, 5776, 5800, 29}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(d*e^3*(c + d*x))) - (
a + b*ArcSinh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c + d*x])/(d*e^3
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{e^3x^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^3}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{de^3(c+dx)} \\
&\quad -\frac{(a+b\text{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{x}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{de^3(c+dx)} - \frac{(a+b\text{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{a\left(a+2b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)+2b\left(a+b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)\operatorname{arcsinh}(c+dx)+}{2de^3(c+dx)^2}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a*(a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) + 2*b*(a + b*
(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])*ArcSinh[c + d*x] + b^2*ArcSinh
[c + d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x])/(d*e^3*(c + d*x)^2)
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c+\sqrt{1+(dx+c)^2}\right.\right.\right.}{e^3}}{d}$ |
| default | $\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c+\sqrt{1+(dx+c)^2}\right.\right.\right.}{e^3}}{d}$ |
| parts | $-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c+\right.\right.\right.}{e^3d}$ |

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*x+c)*(
-2*(d*x+c)^2+2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2+ln((d*
x+c+(1+(d*x+c)^2)^(1/2))^2-1))+2*a*b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2
/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(81) = 162$.

Time $= 0.31$ (sec) , antiderivative size $= 319$, normalized size of antiderivative $= 3.75$

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^3}\, dx =$$

$$-\frac{2\,abc^2d^2x^2 + 4\,abc^3dx + 2\,abc^4 + b^2c^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^2 + a^2c^2 - 2\left(abd^2x^2 + 2\,a\right.}{}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 + b^2*c^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + a^2*c^2 - 2*(a*b*d^2*x^2 + 2*a*b*c*
d*x - (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
 + b^2*c^4)*log(d*x + c) - 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
 - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^
3)
```

## Sympy [F]

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^3}\, dx$$

$$= \frac{\int \frac{a^2}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\, dx + \int \frac{b^2 \operatorname{asinh}^2(c + dx)}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\, dx + \int \frac{2ab \operatorname{asinh}(c + dx)}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\, dx}{e^3}$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\begin{aligned}
&\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^3}\, dx \\
&= -\left(\frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}d\,\operatorname{arsinh}\left(dx + c\right)}{d^3e^3x + cd^2e^3} - \frac{\log\left(dx + c\right)}{de^3}\right)b^2 \\
&\quad - ab\left(\frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}d}{d^3e^3x + cd^2e^3} + \frac{\operatorname{arsinh}\left(dx + c\right)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right) \\
&\quad - \frac{b^2\,\operatorname{arsinh}\left(dx + c\right)^2}{2\left(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3\right)} - \frac{a^2}{2\left(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3\right)}
\end{aligned}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2*e
^3) - log(d*x + c)/(d*e^3))*b^2 - a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/
(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c
^2*d*e^3)) - 1/2*b^2*arcsinh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*
d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

## Giac [F]

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^3}\, dx = \int \frac{\left(b\,\operatorname{arsinh}\left(dx + c\right) + a\right)^2}{\left(dex + ce\right)^3}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^3}\, dx = \int \frac{(a + b\,\operatorname{asinh}(c + d\,x))^2}{(c\,e + d\,e\,x)^3}\, dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

# 3.135 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal result . . . 1043
Rubi [A] (verified) . . . 1043
Mathematica [A] (verified) . . . 1046
Maple [A] (verified) . . . 1047
Fricas [F] . . . 1048
Sympy [F] . . . 1048
Maxima [F] . . . 1048
Giac [F] . . . 1049
Mupad [F(-1)] . . . 1049

## Optimal result

Integrand size = 23, antiderivative size = 169

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx &= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2} \\ &\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} \\ &\quad + \frac{2b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\ &\quad + \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\ &\quad - \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}\end{aligned}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+
b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+1/3*b^2*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))/d/e^4-1/3*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{5859, 12, 5776, 5809, 5816, 4267, 2317, 2438, 30\}$

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = \frac{2b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{3de^4} - \frac{b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{b^2}{3de^4(c+dx)}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*b^2/(d*e^4*(c + d*x)) - (b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*
x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^2/(3*d*e^4*(c + d*x)^
3) + (2*b*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) +
 (b^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(3*d*e^4) - (b^2*PolyLog[2, E^ArcSin
h[c + d*x]])/(3*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{e^4x^4}\,dx, x, c+dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^4}\,dx, x, c+dx\right)}{de^4} \\ &= -\frac{(a+b\text{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4}\end{aligned}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3}$$
$$- \frac{b\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x^2}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} - \frac{b\operatorname{Subst}(\int (a+bx)\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(c+dx))}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{2b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$
$$+ \frac{b^2\operatorname{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{3de^4}$$
$$- \frac{b^2\operatorname{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{2b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$
$$+ \frac{b^2\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{b^2\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{2b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$
$$+ \frac{b^2\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{b^2\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$

## Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx =$$
$$-\frac{4a^2+ab\left(8\operatorname{arcsinh}(c+dx)+2\sinh(2\operatorname{arcsinh}(c+dx))+\left(\log\left(\cosh\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}\operatorname{ar}\right.\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/12*(4*a^2 + a*b*(8*ArcSinh[c + d*x] + 2*Sinh[2*ArcSinh[c + d*x]] + (Log[
Cosh[ArcSinh[c + d*x]/2]] - Log[Sinh[ArcSinh[c + d*x]/2]])*(3*c + 3*d*x - S
inh[3*ArcSinh[c + d*x]])) + b^2*(4*(c + d*x)^2 + 4*ArcSinh[c + d*x]^2 + 4*(
c + d*x)^3*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^
(-ArcSinh[c + d*x])] + ArcSinh[c + d*x]*(2*Sinh[2*ArcSinh[c + d*x]] + (Log[
1 - E^(-ArcSinh[c + d*x])] - Log[1 + E^(-ArcSinh[c + d*x])])*(-3*(c + d*x)
+ Sinh[3*ArcSinh[c + d*x]])))/(d*e^4*(c + d*x)^3)
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{3}+$ |
| default | $-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{3}+$ |
| parts | $-\frac{a^2}{3e^4(dx+c)^3d}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{3}+$ |

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3*((1+(d*x+c)^2)^(1/2)*(d*x+c)*arcs
inh(d*x+c)+arcsinh(d*x+c)^2+(d*x+c)^2)/(d*x+c)^3+1/3*arcsinh(d*x+c)*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+1/3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*arcs
inh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*polylog(2,d*x+c+(1+(d*x+c)^2
)^(1/2)))+2*a*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1+(d*x+c)
^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^4*e^4*x
^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\,\text{asinh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\,\text{asinh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*asinh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 + b
^2*d^3)*x^3 + 3*(c^3 + c)*a*b + (c^3 + c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^2)
*x^2 + (3*(3*c^2*d + d)*a*b + (3*c^2*d + d)*b^2)*x + (b^2*c^2 + 3*(c^2 + 1)
*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*c*d + b^2*c*d)*x)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^7
*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5*e^4)
*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4
)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d*e^4)
*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 +
 d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^
2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4,x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

# 3.136 $\int (ce+dex)^m(a+b\textbf{arcsinh}(c+dx))^3\,dx$

Optimal result . . . . . 1050
Rubi [N/A] . . . . . 1050
Mathematica [N/A] . . . . . 1051
Maple [N/A] (verified) . . . . . 1051
Fricas [N/A] . . . . . 1051
Sympy [N/A] . . . . . 1051
Maxima [N/A] . . . . . 1052
Giac [N/A] . . . . . 1052
Mupad [N/A] . . . . . 1053

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^3}{de(1+m)} - \frac{3b\text{Int}\left(\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

## Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^m(a+b\text{arcsinh}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^3}{de(1+m)} - \frac{(3b)\text{Subst}\left(\int \frac{(ex)^{1+m}(a+b\text{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de(1+m)}$$

## Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex+ce)^m\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^3\,dx$$

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \int \left(b\,\text{arsinh}\left(dx+c\right)+a\right)^3(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

## Sympy [N/A]

Not integrable

Time = 17.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \int \left(e(c+dx)\right)^m\left(a+b\,\text{asinh}\left(c+dx\right)\right)^3\,dx$$

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**3,x)
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**3, x)
```

## Maxima [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 716, normalized size of antiderivative = 31.13

$$\int (ce+dex)^m (a+b\text{arcsinh}(c+dx))^3\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^3 (dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (b^3*d*e^m*x + b^3*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^3/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^3/(d*e*(m + 1)) + integ
rate(-3*(((b^3*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^2 - (a*b^2*d^2
*e^m*(m + 1) - b^3*d^2*e^m)*x^2 - 2*(a*b^2*c*d*e^m*(m + 1) - b^3*c*d*e^m)*x
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) + c*e^m
*(m + 1))*a*b^2 - (c^3*e^m + c*e^m)*b^3 + (a*b^2*d^3*e^m*(m + 1) - b^3*d^3*
e^m)*x^3 + 3*(a*b^2*c*d^2*e^m*(m + 1) - b^3*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*
(m + 1) + d*e^m*(m + 1))*a*b^2 - (3*c^2*d*e^m + d*e^m)*b^3)*x)*(d*x + c)^m)
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - ((a^2*b*d^2*e^m*(m +
1)*x^2 + 2*a^2*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1))*a^2*b)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*b*d^3*e^m*(m + 1)*x^3
 + 3*a^2*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e^m*(m + 1))*a^
2*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*b)*(d*x + c)^m)*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2
 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)
*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)), x)
```

## Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m (a+b\text{arcsinh}(c+dx))^3\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^3 (dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3, x)
```

# 3.137 $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . . . 1054
Rubi [A] (verified) . . . . . 1055
Mathematica [A] (verified) . . . . . 1060
Maple [A] (verified) . . . . . 1060
Fricas [B] (verification not implemented) . . . . . 1061
Sympy [B] (verification not implemented) . . . . . 1062
Maxima [F] . . . . . 1063
Giac [F] . . . . . 1064
Mupad [F(-1)] . . . . . 1065

## Optimal result

Integrand size = 23, antiderivative size = 326

$$\begin{aligned}
&\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{16}{25}ab^2e^4x - \frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d} + \frac{76b^3e^4(1+(c+dx)^2)^{3/2}}{1125d} - \frac{6b^3e^4(1+(c+dx)^2)^{5/2}}{625d} \\
&\quad + \frac{16b^3e^4(c+dx)\operatorname{arcsinh}(c+dx)}{25d} - \frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} - \frac{8be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d}
\end{aligned}$$

```
[Out] 16/25*a*b^2*e^4*x+76/1125*b^3*e^4*(1+(d*x+c)^2)^(3/2)/d-6/625*b^3*e^4*(1+(d
*x+c)^2)^(5/2)/d+16/25*b^3*e^4*(d*x+c)*arcsinh(d*x+c)/d-8/75*b^2*e^4*(d*x+c
)^3*(a+b*arcsinh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))/d+1
/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^3/d-298/375*b^3*e^4*(1+(d*x+c)^2)^(1/
2)/d-8/25*b*e^4*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d+4/25*b*e^4*(d*
x+c)^2*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/25*b*e^4*(d*x+c)^4*(a
+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 267, 272, 45}

$$\begin{aligned}
&\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} - \frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad - \frac{3be^4\sqrt{(c+dx)^2+1}(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{8be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} + \frac{16}{25}ab^2e^4x \\
&\quad + \frac{16b^3e^4(c+dx)\operatorname{arcsinh}(c+dx)}{25d} - \frac{6b^3e^4\left((c+dx)^2+1\right)^{5/2}}{625d} \\
&\quad + \frac{76b^3e^4\left((c+dx)^2+1\right)^{3/2}}{1125d} - \frac{298b^3e^4\sqrt{(c+dx)^2+1}}{375d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (16*a*b^2*e^4*x)/25 - (298*b^3*e^4*Sqrt[1 + (c + d*x)^2])/(375*d) + (76*b^3
*e^4*(1 + (c + d*x)^2)^(3/2))/(1125*d) - (6*b^3*e^4*(1 + (c + d*x)^2)^(5/2)
)/(625*d) + (16*b^3*e^4*(c + d*x)*ArcSinh[c + d*x])/(25*d) - (8*b^2*e^4*(c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*A
rcSinh[c + d*x]))/(125*d) - (8*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c
 + d*x])^2)/(25*d) + (4*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcS
inh[c + d*x])^2)/(25*d) - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b
*ArcSinh[c + d*x])^2)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)
/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int e^4 x^4 (a+b\operatorname{arcsinh}(x))^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \operatorname{Subst}\left(\int x^4 (a+b\operatorname{arcsinh}(x))^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^3}{5d} - \frac{(3be^4)\operatorname{Subst}\left(\int \frac{x^5 (a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5d} \\
&= -\frac{3be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad + \frac{(12be^4)\operatorname{Subst}\left(\int \frac{x^3 (a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d} \\
&\quad + \frac{(6b^2e^4)\operatorname{Subst}\left(\int x^4 (a+b\operatorname{arcsinh}(x))\,dx, x, c+dx\right)}{25d} \\
&= \frac{6b^2e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))}{125d} \\
&\quad + \frac{4be^4 (c+dx)^2 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad - \frac{(8be^4)\operatorname{Subst}\left(\int \frac{x (a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d} \\
&\quad - \frac{(8b^2e^4)\operatorname{Subst}\left(\int x^2 (a+b\operatorname{arcsinh}(x))\,dx, x, c+dx\right)}{25d} \\
&\quad - \frac{(6b^3e^4)\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{125d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} + \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} \\
&\quad - \frac{8be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad + \frac{(16b^2e^4)\operatorname{Subst}(\int (a+b\operatorname{arcsinh}(x))\,dx, x, c+dx)}{25d} \\
&\quad - \frac{(3b^3e^4)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{125d} + \frac{(8b^3e^4)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{75d} \\
&= \frac{16}{25}ab^2e^4x - \frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} \\
&\quad - \frac{8be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad - \frac{(3b^3e^4)\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}} - 2\sqrt{1+x} + (1+x)^{3/2}\right)\,dx, x, (c+dx)^2\right)}{125d} \\
&\quad + \frac{(4b^3e^4)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{75d} \\
&\quad + \frac{(16b^3e^4)\operatorname{Subst}(\int \operatorname{arcsinh}(x)\,dx, x, c+dx)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{16}{25}ab^2e^4x - \frac{6b^3e^4\sqrt{1+(c+dx)^2}}{125d} + \frac{4b^3e^4(1+(c+dx)^2)^{3/2}}{125d} \\
&\quad - \frac{6b^3e^4(1+(c+dx)^2)^{5/2}}{625d} + \frac{16b^3e^4(c+dx)\operatorname{arcsinh}(c+dx)}{25d} \\
&\quad - \frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} + \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} \\
&\quad - \frac{8be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d} \\
&\quad + \frac{(4b^3e^4)\operatorname{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right)\,dx, x, (c+dx)^2\right)}{75d} \\
&\quad - \frac{(16b^3e^4)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{16}{25}ab^2e^4x - \frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d} + \frac{76b^3e^4(1+(c+dx)^2)^{3/2}}{1125d} \\
&\quad - \frac{6b^3e^4(1+(c+dx)^2)^{5/2}}{625d} + \frac{16b^3e^4(c+dx)\operatorname{arcsinh}(c+dx)}{25d} \\
&\quad - \frac{8b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{75d} + \frac{6b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))}{125d} \\
&\quad - \frac{8be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^3}{5d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.09

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

$$= \frac{e^4\Big(240ab^2(c+dx) - 40ab^2(c+dx)^3 + 3a(25a^2+6b^2)(c+dx)^5 + \frac{1}{15}b\sqrt{1+(c+dx)^2}(-8(225a^2+518b^2) +$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) - 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[1 + (c + d*x)^2]*(-8*(225*a^2 + 518*b^2) + 4*(225*a^2 +
68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 - b*(-240*b^2*(c
 + d*x) + 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*(c + d*x)^5 + 2
40*a*b*Sqrt[1 + (c + d*x)^2] - 120*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] +
90*a*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 15*b^2*(-15*a*
(c + d*x)^5 + 8*b*Sqrt[1 + (c + d*x)^2] - 4*b*(c + d*x)^2*Sqrt[1 + (c + d*x
)^2] + 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 75*b^3*(
c + d*x)^5*ArcSinh[c + d*x]^3))/(375*d)
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| derivativedivides | $\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2}{}\right.$ |
| default | $\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2}{}\right.$ |
| parts | $\frac{e^4a^3(dx+c)^5}{5d}+\frac{e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c}{}\right.}{}$ |

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*e^4*a^3*(d*x+c)^5+e^4*b^3*(1/5*(d*x+c)^5*arcsinh(d*x+c)^3-8/25*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/25*(d*x+c)^4*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+4/25*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+16/25*(d*x+c
)*arcsinh(d*x+c)-4144/5625*(1+(d*x+c)^2)^(1/2)+6/125*(d*x+c)^5*arcsinh(d*x+
c)-6/625*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+272/5625*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)-8/75*(d*x+c)^3*arcsinh(d*x+c))+3*e^4*a*b^2*(1/5*(d*x+c)^5*arcsinh(d*x+c)
^2-16/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d*x+c)*(
1+(d*x+c)^2)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+16/75*
```

```
d*x+16/75*c+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+3*e^4*a^2*b*(1/5*(d*x+c)^5*arc
sinh(d*x+c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)
^(1/2)-8/75*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(292) = 584$.

Time = 0.30 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.30

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e^
4*x^4 - 150*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 - 450*(4*a*b^2
*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 - 225*(8*a*b^2*c^2 - (25*a^3 + 6*a
*b^2)*c^4 - 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4*x^4
 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x + b^
3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 225*(15*a*b
^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3 + 150*a
*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3*b^3*d^4
*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 - 2*b^3)*d^2*e^4*x^2 + 4*(3*
b^3*c^3 - 2*b^3*c)*d*e^4*x + (3*b^3*c^4 - 4*b^3*c^2 + 8*b^3)*e^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^
2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*e^4*
x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 - 30*(4*b^3*c - 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 - 15*(8*b^3*c^2 - 3*(25*a^2*b + 2*b^3)*c
^4 - 16*b^3)*d*e^4*x - (40*b^3*c^3 - 9*(25*a^2*b + 2*b^3)*c^5 - 240*b^3*c)*
e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*(9*a*b^2*c^2 - 2
*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 - 2*a*b^2*c)*d*e^4*x + (3*a*b^2*c^4 -
4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (27*(25*a^2*b + 2*b^3)*d^4*e^4*x^4 +
 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 - 2*(450*a^2*b + 136*b^3 - 81*(25*a^2
*b + 2*b^3)*c^2)*d^2*e^4*x^2 + 4*(27*(25*a^2*b + 2*b^3)*c^3 - 2*(225*a^2*b
+ 68*b^3)*c)*d*e^4*x + (27*(25*a^2*b + 2*b^3)*c^4 + 1800*a^2*b + 4144*b^3 -
 4*(225*a^2*b + 68*b^3)*c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(306) = 612$.

Time = 1.02 (sec) , antiderivative size = 2518, normalized size of antiderivative = 7.72

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
**4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*asinh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*asinh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*asinh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*asinh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25
 + 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 3*a**2*b*d**4
*e**4*x**5*asinh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/25 + 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a*b*
*2*c**5*e**4*asinh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*asinh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)*asinh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*asinh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*asi
nh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c**2*
e**4*x/25 + 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c*d*e**4*x**2/25 + 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*asinh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*d*
*2*e**4*x**3/75 + 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + b**3*c**5*e**4*asinh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*asinh(c + d*x)/(125*d) + b**3*c**4*e**4*x*asinh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*asinh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*asi
nh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*asinh(c + d*x)/25 - 12*b**3*c**3*
```

```
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**3*e**4*asi
nh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*asinh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*asinh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**2*e**4*x*asinh(c + d*
x)/25 + 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x
)**2/(25*d) + 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*asinh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*asi
nh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)*asinh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*c*d*e**4*x**2*asinh(c + d*x)/25 + 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*c*e**4*asinh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*asinh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*asinh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)*asinh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*d**2*e**4*x**3*asinh(c + d*x)/75 + 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*e**4*x*asin
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**3, True))
```

## Maxima [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*d^4*e^4*x^5 + a^3*c*d^3*e^4*x^4 + 2*a^3*c^2*d^2*e^4*x^3 + 2*a^3*c^3
*d*e^4*x^2 + 3*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^3*d*e^4 + (6*x
^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c
^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*
d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*
c^2*d^2*e^4 + 1/8*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*a
rcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^
```

```
2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9
*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^
5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*c*d^3*e^
4 + 1/200*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c
^2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/
d^6)*d)*a^2*b*d^4*e^4 + a^3*c^4*e^4*x + 3*((d*x + c)*arcsinh(d*x + c) - sqr
t((d*x + c)^2 + 1))*a^2*b*c^4*e^4/d + 1/5*(b^3*d^4*e^4*x^5 + 5*b^3*c*d^3*e^
4*x^4 + 10*b^3*c^2*d^2*e^4*x^3 + 10*b^3*c^3*d*e^4*x^2 + 5*b^3*c^4*e^4*x)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/5*((5*a*b^2*
d^7*e^4 - b^3*d^7*e^4)*x^7 + 7*(5*a*b^2*c*d^6*e^4 - b^3*c*d^6*e^4)*x^6 + (5
*(21*c^2*d^5*e^4 + d^5*e^4)*a*b^2 - (21*c^2*d^5*e^4 + d^5*e^4)*b^3)*x^5 + 5
*(5*(7*c^3*d^4*e^4 + c*d^4*e^4)*a*b^2 - (7*c^3*d^4*e^4 + c*d^4*e^4)*b^3)*x^
4 + 5*(c^7*e^4 + c^5*e^4)*a*b^2 + 5*(5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*a*b^
2 - (7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*b^3)*x^3 + 5*((21*c^5*d^2*e^4 + 10*c^3*
d^2*e^4)*a*b^2 - 2*(2*c^5*d^2*e^4 + c^3*d^2*e^4)*b^3)*x^2 + 5*((7*c^6*d*e^4
 + 5*c^4*d*e^4)*a*b^2 - (c^6*d*e^4 + c^4*d*e^4)*b^3)*x + ((5*a*b^2*d^6*e^4
 - b^3*d^6*e^4)*x^6 + 6*(5*a*b^2*c*d^5*e^4 - b^3*c*d^5*e^4)*x^5 - 5*(3*b^3*c
^2*d^4*e^4 - (15*c^2*d^4*e^4 + d^4*e^4)*a*b^2)*x^4 + 5*(c^6*e^4 + c^4*e^4)*
a*b^2 - 20*(b^3*c^3*d^3*e^4 - (5*c^3*d^3*e^4 + c*d^3*e^4)*a*b^2)*x^3 - 15*(
b^3*c^4*d^2*e^4 - (5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*a*b^2)*x^2 - 5*(b^3*c^5*d
*e^4 - 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^
2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x
)
```

## Giac [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^4(b\,\operatorname{arsinh}(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3, x)
```

# 3.138 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . . . 1066
Rubi [A] (verified) . . . . . 1067
Mathematica [A] (verified) . . . . . 1070
Maple [A] (verified) . . . . . 1071
Fricas [B] (verification not implemented) . . . . . 1071
Sympy [B] (verification not implemented) . . . . . 1072
Maxima [F] . . . . . 1073
Giac [F] . . . . . 1074
Mupad [F(-1)] . . . . . 1074

## Optimal result

Integrand size = 23, antiderivative size = 279

$$\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{45b^3e^3\operatorname{arcsinh}(c+dx)}{256d} \\
&\quad - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{32d} + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^3}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d}
\end{aligned}$$

```
[Out] -45/256*b^3*e^3*arcsinh(d*x+c)/d-9/32*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c)
)/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d-3/32*e^3*(a+b*arcsinh(d*x
+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^3/d+45/256*b^3*e^3*(d*x+c)*
(1+(d*x+c)^2)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/d+9/32*b*
e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/16*b*e^3*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5812, 5783, 327, 221}

$$\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{9be^3\sqrt{(c+dx)^2+1}(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^3}{32d} - \frac{45b^3e^3\operatorname{arcsinh}(c+dx)}{256d} \\
&\quad - \frac{3b^3e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{128d} + \frac{45b^3e^3\sqrt{(c+dx)^2+1}(c+dx)}{256d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (45*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(256*d) - (3*b^3*e^3*(c + d*x)
^3*Sqrt[1 + (c + d*x)^2])/(128*d) - (45*b^3*e^3*ArcSinh[c + d*x])/(256*d) -
 (9*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(32*d) + (3*b^2*e^3*(c +
d*x)^4*(a + b*ArcSinh[c + d*x]))/(32*d) + (9*b*e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(32*d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^3)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b\text{arcsinh}(x))^3 \, dx, x, c + dx\right)}{d} \\
&= \frac{e^3 \text{Subst}\left(\int x^3 (a + b\text{arcsinh}(x))^3 \, dx, x, c + dx\right)}{d} \\
&= \frac{e^3 (c + dx)^4 (a + b\text{arcsinh}(c + dx))^3}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4 (a + b\text{arcsinh}(x))^2}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{4d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{16d} \\
&\quad + \frac{(3b^2e^3)\operatorname{Subst}\left(\int x^3(a+b\operatorname{arcsinh}(x))\,dx, x, c+dx\right)}{8d} \\
&= \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{(9be^3)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} \\
&\quad - \frac{(9b^2e^3)\operatorname{Subst}(\int x(a+b\operatorname{arcsinh}(x))\,dx, x, c+dx)}{16d} \\
&\quad - \frac{(3b^3e^3)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} \\
&= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^3}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{128d} + \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} \\
&\quad - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{32d} + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^3}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{256d} - \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d} \\
&= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} \\
&\quad - \frac{45b^3e^3\operatorname{arcsinh}(c+dx)}{256d} - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{32d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^3}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3}{4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{e^3\Big(-72ab^2(c+dx)^2 + 8a(8a^2+3b^2)(c+dx)^4 + 3b(c+dx)\sqrt{1+(c+dx)^2}(3(8a^2+5b^2) - 2(8a^2+b^2)(c+
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]

[Out] (e^3*(-72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*(c + d*
x)*Sqrt[1 + (c + d*x)^2]*(3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2)*(c + d*x)^2)
- 9*b*(8*a^2 + 5*b^2)*ArcSinh[c + d*x] - 24*b*(c + d*x)*(3*b^2*(c + d*x) -
8*a^2*(c + d*x)^3 - b^2*(c + d*x)^3 - 6*a*b*Sqrt[1 + (c + d*x)^2] + 4*a*b*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 24*b^2*(-3*a + 8*a*(c
+ d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (
c + d*x)^2])*ArcSinh[c + d*x]^2 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*
x]^3))/(256*d)
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| derivativedivides | $\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}-\frac{3}{}\right.$ |
| default | $\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}-\frac{3}{}\right.$ |
| parts | $\frac{e^3a^3(dx+c)^4}{4d}+\frac{e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}\right.}{}$ |

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a^3*(d*x+c)^4+e^3*b^3*(1/4*(d*x+c)^4*arcsinh(d*x+c)^3-3/16*(d*
x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9/32*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/32*(d*x+c)^4*arcsinh(d*x+c)-3/12
8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x+c)*(1+(d*x+c)^2)^(1/2)+27/256*a
rcsinh(d*x+c)-9/32*(1+(d*x+c)^2)*arcsinh(d*x+c))+3*e^3*a*b^2*(1/4*(d*x+c)^4
*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*(1+
(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^
4-3/32*(d*x+c)^2-3/32)+3*e^3*a^2*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+
c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+
c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(253) = 506$.

Time = 0.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.98

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

$$=\frac{8\,(8\,a^3+3\,ab^2)d^4e^3x^4+32\,(8\,a^3+3\,ab^2)cd^3e^3x^3-24\,(3\,ab^2-2\,(8\,a^3+3\,ab^2)c^2)d^2e^3x^2-16\,(9\,ab^2c-2\,(8}{}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^3
 - 24*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^2*c - 2*(
8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*x^3
 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 24*(8*a*b^2*d^4*e^3*x^4
 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d*e^3*x
```

```
 + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^3*x^2 +
 3*(2*b^3*c^2 - b^3)*d*e^3*x + (2*b^3*c^3 - 3*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(8
*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 - 24*(b^3 -
 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 - 16*(3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*
d*e^3*x - (24*b^3*c^2 - 8*(8*a^2*b + b^3)*c^4 + 24*a^2*b + 15*b^3)*e^3 - 16
*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2 - a*b^2)*d*e
^3*x + (2*a*b^2*c^3 - 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(2*(8*a^2*b + b^3)*d^3*e
^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*b
+ b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3)*c)*e^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1828 vs. $2(260) = 520$.

Time = 0.72 (sec) , antiderivative size = 1828, normalized size of antiderivative = 6.55

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*asinh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*asinh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*asinh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*asinh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/16 + 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*asinh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/32 - 9*a**2*b*e**3*asinh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*asinh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*c*e**3*x/16 + 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*asinh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*d
*e**3*x**2/32 + 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)/16 - 9*a*b**2*e**3*asinh(c + d*x)**2/(32*d) + b**3*c**4*e**3*asinh
```

```
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*asinh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*asinh(c + d*x)**3 + 3*b**3*c**3*e**3*x*asinh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*asinh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*asinh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c**2*
e**3*asinh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*asinh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c*e**3*x*asinh(c + d*x)/16 + 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(32*d) + 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(256*d) + b**3*d**3*e**3*
x**4*asinh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*asinh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*d*
e**3*x**2*asinh(c + d*x)/32 + 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**2/32 + 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/256 - 3*b**3*e**3*asinh(c + d*x)**3/(32*d) - 45*b**3*e**3*asinh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**3, True))
```

## Maxima [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^3(b\,\operatorname{arsinh}\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x^2 + 9/4*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arcsinh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^
2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d
^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 -
 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*c*d^2*e^3 + 1/32
*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 -
14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*
d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5
 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcs
```

```
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*d^3*e^3 + a^3*c^3*e^3*x +
 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^3*e^3/d + 1
/4*(b^3*d^3*e^3*x^4 + 4*b^3*c*d^2*e^3*x^3 + 6*b^3*c^2*d*e^3*x^2 + 4*b^3*c^3
*e^3*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/4*
((4*a*b^2*d^6*e^3 - b^3*d^6*e^3)*x^6 + 6*(4*a*b^2*c*d^5*e^3 - b^3*c*d^5*e^3
)*x^5 + (4*(15*c^2*d^4*e^3 + d^4*e^3)*a*b^2 - (15*c^2*d^4*e^3 + d^4*e^3)*b^
3)*x^4 + 4*(c^6*e^3 + c^4*e^3)*a*b^2 + 4*(4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a*b
^2 - (5*c^3*d^3*e^3 + c*d^3*e^3)*b^3)*x^3 + 2*(6*(5*c^4*d^2*e^3 + 2*c^2*d^2
*e^3)*a*b^2 - (7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^3)*x^2 + 4*(2*(3*c^5*d*e^3
+ 2*c^3*d*e^3)*a*b^2 - (c^5*d*e^3 + c^3*d*e^3)*b^3)*x + ((4*a*b^2*d^5*e^3 -
 b^3*d^5*e^3)*x^5 + 5*(4*a*b^2*c*d^4*e^3 - b^3*c*d^4*e^3)*x^4 + 4*(c^5*e^3
+ c^3*e^3)*a*b^2 - 2*(5*b^3*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 + d^3*e^3)*a*b^
2)*x^3 - 2*(5*b^3*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a*b^2)*x^2
 - 4*(b^3*c^4*d*e^3 - (5*c^4*d*e^3 + 3*c^2*d*e^3)*a*b^2)*x)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3
*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^
(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3, x)
```

# 3.139 $\int (ce+dex)^2(a+b\mathbf{arcsinh}(c+dx))^3\,dx$

Optimal result . . . . . 1075
Rubi [A] (verified) . . . . . 1076
Mathematica [A] (verified) . . . . . 1079
Maple [A] (verified) . . . . . 1080
Fricas [B] (verification not implemented) . . . . . 1080
Sympy [B] (verification not implemented) . . . . . 1081
Maxima [F] . . . . . 1082
Giac [F] . . . . . 1082
Mupad [F(-1)] . . . . . 1083

## Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= -\frac{4}{3}ab^2e^2x + \frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d} - \frac{2b^3e^2(1+(c+dx)^2)^{3/2}}{27d} \\
&\quad - \frac{4b^3e^2(c+dx)\operatorname{arcsinh}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{2be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3}{3d}
\end{aligned}$$

```
[Out] -4/3*a*b^2*e^2*x-2/27*b^3*e^2*(1+(d*x+c)^2)^(3/2)/d-4/3*b^3*e^2*(d*x+c)*arc
sinh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1/3*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^3/d+14/9*b^3*e^2*(1+(d*x+c)^2)^(1/2)/d+2/3*b*e^2*(a+
b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 267, 272, 45}

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{2b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{9d} + \frac{2be^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3}{3d} - \frac{4}{3}ab^2e^2x - \frac{4b^3e^2(c+dx)\operatorname{arcsinh}(c+dx)}{3d} \\
&\quad - \frac{2b^3e^2\left((c+dx)^2+1\right)^{3/2}}{27d} + \frac{14b^3e^2\sqrt{(c+dx)^2+1}}{9d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-4*a*b^2*e^2*x)/3 + (14*b^3*e^2*Sqrt[1 + (c + d*x)^2])/(9*d) - (2*b^3*e^2*
(1 + (c + d*x)^2)^(3/2))/(27*d) - (4*b^3*e^2*(c + d*x)*ArcSinh[c + d*x])/(3
*d) + (2*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(9*d) + (2*b*e^2*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) - (b*e^2*(c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(
a + b*ArcSinh[c + d*x])^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^2 x^2 (a + b\text{arcsinh}(x))^3 \, dx, x, c + dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{e^2 \operatorname{Subst}\left(\int x^2 (a + b \operatorname{arcsinh}(x))^3 \, dx, x, c + dx\right)}{d} \\
&= \frac{e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))^3}{3d} - \frac{(be^2) \operatorname{Subst}\left(\int \frac{x^3 (a + b \operatorname{arcsinh}(x))^2}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{d} \\
&= -\frac{be^2 (c + dx)^2 \sqrt{1 + (c + dx)^2} (a + b \operatorname{arcsinh}(c + dx))^2}{3d} \\
&\quad + \frac{e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))^3}{3d} \\
&\quad + \frac{(2be^2) \operatorname{Subst}\left(\int \frac{x (a + b \operatorname{arcsinh}(x))^2}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{3d} \\
&\quad + \frac{(2b^2 e^2) \operatorname{Subst}\left(\int x^2 (a + b \operatorname{arcsinh}(x)) \, dx, x, c + dx\right)}{3d} \\
&= \frac{2b^2 e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))}{9d} + \frac{2be^2 \sqrt{1 + (c + dx)^2} (a + b \operatorname{arcsinh}(c + dx))^2}{3d} \\
&\quad - \frac{be^2 (c + dx)^2 \sqrt{1 + (c + dx)^2} (a + b \operatorname{arcsinh}(c + dx))^2}{3d} \\
&\quad + \frac{e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))^3}{3d} \\
&\quad - \frac{(4b^2 e^2) \operatorname{Subst}(\int (a + b \operatorname{arcsinh}(x)) \, dx, x, c + dx)}{3d} \\
&\quad - \frac{(2b^3 e^2) \operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1 + x^2}} \, dx, x, c + dx\right)}{9d} \\
&= -\frac{4}{3} ab^2 e^2 x + \frac{2b^2 e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))}{9d} \\
&\quad + \frac{2be^2 \sqrt{1 + (c + dx)^2} (a + b \operatorname{arcsinh}(c + dx))^2}{3d} \\
&\quad - \frac{be^2 (c + dx)^2 \sqrt{1 + (c + dx)^2} (a + b \operatorname{arcsinh}(c + dx))^2}{3d} \\
&\quad + \frac{e^2 (c + dx)^3 (a + b \operatorname{arcsinh}(c + dx))^3}{3d} - \frac{(b^3 e^2) \operatorname{Subst}\left(\int \frac{x}{\sqrt{1 + x}} \, dx, x, (c + dx)^2\right)}{9d} \\
&\quad - \frac{(4b^3 e^2) \operatorname{Subst}(\int \operatorname{arcsinh}(x) \, dx, x, c + dx)}{3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4}{3}ab^2e^2x - \frac{4b^3e^2(c+dx)\operatorname{arcsinh}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{2be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3}{3d} \\
&\quad - \frac{(b^3e^2)\operatorname{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right)\,dx, x, (c+dx)^2\right)}{9d} \\
&\quad + \frac{(4b^3e^2)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3d} \\
&= -\frac{4}{3}ab^2e^2x + \frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d} - \frac{2b^3e^2(1+(c+dx)^2)^{3/2}}{27d} \\
&\quad - \frac{4b^3e^2(c+dx)\operatorname{arcsinh}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))}{9d} \\
&\quad + \frac{2be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3}{3d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$
\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{e^2\Big(-12ab^2(c+dx) + a(3a^2+2b^2)(c+dx)^3 + \frac{1}{3}b\sqrt{1+(c+dx)^2}(18a^2+40b^2-(9a^2+2b^2)(c+dx)^2) -}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*(-12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[1 + (c
+ d*x)^2]*(18*a^2 + 40*b^2 - (9*a^2 + 2*b^2)*(c + d*x)^2))/3 - b*(12*b^2*(c
 + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[1 + (c + d*x)
^2] + 6*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 3*b^2*(-3
*a*(c + d*x)^3 - 2*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*
x)^2])*ArcSinh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcSinh[c + d*x]^3))/(9*d)
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| derivativedivides | $\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+c)\operatorname{arc}}{3}\right.$ |
| default | $\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+c)\operatorname{arc}}{3}\right.$ |
| parts | $\frac{e^2a^3(dx+c)^3}{3d}+\frac{e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+c)}{d}\right.$ |

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arcsinh(d*x+c)^3+2/3*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1+(d*x+c)^2)^(1/2)+2/9*(d*x+c)^3
*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1/2))+3*e^2*a*b^2*(1/3*(d*x+c
)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x
+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+3*e^2*a^2*b
*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*
x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(203) = 406$.

Time = 0.29 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.70

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

$$=\frac{3\,(3\,a^3+2\,ab^2)d^3e^2x^3+9\,(3\,a^3+2\,ab^2)cd^2e^2x^2-9\,(4\,ab^2-(3\,a^3+2\,ab^2)c^2)de^2x+9\,(b^3d^3e^2x^3+3\,b^3cd^2e^2}{}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2 -
 9*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3*c
*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^2 +
9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x
+ (b^3*c^2 - 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x^3 + 3*
(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 - 3*(4*b^3 - (9*a^2*b + 2*b^3)*c^2)*d*e^2*x
```

```
 - (12*b^3*c - (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*
c*d*e^2*x + (a*b^2*c^2 - 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - ((9*a^2*b + 2*b^3)*d^2*e^
2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x - (18*a^2*b + 40*b^3 - (9*a^2*b + 2*b
^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(211) = 422$.

Time = 0.47 (sec) , antiderivative size = 1173, normalized size of antiderivative = 5.17

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*asinh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*asinh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(3*d) + 3*a**2*b*c*d*e
**2*x**2*asinh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/3 + a**2*b*d**2*e**2*x**3*asinh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/3 + 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(3*d) + a*b**2*c**3*e**2*asinh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*asinh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
asinh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*asinh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 4*a*b**2*e**2*x/3 + 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + b**3*c**3*e**
2*asinh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*asinh(c + d*x)**3 + 2*b**3*c**2*e**2*x*asinh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**3*c*d*e**2*x*
*2*asinh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 4*b**3*c*e**2*asinh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*asi
nh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27
 - 4*b**3*e**2*x*asinh(c + d*x)/3 + 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)**2/(3*d) + 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**3, True))
```

## Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arcsinh(d*x + c) - d*(3*
c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt
(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 +
 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c
^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 +
 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3*((d*x + c)*arcsi
nh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*d^2*e^2*x^3
 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^3 + integrate(((3*a*b^2*d^5*e^2 - b^3*d^5*e^2)*x^5 + 5*(3*a*b
^2*c*d^4*e^2 - b^3*c*d^4*e^2)*x^4 + 3*(c^5*e^2 + c^3*e^2)*a*b^2 + (3*(10*c^
2*d^3*e^2 + d^3*e^2)*a*b^2 - (10*c^2*d^3*e^2 + d^3*e^2)*b^3)*x^3 + 3*((10*c
^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^2 - (3*c^3*d^2*e^2 + c*d^2*e^2)*b^3)*x^2 + 3*
((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^2 - (c^4*d*e^2 + c^2*d*e^2)*b^3)*x + ((3*a
*b^2*d^4*e^2 - b^3*d^4*e^2)*x^4 + 3*(c^4*e^2 + c^2*e^2)*a*b^2 + 4*(3*a*b^2*
c*d^3*e^2 - b^3*c*d^3*e^2)*x^3 - 3*(2*b^3*c^2*d^2*e^2 - (6*c^2*d^2*e^2 + d^
2*e^2)*a*b^2)*x^2 - 3*(b^3*c^3*d*e^2 - 2*(2*c^3*d*e^2 + c*d*e^2)*a*b^2)*x)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*
d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3, x)
```

# 3.140 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . 1084
Rubi [A] (verified) . . . 1084
Mathematica [A] (verified) . . . 1087
Maple [A] (verified) . . . 1088
Fricas [B] (verification not implemented) . . . 1088
Sympy [B] (verification not implemented) . . . 1089
Maxima [F] . . . 1089
Giac [F] . . . 1090
Mupad [F(-1)] . . . 1090

## Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d} + \frac{3b^3e\operatorname{arcsinh}(c+dx)}{8d} \\
&\quad + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{4d} \\
&\quad - \frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{4d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^3}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^3}{2d}
\end{aligned}$$

```
[Out] 3/8*b^3*e*arcsinh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d+1/4*e
*(a+b*arcsinh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^3/d-3/8*b^3*
e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(
d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5859, 12, 5776, 5812, 5783, 327, 221\}$

$$\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{4d} \\
&\quad - \frac{3be(c+dx)\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{4d} \\
&\quad + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^3}{2d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{3b^3e\operatorname{arcsinh}(c+dx)}{8d} - \frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}}{8d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(8*d) + (3*b^3*e*ArcSinh[c + d*x
])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(4*d) - (3*b*e*(c
 + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(a + b
*ArcSinh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/(2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\text{arcsinh}(x))^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^3}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\text{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{4d} \\
&\quad + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^3}{2d} \\
&\quad + \frac{(3be)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&\quad + \frac{(3b^2e)\,\text{Subst}(\int x(a+b\text{arcsinh}(x))\,dx, x, c+dx)}{2d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{4d} \\
&\quad - \frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{4d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^3}{2d} - \frac{(3b^3e)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d} + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{4d} \\
&\quad - \frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{4d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^3}{2d} + \frac{(3b^3e)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d} + \frac{3b^3e\operatorname{arcsinh}(c+dx)}{8d} \\
&\quad + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))}{4d} \\
&\quad - \frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{4d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^3}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^3}{2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx \\
&= \frac{e\Big(2a(2a^2+3b^2)(c+dx)^2 - 3b(2a^2+b^2)(c+dx)\sqrt{1+(c+dx)^2} + 3b(2a^2+b^2)\operatorname{arcsinh}(c+dx) - 6b(c+
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]

[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*(c + d*x)*Sqrt[1 +
(c + d*x)^2] + 3*b*(2*a^2 + b^2)*ArcSinh[c + d*x] - 6*b*(c + d*x)*(-2*a^2*(
c + d*x) - b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] +
6*b^2*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c + d*x)^2])*ArcSinh[c +
 d*x]^2 + 2*b^3*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^3))/(8*d)
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{e\,a^3(dx+c)^2}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx+c)^2\right)\operatorname{arcsinh}}{4}\right.}{d}$ |
| default | $\frac{\frac{e\,a^3(dx+c)^2}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx+c)^2\right)\operatorname{arcsinh}}{4}\right.}{d}$ |
| parts | $e\,a^3\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx\right.}{d}$ |

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a^3*(d*x+c)^2+e*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/4*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*(1+(d*x+c
)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*x+c))+3*e
*a*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+(d*x+c)^2)^(1/2)*(d*x+c)*
arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+3*e*a^2*b*(1/2*(d*x+
c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.43

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

$$=\frac{2\left(2\,a^3+3\,ab^2\right)d^2ex^2+4\left(2\,a^3+3\,ab^2\right)cdex+2\left(2\,b^3d^2ex^2+4\,b^3cdex+\left(2\,b^3c^2+b^3\right)e\right)\log\left(dx+c+\sqrt{d^2x^2}\right.}{}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^3
*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 + b^3)*e)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*a*b
^2*c^2 + a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(2*(2*a^2*b + b^3)
*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x + (2*a^2*b + b^3 + 2*(2*a^2*b + b^3)
*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*((2*a^2*b + b^3)*d*e*x +
 (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(148) = 296$.

Time $= 0.31$ (sec) , antiderivative size $= 685$, normalized size of antiderivative $= 4.25$

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^3\,dx$$

$$= \begin{cases} a^3cex + \frac{a^3dex^2}{2} + \frac{3a^2bc^2e\,\operatorname{asinh}(c+dx)}{2d} + 3a^2bcex\,\operatorname{asinh}(c + dx) - \frac{3a^2bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{3a^2bdex^2\,\operatorname{asinh}(c+dx)}{2} - \\ cex(a + b\,\operatorname{asinh}(c))^3 \end{cases}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*asinh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*asinh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/(4*d) + 3*a**2*b*d*e*x**2*asinh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 3*a**2*b*e*asinh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*asinh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*asinh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*asinh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/2 + 3*a*
b**2*e*asinh(c + d*x)**2/(4*d) + b**3*c**2*e*asinh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*asinh(c + d*x)/(4*d) + b**3*c*e*x*asinh(c + d*x)**3 + 3*b**3*c*e*
x*asinh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) +
 b**3*d*e*x**2*asinh(c + d*x)**3/2 + 3*b**3*d*e*x**2*asinh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + b**3*e*asinh(c + d*x)**3/(4*d)
+ 3*b**3*e*asinh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**3, Tru
e))
```

## Maxima [F]

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^3\,dx = \int (dex + ce)(b\,\operatorname{arsinh}(dx + c) + a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*d*e + a^3
*c*e*x + 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c*e/d
 + 1/2*(b^3*d*e*x^2 + 2*b^3*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
```

```
^2 + 1))^3 + integrate(3/2*((2*a*b^2*d^4*e - b^3*d^4*e)*x^4 + 2*(c^4*e + c^
2*e)*a*b^2 + 4*(2*a*b^2*c*d^3*e - b^3*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e + d^2*
e)*a*b^2 - (5*c^2*d^2*e + d^2*e)*b^3)*x^2 + 2*(2*(2*c^3*d*e + c*d*e)*a*b^2
 - (c^3*d*e + c*d*e)*b^3)*x + (2*(c^3*e + c*e)*a*b^2 + (2*a*b^2*d^3*e - b^3*
d^3*e)*x^3 + 3*(2*a*b^2*c*d^2*e - b^3*c*d^2*e)*x^2 - 2*(b^3*c^2*d*e - (3*c^
2*d*e + d*e)*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
 d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^3\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3, x)
```

# 3.141 $\int (a + b\text{arcsinh}(c + dx))^3\, dx$

Optimal result . . . . . 1091
Rubi [A] (verified) . . . . . 1091
Mathematica [A] (verified) . . . . . 1093
Maple [A] (verified) . . . . . 1093
Fricas [B] (verification not implemented) . . . . . 1094
Sympy [B] (verification not implemented) . . . . . 1094
Maxima [F] . . . . . 1095
Giac [F] . . . . . 1095
Mupad [F(-1)] . . . . . 1095

## Optimal result

Integrand size = 12, antiderivative size = 100

$$\int (a + b\text{arcsinh}(c + dx))^3\, dx = 6ab^2x - \frac{6b^3\sqrt{1 + (c + dx)^2}}{d} + \frac{6b^3(c + dx)\text{arcsinh}(c + dx)}{d} - \frac{3b\sqrt{1 + (c + dx)^2}(a + b\text{arcsinh}(c + dx))^2}{d} + \frac{(c + dx)(a + b\text{arcsinh}(c + dx))^3}{d}$$

```
[Out] 6*a*b^2*x+6*b^3*(d*x+c)*arcsinh(d*x+c)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^3/d-6
*b^3*(1+(d*x+c)^2)^(1/2)/d-3*b*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5772, 5798, 267}

$$\int (a + b\text{arcsinh}(c + dx))^3\, dx = -\frac{3b\sqrt{(c + dx)^2 + 1}(a + b\text{arcsinh}(c + dx))^2}{d} + \frac{(c + dx)(a + b\text{arcsinh}(c + dx))^3}{d} + 6ab^2x + \frac{6b^3(c + dx)\text{arcsinh}(c + dx)}{d} - \frac{6b^3\sqrt{(c + dx)^2 + 1}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[1 + (c + d*x)^2])/d + (6*b^3*(c + d*x)*ArcSinh[c +
d*x])/d - (3*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/d + ((c +
d*x)*(a + b*ArcSinh[c + d*x])^3)/d
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^3\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3}{d} - \frac{(3b)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{(6b^2)\operatorname{Subst}(\int (a+b\operatorname{arcsinh}(x))\,dx, x, c+dx)}{d} \\
&= 6ab^2x - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3}{d} + \frac{(6b^3)\operatorname{Subst}(\int \operatorname{arcsinh}(x)\,dx, x, c+dx)}{d} \\
&= 6ab^2x + \frac{6b^3(c+dx)\operatorname{arcsinh}(c+dx)}{d} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3}{d} - \frac{(6b^3)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d}
\end{aligned}
$$

$$= 6ab^2x - \frac{6b^3\sqrt{1+(c+dx)^2}}{d} + \frac{6b^3(c+dx)\operatorname{arcsinh}(c+dx)}{d}$$
$$- \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3}{d}$$

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int (a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$= \frac{a(a^2+6b^2)(c+dx) - 3b(a^2+2b^2)\sqrt{1+(c+dx)^2} - 3b\Big(-a^2(c+dx) - 2b^2(c+dx) + 2ab\sqrt{1+(c+dx}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[1 + (c + d*x)^2] - 3*b*
(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[
c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x
]^2 + b^3*(c + d*x)*ArcSinh[c + d*x]^3)/d
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)+3a$ |
| default | $\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)+3a$ |
| parts | $x\,a^3+\frac{b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)}{d}+\frac{3}{}$ |

```
[In] int((a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+3*a*b^2*((d*x+c)
*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+3*a^2*b*(
(d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(96) = 192$.

Time $= 0.26$ (sec) , antiderivative size $= 239$, normalized size of antiderivative $= 2.39$

$$\int (a + b\operatorname{arcsinh}(c + dx))^3\,dx$$

$$= \frac{(b^3dx + b^3c)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^3 + (a^3 + 6\,ab^2)dx + 3\left(ab^2dx + ab^2c - \sqrt{d^2x^2 + 2\,cdx +}\right.$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*(a^2*b + 2*b^3))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.

Time $= 0.17$ (sec) , antiderivative size $= 282$, normalized size of antiderivative $= 2.82$

$$\int (a + b\operatorname{arcsinh}(c + dx))^3\,dx$$

$$= \begin{cases} a^3x + \frac{3a^2bc\operatorname{asinh}(c+dx)}{d} + 3a^2bx\operatorname{asinh}(c + dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{3ab^2c\operatorname{asinh}^2(c+dx)}{d} + 3ab^2x\operatorname{asinh}^2(c+ \\ x(a + b\operatorname{asinh}(c))^3 \end{cases}$$

```
[In] integrate((a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*asinh(c + d*x)/d + 3*a**2*b*x*asinh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 3*a*b**2*c*asinh(c + d
*x)**2/d + 3*a*b**2*x*asinh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + b**3*c*asinh(c + d*x)**3/d + 6
*b**3*c*asinh(c + d*x)/d + b**3*x*asinh(c + d*x)**3 + 6*b**3*x*asinh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*(a + b*asinh(c))**
3, True))
```

## Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^3\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^3\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + a^3*x + 3*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b/d + integrate(3*((c^3
+ c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2
+ ((3*c^2*d + d)*a*b^2 - (c^2*d + d)*b^3)*x + ((c^2 + 1)*a*b^2 + (a*b^2*d^2
 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*
x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^3\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^3\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^3\,dx = \int (a + b\,\text{asinh}(c + d\,x))^3\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3, x)
```

# 3.142 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx$

Optimal result . . . 1096
Rubi [A] (verified) . . . 1096
Mathematica [A] (verified) . . . 1100
Maple [B] (verified) . . . 1100
Fricas [F] . . . 1101
Sympy [F] . . . 1101
Maxima [F] . . . 1102
Giac [F] . . . 1102
Mupad [F(-1)] . . . 1102

## Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx &= \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{4bde}\\ &+\frac{(a+b\operatorname{arcsinh}(c+dx))^3\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de}\\ &-\frac{3b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}\\ &-\frac{3b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}\\ &-\frac{3b^3\operatorname{PolyLog}\left(4,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{4de}\end{aligned}$$

```
[Out] 1/4*(a+b*arcsinh(d*x+c))^4/b/d/e+(a+b*arcsinh(d*x+c))^3*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3/2*b*(a+b*arcsinh(d*x+c))^2*polylog(2,1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-3/2*b^2*(a+b*arcsinh(d*x+c))*polylog(3,1/(d*x+c+(1+
(d*x+c)^2)^(1/2))^2)/d/e-3/4*b^3*polylog(4,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)
/d/e
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{5859, 12, 5775, 3797, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx = -\frac{3b^2\operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{2de} - \frac{3b\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{2de} + \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{4bde} + \frac{\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^3}{de} - \frac{3b^3\operatorname{PolyLog}\left(4, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{4de}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcSinh[c + d*x])^3*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^3*PolyLog[4, E^(-2*ArcSinh[c
 + d*x])])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= -\frac{\text{Subst}\left(\int x^3 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\, dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^3}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\, dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de}$$
$$- \frac{3\text{Subst}\left(\int x^2 \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\, dx, x, a+b\text{arcsinh}(c+dx)\right)}{de}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de}$$
$$- \frac{3b(a+b\text{arcsinh}(c+dx))^2 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$
$$+ \frac{(3b)\text{Subst}\left(\int x \,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\, dx, x, a+b\text{arcsinh}(c+dx)\right)}{de}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de}$$
$$- \frac{3b(a+b\text{arcsinh}(c+dx))^2 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$
$$- \frac{3b^2(a+b\text{arcsinh}(c+dx)) \,\text{PolyLog}\left(3, e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$
$$+ \frac{(3b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(3, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\, dx, x, a+b\text{arcsinh}(c+dx)\right)}{2de}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de}$$
$$- \frac{3b(a+b\text{arcsinh}(c+dx))^2 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$
$$- \frac{3b^2(a+b\text{arcsinh}(c+dx)) \,\text{PolyLog}\left(3, e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$
$$- \frac{(3b^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\, dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}\right)}{4de}$$

$$= \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{3b(a+b\text{arcsinh}(c+dx))^2 \text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{2de} - \frac{3b^2(a+b\text{arcsinh}(c+dx)) \text{PolyLog}\left(3, e^{-2\text{arcsinh}(c+dx)}\right)}{2de} - \frac{3b^3 \text{PolyLog}\left(4, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}\right)}{4de}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{b} + 4(a+b\text{arcsinh}(c+dx))^3 \log\left(1-e^{2\text{arcsinh}(c+dx)}\right) + 6b(a+b\text{arcsinh}(c+dx))^2 \text{PolyLo}}{4d}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])^4/b) + 4*(a + b*ArcSinh[c + d*x])^3*Log[1 - E^(
2*ArcSinh[c + d*x])] + 6*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^(2*ArcSinh[c +
d*x])] + 3*b^3*PolyLog[4, E^(2*ArcSinh[c + d*x])])/(4*d*e)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(179) = 358.

Time = 0.47 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.59

| method | result |
|---|---|
| derivativedivides | $\frac{a^3\ln(dx+c)}{e} + \frac{b^3\left(-\frac{\text{arcsinh}(dx+c)^4}{4} + \text{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right) + 3\,\text{arcsinh}(dx+c)^2\,\text{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.\right.$ |
| default | $\frac{a^3\ln(dx+c)}{e} + \frac{b^3\left(-\frac{\text{arcsinh}(dx+c)^4}{4} + \text{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right) + 3\,\text{arcsinh}(dx+c)^2\,\text{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.\right.$ |
| parts | $\frac{a^3\ln(dx+c)}{ed} + \frac{b^3\left(-\frac{\text{arcsinh}(dx+c)^4}{4} + \text{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right) + 3\,\text{arcsinh}(dx+c)^2\,\text{polylog}\left(2,-dx-\right.\right.$ |

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/e*ln(d*x+c)+b^3/e*(-1/4*arcsinh(d*x+c)^4+arcsinh(d*x+c)^3*ln(1+d*x
+c+(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(
```

```
1/2))-6*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+6*polylog(4,-d
*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+
3*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*po
lylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+6*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2)))+
3*a*b^2/e*(-1/3*arcsinh(d*x+c)^3+arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^
(1/2))+2*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*polylog(3,-
d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))
+2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(3,d*x+c+(1
+(d*x+c)^2)^(1/2)))+3*a^2*b/e*(-1/2*arcsinh(d*x+c)^2+arcsinh(d*x+c)*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+
c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3\,\text{asinh}^3\,(c+dx)}{c+dx}\,dx + \int \frac{3ab^2\,\text{asinh}^2\,(c+dx)}{c+dx}\,dx + \int \frac{3a^2b\,\text{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*asinh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
sinh(c + d*x)/(c + d*x), x))/e
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(b^3*log(d*x + c + sqrt((d*x + c)^2 +
 1))^3/(d*e*x + c*e) + 3*a*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*e*
x + c*e) + 3*a^2*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x), x)
```

# 3.143 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal result . . . 1103
Rubi [A] (verified) . . . 1104
Mathematica [A] (verified) . . . 1107
Maple [A] (verified) . . . 1107
Fricas [F] . . . 1108
Sympy [F] . . . 1108
Maxima [F(-2)] . . . 1108
Giac [F] . . . 1109
Mupad [F(-1)] . . . 1109

## Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = &-\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)}\\ &-\frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{6b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{6b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\end{aligned}$$

```
[Out] -(a+b*arcsinh(d*x+c))^3/d/e^2/(d*x+c)-6*b*(a+b*arcsinh(d*x+c))^2*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))/d/e^2+6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^2+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2-6*b
^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5776, 5816, 4267, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = &-\frac{6b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^2}\\ &-\frac{6b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2}\\ &+\frac{6b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2}\\ &-\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)}+\frac{6b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{6b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^3/(d*e^2*(c + d*x))) - (6*b*(a + b*ArcSinh[c + d
*x])^2*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^3*PolyLog[3, -E^ArcSinh[c
 + d*x]])/(d*e^2) - (6*b^3*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{e^2x^2}\,dx, x, c+dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x^2}\,dx, x, c+dx\right)}{de^2} \\ &= -\frac{(a+b\text{arcsinh}(c+dx))^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}\end{aligned}$$

$$= -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} + \frac{(3b)\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^2}$$

$$= -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} - \frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$- \frac{(6b^2)\operatorname{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^2}$$
$$+ \frac{(6b^2)\operatorname{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^2}$$

$$= -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} - \frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$- \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{(6b^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^2}$$
$$- \frac{(6b^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^2}$$

$$= -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} - \frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$- \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{(6b^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$- \frac{(6b^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$

$$= -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} - \frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$- \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$
$$+ \frac{6b^3\operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} - \frac{6b^3\operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.90

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$$
$$= \frac{-\frac{a^3}{c+dx} - \frac{3a^2b\operatorname{arcsinh}(c+dx)}{c+dx} + 3a^2b\log(c+dx) - 3a^2b\log\left(1+\sqrt{1+c^2+2cdx+d^2x^2}\right) + 3ab^2\Big(\operatorname{arcsinh}(c+$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^3/(c + d*x)) - (3*a^2*b*ArcSinh[c + d*x])/(c + d*x) + 3*a^2*b*Log[c +
d*x] - 3*a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] + 3*a*b^2*(ArcSin
h[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])
] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + b^3*(-(ArcSinh[c + d*x]^3/(c + d
*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c +
d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-
ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] +
 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]
))/(d*e^2)
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.27

| method | result |
|---|---|
| derivativedivides | $-\frac{a^3}{e^2(dx+c)}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c$ |
| default | $-\frac{a^3}{e^2(dx+c)}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c$ |
| parts | $-\frac{a^3}{e^2(dx+c)d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-$ |

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/e^2/(d*x+c)+b^3/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)
^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*
x+c)^2)^(1/2))+6*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*l
n(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^
2)^(1/2))-6*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^2*(-1/(d*x+c)*a
rcsinh(d*x+c)^2-2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(
```

```
2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/
2))+2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a^2*b/e^2*(-1/(d*x+c)*arcsinh
(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\,\text{asinh}^3\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\,\text{asinh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\,\text{asinh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*asinh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*asinh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2,x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

# 3.144 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$

Optimal result . . . . . 1110
Rubi [A] (verified) . . . . . 1110
Mathematica [A] (verified) . . . . . 1114
Maple [A] (verified) . . . . . 1114
Fricas [F] . . . . . 1115
Sympy [F] . . . . . 1115
Maxima [F] . . . . . 1115
Giac [F] . . . . . 1116
Mupad [F(-1)] . . . . . 1116

## Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = &\frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3}\\ &-\frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)}\\ &-\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2}\\ &+\frac{3b^2(a+b\operatorname{arcsinh}(c+dx))\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3}\\ &-\frac{3b^3\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de^3}\end{aligned}$$

```
[Out] 3/2*b*(a+b*arcsinh(d*x+c))^2/d/e^3-1/2*(a+b*arcsinh(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3
/2*b^3*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3/2*b*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{5859, 12, 5776, 5800, 5775, 3797, 2221, 2317, 2438\}$

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \frac{3b^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^3} - \frac{3b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} + \frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} - \frac{3b^3\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de^3}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (3*b*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^3) - (3*b*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^3/(
2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcSinh[c + d*x])*Log[1 - E^(-2*ArcSin
h[c + d*x])])/(d*e^3) - (3*b^3*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*d*e^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{e^3x^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x^3}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^3}
\end{aligned}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3b^2)\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{x}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} - \frac{(3b)\operatorname{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{de^3}$$

$$= \frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{(6b)\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{de^3}$$

$$= \frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\operatorname{arcsinh}(c+dx))\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} - \frac{(3b^2)\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{de^3}$$

$$= \frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\operatorname{arcsinh}(c+dx))\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} + \frac{(3b^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de^3}$$

$$= \frac{3b(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\operatorname{arcsinh}(c+dx))\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} - \frac{3b^3\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de^3}$$

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx =$$
$$-\frac{3b^2\left(a+b(c+dx)\left(-c-dx+\sqrt{1+c^2+2cdx+d^2x^2}\right)\right)\operatorname{arcsinh}(c+dx)^2+b^3\operatorname{arcsinh}(c+dx)^3+3b\operatorname{arcsi}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(3*b^2*(a + b*(c + d*x)*(-c - d*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])
)*ArcSinh[c + d*x]^2 + b^3*ArcSinh[c + d*x]^3 + 3*b*ArcSinh[c + d*x]*(a*(a
+ 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - 2*b^2*(c + d*x)^2*Log[
1 - E^(-2*ArcSinh[c + d*x])]) + a*(a*(a + 3*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*
d*x + d^2*x^2]) - 6*b^2*(c + d*x)^2*Log[c + d*x]) + 3*b^3*(c + d*x)^2*PolyL
og[2, E^(-2*ArcSinh[c + d*x])])/(d*e^3*(c + d*x)^2)
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| derivativedivides | $-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(dx+c)\ln\left(1+c\right.\right.}{}$ |
| default | $-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(dx+c)\ln\left(1+c\right.\right.}{}$ |
| parts | $-\frac{a^3}{2e^3(dx+c)^2d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(dx}{}$ |

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(-1/2*arcsinh(d*x+c)^2*(3*(d*x+c)*(1+(d
*x+c)^2)^(1/2)-3*(d*x+c)^2+arcsinh(d*x+c))/(d*x+c)^2-3*arcsinh(d*x+c)^2+3*a
rcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*polylog(2,-d*x-c-(1+(d*x+c)
^2)^(1/2))+3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3*polylog(2,d*x
+c+(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*x+c)*
(-2*(d*x+c)^2+2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2+ln((d
*x+c+(1+(d*x+c)^2)^(1/2))^2-1))+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-
1/2/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e
^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$= \frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\,\text{asinh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\,\text{asinh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\,\text{asinh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

## Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2
*e^3) - log(d*x + c)/(d*e^3))*a*b^2 - 1/2*(log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2*integrate(
3/2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)
+ 1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^5*e^3*x^5 + 5*c*
d^4*e^3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*
d^2*e^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x + (d^4*e^3*x^4 +
 4*c*d^3*e^3*x^3 + c^4*e^3 + c^2*e^3 + (6*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(2
```

```
*c^3*d*e^3 + c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))*b^3 - 3/2*
a^2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsin
h(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3/2*a*b^2*arcsinh(d
*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(d^3*e^3*x^2
+ 2*c*d^2*e^3*x + c^2*d*e^3)
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\operatorname{arsinh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

# 3.145 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal result . . . 1117
Rubi [A] (verified) . . . 1118
Mathematica [B] (warning: unable to verify) . . . 1123
Maple [A] (verified) . . . 1124
Fricas [F] . . . 1124
Sympy [F] . . . 1125
Maxima [F] . . . 1125
Giac [F] . . . 1126
Mupad [F(-1)] . . . 1126

## Optimal result

Integrand size = 23, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = &-\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)}\\ &-\frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2}\\ &-\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3}\\ &+\frac{b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{b^3\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^4}\\ &+\frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\end{aligned}$$

```
[Out] -b^2*(a+b*arcsinh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^3/d/e^4/(d
*x+c)^3+b*(a+b*arcsinh(d*x+c))^2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-b
^3*arctanh((1+(d*x+c)^2)^(1/2))/d/e^4+b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d
*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))/d/e^4-b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+b^
3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-1/2*b*(a+b*arcsinh(d*x+c))^2*(
1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5859, 12, 5776, 5809, 5816, 4267, 2611, 2320, 6724, 272, 65, 213}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = {} & \frac{b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^4} \\ & + \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^4} \\ & - \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^4} \\ & - \frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} \\ & - \frac{b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2} \\ & - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} - \frac{b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\ & + \frac{b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\ & - \frac{b^3\operatorname{arctanh}\left(\sqrt{(c+dx)^2+1}\right)}{de^4}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcSinh[c + d*x]))/(d*e^4*(c + d*x))) - (b*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c +
d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcSinh[c + d*x])^2*ArcTanh[E^Arc
Sinh[c + d*x]])/(d*e^4) - (b^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4) + (b
^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) - (b^2
*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (b^3*Po
lyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (b^3*PolyLog[3, E^ArcSinh[c + d*x]
])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{e^4x^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x^4}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{2de^4(c+dx)^2} - \frac{(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^4} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x^2}\,dx, x, c+dx\right)}{de^4}
\end{aligned}$$

$$= -\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} - \frac{b\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{2de^4}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}$$
$$+ \frac{b^2\operatorname{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^4}$$
$$- \frac{b^2\operatorname{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^4}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{2de^4}$$

$$= -\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2}$$
$$- \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}$$
$$+ \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}$$
$$- \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{de^4}$$
$$- \frac{b^3\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^4}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^x\right)\,dx, x, \operatorname{arcsinh}(c+dx)\right)}{de^4}$$

$$
\begin{aligned}
&= -\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^4} \\
&\quad + \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{b^3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&= -\frac{b^2(a+b\operatorname{arcsinh}(c+dx))}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^2}{2de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^4} \\
&\quad + \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}
\end{aligned}
$$

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 694 vs. $2(261) = 522$.

Time $= 7.25$ (sec) , antiderivative size $= 694$, normalized size of antiderivative $= 2.66$

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = -\frac{a^3}{3de^4(c+dx)^3} - \frac{a^2b\sqrt{1+c^2+2cdx+d^2x^2}}{2de^4(c+dx)^2}$$
$$- \frac{a^2b\operatorname{arcsinh}(c+dx)}{de^4(c+dx)^3} - \frac{a^2b\log(c+dx)}{2de^4} + \frac{a^2b\log\left(1+\sqrt{1+c^2+2cdx+d^2x^2}\right)}{2de^4}$$
$$+ \frac{ab^2\left(-8\,\mathrm{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(c+dx)}\right) - \frac{2\left(-2+4\operatorname{arcsinh}(c+dx)^2+2\cosh(2\operatorname{arcsinh}(c+dx))-3(c+dx)\operatorname{arcsinh}(c+dx)\log\right.}{}\right.}{}$$
$$+ \frac{b^3\left(-24\operatorname{arcsinh}(c+dx)\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)+4\operatorname{arcsinh}(c+dx)^3\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)-6\operatorname{arcsin}\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^3/(d*e^4*(c + d*x)^3) - (a^2*b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d*e^4*(c + d*x)^2) - (a^2*b*ArcSinh[c + d*x])/(d*e^4*(c + d*x)^3) - (a^2*b
*Log[c + d*x])/(2*d*e^4) + (a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]
])/(2*d*e^4) + (a*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])] - (2*(-2 + 4*A
rcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x)*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c + d*x]*Log[1 + E^
(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + 2*
ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x]*Log[1 - E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x]*Log[1 + E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(8*d*e^4) + (b^3*
(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSinh[c + d*x]^3*Coth[
ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c + d*x]/2]^2 - (c
+ d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24*ArcSinh[c + d*x]^
2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*Log[1 + E^(-ArcSin
h[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcSinh[c + d*x]*PolyL
og[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[
c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[3, E^(-ArcS
inh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*x]/2]^2 - (16*ArcS
inh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x
]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[ArcSinh[c + d*x]/2])
)/(48*d*e^4)
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.86

| method | result |
| --- | --- |
| derivativedivides | $-\frac{a^3}{3e^4(dx+c)^3}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+$ |
| default | $-\frac{a^3}{3e^4(dx+c)^3}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+$ |
| parts | $-\frac{a^3}{3e^4(dx+c)^3d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\ln($ |

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/e^4/(d*x+c)^3+b^3/e^4*(-1/6/(d*x+c)^3*arcsinh(d*x+c)*(3*(1+(d
*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+2*arcsinh(d*x+c)^2+6*(d*x+c)^2)+1/2*a
rcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)*polylog(2,-d
*x-c-(1+(d*x+c)^2)^(1/2))-polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/2*arcsinh
(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-arcsinh(d*x+c)*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))+polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-2*arctanh(d*x+c+(1+
(d*x+c)^2)^(1/2)))+3*a*b^2/e^4*(-1/3*((1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d
*x+c)+arcsinh(d*x+c)^2+(d*x+c)^2)/(d*x+c)^3+1/3*arcsinh(d*x+c)*ln(1+d*x+c+(
1+(d*x+c)^2)^(1/2))+1/3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*arcsinh(d
*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/
2)))+3*a^2*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1+(d*x+c)^2)
^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4
*c^3*d*e^4*x + c^4*e^4), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{}{}}{e^4}$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*asinh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*asinh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 + c)*a*b^2
 + (c^3 + c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c*d
^2)*x^2 + (3*(3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 + 3*(c^2
 + 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*(3*a*b^2*c*d + b^3*c*d)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2*b*x + (
c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d
^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*
d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4
*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4
+ 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^4}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4,x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

# 3.146 $\int (ce+dex)^m(a+b\textbf{arcsinh}(c+dx))^4\,dx$

Optimal result . . . . . 1127
Rubi [N/A] . . . . . 1127
Mathematica [N/A] . . . . . 1128
Maple [N/A] (verified) . . . . . 1128
Fricas [N/A] . . . . . 1128
Sympy [N/A] . . . . . 1128
Maxima [N/A] . . . . . 1129
Giac [N/A] . . . . . 1129
Mupad [N/A] . . . . . 1130

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^4}{de(1+m)} - \frac{4b\text{Int}\left(\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^m(a+b\text{arcsinh}(x))^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^4}{de(1+m)} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{1+m}(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de(1+m)}$$

## Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex+ce)^m \left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^4\,dx$$

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int \left(b\,\text{arsinh}\left(dx+c\right)+a\right)^4(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

## Sympy [N/A]

Not integrable

Time = 35.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int \left(e(c+dx)\right)^m \left(a+b\,\text{asinh}\left(c+dx\right)\right)^4\,dx$$

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**4,x)
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**4, x)
```

## Maxima [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 940, normalized size of antiderivative = 40.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^4(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (b^4*d*e^m*x + b^4*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^4/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^4/(d*e*(m + 1)) + integ
rate(-2*(2*((b^4*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^3 - (a*b^3*d
^2*e^m*(m + 1) - b^4*d^2*e^m)*x^2 - 2*(a*b^3*c*d*e^m*(m + 1) - b^4*c*d*e^m)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) + c*e
^m*(m + 1))*a*b^3 - (c^3*e^m + c*e^m)*b^4 + (a*b^3*d^3*e^m*(m + 1) - b^4*d^
3*e^m)*x^3 + 3*(a*b^3*c*d^2*e^m*(m + 1) - b^4*c*d^2*e^m)*x^2 + ((3*c^2*d*e^
m*(m + 1) + d*e^m*(m + 1))*a*b^3 - (3*c^2*d*e^m + d*e^m)*b^4)*x)*(d*x + c)^
m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 3*((a^2*b^2*d^2*e^m
*(m + 1)*x^2 + 2*a^2*b^2*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1)
)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*b^2*d^3*e^m
*(m + 1)*x^3 + 3*a^2*b^2*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e
^m*(m + 1))*a^2*b^2*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*b^2)*(d*x + c
)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 2*((a^3*b*d^2*e^m
*(m + 1)*x^2 + 2*a^3*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1))*
a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^3*b*d^3*e^m*(m +
1)*x^3 + 3*a^3*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e^m*(m +
1))*a^3*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^3*b)*(d*x + c)^m)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m +
1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(
m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^4(dex+ce)^m\,dx$$

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

## Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m (a+b\text{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4, x)
```

# 3.147 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

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Maple [A] (verified) . . . . . 1136
Fricas [B] (verification not implemented) . . . . . 1137
Sympy [B] (verification not implemented) . . . . . 1138
Maxima [F] . . . . . 1139
Giac [F] . . . . . 1141
Mupad [F(-1)] . . . . . 1141

## Optimal result

Integrand size = 23, antiderivative size = 349

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= -\frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} \\
&\quad + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{64d} \\
&\quad - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{32d} - \frac{45b^2e^3(a+b\operatorname{arcsinh}(c+dx))^2}{128d} \\
&\quad - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d} + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d}
\end{aligned}
$$

```
[Out] -45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*a
rcsinh(d*x+c))^2/d-9/16*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c))^2/d+3/16*b^2
*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d-3/32*e^3*(a+b*arcsinh(d*x+c))^4/d+1
/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^4/d+45/64*b^3*e^3*(d*x+c)*(a+b*arcsin
h(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))
*(1+(d*x+c)^2)^(1/2)/d+3/8*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^
2)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= -\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad + \frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{64d} \\
&\quad + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{45b^2e^3(a+b\operatorname{arcsinh}(c+dx))^2}{128d} - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d} + \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^4e^3(c+dx)^2}{128d}
\end{aligned}
$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) + (45*b
^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(64*d) - (
3*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(32*d
) - (45*b^2*e^3*(a + b*ArcSinh[c + d*x])^2)/(128*d) - (9*b^2*e^3*(c + d*x)^
2*(a + b*ArcSinh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcSin
h[c + d*x])^2)/(16*d) + (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x])^3)/(8*d) - (b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*A
rcSinh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^4)/(32*d) + (e^
3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 (a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\ &= \frac{e^3\text{Subst}\left(\int x^3 (a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\ &= \frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^4}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&\quad + \frac{(3b^2e^3)\operatorname{Subst}\left(\int x^3(a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{4d} \\
&= \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&\quad - \frac{(9b^2e^3)\operatorname{Subst}\left(\int x(a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{8d} \\
&\quad - \frac{(3b^3e^3)\operatorname{Subst}\left(\int \frac{x^4(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d} + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d} \\
&\quad + \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} \\
&\quad + \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&\quad + \frac{(3b^4e^3)\operatorname{Subst}\left(\int x^3\,dx, x, c+dx\right)}{32d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{64d} \\
&\quad - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d} + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d} \\
&\quad - \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d} \\
&\quad - \frac{(9b^3e^3)\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{16d} \\
&\quad - \frac{(9b^4e^3)\operatorname{Subst}(\int x\,dx, x, c+dx)}{64d} - \frac{(9b^4e^3)\operatorname{Subst}(\int x\,dx, x, c+dx)}{16d} \\
&= -\frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} \\
&\quad + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{64d} \\
&\quad - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{32d} \\
&\quad - \frac{45b^2e^3(a+b\operatorname{arcsinh}(c+dx))^2}{128d} - \frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.36

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

$$= \frac{e^3\Big(-9b^2(8a^2+5b^2)\,(c+dx)^2+(32a^4+24a^2b^2+3b^4)\,(c+dx)^4+2ab(c+dx)\sqrt{1+(c+dx)^2}(24a^2+45b^2$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*(-9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
 + d*x)^4 + 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(24*a^2 + 45*b^2 - 2*(8*a
^2 + 3*b^2)*(c + d*x)^2) - 6*a*b*(8*a^2 + 15*b^2)*ArcSinh[c + d*x] + 2*b*(c
 + d*x)*(-72*a*b^2*(c + d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 +
72*a^2*b*Sqrt[1 + (c + d*x)^2] + 45*b^3*Sqrt[1 + (c + d*x)^2] - 48*a^2*b*(c
 + d*x)^2*Sqrt[1 + (c + d*x)^2] - 6*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*
ArcSinh[c + d*x] + 3*b^2*(-24*a^2 - 15*b^2 - 24*b^2*(c + d*x)^2 + 64*a^2*(c
 + d*x)^4 + 8*b^2*(c + d*x)^4 + 48*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 32
*a*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 16*b^3*(-3*a +
 8*a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sq
rt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + 4*b^4*(-3 + 8*(c + d*x)^4)*ArcSin
h[c + d*x]^4))/(128*d)
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| derivativedivides | $\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\Big(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}-\frac{3\,\mathrm{arc}}{}$ |
| default | $\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\Big(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}-\frac{3\,\mathrm{arc}}{}$ |
| parts | $\frac{e^3a^4(dx+c)^4}{4d}+\frac{e^3b^4\Big(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}-$ |

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a^4*(d*x+c)^4+e^3*b^4*(1/4*(d*x+c)^4*arcsinh(d*x+c)^4-1/4*(d*x
+c)^3*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/8*(d*x+c)*arcsinh(d*x+c)^3*(1+
(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)^4+3/16*(d*x+c)^4*arcsinh(d*x+c)^2-3/32
*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+45/64*(1+(d*x+c)^2)^(1/2)*(d*
x+c)*arcsinh(d*x+c)+27/128*arcsinh(d*x+c)^2+3/128*(d*x+c)^4-45/128*(d*x+c)^
```

```
2-45/128-9/16*arcsinh(d*x+c)^2*(1+(d*x+c)^2))+4*e^3*a*b^3*(1/4*(d*x+c)^4*ar
csinh(d*x+c)^3-3/16*(d*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9/32*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/32*(d*x+c
)^4*arcsinh(d*x+c)-3/128*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x+c)*(1+(d
*x+c)^2)^(1/2)+27/256*arcsinh(d*x+c)-9/32*(1+(d*x+c)^2)*arcsinh(d*x+c))+6*e
^3*a^2*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh(d*x+c)*(1+
(d*x+c)^2)^(1/2)+3/16*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-3/32*arcsi
nh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+4*e^3*b*a^3*(1/4*(d*x+c)^4*
arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2
)^(1/2)-3/32*arcsinh(d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(319) = 638$.

Time $= 0.29$ (sec) , antiderivative size $= 1241$, normalized size of antiderivative $= 3.56$

$$\int (ce + dex)^3 (a + b\text{arcsinh}(c + dx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c*d^3*e^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3
*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*
b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 + 48*
b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4 + 32*
a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*x + (8*
a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2 + 3*(2*
b^4*c^2 - b^4)*d*e^3*x + (2*b^4*c^3 - 3*b^4*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(8*(8*a^
2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 - 24*(b^4 - 2
*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3
)*d*e^3*x - (24*b^4*c^2 - 8*(8*a^2*b^2 + b^4)*c^4 + 24*a^2*b^2 + 15*b^4)*e^
3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*b^3*c^2 - a*b^
3)*d*e^3*x + (2*a*b^3*c^3 - 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*(8*(8*a^3*b + 3*
a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3 - 24*(3*a*b^3 - 2
*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^3*c - 2*(8*a^3*b + 3*a*b^
3)*c^3)*d*e^3*x - (72*a*b^3*c^2 - 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*a^3*b + 45
*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6*(8*a^2*b^2 + b^4)*c*d^
2*e^3*x^2 - 3*(8*a^2*b^2 + 5*b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d*e^3*x + (2*(8
*a^2*b^2 + b^4)*c^3 - 3*(8*a^2*b^2 + 5*b^4)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(2*(8*a^3*
b + 3*a*b^3)*d^3*e^3*x^3 + 6*(8*a^3*b + 3*a*b^3)*c*d^2*e^3*x^2 - 3*(8*a^3*b
```

```
 + 15*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*d*e^3*x + (2*(8*a^3*b + 3*a*b^3)*c
^3 - 3*(8*a^3*b + 15*a*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2876 vs. $2(325) = 650$.

Time = 1.12 (sec) , antiderivative size = 2876, normalized size of antiderivative = 8.24

$$\int (ce + dex)^3 (a + b\text{arcsinh}(c + dx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*asinh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*asinh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*as
inh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
4 + 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) + a**3*b*d**
3*e**3*x**4*asinh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/4 + 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 - 3
*a**3*b*e**3*asinh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*asinh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2 + 9*a**2*b**2*c
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/4 - 9*a**2*b**2*c*e**3*x/8 + 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*asinh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/4 - 9*a**2*b**2*d*e**3*x**2/16 + 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a**2*b**2*e**3*asinh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*asinh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*asinh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*asinh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*as
inh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*asinh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*asinh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/32 - 9*a*b**3*c**2*e**3*asinh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*asinh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
```

```
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32
 - 9*a*b**3*c*e**3*x*asinh(c + d*x)/4 + 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)**2/(8*d) + 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/(64*d) + a*b**3*d**3*e**3*x**4*asinh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*asinh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32 - 9*a*b**3*d*e**3*x**2*asinh(c + d*x
)/8 + 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**
2/8 + 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/64 - 3*a*b**3*e
**3*asinh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*asinh(c + d*x)/(64*d) + b**4*c
**4*e**3*asinh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*asinh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*asinh(c + d*x)**4 + 3*b**4*c**3*e**3*x*asinh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*asinh(c + d*x)
**4/2 + 9*b**4*c**2*d*e**3*x**2*asinh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/32 - 9*b**4*c**2*e**3*asinh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*asinh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*asinh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*c*e**3*x*asinh(c + d*x)**2/8 - 45*b**
4*c*e**3*x/64 + 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)**3/(8*d) + 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*asinh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*asinh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*d
*e**3*x**2*asinh(c + d*x)**2/16 - 45*b**4*d*e**3*x**2/128 + 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/8 + 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/64 - 3*b**4*e**3*asinh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*asinh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*asinh(c))**4, True))
```

## Maxima [F]

$$\int (ce + dex)^3 (a + b\text{arcsinh}(c + dx))^4 \, dx = \int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*d^3*e^3*x^4 + a^4*c*d^2*e^3*x^3 + 3/2*a^4*c^2*d*e^3*x^2 + 3*(2*x^2*
arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
```

```
2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arcsinh(d*x + c
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^3*b*c*d^2*e^3 + 1/24*(
24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^
2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 -
 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsin
h(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^3*b*d^3*e^3 + a^4*c^3*e^3*x + 4
*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^3*e^3/d + 1/4
*(b^4*d^3*e^3*x^4 + 4*b^4*c*d^2*e^3*x^3 + 6*b^4*c^2*d*e^3*x^2 + 4*b^4*c^3*e
^3*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + integrate((((4*a
*b^3*d^6*e^3 - b^4*d^6*e^3)*x^6 + 6*(4*a*b^3*c*d^5*e^3 - b^4*c*d^5*e^3)*x^5
 + 4*(c^6*e^3 + c^4*e^3)*a*b^3 + (4*(15*c^2*d^4*e^3 + d^4*e^3)*a*b^3 - (15*
c^2*d^4*e^3 + d^4*e^3)*b^4)*x^4 + 4*(4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a*b^3 -
(5*c^3*d^3*e^3 + c*d^3*e^3)*b^4)*x^3 + 2*(6*(5*c^4*d^2*e^3 + 2*c^2*d^2*e^3)
*a*b^3 - (7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^4)*x^2 + 4*(2*(3*c^5*d*e^3 + 2*c
^3*d*e^3)*a*b^3 - (c^5*d*e^3 + c^3*d*e^3)*b^4)*x + ((4*a*b^3*d^5*e^3 - b^4*
d^5*e^3)*x^5 + 4*(c^5*e^3 + c^3*e^3)*a*b^3 + 5*(4*a*b^3*c*d^4*e^3 - b^4*c*d
^4*e^3)*x^4 - 2*(5*b^4*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 + d^3*e^3)*a*b^3)*x^
3 - 2*(5*b^4*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a*b^3)*x^2 - 4*
(b^4*c^4*d*e^3 - (5*c^4*d*e^3 + 3*c^2*d*e^3)*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 6*(a^2*
b^2*d^6*e^3*x^6 + 6*a^2*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 + d^4*e^3)*a^2*
b^2*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a^2*b^2*x^3 + 3*(5*c^4*d^2*e^3 + 2*
c^2*d^2*e^3)*a^2*b^2*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*a^2*b^2*x + (c^6*e
^3 + c^4*e^3)*a^2*b^2 + (a^2*b^2*d^5*e^3*x^5 + 5*a^2*b^2*c*d^4*e^3*x^4 + (1
0*c^2*d^3*e^3 + d^3*e^3)*a^2*b^2*x^3 + (10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a^2*b
^2*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*a^2*b^2*x + (c^5*e^3 + c^3*e^3)*a^2*b^
2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4, x)
```

# 3.148 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . 1142
Rubi [A] (verified) . . . 1143
Mathematica [A] (verified) . . . 1146
Maple [A] (verified) . . . 1147
Fricas [B] (verification not implemented) . . . 1147
Sympy [B] (verification not implemented) . . . 1148
Maxima [F] . . . 1149
Giac [F] . . . 1150
Mupad [F(-1)] . . . 1150

## Optimal result

Integrand size = 23, antiderivative size = 281

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= -\frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{3d} + \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} \\
&\quad + \frac{8be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d}
\end{aligned}$$

```
[Out] -160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d-8/3*b^2*e^2*(d*x+c)*(a+b*arcsinh
(d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^2/d+1/3*e^2*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^4/d+160/27*b^3*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^
2)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/
d+8/9*b*e^2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d-4/9*b*e^2*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 8, 30}

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= \frac{160b^3e^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad + \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} - \frac{8b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{3d} \\
&\quad + \frac{8be^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d} + \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160}{27}b^4e^2x
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) + (160*b^3*e^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^3*e^2*(c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^2*e^2*(c + d*x)
*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (4*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[
c + d*x])^2)/(9*d) + (8*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
)^3)/(9*d) - (4*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^3)/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^2x^2(a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\ &= \frac{e^2\text{Subst}\left(\int x^2(a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\ &= \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^4}{3d} - \frac{(4be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3d}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d} \\
&\quad + \frac{(8be^2)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&\quad + \frac{(4b^2e^2)\operatorname{Subst}\left(\int x^2(a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{3d} \\
&= \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} \\
&\quad + \frac{8be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d} \\
&\quad - \frac{(8b^2e^2)\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{3d} \\
&\quad - \frac{(8b^3e^2)\operatorname{Subst}\left(\int \frac{x^3(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&= -\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{3d} + \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} \\
&\quad + \frac{8be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d} \\
&\quad + \frac{(16b^3e^2)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{27d} \\
&\quad + \frac{(16b^3e^2)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3d} \\
&\quad + \frac{(8b^4e^2)\operatorname{Subst}\left(\int x^2\,dx, x, c+dx\right)}{27d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{3d} + \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} \\
&\quad + \frac{8be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d} \\
&\quad - \frac{(16b^4e^2)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{27d} - \frac{(16b^4e^2)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{3d} \\
&= -\frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{27d} \\
&\quad - \frac{8b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{3d} + \frac{4b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^2}{9d} \\
&\quad + \frac{8be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad - \frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{9d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^4}{3d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.47

$$
\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= \frac{e^2\Big(-24b^2(9a^2+20b^2)(c+dx) + (27a^4+36a^2b^2+8b^4)(c+dx)^3 + 12ab\sqrt{1+(c+dx)^2}(6a^2+40b^2-(3a
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]

[Out] (e^2*(-24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
 + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2]*(6*a^2 + 40*b^2 - (3*a^2 + 2*b^2)*
(c + d*x)^2) + 12*b*(-36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3 + 6*a*b^2*(c +
 d*x)^3 + 18*a^2*b*Sqrt[1 + (c + d*x)^2] + 40*b^3*Sqrt[1 + (c + d*x)^2] - 9
*a^2*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 2*b^3*(c + d*x)^2*Sqrt[1 + (c +
```

```
d*x)^2])*ArcSinh[c + d*x] + 18*b^2*(-12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 +
 2*b^2*(c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2] - 6*a*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 - 36*b^3*(-3*a*(c + d*x)^3 - 2*b*Sqrt[
1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3
+ 27*b^4*(c + d*x)^3*ArcSinh[c + d*x]^4))/(81*d)
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.68

| method | result |
|---|---|
| derivativedivides | $\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx+c)}$ |
| default | $\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx+c)}$ |
| parts | $\frac{e^2a^4(dx+c)^3}{3d}+\frac{e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx}$ |

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*a^4*(d*x+c)^3+e^2*b^4*(1/3*(d*x+c)^3*arcsinh(d*x+c)^4+8/9*arcs
inh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-4/9*(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+c)^
2)^(1/2)-8/3*(d*x+c)*arcsinh(d*x+c)^2+160/27*arcsinh(d*x+c)*(1+(d*x+c)^2)^(
1/2)-160/27*d*x-160/27*c+4/9*(d*x+c)^3*arcsinh(d*x+c)^2-8/27*arcsinh(d*x+c)
*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+8/81*(d*x+c)^3)+4*e^2*a*b^3*(1/3*(d*x+c)^3*a
rcsinh(d*x+c)^3+2/3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^2*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1+(d*x+c
)^2)^(1/2)+2/9*(d*x+c)^3*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1/2))
+6*e^2*a^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*(1+(d*x+c
)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d*x-4/9*c+2
/27*(d*x+c)^3)+4*e^2*b*a^3*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(255) = 510$.

Time = 0.29 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.20

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

$$=\frac{(27\,a^4+36\,a^2b^2+8\,b^4)d^3e^2x^3+3\,(27\,a^4+36\,a^2b^2+8\,b^4)cd^2e^2x^2-3\,(72\,a^2b^2+160\,b^4-(27\,a^4+36\,a^2b^2$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
8*b^4)*c*d^2*e^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2*d*
e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 3
6*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x + 3*a*
b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 - 2*b^4)*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d
^2*e^2*x^2 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x - (12*b^4*c - (9*a
^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2*x + (a*b^
3*c^2 - 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^3*e^2*x^3 +
9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 - 9*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)
*d*e^2*x - 3*(12*a*b^3*c - (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9*a^2*b^2 + 2*b
^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x - (18*a^2*b^2 + 40*b^4 -
(9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 12*((3*a^3*b + 2*a*b^3)*d^2*e^2*x^
2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x - (6*a^3*b + 40*a*b^3 - (3*a^3*b + 2*a*
b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(264) = 528$.

Time = 0.70 (sec) , antiderivative size = 1889, normalized size of antiderivative = 6.72

$$\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*asinh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*asinh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*asinh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/9 + 4*a**3*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a**2*b**2*c**3*e**2*asinh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*asinh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*asinh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*asinh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
```

```
nh(c + d*x)/3 - 8*a**2*b**2*e**2*x/3 + 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*asinh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
sinh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*asinh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*asinh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 8*a*b**3*
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 16*a*b**3*c*e**2*asin
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*asinh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/27 - 16*a*b**3*e**2*x*asinh(c + d*x)/3 + 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) + 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**4*c**3*e**2*asinh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*asinh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*asinh(c + d*x)**4 + 4*b**4*c**2*e**2*x*asinh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asin
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*asinh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*asinh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*c*e**2*asinh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*asinh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*asinh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*e**2*x*as
inh(c + d*x)**2/3 - 160*b**4*e**2*x/27 + 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)*asinh(c + d*x)**3/(9*d) + 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asin
h(c))**4, True))
```

## Maxima [F]

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^4 \, dx = \int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*d^2*e^2*x^3 + a^4*c*d*e^2*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^
2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d
^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*
```

```
d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4
*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2
 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 1
5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*(c^2 + 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*e^2*x + 4*((d*x + c)*arcsinh
(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^2*e^2*x^3 +
 3*b^4*c*d*e^2*x^2 + 3*b^4*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^4 + integrate(2/3*(2*((3*a*b^3*d^5*e^2 - b^4*d^5*e^2)*x^5 + 3*(
c^5*e^2 + c^3*e^2)*a*b^3 + 5*(3*a*b^3*c*d^4*e^2 - b^4*c*d^4*e^2)*x^4 + (3*(
10*c^2*d^3*e^2 + d^3*e^2)*a*b^3 - (10*c^2*d^3*e^2 + d^3*e^2)*b^4)*x^3 + 3*(
(10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^3 - (3*c^3*d^2*e^2 + c*d^2*e^2)*b^4)*x^2
 + 3*((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^3 - (c^4*d*e^2 + c^2*d*e^2)*b^4)*x +
(3*(c^4*e^2 + c^2*e^2)*a*b^3 + (3*a*b^3*d^4*e^2 - b^4*d^4*e^2)*x^4 + 4*(3*a
*b^3*c*d^3*e^2 - b^4*c*d^3*e^2)*x^3 - 3*(2*b^4*c^2*d^2*e^2 - (6*c^2*d^2*e^2
 + d^2*e^2)*a*b^3)*x^2 - 3*(b^4*c^3*d*e^2 - 2*(2*c^3*d*e^2 + c*d*e^2)*a*b^3
)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^3 + 9*(a^2*b^2*d^5*e^2*x^5 + 5*a^2*b^2*c*d^4*e^2*x^4 + (10*c^
2*d^3*e^2 + d^3*e^2)*a^2*b^2*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a^2*b^2*x
^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2)*a^2*b^2*x + (c^5*e^2 + c^3*e^2)*a^2*b^2 +
(a^2*b^2*d^4*e^2*x^4 + 4*a^2*b^2*c*d^3*e^2*x^3 + (6*c^2*d^2*e^2 + d^2*e^2)*
a^2*b^2*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*a^2*b^2*x + (c^4*e^2 + c^2*e^2)*a^2
*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4, x)
```

# 3.149 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . . . 1151
Rubi [A] (verified) . . . . . 1151
Mathematica [A] (verified) . . . . . 1154
Maple [B] (verified) . . . . . 1155
Fricas [B] (verification not implemented) . . . . . 1155
Sympy [B] (verification not implemented) . . . . . 1156
Maxima [F] . . . . . 1157
Giac [F] . . . . . 1157
Mupad [F(-1)] . . . . . 1158

## Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{2d} \\
&\quad + \frac{3b^2e(a+b\operatorname{arcsinh}(c+dx))^2}{4d} + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{2d} \\
&\quad - \frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^4}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^4}{2d}
\end{aligned}$$

```
[Out] 3/4*b^4*e*(d*x+c)^2/d+3/4*b^2*e*(a+b*arcsinh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^2/d+1/4*e*(a+b*arcsinh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(
a+b*arcsinh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2
)^(1/2)/d-b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{5859, 12, 5776, 5812, 5783, 30\}$

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^4\,dx$$
$$= -\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}(a + b\operatorname{arcsinh}(c+dx))}{2d}$$
$$+ \frac{3b^2e(c+dx)^2(a + b\operatorname{arcsinh}(c+dx))^2}{2d} + \frac{3b^2e(a + b\operatorname{arcsinh}(c+dx))^2}{4d}$$
$$- \frac{be(c+dx)\sqrt{(c+dx)^2+1}(a + b\operatorname{arcsinh}(c+dx))^3}{d}$$
$$+ \frac{e(c+dx)^2(a + b\operatorname{arcsinh}(c+dx))^4}{2d} + \frac{e(a + b\operatorname{arcsinh}(c+dx))^4}{4d} + \frac{3b^4e(c+dx)^2}{4d}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x]))/(2*d) + (3*b^2*e*(a + b*ArcSinh[c + d*x])^2)/(4*d) +
(3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/(2*d) - (b*e*(c + d*x)*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/d + (e*(a + b*ArcSinh[c + d*
x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^4}{2d} - \frac{(2be)\text{Subst}\left(\int \frac{x^2(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^4}{2d} + \frac{(be)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&\quad + \frac{(3b^2e)\,\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{3b^2e(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{2d} \\
&\quad - \frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^4}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^4}{2d} \\
&\quad - \frac{(3b^3e)\,\text{Subst}\left(\int \frac{x^2(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{2d} \\
&\quad + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{2d} \\
&\quad - \frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^4}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^4}{2d} \\
&\quad + \frac{(3b^3e)\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} + \frac{(3b^4e)\operatorname{Subst}(\int x\,dx, x, c+dx)}{2d} \\
&= \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{2d} \\
&\quad + \frac{3b^2e(a+b\operatorname{arcsinh}(c+dx))^2}{4d} + \frac{3b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{2d} \\
&\quad - \frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^4}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^4}{2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.54

$$
\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= \frac{e\Big((2a^4+6a^2b^2+3b^4)(c+dx)^2 - 2ab(2a^2+3b^2)(c+dx)\sqrt{1+(c+dx)^2} + 2ab(2a^2+3b^2)\operatorname{arcsinh}(c+dx)\Big)}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]

[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*(c + d*
x)*Sqrt[1 + (c + d*x)^2] + 2*a*b*(2*a^2 + 3*b^2)*ArcSinh[c + d*x] - 2*b*(c
+ d*x)*(-4*a^3*(c + d*x) - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[1 + (c + d*x)^2
] + 3*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 3*b^2*(2*a^2 + b^2 + 4*
a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]
)*ArcSinh[c + d*x]^2 + 4*b^3*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c
 + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^4
))/(4*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(179) = 358$.

Time = 0.08 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

| method | result |
|---|---|
| derivativedivides | $\frac{e\,a^4(dx+c)^2}{2}+e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinh}(dx+c)^2}{2}$ |
| default | $\frac{e\,a^4(dx+c)^2}{2}+e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinh}(dx+c)^2}{2}$ |
| parts | $e\,a^4\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\,a}{}\right.}{}$ |

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a^4*(d*x+c)^2+e*b^4*(1/2*(1+(d*x+c)^2)*arcsinh(d*x+c)^4-(d*x+c)*
arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^4+3/2*arcsinh(d*x+c
)^2*(1+(d*x+c)^2)-3/2*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-3/4*arcsin
h(d*x+c)^2+3/4*(d*x+c)^2+3/4)+4*e*a*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)
-3/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*
(1+(d*x+c)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*
x+c))+6*e*a^2*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+(d*x+c)^2)^(1/
2)*(d*x+c)*arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+4*e*b*a^3
*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(
d*x+c)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.94

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

$$=\frac{(2\,a^4+6\,a^2b^2+3\,b^4)d^2ex^2+2\,(2\,a^4+6\,a^2b^2+3\,b^4)cdex+(2\,b^4d^2ex^2+4\,b^4cdex+(2\,b^4c^2+b^4)e)\log\left(dx\right.}{}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)*
c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 + b^4)*e)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b^3*
c*d*e*x + (2*a*b^3*c^2 + a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(2
```

```
*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x + (2*a^2*b^2 + b
^4 + 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 +
2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x + (2*a^3
*b + 3*a*b^3 + 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4)*d*e*x +
(2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x + (2*a^3*b +
 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(178) = 356$.

Time $= 0.45$ (sec) , antiderivative size $= 1027$, normalized size of antiderivative $= 5.27$

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*asinh(c + d*x)/d
+ 4*a**3*b*c*e*x*asinh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/d + 2*a**3*b*d*e*x**2*asinh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1) + a**3*b*e*asinh(c + d*x)/d + 3*a**2*b**2*c**2*e*asinh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*asinh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*asinh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x) + 3*a**2*b**2*e*asinh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*asinh(c + d*x)**3/d + 3*a*b**3*c**2*e*
asinh(c + d*x)/d + 4*a*b**3*c*e*x*asinh(c + d*x)**3 + 6*a*b**3*c*e*x*asinh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d) + 2*a*b**3*
d*e*x**2*asinh(c + d*x)**3 + 3*a*b**3*d*e*x**2*asinh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/2 + a*b**3*e*asinh(c + d*x)**3/d + 3*a*b*
*3*e*asinh(c + d*x)/(2*d) + b**4*c**2*e*asinh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*asinh(c + d*x)**2/(2*d) + b**4*c*e*x*asinh(c + d*x)**4 + 3*b**4*c*e*x*
asinh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/(2*d) + b**4*d*e*x**2*asinh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*asinh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/2 + b**4*e*asinh(c + d*x)**4/(4*d) + 3*b**4*e*asinh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**4, True))
```

## Maxima [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*a^4*d*e*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*
d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*d*e + a^4*c*e
*x + 4*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c*e/d + 1
/2*(b^4*d*e*x^2 + 2*b^4*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4 + integrate(2*((2*(c^4*e + c^2*e)*a*b^3 + (2*a*b^3*d^4*e - b^4*d^4*e
)*x^4 + 4*(2*a*b^3*c*d^3*e - b^4*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e + d^2*e)*a*
b^3 - (5*c^2*d^2*e + d^2*e)*b^4)*x^2 + 2*(2*(2*c^3*d*e + c*d*e)*a*b^3 - (c^
3*d*e + c*d*e)*b^4)*x + (2*(c^3*e + c*e)*a*b^3 + (2*a*b^3*d^3*e - b^4*d^3*e
)*x^3 + 3*(2*a*b^3*c*d^2*e - b^4*c*d^2*e)*x^2 - 2*(b^4*c^2*d*e - (3*c^2*d*e
 + d*e)*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^4*e*x^4 + 4*a^2*b^2*c*d^3*e*x^3
 + (6*c^2*d^2*e + d^2*e)*a^2*b^2*x^2 + 2*(2*c^3*d*e + c*d*e)*a^2*b^2*x + (c
^4*e + c^2*e)*a^2*b^2 + (a^2*b^2*d^3*e*x^3 + 3*a^2*b^2*c*d^2*e*x^2 + (3*c^2
*d*e + d*e)*a^2*b^2*x + (c^3*e + c*e)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c),
x)
```

## Giac [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\text{arcsinh}(c + dx))^4\,dx = \int (c\,e + d\,e\,x)\,(a + b\,\text{asinh}(c + d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4, x)
```

# 3.150 $\int (a + b\operatorname{arcsinh}(c + dx))^4\,dx$

Optimal result . . . 1159
Rubi [A] (verified) . . . 1159
Mathematica [A] (verified) . . . 1161
Maple [B] (verified) . . . 1161
Fricas [B] (verification not implemented) . . . 1162
Sympy [B] (verification not implemented) . . . 1163
Maxima [F] . . . 1163
Giac [F] . . . 1164
Mupad [F(-1)] . . . 1164

## Optimal result

Integrand size = 12, antiderivative size = 115

$$\int (a + b\operatorname{arcsinh}(c + dx))^4\,dx = 24b^4x - \frac{24b^3\sqrt{1 + (c + dx)^2}(a + b\operatorname{arcsinh}(c + dx))}{d} + \frac{12b^2(c + dx)(a + b\operatorname{arcsinh}(c + dx))^2}{d} - \frac{4b\sqrt{1 + (c + dx)^2}(a + b\operatorname{arcsinh}(c + dx))^3}{d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^4}{d}$$

```
[Out] 24*b^4*x+12*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^2/d+(d*x+c)*(a+b*arcsinh(d*x+c
))^4/d-24*b^3*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-4*b*(a+b*arcsinh(d
*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5772, 5798, 8}

$$\int (a + b\operatorname{arcsinh}(c + dx))^4\,dx = -\frac{24b^3\sqrt{(c + dx)^2 + 1}(a + b\operatorname{arcsinh}(c + dx))}{d} + \frac{12b^2(c + dx)(a + b\operatorname{arcsinh}(c + dx))^2}{d} - \frac{4b\sqrt{(c + dx)^2 + 1}(a + b\operatorname{arcsinh}(c + dx))^3}{d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^4}{d} + 24b^4x$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + (12*
b^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^2)/d - (4*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcSinh[c + d*x])^4)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
 + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^4\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4}{d} - \frac{(4b)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{4b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4}{d} \\
&\quad + \frac{(12b^2)\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{12b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d} - \frac{4b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4}{d} - \frac{(24b^3)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{24b^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{d} \\
&\quad + \frac{12b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d} \\
&\quad - \frac{4b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4}{d} + \frac{(24b^4)\operatorname{Subst}(\int 1\,dx, x, c+dx)}{d} \\
&= 24b^4x - \frac{24b^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{d} + \frac{12b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2}{d} \\
&\quad - \frac{4b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4}{d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.97

$$
\begin{aligned}
&\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= \frac{\left(a^4+12a^2b^2+24b^4\right)(c+dx) - 4ab(a^2+6b^2)\sqrt{1+(c+dx)^2} - 4b\Big(-a^3(c+dx) - 6ab^2(c+dx) + 3a^2b\sqrt{}}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[1 + (c +
d*x)^2] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*b*Sqrt[1 + (c +
 d*x)^2] + 6*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 6*b^2*(a^2*(c +
d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 -
4*b^3*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(
c + d*x)*ArcSinh[c + d*x]^4)/d
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(111) = 222$.

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.13

| method | result |
|---|---|
| derivativedivides | $\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\sqrt{}\right.}{}$ |
| default | $\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\sqrt{}\right.}{}$ |
| parts | $a^4x+\frac{b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\sqrt{1+}\right.}{d}$ |

```
[In] int((a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*((d*x+c)*arcsinh(d*x+c)^4-4*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)+12*(d*x+c)*arcsinh(d*x+c)^2-24*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2
)+24*d*x+24*c)+4*a*b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+6*a^2*b^2*((d*
x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+4*b*a
^3*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(111) = 222$.

Time $= 0.28$ (sec) , antiderivative size $= 344$, normalized size of antiderivative $= 2.99$

$$\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

$$=\frac{(b^4dx+b^4c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4+4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2+1}b^4\right)\log\left(d\right.}{}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(105) = 210.

Time = 0.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.86

$$\int (a + b\operatorname{arcsinh}(c + dx))^4 \, dx$$

$$= \begin{cases} a^4 x + \frac{4a^3 bc \operatorname{asinh}(c+dx)}{d} + 4a^3 bx \operatorname{asinh}(c + dx) - \frac{4a^3 b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{6a^2 b^2 c \operatorname{asinh}^2(c+dx)}{d} + 6a^2 b^2 x \operatorname{asinh}^2(c \\ x(a + b\operatorname{asinh}(c))^4 \end{cases}$$

```
[In] integrate((a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*asinh(c + d*x)/d + 4*a**3*b*x*asinh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 6*a**2*b**2*c*asinh(c
+ d*x)**2/d + 6*a**2*b**2*x*asinh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 4*a*b**3*c*asinh
(c + d*x)**3/d + 24*a*b**3*c*asinh(c + d*x)/d + 4*a*b**3*x*asinh(c + d*x)**
3 + 24*a*b**3*x*asinh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d
 + b**4*c*asinh(c + d*x)**4/d + 12*b**4*c*asinh(c + d*x)**2/d + b**4*x*asin
h(c + d*x)**4 + 12*b**4*x*asinh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**4, Tru
e))
```

## Maxima [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^4 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + a^4*x + 4*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b/d + integrate(2*(2*((c
^3 + c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2)*x
^2 + ((3*c^2*d + d)*a*b^3 - (c^2*d + d)*b^4)*x + ((c^2 + 1)*a*b^3 + (a*b^3*
d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^3
*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 + c)*a^2*b^2 +
(a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

## Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^4 \, dx = \int (b \, \text{arsinh}\,(dx + c) + a)^4 \, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^4 \, dx = \int (a + b \, \text{asinh}(c + d\,x))^4 \, dx$$

```
[In] int((a + b*asinh(c + d*x))^4,x)
[Out] int((a + b*asinh(c + d*x))^4, x)
```

# 3.151 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx$

Optimal result . . . 1165
Rubi [A] (verified) . . . 1166
Mathematica [A] (verified) . . . 1169
Maple [B] (verified) . . . 1170
Fricas [F] . . . 1170
Sympy [F] . . . 1171
Maxima [F] . . . 1171
Giac [F] . . . 1171
Mupad [F(-1)] . . . 1172

## Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx &= \frac{(a+b\operatorname{arcsinh}(c+dx))^5}{5bde}\\ &+\frac{(a+b\operatorname{arcsinh}(c+dx))^4\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de}\\ &-\frac{2b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de}\\ &-\frac{3b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de}\\ &-\frac{3b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(4,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de}\\ &-\frac{3b^4\operatorname{PolyLog}\left(5,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}\end{aligned}$$

```
[Out] 1/5*(a+b*arcsinh(d*x+c))^5/b/d/e+(a+b*arcsinh(d*x+c))^4*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-2*b*(a+b*arcsinh(d*x+c))^3*polylog(2,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3*b^2*(a+b*arcsinh(d*x+c))^2*polylog(3,1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3*b^3*(a+b*arcsinh(d*x+c))*polylog(4,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3/2*b^4*polylog(5,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/
e
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx = &-\frac{3b^3\operatorname{PolyLog}\left(4, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de}\\ &-\frac{3b^2\operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de}\\ &-\frac{2b\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^3}{de}\\ &+\frac{(a+b\operatorname{arcsinh}(c+dx))^5}{5bde}\\ &+\frac{\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^4}{de}\\ &-\frac{3b^4\operatorname{PolyLog}\left(5, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{2de}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcSinh[c + d*x])^4*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^3*(a + b*ArcSinh[c + d*x])*Pol
yLog[4, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, E^(-2*ArcSinh[c
 + d*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{x}\,dx, x, c+dx\right)}{de} \\
&= -\frac{\text{Subst}\left(\int x^4 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^4}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bde} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\text{arcsinh}(c+dx))^4 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{4\text{Subst}\left(\int x^3 \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\text{arcsinh}(c+dx))^4 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{2b(a+b\text{arcsinh}(c+dx))^3 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad + \frac{(6b)\text{Subst}\left(\int x^2 \,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\text{arcsinh}(c+dx))^4 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{2b(a+b\text{arcsinh}(c+dx))^3 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{3b^2(a+b\text{arcsinh}(c+dx))^2 \,\text{PolyLog}\left(3, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad + \frac{(6b^2)\,\text{Subst}\left(\int x \,\text{PolyLog}\left(3, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de} \\
&= \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\text{arcsinh}(c+dx))^4 \log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{2b(a+b\text{arcsinh}(c+dx))^3 \,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{3b^2(a+b\text{arcsinh}(c+dx))^2 \,\text{PolyLog}\left(3, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad - \frac{3b^3(a+b\text{arcsinh}(c+dx)) \,\text{PolyLog}\left(4, e^{-2\text{arcsinh}(c+dx)}\right)}{de} \\
&\quad + \frac{(3b^3)\,\text{Subst}\left(\int \text{PolyLog}\left(4, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de}
\end{aligned}
$$

$$= \frac{(a+b\operatorname{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))^4 \log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3 \operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{3b^2(a+b\operatorname{arcsinh}(c+dx))^2 \operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{3b^3(a+b\operatorname{arcsinh}(c+dx)) \operatorname{PolyLog}\left(4, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{(3b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de}$$

$$= \frac{(a+b\operatorname{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\operatorname{arcsinh}(c+dx))^4 \log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3 \operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{3b^2(a+b\operatorname{arcsinh}(c+dx))^2 \operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{3b^3(a+b\operatorname{arcsinh}(c+dx)) \operatorname{PolyLog}\left(4, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de} - \frac{3b^4 \operatorname{PolyLog}\left(5, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{2de}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx$$
$$= \frac{-\frac{(a+b\operatorname{arcsinh}(c+dx))^5}{5b} + (a+b\operatorname{arcsinh}(c+dx))^4 \log\left(1-e^{2\operatorname{arcsinh}(c+dx)}\right) + 2b(a+b\operatorname{arcsinh}(c+dx))^3 \operatorname{PolyLo}}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]

[Out] (-1/5*(a + b*ArcSinh[c + d*x])^5/b + (a + b*ArcSinh[c + d*x])^4*Log[1 - E^(
2*ArcSinh[c + d*x])] + 2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[3, E^(2*ArcSinh[c
+ d*x])] + 3*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[4, E^(2*ArcSinh[c + d*x])
] - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d*x])])/2)/(d*e)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. 2(222) = 444.

Time = 0.54 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.63

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 861 |
| default | Expression too large to display | 861 |
| parts | Expression too large to display | 872 |

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4/e*ln(d*x+c)+b^4/e*(-1/5*arcsinh(d*x+c)^5+arcsinh(d*x+c)^4*ln(1+d*x
+c+(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)^3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(
1/2))-12*arcsinh(d*x+c)^2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+24*arcsinh(
d*x+c)*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))-24*polylog(5,-d*x-c-(1+(d*x+c)
^2)^(1/2))+arcsinh(d*x+c)^4*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c
)^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-12*arcsinh(d*x+c)^2*polylog(3,d*x+
c+(1+(d*x+c)^2)^(1/2))+24*arcsinh(d*x+c)*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2
))-24*polylog(5,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3/e*(-1/4*arcsinh(d*x+c)^
4+arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*polyl
og(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+
c)^2)^(1/2))+6*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^3*ln(1-
d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)
^(1/2))-6*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+6*polylog(4,d
*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2/e*(-1/3*arcsinh(d*x+c)^3+arcsinh(d*x+c
)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d
*x+c)^2)^(1/2))-2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^2*ln
(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2
)^(1/2))-2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/e*(-1/2*arcsinh(d*
x+c)^2+arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(
d*x+c)^2)^(1/2))+arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d
*x+c+(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx$$
$$= \frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*asinh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*asinh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*asinh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*asinh(c + d*x)/(c + d
*x), x))/e
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^4*log(d*e*x + c*e)/(d*e) + integrate(b^4*log(d*x + c + sqrt((d*x + c)^2 +
 1))^4/(d*e*x + c*e) + 4*a*b^3*log(d*x + c + sqrt((d*x + c)^2 + 1))^3/(d*e*
x + c*e) + 6*a^2*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*e*x + c*e) +
 4*a^3*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x)
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{ce+dex}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^4}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x), x)
```

# 3.152 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$

Optimal result . . . 1173
Rubi [A] (verified) . . . 1174
Mathematica [B] (verified) . . . 1178
Maple [B] (verified) . . . 1179
Fricas [F] . . . 1179
Sympy [F] . . . 1180
Maxima [F(-2)] . . . 1180
Giac [F] . . . 1180
Mupad [F(-1)] . . . 1181

## Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = &-\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{de^2(c+dx)}\\ &-\frac{8b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &-\frac{24b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{24b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\end{aligned}$$

```
[Out] -(a+b*arcsinh(d*x+c))^4/d/e^2/(d*x+c)-8*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,-d*x
-c-(1+(d*x+c)^2)^(1/2))/d/e^2+12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c
+(1+(d*x+c)^2)^(1/2))/d/e^2+24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1
+(d*x+c)^2)^(1/2))/d/e^2-24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*
x+c)^2)^(1/2))/d/e^2-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2+24*
b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5816, 4267, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = &-\frac{8b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^3}{de^2}\\ &+\frac{24b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2}\\ &-\frac{24b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2}\\ &-\frac{12b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^2}\\ &+\frac{12b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^2}\\ &-\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{de^2(c+dx)}\\ &-\frac{24b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\\ &+\frac{24b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^4/(d*e^2*(c + d*x))) - (8*b*(a + b*ArcSinh[c + d
*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (12*b^2*(a + b*ArcSinh[c + d*
x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (12*b^2*(a + b*ArcSinh[c +
 d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (24*b^3*(a + b*ArcSinh[c
 + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^3*(a + b*ArcSinh[
c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^4*PolyLog[4, -E^A
rcSinh[c + d*x]])/(d*e^2) + (24*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{de^2(c+dx)} + \frac{(4b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{de^2(c+dx)} + \frac{(4b)\text{Subst}\left(\int (a+bx)^3\text{csch}(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{de^2(c+dx)} - \frac{8b(a+b\text{arcsinh}(c+dx))^3\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{(12b^2)\,\text{Subst}\left(\int (a+bx)^2\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&\quad + \frac{(12b^2)\,\text{Subst}\left(\int (a+bx)^2\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{de^2(c+dx)} - \frac{8b(a+b\text{arcsinh}(c+dx))^3\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{12b^2(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{12b^2(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{(24b^3)\,\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2,-e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2} \\
&\quad - \frac{(24b^3)\,\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2,e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{de^2(c+dx)} - \frac{8b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{(24b^4)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^2} \\
&\quad + \frac{(24b^4)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^2} \\
&= -\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{de^2(c+dx)} - \frac{8b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{(24b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{(24b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&= -\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{de^2(c+dx)} - \frac{8b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{12b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad + \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} \\
&\quad - \frac{24b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} + \frac{24b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 510 vs. $2(234) = 468$.

Time $= 1.32$ (sec) , antiderivative size $= 510$, normalized size of antiderivative $= 2.18$

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^2}\, dx$$

$$= \frac{-\frac{2a^4}{c+dx} - 8a^3b\left(\frac{\text{arcsinh}(c+dx)}{c+dx} + \log\left(\frac{1}{2}(c+dx)\text{csch}\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right) - \log\left(\sinh\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)\right) +}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]

[Out] ((-2*a^4)/(c + d*x) - 8*a^3*b*(ArcSinh[c + d*x]/(c + d*x) + Log[((c + d*x)*
Csch[ArcSinh[c + d*x]/2])/2] - Log[Sinh[ArcSinh[c + d*x]/2]]) + 12*a^2*b^2*
(ArcSinh[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c
 + d*x])] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c
 + d*x])] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + 8*a*b^3*(-(ArcSinh[c + d*
x]^3/(c + d*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*A
rcSinh[c + d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyL
og[2, -E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c
 + d*x])] + 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh
[c + d*x])]) + b^4*(Pi^4 - 2*ArcSinh[c + d*x]^4 - (2*ArcSinh[c + d*x]^4)/(c
 + d*x) - 8*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] + 8*ArcSinh[c
 + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] + 24*ArcSinh[c + d*x]^2*PolyLog[2, -E
^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]
] + 48*ArcSinh[c + d*x]*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 48*ArcSinh[c +
 d*x]*PolyLog[3, E^ArcSinh[c + d*x]] + 48*PolyLog[4, -E^(-ArcSinh[c + d*x])
] + 48*PolyLog[4, E^ArcSinh[c + d*x]]))/(2*d*e^2)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(299) = 598$.

Time $= 0.47$ (sec) , antiderivative size $= 630$, normalized size of antiderivative $= 2.69$

| method | result |
|---|---|
| derivativedivides | $-\frac{a^4}{e^2(dx+c)}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx-\right.\right.}$ |
| default | $-\frac{a^4}{e^2(dx+c)}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx-\right.\right.}$ |
| parts | $-\frac{a^4}{e^2(dx+c)d}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,\right.\right.}$ |

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4/e^2/(d*x+c)+b^4/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^4-4*arcsinh(d*x+c)
^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-12*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+
(d*x+c)^2)^(1/2))+24*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-2
4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d
*x+c)^2)^(1/2))+12*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-24
*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*polylog(4,d*x+c+(1+
(d*x+c)^2)^(1/2)))+4*a*b^3/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c
)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d
*x+c)^2)^(1/2))+6*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*
ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)
^2)^(1/2))-6*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2/e^2*(-1/(d*x+c
)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*polyl
og(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^
(1/2))+2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/e^2*(-1/(d*x+c)*arcs
inh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$$
$$= \frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b \operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*asinh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*asinh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*asinh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2,x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

# 3.153 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal result . . . . . 1182
Rubi [A] (verified) . . . . . 1183
Mathematica [A] (verified) . . . . . 1186
Maple [B] (verified) . . . . . 1187
Fricas [F] . . . . . 1188
Sympy [F] . . . . . 1188
Maxima [F] . . . . . 1188
Giac [F] . . . . . 1189
Mupad [F(-1)] . . . . . 1189

## Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = {}& \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3}{de^3} \\ & - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{de^3(c+dx)} \\ & - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} \\ & + \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\ & - \frac{6b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\ & - \frac{3b^4\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3}\end{aligned}$$

```
[Out] 2*b*(a+b*arcsinh(d*x+c))^3/d/e^3-1/2*(a+b*arcsinh(d*x+c))^4/d/e^3/(d*x+c)^2
+6*b^2*(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-6
*b^3*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-
3*b^4*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-2*b*(a+b*arcsinh(d*x
+c))^3*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5776, 5800, 5775, 3797, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = &-\frac{6b^3\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^3}\\ &+\frac{6b^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^3}\\ &-\frac{2b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{de^3(c+dx)}\\ &+\frac{2b(a+b\operatorname{arcsinh}(c+dx))^3}{de^3}-\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{2de^3(c+dx)^2}\\ &-\frac{3b^4\operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (2*b*(a + b*ArcSinh[c + d*x])^3)/(d*e^3) - (2*b*Sqrt[1 + (c + d*x)^2]*(a +
b*ArcSinh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^4/(2*d*
e^3*(c + d*x)^2) + (6*b^2*(a + b*ArcSinh[c + d*x])^2*Log[1 - E^(-2*ArcSinh[
c + d*x])])/(d*e^3) - (6*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(-2*ArcS
inh[c + d*x])])/(d*e^3) - (3*b^4*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(d*e^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{e^3x^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{x^3}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{(2b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{de^3(c+dx)} - \frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} \\
&\quad + \frac{(6b^2)\,\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x}\,dx, x, c+dx\right)}{de^3} \\
&= -\frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{de^3(c+dx)} - \frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} \\
&\quad - \frac{(6b)\text{Subst}\left(\int x^2 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de^3} \\
&= \frac{2b(a+b\text{arcsinh}(c+dx))^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{de^3(c+dx)} \\
&\quad - \frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{(12b)\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de^3} \\
&= \frac{2b(a+b\text{arcsinh}(c+dx))^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{de^3(c+dx)} \\
&\quad - \frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{6b^2(a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{de^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{de^3(c+dx)} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{6b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad + \frac{(6b^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx,x,a+b\operatorname{arcsinh}(c+dx)\right)}{de^3} \\
&= \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{de^3(c+dx)} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{6b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{(3b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{de^3} \\
&= \frac{2b(a+b\operatorname{arcsinh}(c+dx))^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{de^3(c+dx)} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} + \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))^2\log\left(1-e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{6b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}(c+dx)}\right)}{de^3} \\
&\quad - \frac{3b^4\operatorname{PolyLog}\left(3,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}\right)}{de^3}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.88

$$
\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx =
$$

$$
-\frac{\frac{a^4}{(c+dx)^2} + \frac{4a^3b\sqrt{1+(c+dx)^2}}{c+dx} + \frac{4a^3b\operatorname{arcsinh}(c+dx)}{(c+dx)^2} + \frac{b^4\operatorname{arcsinh}(c+dx)^4}{(c+dx)^2} - 12a^2b^2\left(-\frac{\sqrt{1+(c+dx)^2}\operatorname{arcsinh}(c+dx)}{c+dx} - \frac{\operatorname{arcs}}{2}\right.}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]

[Out] -1/2*(a^4/(c + d*x)^2 + (4*a^3*b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (4*a^3*
b*ArcSinh[c + d*x])/(c + d*x)^2 + (b^4*ArcSinh[c + d*x]^4)/(c + d*x)^2 - 12
*a^2*b^2*(-((Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)) - ArcSinh[c
```

```
 + d*x]^2/(2*(c + d*x)^2) + Log[c + d*x]) - 4*a*b^3*(ArcSinh[c + d*x]*(3*Ar
cSinh[c + d*x] - (3*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) - Arc
Sinh[c + d*x]^2/(c + d*x)^2 + 6*Log[1 - E^(-2*ArcSinh[c + d*x])]) - 3*PolyL
og[2, E^(-2*ArcSinh[c + d*x])]) - 2*b^4*(2*ArcSinh[c + d*x]^2*(ArcSinh[c +
d*x] - (Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) + 3*Log[1 - E^(-2
*ArcSinh[c + d*x])]) - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-2*ArcSinh[c + d*x]
)] - 3*PolyLog[3, E^(-2*ArcSinh[c + d*x])]))/(d*e^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(206) = 412$.

Time = 0.52 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.02

| method | result |
|---|---|
| derivativedivides | $-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}(dx+c)^2\ln$ |
| default | $-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}(dx+c)^2\ln$ |
| parts | $-\frac{a^4}{2e^3(dx+c)^2d}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}($ |

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^4/e^3/(d*x+c)^2+b^4/e^3*(-1/2*arcsinh(d*x+c)^3*(-4*(d*x+c)^2+4*
(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2-4*arcsinh(d*x+c)^3+6*
arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12*arcsinh(d*x+c)*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+6*ar
csinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*arcsinh(d*x+c)*polylog(2,
d*x+c+(1+(d*x+c)^2)^(1/2))-12*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3
/e^3*(-1/2*arcsinh(d*x+c)^2*(3*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3*(d*x+c)^2+arcs
inh(d*x+c))/(d*x+c)^2-3*arcsinh(d*x+c)^2+3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*
x+c)^2)^(1/2))+3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)*ln(
1-d*x-c-(1+(d*x+c)^2)^(1/2))+3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*
b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*x+c)*(-2*(d*x+c)^2+2*(d*x+c)*(1+(d
*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2+ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1
))+4*b*a^3/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*(1+(d*x+c)^2)^(1/
2)))
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$$
$$= \frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^4\,\mathrm{asinh}^4\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4ab^3\,\mathrm{asinh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{6a^2b^2\,\mathrm{asinh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx +}{e^3}$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*asinh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^3*e^3*x^2 +
2*c*d^2*e^3*x + c^2*d*e^3) - 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh
(d*x + c)/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c)/(d*e^3))*a^2*b^2 - 2*a^3*b
*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x
 + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3*a^2*b^2*arcsinh(d*x +
c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^4/(d^3*e^3*x^2 + 2*c
*d^2*e^3*x + c^2*d*e^3) + integrate(2*(2*(c^3 + c)*a*b^3 + (c^3 + c)*b^4 +
```

```
(2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (2*(3*c^2
*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 + 2*(c^2 + 1)*a*b^3 + (2*a*
b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^6*e^3*x
^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^4*e^3)*x^4 +
 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*x^2
+ 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^
3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e^3 + 3*c*d^2*e^
3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)),
 x)
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\operatorname{arsinh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

# 3.154 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

Optimal result . . . . . 1190
Rubi [A] (verified) . . . . . 1191
Mathematica [B] (warning: unable to verify) . . . . . 1197
Maple [A] (verified) . . . . . 1199
Fricas [F] . . . . . 1199
Sympy [F] . . . . . 1200
Maxima [F] . . . . . 1200
Giac [F] . . . . . 1201
Mupad [F(-1)] . . . . . 1201

## Optimal result

Integrand size = 23, antiderivative size = 385

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = &-\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)}\\ &-\frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2}\\ &-\frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3}\\ &-\frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}\\ &-\frac{4b^4\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{4b^4\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &+\frac{4b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\\ &-\frac{4b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}\end{aligned}$$

```
[Out] -2*b^2*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^4/d/e^
4/(d*x+c)^3-8*b^3*(a+b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d
/e^4+4/3*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-
4*b^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+2*b^2*(a+b*arcsinh(d*x+c)
)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^4-2*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c+(1+(d*x+
c)^2)^(1/2))/d/e^4-4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1+(d*x+c)^2
)^(1/2))/d/e^4+4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/
2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*polylog(4
,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3*b*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2
)^(1/2)/d/e^4/(d*x+c)^2
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5859, 12, 5776, 5809, 5816, 4267, 2611, 6744, 2320, 6724, 2317, 2438}

$$\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx &= -\frac{8b^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^4} \\
&+ \frac{4b\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4} \\
&- \frac{4b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^4} \\
&+ \frac{4b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))}{de^4} \\
&+ \frac{2b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^4} \\
&- \frac{2b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)(a+b\operatorname{arcsinh}(c+dx))^2}{de^4} \\
&- \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} \\
&- \frac{2b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&- \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{4b^4\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&+ \frac{4b^4\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&+ \frac{4b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&- \frac{4b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}
\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (-2*b^2*(a + b*ArcSinh[c + d*x])^2)/(d*e^4*(c + d*x)) - (2*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[
c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcSinh[c + d*x])*ArcTanh
[E^ArcSinh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^A
rcSinh[c + d*x]])/(3*d*e^4) - (4*b^4*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^
4) + (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*
e^4) + (4*b^4*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (2*b^2*(a + b*ArcSi
nh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[4, -
E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^
4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{e^4x^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{x^4}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{(a+b\text{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} + \frac{(4b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4} \\
&= -\frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} - \frac{(a+b\text{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} \\
&\quad - \frac{(2b)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4} \\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x^2}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{2b^2(a+b\text{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\text{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{(2b)\text{Subst}\left(\int (a+bx)^3\text{csch}(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{3de^4} \\
&\quad + \frac{(4b^3)\,\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{2b^2(a+b\text{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\text{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} + \frac{4b(a+b\text{arcsinh}(c+dx))^3\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{3de^4} \\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int (a+bx)^2\log\left(1-e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^4} \\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int (a+bx)^2\log\left(1+e^x\right)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^4} \\
&\quad + \frac{(4b^3)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{arcsinh}(c+dx)\right)}{de^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\
&\quad + \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{(4b^3)\operatorname{Subst}\left(\int (a+bx)\operatorname{PolyLog}\left(2,-e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4} \\
&\quad + \frac{(4b^3)\operatorname{Subst}\left(\int (a+bx)\operatorname{PolyLog}\left(2,e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4} \\
&\quad - \frac{(4b^4)\operatorname{Subst}\left(\int \log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4} \\
&\quad + \frac{(4b^4)\operatorname{Subst}\left(\int \log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4} \\
&= -\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\
&\quad + \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{(4b^4)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{(4b^4)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{(4b^4)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,-e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4} \\
&\quad - \frac{(4b^4)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3,e^x\right)\,dx,x,\operatorname{arcsinh}(c+dx)\right)}{de^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\
&\quad - \frac{4b^4\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^4\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{(4b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{(4b^4)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} \\
&\quad - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} \\
&\quad - \frac{4b^4\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^4\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad - \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} \\
&\quad + \frac{4b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} - \frac{4b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}
\end{aligned}
$$

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1198 vs. $2(385) = 770$.

Time $= 8.24$ (sec) , antiderivative size $= 1198$, normalized size of antiderivative $= 3.11$

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx &= -\frac{a^4}{3de^4(c+dx)^3} \\
&\quad + \frac{a^2b^2\left(-8\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(c+dx)}\right) - \frac{2\left(-2+4\operatorname{arcsinh}(c+dx)^2+2\cosh(2\operatorname{arcsinh}(c+dx))-3(c+dx)\operatorname{arcsinh}(c+dx)\right.}{}\right.}{} \\
&\quad + \frac{ab^3\left(-24\operatorname{arcsinh}(c+dx)\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)+4\operatorname{arcsinh}(c+dx)^3\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)-6\operatorname{arcs}\right.}{} \\
&\quad + \frac{b^4\left(-2\pi^4+4\operatorname{arcsinh}(c+dx)^4-24\operatorname{arcsinh}(c+dx)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)+2\operatorname{arcsinh}(c+dx)^4\coth\right.}{} \\
&\quad + \frac{4a^3b\left(\frac{1}{12}\operatorname{arcsinh}(c+dx)\coth\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)-\frac{1}{24}\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(c+dx)\right)-\frac{1}{24}\operatorname{arcsinh}(c+dx)\coth\right.}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^4/(d*e^4*(c + d*x)^3) + (a^2*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x
])] - (2*(-2 + 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d
*x)*ArcSinh[c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c
 + d*x]*Log[1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSi
nh[c + d*x])] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d
*x]*Log[1 - E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d
*x]*Log[1 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))
/(4*d*e^4) + (a*b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcS
inh[c + d*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh
[c + d*x]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 -
24*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^
2*Log[1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*Ar
cSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*Pol
yLog[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48
*PolyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c +
d*x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3
+ 24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[
ArcSinh[c + d*x]/2]))/(12*d*e^4) + (b^4*(-2*Pi^4 + 4*ArcSinh[c + d*x]^4 - 2
4*ArcSinh[c + d*x]^2*Coth[ArcSinh[c + d*x]/2] + 2*ArcSinh[c + d*x]^4*Coth[A
rcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^2 - ((c
+ d*x)*ArcSinh[c + d*x]^4*Csch[ArcSinh[c + d*x]/2]^4)/2 + 96*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] - 96*ArcSinh[c + d*x]*Log[1 + E^(-ArcSinh
[c + d*x])] + 16*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] - 16*Arc
Sinh[c + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] - 48*(-2 + ArcSinh[c + d*x]^2)*
PolyLog[2, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[2, E^(-ArcSinh[c + d*x])] -
 48*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] - 96*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 96*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] - 96*PolyLog[4, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[4, E^Arc
Sinh[c + d*x]] - 4*ArcSinh[c + d*x]^3*Sech[ArcSinh[c + d*x]/2]^2 - (8*ArcSi
nh[c + d*x]^4*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]
^2*Tanh[ArcSinh[c + d*x]/2] - 2*ArcSinh[c + d*x]^4*Tanh[ArcSinh[c + d*x]/2]
))/(24*d*e^4) + (4*a^3*b*((ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2])/12 -
Csch[ArcSinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2]*
Csch[ArcSinh[c + d*x]/2]^2)/24 + Log[Cosh[ArcSinh[c + d*x]/2]]/6 - Log[Sinh
[ArcSinh[c + d*x]/2]]/6 - Sech[ArcSinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]
*Tanh[ArcSinh[c + d*x]/2])/12 - (ArcSinh[c + d*x]*Sech[ArcSinh[c + d*x]/2]^
2*Tanh[ArcSinh[c + d*x]/2])/24))/(d*e^4)
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 883, normalized size of antiderivative = 2.29

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 883 |
| default | Expression too large to display | 883 |
| parts | Expression too large to display | 894 |

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^4/e^4/(d*x+c)^3+b^4/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)^2*(2*(1+
(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+arcsinh(d*x+c)^2+6*(d*x+c)^2)+2/3*a
rcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)^2*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))-4*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^
2)^(1/2))+4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3*arcsinh(d*x+c)^3*ln(1
-d*x-c-(1+(d*x+c)^2)^(1/2))-2*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2
)^(1/2))+4*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-4*polylog(4,
d*x+c+(1+(d*x+c)^2)^(1/2))-4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))
-4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*
x+c)^2)^(1/2))+4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3/e^4*(-1/6/(d
*x+c)^3*arcsinh(d*x+c)*(3*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+2*arcs
inh(d*x+c)^2+6*(d*x+c)^2)+1/2*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/
2))+arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-polylog(3,-d*x-c-(
1+(d*x+c)^2)^(1/2))-1/2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-ar
csinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(3,d*x+c+(1+(d*x+c
)^2)^(1/2))-2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2/e^4*(-1/3*((1+(
d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+arcsinh(d*x+c)^2+(d*x+c)^2)/(d*x+c)^
3+1/3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*polylog(2,-d*x-c-(
1+(d*x+c)^2)^(1/2))-1/3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*
polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/e^4*(-1/3/(d*x+c)^3*arcsinh(d
*x+c)-1/6/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2)))
)
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\,\operatorname{asinh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\,\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{}{c^4}}{e^4}$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*asinh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*asinh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*asinh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

## Maxima [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 + c)
*a*b^3 + (c^3 + c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 + b
^4*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 +
3*(c^2 + 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(3*a*b^3*c*d + b^4*c*d)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a
^2*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 +
1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (3*c^2*d +
d)*a^3*b*x + (c^3 + c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 + 1)*a
^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^
2*d^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3
*e^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*
d*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4
 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(
5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{asinh}(c+d\,x))^4}{(ce+dex)^4}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^4}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4,x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

# 3.155 $\int \frac{(ce+dex)^m}{a+b\mathbf{arcsinh}(c+dx)}\,dx$

Optimal result . . . . . 1202
Rubi [N/A] . . . . . 1202
Mathematica [N/A] . . . . . 1203
Maple [N/A] (verified) . . . . . 1203
Fricas [N/A] . . . . . 1203
Sympy [N/A] . . . . . 1203
Maxima [N/A] . . . . . 1204
Giac [N/A] . . . . . 1204
Mupad [N/A] . . . . . 1204

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \text{Int}\left(\frac{(e(c+dx))^m}{a+b\text{arcsinh}(c+dx)}, x\right)$$

```
[Out] Unintegrable((e*(d*x+c))^m/(a+b*arcsinh(d*x+c)),x)
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx$$

```
[In] Int[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcSinh[x]), x], x, c + d*x]/d
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d}$$

## Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx$$

```
[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(dex+ce)^m}{a+b\,\text{arcsinh}\left(dx+c\right)}dx$$

```
[In] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
[Out] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^m}{b\,\text{arsinh}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(e(c+dx))^m}{a+b\,\text{asinh}\left(c+dx\right)}\,dx$$

```
[In] integrate((d*e*x+c*e)**m/(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m/(a + b*asinh(c + d*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(c\,e+d\,e\,x)^m}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)), x)
```

# 3.156 $\int \frac{(ce+dex)^4}{a+b\mathbf{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1205
Rubi [A] (verified) . . . 1206
Mathematica [A] (verified) . . . 1208
Maple [A] (verified) . . . 1209
Fricas [F] . . . 1209
Sympy [F] . . . 1209
Maxima [F] . . . 1210
Giac [F] . . . 1210
Mupad [F(-1)] . . . 1210

## Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8bd} - \frac{3e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16bd} + \frac{e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8bd} + \frac{3e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16bd}$$

```
[Out] 1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-3/16*e^4*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/16*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*cosh(5
*a/b)/b/d-1/8*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b/d+3/16*e^4*Shi(3*
(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Shi(5*(a+b*arcsinh(d*x+c))
/b)*sinh(5*a/b)/b/d
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} - \frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} + \frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} + \frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) - (3*e^4*C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) + (e^4*
Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4
*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) + (3*e^4*Sinh[
(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4*Sinh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{x^4}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^4\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4 \text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad - \frac{(3e^4)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&= \frac{\left(e^4 \cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad - \frac{\left(3e^4 \cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{\left(e^4 \cosh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad - \frac{\left(e^4 \sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{\left(3e^4 \sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad - \frac{\left(e^4 \sinh\left(\frac{5a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&= \frac{e^4 \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} - \frac{3e^4 \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} \\
&\quad + \frac{e^4 \cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4 \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} \\
&\quad + \frac{3e^4 \sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4 \sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$
\begin{aligned}
&\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx \\
&= \frac{e^4\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right) - 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\right.\right.\right.
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]

[Out] (e^4*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b
```

```
+ ArcSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3
*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*Sin
hIntegral[5*(a/b + ArcSinh[c + d*x])]))/(16*b*d)
```

## Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| derivativedivides | $-\frac{e^4 \mathrm{e}^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 \mathrm{e}^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 \mathrm{e}^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 \mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(}{16b}$ $d$ |
| default | $-\frac{e^4 \mathrm{e}^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 \mathrm{e}^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 \mathrm{e}^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 \mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(}{16b}$ $d$ |

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32*e^4/b*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a
/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/16*e^4/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/
b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4/b*exp(-3*a/b)*Ei
(1,-3*arcsinh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5
*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\operatorname{arsinh}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b*arcsinh(d*x + c) + a), x)
```

## Sympy [F]

$$\begin{aligned}\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = e^4\Bigg(&\int \frac{c^4}{a+b\,\operatorname{asinh}\left(c+dx\right)}\,dx + \int \frac{d^4x^4}{a+b\,\operatorname{asinh}\left(c+dx\right)}\,dx \\ &+ \int \frac{4cd^3x^3}{a+b\,\operatorname{asinh}\left(c+dx\right)}\,dx + \int \frac{6c^2d^2x^2}{a+b\,\operatorname{asinh}\left(c+dx\right)}\,dx \\ &+ \int \frac{4c^3dx}{a+b\,\operatorname{asinh}\left(c+dx\right)}\,dx\Bigg)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**4*(Integral(c**4/(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/(a + b*
asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*asinh(c + d*x)), x) + I
ntegral(6*c**2*d**2*x**2/(a + b*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(
a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{a+b\,\operatorname{asinh}(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)), x)
```

# 3.157 $\int \frac{(ce+dex)^3}{a+b\mathbf{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1211
Rubi [A] (verified) . . . 1211
Mathematica [A] (verified) . . . 1214
Maple [A] (verified) . . . 1214
Fricas [F] . . . 1215
Sympy [F] . . . 1215
Maxima [F] . . . 1215
Giac [F] . . . 1215
Mupad [F(-1)] . . . 1216

## Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e^3\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8bd}$$

```
[Out] -1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*
Shi(4*(a+b*arcsinh(d*x+c))/b)/b/d+1/4*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*sin
h(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b/d
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{5859, 12, 5780, 5556, 3384, 3379, 3382\}$

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx = \frac{e^3 \sinh\left(\frac{2a}{b}\right) \text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd} - \frac{e^3 \sinh\left(\frac{4a}{b}\right) \text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8bd} - \frac{e^3 \cosh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^3 \cosh\left(\frac{4a}{b}\right) \text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8bd}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(4*b*d) -
(e^3*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(4*a)/b])/(8*b*d) -
(e^3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) +
(e^3*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(8*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e^3\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e^3\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&= -\frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad + \frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{\left(e^3\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad - \frac{\left(e^3\sinh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd}
\end{aligned}
$$

$$= \frac{e^3 \mathrm{Chi}\left(\frac{2(a+b\mathrm{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3 \mathrm{Chi}\left(\frac{4(a+b\mathrm{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd}$$
$$- \frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\mathrm{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\mathrm{arcsinh}(c+dx))}{b}\right)}{8bd}$$

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{(ce+dex)^3}{a+b\mathrm{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^3\left(2\mathrm{Chi}\left(2\left(\frac{a}{b}+\mathrm{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \mathrm{Chi}\left(4\left(\frac{a}{b}+\mathrm{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right) - 2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}\right.\right.\right.}{8bd}$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - CoshIntegr
al[4*(a/b + ArcSinh[c + d*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c + d*x])]))/(8*b*d)
```

## Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| derivativedivides | $\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b} - \frac{e^3 e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b} + \frac{e^3 e^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{8b} - \frac{e^3 e^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arc}\right.}{16b}$ |
| default | $\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b} - \frac{e^3 e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b} + \frac{e^3 e^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{8b} - \frac{e^3 e^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arc}\right.}{16b}$ |

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^3}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcsi
nh(d*x + c) + a), x)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx = e^3\left(\int \frac{c^3}{a+b\,\operatorname{asinh}(c+dx)}\,dx + \int \frac{d^3x^3}{a+b\,\operatorname{asinh}(c+dx)}\,dx + \int \frac{3cd^2x^2}{a+b\,\operatorname{asinh}(c+dx)}\,dx + \int \frac{3c^2dx}{a+b\,\operatorname{asinh}(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*asinh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^3}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^3}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)), x)
```

# 3.158 $\int \frac{(ce+dex)^2}{a+b\mathbf{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1217
Rubi [A] (verified) . . . 1217
Mathematica [A] (verified) . . . 1220
Maple [A] (verified) . . . 1220
Fricas [F] . . . 1221
Sympy [F] . . . 1221
Maxima [F] . . . 1221
Giac [F] . . . 1221
Mupad [F(-1)] . . . 1222

## Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx = -\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4bd} + \frac{e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4bd} - \frac{e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4bd}$$

```
[Out] -1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d+1/4*e^2*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/4*e^2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)
/b/d-1/4*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{5859, 12, 5780, 5556, 3384, 3379, 3382\}$

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4bd} + \frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4bd} - \frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/4*(e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) + (e^2*
Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) + (e^2*
Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b*d) - (e^2*Sinh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2 x^2}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^2\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&= -\frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad + \frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad + \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad - \frac{\left(e^2\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd}
\end{aligned}
$$

$$= -\frac{e^2 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4bd} + \frac{e^2 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^2 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4bd} - \frac{e^2 \sinh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4bd}$$

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^2\left(-\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcs}\right.\right.}{4bd}$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*Cos
hIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b
*d)
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{e^2 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3\operatorname{arcsinh}(d\right.}{8b}}{d}$ |
| default | $-\frac{\frac{e^2 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3\operatorname{arcsinh}(d\right.}{8b}}{d}$ |

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/8*
e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arcsinh(d*x + c) + a), x)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx = e^2\left(\int \frac{c^2}{a+b\,\operatorname{asinh}(c+dx)}\,dx + \int \frac{d^2x^2}{a+b\,\operatorname{asinh}(c+dx)}\,dx + \int \frac{2cdx}{a+b\,\operatorname{asinh}(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c)),x)
[Out] e**2*(Integral(c**2/(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
asinh(c + d*x)), x) + Integral(2*c*d*x/(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)), x)
```

# 3.159 $\int \frac{ce+dex}{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1223
Rubi [A] (verified) . . . 1223
Mathematica [A] (verified) . . . 1225
Maple [A] (verified) . . . 1225
Fricas [F] . . . 1226
Sympy [F] . . . 1226
Maxima [F] . . . 1226
Giac [F] . . . 1226
Mupad [F(-1)] . . . 1227

## Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{ce+dex}{a+b\operatorname{arcsinh}(c+dx)}\,dx = -\frac{e\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2bd}$$

```
[Out] 1/2*e*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arcsin
h(d*x+c))/b)*sinh(2*a/b)/b/d
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$\int \frac{ce+dex}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2bd} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2bd}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/2*(e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(b*d) +
 (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{ex}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{e\text{Subst}\left(\int \frac{x}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&\quad - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&= -\frac{e\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2bd}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$
\begin{aligned}
&\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx \\
&= -\frac{e\left(\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(c+dx)\right)\sinh\left(\frac{2a}{b}\right) - \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(c+dx)\right)\right)}{2bd}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]))/(b*d)
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{e\,e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b} - \frac{e\,e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$ | 66 |
| default | $\frac{\frac{e\,e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b} - \frac{e\,e^{-\frac{2a}{b}}\,\text{Ei}_1\left(-2\,\text{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$ | 66 |

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

## Fricas [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

## Sympy [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = e\left(\int \frac{c}{a+b\,\text{asinh}\,(c+dx)}\,dx + \int \frac{dx}{a+b\,\text{asinh}\,(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
[Out] e*(Integral(c/(a + b*asinh(c + d*x)), x) + Integral(d*x/(a + b*asinh(c + d*
x)), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{a + b\text{arcsinh}(c + dx)}\,dx = \int \frac{c\,e + d\,e\,x}{a + b\,\text{asinh}\,(c + d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x)), x)
```

# 3.160 $\int \frac{1}{a+b\mathbf{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1228
Rubi [A] (verified) . . . 1228
Mathematica [A] (verified) . . . 1230
Maple [A] (verified) . . . 1230
Fricas [F] . . . 1230
Sympy [F] . . . 1231
Maxima [F] . . . 1231
Giac [F] . . . 1231
Mupad [F(-1)] . . . 1231

## Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{bd}$$

```
[Out] Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b/d
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5858, 5774, 3384, 3379, 3382}

$$\int \frac{1}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{bd}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) - (Sinh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{bd}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}{bd}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c + d*x]])/(b*d)
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$ | 60 |
| default | $\frac{-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$ | 60 |

```
[In] int(1/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(d*x+c)-a/b))
```

## Fricas [F]

$$\int \frac{1}{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \frac{1}{b\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(d*x + c) + a), x)
```

## Sympy [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{1}{a+b\,\text{asinh}\,(c+dx)}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c)),x)
[Out] Integral(1/(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{1}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x)),x)
[Out] int(1/(a + b*asinh(c + d*x)), x)
```

# 3.161 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))}\,dx$

Optimal result . . . 1232
Rubi [N/A] . . . 1232
Mathematica [N/A] . . . 1233
Maple [N/A] (verified) . . . 1233
Fricas [N/A] . . . 1233
Sympy [N/A] . . . 1233
Maxima [N/A] . . . 1234
Giac [N/A] . . . 1234
Mupad [N/A] . . . 1234

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c)),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\left(dx+c\right)+a)}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \frac{\int \frac{1}{ac+adx+bc\,\text{asinh}\left(c+dx\right)+bdx\,\text{asinh}\left(c+dx\right)}\,dx}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
[Out] Integral(1/(a*c + a*d*x + b*c*asinh(c + d*x) + b*d*x*asinh(c + d*x)), x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}\,(c+d\,x)\right)}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))), x)
```

# 3.162 $\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . 1235
Rubi [A] (verified) . . . 1236
Mathematica [A] (verified) . . . 1239
Maple [B] (verified) . . . 1239
Fricas [F] . . . 1240
Sympy [F] . . . 1240
Maxima [F] . . . 1241
Giac [F] . . . 1242
Mupad [F(-1)] . . . 1242

## Optimal result

Integrand size = 23, antiderivative size = 256

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = &-\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{e^4\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2d}\\ &+\frac{9e^4\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2d}\\ &-\frac{5e^4\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2d}\\ &+\frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^2d}\\ &-\frac{9e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d}\\ &+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d}\end{aligned}$$

```
[Out] 1/8*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-9/16*e^4*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+5/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arcsinh(d
*x+c))/b)/b^2/d-1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^2/d+9/16*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Chi(5*(a+b*arcsi
nh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*
arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = &-\frac{e^4 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^2d} \\ &+\frac{9e^4 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\ &-\frac{5e^4 \sinh\left(\frac{5a}{b}\right) \operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\ &+\frac{e^4 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^2d} \\ &-\frac{9e^4 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\ &+\frac{5e^4 \cosh\left(\frac{5a}{b}\right) \operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\ &-\frac{e^4(c+dx)^4\sqrt{(c+dx)^2+1}}{bd(a+b\operatorname{arcsinh}(c+dx))}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^4*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(8*b^2*d) + (9*e^
4*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(16*b^2*d) -
(5*e^4*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(16*b^2*
d) + (e^4*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b^2*d) - (
9*e^4*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b^2*d
) + (5*e^4*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*
b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} + \frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} - \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^4\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d} \\
&\quad - \frac{(5e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&\quad + \frac{(9e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d} \\
&\quad - \frac{\left(9e^4\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&\quad + \frac{\left(5e^4\cosh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&\quad - \frac{\left(e^4\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d} \\
&\quad + \frac{\left(9e^4\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&\quad - \frac{\left(5e^4\sinh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^4\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2d} \\
&\quad + \frac{9e^4\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2d} - \frac{5e^4\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2d} \\
&\quad + \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^2d} - \frac{9e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\
&\quad + \frac{5e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{16b^2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.10

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$$
$$= \frac{e^4\left(-\frac{16b(c+dx)^4\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)}+16\left(3\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\sinh\left(\right.\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
16*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - CoshIntegral[3*(a/b
+ ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh
[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) - 5*(1
0*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b +
ArcSinh[c + d*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]
*Sinh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcSinh[c + d*x])])))/(16*b^2*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(242) = 484$.

Time = 0.97 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.35

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx+5c\right)e^4}{32b(a+b\ \operatorname{arcsinh}(dx+c))}+\frac{5e^4\mathrm{e}^{\frac{5a}{b}}\ \mathrm{Ei}_1\left(5\ \operatorname{arcsinh}(d$ |
| default | $\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx+5c\right)e^4}{32b(a+b\ \operatorname{arcsinh}(dx+c))}+\frac{5e^4\mathrm{e}^{\frac{5a}{b}}\ \mathrm{Ei}_1\left(5\ \operatorname{arcsinh}(d$ |

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4/b/(a+b*arcs
inh(d*x+c))+5/32*e^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-3/32*(-4*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4
/b/(a+b*arcsinh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b
)+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4/b/(a+b*arcsinh(d*x+c))+1/16*e^4/b^2
*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(
a+b*arcsinh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32/b
*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(
```

```
1/2))/(a+b*arcsinh(d*x+c))+9/32/b^2*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-
3*a/b)-1/32/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/
2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arc
sinh(d*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

## Sympy [F]

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = e^4\Bigg(&\int \frac{c^4}{a^2+2ab\,\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\\ &+\int \frac{d^4x^4}{a^2+2ab\,\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\\ &+\int \frac{4cd^3x^3}{a^2+2ab\,\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\\ &+\int \frac{6c^2d^2x^2}{a^2+2ab\,\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\\ &+\int \frac{4c^3dx}{a^2+2ab\,\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\Bigg)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x)
+ b**2*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*asinh(c +
 d*x) + b**2*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5
*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^
3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d
*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*
e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 +
 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b
*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((5*d^8*e^4
*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 + 10*c^6*e^4 + 5*c^4*e^4 + 10*(14*c^2*d
^6*e^4 + d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 + 3*c*d^5*e^4)*x^5 + 5*(70*c^4*d
^4*e^4 + 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 + 10*c^3*d^3*e^
4 + c*d^3*e^4)*x^3 + 10*(14*c^6*d^2*e^4 + 15*c^4*d^2*e^4 + 3*c^2*d^2*e^4)*x
^2 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4 + 3*c^4*e^4 + 3*(25*c^2*
d^4*e^4 + d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 + 3*c*d^3*e^4)*x^3 + 3*(25*c^4*d
^2*e^4 + 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 + 2*c^3*d*e^4)*x)*(d^2*x^2 + 2
*c*d*x + c^2 + 1) + 20*(2*c^7*d*e^4 + 3*c^5*d*e^4 + c^3*d*e^4)*x + (10*d^7*
e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 + 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2
*d^5*e^4 + 13*d^5*e^4)*x^5 + 5*(70*c^3*d^4*e^4 + 13*c*d^4*e^4)*x^4 + 2*(175
*c^4*d^3*e^4 + 65*c^2*d^3*e^4 + 2*d^3*e^4)*x^3 + 2*(105*c^5*d^2*e^4 + 65*c^
3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (70*c^6*d*e^4 + 65*c^4*d*e^4 + 12*c^2*d*e^4)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2, x)
```

# 3.163 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . 1243
Rubi [A] (verified) . . . 1244
Mathematica [A] (verified) . . . 1246
Maple [B] (verified) . . . 1247
Fricas [F] . . . 1247
Sympy [F] . . . 1247
Maxima [F] . . . 1248
Giac [F] . . . 1249
Mupad [F(-1)] . . . 1249

## Optimal result

Integrand size = 23, antiderivative size = 188

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} + \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} - \frac{e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d}$$

```
[Out] -1/2*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+b
*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^2/d+1/2*e^3*Shi(2*(a+b*arcsinh(d*x+c))/b)
*sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b^2/d-
e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = &-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d}\\ &+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d}\\ &+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d}\\ &-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d}\\ &-\frac{e^3(c+dx)^3\sqrt{(c+dx)^2+1}}{bd(a+b\text{arcsinh}(c+dx))}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*d)
 + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*
d) + (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^
2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*
b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{e^3\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{2x} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^2 d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^2 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad + \frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad + \frac{\left(e^3\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad - \frac{\left(e^3\sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} \\
&\quad + \frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} \\
&\quad + \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d} - \frac{e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.03

$$
\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx =
$$

$$
-\frac{e^3\left(\frac{2b(c+dx)^3\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)} + \cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right) - \cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(4\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\right.
$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]

[Out] -1/2*(e^3*((2*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])
 + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c + d*x])] - 3*Log[a + b*ArcSinh[c + d*x]] - 4
*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])] + 3*(Log[a + b*ArcS
inh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]) + S
inh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(178) = 356.

Time = 0.91 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.06

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\ \mathrm{arcsinh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\ \mathrm{Ei}_1\left(4\ \mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{4b^2}-\frac{\left(-2(dx+c)\sqrt{1}\right.}{8b(a}$ |
| default | $\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\ \mathrm{arcsinh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\ \mathrm{Ei}_1\left(4\ \mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{4b^2}-\frac{\left(-2(dx+c)\sqrt{1}\right.}{8b(a}$ |

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+c
)^2)^(1/2)+8*(d*x+c)^2+1)*e^3/b/(a+b*arcsinh(d*x+c))-1/4*e^3/b^2*exp(4*a/b)
*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)
^2+1)*e^3/b/(a+b*arcsinh(d*x+c))+1/4*e^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+
c)+2*a/b)+1/8/b*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcs
inh(d*x+c))+1/4/b^2*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/16/b*e^
3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4*arcsi
nh(d*x+c)-4*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*arc
sinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = e^3\left(\int \frac{c^3}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{d^3x^3}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{3cd^2x^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{3c^2dx}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2
*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\operatorname{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^4
*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*d^
2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^3*x
^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e^3 +
 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 +
2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2
*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((4*d^7*e^3*x^7
 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c^2*d^5*e^3
 + 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 + 2*c*d^4*e^3)*x^4 + 4*(35*c^4*d^3*e^
3 + 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(21*c^5*d^2*e^3 + 20*c^3*d^2*e^3 + 3*
c*d^2*e^3)*x^2 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3*x^4 + 2*c^5*e^3 + c^3*e^3
+ (20*c^2*d^3*e^3 + d^3*e^3)*x^3 + (20*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (10
*c^4*d*e^3 + 3*c^2*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(7*c^6*d*e^3
 + 10*c^4*d*e^3 + 3*c^2*d*e^3)*x + (8*d^6*e^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^
6*e^3 + 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2*d^4*e^3 + d^4*e^3)*x^4 + 40*(4*
c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(40*c^4*d^2*e^3 + 20*c^2*d^2*e^3 + d^2*e^3
)*x^2 + 2*(24*c^5*d*e^3 + 20*c^3*d*e^3 + 3*c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2
 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)
*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c
)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2, x)
```

# 3.164 $\int \frac{(ce+dex)^2}{(a+b\mathbf{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . 1250
Rubi [A] (verified) . . . 1251
Mathematica [A] (verified) . . . 1253
Maple [A] (verified) . . . 1254
Fricas [F] . . . 1254
Sympy [F] . . . 1254
Maxima [F] . . . 1255
Giac [F] . . . 1256
Mupad [F(-1)] . . . 1256

## Optimal result

Integrand size = 23, antiderivative size = 184

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx &= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))}\\ &+\frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2d}\\ &-\frac{3e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2d}\\ &-\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4b^2d}\\ &+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4b^2d}\end{aligned}$$

```
[Out] -1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d+3/4*e^2*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^2/d-3/4*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = &\frac{e^2 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4b^2d}\\ &-\frac{3e^2 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4b^2d}\\ &-\frac{e^2 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4b^2d}\\ &+\frac{3e^2 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4b^2d}\\ &-\frac{e^2(c+dx)^2\sqrt{(c+dx)^2+1}}{bd(a+b\operatorname{arcsinh}(c+dx))}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) +
 (e^2*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(4*b^2*d) - (3*e^
2*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(4*b^2*d) - (
e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b^2*d) + (3*e^2*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2 x^2}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{e^2\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{e^2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4b^2 d} \\
&\quad - \frac{(3e^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4b^2 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad + \frac{\left(3e^2\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad + \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad - \frac{\left(3e^2\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2d} \\
&\quad - \frac{3e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2d} - \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{4b^2d} \\
&\quad + \frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{4b^2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$
\begin{aligned}
&\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx \\
&= \frac{e^2\left(-\frac{4b(c+dx)^2\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)} + \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right) - 3\operatorname{Chi}\left(3\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right) -\right.}{4b^2d}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]

[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Arc
Sinh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b^2*d)
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.86

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{3e^2e^{\frac{3a}{b}}\ \mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b^2}-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}$ |
| default | $\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{3e^2e^{\frac{3a}{b}}\ \mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b^2}-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}$ |

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+
3*d*x+3*c)*e^2/b/(a+b*arcsinh(d*x+c))+3/8*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh
(d*x+c)+3*a/b)-1/8*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2/b/(a+b*arcsinh(d*x+c))-
1/8*e^2/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/8/b*e^2*(d*x+c+(1+(d*x+c)^2
)^(1/2))/(a+b*arcsinh(d*x+c))+1/8/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/
b)-1/8/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x
+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(
d*x+c)-3*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arcsinh(d*x + c)^2 + 2*
a*b*arcsinh(d*x + c) + a^2), x)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = e^2\left(\int \frac{c^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{d^2x^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{2cdx}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d
*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e^2 + d^3
*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2
)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^2 +
 d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*
c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a
*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((3*d^6*e^2*x^6 + 18*
c*d^5*e^2*x^5 + 3*c^6*e^2 + 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 + 2*d^4*e^2)*x^4
+ 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + 3*(15*c^4*d^2*e^2 + 12
*c^2*d^2*e^2 + d^2*e^2)*x^2 + (3*d^4*e^2*x^4 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2
 + c^2*e^2 + (18*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(6*c^3*d*e^2 + c*d*e^2)*x)*
(d^2*x^2 + 2*c*d*x + c^2 + 1) + 6*(3*c^5*d*e^2 + 4*c^3*d*e^2 + c*d*e^2)*x +
 (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6*c^5*e^2 + 7*c^3*e^2 + (60*c^2*d^3*e^
2 + 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c^3*d^2*e^2 + 7*c*d^2*e^2)*x^2 + (30*c
^4*d*e^2 + 21*c^2*d*e^2 + 2*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a
*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d
)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^
2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2
+ d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2, x)
```

# 3.165 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . 1257
Rubi [A] (verified) . . . 1257
Mathematica [A] (verified) . . . 1259
Maple [A] (verified) . . . 1260
Fricas [F] . . . 1260
Sympy [F] . . . 1260
Maxima [F] . . . 1261
Giac [F] . . . 1261
Mupad [F(-1)] . . . 1262

## Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d}$$

```
[Out] e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arcsinh(d*x+
c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x
+c))
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d} - \frac{e\sqrt{(c+dx)^2+1}(c+dx)}{bd(a+b\operatorname{arcsinh}(c+dx))}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) + (e*
Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d) - (e*Si
nh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&\quad - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^2d} \\
&\quad - \frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^2d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}\,dx \\
&= \frac{e\left(-\frac{b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}}{a+b\text{arcsinh}(c+dx)} + \cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx\right.\right.\right.}{b^2d}
\end{aligned}$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(a + b*ArcSinh[c + d*
x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\operatorname{arcsinh}(dx+c))}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\operatorname{Ei}_1\left(2\,\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{4b(a+b\,\operatorname{arcsinh}(dx+c))}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\operatorname{Ei}_1}{d}$ |
| default | $\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\operatorname{arcsinh}(dx+c))}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\operatorname{Ei}_1\left(2\,\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{4b(a+b\,\operatorname{arcsinh}(dx+c))}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\operatorname{Ei}_1}{d}$ |

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e/b/(a+b*arcsinh(d*
x+c))-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4/b*e*(2*(d*x+c)^
2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^2*e*exp(-2*a/
b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

## Fricas [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{dex+ce}{\left(b\,\operatorname{arsinh}\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a
^2), x)
```

## Sympy [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = e\left(\int \frac{c}{a^2+2ab\,\operatorname{asinh}\left(c+dx\right)+b^2\,\operatorname{asinh}^2\left(c+dx\right)}\,dx + \int \frac{dx}{a^2+2ab\,\operatorname{asinh}\left(c+dx\right)+b^2\,\operatorname{asinh}^2\left(c+dx\right)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^2}\,dx = \int \frac{dex + ce}{\left(b\,\operatorname{arsinh}\left(dx + c\right) + a\right)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e + c^2*e + (6*c^2*d^2*e + d^2*e)*x^2 + 2
*(2*c^3*d*e + c*d*e)*x + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + c*e + (3*c^2*
d*e + d*e)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2
*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^
2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e + 4*c^3*e +
 4*(5*c^2*d^3*e + d^3*e)*x^3 + 4*(5*c^3*d^2*e + 3*c*d^2*e)*x^2 + 2*(d^3*e*x
^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2
*c*e + 2*(5*c^4*d*e + 6*c^2*d*e + d*e)*x + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 +
4*c^4*e + 4*c^2*e + 4*(6*c^2*d^2*e + d^2*e)*x^2 + 8*(2*c^3*d*e + c*d*e)*x +
 e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^2}\,dx = \int \frac{dex + ce}{\left(b\,\operatorname{arsinh}\left(dx + c\right) + a\right)^2}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2, x)
```

# 3.166 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . 1263
Rubi [A] (verified) . . . 1263
Mathematica [A] (verified) . . . 1265
Maple [A] (verified) . . . 1266
Fricas [F] . . . 1266
Sympy [F] . . . 1266
Maxima [F] . . . 1267
Giac [F] . . . 1267
Mupad [F(-1)] . . . 1267

## Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} - \frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2 d} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^2 d}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^2/d-(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3384, 3379, 3382}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = -\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^2 d} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^2 d} - \frac{\sqrt{(c+dx)^2+1}}{bd(a+b\operatorname{arcsinh}(c+dx))}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x]))) - (CoshIntegral[(a
+ b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(b^2*d) + (Cosh[a/b]*SinhIntegral[(a +
b*ArcSinh[c + d*x])/b])/(b^2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2d} \\
&\quad + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{b^2d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}\,dx \\
&= \frac{-\frac{b\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{b^2d}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])) - CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x
]])/(b^2*d)
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.41

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{e^{\frac{a}{b}}\ \mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\ \mathrm{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\ \mathrm{Ei}_1\left(-\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b^2}}{d}$ | 128 |
| default | $\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{e^{\frac{a}{b}}\ \mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\ \mathrm{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\ \mathrm{Ei}_1\left(-\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b^2}}{d}$ | 128 |

```
[In] int(1/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-(1+(d*x+c)^2)^(1/2)+d*x+c)/b/(a+b*arcsinh(d*x+c))+1/2/b^2*exp(a/
b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(
d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b))
```

## Fricas [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^2}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x
^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*
b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((d^4*x^4
+ 4*c*d^3*x^3 + c^4 + 2*(3*c^2*d^2 + d^2)*x^2 + (d^2*x^2 + 2*c*d*x + c^2 +
1)*(d^2*x^2 + 2*c*d*x + c^2 - 1) + 2*c^2 + 4*(c^3*d + c*d)*x + (2*d^3*x^3 +
 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d + d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1) + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c
^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*
b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}\,(c+d\,x))^2}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^2, x)
```

# 3.167 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))^2}\,dx$

Optimal result . . . . . 1268
Rubi [N/A] . . . . . 1268
Mathematica [N/A] . . . . . 1269
Maple [N/A] (verified) . . . . . 1269
Fricas [N/A] . . . . . 1269
Sympy [N/A] . . . . . 1270
Maxima [N/A] . . . . . 1270
Giac [N/A] . . . . . 1271
Mupad [N/A] . . . . . 1271

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^2},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^2,x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^2}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^2}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\left(dx+c\right)+a)^2}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2
+ 2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \frac{\int \frac{1}{a^2c+a^2dx+2abc\,\text{asinh}\,(c+dx)+2abdx\,\text{asinh}\,(c+dx)+b^2c\,\text{asinh}^2\,(c+dx)+b^2dx\,\text{asinh}^2\,(c+dx)}\,dx}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*asinh(c + d*x) + 2*a*b*d*x*asinh(c
+ d*x) + b**2*c*asinh(c + d*x)**2 + b**2*d*x*asinh(c + d*x)**2), x)/e
```

## Maxima [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 1086, normalized size of antiderivative = 47.22

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + (3*c^2*d^2*e + d^2*e)*
a*b*x + (c^3*d*e + c*d*e)*a*b + (b^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2
*d^2*e + d^2*e)*b^2*x + (c^3*d*e + c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^
2*e*x + b^2*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^3*e*x^2 + 2*a*b*c*d^2*e*x + a*b*c^2*
d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - integrate((2*(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(d*x + c) + (2*d^2*x^2 + 4*c*d*x + 2*c^2 + 1)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(a*b*d^6*e*x^6 + 6*a*b*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*
e)*a*b*x^4 + 4*(5*c^3*d^3*e + 2*c*d^3*e)*a*b*x^3 + (15*c^4*d^2*e + 12*c^2*d
^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*a*b*x + (c^6*e +
2*c^4*e + c^2*e)*a*b + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a*b*c^2*d^2*e
*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^
6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*e)*b^2*x^4 + 4*(5*c^3*d
^3*e + 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*b^2*x^2 +
 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*b^2*x + (c^6*e + 2*c^4*e + c^2*e)*b^2 +
(b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x
+ b^2*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^5*e*x^5 + 5*b^2*c*d^4
*e*x^4 + (10*c^2*d^3*e + d^3*e)*b^2*x^3 + (10*c^3*d^2*e + 3*c*d^2*e)*b^2*x^
```

```
2 + (5*c^4*d*e + 3*c^2*d*e)*b^2*x + (c^5*e + c^3*e)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^5*e*x^5 + 5*a*b*c*d^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*a*b*x^3 + (10*c^3*d^
2*e + 3*c*d^2*e)*a*b*x^2 + (5*c^4*d*e + 3*c^2*d*e)*a*b*x + (c^5*e + c^3*e)*
a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{\left(c\,e+d\,e\,x\right)\,\left(a+b\,\text{asinh}\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2), x)
```

# 3.168 $\int \frac{(ce+dex)^4}{(a+b\mathbf{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1272
Rubi [A] (verified) . . . 1273
Mathematica [A] (verified) . . . 1278
Maple [B] (verified) . . . 1278
Fricas [F] . . . 1279
Sympy [F] . . . 1279
Maxima [F] . . . 1280
Giac [F] . . . 1284
Mupad [F(-1)] . . . 1285

## Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = &-\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &+\frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ &-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &-\frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ &+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\end{aligned}$$

```
[Out] -2*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*ar
csinh(d*x+c))+1/16*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d-27/32*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+25/32*e^4*Chi(5*(a+b*arcs
inh(d*x+c))/b)*cosh(5*a/b)/b^3/d-1/16*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^3/d+27/32*e^4*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*
```

```
e^4*Shi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4*(1+(d
*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = &\frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ &-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &-\frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ &+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ &-\frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}-\frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcSinh[c + d*x])) - (5*e^4*(c + d*x
)^5)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[(a +
b*ArcSinh[c + d*x])/b])/(16*b^3*d) - (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*
(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*CoshIntegr
al[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (e^4*Sinh[a/b]*SinhIntegra
l[(a + b*ArcSinh[c + d*x])/b])/(16*b^3*d) + (27*e^4*Sinh[(3*a)/b]*SinhInteg
ral[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (25*e^4*Sinh[(5*a)/b]*Sin
hIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
```

```
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} + \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{bd} \\
&\quad + \frac{(5e^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2bd} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2 d(a+b\text{arcsinh}(c+dx))} - \frac{5e^4(c+dx)^5}{2b^2 d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{(6e^4)\,\text{Subst}\left(\int \frac{x^2}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{b^2 d} + \frac{(25e^4)\,\text{Subst}\left(\int \frac{x^4}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{2b^2 d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2 d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{5e^4(c+dx)^5}{2b^2 d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{(6e^4)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3 d} \\
&\quad + \frac{(25e^4)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3 d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2 d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{5e^4(c+dx)^5}{2b^2 d(a+b\text{arcsinh}(c+dx))} + \frac{(6e^4)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+\right.}{b^3 d} \\
&\quad + \frac{(25e^4)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d} \\
&\quad + \frac{(3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{(3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^3d} \\
&\quad - \frac{(75e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{\left(3e^4\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{\left(25e^4\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^3d} \\
&\quad + \frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{\left(75e^4\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d} \\
&\quad + \frac{\left(25e^4\cosh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d} \\
&\quad + \frac{\left(3e^4\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{\left(25e^4\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^3d} \\
&\quad - \frac{\left(3e^4\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{\left(75e^4\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d} \\
&\quad - \frac{\left(25e^4\sinh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^3d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d} - \frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d} \\
&\quad + \frac{25e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d} - \frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d} \\
&\quad + \frac{27e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d} - \frac{25e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$$
$$= \frac{e^4\Big(-\frac{16b^2(c+dx)^4\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^2}+\frac{16b\left(-4(c+dx)^3-5(c+dx)^5\right)}{a+b\text{arcsinh}(c+dx)}+48\Big(-\cosh\left(\frac{a}{b}\right)\text{Chi}\Big(\frac{a}{b}+\text{arcsinh}(c+dx)\Big)+\cosh\left(\frac{3a}{b}\right)\text{C}}$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
2 + (16*b*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x]) + 48*(
-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c +
d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 25*(2*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c
 + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c + d*x])])))/(32*b^3*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(302) = 604.

Time = 1.05 (sec) , antiderivative size = 896, normalized size of antiderivative = 2.80

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 896 |
| default | Expression too large to display | 896 |

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/64*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(5*b*arcsi
nh(d*x+c)+5*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-25/64*
e^4/b^3*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/64*(-4*(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(3*b*arcsinh(d*x
+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+27/64*e^4/b^
3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/32*(-(1+(d*x+c)^2)^(1/2)+d*x+c)
*e^4*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+
a^2)-1/32*e^4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/32/b*e^4*(d*x+c+(1+(d
*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1
/2))/(a+b*arcsinh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+
```

```
3/64/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c
)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64/b^2*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64
/b^3*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/64/b*e^4*(16*(d*x+c)^5
+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d
*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-5/64/b^2*e^4*(16
*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+
c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-25/64/b^
3*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*
arcsinh(d*x + c) + a^3), x)
```

## Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx \\
&= e^4\left(\int \frac{c^4}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx\right. \\
&\qquad + \int \frac{d^4x^4}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx \\
&\qquad + \int \frac{4cd^3x^3}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx \\
&\qquad + \int \frac{6c^2d^2x^2}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx \\
&\qquad \left. + \int \frac{4c^3dx}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx\right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**4*(Integral(c**4/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
```

```
Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a**3 +
3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)*
*3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*as
inh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((5*a*d^11*e^4 + b*d^11*e^4)*x^11 + 11*(5*a*c*d^10*e^4 + b*c*d^10*e^4)
*x^10 + (5*(55*c^2*d^9*e^4 + 3*d^9*e^4)*a + (55*c^2*d^9*e^4 + 3*d^9*e^4)*b)
*x^9 + 3*(5*(55*c^3*d^8*e^4 + 9*c*d^8*e^4)*a + (55*c^3*d^8*e^4 + 9*c*d^8*e^
4)*b)*x^8 + 3*(5*(110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*a + (110*c^4*
d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*b)*x^7 + 21*(5*(22*c^5*d^6*e^4 + 12*c^3
*d^6*e^4 + c*d^6*e^4)*a + (22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^4)*b)*
x^6 + (5*(462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*a +
 (462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*b)*x^5 + (5
*(330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*a + (3
30*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*b)*x^4 +
(5*(165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*a
 + (165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*b
)*x^3 + (5*(55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*
e^4)*a + (55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*e^
4)*b)*x^2 + ((5*a*d^8*e^4 + b*d^8*e^4)*x^8 + 8*(5*a*c*d^7*e^4 + b*c*d^7*e^4
)*x^7 + (4*(35*c^2*d^6*e^4 + 2*d^6*e^4)*a + (28*c^2*d^6*e^4 + d^6*e^4)*b)*x
^6 + 2*(4*(35*c^3*d^5*e^4 + 6*c*d^5*e^4)*a + (28*c^3*d^5*e^4 + 3*c*d^5*e^4)
*b)*x^5 + ((350*c^4*d^4*e^4 + 120*c^2*d^4*e^4 + 3*d^4*e^4)*a + 5*(14*c^4*d^
4*e^4 + 3*c^2*d^4*e^4)*b)*x^4 + 4*((70*c^5*d^3*e^4 + 40*c^3*d^3*e^4 + 3*c*d
^3*e^4)*a + (14*c^5*d^3*e^4 + 5*c^3*d^3*e^4)*b)*x^3 + (2*(70*c^6*d^2*e^4 +
60*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*a + (28*c^6*d^2*e^4 + 15*c^4*d^2*e^4)*b)*x^
2 + (5*c^8*e^4 + 8*c^6*e^4 + 3*c^4*e^4)*a + (c^8*e^4 + c^6*e^4)*b + 2*(2*(1
0*c^7*d*e^4 + 12*c^5*d*e^4 + 3*c^3*d*e^4)*a + (4*c^7*d*e^4 + 3*c^5*d*e^4)*b
)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(5*a*d^9*e^4 + b*d^9*e^4)*x^9
 + 27*(5*a*c*d^8*e^4 + b*c*d^8*e^4)*x^8 + ((540*c^2*d^7*e^4 + 31*d^7*e^4)*a
 + (108*c^2*d^7*e^4 + 5*d^7*e^4)*b)*x^7 + 7*((180*c^3*d^6*e^4 + 31*c*d^6*e^
4)*a + (36*c^3*d^6*e^4 + 5*c*d^6*e^4)*b)*x^6 + ((1890*c^4*d^5*e^4 + 651*c^2
*d^5*e^4 + 20*d^5*e^4)*a + (378*c^4*d^5*e^4 + 105*c^2*d^5*e^4 + 2*d^5*e^4)*
b)*x^5 + (5*(378*c^5*d^4*e^4 + 217*c^3*d^4*e^4 + 20*c*d^4*e^4)*a + (378*c^5
*d^4*e^4 + 175*c^3*d^4*e^4 + 10*c*d^4*e^4)*b)*x^4 + ((1260*c^6*d^3*e^4 + 10
85*c^4*d^3*e^4 + 200*c^2*d^3*e^4 + 4*d^3*e^4)*a + (252*c^6*d^3*e^4 + 175*c^
```

```
4*d^3*e^4 + 20*c^2*d^3*e^4)*b)*x^3 + ((540*c^7*d^2*e^4 + 651*c^5*d^2*e^4 +
200*c^3*d^2*e^4 + 12*c*d^2*e^4)*a + (108*c^7*d^2*e^4 + 105*c^5*d^2*e^4 + 20
*c^3*d^2*e^4)*b)*x^2 + (15*c^9*e^4 + 31*c^7*e^4 + 20*c^5*e^4 + 4*c^3*e^4)*a
 + (3*c^9*e^4 + 5*c^7*e^4 + 2*c^5*e^4)*b + ((135*c^8*d*e^4 + 217*c^6*d*e^4
+ 100*c^4*d*e^4 + 12*c^2*d*e^4)*a + (27*c^8*d*e^4 + 35*c^6*d*e^4 + 10*c^4*d
*e^4)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(c^11*e^4 + 3*c^9*e^4 + 3*c^7
*e^4 + c^5*e^4)*a + (c^11*e^4 + 3*c^9*e^4 + 3*c^7*e^4 + c^5*e^4)*b + (5*(11
*c^10*d*e^4 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*a + (11*c^10*d*e^4
 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*b)*x + (5*b*d^11*e^4*x^11 + 5
5*b*c*d^10*e^4*x^10 + 5*(55*c^2*d^9*e^4 + 3*d^9*e^4)*b*x^9 + 15*(55*c^3*d^8
*e^4 + 9*c*d^8*e^4)*b*x^8 + 15*(110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)
*b*x^7 + 105*(22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^4)*b*x^6 + 5*(462*c
^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*b*x^5 + 5*(330*c^7
*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*b*x^4 + 5*(165*
c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*b*x^3 + 5
*(55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*b*x^2
 + 5*(11*c^10*d*e^4 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*b*x + (5*b
*d^8*e^4*x^8 + 40*b*c*d^7*e^4*x^7 + 4*(35*c^2*d^6*e^4 + 2*d^6*e^4)*b*x^6 +
8*(35*c^3*d^5*e^4 + 6*c*d^5*e^4)*b*x^5 + (350*c^4*d^4*e^4 + 120*c^2*d^4*e^4
 + 3*d^4*e^4)*b*x^4 + 4*(70*c^5*d^3*e^4 + 40*c^3*d^3*e^4 + 3*c*d^3*e^4)*b*x
^3 + 2*(70*c^6*d^2*e^4 + 60*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*b*x^2 + 4*(10*c^7*
d*e^4 + 12*c^5*d*e^4 + 3*c^3*d*e^4)*b*x + (5*c^8*e^4 + 8*c^6*e^4 + 3*c^4*e^
4)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (15*b*d^9*e^4*x^9 + 135*b*c*d^8
*e^4*x^8 + (540*c^2*d^7*e^4 + 31*d^7*e^4)*b*x^7 + 7*(180*c^3*d^6*e^4 + 31*c
*d^6*e^4)*b*x^6 + (1890*c^4*d^5*e^4 + 651*c^2*d^5*e^4 + 20*d^5*e^4)*b*x^5 +
 5*(378*c^5*d^4*e^4 + 217*c^3*d^4*e^4 + 20*c*d^4*e^4)*b*x^4 + (1260*c^6*d^3
*e^4 + 1085*c^4*d^3*e^4 + 200*c^2*d^3*e^4 + 4*d^3*e^4)*b*x^3 + (540*c^7*d^2
*e^4 + 651*c^5*d^2*e^4 + 200*c^3*d^2*e^4 + 12*c*d^2*e^4)*b*x^2 + (135*c^8*d
*e^4 + 217*c^6*d*e^4 + 100*c^4*d*e^4 + 12*c^2*d*e^4)*b*x + (15*c^9*e^4 + 31
*c^7*e^4 + 20*c^5*e^4 + 4*c^3*e^4)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(c^
11*e^4 + 3*c^9*e^4 + 3*c^7*e^4 + c^5*e^4)*b + (15*b*d^10*e^4*x^10 + 150*b*c
*d^9*e^4*x^9 + (675*c^2*d^8*e^4 + 38*d^8*e^4)*b*x^8 + 8*(225*c^3*d^7*e^4 +
38*c*d^7*e^4)*b*x^7 + 2*(1575*c^4*d^6*e^4 + 532*c^2*d^6*e^4 + 16*d^6*e^4)*b
*x^6 + 4*(945*c^5*d^5*e^4 + 532*c^3*d^5*e^4 + 48*c*d^5*e^4)*b*x^5 + (3150*c
^6*d^4*e^4 + 2660*c^4*d^4*e^4 + 480*c^2*d^4*e^4 + 9*d^4*e^4)*b*x^4 + 4*(450
*c^7*d^3*e^4 + 532*c^5*d^3*e^4 + 160*c^3*d^3*e^4 + 9*c*d^3*e^4)*b*x^3 + (67
5*c^8*d^2*e^4 + 1064*c^6*d^2*e^4 + 480*c^4*d^2*e^4 + 54*c^2*d^2*e^4)*b*x^2
+ 2*(75*c^9*d*e^4 + 152*c^7*d*e^4 + 96*c^5*d*e^4 + 18*c^3*d*e^4)*b*x + (15*
c^10*e^4 + 38*c^8*e^4 + 32*c^6*e^4 + 9*c^4*e^4)*b)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(5*a*d^10*
e^4 + b*d^10*e^4)*x^10 + 30*(5*a*c*d^9*e^4 + b*c*d^9*e^4)*x^9 + ((675*c^2*d
^8*e^4 + 38*d^8*e^4)*a + (135*c^2*d^8*e^4 + 7*d^8*e^4)*b)*x^8 + 8*((225*c^3
*d^7*e^4 + 38*c*d^7*e^4)*a + (45*c^3*d^7*e^4 + 7*c*d^7*e^4)*b)*x^7 + (2*(15
75*c^4*d^6*e^4 + 532*c^2*d^6*e^4 + 16*d^6*e^4)*a + (630*c^4*d^6*e^4 + 196*c
^2*d^6*e^4 + 5*d^6*e^4)*b)*x^6 + 2*(2*(945*c^5*d^5*e^4 + 532*c^3*d^5*e^4 +
```

```
48*c*d^5*e^4)*a + (378*c^5*d^5*e^4 + 196*c^3*d^5*e^4 + 15*c*d^5*e^4)*b)*x^5
 + ((3150*c^6*d^4*e^4 + 2660*c^4*d^4*e^4 + 480*c^2*d^4*e^4 + 9*d^4*e^4)*a +
 (630*c^6*d^4*e^4 + 490*c^4*d^4*e^4 + 75*c^2*d^4*e^4 + d^4*e^4)*b)*x^4 + 4*
((450*c^7*d^3*e^4 + 532*c^5*d^3*e^4 + 160*c^3*d^3*e^4 + 9*c*d^3*e^4)*a + (9
0*c^7*d^3*e^4 + 98*c^5*d^3*e^4 + 25*c^3*d^3*e^4 + c*d^3*e^4)*b)*x^3 + ((675
*c^8*d^2*e^4 + 1064*c^6*d^2*e^4 + 480*c^4*d^2*e^4 + 54*c^2*d^2*e^4)*a + (13
5*c^8*d^2*e^4 + 196*c^6*d^2*e^4 + 75*c^4*d^2*e^4 + 6*c^2*d^2*e^4)*b)*x^2 +
(15*c^10*e^4 + 38*c^8*e^4 + 32*c^6*e^4 + 9*c^4*e^4)*a + (3*c^10*e^4 + 7*c^8
*e^4 + 5*c^6*e^4 + c^4*e^4)*b + 2*((75*c^9*d*e^4 + 152*c^7*d*e^4 + 96*c^5*d
*e^4 + 18*c^3*d*e^4)*a + (15*c^9*d*e^4 + 28*c^7*d*e^4 + 15*c^5*d*e^4 + 2*c^
3*d*e^4)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*
b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a
^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c
^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*
d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*
c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c
^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3
+ 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 +
2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*
(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*
d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2
 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
 + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2
 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*
c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^
5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3
 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^
4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2
*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^
4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)
*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*
d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3
+ 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c
^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*
(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^
4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(25*d^12*e^4*x^12 + 300*c*d^1
1*e^4*x^11 + 25*c^12*e^4 + 100*c^10*e^4 + 150*c^8*e^4 + 50*(33*c^2*d^10*e^4
 + 2*d^10*e^4)*x^10 + 100*c^6*e^4 + 500*(11*c^3*d^9*e^4 + 2*c*d^9*e^4)*x^9
+ 75*(165*c^4*d^8*e^4 + 60*c^2*d^8*e^4 + 2*d^8*e^4)*x^8 + 25*c^4*e^4 + 600*
(33*c^5*d^7*e^4 + 20*c^3*d^7*e^4 + 2*c*d^7*e^4)*x^7 + 100*(231*c^6*d^6*e^4
```

```
+ 210*c^4*d^6*e^4 + 42*c^2*d^6*e^4 + d^6*e^4)*x^6 + 600*(33*c^7*d^5*e^4 + 4
2*c^5*d^5*e^4 + 14*c^3*d^5*e^4 + c*d^5*e^4)*x^5 + 25*(495*c^8*d^4*e^4 + 840
*c^6*d^4*e^4 + 420*c^4*d^4*e^4 + 60*c^2*d^4*e^4 + d^4*e^4)*x^4 + 100*(55*c^
9*d^3*e^4 + 120*c^7*d^3*e^4 + 84*c^5*d^3*e^4 + 20*c^3*d^3*e^4 + c*d^3*e^4)*
x^3 + (25*d^8*e^4*x^8 + 200*c*d^7*e^4*x^7 + 25*c^8*e^4 + 24*c^6*e^4 + 3*c^4
*e^4 + 4*(175*c^2*d^6*e^4 + 6*d^6*e^4)*x^6 + 8*(175*c^3*d^5*e^4 + 18*c*d^5*
e^4)*x^5 + (1750*c^4*d^4*e^4 + 360*c^2*d^4*e^4 + 3*d^4*e^4)*x^4 + 4*(350*c^
5*d^3*e^4 + 120*c^3*d^3*e^4 + 3*c*d^3*e^4)*x^3 + 2*(350*c^6*d^2*e^4 + 180*c
^4*d^2*e^4 + 9*c^2*d^2*e^4)*x^2 + 4*(50*c^7*d*e^4 + 36*c^5*d*e^4 + 3*c^3*d*
e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 150*(11*c^10*d^2*e^4 + 30*c^8*d^2
*e^4 + 28*c^6*d^2*e^4 + 10*c^4*d^2*e^4 + c^2*d^2*e^4)*x^2 + (100*d^9*e^4*x^
9 + 900*c*d^8*e^4*x^8 + 100*c^9*e^4 + 172*c^7*e^4 + 87*c^5*e^4 + 4*(900*c^2
*d^7*e^4 + 43*d^7*e^4)*x^7 + 12*c^3*e^4 + 28*(300*c^3*d^6*e^4 + 43*c*d^6*e^
4)*x^6 + 3*(4200*c^4*d^5*e^4 + 1204*c^2*d^5*e^4 + 29*d^5*e^4)*x^5 + 5*(2520
*c^5*d^4*e^4 + 1204*c^3*d^4*e^4 + 87*c*d^4*e^4)*x^4 + 2*(4200*c^6*d^3*e^4 +
 3010*c^4*d^3*e^4 + 435*c^2*d^3*e^4 + 6*d^3*e^4)*x^3 + 6*(600*c^7*d^2*e^4 +
 602*c^5*d^2*e^4 + 145*c^3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (900*c^8*d*e^4 + 12
04*c^6*d*e^4 + 435*c^4*d*e^4 + 36*c^2*d*e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^(3/2) + 3*(50*d^10*e^4*x^10 + 500*c*d^9*e^4*x^9 + 50*c^10*e^4 + 124*c^8*
e^4 + 105*c^6*e^4 + 2*(1125*c^2*d^8*e^4 + 62*d^8*e^4)*x^8 + 35*c^4*e^4 + 16
*(375*c^3*d^7*e^4 + 62*c*d^7*e^4)*x^7 + 7*(1500*c^4*d^6*e^4 + 496*c^2*d^6*e
^4 + 15*d^6*e^4)*x^6 + 4*c^2*e^4 + 14*(900*c^5*d^5*e^4 + 496*c^3*d^5*e^4 +
45*c*d^5*e^4)*x^5 + 35*(300*c^6*d^4*e^4 + 248*c^4*d^4*e^4 + 45*c^2*d^4*e^4
+ d^4*e^4)*x^4 + 4*(1500*c^7*d^3*e^4 + 1736*c^5*d^3*e^4 + 525*c^3*d^3*e^4 +
 35*c*d^3*e^4)*x^3 + (2250*c^8*d^2*e^4 + 3472*c^6*d^2*e^4 + 1575*c^4*d^2*e^
4 + 210*c^2*d^2*e^4 + 4*d^2*e^4)*x^2 + 2*(250*c^9*d*e^4 + 496*c^7*d*e^4 + 3
15*c^5*d*e^4 + 70*c^3*d*e^4 + 4*c*d*e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
 100*(3*c^11*d*e^4 + 10*c^9*d*e^4 + 12*c^7*d*e^4 + 6*c^5*d*e^4 + c^3*d*e^4)
*x + (100*d^11*e^4*x^11 + 1100*c*d^10*e^4*x^10 + 100*c^11*e^4 + 324*c^9*e^4
 + 381*c^7*e^4 + 4*(1375*c^2*d^9*e^4 + 81*d^9*e^4)*x^9 + 193*c^5*e^4 + 12*(
1375*c^3*d^8*e^4 + 243*c*d^8*e^4)*x^8 + 3*(11000*c^4*d^7*e^4 + 3888*c^2*d^7
*e^4 + 127*d^7*e^4)*x^7 + 36*c^3*e^4 + 21*(2200*c^5*d^6*e^4 + 1296*c^3*d^6*
e^4 + 127*c*d^6*e^4)*x^6 + (46200*c^6*d^5*e^4 + 40824*c^4*d^5*e^4 + 8001*c^
2*d^5*e^4 + 193*d^5*e^4)*x^5 + (33000*c^7*d^4*e^4 + 40824*c^5*d^4*e^4 + 133
35*c^3*d^4*e^4 + 965*c*d^4*e^4)*x^4 + (16500*c^8*d^3*e^4 + 27216*c^6*d^3*e^
4 + 13335*c^4*d^3*e^4 + 1930*c^2*d^3*e^4 + 36*d^3*e^4)*x^3 + (5500*c^9*d^2*
e^4 + 11664*c^7*d^2*e^4 + 8001*c^5*d^2*e^4 + 1930*c^3*d^2*e^4 + 108*c*d^2*e
^4)*x^2 + (1100*c^10*d*e^4 + 2916*c^8*d*e^4 + 2667*c^6*d*e^4 + 965*c^4*d*e^
4 + 108*c^2*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8
*a*b^2*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*
a*b^2*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 +
10*c^3*d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d
^2)*a*b^2*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6
+ 6*c^4 + 4*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c
^2*d^2*x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
```

```
 4*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*
c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*
b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5
*x^5 + (15*c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 +
 (15*c^4*d^2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*
b^2*x + (c^6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8
*x^8 + 8*b^3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d
^5)*b^3*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 +
10*c^3*d^3 + 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2
)*b^3*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^
4 + 4*c^2 + 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4
*b^3*c^3*d*x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 +
5*b^3*c*d^4*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x
^2 + (5*c^4*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^
4 + 4*(5*c^3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x
^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2
+ 2*c*d*x + c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^
5)*b^3*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3
*d^3)*b^3*x^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 1
5*c^4*d + 9*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*
(a*b^2*d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c
^3*d^4 + 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 +
 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c
^2*d + d)*a*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{asinh}\,(c+d\,x))^3}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3, x)
```

# 3.169 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1286
Rubi [A] (verified) . . . 1287
Mathematica [A] (verified) . . . 1291
Maple [B] (verified) . . . 1291
Fricas [F] . . . 1292
Sympy [F] . . . 1292
Maxima [F] . . . 1293
Giac [F] . . . 1297
Mupad [F(-1)] . . . 1297

## Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{3e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^3\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2b^3d} - \frac{e^3\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{b^3d} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^3d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}$$

```
[Out] -3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*ar
csinh(d*x+c))-1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d+e^3*c
osh(4*a/b)*Shi(4*(a+b*arcsinh(d*x+c))/b)/b^3/d+1/2*e^3*Chi(2*(a+b*arcsinh(d
*x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b
^3/d-1/2*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = &\frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^3d}\\ &-\frac{e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}\\ &-\frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^3d}\\ &+\frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}\\ &-\frac{2e^3(c+dx)^4}{b^2d(a+b\operatorname{arcsinh}(c+dx))}-\frac{3e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (2*e^3*(c + d
*x)^4)/(b^2*d*(a + b*ArcSinh[c + d*x])) + (e^3*CoshIntegral[(2*(a + b*ArcSi
nh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4*(a + b*Arc
Sinh[c + d*x]))/b]*Sinh[(4*a)/b])/(b^3*d) - (e^3*Cosh[(2*a)/b]*SinhIntegral
[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*SinhIntegr
al[(4*(a + b*ArcSinh[c + d*x]))/b])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} + \frac{(3e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2bd} \\
&\quad + \frac{(2e^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{3e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{(3e^3)\,\text{Subst}\left(\int \frac{x}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{b^2d} + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x^3}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{b^2d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{3e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{(3e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(8e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{3e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{(3e^3)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(8e^3)\,\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8x} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{3e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{2e^3(c+dx)^4}{b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^3\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{(2e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{3e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{\left(3e^3\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{\left(2e^3\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{\left(3e^3\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{\left(2e^3\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{\left(e^3\sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{3e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{e^3\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2b^3d} - \frac{e^3\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{b^3d} \\
&\quad - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{2b^3d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.72

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$$
$$= \frac{e^3\left(-\frac{b^2(c+dx)^3\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{b\left(-3(c+dx)^2-4(c+dx)^4\right)}{a+b\operatorname{arcsinh}(c+dx)}+\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2\operatorname{Chi}\left(4\left(\frac{a}{b}+\text{ar}\right.\right.\right.}$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-((b^2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2)
 + (b*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x]) + CoshInte
gral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegral[4*(a/b + Ar
cSinh[c + d*x])]*Sinh[(4*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(
2*b^3*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(237) = 474.

Time = 1.02 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\ \operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e^3\mathrm{e}^{\frac{4a}{b}}\ \operatorname{Ei}_1\left(4\ \operatorname{arcsinh}(d\right.}{2b^3}$ |
| default | $-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\ \operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e^3\mathrm{e}^{\frac{4a}{b}}\ \operatorname{Ei}_1\left(4\ \operatorname{arcsinh}(d\right.}{2b^3}$ |

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(4*b*arcsinh(d*x+c)+4*a-b)/b^2/(b^2*arcsinh(
d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e^3/b^3*exp(4*a/b)*Ei(1,4*arcsinh(d*
x+c)+4*a/b)+1/16*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e^3*(2*b*ar
csinh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4
*e^3/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/16/b*e^3*(2*(d*x+c)^2+1+
2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/8/b^2*e^3*(2*(d*x+c
)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/4/b^3*e^3*exp(-
2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*
(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh
(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(
```

```
1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/2/b^3*e^3*exp(
-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*arc
sinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a
^3), x)
```

## Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx \\
&= e^3\left(\int \frac{c^3}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right. \\
&\qquad + \int \frac{d^3x^3}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \\
&\qquad + \int \frac{3cd^2x^2}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \\
&\qquad \left. + \int \frac{3c^2dx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x
))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*d^10*e^3 + b*d^10*e^3)*x^10 + 10*(4*a*c*d^9*e^3 + b*c*d^9*e^3)*x
^9 + 3*(4*(15*c^2*d^8*e^3 + d^8*e^3)*a + (15*c^2*d^8*e^3 + d^8*e^3)*b)*x^8
+ 24*(4*(5*c^3*d^7*e^3 + c*d^7*e^3)*a + (5*c^3*d^7*e^3 + c*d^7*e^3)*b)*x^7
+ 3*(4*(70*c^4*d^6*e^3 + 28*c^2*d^6*e^3 + d^6*e^3)*a + (70*c^4*d^6*e^3 + 28
*c^2*d^6*e^3 + d^6*e^3)*b)*x^6 + 6*(4*(42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*
c*d^5*e^3)*a + (42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b)*x^5 + (4*
(210*c^6*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*a + (210*c^6
*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*b)*x^4 + 4*(4*(30*c^
7*d^3*e^3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*a + (30*c^7*d^3*e^
3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*b)*x^3 + 3*(4*(15*c^8*d^2*
e^3 + 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*a + (15*c^8*d^2*e^3
+ 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*b)*x^2 + ((4*a*d^7*e^3 +
 b*d^7*e^3)*x^7 + 7*(4*a*c*d^6*e^3 + b*c*d^6*e^3)*x^6 + (6*(14*c^2*d^5*e^3
+ d^5*e^3)*a + (21*c^2*d^5*e^3 + d^5*e^3)*b)*x^5 + 5*(2*(14*c^3*d^4*e^3 + 3
*c*d^4*e^3)*a + (7*c^3*d^4*e^3 + c*d^4*e^3)*b)*x^4 + (2*(70*c^4*d^3*e^3 + 3
0*c^2*d^3*e^3 + d^3*e^3)*a + 5*(7*c^4*d^3*e^3 + 2*c^2*d^3*e^3)*b)*x^3 + (6*
(14*c^5*d^2*e^3 + 10*c^3*d^2*e^3 + c*d^2*e^3)*a + (21*c^5*d^2*e^3 + 10*c^3*
d^2*e^3)*b)*x^2 + 2*(2*c^7*e^3 + 3*c^5*e^3 + c^3*e^3)*a + (c^7*e^3 + c^5*e^
3)*b + (2*(14*c^6*d*e^3 + 15*c^4*d*e^3 + 3*c^2*d*e^3)*a + (7*c^6*d*e^3 + 5*
c^4*d*e^3)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(4*a*d^8*e^3 + b*
d^8*e^3)*x^8 + 24*(4*a*c*d^7*e^3 + b*c*d^7*e^3)*x^7 + (24*(14*c^2*d^6*e^3 +
 d^6*e^3)*a + (84*c^2*d^6*e^3 + 5*d^6*e^3)*b)*x^6 + 6*(8*(14*c^3*d^5*e^3 +
3*c*d^5*e^3)*a + (28*c^3*d^5*e^3 + 5*c*d^5*e^3)*b)*x^5 + (15*(56*c^4*d^4*e^
3 + 24*c^2*d^4*e^3 + d^4*e^3)*a + (210*c^4*d^4*e^3 + 75*c^2*d^4*e^3 + 2*d^4
*e^3)*b)*x^4 + 4*(3*(56*c^5*d^3*e^3 + 40*c^3*d^3*e^3 + 5*c*d^3*e^3)*a + (42
*c^5*d^3*e^3 + 25*c^3*d^3*e^3 + 2*c*d^3*e^3)*b)*x^3 + 3*((112*c^6*d^2*e^3 +
 120*c^4*d^2*e^3 + 30*c^2*d^2*e^3 + d^2*e^3)*a + (28*c^6*d^2*e^3 + 25*c^4*d
^2*e^3 + 4*c^2*d^2*e^3)*b)*x^2 + 3*(4*c^8*e^3 + 8*c^6*e^3 + 5*c^4*e^3 + c^2
*e^3)*a + (3*c^8*e^3 + 5*c^6*e^3 + 2*c^4*e^3)*b + 2*(3*(16*c^7*d*e^3 + 24*c
^5*d*e^3 + 10*c^3*d*e^3 + c*d*e^3)*a + (12*c^7*d*e^3 + 15*c^5*d*e^3 + 4*c^3
*d*e^3)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(c^10*e^3 + 3*c^8*e^3 + 3*c
^6*e^3 + c^4*e^3)*a + (c^10*e^3 + 3*c^8*e^3 + 3*c^6*e^3 + c^4*e^3)*b + 2*(4
*(5*c^9*d*e^3 + 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*a + (5*c^9*d*e^3
+ 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*b)*x + (4*b*d^10*e^3*x^10 + 40*
b*c*d^9*e^3*x^9 + 12*(15*c^2*d^8*e^3 + d^8*e^3)*b*x^8 + 96*(5*c^3*d^7*e^3 +
 c*d^7*e^3)*b*x^7 + 12*(70*c^4*d^6*e^3 + 28*c^2*d^6*e^3 + d^6*e^3)*b*x^6 +
24*(42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b*x^5 + 4*(210*c^6*d^4*e
```

```
^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*b*x^4 + 16*(30*c^7*d^3*e^3
 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*b*x^3 + 12*(15*c^8*d^2*e^3
+ 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*b*x^2 + 8*(5*c^9*d*e^3 +
 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*b*x + 2*(2*b*d^7*e^3*x^7 + 14*b*
c*d^6*e^3*x^6 + 3*(14*c^2*d^5*e^3 + d^5*e^3)*b*x^5 + 5*(14*c^3*d^4*e^3 + 3*
c*d^4*e^3)*b*x^4 + (70*c^4*d^3*e^3 + 30*c^2*d^3*e^3 + d^3*e^3)*b*x^3 + 3*(1
4*c^5*d^2*e^3 + 10*c^3*d^2*e^3 + c*d^2*e^3)*b*x^2 + (14*c^6*d*e^3 + 15*c^4*
d*e^3 + 3*c^2*d*e^3)*b*x + (2*c^7*e^3 + 3*c^5*e^3 + c^3*e^3)*b)*(d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + 3*(4*b*d^8*e^3*x^8 + 32*b*c*d^7*e^3*x^7 + 8*(14*
c^2*d^6*e^3 + d^6*e^3)*b*x^6 + 16*(14*c^3*d^5*e^3 + 3*c*d^5*e^3)*b*x^5 + 5*
(56*c^4*d^4*e^3 + 24*c^2*d^4*e^3 + d^4*e^3)*b*x^4 + 4*(56*c^5*d^3*e^3 + 40*
c^3*d^3*e^3 + 5*c*d^3*e^3)*b*x^3 + (112*c^6*d^2*e^3 + 120*c^4*d^2*e^3 + 30*
c^2*d^2*e^3 + d^2*e^3)*b*x^2 + 2*(16*c^7*d*e^3 + 24*c^5*d*e^3 + 10*c^3*d*e^
3 + c*d*e^3)*b*x + (4*c^8*e^3 + 8*c^6*e^3 + 5*c^4*e^3 + c^2*e^3)*b)*(d^2*x^
2 + 2*c*d*x + c^2 + 1) + 4*(c^10*e^3 + 3*c^8*e^3 + 3*c^6*e^3 + c^4*e^3)*b +
 (12*b*d^9*e^3*x^9 + 108*b*c*d^8*e^3*x^8 + 6*(72*c^2*d^7*e^3 + 5*d^7*e^3)*b
*x^7 + 42*(24*c^3*d^6*e^3 + 5*c*d^6*e^3)*b*x^6 + (1512*c^4*d^5*e^3 + 630*c^
2*d^5*e^3 + 25*d^5*e^3)*b*x^5 + (1512*c^5*d^4*e^3 + 1050*c^3*d^4*e^3 + 125*
c*d^4*e^3)*b*x^4 + (1008*c^6*d^3*e^3 + 1050*c^4*d^3*e^3 + 250*c^2*d^3*e^3 +
 7*d^3*e^3)*b*x^3 + (432*c^7*d^2*e^3 + 630*c^5*d^2*e^3 + 250*c^3*d^2*e^3 +
21*c*d^2*e^3)*b*x^2 + (108*c^8*d*e^3 + 210*c^6*d*e^3 + 125*c^4*d*e^3 + 21*c
^2*d*e^3)*b*x + (12*c^9*e^3 + 30*c^7*e^3 + 25*c^5*e^3 + 7*c^3*e^3)*b)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)) + (3*(4*a*d^9*e^3 + b*d^9*e^3)*x^9 + 27*(4*a*c*d^8*e^3 + b*c*d^8*e^3)*x
^8 + (6*(72*c^2*d^7*e^3 + 5*d^7*e^3)*a + (108*c^2*d^7*e^3 + 7*d^7*e^3)*b)*x
^7 + 7*(6*(24*c^3*d^6*e^3 + 5*c*d^6*e^3)*a + (36*c^3*d^6*e^3 + 7*c*d^6*e^3)
*b)*x^6 + ((1512*c^4*d^5*e^3 + 630*c^2*d^5*e^3 + 25*d^5*e^3)*a + (378*c^4*d
^5*e^3 + 147*c^2*d^5*e^3 + 5*d^5*e^3)*b)*x^5 + ((1512*c^5*d^4*e^3 + 1050*c^
3*d^4*e^3 + 125*c*d^4*e^3)*a + (378*c^5*d^4*e^3 + 245*c^3*d^4*e^3 + 25*c*d^
4*e^3)*b)*x^4 + ((1008*c^6*d^3*e^3 + 1050*c^4*d^3*e^3 + 250*c^2*d^3*e^3 + 7
*d^3*e^3)*a + (252*c^6*d^3*e^3 + 245*c^4*d^3*e^3 + 50*c^2*d^3*e^3 + d^3*e^3
)*b)*x^3 + ((432*c^7*d^2*e^3 + 630*c^5*d^2*e^3 + 250*c^3*d^2*e^3 + 21*c*d^2
*e^3)*a + (108*c^7*d^2*e^3 + 147*c^5*d^2*e^3 + 50*c^3*d^2*e^3 + 3*c*d^2*e^3
)*b)*x^2 + (12*c^9*e^3 + 30*c^7*e^3 + 25*c^5*e^3 + 7*c^3*e^3)*a + (3*c^9*e^
3 + 7*c^7*e^3 + 5*c^5*e^3 + c^3*e^3)*b + ((108*c^8*d*e^3 + 210*c^6*d*e^3 +
125*c^4*d*e^3 + 21*c^2*d*e^3)*a + (27*c^8*d*e^3 + 49*c^6*d*e^3 + 25*c^4*d*e
^3 + 3*c^2*d*e^3)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6
 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3
*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*
d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^
2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3
*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*
d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4
*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b
```

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^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4
*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*
x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*
c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^
2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2
*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2
*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 +
6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6
*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*
a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b
^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2
 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^
5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5
*x^4 + 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^
2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b
^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(16*d^11*e^3*x^11 +
176*c*d^10*e^3*x^10 + 16*c^11*e^3 + 64*c^9*e^3 + 96*c^7*e^3 + 16*(55*c^2*d^
9*e^3 + 4*d^9*e^3)*x^9 + 48*(55*c^3*d^8*e^3 + 12*c*d^8*e^3)*x^8 + 64*c^5*e^
3 + 96*(55*c^4*d^7*e^3 + 24*c^2*d^7*e^3 + d^7*e^3)*x^7 + 672*(11*c^5*d^6*e^
3 + 8*c^3*d^6*e^3 + c*d^6*e^3)*x^6 + 16*c^3*e^3 + 32*(231*c^6*d^5*e^3 + 252
*c^4*d^5*e^3 + 63*c^2*d^5*e^3 + 2*d^5*e^3)*x^5 + 32*(165*c^7*d^4*e^3 + 252*
c^5*d^4*e^3 + 105*c^3*d^4*e^3 + 10*c*d^4*e^3)*x^4 + 16*(165*c^8*d^3*e^3 + 3
36*c^6*d^3*e^3 + 210*c^4*d^3*e^3 + 40*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(4*d^7
*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 3*c^5*e^3 + 3*(28*c^2*d^5*e^3 + d
^5*e^3)*x^5 + 5*(28*c^3*d^4*e^3 + 3*c*d^4*e^3)*x^4 + 10*(14*c^4*d^3*e^3 + 3
*c^2*d^3*e^3)*x^3 + 6*(14*c^5*d^2*e^3 + 5*c^3*d^2*e^3)*x^2 + (28*c^6*d*e^3
+ 15*c^4*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 16*(55*c^9*d^2*e^3 + 1
44*c^7*d^2*e^3 + 126*c^5*d^2*e^3 + 40*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (64*
d^8*e^3*x^8 + 512*c*d^7*e^3*x^7 + 64*c^8*e^3 + 100*c^6*e^3 + 42*c^4*e^3 + 4
*(448*c^2*d^6*e^3 + 25*d^6*e^3)*x^6 + 8*(448*c^3*d^5*e^3 + 75*c*d^5*e^3)*x^
5 + 3*c^2*e^3 + 2*(2240*c^4*d^4*e^3 + 750*c^2*d^4*e^3 + 21*d^4*e^3)*x^4 + 8
*(448*c^5*d^3*e^3 + 250*c^3*d^3*e^3 + 21*c*d^3*e^3)*x^3 + (1792*c^6*d^2*e^3
 + 1500*c^4*d^2*e^3 + 252*c^2*d^2*e^3 + 3*d^2*e^3)*x^2 + 2*(256*c^7*d*e^3 +
 300*c^5*d*e^3 + 84*c^3*d*e^3 + 3*c*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + 6*(16*d^9*e^3*x^9 + 144*c*d^8*e^3*x^8 + 16*c^9*e^3 + 38*c^7*e^3 +
30*c^5*e^3 + 2*(288*c^2*d^7*e^3 + 19*d^7*e^3)*x^7 + 14*(96*c^3*d^6*e^3 + 19
*c*d^6*e^3)*x^6 + 9*c^3*e^3 + 6*(336*c^4*d^5*e^3 + 133*c^2*d^5*e^3 + 5*d^5*
e^3)*x^5 + 2*(1008*c^5*d^4*e^3 + 665*c^3*d^4*e^3 + 75*c*d^4*e^3)*x^4 + c*e^
3 + (1344*c^6*d^3*e^3 + 1330*c^4*d^3*e^3 + 300*c^2*d^3*e^3 + 9*d^3*e^3)*x^3
```

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 + 3*(192*c^7*d^2*e^3 + 266*c^5*d^2*e^3 + 100*c^3*d^2*e^3 + 9*c*d^2*e^3)*x^
2 + (144*c^8*d*e^3 + 266*c^6*d*e^3 + 150*c^4*d*e^3 + 27*c^2*d*e^3 + d*e^3)*
x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 16*(11*c^10*d*e^3 + 36*c^8*d*e^3 + 42*c^
6*d*e^3 + 20*c^4*d*e^3 + 3*c^2*d*e^3)*x + (64*d^10*e^3*x^10 + 640*c*d^9*e^3
*x^9 + 64*c^10*e^3 + 204*c^8*e^3 + 234*c^6*e^3 + 12*(240*c^2*d^8*e^3 + 17*d
^8*e^3)*x^8 + 96*(80*c^3*d^7*e^3 + 17*c*d^7*e^3)*x^7 + 115*c^4*e^3 + 6*(224
0*c^4*d^6*e^3 + 952*c^2*d^6*e^3 + 39*d^6*e^3)*x^6 + 12*(1344*c^5*d^5*e^3 +
952*c^3*d^5*e^3 + 117*c*d^5*e^3)*x^5 + 21*c^2*e^3 + 5*(2688*c^6*d^4*e^3 + 2
856*c^4*d^4*e^3 + 702*c^2*d^4*e^3 + 23*d^4*e^3)*x^4 + 4*(1920*c^7*d^3*e^3 +
 2856*c^5*d^3*e^3 + 1170*c^3*d^3*e^3 + 115*c*d^3*e^3)*x^3 + 3*(960*c^8*d^2*
e^3 + 1904*c^6*d^2*e^3 + 1170*c^4*d^2*e^3 + 230*c^2*d^2*e^3 + 7*d^2*e^3)*x^
2 + 2*(320*c^9*d*e^3 + 816*c^7*d*e^3 + 702*c^5*d*e^3 + 230*c^3*d*e^3 + 21*c
*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^
7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 +
 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3
+ 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x
^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4
*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2
+ 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d
^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3
*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*
c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^
2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^
6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^
3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5
 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3
+ 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 +
 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 +
 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4
*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4
*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^
3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c
^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x +
 c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5
+ 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x
^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9
*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*
x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*
c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a
*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
```

```
), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^3}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3, x)
```

# 3.170 $\int \frac{(ce+dex)^2}{(a+b\mathbf{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1298
Rubi [A] (verified) . . . 1299
Mathematica [A] (verified) . . . 1303
Maple [B] (verified) . . . 1303
Fricas [F] . . . 1304
Sympy [F] . . . 1304
Maxima [F] . . . 1304
Giac [F] . . . 1308
Mupad [F(-1)] . . . 1308

## Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = &-\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{e^2(c+dx)}{b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{3e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d}\\ &+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d}\\ &+\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d}\\ &-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d}\end{aligned}$$

```
[Out] -e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arcsin
h(d*x+c))-1/8*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d+9/8*e^2*Chi(3
*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+1/8*e^2*Shi((a+b*arcsinh(d*x+c))
/b)*sinh(a/b)/b^3/d-9/8*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d
-1/2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382, 5774}

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = &-\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d}\\ &+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d}\\ &+\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d}\\ &-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d}\\ &-\frac{3e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}-\frac{e^2(c+dx)}{b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ &-\frac{e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (e^2*(c + d*x))/(b^2*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^3)
/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e^2*Cosh[a/b]*CoshIntegral[(a + b*Ar
cSinh[c + d*x])/b])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b
*ArcSinh[c + d*x]))/b])/(8*b^3*d) + (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcS
inh[c + d*x])/b])/(8*b^3*d) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c + d*x]))/b])/(8*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} + \frac{e^2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{bd} \\
&\quad + \frac{(3e^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2bd} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\text{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{e^2\text{Subst}\left(\int \frac{1}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{b^2d} + \frac{(9e^2)\,\text{Subst}\left(\int \frac{x^2}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{2b^2d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))} + \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(9e^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{(9e^2)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^3d} \\
&\quad - \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^3d} \\
&\quad + \frac{(9e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&\quad - \frac{(9e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^3d} \\
&\quad - \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{b^3d} \\
&\quad - \frac{\left(9e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&\quad + \frac{\left(9e^2\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&\quad + \frac{\left(9e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&\quad - \frac{\left(9e^2\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d} + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d} \\
&\quad + \frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{8b^3d} - \frac{9e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^3d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$$
$$= \frac{e^2\left(-\frac{4b^2(c+dx)^2\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{4b(-2(c+dx)-3(c+dx)^3)}{a+b\operatorname{arcsinh}(c+dx)}+8\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)-8\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}\right.\right.}$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*((-4*b^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2
 + (4*b*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x]) + 8*Cosh[a
/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 8*Sinh[a/b]*SinhIntegral[a/b + A
rcSinh[c + d*x]] + 9*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + C
osh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhInteg
ral[a/b + ArcSinh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c
 + d*x])])))/(8*b^3*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(232) = 464.

Time = 0.48 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.06

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\ \operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\ \operatorname{Ei}_1\left(3\ \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{16b^3}$ |
| default | $-\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\ \operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\ \operatorname{Ei}_1\left(3\ \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{16b^3}$ |

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2
)+3*d*x+3*c)*e^2*(3*b*arcsinh(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b
*arcsinh(d*x+c)+a^2)-9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1
/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsin
h(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arcsinh(d*x
+c)+a/b)+1/16/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/16
/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/16/b^3*e^2*exp(
-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)
^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^2
*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(
```

```
1/2))/(a+b*arcsinh(d*x+c))-9/16/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-
3*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arcsinh(d*x + c)^3 + 3*
a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

## Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx \\
&= e^2\left(\int \frac{c^2}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx\right. \\
&\qquad + \int \frac{d^2x^2}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx \\
&\qquad \left. + \int \frac{2cdx}{a^3+3a^2b\,\operatorname{asinh}(c+dx)+3ab^2\,\operatorname{asinh}^2(c+dx)+b^3\,\operatorname{asinh}^3(c+dx)}\,dx\right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)*
*2 + b**3*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*d^9*e^2 + b*d^9*e^2)*x^9 + 9*(3*a*c*d^8*e^2 + b*c*d^8*e^2)*x^8 +
 3*(3*(12*c^2*d^7*e^2 + d^7*e^2)*a + (12*c^2*d^7*e^2 + d^7*e^2)*b)*x^7 + 21
```

```
*(3*(4*c^3*d^6*e^2 + c*d^6*e^2)*a + (4*c^3*d^6*e^2 + c*d^6*e^2)*b)*x^6 + 3*
(3*(42*c^4*d^5*e^2 + 21*c^2*d^5*e^2 + d^5*e^2)*a + (42*c^4*d^5*e^2 + 21*c^2
*d^5*e^2 + d^5*e^2)*b)*x^5 + 3*(3*(42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5*c*d^
4*e^2)*a + (42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5*c*d^4*e^2)*b)*x^4 + (3*(84*
c^6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*a + (84*c^6*d^3*e
^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*b)*x^3 + 3*(3*(12*c^7*d^2*
e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*a + (12*c^7*d^2*e^2 + 21
*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*b)*x^2 + ((3*a*d^6*e^2 + b*d^6*e
^2)*x^6 + 6*(3*a*c*d^5*e^2 + b*c*d^5*e^2)*x^5 + ((45*c^2*d^4*e^2 + 4*d^4*e^
2)*a + (15*c^2*d^4*e^2 + d^4*e^2)*b)*x^4 + 4*((15*c^3*d^3*e^2 + 4*c*d^3*e^2
)*a + (5*c^3*d^3*e^2 + c*d^3*e^2)*b)*x^3 + ((45*c^4*d^2*e^2 + 24*c^2*d^2*e^
2 + d^2*e^2)*a + 3*(5*c^4*d^2*e^2 + 2*c^2*d^2*e^2)*b)*x^2 + (3*c^6*e^2 + 4*
c^4*e^2 + c^2*e^2)*a + (c^6*e^2 + c^4*e^2)*b + 2*((9*c^5*d*e^2 + 8*c^3*d*e^
2 + c*d*e^2)*a + (3*c^5*d*e^2 + 2*c^3*d*e^2)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + (3*(3*a*d^7*e^2 + b*d^7*e^2)*x^7 + 21*(3*a*c*d^6*e^2 + b*c*d^
6*e^2)*x^6 + ((189*c^2*d^5*e^2 + 17*d^5*e^2)*a + (63*c^2*d^5*e^2 + 5*d^5*e^
2)*b)*x^5 + 5*((63*c^3*d^4*e^2 + 17*c*d^4*e^2)*a + (21*c^3*d^4*e^2 + 5*c*d^
4*e^2)*b)*x^4 + (5*(63*c^4*d^3*e^2 + 34*c^2*d^3*e^2 + 2*d^3*e^2)*a + (105*c
^4*d^3*e^2 + 50*c^2*d^3*e^2 + 2*d^3*e^2)*b)*x^3 + ((189*c^5*d^2*e^2 + 170*c
^3*d^2*e^2 + 30*c*d^2*e^2)*a + (63*c^5*d^2*e^2 + 50*c^3*d^2*e^2 + 6*c*d^2*e
^2)*b)*x^2 + (9*c^7*e^2 + 17*c^5*e^2 + 10*c^3*e^2 + 2*c*e^2)*a + (3*c^7*e^2
 + 5*c^5*e^2 + 2*c^3*e^2)*b + ((63*c^6*d*e^2 + 85*c^4*d*e^2 + 30*c^2*d*e^2
+ 2*d*e^2)*a + (21*c^6*d*e^2 + 25*c^4*d*e^2 + 6*c^2*d*e^2)*b)*x)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 3*(c^9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*a + (c^
9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*b + 3*(3*(3*c^8*d*e^2 + 7*c^6*d*e^
2 + 5*c^4*d*e^2 + c^2*d*e^2)*a + (3*c^8*d*e^2 + 7*c^6*d*e^2 + 5*c^4*d*e^2 +
 c^2*d*e^2)*b)*x + (3*b*d^9*e^2*x^9 + 27*b*c*d^8*e^2*x^8 + 9*(12*c^2*d^7*e^
2 + d^7*e^2)*b*x^7 + 63*(4*c^3*d^6*e^2 + c*d^6*e^2)*b*x^6 + 9*(42*c^4*d^5*e
^2 + 21*c^2*d^5*e^2 + d^5*e^2)*b*x^5 + 9*(42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 +
 5*c*d^4*e^2)*b*x^4 + 3*(84*c^6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2
+ d^3*e^2)*b*x^3 + 9*(12*c^7*d^2*e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*
d^2*e^2)*b*x^2 + 9*(3*c^8*d*e^2 + 7*c^6*d*e^2 + 5*c^4*d*e^2 + c^2*d*e^2)*b*
x + (3*b*d^6*e^2*x^6 + 18*b*c*d^5*e^2*x^5 + (45*c^2*d^4*e^2 + 4*d^4*e^2)*b*
x^4 + 4*(15*c^3*d^3*e^2 + 4*c*d^3*e^2)*b*x^3 + (45*c^4*d^2*e^2 + 24*c^2*d^2
*e^2 + d^2*e^2)*b*x^2 + 2*(9*c^5*d*e^2 + 8*c^3*d*e^2 + c*d*e^2)*b*x + (3*c^
6*e^2 + 4*c^4*e^2 + c^2*e^2)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (9*b*
d^7*e^2*x^7 + 63*b*c*d^6*e^2*x^6 + (189*c^2*d^5*e^2 + 17*d^5*e^2)*b*x^5 + 5
*(63*c^3*d^4*e^2 + 17*c*d^4*e^2)*b*x^4 + 5*(63*c^4*d^3*e^2 + 34*c^2*d^3*e^2
 + 2*d^3*e^2)*b*x^3 + (189*c^5*d^2*e^2 + 170*c^3*d^2*e^2 + 30*c*d^2*e^2)*b*
x^2 + (63*c^6*d*e^2 + 85*c^4*d*e^2 + 30*c^2*d*e^2 + 2*d*e^2)*b*x + (9*c^7*e
^2 + 17*c^5*e^2 + 10*c^3*e^2 + 2*c*e^2)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
3*(c^9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*b + (9*b*d^8*e^2*x^8 + 72*b*c
*d^7*e^2*x^7 + 2*(126*c^2*d^6*e^2 + 11*d^6*e^2)*b*x^6 + 12*(42*c^3*d^5*e^2
+ 11*c*d^5*e^2)*b*x^5 + 6*(105*c^4*d^4*e^2 + 55*c^2*d^4*e^2 + 3*d^4*e^2)*b*
x^4 + 8*(63*c^5*d^3*e^2 + 55*c^3*d^3*e^2 + 9*c*d^3*e^2)*b*x^3 + (252*c^6*d^
```

```
2*e^2 + 330*c^4*d^2*e^2 + 108*c^2*d^2*e^2 + 5*d^2*e^2)*b*x^2 + 2*(36*c^7*d*
e^2 + 66*c^5*d*e^2 + 36*c^3*d*e^2 + 5*c*d*e^2)*b*x + (9*c^8*e^2 + 22*c^6*e^
2 + 18*c^4*e^2 + 5*c^2*e^2)*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(3*a*d^8*e^2 + b*d^8*e^2)*x^8
+ 24*(3*a*c*d^7*e^2 + b*c*d^7*e^2)*x^7 + (2*(126*c^2*d^6*e^2 + 11*d^6*e^2)*
a + 7*(12*c^2*d^6*e^2 + d^6*e^2)*b)*x^6 + 6*(2*(42*c^3*d^5*e^2 + 11*c*d^5*e
^2)*a + 7*(4*c^3*d^5*e^2 + c*d^5*e^2)*b)*x^5 + (6*(105*c^4*d^4*e^2 + 55*c^2
*d^4*e^2 + 3*d^4*e^2)*a + 5*(42*c^4*d^4*e^2 + 21*c^2*d^4*e^2 + d^4*e^2)*b)*
x^4 + 4*(2*(63*c^5*d^3*e^2 + 55*c^3*d^3*e^2 + 9*c*d^3*e^2)*a + (42*c^5*d^3*
e^2 + 35*c^3*d^3*e^2 + 5*c*d^3*e^2)*b)*x^3 + ((252*c^6*d^2*e^2 + 330*c^4*d^
2*e^2 + 108*c^2*d^2*e^2 + 5*d^2*e^2)*a + (84*c^6*d^2*e^2 + 105*c^4*d^2*e^2
+ 30*c^2*d^2*e^2 + d^2*e^2)*b)*x^2 + (9*c^8*e^2 + 22*c^6*e^2 + 18*c^4*e^2 +
 5*c^2*e^2)*a + (3*c^8*e^2 + 7*c^6*e^2 + 5*c^4*e^2 + c^2*e^2)*b + 2*((36*c^
7*d*e^2 + 66*c^5*d*e^2 + 36*c^3*d*e^2 + 5*c*d*e^2)*a + (12*c^7*d*e^2 + 21*c
^5*d*e^2 + 10*c^3*d*e^2 + c*d*e^2)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 +
 4*(5*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*
b^2*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*
c^2*d + d)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b
^4*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*
b^4*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*
d + d)*b^4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*
(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6
*c^2*d^3 + d^3)*b^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4
)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c
^2*d^4 + d^4)*b^4*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^
2*d^2 + d^2)*b^4*x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4
*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2
+ 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*
c^2*d^3 + d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2
*d)*a^2*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d
^6*x^5 + 3*(5*c^2*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3
+ 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^
2)*a*b^3*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b
^3*c*d^3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x
^2 + 2*(2*c^3*d^2 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)
*a*b^3*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d
^2)*a*b^3*x + (c^5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5
+ 5*a^2*b^2*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*
c*d^3)*a^2*b^2*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c
^3*d + c*d)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(9*
```

```
d^10*e^2*x^10 + 90*c*d^9*e^2*x^9 + 9*c^10*e^2 + 36*c^8*e^2 + 9*(45*c^2*d^8*
e^2 + 4*d^8*e^2)*x^8 + 54*c^6*e^2 + 72*(15*c^3*d^7*e^2 + 4*c*d^7*e^2)*x^7 +
 18*(105*c^4*d^6*e^2 + 56*c^2*d^6*e^2 + 3*d^6*e^2)*x^6 + 36*c^4*e^2 + 36*(6
3*c^5*d^5*e^2 + 56*c^3*d^5*e^2 + 9*c*d^5*e^2)*x^5 + 18*(105*c^6*d^4*e^2 + 1
40*c^4*d^4*e^2 + 45*c^2*d^4*e^2 + 2*d^4*e^2)*x^4 + 9*c^2*e^2 + 72*(15*c^7*d
^3*e^2 + 28*c^5*d^3*e^2 + 15*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + (9*d^6*e^2*x^
6 + 54*c*d^5*e^2*x^5 + 9*c^6*e^2 + 4*c^4*e^2 + (135*c^2*d^4*e^2 + 4*d^4*e^2
)*x^4 - c^2*e^2 + 4*(45*c^3*d^3*e^2 + 4*c*d^3*e^2)*x^3 + (135*c^4*d^2*e^2 +
 24*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(27*c^5*d*e^2 + 8*c^3*d*e^2 - c*d*e^2)*x
)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 9*(45*c^8*d^2*e^2 + 112*c^6*d^2*e^2 + 9
0*c^4*d^2*e^2 + 24*c^2*d^2*e^2 + d^2*e^2)*x^2 + (36*d^7*e^2*x^7 + 252*c*d^6
*e^2*x^6 + 36*c^7*e^2 + 48*c^5*e^2 + 12*(63*c^2*d^5*e^2 + 4*d^5*e^2)*x^5 +
13*c^3*e^2 + 60*(21*c^3*d^4*e^2 + 4*c*d^4*e^2)*x^4 + (1260*c^4*d^3*e^2 + 48
0*c^2*d^3*e^2 + 13*d^3*e^2)*x^3 - 2*c*e^2 + 3*(252*c^5*d^2*e^2 + 160*c^3*d^
2*e^2 + 13*c*d^2*e^2)*x^2 + (252*c^6*d*e^2 + 240*c^4*d*e^2 + 39*c^2*d*e^2 -
 2*d*e^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (54*d^8*e^2*x^8 + 432*c*
d^7*e^2*x^7 + 54*c^8*e^2 + 120*c^6*e^2 + 24*(63*c^2*d^6*e^2 + 5*d^6*e^2)*x^
6 + 83*c^4*e^2 + 144*(21*c^3*d^5*e^2 + 5*c*d^5*e^2)*x^5 + (3780*c^4*d^4*e^2
 + 1800*c^2*d^4*e^2 + 83*d^4*e^2)*x^4 + 19*c^2*e^2 + 4*(756*c^5*d^3*e^2 + 6
00*c^3*d^3*e^2 + 83*c*d^3*e^2)*x^3 + (1512*c^6*d^2*e^2 + 1800*c^4*d^2*e^2 +
 498*c^2*d^2*e^2 + 19*d^2*e^2)*x^2 + 2*e^2 + 2*(216*c^7*d*e^2 + 360*c^5*d*e
^2 + 166*c^3*d*e^2 + 19*c*d*e^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 18*(5*c
^9*d*e^2 + 16*c^7*d*e^2 + 18*c^5*d*e^2 + 8*c^3*d*e^2 + c*d*e^2)*x + (36*d^9
*e^2*x^9 + 324*c*d^8*e^2*x^8 + 36*c^9*e^2 + 112*c^7*e^2 + 16*(81*c^2*d^7*e^
2 + 7*d^7*e^2)*x^7 + 123*c^5*e^2 + 112*(27*c^3*d^6*e^2 + 7*c*d^6*e^2)*x^6 +
 3*(1512*c^4*d^5*e^2 + 784*c^2*d^5*e^2 + 41*d^5*e^2)*x^5 + 57*c^3*e^2 + (45
36*c^5*d^4*e^2 + 3920*c^3*d^4*e^2 + 615*c*d^4*e^2)*x^4 + (3024*c^6*d^3*e^2
+ 3920*c^4*d^3*e^2 + 1230*c^2*d^3*e^2 + 57*d^3*e^2)*x^3 + 10*c*e^2 + 3*(432
*c^7*d^2*e^2 + 784*c^5*d^2*e^2 + 410*c^3*d^2*e^2 + 57*c*d^2*e^2)*x^2 + (324
*c^8*d*e^2 + 784*c^6*d*e^2 + 615*c^4*d*e^2 + 171*c^2*d*e^2 + 10*d*e^2)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7
*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*a
*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x^2 + 8*(c^7*
d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*a*
b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^
3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d^5*x^5 + 5*a
*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3*c*d^2)*a*b^
2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 + 2*
d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 + 12*c^2*d
^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^6 + 2*c^4 +
c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7
+ 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5 + 2*(35*c^4
*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*b
```

```
^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 + 8*(c^7*d +
3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*b^3 + (b
^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)
*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + (10*c
^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4*d + 3*c^2*d
)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(b^3*d^6
*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^3*d^3 + 2*c*
d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c^5*d + 4*c^3
*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x^3 + 3*(7*c^
5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*
b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*x^7 + 7*a*b^
2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*a*b^2
*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d^2 + 10*c^3*
d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a*b^2*x + (c^
7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\operatorname{asinh}(c+d\,x))^3}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3, x)
```

# 3.171 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1309
Rubi [A] (verified) . . . 1309
Mathematica [A] (verified) . . . 1313
Maple [A] (verified) . . . 1313
Fricas [F] . . . 1314
Sympy [F] . . . 1314
Maxima [F] . . . 1314
Giac [F] . . . 1317
Mupad [F(-1)] . . . 1318

## Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e(c+dx)^2}{b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^3d} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}$$

```
[Out] -1/2*e/b^2/d/(a+b*arcsinh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))+e*
cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arcsinh(d*x+c)
)/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382, 5783}

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx = -\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^3d} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^3d} - \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))} - \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{e\sqrt{(c+dx)^2+1}(c+dx)}{2bd(a+b\text{arcsinh}(c+dx))^2}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^2) -
 e/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d*(a + b*ArcSi
nh[c + d*x])) - (e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/
b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]
)/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2bd}$$
$$+ \frac{e\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))} + \frac{(2e)\text{Subst}\left(\int \frac{x}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{b^2d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{(2e)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{(2e)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))}$$
$$- \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))} - \frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2}$$
$$- \frac{e}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{e(c+dx)^2}{b^2d(a+b\text{arcsinh}(c+dx))}$$
$$+ \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}$$
$$- \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e(c+dx)^2}{b^2d(a+b\operatorname{arcsinh}(c+dx))}$$
$$- \frac{e\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^3d} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}$$

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$$
$$= \frac{e\left(-\frac{b^2(c+dx)\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^2} + \frac{b\left(-1-2(c+dx)^2\right)}{a+b\operatorname{arcsinh}(c+dx)} - 2\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right) + 2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\right.\right.}{2b^3d}$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2) + (
b*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 2*CoshIntegral[2*(a/b +
ArcSinh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c + d*x])]))/(2*b^3*d)
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\ \operatorname{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\ \operatorname{Ei}_1\left(2\ \operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2+1+2(dx+c\right.}{8b(a+b\ \operatorname{arcsinh}(}$ $d$ |
| default | $-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\ \operatorname{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\ \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\ \operatorname{Ei}_1\left(2\ \operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2+1+2(dx+c\right.}{8b(a+b\ \operatorname{arcsinh}(}$ $d$ |

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b*arcsinh(d*x
+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e/b^3*ex
p(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*e*(2*(d*x+c)^2+1+2*(d*x+c)*
(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arc
sinh(d*x+c)-2*a/b))
```

## Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2
 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

## Sympy [F]

$$\begin{aligned}&\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx \\ &= e\left(\int \frac{c}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \right.\\ &\qquad \left. + \int \frac{dx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2
+ b**3*asinh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*asinh(c + d*x
) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*d^8*e + b*d^8*e)*x^8 + 8*(2*a*c*d^7*e + b*c*d^7*e)*x^7 + (2*(28*
c^2*d^6*e + 3*d^6*e)*a + (28*c^2*d^6*e + 3*d^6*e)*b)*x^6 + 2*(2*(28*c^3*d^5
*e + 9*c*d^5*e)*a + (28*c^3*d^5*e + 9*c*d^5*e)*b)*x^5 + (2*(70*c^4*d^4*e +
45*c^2*d^4*e + 3*d^4*e)*a + (70*c^4*d^4*e + 45*c^2*d^4*e + 3*d^4*e)*b)*x^4
+ 4*(2*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3*e)*a + (14*c^5*d^3*e + 15*c^3
*d^3*e + 3*c*d^3*e)*b)*x^3 + (2*(28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e
 + d^2*e)*a + (28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e + d^2*e)*b)*x^2 +
 ((2*a*d^5*e + b*d^5*e)*x^5 + 5*(2*a*c*d^4*e + b*c*d^4*e)*x^4 + (2*(10*c^2*
d^3*e + d^3*e)*a + (10*c^2*d^3*e + d^3*e)*b)*x^3 + (2*(10*c^3*d^2*e + 3*c*d
```

```
^2*e)*a + (10*c^3*d^2*e + 3*c*d^2*e)*b)*x^2 + 2*(c^5*e + c^3*e)*a + (c^5*e
+ c^3*e)*b + (2*(5*c^4*d*e + 3*c^2*d*e)*a + (5*c^4*d*e + 3*c^2*d*e)*b)*x)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(2*a*d^6*e + b*d^6*e)*x^6 + 18*(2*a
*c*d^5*e + b*c*d^5*e)*x^5 + 5*(2*(9*c^2*d^4*e + d^4*e)*a + (9*c^2*d^4*e + d
^4*e)*b)*x^4 + 20*(2*(3*c^3*d^3*e + c*d^3*e)*a + (3*c^3*d^3*e + c*d^3*e)*b)
*x^3 + (5*(18*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*a + (45*c^4*d^2*e + 30*c^2*
d^2*e + 2*d^2*e)*b)*x^2 + (6*c^6*e + 10*c^4*e + 5*c^2*e + e)*a + (3*c^6*e +
 5*c^4*e + 2*c^2*e)*b + 2*((18*c^5*d*e + 20*c^3*d*e + 5*c*d*e)*a + (9*c^5*d
*e + 10*c^3*d*e + 2*c*d*e)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(c^8*e +
 3*c^6*e + 3*c^4*e + c^2*e)*a + (c^8*e + 3*c^6*e + 3*c^4*e + c^2*e)*b + 2*(
2*(4*c^7*d*e + 9*c^5*d*e + 6*c^3*d*e + c*d*e)*a + (4*c^7*d*e + 9*c^5*d*e +
6*c^3*d*e + c*d*e)*b)*x + (2*b*d^8*e*x^8 + 16*b*c*d^7*e*x^7 + 2*(28*c^2*d^6
*e + 3*d^6*e)*b*x^6 + 4*(28*c^3*d^5*e + 9*c*d^5*e)*b*x^5 + 2*(70*c^4*d^4*e
+ 45*c^2*d^4*e + 3*d^4*e)*b*x^4 + 8*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3*
e)*b*x^3 + 2*(28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e + d^2*e)*b*x^2 + 4
*(4*c^7*d*e + 9*c^5*d*e + 6*c^3*d*e + c*d*e)*b*x + 2*(b*d^5*e*x^5 + 5*b*c*d
^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*b*x^3 + (10*c^3*d^2*e + 3*c*d^2*e)*b*x^2
+ (5*c^4*d*e + 3*c^2*d*e)*b*x + (c^5*e + c^3*e)*b)*(d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + (6*b*d^6*e*x^6 + 36*b*c*d^5*e*x^5 + 10*(9*c^2*d^4*e + d^4*e)*
b*x^4 + 40*(3*c^3*d^3*e + c*d^3*e)*b*x^3 + 5*(18*c^4*d^2*e + 12*c^2*d^2*e +
 d^2*e)*b*x^2 + 2*(18*c^5*d*e + 20*c^3*d*e + 5*c*d*e)*b*x + (6*c^6*e + 10*c
^4*e + 5*c^2*e + e)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(c^8*e + 3*c^6*e +
 3*c^4*e + c^2*e)*b + (6*b*d^7*e*x^7 + 42*b*c*d^6*e*x^6 + 14*(9*c^2*d^5*e +
 d^5*e)*b*x^5 + 70*(3*c^3*d^4*e + c*d^4*e)*b*x^4 + (210*c^4*d^3*e + 140*c^2
*d^3*e + 11*d^3*e)*b*x^3 + (126*c^5*d^2*e + 140*c^3*d^2*e + 33*c*d^2*e)*b*x
^2 + (42*c^6*d*e + 70*c^4*d*e + 33*c^2*d*e + 3*d*e)*b*x + (6*c^7*e + 14*c^5
*e + 11*c^3*e + 3*c*e)*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(2*a*d^7*e + b*d^7*e)*x^7 + 21*(2*a
*c*d^6*e + b*c*d^6*e)*x^6 + 7*(2*(9*c^2*d^5*e + d^5*e)*a + (9*c^2*d^5*e + d
^5*e)*b)*x^5 + 35*(2*(3*c^3*d^4*e + c*d^4*e)*a + (3*c^3*d^4*e + c*d^4*e)*b)
*x^4 + ((210*c^4*d^3*e + 140*c^2*d^3*e + 11*d^3*e)*a + 5*(21*c^4*d^3*e + 14
*c^2*d^3*e + d^3*e)*b)*x^3 + ((126*c^5*d^2*e + 140*c^3*d^2*e + 33*c*d^2*e)*
a + (63*c^5*d^2*e + 70*c^3*d^2*e + 15*c*d^2*e)*b)*x^2 + (6*c^7*e + 14*c^5*e
 + 11*c^3*e + 3*c*e)*a + (3*c^7*e + 7*c^5*e + 5*c^3*e + c*e)*b + ((42*c^6*d
*e + 70*c^4*d*e + 33*c^2*d*e + 3*d*e)*a + (21*c^6*d*e + 35*c^4*d*e + 15*c^2
*d*e + d*e)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a
^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4
)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 +
2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b
^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 +
 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 +
2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x
^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2
 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x +
```

```
c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 +
 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c
^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*
x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3
/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*
x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 +
 d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*
d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3
*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*
c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5
*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d
^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b
^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d
^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d +
2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 +
 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5
*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(4*d^9*e*x^9 + 36*c*d^8*e*
x^8 + 4*c^9*e + 16*c^7*e + 16*(9*c^2*d^7*e + d^7*e)*x^7 + 112*(3*c^3*d^6*e
+ c*d^6*e)*x^6 + 24*c^5*e + 24*(21*c^4*d^5*e + 14*c^2*d^5*e + d^5*e)*x^5 +
8*(63*c^5*d^4*e + 70*c^3*d^4*e + 15*c*d^4*e)*x^4 + 16*c^3*e + 16*(21*c^6*d^
3*e + 35*c^4*d^3*e + 15*c^2*d^3*e + d^3*e)*x^3 + 4*(d^5*e*x^5 + 5*c*d^4*e*x
^4 + 10*c^2*d^3*e*x^3 + 10*c^3*d^2*e*x^2 + 5*c^4*d*e*x + c^5*e)*(d^2*x^2 +
2*c*d*x + c^2 + 1)^2 + 48*(3*c^7*d^2*e + 7*c^5*d^2*e + 5*c^3*d^2*e + c*d^2*
e)*x^2 + (16*d^6*e*x^6 + 96*c*d^5*e*x^5 + 16*c^6*e + 16*c^4*e + 16*(15*c^2*
d^4*e + d^4*e)*x^4 + 64*(5*c^3*d^3*e + c*d^3*e)*x^3 + 48*(5*c^4*d^2*e + 2*c
^2*d^2*e)*x^2 + 32*(3*c^5*d*e + 2*c^3*d*e)*x - 3*e)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + 24*(d^7*e*x^7 + 7*c*d^6*e*x^6 + c^7*e + 2*c^5*e + (21*c^2*d^
5*e + 2*d^5*e)*x^5 + 5*(7*c^3*d^4*e + 2*c*d^4*e)*x^4 + c^3*e + (35*c^4*d^3*
e + 20*c^2*d^3*e + d^3*e)*x^3 + (21*c^5*d^2*e + 20*c^3*d^2*e + 3*c*d^2*e)*x
^2 + (7*c^6*d*e + 10*c^4*d*e + 3*c^2*d*e)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 4*c*e + 4*(9*c^8*d*e + 28*c^6*d*e + 30*c^4*d*e + 12*c^2*d*e + d*e)*x + (1
6*d^8*e*x^8 + 128*c*d^7*e*x^7 + 16*c^8*e + 48*c^6*e + 16*(28*c^2*d^6*e + 3*
d^6*e)*x^6 + 32*(28*c^3*d^5*e + 9*c*d^5*e)*x^5 + 48*c^4*e + 16*(70*c^4*d^4*
e + 45*c^2*d^4*e + 3*d^4*e)*x^4 + 64*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3
*e)*x^3 + 19*c^2*e + (448*c^6*d^2*e + 720*c^4*d^2*e + 288*c^2*d^2*e + 19*d^
2*e)*x^2 + 2*(64*c^7*d*e + 144*c^5*d*e + 96*c^3*d*e + 19*c*d*e)*x + 3*e)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7*
c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d^
4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*a*
b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x^2 + 8*(c^7*d
 + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*a*b
```

```
^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^3
*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d^5*x^5 + 5*a*
b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3*c*d^2)*a*b^2
*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 + 2*d
^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 + 12*c^2*d^
2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^6 + 2*c^4 + c
^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7 +
 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5 + 2*(35*c^4*
d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*b^
3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 + 8*(c^7*d + 3
*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*b^3 + (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + (10*c^
2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4*d + 3*c^2*d)
*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(b^3*d^6*
x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^3*d^3 + 2*c*d
^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c^5*d + 4*c^3*
d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4
*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5 + 5*(7*c^3*d^
4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x^3 + 3*(7*c^5
*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b
^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*x^7 + 7*a*b^2
*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*a*b^2*
x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d^2 + 10*c^3*d
^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a*b^2*x + (c^7
 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^3}\,dx = \int \frac{dex + ce}{(b\,\mathrm{arsinh}\,(dx + c) + a)^3}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\,\operatorname{asinh}(c+d\,x))^3}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3, x)
```

# 3.172 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1319
Rubi [A] (verified) . . . 1319
Mathematica [A] (verified) . . . 1322
Maple [A] (verified) . . . 1322
Fricas [F] . . . 1322
Sympy [F] . . . 1323
Maxima [F] . . . 1323
Giac [F] . . . 1325
Mupad [F(-1)] . . . 1326

## Optimal result

Integrand size = 12, antiderivative size = 125

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{c+dx}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2b^3d} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2b^3d}$$

```
[Out] 1/2*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))+1/2*Chi((a+b*arcsinh(d*x+c))/b)*cos
h(a/b)/b^3/d-1/2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5858, 5773, 5818, 5774, 3384, 3379, 3382}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2b^3d} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{2b^3d} - \frac{c+dx}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{\sqrt{(c+dx)^2+1}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -1/2*Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x])^2) - (c + d*x)/(2*
b^2*d*(a + b*ArcSinh[c + d*x])) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c
+ d*x])/b])/(2*b^3*d) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b]
)/(2*b^3*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2bd} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{c+dx}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{2b^2d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{c+dx}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{c+dx}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^3d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{c+dx}{2b^2d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2b^3d} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2b^3d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = -\frac{\frac{b^2\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{b(c+dx)}{a+b\operatorname{arcsinh}(c+dx)}-\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsinh}(c\right.}{2b^3d}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -1/2*((b^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (b*(c + d*x)
)/(a + b*ArcSinh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]
 + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^3*d)
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\,\operatorname{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\,\operatorname{arcsinh}(dx+c)^2+2ab\,\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\,\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b(a+b\,\operatorname{arcsinh}(dx+c))^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b^2(a+b\,\operatorname{arcsinh}(dx+c))}}{d}$ |
| default | $\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\,\operatorname{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\,\operatorname{arcsinh}(dx+c)^2+2ab\,\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\,\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b(a+b\,\operatorname{arcsinh}(dx+c))^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b^2(a+b\,\operatorname{arcsinh}(dx+c))}}{d}$ |

```
[In] int(1/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcs
inh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)
+a/b)-1/4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*(d*x
+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

## Fricas [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*a
rcsinh(d*x + c) + a^3), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}(c+dx))^3}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**3,x)
[Out] Integral((a + b*asinh(c + d*x))**(-3), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 + d^5
)*a + (7*c^2*d^5 + d^5)*b)*x^5 + 5*((7*c^3*d^4 + 3*c*d^4)*a + (7*c^3*d^4 +
3*c*d^4)*b)*x^4 + ((35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 + 30*c
^2*d^3 + 3*d^3)*b)*x^3 + 3*((7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*b)*x^2 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c
*d^3)*x^3 + (6*a*c^2*d^2 + (6*c^2*d^2 + d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 +
c^2)*b + 2*(2*a*c^3*d + (2*c^3*d + c*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d
^3 + d^3)*a + 5*(6*c^2*d^3 + d^3)*b)*x^3 + 3*((10*c^3*d^2 + 3*c*d^2)*a + 5*
(2*c^3*d^2 + c*d^2)*b)*x^2 + 3*(c^5 + c^3)*a + (3*c^5 + 5*c^3 + 2*c)*b + (3
*(5*c^4*d + 3*c^2*d)*a + (15*c^4*d + 15*c^2*d + 2*d)*b)*x)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*a + (c^7 + 3*c^5 + 3*c^3 + c)*b +
 ((7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)
*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b*x^3 + 3*(7*c^5*d^
2 + 10*c^3*d^2 + 3*c*d^2)*b*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b*x +
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b*d^5*x^5 + 5*b*c*d^4*x^4 + (10*c^2
*d^3 + d^3)*b*x^3 + (10*c^3*d^2 + 3*c*d^2)*b*x^2 + (5*c^4*d + 3*c^2*d)*b*x
+ (c^5 + c^3)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*
b + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*d^4 + 2*d^4)*b*x^4 + 12*(5*c^
3*d^3 + 2*c*d^3)*b*x^3 + (45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*b*x^2 + 2*(9*c^5
*d + 12*c^3*d + 4*c*d)*b*x + (3*c^6 + 6*c^4 + 4*c^2 + 1)*b)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(
a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*(15*c^2*d^4 + 2*d^4)*a
 + (45*c^2*d^4 + 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 + 2*c*d^3)*a + (15*c^3*d^3
 + 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*a + (45*c^4*d^2 + 4
```

```
2*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 + 6*c^4 + 4*c^2 + 1)*a + (3*c^6 + 7*c^4
+ 5*c^2 + 1)*b + 2*((9*c^5*d + 12*c^3*d + 4*c*d)*a + (9*c^5*d + 14*c^3*d +
5*c*d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^
2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2
*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^
7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*
d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3 +
3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1
)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*
(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 +
 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*d
+ 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 +
 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2 +
 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)
*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 +
 d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^4*
d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d
^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^4
+ 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)*a
*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*d^
6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3 +
3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c^3
*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*(5
*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^4*
d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(d^8*x^8 + 8*c*d^7*x^7 + c^8 +
4*(7*c^2*d^6 + d^6)*x^6 + 4*c^6 + 8*(7*c^3*d^5 + 3*c*d^5)*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*x^4 + 6*c^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)
*x^3 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*x^
2 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 + d^3)*x^3 + 4*c^3 +
4*(10*c^3*d^2 + 3*c*d^2)*x^2 + (20*c^4*d + 12*c^2*d + 3*d)*x + 3*c)*(d^2*x^
2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(2*d^6*x^6 + 12*c*d^5*x^5 + 2*c^6 + 2*(15*
c^2*d^4 + 2*d^4)*x^4 + 4*c^4 + 8*(5*c^3*d^3 + 2*c*d^3)*x^3 + (30*c^4*d^2 +
24*c^2*d^2 + d^2)*x^2 + c^2 + 2*(6*c^5*d + 8*c^3*d + c*d)*x - 1)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 4*c^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*x + (4*d^
7*x^7 + 28*c*d^6*x^6 + 4*c^7 + 12*(7*c^2*d^5 + d^5)*x^5 + 12*c^5 + 20*(7*c^
3*d^4 + 3*c*d^4)*x^4 + (140*c^4*d^3 + 120*c^2*d^3 + 9*d^3)*x^3 + 9*c^3 + 3*
```

```
(28*c^5*d^2 + 40*c^3*d^2 + 9*c*d^2)*x^2 + (28*c^6*d + 60*c^4*d + 27*c^2*d +
 d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)/(a*b^2*d^8*x^8 + 8*a*b^2*
c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x
^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b
^2*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4
 + 4*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*
x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b
^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2
 + 3*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d
^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 +
(15*c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^
4*d^2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x +
 (c^6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 +
8*b^3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3
*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x
^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c
^2 + 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^
3*d*x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c
*d^4*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5
*c^4*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3
/2) + 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(
5*c^3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*
(3*c^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*
x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b
^3*x^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d
 + 9*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*
d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4
+ 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c
^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d +
d)*a*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{\left(b\,\operatorname{arsinh}\left(dx+c\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^3,x)
[Out] int(1/(a + b*asinh(c + d*x))^3, x)
```

# 3.173 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))^3}\,dx$

Optimal result . . . 1327
Rubi [N/A] . . . 1327
Mathematica [N/A] . . . 1328
Maple [N/A] (verified) . . . 1328
Fricas [N/A] . . . 1328
Sympy [N/A] . . . 1329
Maxima [N/A] . . . 1329
Giac [N/A] . . . 1332
Mupad [N/A] . . . 1333

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^3},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^3,x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^3), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^3}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^3}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\left(dx+c\right)+a)^3}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3
+ 3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arcsinh(d*x + c)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.87

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$$
$$= \frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\,\operatorname{asinh}(c+dx)+3a^2bdx\,\operatorname{asinh}(c+dx)+3ab^2c\,\operatorname{asinh}^2(c+dx)+3ab^2dx\,\operatorname{asinh}^2(c+dx)+b^3c\,\operatorname{asinh}^3(c+dx)+b^3dx\,\operatorname{asinh}^3(c+dx)}\,d}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*asinh(c + d*x) + 3*a**2*b*d*x*as
inh(c + d*x) + 3*a*b**2*c*asinh(c + d*x)**2 + 3*a*b**2*d*x*asinh(c + d*x)**
2 + b**3*c*asinh(c + d*x)**3 + b**3*d*x*asinh(c + d*x)**3), x)/e
```

## Maxima [N/A]

Not integrable

Time = 68.05 (sec) , antiderivative size = 6716, normalized size of antiderivative = 292.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)\left(b\,\operatorname{arsinh}(dx+c)+a\right)^3}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 + 3*d^6)*b*x^6 + 2*(28*c^3*d^
5 + 9*c*d^5)*b*x^5 + (70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*d^
3 + 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d
^2)*b*x^2 + 2*(4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*b*x + (b*d^5*x^5 + 5*b*c*
d^4*x^4 - (2*a*d^3 - (10*c^2*d^3 + d^3)*b)*x^3 - (6*a*c*d^2 - (10*c^3*d^2 +
 3*c*d^2)*b)*x^2 - 2*(c^3 + c)*a + (c^5 + c^3)*b - (2*(3*c^2*d + d)*a - (5*
c^4*d + 3*c^2*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*b*d^6*x^6 +
 18*b*c*d^5*x^5 - (4*a*d^4 - 5*(9*c^2*d^4 + d^4)*b)*x^4 - 4*(4*a*c*d^3 - 5*
(3*c^3*d^3 + c*d^3)*b)*x^3 - ((24*c^2*d^2 + 5*d^2)*a - (45*c^4*d^2 + 30*c^2
*d^2 + 2*d^2)*b)*x^2 - (4*c^4 + 5*c^2 + 1)*a + (3*c^6 + 5*c^4 + 2*c^2)*b -
2*((8*c^3*d + 5*c*d)*a - (9*c^5*d + 10*c^3*d + 2*c*d)*b)*x)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + (c^8 + 3*c^6 + 3*c^4 + c^2)*b - (2*(b*d^3*x^3 + 3*b*c*d^2*
x^2 + (3*c^2*d + d)*b*x + (c^3 + c)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ (4*b*d^4*x^4 + 16*b*c*d^3*x^3 + (24*c^2*d^2 + 5*d^2)*b*x^2 + 2*(8*c^3*d +
 5*c*d)*b*x + (4*c^4 + 5*c^2 + 1)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*b*d
^5*x^5 + 10*b*c*d^4*x^4 + (20*c^2*d^3 + 3*d^3)*b*x^3 + (20*c^3*d^2 + 9*c*d^
2)*b*x^2 + (10*c^4*d + 9*c^2*d + d)*b*x + (2*c^5 + 3*c^3 + c)*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
```

```
 (3*b*d^7*x^7 + 21*b*c*d^6*x^6 - (2*a*d^5 - 7*(9*c^2*d^5 + d^5)*b)*x^5 - 5*
(2*a*c*d^4 - 7*(3*c^3*d^4 + c*d^4)*b)*x^4 - ((20*c^2*d^3 + 3*d^3)*a - 5*(21
*c^4*d^3 + 14*c^2*d^3 + d^3)*b)*x^3 - ((20*c^3*d^2 + 9*c*d^2)*a - (63*c^5*d
^2 + 70*c^3*d^2 + 15*c*d^2)*b)*x^2 - (2*c^5 + 3*c^3 + c)*a + (3*c^7 + 7*c^5
 + 5*c^3 + c)*b - ((10*c^4*d + 9*c^2*d + d)*a - (21*c^6*d + 35*c^4*d + 15*c
^2*d + d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^9*e*x^8 + 8*a
^2*b^2*c*d^8*e*x^7 + (28*c^2*d^7*e + 3*d^7*e)*a^2*b^2*x^6 + 2*(28*c^3*d^6*e
 + 9*c*d^6*e)*a^2*b^2*x^5 + (70*c^4*d^5*e + 45*c^2*d^5*e + 3*d^5*e)*a^2*b^2
*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*d^4*e)*a^2*b^2*x^3 + (28*c^6*d^
3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*a^2*b^2*x^2 + 2*(4*c^7*d^2*e + 9
*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*a^2*b^2*x + (c^8*d*e + 3*c^6*d*e + 3*c^
4*d*e + c^2*d*e)*a^2*b^2 + (b^4*d^9*e*x^8 + 8*b^4*c*d^8*e*x^7 + (28*c^2*d^7
*e + 3*d^7*e)*b^4*x^6 + 2*(28*c^3*d^6*e + 9*c*d^6*e)*b^4*x^5 + (70*c^4*d^5*
e + 45*c^2*d^5*e + 3*d^5*e)*b^4*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*
d^4*e)*b^4*x^3 + (28*c^6*d^3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*b^4*x
^2 + 2*(4*c^7*d^2*e + 9*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*b^4*x + (c^8*d*e
 + 3*c^6*d*e + 3*c^4*d*e + c^2*d*e)*b^4 + (b^4*d^6*e*x^5 + 5*b^4*c*d^5*e*x^
4 + 10*b^4*c^2*d^4*e*x^3 + 10*b^4*c^3*d^3*e*x^2 + 5*b^4*c^4*d^2*e*x + b^4*c
^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^7*e*x^6 + 6*b^4*c*d^
6*e*x^5 + (15*c^2*d^5*e + d^5*e)*b^4*x^4 + 4*(5*c^3*d^4*e + c*d^4*e)*b^4*x^
3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*b^4*x^2 + 2*(3*c^5*d^2*e + 2*c^3*d^2*e)*b
^4*x + (c^6*d*e + c^4*d*e)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^8*
e*x^7 + 7*b^4*c*d^7*e*x^6 + (21*c^2*d^6*e + 2*d^6*e)*b^4*x^5 + 5*(7*c^3*d^5
*e + 2*c*d^5*e)*b^4*x^4 + (35*c^4*d^4*e + 20*c^2*d^4*e + d^4*e)*b^4*x^3 + (
21*c^5*d^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*b^4*x^2 + (7*c^6*d^2*e + 10*c^4*d^
2*e + 3*c^2*d^2*e)*b^4*x + (c^7*d*e + 2*c^5*d*e + c^3*d*e)*b^4)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
 + (a^2*b^2*d^6*e*x^5 + 5*a^2*b^2*c*d^5*e*x^4 + 10*a^2*b^2*c^2*d^4*e*x^3 + 1
0*a^2*b^2*c^3*d^3*e*x^2 + 5*a^2*b^2*c^4*d^2*e*x + a^2*b^2*c^5*d*e)*(d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^7*e*x^6 + 6*a^2*b^2*c*d^6*e*x^5
 + (15*c^2*d^5*e + d^5*e)*a^2*b^2*x^4 + 4*(5*c^3*d^4*e + c*d^4*e)*a^2*b^2*x^
3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*a^2*b^2*x^2 + 2*(3*c^5*d^2*e + 2*c^3*d^2*
e)*a^2*b^2*x + (c^6*d*e + c^4*d*e)*a^2*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
 2*(a*b^3*d^9*e*x^8 + 8*a*b^3*c*d^8*e*x^7 + (28*c^2*d^7*e + 3*d^7*e)*a*b^3*
x^6 + 2*(28*c^3*d^6*e + 9*c*d^6*e)*a*b^3*x^5 + (70*c^4*d^5*e + 45*c^2*d^5*e
 + 3*d^5*e)*a*b^3*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*d^4*e)*a*b^3*x
^3 + (28*c^6*d^3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*a*b^3*x^2 + 2*(4*
c^7*d^2*e + 9*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*a*b^3*x + (c^8*d*e + 3*c^6
*d*e + 3*c^4*d*e + c^2*d*e)*a*b^3 + (a*b^3*d^6*e*x^5 + 5*a*b^3*c*d^5*e*x^4
 + 10*a*b^3*c^2*d^4*e*x^3 + 10*a*b^3*c^3*d^3*e*x^2 + 5*a*b^3*c^4*d^2*e*x + a
*b^3*c^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^7*e*x^6 + 6*
a*b^3*c*d^6*e*x^5 + (15*c^2*d^5*e + d^5*e)*a*b^3*x^4 + 4*(5*c^3*d^4*e + c*d
^4*e)*a*b^3*x^3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*a*b^3*x^2 + 2*(3*c^5*d^2*e
 + 2*c^3*d^2*e)*a*b^3*x + (c^6*d*e + c^4*d*e)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^
2 + 1) + 3*(a*b^3*d^8*e*x^7 + 7*a*b^3*c*d^7*e*x^6 + (21*c^2*d^6*e + 2*d^6*e
```

```
)*a*b^3*x^5 + 5*(7*c^3*d^5*e + 2*c*d^5*e)*a*b^3*x^4 + (35*c^4*d^4*e + 20*c^
2*d^4*e + d^4*e)*a*b^3*x^3 + (21*c^5*d^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*a*b^
3*x^2 + (7*c^6*d^2*e + 10*c^4*d^2*e + 3*c^2*d^2*e)*a*b^3*x + (c^7*d*e + 2*c
^5*d*e + c^3*d*e)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^8*e*x^7 + 7*a^2*b^2*c*d^7*
e*x^6 + (21*c^2*d^6*e + 2*d^6*e)*a^2*b^2*x^5 + 5*(7*c^3*d^5*e + 2*c*d^5*e)*
a^2*b^2*x^4 + (35*c^4*d^4*e + 20*c^2*d^4*e + d^4*e)*a^2*b^2*x^3 + (21*c^5*d
^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*a^2*b^2*x^2 + (7*c^6*d^2*e + 10*c^4*d^2*e
+ 3*c^2*d^2*e)*a^2*b^2*x + (c^7*d*e + 2*c^5*d*e + c^3*d*e)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(4*(d^4*x^4 + 4*c*d^3*x^3 + c^4
 + 2*(3*c^2*d^2 + d^2)*x^2 + 2*c^2 + 4*(c^3*d + c*d)*x)*(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 + (12*d^5*x^5 + 60*c*d^4*x^4 + 12*c^5 + 2*(60*c^2*d^3 + 11*d^3
)*x^3 + 22*c^3 + 6*(20*c^3*d^2 + 11*c*d^2)*x^2 + (60*c^4*d + 66*c^2*d + 7*d
)*x + 7*c)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 2*(6*d^6*x^6 + 36*c*d^5*x^
5 + 6*c^6 + 10*(9*c^2*d^4 + d^4)*x^4 + 10*c^4 + 40*(3*c^3*d^3 + c*d^3)*x^3
+ 5*(18*c^4*d^2 + 12*c^2*d^2 + d^2)*x^2 + 5*c^2 + 2*(18*c^5*d + 20*c^3*d +
5*c*d)*x + 1)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (4*d^7*x^7 + 28*c*d^6*x^6 + 4
*c^7 + 6*(14*c^2*d^5 + d^5)*x^5 + 6*c^5 + 10*(14*c^3*d^4 + 3*c*d^4)*x^4 + (
140*c^4*d^3 + 60*c^2*d^3 + 3*d^3)*x^3 + 3*c^3 + 3*(28*c^5*d^2 + 20*c^3*d^2
+ 3*c*d^2)*x^2 + (28*c^6*d + 30*c^4*d + 9*c^2*d + d)*x + c)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))/(a*b^2*d^11*e*x^11 + 11*a*b^2*c*d^10*e*x^10 + (55*c^2*d
^9*e + 4*d^9*e)*a*b^2*x^9 + 3*(55*c^3*d^8*e + 12*c*d^8*e)*a*b^2*x^8 + 6*(55
*c^4*d^7*e + 24*c^2*d^7*e + d^7*e)*a*b^2*x^7 + 42*(11*c^5*d^6*e + 8*c^3*d^6
*e + c*d^6*e)*a*b^2*x^6 + 2*(231*c^6*d^5*e + 252*c^4*d^5*e + 63*c^2*d^5*e +
 2*d^5*e)*a*b^2*x^5 + 2*(165*c^7*d^4*e + 252*c^5*d^4*e + 105*c^3*d^4*e + 10
*c*d^4*e)*a*b^2*x^4 + (165*c^8*d^3*e + 336*c^6*d^3*e + 210*c^4*d^3*e + 40*c
^2*d^3*e + d^3*e)*a*b^2*x^3 + (55*c^9*d^2*e + 144*c^7*d^2*e + 126*c^5*d^2*e
 + 40*c^3*d^2*e + 3*c*d^2*e)*a*b^2*x^2 + (11*c^10*d*e + 36*c^8*d*e + 42*c^6
*d*e + 20*c^4*d*e + 3*c^2*d*e)*a*b^2*x + (c^11*e + 4*c^9*e + 6*c^7*e + 4*c^
5*e + c^3*e)*a*b^2 + (a*b^2*d^7*e*x^7 + 7*a*b^2*c*d^6*e*x^6 + 21*a*b^2*c^2*
d^5*e*x^5 + 35*a*b^2*c^3*d^4*e*x^4 + 35*a*b^2*c^4*d^3*e*x^3 + 21*a*b^2*c^5*
d^2*e*x^2 + 7*a*b^2*c^6*d*e*x + a*b^2*c^7*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 + 4*(a*b^2*d^8*e*x^8 + 8*a*b^2*c*d^7*e*x^7 + (28*c^2*d^6*e + d^6*e)*a*b^2
*x^6 + 2*(28*c^3*d^5*e + 3*c*d^5*e)*a*b^2*x^5 + 5*(14*c^4*d^4*e + 3*c^2*d^4
*e)*a*b^2*x^4 + 4*(14*c^5*d^3*e + 5*c^3*d^3*e)*a*b^2*x^3 + (28*c^6*d^2*e +
15*c^4*d^2*e)*a*b^2*x^2 + 2*(4*c^7*d*e + 3*c^5*d*e)*a*b^2*x + (c^8*e + c^6*
e)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^9*e*x^9 + 9*a*b^
2*c*d^8*e*x^8 + 2*(18*c^2*d^7*e + d^7*e)*a*b^2*x^7 + 14*(6*c^3*d^6*e + c*d^
6*e)*a*b^2*x^6 + (126*c^4*d^5*e + 42*c^2*d^5*e + d^5*e)*a*b^2*x^5 + (126*c^
5*d^4*e + 70*c^3*d^4*e + 5*c*d^4*e)*a*b^2*x^4 + 2*(42*c^6*d^3*e + 35*c^4*d^
3*e + 5*c^2*d^3*e)*a*b^2*x^3 + 2*(18*c^7*d^2*e + 21*c^5*d^2*e + 5*c^3*d^2*e
)*a*b^2*x^2 + (9*c^8*d*e + 14*c^6*d*e + 5*c^4*d*e)*a*b^2*x + (c^9*e + 2*c^7
*e + c^5*e)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^11*e*x^11 + 11*b^
3*c*d^10*e*x^10 + (55*c^2*d^9*e + 4*d^9*e)*b^3*x^9 + 3*(55*c^3*d^8*e + 12*c
*d^8*e)*b^3*x^8 + 6*(55*c^4*d^7*e + 24*c^2*d^7*e + d^7*e)*b^3*x^7 + 42*(11*
```

```
c^5*d^6*e + 8*c^3*d^6*e + c*d^6*e)*b^3*x^6 + 2*(231*c^6*d^5*e + 252*c^4*d^5
*e + 63*c^2*d^5*e + 2*d^5*e)*b^3*x^5 + 2*(165*c^7*d^4*e + 252*c^5*d^4*e + 1
05*c^3*d^4*e + 10*c*d^4*e)*b^3*x^4 + (165*c^8*d^3*e + 336*c^6*d^3*e + 210*c
^4*d^3*e + 40*c^2*d^3*e + d^3*e)*b^3*x^3 + (55*c^9*d^2*e + 144*c^7*d^2*e +
126*c^5*d^2*e + 40*c^3*d^2*e + 3*c*d^2*e)*b^3*x^2 + (11*c^10*d*e + 36*c^8*d
*e + 42*c^6*d*e + 20*c^4*d*e + 3*c^2*d*e)*b^3*x + (c^11*e + 4*c^9*e + 6*c^7
*e + 4*c^5*e + c^3*e)*b^3 + (b^3*d^7*e*x^7 + 7*b^3*c*d^6*e*x^6 + 21*b^3*c^2
*d^5*e*x^5 + 35*b^3*c^3*d^4*e*x^4 + 35*b^3*c^4*d^3*e*x^3 + 21*b^3*c^5*d^2*e
*x^2 + 7*b^3*c^6*d*e*x + b^3*c^7*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^
3*d^8*e*x^8 + 8*b^3*c*d^7*e*x^7 + (28*c^2*d^6*e + d^6*e)*b^3*x^6 + 2*(28*c^
3*d^5*e + 3*c*d^5*e)*b^3*x^5 + 5*(14*c^4*d^4*e + 3*c^2*d^4*e)*b^3*x^4 + 4*(
14*c^5*d^3*e + 5*c^3*d^3*e)*b^3*x^3 + (28*c^6*d^2*e + 15*c^4*d^2*e)*b^3*x^2
 + 2*(4*c^7*d*e + 3*c^5*d*e)*b^3*x + (c^8*e + c^6*e)*b^3)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^(3/2) + 6*(b^3*d^9*e*x^9 + 9*b^3*c*d^8*e*x^8 + 2*(18*c^2*d^7*e
 + d^7*e)*b^3*x^7 + 14*(6*c^3*d^6*e + c*d^6*e)*b^3*x^6 + (126*c^4*d^5*e + 4
2*c^2*d^5*e + d^5*e)*b^3*x^5 + (126*c^5*d^4*e + 70*c^3*d^4*e + 5*c*d^4*e)*b
^3*x^4 + 2*(42*c^6*d^3*e + 35*c^4*d^3*e + 5*c^2*d^3*e)*b^3*x^3 + 2*(18*c^7*
d^2*e + 21*c^5*d^2*e + 5*c^3*d^2*e)*b^3*x^2 + (9*c^8*d*e + 14*c^6*d*e + 5*c
^4*d*e)*b^3*x + (c^9*e + 2*c^7*e + c^5*e)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1
) + 4*(b^3*d^10*e*x^10 + 10*b^3*c*d^9*e*x^9 + 3*(15*c^2*d^8*e + d^8*e)*b^3*
x^8 + 24*(5*c^3*d^7*e + c*d^7*e)*b^3*x^7 + 3*(70*c^4*d^6*e + 28*c^2*d^6*e +
 d^6*e)*b^3*x^6 + 6*(42*c^5*d^5*e + 28*c^3*d^5*e + 3*c*d^5*e)*b^3*x^5 + (21
0*c^6*d^4*e + 210*c^4*d^4*e + 45*c^2*d^4*e + d^4*e)*b^3*x^4 + 4*(30*c^7*d^3
*e + 42*c^5*d^3*e + 15*c^3*d^3*e + c*d^3*e)*b^3*x^3 + 3*(15*c^8*d^2*e + 28*
c^6*d^2*e + 15*c^4*d^2*e + 2*c^2*d^2*e)*b^3*x^2 + 2*(5*c^9*d*e + 12*c^7*d*e
 + 9*c^5*d*e + 2*c^3*d*e)*b^3*x + (c^10*e + 3*c^8*e + 3*c^6*e + c^4*e)*b^3)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) + 4*(a*b^2*d^10*e*x^10 + 10*a*b^2*c*d^9*e*x^9 + 3*(15*c^2*d^8*e +
 d^8*e)*a*b^2*x^8 + 24*(5*c^3*d^7*e + c*d^7*e)*a*b^2*x^7 + 3*(70*c^4*d^6*e
+ 28*c^2*d^6*e + d^6*e)*a*b^2*x^6 + 6*(42*c^5*d^5*e + 28*c^3*d^5*e + 3*c*d^
5*e)*a*b^2*x^5 + (210*c^6*d^4*e + 210*c^4*d^4*e + 45*c^2*d^4*e + d^4*e)*a*b
^2*x^4 + 4*(30*c^7*d^3*e + 42*c^5*d^3*e + 15*c^3*d^3*e + c*d^3*e)*a*b^2*x^3
 + 3*(15*c^8*d^2*e + 28*c^6*d^2*e + 15*c^4*d^2*e + 2*c^2*d^2*e)*a*b^2*x^2 +
 2*(5*c^9*d*e + 12*c^7*d*e + 9*c^5*d*e + 2*c^3*d*e)*a*b^2*x + (c^10*e + 3*c
^8*e + 3*c^6*e + c^4*e)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)(b\,\operatorname{arsinh}\left(dx+c\right)+a)^3}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}\left(c+d\,x\right)\right)^3}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3), x)
```

# 3.174 $\int \frac{(ce+dex)^4}{(a+b\mathbf{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1334
Rubi [A] (verified) . . . 1335
Mathematica [A] (verified) . . . 1340
Maple [B] (verified) . . . 1341
Fricas [F] . . . 1342
Sympy [F] . . . 1342
Maxima [F(-1)] . . . 1343
Giac [F] . . . 1343
Mupad [F(-1)] . . . 1343

## Optimal result

Integrand size = 23, antiderivative size = 410

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^4\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{48b^4d} + \frac{27e^4\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32b^4d} - \frac{125e^4\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{96b^4d} + \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{48b^4d} - \frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^4d} + \frac{125e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{96b^4d}$$

```
[Out] -2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a+
b*arcsinh(d*x+c))^2+1/48*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d-27
```

```
/32*e^4*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+125/96*e^4*cosh(5*a
/b)*Shi(5*(a+b*arcsinh(d*x+c))/b)/b^4/d-1/48*e^4*Chi((a+b*arcsinh(d*x+c))/b
)*sinh(a/b)/b^4/d+27/32*e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^4/d
-125/96*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-2*e^4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-25/6*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2
)/b^3/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx &= -\frac{e^4 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{48b^4d} \\ &+ \frac{27e^4 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^4d} \\ &- \frac{125e^4 \sinh\left(\frac{5a}{b}\right) \operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{96b^4d} \\ &+ \frac{e^4 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{48b^4d} \\ &- \frac{27e^4 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^4d} \\ &+ \frac{125e^4 \cosh\left(\frac{5a}{b}\right) \operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{96b^4d} \\ &- \frac{25e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} \\ &- \frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^2}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\ &- \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\ &- \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\ &- \frac{e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3bd(a+b\operatorname{arcsinh}(c+dx))^3}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (5*e^4*(c +
 d*x)^5)/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^4*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (25*e^4*(c + d*x)^4*Sqrt[
1 + (c + d*x)^2])/(6*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^4*CoshIntegral[(a
 + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(48*b^4*d) + (27*e^4*CoshIntegral[(3*(
a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(32*b^4*d) - (125*e^4*CoshIntegr
al[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(96*b^4*d) + (e^4*Cosh[a/
b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(48*b^4*d) - (27*e^4*Cosh[(3*a
)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^4*d) + (125*e^4*Co
sh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(96*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} + \frac{(4e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&\quad + \frac{(5e^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{5e^4(c+dx)^5}{6b^2d(a+b\text{arcsinh}(c+dx))^2} + \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{b^2d} \\
&\quad + \frac{(25e^4)\,\text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{6b^2d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{(2e^4)\operatorname{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^4d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16x} + \frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16x} - \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8x}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d} \\
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48b^4d} \\
&\quad - \frac{(125e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96b^4d} \\
&\quad - \frac{(3e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad + \frac{(75e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad + \frac{\left(25e^4\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48b^4d} \\
&\quad + \frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad - \frac{\left(75e^4\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^4d} \\
&\quad + \frac{\left(125e^4\cosh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96b^4d} \\
&\quad + \frac{\left(e^4\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad - \frac{\left(25e^4\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48b^4d} \\
&\quad - \frac{\left(3e^4\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad + \frac{\left(75e^4\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32b^4d} \\
&\quad - \frac{\left(125e^4\sinh\left(\frac{5a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{2e^4(c+dx)^3}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{5e^4(c+dx)^5}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^4\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{48b^4d} \\
&\quad + \frac{27e^4\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32b^4d} \\
&\quad - \frac{125e^4\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{96b^4d} \\
&\quad + \frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{48b^4d} - \frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^4d} \\
&\quad + \frac{125e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{96b^4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00

$$
\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx =
$$

$$
-\frac{e^4\left(\frac{32b^3(c+dx)^4\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{16b^2\left(-4(c+dx)^3-5(c+dx)^5\right)}{(a+b\operatorname{arcsinh}(c+dx))^2} + \frac{16b\sqrt{1+(c+dx)^2}\left(12(c+dx)^2+25(c+dx)^4\right)}{a+b\operatorname{arcsinh}(c+dx)} + 384\left(\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsi}\right.\right.\right.\right.
$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]

[Out] -1/96*(e^4*((32*b^3*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d
*x])^3 - (16*b^2*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x])
^2 + (16*b*Sqrt[1 + (c + d*x)^2]*(12*(c + d*x)^2 + 25*(c + d*x)^4))/(a + b*
ArcSinh[c + d*x]) + 384*(CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - C
osh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]) + 544*(-3*CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Cosh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 125*(10*CoshIntegral[a/b + A
rcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]*Sinh[(5*a)/b] - 10*Co
sh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh[(3*a)/b]*SinhIntegral
[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c + d*x])])))/(b^4*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. 2(384) = 768.

Time = 0.90 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.03

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1244 |
| default | Expression too large to display | 1244 |

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/192*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(25*b^2*ar
csinh(d*x+c)^2+50*a*b*arcsinh(d*x+c)-5*b^2*arcsinh(d*x+c)+25*a^2-5*a*b+2*b^
2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c
)+a^3)+125/192*e^4/b^4*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-1/64*(-4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(
9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsinh(d*x+c)+9*a^2-3*a
*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsin
h(d*x+c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/96*(-
(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)-b
^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(
d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/96*e^4/b^4*exp(a/b)*Ei(1,arcsinh(d*x
+c)+a/b)-1/48/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/96
/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/96/b^3*e^4*(d
*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,
-arcsinh(d*x+c)-a/b)+1/32/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+3/64/b^2*e^4*(4*(d*
x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*
arcsinh(d*x+c))^2+9/64/b^3*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64/b^4*e^4*exp(-3*a
/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/96/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*
(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1
+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-5/192/b^2*e^4*(16*(d*x+c)^5+20*(d
*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-25/192/b^3*e^4*(16*(d*
x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2
*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-125/192/b^4*
e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^
2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

## Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx \\
&= e^4\left(\int \frac{c^4}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right. \\
&\quad + \int \frac{d^4x^4}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx \\
&\quad + \int \frac{4cd^3x^3}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx \\
&\quad + \int \frac{6c^2d^2x^2}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx \\
&\quad \left. + \int \frac{4c^3dx}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**4*(Integral(c**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**4*x**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*
c*d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(6*c*
*2*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**
2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*c
**3*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4
*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\operatorname{asinh}(c+d\,x))^4}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4, x)
```

# 3.175 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1344
Rubi [A] (verified) . . . 1345
Mathematica [A] (verified) . . . 1349
Maple [B] (verified) . . . 1349
Fricas [F] . . . 1350
Sympy [F] . . . 1351
Maxima [F(-1)] . . . 1351
Giac [F] . . . 1352
Mupad [F(-1)] . . . 1352

## Optimal result

Integrand size = 23, antiderivative size = 340

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e^3(c+dx)^4}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} + \frac{4e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} + \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{4e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d}$$

```
[Out] -1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a+
b*arcsinh(d*x+c))^2-1/3*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d
+4/3*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^4/d+1/3*e^3*Shi(2*(a+b
*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)
*sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^3-e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-8/3*e^3
*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx &= -\frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\ &+ \frac{4e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\ &+ \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\ &- \frac{4e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\ &- \frac{8e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} \\ &- \frac{e^3\sqrt{(c+dx)^2+1}(c+dx)}{b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\ &- \frac{e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\ &- \frac{e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{3bd(a+b\operatorname{arcsinh}(c+dx))^3}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^3*(c + d
*x)^4)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (8*e^3*(c + d*x)^3*Sqrt[1 + (c
+ d*x)^2])/(3*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^3*Cosh[(2*a)/b]*CoshInte
gral[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*Cosh
Integral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (e^3*Sinh[(2*a)/b]*Si
nhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b
]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} + \frac{e^3\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{bd} \\
&\quad + \frac{(4e^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{2e^3(c+dx)^4}{3b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{b^2d} + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{2e^3(c+dx)^4}{3b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\text{arcsinh}(c+dx))} \\
&\quad - \frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^4d} \\
&\quad + \frac{(8e^3)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{2x} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^4d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{2e^3(c+dx)^4}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{(4e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad - \frac{(4e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad + \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^4d} \\
&\quad - \frac{\left(e^3\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^4d} \\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{2e^3(c+dx)^4}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^4d} \\
&\quad - \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^4d} \\
&\quad - \frac{\left(4e^3\cosh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad + \frac{\left(4e^3\cosh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad + \frac{\left(4e^3\sinh\left(\frac{2a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad - \frac{\left(4e^3\sinh\left(\frac{4a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^3(c+dx)^2}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{2e^3(c+dx)^4}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\
&\quad + \frac{4e^3\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} \\
&\quad + \frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{4e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx \\
&= \frac{e^3\left(-\frac{2b^3(c+dx)^3\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3} + \frac{b^2(-3(c+dx)^2-4(c+dx)^4)}{(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2b\sqrt{1+(c+dx)^2}(3(c+dx)+8(c+dx)^3)}{a+b\operatorname{arcsinh}(c+dx)} + 6\log(a+b\operatorname{arcsinh}(c+d\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (b^2*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x])^2 - (2*b
*Sqrt[1 + (c + d*x)^2]*(3*(c + d*x) + 8*(c + d*x)^3))/(a + b*ArcSinh[c + d*
x]) + 6*Log[a + b*ArcSinh[c + d*x]] + 30*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b
 + ArcSinh[c + d*x])] - Log[a + b*ArcSinh[c + d*x]] - Sinh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcSinh[c + d*x])]) + 8*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/
b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c + d*
x])] + 3*Log[a + b*ArcSinh[c + d*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c + d*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x]
)])))/(6*b^4*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(318) = 636.

Time = 0.89 (sec) , antiderivative size = 800, normalized size of antiderivative = 2.35

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3\left(8b^2\operatorname{arcsinh}(dx+c)^2+16ab\operatorname{arcsinh}(dx+c)-2b^2\operatorname{arcsinh}(}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$ |
| default | $\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3\left(8b^2\operatorname{arcsinh}(dx+c)^2+16ab\operatorname{arcsinh}(dx+c)-2b^2\operatorname{arcsinh}(}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$ |

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+c
)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(8*b^2*arcsinh(d*x+c)^2+16*a*b*arcsinh(d*x+c)
-2*b^2*arcsinh(d*x+c)+8*a^2-2*a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*ar
csinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-2/3*e^3/b^4*exp(4*a/b)*Ei(1,4*ar
csinh(d*x+c)+4*a/b)-1/24*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e^3
*(2*b^2*arcsinh(d*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^2-a*b+
b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x
+c)+a^3)+1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/24/b*e^3*(2*
(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+1/24/b^2*
e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/
12/b^3*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c
))+1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/48/b*e^3*(8*(d*x
+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(
1/2)+1)/(a+b*arcsinh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x
+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x
+c))^2-1/6/b^3*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)
+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-2/3/b^4*e^3*exp(-4*a
/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^3}{(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*arc
sinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2
 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= e^3\left(\int \frac{c^3}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right.$$
$$+\int \frac{d^3x^3}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx$$
$$+\int \frac{3cd^2x^2}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx$$
$$\left.+\int \frac{3c^2dx}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**3*(Integral(c**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*
c*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*c*
*2*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*
a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^4}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4, x)
```

# 3.176 $\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1353
Rubi [A] (verified) . . . 1354
Mathematica [A] (verified) . . . 1359
Maple [B] (verified) . . . 1359
Fricas [F] . . . 1360
Sympy [F] . . . 1360
Maxima [F(-1)] . . . 1361
Giac [F] . . . 1361
Mupad [F(-1)] . . . 1361

## Optimal result

Integrand size = 23, antiderivative size = 331

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{24b^4d} - \frac{9e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{8b^4d} - \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{24b^4d} + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^4d}$$

```
[Out] -1/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b*
arcsinh(d*x+c))^2-1/24*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d+9/8*
e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+1/24*e^2*Chi((a+b*arcsi
nh(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*
a/b)/b^4/d-1/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3
-1/3*e^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382, 5773, 5819}

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = &\frac{e^2 \sinh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{24b^4d} \\ &- \frac{9e^2 \sinh\left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8b^4d} \\ &- \frac{e^2 \cosh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{24b^4d} \\ &+ \frac{9e^2 \cosh\left(\frac{3a}{b}\right) \text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8b^4d} \\ &- \frac{3e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{2b^3d(a+b\text{arcsinh}(c+dx))} \\ &- \frac{e^2\sqrt{(c+dx)^2+1}}{3b^3d(a+b\text{arcsinh}(c+dx))} \\ &- \frac{e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))^2} \\ &- \frac{e^2(c+dx)}{3b^2d(a+b\text{arcsinh}(c+dx))^2} \\ &- \frac{e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{3bd(a+b\text{arcsinh}(c+dx))^3}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*(c + d*x)^
3)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*Sqrt[1 + (c + d*x)^2])/(3*b^
3*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(
2*b^3*d*(a + b*ArcSinh[c + d*x])) + (e^2*CoshIntegral[(a + b*ArcSinh[c + d*
x])/b]*Sinh[a/b])/(24*b^4*d) - (9*e^2*CoshIntegral[(3*(a + b*ArcSinh[c + d*
x]))/b]*Sinh[(3*a)/b])/(8*b^4*d) - (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c + d*x])/b])/(24*b^4*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c + d*x]))/b])/(8*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^2 x^2}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2 \text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} + \frac{(2e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd}$$
$$+ \frac{e^2 \text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2 d(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)^3}{2b^2 d(a+b\text{arcsinh}(c+dx))^2}$$
$$+ \frac{e^2 \text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{3b^2 d} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{2b^2 d}$$

$$
\begin{aligned}
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d} \\
&\quad + \frac{e^2\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\operatorname{arcsinh}(x))}\,dx, x, c+dx\right)}{3b^3d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d} \\
&\quad - \frac{(9e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad - \frac{\left(3e^2\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d} \\
&\quad + \frac{\left(9e^2\cosh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d} \\
&\quad - \frac{\left(e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad + \frac{\left(3e^2\sinh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d} \\
&\quad - \frac{\left(9e^2\sinh\left(\frac{3a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^4d} \\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} \\
&\quad - \frac{e^2(c+dx)^3}{2b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} \\
&\quad - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\operatorname{arcsinh}(c+dx))} + \frac{e^2\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{24b^4d} \\
&\quad - \frac{9e^2\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{8b^4d} - \frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{24b^4d} \\
&\quad + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{8b^4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.78

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{e^2\left(-\frac{8b^3(c+dx)^2\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3}+\frac{4b^2(-2(c+dx)-3(c+dx)^3)}{(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{4b\sqrt{1+(c+dx)^2}(2+9(c+dx)^2)}{a+b\operatorname{arcsinh}(c+dx)}-80\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)s\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (4*b^2*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x])^2 - (4*b
*Sqrt[1 + (c + d*x)^2]*(2 + 9*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 80*C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + 80*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c + d*x]] + 27*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/
b] - CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + A
rcSinh[c + d*x])])))/(24*b^4*d)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(307) = 614.

Time = 0.47 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.14

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3b^2\operatorname{arcsinh}(dx+c)+9a^2\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$ |
| default | $\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3b^2\operatorname{arcsinh}(dx+c)+9a^2\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$ |

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)
+3*d*x+3*c)*e^2*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsinh
(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)
^2+3*a^2*b*arcsinh(d*x+c)+a^3)+9/16*e^2/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c
)+3*a/b)-1/48*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b^2*arcsinh(d*x+c)^2+2*a*b*
arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+
3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/24/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))^3+1/48/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2
+1/48/b^3*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/48/b^4*e^2
```

```
*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/1
6/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)
^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^3*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16/b
^4*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

## Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arcsinh(d*x + c)^4 + 4*
a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d
*x + c) + a^4), x)
```

## Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^4}\,dx \\
&= e^2\left(\int \frac{c^2}{a^4+4a^3b\,\mathrm{asinh}\,(c+dx)+6a^2b^2\,\mathrm{asinh}^2\,(c+dx)+4ab^3\,\mathrm{asinh}^3\,(c+dx)+b^4\,\mathrm{asinh}^4\,(c+dx)}\,dx\right. \\
&\quad + \int \frac{d^2x^2}{a^4+4a^3b\,\mathrm{asinh}\,(c+dx)+6a^2b^2\,\mathrm{asinh}^2\,(c+dx)+4ab^3\,\mathrm{asinh}^3\,(c+dx)+b^4\,\mathrm{asinh}^4\,(c+dx)}\,dx \\
&\quad \left. + \int \frac{2cdx}{a^4+4a^3b\,\mathrm{asinh}\,(c+dx)+6a^2b^2\,\mathrm{asinh}^2\,(c+dx)+4ab^3\,\mathrm{asinh}^3\,(c+dx)+b^4\,\mathrm{asinh}^4\,(c+dx)}\,dx\right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**2*(Integral(c**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(2*
c*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a
*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\operatorname{asinh}(c+d\,x))^4}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4, x)
```

# 3.177 $\int \frac{ce+dex}{(a+b\mathbf{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1362
Rubi [A] (verified) . . . 1363
Mathematica [A] (verified) . . . 1366
Maple [A] (verified) . . . 1366
Fricas [F] . . . 1367
Sympy [F] . . . 1367
Maxima [F(-1)] . . . 1367
Giac [F] . . . 1368
Mupad [F(-1)] . . . 1368

## Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{e}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e(c+dx)^2}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3b^3d(a+b\operatorname{arcsinh}(c+dx))} + \frac{2e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{2e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{3b^4d}$$

```
[Out] -1/6*e/b^2/d/(a+b*arcsinh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+
c))^2+2/3*e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(a+
b*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/
d/(a+b*arcsinh(d*x+c))^3-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsi
nh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382, 5783}

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{3b^3d(a+b\text{arcsinh}(c+dx))} - \frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e\sqrt{(c+dx)^2+1}(c+dx)}{3bd(a+b\text{arcsinh}(c+dx))^3}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^3) -
 e/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b^2*d*(a + b*A
rcSinh[c + d*x])^2) - (2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*(a + b
*ArcSinh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c +
 d*x]))/b])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c
 + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&\quad + \frac{(2e)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2} + \frac{(2e)\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))} + \frac{(2e)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^4d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{\left(2e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad - \frac{\left(2e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^4d} \\
&= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))} \\
&\quad + \frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d} - \frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{e\left(-\frac{2b^3(c+dx)\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3}+\frac{b^2(-1-2(c+dx)^2)}{(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{4b(c+dx)\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)}+4\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right)\right.}{6b^4}$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (
b^2*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x])^2 - (4*b*(c + d*x)*Sqrt[
1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2
*(a/b + ArcSinh[c + d*x])] + 4*Log[a + b*ArcSinh[c + d*x]] - 4*(Log[a + b*A
rcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))
)/(6*b^4*d)
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3ab^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcs}\right.}{3}$ |
| default | $\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3ab^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcs}\right.}{3}$ |

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b^2*arcsinh(d
*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^2-a*b+b^2)/b^3/(b^3*arc
sinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/3*e/b^
4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2+1+2*(d*x+c)
*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/b^2*e*(2*(d*x+c)^2+1+2*(d
*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2+1+
2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*E
i(1,-2*arcsinh(d*x+c)-2*a/b))
```

## Fricas [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{dex+ce}{(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3
 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

## Sympy [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= e\left(\int \frac{c}{a^4+4a^3b\,\operatorname{asinh}(c+dx)+6a^2b^2\,\operatorname{asinh}^2(c+dx)+4ab^3\,\operatorname{asinh}^3(c+dx)+b^4\,\operatorname{asinh}^4(c+dx)}\,dx\right.$$
$$\left.+\int \frac{dx}{a^4+4a^3b\,\operatorname{asinh}(c+dx)+6a^2b^2\,\operatorname{asinh}^2(c+dx)+4ab^3\,\operatorname{asinh}^3(c+dx)+b^4\,\operatorname{asinh}^4(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e*(Integral(c/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(d*
x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a*b**
3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

## Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\,\text{asinh}\,(c+d\,x))^4}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4, x)
```

# 3.178 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1369
Rubi [A] (verified) . . . 1369
Mathematica [A] (verified) . . . 1372
Maple [A] (verified) . . . 1372
Fricas [F] . . . 1373
Sympy [F] . . . 1373
Maxima [F] . . . 1374
Giac [F] . . . 1383
Mupad [F(-1)] . . . 1384

## Optimal result

Integrand size = 12, antiderivative size = 160

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{6b^4d} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{6b^4d}$$

```
[Out] 1/6*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))^2+1/6*cosh(a/b)*Shi((a+b*arcsinh(d*
x+c))/b)/b^4/d-1/6*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-1/6*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*a
rcsinh(d*x+c))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used

$= \{5858, 5773, 5818, 5819, 3384, 3379, 3382\}$

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{6b^4d} + \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{6b^4d} - \frac{\sqrt{(c+dx)^2+1}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{c+dx}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{\sqrt{(c+dx)^2+1}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -1/3*Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x])^3) - (c + d*x)/(6*
b^2*d*(a + b*ArcSinh[c + d*x])^2) - Sqrt[1 + (c + d*x)^2]/(6*b^3*d*(a + b*A
rcSinh[c + d*x])) - (CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(6
*b^4*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(6*b^4*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^4}\,dx, x, c+dx\right)}{d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^3}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^2}\,dx, x, c+dx\right)}{6b^2d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\text{arcsinh}(c+dx))} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))}\,dx, x, c+dx\right)}{6b^3d} \\
&= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\text{arcsinh}(c+dx))^2} \\
&\quad - \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\text{arcsinh}(c+dx))} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{6b^4d}
\end{aligned}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2}$$
$$- \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))}$$
$$+ \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d}$$
$$- \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d}$$
$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\operatorname{arcsinh}(c+dx))^2}$$
$$- \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\operatorname{arcsinh}(c+dx))} - \frac{\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{6b^4d}$$
$$+ \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{6b^4d}$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx =$$
$$-\frac{\frac{2b^3\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3} + \frac{b^2(c+dx)}{(a+b\operatorname{arcsinh}(c+dx))^2} + \frac{b\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)} + \operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right) - \cosh($$
$$6b^4d$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -1/6*((2*b^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (b^2*(c +
d*x))/(a + b*ArcSinh[c + d*x])^2 + (b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh
[c + d*x]) + CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*Sin
hIntegral[a/b + ArcSinh[c + d*x]])/(b^4*d)
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{12b^4}-\frac{}{d}$ |
| default | $\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{12b^4}-\frac{}{d}$ |

```
[In] int(1/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(
d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*
arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/12/b^4*exp(a/b)*Ei(1,arcsinh
(d*x+c)+a/b)-1/6/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/
b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/12/b^3*(d*x+c+(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arcsinh(d*x+
c)-a/b))
```

## Fricas [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2
*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

## Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}(c+dx))^4}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-4), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*((a^2*d^11 + a*b*d^11 + 2*b^2*d^11)*x^11 + 11*(a^2*c*d^10 + a*b*c*d^10
 + 2*b^2*c*d^10)*x^10 + 5*((11*c^2*d^9 + d^9)*a^2 + (11*c^2*d^9 + d^9)*a*b
+ 2*(11*c^2*d^9 + d^9)*b^2)*x^9 + 15*((11*c^3*d^8 + 3*c*d^8)*a^2 + (11*c^3*
d^8 + 3*c*d^8)*a*b + 2*(11*c^3*d^8 + 3*c*d^8)*b^2)*x^8 + 10*((33*c^4*d^7 +
18*c^2*d^7 + d^7)*a^2 + (33*c^4*d^7 + 18*c^2*d^7 + d^7)*a*b + 2*(33*c^4*d^7
 + 18*c^2*d^7 + d^7)*b^2)*x^7 + 14*((33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a^2
 + (33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a*b + 2*(33*c^5*d^6 + 30*c^3*d^6 + 5
*c*d^6)*b^2)*x^6 + 2*((231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a^2
 + (231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a*b + 2*(231*c^6*d^5 +
 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2)*x^5 + 10*((33*c^7*d^4 + 63*c^5*d^4
 + 35*c^3*d^4 + 5*c*d^4)*a^2 + (33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*
d^4)*a*b + 2*(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b^2)*x^4 + 5*
((33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a^2 + (33*c^8*d^
3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a*b + 2*(33*c^8*d^3 + 84*c^
6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2)*x^3 + ((a^2*d^6 + a*b*d^6 + 2*b
^2*d^6)*x^6 + 6*(a^2*c*d^5 + a*b*c*d^5 + 2*b^2*c*d^5)*x^5 + (15*a*b*c^2*d^4
 + (15*c^2*d^4 + d^4)*a^2 + 2*(15*c^2*d^4 + d^4)*b^2)*x^4 + 4*(5*a*b*c^3*d^
3 + (5*c^3*d^3 + c*d^3)*a^2 + 2*(5*c^3*d^3 + c*d^3)*b^2)*x^3 + (c^6 + c^4 +
 3*c^2 + 3)*a^2 + (c^6 - c^2)*a*b + 2*(c^6 + c^4)*b^2 + (3*(5*c^4*d^2 + 2*c
^2*d^2 + d^2)*a^2 + (15*c^4*d^2 - d^2)*a*b + 6*(5*c^4*d^2 + 2*c^2*d^2)*b^2)
*x^2 + 2*((3*c^5*d + 2*c^3*d + 3*c*d)*a^2 + (3*c^5*d - c*d)*a*b + 2*(3*c^5*
d + 2*c^3*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + (5*(a^2*d^7 + a*
b*d^7 + 2*b^2*d^7)*x^7 + 35*(a^2*c*d^6 + a*b*c*d^6 + 2*b^2*c*d^6)*x^6 + (3*
(35*c^2*d^5 + 3*d^5)*a^2 + 5*(21*c^2*d^5 + d^5)*a*b + 6*(35*c^2*d^5 + 3*d^5
)*b^2)*x^5 + 5*((35*c^3*d^4 + 9*c*d^4)*a^2 + 5*(7*c^3*d^4 + c*d^4)*a*b + 2*
(35*c^3*d^4 + 9*c*d^4)*b^2)*x^4 + (5*(35*c^4*d^3 + 18*c^2*d^3 + 2*d^3)*a^2
+ (175*c^4*d^3 + 50*c^2*d^3 - 2*d^3)*a*b + 2*(175*c^4*d^3 + 90*c^2*d^3 + 4*
d^3)*b^2)*x^3 + (5*c^7 + 9*c^5 + 10*c^3 + 6*c)*a^2 + (5*c^7 + 5*c^5 - 2*c^3
 - 2*c)*a*b + 2*(5*c^7 + 9*c^5 + 4*c^3)*b^2 + (15*(7*c^5*d^2 + 6*c^3*d^2 +
2*c*d^2)*a^2 + (105*c^5*d^2 + 50*c^3*d^2 - 6*c*d^2)*a*b + 6*(35*c^5*d^2 + 3
0*c^3*d^2 + 4*c*d^2)*b^2)*x^2 + ((35*c^6*d + 45*c^4*d + 30*c^2*d + 6*d)*a^2
 + (35*c^6*d + 25*c^4*d - 6*c^2*d - 2*d)*a*b + 2*(35*c^6*d + 45*c^4*d + 12*
c^2*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + (c^11 + 5*c^9 + 10*c^7 + 1
0*c^5 + 5*c^3 + c)*a^2 + (c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*a*b +
 2*(c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*b^2 + 5*((11*c^9*d^2 + 36*c
^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*a^2 + (11*c^9*d^2 + 36*c^7*d^2
+ 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*a*b + 2*(11*c^9*d^2 + 36*c^7*d^2 + 42*
```

```
c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*b^2)*x^2 + (b^2*d^11*x^11 + 11*b^2*c*d^10*x
^10 + 5*(11*c^2*d^9 + d^9)*b^2*x^9 + 15*(11*c^3*d^8 + 3*c*d^8)*b^2*x^8 + 10
*(33*c^4*d^7 + 18*c^2*d^7 + d^7)*b^2*x^7 + 14*(33*c^5*d^6 + 30*c^3*d^6 + 5*
c*d^6)*b^2*x^6 + 2*(231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2*x^
5 + 10*(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b^2*x^4 + 5*(33*c^8
*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2*x^3 + 5*(11*c^9*d^2
+ 36*c^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*b^2*x^2 + (11*c^10*d + 45
*c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*b^2*x + (b^2*d^6*x^6 + 6*b^2*c
*d^5*x^5 + (15*c^2*d^4 + d^4)*b^2*x^4 + 4*(5*c^3*d^3 + c*d^3)*b^2*x^3 + 3*(
5*c^4*d^2 + 2*c^2*d^2 + d^2)*b^2*x^2 + 2*(3*c^5*d + 2*c^3*d + 3*c*d)*b^2*x
+ (c^6 + c^4 + 3*c^2 + 3)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + (5*b^2
*d^7*x^7 + 35*b^2*c*d^6*x^6 + 3*(35*c^2*d^5 + 3*d^5)*b^2*x^5 + 5*(35*c^3*d^
4 + 9*c*d^4)*b^2*x^4 + 5*(35*c^4*d^3 + 18*c^2*d^3 + 2*d^3)*b^2*x^3 + 15*(7*
c^5*d^2 + 6*c^3*d^2 + 2*c*d^2)*b^2*x^2 + (35*c^6*d + 45*c^4*d + 30*c^2*d +
6*d)*b^2*x + (5*c^7 + 9*c^5 + 10*c^3 + 6*c)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 + (c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*b^2 + (10*b^2*d^8*x^8
+ 80*b^2*c*d^7*x^7 + 2*(140*c^2*d^6 + 13*d^6)*b^2*x^6 + 4*(140*c^3*d^5 + 39
*c*d^5)*b^2*x^5 + 2*(350*c^4*d^4 + 195*c^2*d^4 + 11*d^4)*b^2*x^4 + 8*(70*c^
5*d^3 + 65*c^3*d^3 + 11*c*d^3)*b^2*x^3 + (280*c^6*d^2 + 390*c^4*d^2 + 132*c
^2*d^2 + 3*d^2)*b^2*x^2 + 2*(40*c^7*d + 78*c^5*d + 44*c^3*d + 3*c*d)*b^2*x
+ (10*c^8 + 26*c^6 + 22*c^4 + 3*c^2 - 3)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + 2*(5*b^2*d^9*x^9 + 45*b^2*c*d^8*x^8 + (180*c^2*d^7 + 17*d^7)*b^2*x
^7 + 7*(60*c^3*d^6 + 17*c*d^6)*b^2*x^6 + 3*(210*c^4*d^5 + 119*c^2*d^5 + 6*d
^5)*b^2*x^5 + 5*(126*c^5*d^4 + 119*c^3*d^4 + 18*c*d^4)*b^2*x^4 + 5*(84*c^6*
d^3 + 119*c^4*d^3 + 36*c^2*d^3 + d^3)*b^2*x^3 + 3*(60*c^7*d^2 + 119*c^5*d^2
 + 60*c^3*d^2 + 5*c*d^2)*b^2*x^2 + (45*c^8*d + 119*c^6*d + 90*c^4*d + 15*c^
2*d - d)*b^2*x + (5*c^9 + 17*c^7 + 18*c^5 + 5*c^3 - c)*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + (5*b^2*d^10*x^10 + 50*b^2*c*d^9*x^9 + 3*(75*c^2*d^8 + 7*d^
8)*b^2*x^8 + 24*(25*c^3*d^7 + 7*c*d^7)*b^2*x^7 + (1050*c^4*d^6 + 588*c^2*d^
6 + 31*d^6)*b^2*x^6 + 6*(210*c^5*d^5 + 196*c^3*d^5 + 31*c*d^5)*b^2*x^5 + 5*
(210*c^6*d^4 + 294*c^4*d^4 + 93*c^2*d^4 + 4*d^4)*b^2*x^4 + 4*(150*c^7*d^3 +
 294*c^5*d^3 + 155*c^3*d^3 + 20*c*d^3)*b^2*x^3 + 3*(75*c^8*d^2 + 196*c^6*d^
2 + 155*c^4*d^2 + 40*c^2*d^2 + 2*d^2)*b^2*x^2 + 2*(25*c^9*d + 84*c^7*d + 93
*c^5*d + 40*c^3*d + 6*c*d)*b^2*x + (5*c^10 + 21*c^8 + 31*c^6 + 20*c^4 + 6*c
^2 + 1)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^2 + (10*(a^2*d^8 + a*b*d^8 + 2*b^2*d^8)*x^8 + 80*(a^2
*c*d^7 + a*b*c*d^7 + 2*b^2*c*d^7)*x^7 + 2*((140*c^2*d^6 + 13*d^6)*a^2 + 10*
(14*c^2*d^6 + d^6)*a*b + 2*(140*c^2*d^6 + 13*d^6)*b^2)*x^6 + 4*((140*c^3*d^
5 + 39*c*d^5)*a^2 + 10*(14*c^3*d^5 + 3*c*d^5)*a*b + 2*(140*c^3*d^5 + 39*c*d
^5)*b^2)*x^5 + 2*((350*c^4*d^4 + 195*c^2*d^4 + 11*d^4)*a^2 + 5*(70*c^4*d^4
+ 30*c^2*d^4 + d^4)*a*b + 2*(350*c^4*d^4 + 195*c^2*d^4 + 11*d^4)*b^2)*x^4 +
 8*((70*c^5*d^3 + 65*c^3*d^3 + 11*c*d^3)*a^2 + 5*(14*c^5*d^3 + 10*c^3*d^3 +
 c*d^3)*a*b + 2*(70*c^5*d^3 + 65*c^3*d^3 + 11*c*d^3)*b^2)*x^3 + (10*c^8 + 2
6*c^6 + 22*c^4 + 3*c^2 - 3)*a^2 + (10*c^8 + 20*c^6 + 10*c^4 - c^2 - 1)*a*b
+ 4*(5*c^8 + 13*c^6 + 11*c^4 + 3*c^2)*b^2 + ((280*c^6*d^2 + 390*c^4*d^2 + 1
```

```
32*c^2*d^2 + 3*d^2)*a^2 + (280*c^6*d^2 + 300*c^4*d^2 + 60*c^2*d^2 - d^2)*a*
b + 4*(140*c^6*d^2 + 195*c^4*d^2 + 66*c^2*d^2 + 3*d^2)*b^2)*x^2 + 2*((40*c^
7*d + 78*c^5*d + 44*c^3*d + 3*c*d)*a^2 + (40*c^7*d + 60*c^5*d + 20*c^3*d -
c*d)*a*b + 4*(20*c^7*d + 39*c^5*d + 22*c^3*d + 3*c*d)*b^2)*x)*(d^2*x^2 + 2*
c*d*x + c^2 + 1)^(3/2) + 2*(5*(a^2*d^9 + a*b*d^9 + 2*b^2*d^9)*x^9 + 45*(a^2
*c*d^8 + a*b*c*d^8 + 2*b^2*c*d^8)*x^8 + ((180*c^2*d^7 + 17*d^7)*a^2 + 15*(1
2*c^2*d^7 + d^7)*a*b + 2*(180*c^2*d^7 + 17*d^7)*b^2)*x^7 + 7*((60*c^3*d^6 +
 17*c*d^6)*a^2 + 15*(4*c^3*d^6 + c*d^6)*a*b + 2*(60*c^3*d^6 + 17*c*d^6)*b^2
)*x^6 + (3*(210*c^4*d^5 + 119*c^2*d^5 + 6*d^5)*a^2 + (630*c^4*d^5 + 315*c^2
*d^5 + 16*d^5)*a*b + 42*(30*c^4*d^5 + 17*c^2*d^5 + d^5)*b^2)*x^5 + 5*((126*
c^5*d^4 + 119*c^3*d^4 + 18*c*d^4)*a^2 + (126*c^5*d^4 + 105*c^3*d^4 + 16*c*d
^4)*a*b + 14*(18*c^5*d^4 + 17*c^3*d^4 + 3*c*d^4)*b^2)*x^4 + (5*(84*c^6*d^3
+ 119*c^4*d^3 + 36*c^2*d^3 + d^3)*a^2 + (420*c^6*d^3 + 525*c^4*d^3 + 160*c^
2*d^3 + 7*d^3)*a*b + 2*(420*c^6*d^3 + 595*c^4*d^3 + 210*c^2*d^3 + 11*d^3)*b
^2)*x^3 + (5*c^9 + 17*c^7 + 18*c^5 + 5*c^3 - c)*a^2 + (5*c^9 + 15*c^7 + 16*
c^5 + 7*c^3 + c)*a*b + 2*(5*c^9 + 17*c^7 + 21*c^5 + 11*c^3 + 2*c)*b^2 + (3*
(60*c^7*d^2 + 119*c^5*d^2 + 60*c^3*d^2 + 5*c*d^2)*a^2 + (180*c^7*d^2 + 315*
c^5*d^2 + 160*c^3*d^2 + 21*c*d^2)*a*b + 6*(60*c^7*d^2 + 119*c^5*d^2 + 70*c^
3*d^2 + 11*c*d^2)*b^2)*x^2 + ((45*c^8*d + 119*c^6*d + 90*c^4*d + 15*c^2*d -
 d)*a^2 + (45*c^8*d + 105*c^6*d + 80*c^4*d + 21*c^2*d + d)*a*b + 2*(45*c^8*
d + 119*c^6*d + 105*c^4*d + 33*c^2*d + 2*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^
2 + 1) + ((11*c^10*d + 45*c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*a^2 +
 (11*c^10*d + 45*c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*a*b + 2*(11*c^
10*d + 45*c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*b^2)*x + ((2*a*b*d^11
 + b^2*d^11)*x^11 + 11*(2*a*b*c*d^10 + b^2*c*d^10)*x^10 + 5*(2*(11*c^2*d^9
+ d^9)*a*b + (11*c^2*d^9 + d^9)*b^2)*x^9 + 15*(2*(11*c^3*d^8 + 3*c*d^8)*a*b
 + (11*c^3*d^8 + 3*c*d^8)*b^2)*x^8 + 10*(2*(33*c^4*d^7 + 18*c^2*d^7 + d^7)*
a*b + (33*c^4*d^7 + 18*c^2*d^7 + d^7)*b^2)*x^7 + 14*(2*(33*c^5*d^6 + 30*c^3
*d^6 + 5*c*d^6)*a*b + (33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*b^2)*x^6 + 2*(2*(
231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a*b + (231*c^6*d^5 + 315*c
^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2)*x^5 + 10*(2*(33*c^7*d^4 + 63*c^5*d^4 + 3
5*c^3*d^4 + 5*c*d^4)*a*b + (33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)
*b^2)*x^4 + 5*(2*(33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*
a*b + (33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2)*x^3 +
((2*a*b*d^6 + b^2*d^6)*x^6 + 6*(2*a*b*c*d^5 + b^2*c*d^5)*x^5 + (15*b^2*c^2*
d^4 + 2*(15*c^2*d^4 + d^4)*a*b)*x^4 + 4*(5*b^2*c^3*d^3 + 2*(5*c^3*d^3 + c*d
^3)*a*b)*x^3 + 2*(c^6 + c^4 + 3*c^2 + 3)*a*b + (c^6 - c^2)*b^2 + (6*(5*c^4*
d^2 + 2*c^2*d^2 + d^2)*a*b + (15*c^4*d^2 - d^2)*b^2)*x^2 + 2*(2*(3*c^5*d +
2*c^3*d + 3*c*d)*a*b + (3*c^5*d - c*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1
)^(5/2) + (5*(2*a*b*d^7 + b^2*d^7)*x^7 + 35*(2*a*b*c*d^6 + b^2*c*d^6)*x^6 +
 (6*(35*c^2*d^5 + 3*d^5)*a*b + 5*(21*c^2*d^5 + d^5)*b^2)*x^5 + 5*(2*(35*c^3
*d^4 + 9*c*d^4)*a*b + 5*(7*c^3*d^4 + c*d^4)*b^2)*x^4 + (10*(35*c^4*d^3 + 18
*c^2*d^3 + 2*d^3)*a*b + (175*c^4*d^3 + 50*c^2*d^3 - 2*d^3)*b^2)*x^3 + 2*(5*
c^7 + 9*c^5 + 10*c^3 + 6*c)*a*b + (5*c^7 + 5*c^5 - 2*c^3 - 2*c)*b^2 + (30*(
7*c^5*d^2 + 6*c^3*d^2 + 2*c*d^2)*a*b + (105*c^5*d^2 + 50*c^3*d^2 - 6*c*d^2)
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*b^2)*x^2 + (2*(35*c^6*d + 45*c^4*d + 30*c^2*d + 6*d)*a*b + (35*c^6*d + 25*
c^4*d - 6*c^2*d - 2*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 2*(c^11 +
5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*a*b + (c^11 + 5*c^9 + 10*c^7 + 10*c^5
+ 5*c^3 + c)*b^2 + 5*(2*(11*c^9*d^2 + 36*c^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2
+ 3*c*d^2)*a*b + (11*c^9*d^2 + 36*c^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d
^2)*b^2)*x^2 + (10*(2*a*b*d^8 + b^2*d^8)*x^8 + 80*(2*a*b*c*d^7 + b^2*c*d^7)
*x^7 + 4*((140*c^2*d^6 + 13*d^6)*a*b + 5*(14*c^2*d^6 + d^6)*b^2)*x^6 + 8*((
140*c^3*d^5 + 39*c*d^5)*a*b + 5*(14*c^3*d^5 + 3*c*d^5)*b^2)*x^5 + 2*(2*(350
*c^4*d^4 + 195*c^2*d^4 + 11*d^4)*a*b + 5*(70*c^4*d^4 + 30*c^2*d^4 + d^4)*b^
2)*x^4 + 8*(2*(70*c^5*d^3 + 65*c^3*d^3 + 11*c*d^3)*a*b + 5*(14*c^5*d^3 + 10
*c^3*d^3 + c*d^3)*b^2)*x^3 + 2*(10*c^8 + 26*c^6 + 22*c^4 + 3*c^2 - 3)*a*b +
 (10*c^8 + 20*c^6 + 10*c^4 - c^2 - 1)*b^2 + (2*(280*c^6*d^2 + 390*c^4*d^2 +
 132*c^2*d^2 + 3*d^2)*a*b + (280*c^6*d^2 + 300*c^4*d^2 + 60*c^2*d^2 - d^2)*
b^2)*x^2 + 2*(2*(40*c^7*d + 78*c^5*d + 44*c^3*d + 3*c*d)*a*b + (40*c^7*d +
60*c^5*d + 20*c^3*d - c*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 2*
(5*(2*a*b*d^9 + b^2*d^9)*x^9 + 45*(2*a*b*c*d^8 + b^2*c*d^8)*x^8 + (2*(180*c
^2*d^7 + 17*d^7)*a*b + 15*(12*c^2*d^7 + d^7)*b^2)*x^7 + 7*(2*(60*c^3*d^6 +
17*c*d^6)*a*b + 15*(4*c^3*d^6 + c*d^6)*b^2)*x^6 + (6*(210*c^4*d^5 + 119*c^2
*d^5 + 6*d^5)*a*b + (630*c^4*d^5 + 315*c^2*d^5 + 16*d^5)*b^2)*x^5 + 5*(2*(1
26*c^5*d^4 + 119*c^3*d^4 + 18*c*d^4)*a*b + (126*c^5*d^4 + 105*c^3*d^4 + 16*
c*d^4)*b^2)*x^4 + (10*(84*c^6*d^3 + 119*c^4*d^3 + 36*c^2*d^3 + d^3)*a*b + (
420*c^6*d^3 + 525*c^4*d^3 + 160*c^2*d^3 + 7*d^3)*b^2)*x^3 + 2*(5*c^9 + 17*c
^7 + 18*c^5 + 5*c^3 - c)*a*b + (5*c^9 + 15*c^7 + 16*c^5 + 7*c^3 + c)*b^2 +
(6*(60*c^7*d^2 + 119*c^5*d^2 + 60*c^3*d^2 + 5*c*d^2)*a*b + (180*c^7*d^2 + 3
15*c^5*d^2 + 160*c^3*d^2 + 21*c*d^2)*b^2)*x^2 + (2*(45*c^8*d + 119*c^6*d +
90*c^4*d + 15*c^2*d - d)*a*b + (45*c^8*d + 105*c^6*d + 80*c^4*d + 21*c^2*d
+ d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*(11*c^10*d + 45*c^8*d + 70*
c^6*d + 50*c^4*d + 15*c^2*d + d)*a*b + (11*c^10*d + 45*c^8*d + 70*c^6*d + 5
0*c^4*d + 15*c^2*d + d)*b^2)*x + (5*(2*a*b*d^10 + b^2*d^10)*x^10 + 50*(2*a*
b*c*d^9 + b^2*c*d^9)*x^9 + (6*(75*c^2*d^8 + 7*d^8)*a*b + 5*(45*c^2*d^8 + 4*
d^8)*b^2)*x^8 + 8*(6*(25*c^3*d^7 + 7*c*d^7)*a*b + 5*(15*c^3*d^7 + 4*c*d^7)*
b^2)*x^7 + (2*(1050*c^4*d^6 + 588*c^2*d^6 + 31*d^6)*a*b + (1050*c^4*d^6 + 5
60*c^2*d^6 + 31*d^6)*b^2)*x^6 + 2*(6*(210*c^5*d^5 + 196*c^3*d^5 + 31*c*d^5)
*a*b + (630*c^5*d^5 + 560*c^3*d^5 + 93*c*d^5)*b^2)*x^5 + (10*(210*c^6*d^4 +
 294*c^4*d^4 + 93*c^2*d^4 + 4*d^4)*a*b + (1050*c^6*d^4 + 1400*c^4*d^4 + 465
*c^2*d^4 + 23*d^4)*b^2)*x^4 + 4*(2*(150*c^7*d^3 + 294*c^5*d^3 + 155*c^3*d^3
 + 20*c*d^3)*a*b + (150*c^7*d^3 + 280*c^5*d^3 + 155*c^3*d^3 + 23*c*d^3)*b^2
)*x^3 + 2*(5*c^10 + 21*c^8 + 31*c^6 + 20*c^4 + 6*c^2 + 1)*a*b + (5*c^10 + 2
0*c^8 + 31*c^6 + 23*c^4 + 8*c^2 + 1)*b^2 + (6*(75*c^8*d^2 + 196*c^6*d^2 + 1
55*c^4*d^2 + 40*c^2*d^2 + 2*d^2)*a*b + (225*c^8*d^2 + 560*c^6*d^2 + 465*c^4
*d^2 + 138*c^2*d^2 + 8*d^2)*b^2)*x^2 + 2*(2*(25*c^9*d + 84*c^7*d + 93*c^5*d
 + 40*c^3*d + 6*c*d)*a*b + (25*c^9*d + 80*c^7*d + 93*c^5*d + 46*c^3*d + 8*c
*d)*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)) + (5*(a^2*d^10 + a*b*d^10 + 2*b^2*d^10)*x^10 + 50*(a^2
*c*d^9 + a*b*c*d^9 + 2*b^2*c*d^9)*x^9 + (3*(75*c^2*d^8 + 7*d^8)*a^2 + 5*(45
```

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*c^2*d^8 + 4*d^8)*a*b + 6*(75*c^2*d^8 + 7*d^8)*b^2)*x^8 + 8*(3*(25*c^3*d^7
+ 7*c*d^7)*a^2 + 5*(15*c^3*d^7 + 4*c*d^7)*a*b + 6*(25*c^3*d^7 + 7*c*d^7)*b^
2)*x^7 + ((1050*c^4*d^6 + 588*c^2*d^6 + 31*d^6)*a^2 + (1050*c^4*d^6 + 560*c
^2*d^6 + 31*d^6)*a*b + 4*(525*c^4*d^6 + 294*c^2*d^6 + 17*d^6)*b^2)*x^6 + 2*
(3*(210*c^5*d^5 + 196*c^3*d^5 + 31*c*d^5)*a^2 + (630*c^5*d^5 + 560*c^3*d^5
+ 93*c*d^5)*a*b + 12*(105*c^5*d^5 + 98*c^3*d^5 + 17*c*d^5)*b^2)*x^5 + (5*(2
10*c^6*d^4 + 294*c^4*d^4 + 93*c^2*d^4 + 4*d^4)*a^2 + (1050*c^6*d^4 + 1400*c
^4*d^4 + 465*c^2*d^4 + 23*d^4)*a*b + 4*(525*c^6*d^4 + 735*c^4*d^4 + 255*c^2
*d^4 + 13*d^4)*b^2)*x^4 + 4*((150*c^7*d^3 + 294*c^5*d^3 + 155*c^3*d^3 + 20*
c*d^3)*a^2 + (150*c^7*d^3 + 280*c^5*d^3 + 155*c^3*d^3 + 23*c*d^3)*a*b + 4*(
75*c^7*d^3 + 147*c^5*d^3 + 85*c^3*d^3 + 13*c*d^3)*b^2)*x^3 + (5*c^10 + 21*c
^8 + 31*c^6 + 20*c^4 + 6*c^2 + 1)*a^2 + (5*c^10 + 20*c^8 + 31*c^6 + 23*c^4
+ 8*c^2 + 1)*a*b + 2*(5*c^10 + 21*c^8 + 34*c^6 + 26*c^4 + 9*c^2 + 1)*b^2 +
(3*(75*c^8*d^2 + 196*c^6*d^2 + 155*c^4*d^2 + 40*c^2*d^2 + 2*d^2)*a^2 + (225
*c^8*d^2 + 560*c^6*d^2 + 465*c^4*d^2 + 138*c^2*d^2 + 8*d^2)*a*b + 6*(75*c^8
*d^2 + 196*c^6*d^2 + 170*c^4*d^2 + 52*c^2*d^2 + 3*d^2)*b^2)*x^2 + 2*((25*c^
9*d + 84*c^7*d + 93*c^5*d + 40*c^3*d + 6*c*d)*a^2 + (25*c^9*d + 80*c^7*d +
93*c^5*d + 46*c^3*d + 8*c*d)*a*b + 2*(25*c^9*d + 84*c^7*d + 102*c^5*d + 52*
c^3*d + 9*c*d)*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^3*b^3*d^11*x^1
0 + 10*a^3*b^3*c*d^10*x^9 + 5*(9*c^2*d^9 + d^9)*a^3*b^3*x^8 + 40*(3*c^3*d^8
 + c*d^8)*a^3*b^3*x^7 + 10*(21*c^4*d^7 + 14*c^2*d^7 + d^7)*a^3*b^3*x^6 + 4*
(63*c^5*d^6 + 70*c^3*d^6 + 15*c*d^6)*a^3*b^3*x^5 + 10*(21*c^6*d^5 + 35*c^4*
d^5 + 15*c^2*d^5 + d^5)*a^3*b^3*x^4 + 40*(3*c^7*d^4 + 7*c^5*d^4 + 5*c^3*d^4
 + c*d^4)*a^3*b^3*x^3 + 5*(9*c^8*d^3 + 28*c^6*d^3 + 30*c^4*d^3 + 12*c^2*d^3
 + d^3)*a^3*b^3*x^2 + 10*(c^9*d^2 + 4*c^7*d^2 + 6*c^5*d^2 + 4*c^3*d^2 + c*d
^2)*a^3*b^3*x + (c^10*d + 5*c^8*d + 10*c^6*d + 10*c^4*d + 5*c^2*d + d)*a^3*
b^3 + (b^6*d^11*x^10 + 10*b^6*c*d^10*x^9 + 5*(9*c^2*d^9 + d^9)*b^6*x^8 + 40
*(3*c^3*d^8 + c*d^8)*b^6*x^7 + 10*(21*c^4*d^7 + 14*c^2*d^7 + d^7)*b^6*x^6 +
 4*(63*c^5*d^6 + 70*c^3*d^6 + 15*c*d^6)*b^6*x^5 + 10*(21*c^6*d^5 + 35*c^4*d
^5 + 15*c^2*d^5 + d^5)*b^6*x^4 + 40*(3*c^7*d^4 + 7*c^5*d^4 + 5*c^3*d^4 + c*
d^4)*b^6*x^3 + 5*(9*c^8*d^3 + 28*c^6*d^3 + 30*c^4*d^3 + 12*c^2*d^3 + d^3)*b
^6*x^2 + 10*(c^9*d^2 + 4*c^7*d^2 + 6*c^5*d^2 + 4*c^3*d^2 + c*d^2)*b^6*x + (
c^10*d + 5*c^8*d + 10*c^6*d + 10*c^4*d + 5*c^2*d + d)*b^6 + (b^6*d^6*x^5 +
5*b^6*c*d^5*x^4 + 10*b^6*c^2*d^4*x^3 + 10*b^6*c^3*d^3*x^2 + 5*b^6*c^4*d^2*x
 + b^6*c^5*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + 5*(b^6*d^7*x^6 + 6*b^6*
c*d^6*x^5 + (15*c^2*d^5 + d^5)*b^6*x^4 + 4*(5*c^3*d^4 + c*d^4)*b^6*x^3 + 3*
(5*c^4*d^3 + 2*c^2*d^3)*b^6*x^2 + 2*(3*c^5*d^2 + 2*c^3*d^2)*b^6*x + (c^6*d
+ c^4*d)*b^6)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 10*(b^6*d^8*x^7 + 7*b^6*c*d
^7*x^6 + (21*c^2*d^6 + 2*d^6)*b^6*x^5 + 5*(7*c^3*d^5 + 2*c*d^5)*b^6*x^4 + (
35*c^4*d^4 + 20*c^2*d^4 + d^4)*b^6*x^3 + (21*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3
)*b^6*x^2 + (7*c^6*d^2 + 10*c^4*d^2 + 3*c^2*d^2)*b^6*x + (c^7*d + 2*c^5*d +
 c^3*d)*b^6)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 10*(b^6*d^9*x^8 + 8*b^6*
c*d^8*x^7 + (28*c^2*d^7 + 3*d^7)*b^6*x^6 + 2*(28*c^3*d^6 + 9*c*d^6)*b^6*x^5
 + (70*c^4*d^5 + 45*c^2*d^5 + 3*d^5)*b^6*x^4 + 4*(14*c^5*d^4 + 15*c^3*d^4 +
 3*c*d^4)*b^6*x^3 + (28*c^6*d^3 + 45*c^4*d^3 + 18*c^2*d^3 + d^3)*b^6*x^2 +
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2*(4*c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2 + c*d^2)*b^6*x + (c^8*d + 3*c^6*d + 3*
c^4*d + c^2*d)*b^6)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(b^6*d^10*x^9 + 9*b^6
*c*d^9*x^8 + 4*(9*c^2*d^8 + d^8)*b^6*x^7 + 28*(3*c^3*d^7 + c*d^7)*b^6*x^6 +
 6*(21*c^4*d^6 + 14*c^2*d^6 + d^6)*b^6*x^5 + 2*(63*c^5*d^5 + 70*c^3*d^5 + 1
5*c*d^5)*b^6*x^4 + 4*(21*c^6*d^4 + 35*c^4*d^4 + 15*c^2*d^4 + d^4)*b^6*x^3 +
 12*(3*c^7*d^3 + 7*c^5*d^3 + 5*c^3*d^3 + c*d^3)*b^6*x^2 + (9*c^8*d^2 + 28*c
^6*d^2 + 30*c^4*d^2 + 12*c^2*d^2 + d^2)*b^6*x + (c^9*d + 4*c^7*d + 6*c^5*d
+ 4*c^3*d + c*d)*b^6)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^3*b^3*d^6*x^5 + 5*a^3*b^3*c*d^5*x^4 +
 10*a^3*b^3*c^2*d^4*x^3 + 10*a^3*b^3*c^3*d^3*x^2 + 5*a^3*b^3*c^4*d^2*x + a^
3*b^3*c^5*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + 5*(a^3*b^3*d^7*x^6 + 6*a
^3*b^3*c*d^6*x^5 + (15*c^2*d^5 + d^5)*a^3*b^3*x^4 + 4*(5*c^3*d^4 + c*d^4)*a
^3*b^3*x^3 + 3*(5*c^4*d^3 + 2*c^2*d^3)*a^3*b^3*x^2 + 2*(3*c^5*d^2 + 2*c^3*d
^2)*a^3*b^3*x + (c^6*d + c^4*d)*a^3*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
3*(a*b^5*d^11*x^10 + 10*a*b^5*c*d^10*x^9 + 5*(9*c^2*d^9 + d^9)*a*b^5*x^8 +
40*(3*c^3*d^8 + c*d^8)*a*b^5*x^7 + 10*(21*c^4*d^7 + 14*c^2*d^7 + d^7)*a*b^5
*x^6 + 4*(63*c^5*d^6 + 70*c^3*d^6 + 15*c*d^6)*a*b^5*x^5 + 10*(21*c^6*d^5 +
35*c^4*d^5 + 15*c^2*d^5 + d^5)*a*b^5*x^4 + 40*(3*c^7*d^4 + 7*c^5*d^4 + 5*c^
3*d^4 + c*d^4)*a*b^5*x^3 + 5*(9*c^8*d^3 + 28*c^6*d^3 + 30*c^4*d^3 + 12*c^2*
d^3 + d^3)*a*b^5*x^2 + 10*(c^9*d^2 + 4*c^7*d^2 + 6*c^5*d^2 + 4*c^3*d^2 + c*
d^2)*a*b^5*x + (c^10*d + 5*c^8*d + 10*c^6*d + 10*c^4*d + 5*c^2*d + d)*a*b^5
 + (a*b^5*d^6*x^5 + 5*a*b^5*c*d^5*x^4 + 10*a*b^5*c^2*d^4*x^3 + 10*a*b^5*c^3
*d^3*x^2 + 5*a*b^5*c^4*d^2*x + a*b^5*c^5*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(
5/2) + 5*(a*b^5*d^7*x^6 + 6*a*b^5*c*d^6*x^5 + (15*c^2*d^5 + d^5)*a*b^5*x^4
+ 4*(5*c^3*d^4 + c*d^4)*a*b^5*x^3 + 3*(5*c^4*d^3 + 2*c^2*d^3)*a*b^5*x^2 + 2
*(3*c^5*d^2 + 2*c^3*d^2)*a*b^5*x + (c^6*d + c^4*d)*a*b^5)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^2 + 10*(a*b^5*d^8*x^7 + 7*a*b^5*c*d^7*x^6 + (21*c^2*d^6 + 2*d^
6)*a*b^5*x^5 + 5*(7*c^3*d^5 + 2*c*d^5)*a*b^5*x^4 + (35*c^4*d^4 + 20*c^2*d^4
 + d^4)*a*b^5*x^3 + (21*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*a*b^5*x^2 + (7*c^6*
d^2 + 10*c^4*d^2 + 3*c^2*d^2)*a*b^5*x + (c^7*d + 2*c^5*d + c^3*d)*a*b^5)*(d
^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 10*(a*b^5*d^9*x^8 + 8*a*b^5*c*d^8*x^7 +
 (28*c^2*d^7 + 3*d^7)*a*b^5*x^6 + 2*(28*c^3*d^6 + 9*c*d^6)*a*b^5*x^5 + (70*
c^4*d^5 + 45*c^2*d^5 + 3*d^5)*a*b^5*x^4 + 4*(14*c^5*d^4 + 15*c^3*d^4 + 3*c*
d^4)*a*b^5*x^3 + (28*c^6*d^3 + 45*c^4*d^3 + 18*c^2*d^3 + d^3)*a*b^5*x^2 + 2
*(4*c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2 + c*d^2)*a*b^5*x + (c^8*d + 3*c^6*d + 3
*c^4*d + c^2*d)*a*b^5)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(a*b^5*d^10*x^9 +
9*a*b^5*c*d^9*x^8 + 4*(9*c^2*d^8 + d^8)*a*b^5*x^7 + 28*(3*c^3*d^7 + c*d^7)*
a*b^5*x^6 + 6*(21*c^4*d^6 + 14*c^2*d^6 + d^6)*a*b^5*x^5 + 2*(63*c^5*d^5 + 7
0*c^3*d^5 + 15*c*d^5)*a*b^5*x^4 + 4*(21*c^6*d^4 + 35*c^4*d^4 + 15*c^2*d^4 +
 d^4)*a*b^5*x^3 + 12*(3*c^7*d^3 + 7*c^5*d^3 + 5*c^3*d^3 + c*d^3)*a*b^5*x^2
+ (9*c^8*d^2 + 28*c^6*d^2 + 30*c^4*d^2 + 12*c^2*d^2 + d^2)*a*b^5*x + (c^9*d
 + 4*c^7*d + 6*c^5*d + 4*c^3*d + c*d)*a*b^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 10*(a^3*b^3*d^8*x
^7 + 7*a^3*b^3*c*d^7*x^6 + (21*c^2*d^6 + 2*d^6)*a^3*b^3*x^5 + 5*(7*c^3*d^5
+ 2*c*d^5)*a^3*b^3*x^4 + (35*c^4*d^4 + 20*c^2*d^4 + d^4)*a^3*b^3*x^3 + (21*
```

```
c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*a^3*b^3*x^2 + (7*c^6*d^2 + 10*c^4*d^2 + 3*c
^2*d^2)*a^3*b^3*x + (c^7*d + 2*c^5*d + c^3*d)*a^3*b^3)*(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^(3/2) + 10*(a^3*b^3*d^9*x^8 + 8*a^3*b^3*c*d^8*x^7 + (28*c^2*d^7 +
 3*d^7)*a^3*b^3*x^6 + 2*(28*c^3*d^6 + 9*c*d^6)*a^3*b^3*x^5 + (70*c^4*d^5 +
45*c^2*d^5 + 3*d^5)*a^3*b^3*x^4 + 4*(14*c^5*d^4 + 15*c^3*d^4 + 3*c*d^4)*a^3
*b^3*x^3 + (28*c^6*d^3 + 45*c^4*d^3 + 18*c^2*d^3 + d^3)*a^3*b^3*x^2 + 2*(4*
c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2 + c*d^2)*a^3*b^3*x + (c^8*d + 3*c^6*d + 3*c
^4*d + c^2*d)*a^3*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a^2*b^4*d^11*x^10
 + 10*a^2*b^4*c*d^10*x^9 + 5*(9*c^2*d^9 + d^9)*a^2*b^4*x^8 + 40*(3*c^3*d^8
+ c*d^8)*a^2*b^4*x^7 + 10*(21*c^4*d^7 + 14*c^2*d^7 + d^7)*a^2*b^4*x^6 + 4*(
63*c^5*d^6 + 70*c^3*d^6 + 15*c*d^6)*a^2*b^4*x^5 + 10*(21*c^6*d^5 + 35*c^4*d
^5 + 15*c^2*d^5 + d^5)*a^2*b^4*x^4 + 40*(3*c^7*d^4 + 7*c^5*d^4 + 5*c^3*d^4
+ c*d^4)*a^2*b^4*x^3 + 5*(9*c^8*d^3 + 28*c^6*d^3 + 30*c^4*d^3 + 12*c^2*d^3
+ d^3)*a^2*b^4*x^2 + 10*(c^9*d^2 + 4*c^7*d^2 + 6*c^5*d^2 + 4*c^3*d^2 + c*d^
2)*a^2*b^4*x + (c^10*d + 5*c^8*d + 10*c^6*d + 10*c^4*d + 5*c^2*d + d)*a^2*b
^4 + (a^2*b^4*d^6*x^5 + 5*a^2*b^4*c*d^5*x^4 + 10*a^2*b^4*c^2*d^4*x^3 + 10*a
^2*b^4*c^3*d^3*x^2 + 5*a^2*b^4*c^4*d^2*x + a^2*b^4*c^5*d)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^(5/2) + 5*(a^2*b^4*d^7*x^6 + 6*a^2*b^4*c*d^6*x^5 + (15*c^2*d^5
 + d^5)*a^2*b^4*x^4 + 4*(5*c^3*d^4 + c*d^4)*a^2*b^4*x^3 + 3*(5*c^4*d^3 + 2*
c^2*d^3)*a^2*b^4*x^2 + 2*(3*c^5*d^2 + 2*c^3*d^2)*a^2*b^4*x + (c^6*d + c^4*d
)*a^2*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 10*(a^2*b^4*d^8*x^7 + 7*a^2*b^
4*c*d^7*x^6 + (21*c^2*d^6 + 2*d^6)*a^2*b^4*x^5 + 5*(7*c^3*d^5 + 2*c*d^5)*a^
2*b^4*x^4 + (35*c^4*d^4 + 20*c^2*d^4 + d^4)*a^2*b^4*x^3 + (21*c^5*d^3 + 20*
c^3*d^3 + 3*c*d^3)*a^2*b^4*x^2 + (7*c^6*d^2 + 10*c^4*d^2 + 3*c^2*d^2)*a^2*b
^4*x + (c^7*d + 2*c^5*d + c^3*d)*a^2*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + 10*(a^2*b^4*d^9*x^8 + 8*a^2*b^4*c*d^8*x^7 + (28*c^2*d^7 + 3*d^7)*a^2*b
^4*x^6 + 2*(28*c^3*d^6 + 9*c*d^6)*a^2*b^4*x^5 + (70*c^4*d^5 + 45*c^2*d^5 +
3*d^5)*a^2*b^4*x^4 + 4*(14*c^5*d^4 + 15*c^3*d^4 + 3*c*d^4)*a^2*b^4*x^3 + (2
8*c^6*d^3 + 45*c^4*d^3 + 18*c^2*d^3 + d^3)*a^2*b^4*x^2 + 2*(4*c^7*d^2 + 9*c
^5*d^2 + 6*c^3*d^2 + c*d^2)*a^2*b^4*x + (c^8*d + 3*c^6*d + 3*c^4*d + c^2*d)
*a^2*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(a^2*b^4*d^10*x^9 + 9*a^2*b^4*c
*d^9*x^8 + 4*(9*c^2*d^8 + d^8)*a^2*b^4*x^7 + 28*(3*c^3*d^7 + c*d^7)*a^2*b^4
*x^6 + 6*(21*c^4*d^6 + 14*c^2*d^6 + d^6)*a^2*b^4*x^5 + 2*(63*c^5*d^5 + 70*c
^3*d^5 + 15*c*d^5)*a^2*b^4*x^4 + 4*(21*c^6*d^4 + 35*c^4*d^4 + 15*c^2*d^4 +
d^4)*a^2*b^4*x^3 + 12*(3*c^7*d^3 + 7*c^5*d^3 + 5*c^3*d^3 + c*d^3)*a^2*b^4*x
^2 + (9*c^8*d^2 + 28*c^6*d^2 + 30*c^4*d^2 + 12*c^2*d^2 + d^2)*a^2*b^4*x + (
c^9*d + 4*c^7*d + 6*c^5*d + 4*c^3*d + c*d)*a^2*b^4)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 5*(a^3*b^3*d
^10*x^9 + 9*a^3*b^3*c*d^9*x^8 + 4*(9*c^2*d^8 + d^8)*a^3*b^3*x^7 + 28*(3*c^3
*d^7 + c*d^7)*a^3*b^3*x^6 + 6*(21*c^4*d^6 + 14*c^2*d^6 + d^6)*a^3*b^3*x^5 +
 2*(63*c^5*d^5 + 70*c^3*d^5 + 15*c*d^5)*a^3*b^3*x^4 + 4*(21*c^6*d^4 + 35*c^
4*d^4 + 15*c^2*d^4 + d^4)*a^3*b^3*x^3 + 12*(3*c^7*d^3 + 7*c^5*d^3 + 5*c^3*d
^3 + c*d^3)*a^3*b^3*x^2 + (9*c^8*d^2 + 28*c^6*d^2 + 30*c^4*d^2 + 12*c^2*d^2
 + d^2)*a^3*b^3*x + (c^9*d + 4*c^7*d + 6*c^5*d + 4*c^3*d + c*d)*a^3*b^3)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/6*(d^12*x^12 + 12*c*d^11*x^1
```

```
1 + c^12 + 6*(11*c^2*d^10 + d^10)*x^10 + 6*c^10 + 20*(11*c^3*d^9 + 3*c*d^9)
*x^9 + 15*(33*c^4*d^8 + 18*c^2*d^8 + d^8)*x^8 + 15*c^8 + 24*(33*c^5*d^7 + 3
0*c^3*d^7 + 5*c*d^7)*x^7 + 4*(231*c^6*d^6 + 315*c^4*d^6 + 105*c^2*d^6 + 5*d
^6)*x^6 + 20*c^6 + 24*(33*c^7*d^5 + 63*c^5*d^5 + 35*c^3*d^5 + 5*c*d^5)*x^5
+ 15*(33*c^8*d^4 + 84*c^6*d^4 + 70*c^4*d^4 + 20*c^2*d^4 + d^4)*x^4 + (d^6*x
^6 + 6*c*d^5*x^5 + c^6 + (15*c^2*d^4 - d^4)*x^4 - c^4 + 4*(5*c^3*d^3 - c*d^
3)*x^3 + 3*(5*c^4*d^2 - 2*c^2*d^2 - 3*d^2)*x^2 - 9*c^2 + 2*(3*c^5*d - 2*c^3
*d - 9*c*d)*x - 15)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^3 + 15*c^4 + 20*(11*c^9*d
^3 + 36*c^7*d^3 + 42*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*x^3 + (6*d^7*x^7 + 42*
c*d^6*x^6 + 6*c^7 + (126*c^2*d^5 + d^5)*x^5 + c^5 + 5*(42*c^3*d^4 + c*d^4)*
x^4 + (210*c^4*d^3 + 10*c^2*d^3 - 31*d^3)*x^3 - 31*c^3 + (126*c^5*d^2 + 10*
c^3*d^2 - 93*c*d^2)*x^2 + (42*c^6*d + 5*c^4*d - 93*c^2*d - 33*d)*x - 33*c)*
(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + (15*d^8*x^8 + 120*c*d^7*x^7 + 15*c^8
+ 20*(21*c^2*d^6 + d^6)*x^6 + 20*c^6 + 120*(7*c^3*d^5 + c*d^5)*x^5 + (1050*
c^4*d^4 + 300*c^2*d^4 - 19*d^4)*x^4 - 19*c^4 + 4*(210*c^5*d^3 + 100*c^3*d^3
 - 19*c*d^3)*x^3 + 3*(140*c^6*d^2 + 100*c^4*d^2 - 38*c^2*d^2 - d^2)*x^2 - 3
*c^2 + 2*(60*c^7*d + 60*c^5*d - 38*c^3*d - 3*c*d)*x + 21)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^2 + 6*(11*c^10*d^2 + 45*c^8*d^2 + 70*c^6*d^2 + 50*c^4*d^2 + 15
*c^2*d^2 + d^2)*x^2 + (20*d^9*x^9 + 180*c*d^8*x^8 + 20*c^9 + 10*(72*c^2*d^7
 + 5*d^7)*x^7 + 50*c^7 + 70*(24*c^3*d^6 + 5*c*d^6)*x^6 + 6*(420*c^4*d^5 + 1
75*c^2*d^5 + 9*d^5)*x^5 + 54*c^5 + 10*(252*c^5*d^4 + 175*c^3*d^4 + 27*c*d^4
)*x^4 + (1680*c^6*d^3 + 1750*c^4*d^3 + 540*c^2*d^3 + 59*d^3)*x^3 + 59*c^3 +
 3*(240*c^7*d^2 + 350*c^5*d^2 + 180*c^3*d^2 + 59*c*d^2)*x^2 + (180*c^8*d +
350*c^6*d + 270*c^4*d + 177*c^2*d + 35*d)*x + 35*c)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + (15*d^10*x^10 + 150*c*d^9*x^9 + 15*c^10 + 5*(135*c^2*d^8 + 1
1*d^8)*x^8 + 55*c^8 + 40*(45*c^3*d^7 + 11*c*d^7)*x^7 + (3150*c^4*d^6 + 1540
*c^2*d^6 + 101*d^6)*x^6 + 101*c^6 + 2*(1890*c^5*d^5 + 1540*c^3*d^5 + 303*c*
d^5)*x^5 + 5*(630*c^6*d^4 + 770*c^4*d^4 + 303*c^2*d^4 + 18*d^4)*x^4 + 90*c^
4 + 20*(90*c^7*d^3 + 154*c^5*d^3 + 101*c^3*d^3 + 18*c*d^3)*x^3 + (675*c^8*d
^2 + 1540*c^6*d^2 + 1515*c^4*d^2 + 540*c^2*d^2 + 22*d^2)*x^2 + 22*c^2 + 2*(
75*c^9*d + 220*c^7*d + 303*c^5*d + 180*c^3*d + 22*c*d)*x - 7)*(d^2*x^2 + 2*
c*d*x + c^2 + 1) + 6*c^2 + 12*(c^11*d + 5*c^9*d + 10*c^7*d + 10*c^5*d + 5*c
^3*d + c*d)*x + (6*d^11*x^11 + 66*c*d^10*x^10 + 6*c^11 + (330*c^2*d^9 + 29*
d^9)*x^9 + 29*c^9 + 9*(110*c^3*d^8 + 29*c*d^8)*x^8 + (1980*c^4*d^7 + 1044*c
^2*d^7 + 65*d^7)*x^7 + 65*c^7 + 7*(396*c^5*d^6 + 348*c^3*d^6 + 65*c*d^6)*x^
6 + 3*(924*c^6*d^5 + 1218*c^4*d^5 + 455*c^2*d^5 + 22*d^5)*x^5 + 66*c^5 + (1
980*c^7*d^4 + 3654*c^5*d^4 + 2275*c^3*d^4 + 330*c*d^4)*x^4 + (990*c^8*d^3 +
 2436*c^6*d^3 + 2275*c^4*d^3 + 660*c^2*d^3 + 23*d^3)*x^3 + 23*c^3 + 3*(110*
c^9*d^2 + 348*c^7*d^2 + 455*c^5*d^2 + 220*c^3*d^2 + 23*c*d^2)*x^2 + (66*c^1
0*d + 261*c^8*d + 455*c^6*d + 330*c^4*d + 69*c^2*d - d)*x - c)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1) + 1)/(a*b^3*d^12*x^12 + 12*a*b^3*c*d^11*x^11 + 6*(11*
c^2*d^10 + d^10)*a*b^3*x^10 + 20*(11*c^3*d^9 + 3*c*d^9)*a*b^3*x^9 + 15*(33*
c^4*d^8 + 18*c^2*d^8 + d^8)*a*b^3*x^8 + 24*(33*c^5*d^7 + 30*c^3*d^7 + 5*c*d
^7)*a*b^3*x^7 + 4*(231*c^6*d^6 + 315*c^4*d^6 + 105*c^2*d^6 + 5*d^6)*a*b^3*x
^6 + 24*(33*c^7*d^5 + 63*c^5*d^5 + 35*c^3*d^5 + 5*c*d^5)*a*b^3*x^5 + 15*(33
```

```
*c^8*d^4 + 84*c^6*d^4 + 70*c^4*d^4 + 20*c^2*d^4 + d^4)*a*b^3*x^4 + 20*(11*c
^9*d^3 + 36*c^7*d^3 + 42*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*a*b^3*x^3 + 6*(11*
c^10*d^2 + 45*c^8*d^2 + 70*c^6*d^2 + 50*c^4*d^2 + 15*c^2*d^2 + d^2)*a*b^3*x
^2 + 12*(c^11*d + 5*c^9*d + 10*c^7*d + 10*c^5*d + 5*c^3*d + c*d)*a*b^3*x +
(c^12 + 6*c^10 + 15*c^8 + 20*c^6 + 15*c^4 + 6*c^2 + 1)*a*b^3 + (a*b^3*d^6*x
^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + 20*a*b^3*c^3*d^3*x^3 + 15*a
*b^3*c^4*d^2*x^2 + 6*a*b^3*c^5*d*x + a*b^3*c^6)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^3 + 6*(a*b^3*d^7*x^7 + 7*a*b^3*c*d^6*x^6 + (21*c^2*d^5 + d^5)*a*b^3*x^5
+ 5*(7*c^3*d^4 + c*d^4)*a*b^3*x^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*a*b^3*x^3 + (
21*c^5*d^2 + 10*c^3*d^2)*a*b^3*x^2 + (7*c^6*d + 5*c^4*d)*a*b^3*x + (c^7 + c
^5)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + 15*(a*b^3*d^8*x^8 + 8*a*b^
3*c*d^7*x^7 + 2*(14*c^2*d^6 + d^6)*a*b^3*x^6 + 4*(14*c^3*d^5 + 3*c*d^5)*a*b
^3*x^5 + (70*c^4*d^4 + 30*c^2*d^4 + d^4)*a*b^3*x^4 + 4*(14*c^5*d^3 + 10*c^3
*d^3 + c*d^3)*a*b^3*x^3 + 2*(14*c^6*d^2 + 15*c^4*d^2 + 3*c^2*d^2)*a*b^3*x^2
 + 4*(2*c^7*d + 3*c^5*d + c^3*d)*a*b^3*x + (c^8 + 2*c^6 + c^4)*a*b^3)*(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2 + 20*(a*b^3*d^9*x^9 + 9*a*b^3*c*d^8*x^8 + 3*(12*
c^2*d^7 + d^7)*a*b^3*x^7 + 21*(4*c^3*d^6 + c*d^6)*a*b^3*x^6 + 3*(42*c^4*d^5
 + 21*c^2*d^5 + d^5)*a*b^3*x^5 + 3*(42*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*a*b^
3*x^4 + (84*c^6*d^3 + 105*c^4*d^3 + 30*c^2*d^3 + d^3)*a*b^3*x^3 + 3*(12*c^7
*d^2 + 21*c^5*d^2 + 10*c^3*d^2 + c*d^2)*a*b^3*x^2 + 3*(3*c^8*d + 7*c^6*d +
5*c^4*d + c^2*d)*a*b^3*x + (c^9 + 3*c^7 + 3*c^5 + c^3)*a*b^3)*(d^2*x^2 + 2*
c*d*x + c^2 + 1)^(3/2) + 15*(a*b^3*d^10*x^10 + 10*a*b^3*c*d^9*x^9 + (45*c^2
*d^8 + 4*d^8)*a*b^3*x^8 + 8*(15*c^3*d^7 + 4*c*d^7)*a*b^3*x^7 + 2*(105*c^4*d
^6 + 56*c^2*d^6 + 3*d^6)*a*b^3*x^6 + 4*(63*c^5*d^5 + 56*c^3*d^5 + 9*c*d^5)*
a*b^3*x^5 + 2*(105*c^6*d^4 + 140*c^4*d^4 + 45*c^2*d^4 + 2*d^4)*a*b^3*x^4 +
8*(15*c^7*d^3 + 28*c^5*d^3 + 15*c^3*d^3 + 2*c*d^3)*a*b^3*x^3 + (45*c^8*d^2
+ 112*c^6*d^2 + 90*c^4*d^2 + 24*c^2*d^2 + d^2)*a*b^3*x^2 + 2*(5*c^9*d + 16*
c^7*d + 18*c^5*d + 8*c^3*d + c*d)*a*b^3*x + (c^10 + 4*c^8 + 6*c^6 + 4*c^4 +
 c^2)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^4*d^12*x^12 + 12*b^4*c*d^11
*x^11 + 6*(11*c^2*d^10 + d^10)*b^4*x^10 + 20*(11*c^3*d^9 + 3*c*d^9)*b^4*x^9
 + 15*(33*c^4*d^8 + 18*c^2*d^8 + d^8)*b^4*x^8 + 24*(33*c^5*d^7 + 30*c^3*d^7
 + 5*c*d^7)*b^4*x^7 + 4*(231*c^6*d^6 + 315*c^4*d^6 + 105*c^2*d^6 + 5*d^6)*b
^4*x^6 + 24*(33*c^7*d^5 + 63*c^5*d^5 + 35*c^3*d^5 + 5*c*d^5)*b^4*x^5 + 15*(
33*c^8*d^4 + 84*c^6*d^4 + 70*c^4*d^4 + 20*c^2*d^4 + d^4)*b^4*x^4 + 20*(11*c
^9*d^3 + 36*c^7*d^3 + 42*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*b^4*x^3 + 6*(11*c^
10*d^2 + 45*c^8*d^2 + 70*c^6*d^2 + 50*c^4*d^2 + 15*c^2*d^2 + d^2)*b^4*x^2 +
 12*(c^11*d + 5*c^9*d + 10*c^7*d + 10*c^5*d + 5*c^3*d + c*d)*b^4*x + (c^12
+ 6*c^10 + 15*c^8 + 20*c^6 + 15*c^4 + 6*c^2 + 1)*b^4 + (b^4*d^6*x^6 + 6*b^4
*c*d^5*x^5 + 15*b^4*c^2*d^4*x^4 + 20*b^4*c^3*d^3*x^3 + 15*b^4*c^4*d^2*x^2 +
 6*b^4*c^5*d*x + b^4*c^6)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^3 + 6*(b^4*d^7*x^7
+ 7*b^4*c*d^6*x^6 + (21*c^2*d^5 + d^5)*b^4*x^5 + 5*(7*c^3*d^4 + c*d^4)*b^4*
x^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*b^4*x^3 + (21*c^5*d^2 + 10*c^3*d^2)*b^4*x^2
 + (7*c^6*d + 5*c^4*d)*b^4*x + (c^7 + c^5)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^(5/2) + 15*(b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 2*(14*c^2*d^6 + d^6)*b^4*x^6
 + 4*(14*c^3*d^5 + 3*c*d^5)*b^4*x^5 + (70*c^4*d^4 + 30*c^2*d^4 + d^4)*b^4*x
```

```
^4 + 4*(14*c^5*d^3 + 10*c^3*d^3 + c*d^3)*b^4*x^3 + 2*(14*c^6*d^2 + 15*c^4*d
^2 + 3*c^2*d^2)*b^4*x^2 + 4*(2*c^7*d + 3*c^5*d + c^3*d)*b^4*x + (c^8 + 2*c^
6 + c^4)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 20*(b^4*d^9*x^9 + 9*b^4*c*d
^8*x^8 + 3*(12*c^2*d^7 + d^7)*b^4*x^7 + 21*(4*c^3*d^6 + c*d^6)*b^4*x^6 + 3*
(42*c^4*d^5 + 21*c^2*d^5 + d^5)*b^4*x^5 + 3*(42*c^5*d^4 + 35*c^3*d^4 + 5*c*
d^4)*b^4*x^4 + (84*c^6*d^3 + 105*c^4*d^3 + 30*c^2*d^3 + d^3)*b^4*x^3 + 3*(1
2*c^7*d^2 + 21*c^5*d^2 + 10*c^3*d^2 + c*d^2)*b^4*x^2 + 3*(3*c^8*d + 7*c^6*d
 + 5*c^4*d + c^2*d)*b^4*x + (c^9 + 3*c^7 + 3*c^5 + c^3)*b^4)*(d^2*x^2 + 2*c
*d*x + c^2 + 1)^(3/2) + 15*(b^4*d^10*x^10 + 10*b^4*c*d^9*x^9 + (45*c^2*d^8
+ 4*d^8)*b^4*x^8 + 8*(15*c^3*d^7 + 4*c*d^7)*b^4*x^7 + 2*(105*c^4*d^6 + 56*c
^2*d^6 + 3*d^6)*b^4*x^6 + 4*(63*c^5*d^5 + 56*c^3*d^5 + 9*c*d^5)*b^4*x^5 + 2
*(105*c^6*d^4 + 140*c^4*d^4 + 45*c^2*d^4 + 2*d^4)*b^4*x^4 + 8*(15*c^7*d^3 +
 28*c^5*d^3 + 15*c^3*d^3 + 2*c*d^3)*b^4*x^3 + (45*c^8*d^2 + 112*c^6*d^2 + 9
0*c^4*d^2 + 24*c^2*d^2 + d^2)*b^4*x^2 + 2*(5*c^9*d + 16*c^7*d + 18*c^5*d +
8*c^3*d + c*d)*b^4*x + (c^10 + 4*c^8 + 6*c^6 + 4*c^4 + c^2)*b^4)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 6*(b^4*d^11*x^11 + 11*b^4*c*d^10*x^10 + 5*(11*c^2*d^9
 + d^9)*b^4*x^9 + 15*(11*c^3*d^8 + 3*c*d^8)*b^4*x^8 + 10*(33*c^4*d^7 + 18*c
^2*d^7 + d^7)*b^4*x^7 + 14*(33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*b^4*x^6 + 2*
(231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^4*x^5 + 10*(33*c^7*d^4
+ 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b^4*x^4 + 5*(33*c^8*d^3 + 84*c^6*d^3 +
 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^4*x^3 + 5*(11*c^9*d^2 + 36*c^7*d^2 + 42*c
^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*b^4*x^2 + (11*c^10*d + 45*c^8*d + 70*c^6*d +
 50*c^4*d + 15*c^2*d + d)*b^4*x + (c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 +
 c)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)) + 6*(a*b^3*d^11*x^11 + 11*a*b^3*c*d^10*x^10 + 5*(11*c^2*d
^9 + d^9)*a*b^3*x^9 + 15*(11*c^3*d^8 + 3*c*d^8)*a*b^3*x^8 + 10*(33*c^4*d^7
+ 18*c^2*d^7 + d^7)*a*b^3*x^7 + 14*(33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a*b^
3*x^6 + 2*(231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a*b^3*x^5 + 10*
(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*a*b^3*x^4 + 5*(33*c^8*d^3
+ 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a*b^3*x^3 + 5*(11*c^9*d^2 + 3
6*c^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*a*b^3*x^2 + (11*c^10*d + 45*
c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*a*b^3*x + (c^11 + 5*c^9 + 10*c^
7 + 10*c^5 + 5*c^3 + c)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+d\,x))^4}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^4,x)
[Out] int(1/(a + b*asinh(c + d*x))^4, x)
```

# 3.179 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))^4}\,dx$

Optimal result . . . 1385
Rubi [N/A] . . . 1385
Mathematica [N/A] . . . 1386
Maple [N/A] (verified) . . . 1386
Fricas [N/A] . . . 1386
Sympy [N/A] . . . 1387
Maxima [F(-1)] . . . 1387
Giac [N/A] . . . 1387
Mupad [N/A] . . . 1388

## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^4,x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^4), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^4}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^4}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.26

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(dex+ce)\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^4}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4
+ 4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b
^2*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c)),
 x)
```

## Sympy [N/A]

Not integrable

Time = 7.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.57

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{\int \frac{1}{a^4c+a^4dx+4a^3bc\,\operatorname{asinh}(c+dx)+4a^3bdx\,\operatorname{asinh}(c+dx)+6a^2b^2c\,\operatorname{asinh}^2(c+dx)+6a^2b^2dx\,\operatorname{asinh}^2(c+dx)+4ab^3c\,\operatorname{asinh}^3(c+dx)+4ab^3dx\,\operatorname{asinh}^3(c}}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*asinh(c + d*x) + 4*a**3*b*d*x*as
inh(c + d*x) + 6*a**2*b**2*c*asinh(c + d*x)**2 + 6*a**2*b**2*d*x*asinh(c +
d*x)**2 + 4*a*b**3*c*asinh(c + d*x)**3 + 4*a*b**3*d*x*asinh(c + d*x)**3 + b
**4*c*asinh(c + d*x)**4 + b**4*d*x*asinh(c + d*x)**4), x)/e
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

## Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(dex+ce)(b\,\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4), x)
```

## Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}\left(c+d\,x\right)\right)^4}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4), x)
```

# 3.180 $\int (ce+dex)^4 \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1389
Rubi [A] (verified) . . . 1390
Mathematica [A] (verified) . . . 1394
Maple [F] . . . 1395
Fricas [F(-2)] . . . 1395
Sympy [F] . . . 1395
Maxima [F] . . . 1396
Giac [F] . . . 1396
Mupad [F(-1)] . . . 1396

## Optimal result

Integrand size = 25, antiderivative size = 361

$$\int (ce+dex)^4 \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} + \frac{\sqrt{b}e^4 e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} - \frac{\sqrt{b}e^4 e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} + \frac{\sqrt{b}e^4 e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d} - \frac{\sqrt{b}e^4 e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{\sqrt{b}e^4 e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{\sqrt{b}e^4 e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

```
[Out] 1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^(1/
2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d+1/192*e^4
*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/
```

```
d/exp(3*a/b)+1/32*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi
^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3389, 2211, 2236, 2235}

$$\begin{aligned}\int (ce+dex)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = {} & \frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} \\ & - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\ & + \frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d} \\ & - \frac{\sqrt{\pi}\sqrt{b}e^4e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} \\ & + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\ & - \frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d} \\ & + \frac{e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) + (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(64*d) + (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^4 x^4 \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int x^4 \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{10d} \\
&= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d} + \frac{e^4 \text{Subst}\left(\int \frac{\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{10d} \\
&= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d} \\
&\quad - \frac{(ie^4)\,\text{Subst}\left(\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{5i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{5i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{10d} \\
&= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{160d} \\
&\quad - \frac{e^4 \text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{320d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{320d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32d} \\
&= \frac{e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{160d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{160d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d} \\
&\quad + \frac{e^4\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d} \\
&\quad - \frac{e^4\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d}
\end{aligned}
$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d} + \frac{\sqrt{b}e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$
$$- \frac{\sqrt{b}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} + \frac{\sqrt{b}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d}$$
$$- \frac{\sqrt{b}e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{\sqrt{b}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}$$
$$- \frac{\sqrt{b}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\text{arcsin}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(-150*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcS
inh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 25*Sqrt[3]*E^((
2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]
))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*Ar
cSinh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[5]*E^((10*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

## Maple [F]

$$\int (dex + ce)^4 \sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^4 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}\int (ce + dex)^4 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = e^4 \Bigg( & \int c^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int d^4 x^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int 4cd^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int 6c^2 d^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int 4c^3 dx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \Bigg)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(1/2),x)
[Out] e**4*(Integral(c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^4\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^4\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)^4\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.181 $\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1397
Rubi [A] (verified) . . . 1398
Mathematica [A] (verified) . . . 1401
Maple [F] . . . 1402
Fricas [F(-2)] . . . 1402
Sympy [F] . . . 1402
Maxima [F] . . . 1403
Giac [F] . . . 1403
Mupad [F(-1)] . . . 1403

## Optimal result

Integrand size = 25, antiderivative size = 272

$$\begin{aligned}\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = &-\frac{3e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d}\\ &+\frac{e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d}\\ &-\frac{\sqrt{b}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}\\ &+\frac{\sqrt{b}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\ &-\frac{\sqrt{b}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}\\ &+\frac{\sqrt{b}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\end{aligned}$$

```
[Out] 1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)
*2^(1/2)*Pi^(1/2)/d+1/64*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*arcsi
nh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3388, 2211, 2236, 2235}

$$\begin{aligned}\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = &-\frac{\sqrt{\pi}\sqrt{b}e^3e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}\\ &+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\ &-\frac{\sqrt{\pi}\sqrt{b}e^3e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}\\ &+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\ &+\frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d}\\ &-\frac{3e^3\sqrt{a+b\text{arcsinh}(c+dx)}}{32d}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcSinh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d) + (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) + (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int x^3 \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{8d} \\
&= \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} - \frac{e^3\text{Subst}\left(\int \frac{\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8d} \\
&= \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8d} \\
&= -\frac{3e^3\sqrt{a+b\text{arcsinh}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{64d} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16d} \\
&= -\frac{3e^3\sqrt{a+b\text{arcsinh}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{128d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{128d} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32d} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{64d} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{64d} \\
&\quad + \frac{e^3\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d} \\
&\quad + \frac{e^3\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d} \\
&= -\frac{3e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} \\
&\quad - \frac{\sqrt{b}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} + \frac{\sqrt{b}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} \\
&\quad - \frac{\sqrt{b}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&\quad + \frac{\sqrt{b}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx \\
&= \frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{3}{2}, -\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right) - 4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}}\right.}{}
\end{aligned}
$$



```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]

[Out] (e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcSinh[c + d*x]))/b] - 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcSinh[c + d*x])/b)]*(-4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcSinh[c +
d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(128*
d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

## Maple [F]

$$\int (dex + ce)^3 \sqrt{a + b \operatorname{arcsinh}(dx + c)}dx$$

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}\int (ce + dex)^3 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = e^3 \Bigg( & \int c^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int d^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int 3cd^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\ & + \int 3c^2 dx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \Bigg)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int (dex+ce)^3\sqrt{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int (dex+ce)^3\sqrt{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\operatorname{asinh}(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.182 $\int (ce+dex)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1404
Rubi [A] (verified) . . . 1405
Mathematica [A] (verified) . . . 1408
Maple [F] . . . 1409
Fricas [F(-2)] . . . 1409
Sympy [F] . . . 1409
Maxima [F] . . . 1410
Giac [F] . . . 1410
Mupad [F(-1)] . . . 1410

## Optimal result

Integrand size = 25, antiderivative size = 245

$$\int (ce+dex)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} - \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d} + \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d}$$

```
[Out] 1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d+1/16*e^2*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b*arcs
inh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3389, 2211, 2236, 2235}

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d} + \frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d} + \frac{e^2(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{3d}$$

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d) + (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(48*d) + (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^2 x^2 \sqrt{a + b\text{arcsinh}(x)}\, dx, x, c + dx\right)}{d}$$

$$= \frac{e^2 \text{Subst}\left(\int x^2 \sqrt{a + b\text{arcsinh}(x)}\, dx, x, c + dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} - \frac{(be^2)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} + \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} + \frac{(ie^2)\operatorname{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} + \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24d} - \frac{e^2\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} + \frac{e^2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d} - \frac{e^2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d} - \frac{e^2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} + \frac{e^2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d}$$

$$
\begin{aligned}
&= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad + \frac{e^2\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d} \\
&\quad - \frac{e^2\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d} \\
&\quad - \frac{e^2\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&\quad + \frac{e^2\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&= \frac{e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} - \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} \\
&\quad + \frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d} + \frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} \\
&\quad - \frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

$$
\begin{aligned}
&\int (ce+dex)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx \\
&= \frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2}, \frac{a}{b}+\operatorname{arcsinh}(c+dx)\right) + \sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]

[Out] (e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 9*E^((2*a)/b)*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcSin
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)]
)
```

## Maple [F]

$$\int (dex + ce)^2 \sqrt{a + b \operatorname{arcsinh}(dx + c)} dx$$

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (ce + dex)^2 \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = e^2 \left( \int c^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int d^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 2cdx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \right)$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(1/2),x)
[Out] e**2*(Integral(c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*asinh(c + d*x)),
x))
```

## Maxima [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^2\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^2\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.183 $\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . . . 1411
Rubi [A] (verified) . . . . . 1411
Mathematica [A] (verified) . . . . . 1414
Maple [F] . . . . . 1415
Fricas [F(-2)] . . . . . 1415
Sympy [F] . . . . . 1415
Maxima [F] . . . . . 1416
Giac [F] . . . . . 1416
Mupad [F(-1)] . . . . . 1416

## Optimal result

Integrand size = 23, antiderivative size = 164

$$\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

```
[Out] -1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)+1/4*e*(a+b*arcsinh(d*x+c))^(1/2)/d+1/
2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

$= \{5859, 12, 5777, 5819, 3393, 3388, 2211, 2236, 2235\}$

$$\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d}$$

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcSinh[
c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex\sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x\sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{4d} \\
&= \frac{e(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} - \frac{e\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d}
\end{aligned}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{e\operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4d}$$

$$= \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8d}$$

$$= \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} - \frac{e\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d}$$

$$= \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{e\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} - \frac{e\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d}$$

$$= \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.74

$$\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$$

$$= \frac{ee^{-\frac{2a}{b}}\left(-b\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2}, -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right) + be^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{3}{2}, \frac{2(a+b\operatorname{arcsinh}(c+dx)}{b}\right.\right.}{8\sqrt{2}d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*(-(b*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b]) + b*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (2*(a
 + b*ArcSinh[c + d*x]))/b]))/(8*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c
+ d*x]])
```

## Maple [F]

$$\int (dex + ce)\sqrt{a + b\,\mathrm{arcsinh}\,(dx + c)}dx$$

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)\sqrt{a + b\mathrm{arcsinh}(c + dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (ce + dex)\sqrt{a + b\mathrm{arcsinh}(c + dx)}\,dx = e\left(\int c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x*sqrt(a + b*asin
h(c + d*x)), x))
```

## Maxima [F]

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.184 $\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

Optimal result . . . 1417
Rubi [A] (verified) . . . 1417
Mathematica [A] (verified) . . . 1419
Maple [F] . . . 1420
Fricas [F(-2)] . . . 1420
Sympy [F] . . . 1420
Maxima [F] . . . 1420
Giac [F] . . . 1421
Mupad [F(-1)] . . . 1421

## Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5819, 3389, 2211, 2236, 2235}

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} + \frac{(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\text{arcsinh}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d} \\
&\quad - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2d} \\
&= \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} \\
&\quad - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\begin{aligned}
&\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx \\
&= \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}
\end{aligned}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

## Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(dx + c)}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{a + b\operatorname{asinh}(c + dx)}\, dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \sqrt{a + b\operatorname{arcsinh}(c + dx)}\, dx = \int \sqrt{b\operatorname{arsinh}(dx + c) + a}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{arsinh}\,(dx+c)+a}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \int \sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

# 3.185 $\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx$

Optimal result . . . 1422
Rubi [N/A] . . . 1422
Mathematica [N/A] . . . 1423
Maple [N/A] (verified) . . . 1423
Fricas [F(-2)] . . . 1423
Sympy [N/A] . . . 1423
Maxima [N/A] . . . 1424
Giac [N/A] . . . 1424
Mupad [N/A] . . . 1424

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx = \frac{\operatorname{Int}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(1/2)/(d*x+c),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx = \int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx$$

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcSinh[x]]/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{x}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
[Out] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\ \text{arcsinh}\left(dx+c\right)}}{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \frac{\int \frac{\sqrt{a+b\,\text{asinh}\,(c+dx)}}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**(1/2)/(d*e*x+c*e),x)
[Out] Integral(sqrt(a + b*asinh(c + d*x))/(c + d*x), x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

## Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \int \frac{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \int \frac{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

# 3.186 $\int (ce+dex)^4(a+b\mathbf{arcsinh}(c+dx))^{3/2}\,dx$

Optimal result . . . . . 1426
Rubi [A] (verified) . . . . . 1427
Mathematica [A] (verified) . . . . . 1436
Maple [F] . . . . . 1437
Fricas [F(-2)] . . . . . 1437
Sympy [F] . . . . . 1438
Maxima [F] . . . . . 1438
Giac [F] . . . . . 1439
Mupad [F(-1)] . . . . . 1439

## Optimal result

Integrand size = 25, antiderivative size = 601

$$\int (ce+dex)^4 (a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx =$$
$$-\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d}$$
$$+\frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d}$$
$$-\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d}$$
$$+\frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d}+\frac{3b^{3/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}$$
$$-\frac{b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d}$$
$$-\frac{3b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}$$
$$+\frac{3b^{3/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}$$
$$+\frac{3b^{3/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}$$
$$-\frac{b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d}$$
$$-\frac{3b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}$$
$$+\frac{3b^{3/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3/2)/d+3/16000*b^(3/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d+3/160
00*b^(3/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-1/384*b^(3/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-1/384*b^(3/2)*e^4*erfi(3^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+3/64*b^(3/2)*
e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/64*b^(3/2
```

```
)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/25*b*e
^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+2/25*b*e^4*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/50*b*e^4*(d*x+c)^4*(1+(d*x
+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules

used = {5859, 12, 5777, 5812, 5798, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{3\sqrt{\pi}b^{3/2}e^4e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{3\sqrt{3\pi}b^{3/2}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} - \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} + \frac{3\sqrt{\frac{\pi}{5}}b^{3/2}e^4e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} + \frac{3\sqrt{\pi}b^{3/2}e^4e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{3\sqrt{3\pi}b^{3/2}e^4e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} - \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} + \frac{3\sqrt{\frac{\pi}{5}}b^{3/2}e^4e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} + \frac{e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}}{5d} - \frac{3be^4\sqrt{(c+dx)^2+1}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{50d} + \frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{25d} - \frac{4be^4\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}{25d}$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d) + (2*b
*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d)
 - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(5*d) + (3*b^(3/
2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) -
```

```
 (b^(3/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(200*d) - (3*b^(3/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d) + (3*b^(3/2)*e^4*E^((5*a)
/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d
) + (3*b^(3/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64
*d*E^(a/b)) - (b^(3/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(200*d*E^((3*a)/b)) - (3*b^(3/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d*E^((3*a)/b)) + (3*b^(3/
2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
00*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^4 x^4 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int x^4 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 (c+dx)^5 (a+b\text{arcsinh}(c+dx))^{3/2}}{5d} - \frac{(3be^4)\,\text{Subst}\left(\int \frac{x^5\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{10d} \\
&= -\frac{3be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\text{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(6be^4)\,\text{Subst}\left(\int \frac{x^3\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d} \\
&\quad + \frac{(3b^2e^4)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{100d} \\
&= \frac{2be^4 (c+dx)^2 \sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\text{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(3be^4)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{100d} \\
&\quad - \frac{(4be^4)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d} \\
&\quad - \frac{(b^2e^4)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{100d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{25d} \\
&\quad + \frac{(2b^2e^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{25d} \\
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1600d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{800d} \\
&\quad - \frac{(9be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1600d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{25d} \\
&\quad + \frac{(2be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3200d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3200d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1600d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1600d} \\
&\quad - \frac{(9be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3200d} \\
&\quad - \frac{(9be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3200d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{100d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{100d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{25d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1600d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1600d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{800d} \\
&\quad + \frac{(3be^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{800d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad - \frac{(9be^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1600d} \\
&\quad - \frac{(9be^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1600d} \\
&\quad + \frac{(2be^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{25d} \\
&\quad + \frac{(2be^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{25d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} + \frac{67b^{3/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1600d} \\
&\quad - \frac{3b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{3b^{3/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{67b^{3/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1600d} \\
&\quad - \frac{3b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{3b^{3/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{100d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{100d} \\
&\quad + \frac{(be^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{100d} \\
&\quad - \frac{(be^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{100d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{25d} \\
&\quad - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{5d} + \frac{3b^{3/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&\quad - \frac{b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} \\
&\quad - \frac{3b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{3b^{3/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{3b^{3/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&\quad - \frac{b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} \\
&\quad - \frac{3b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} \\
&\quad + \frac{3b^{3/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.57

$$
\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx =
$$

$$
-\frac{be^4e^{-\frac{5a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(2250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+9\sqrt{5}\sqrt{\frac{a}{b}+\operatorname{arcsin}}\right.
$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -1/36000*(b*e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(2250*E^((6*a)/b)*Sqrt[-((a +
b*ArcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + 9*Sqrt[5]*Sqrt
[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 125*
Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcS
inh[c + d*x]))/b] + 2250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, -((a + b*ArcSinh[c + d*x])/b)] - 125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 9*Sqrt[5]*
E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (5*(a + b*ArcSi
nh[c + d*x]))/b]))/(d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

## Maple [F]

$$\int (dex + ce)^4 \left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{3}{2}} dx$$

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = e^4\Bigg(\int ac^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int ad^4x^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx+\int bc^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 4acd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx+\int 6ac^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int 4ac^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx+\int bd^4x^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 4bcd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 6bc^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 4bc^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx\Bigg)$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(a*c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**4*x**4
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**4*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(4*a*c*d**3*x**3*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(6*a*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral
(4*a*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**4*x**4*sqrt(a
+ b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c*d**3*x**3*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(6*b*c**2*d**2*x**2*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c**3*d*x*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.187 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$

Optimal result . . . 1440
Rubi [A] (verified) . . . 1441
Mathematica [A] (verified) . . . 1447
Maple [F] . . . 1447
Fricas [F(-2)] . . . 1447
Sympy [F] . . . 1448
Maxima [F] . . . 1448
Giac [F] . . . 1449
Mupad [F(-1)] . . . 1449

## Optimal result

Integrand size = 25, antiderivative size = 360

$$\int (ce+dex)^3(a + b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} - \frac{3b^{3/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} + \frac{3b^{3/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} + \frac{3b^{3/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} - \frac{3b^{3/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(3/2)/d+3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^3*ex
p(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b^(3/2
)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)+9/64
```

```
*b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/32*b*e^3*
(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$
$$+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}$$
$$+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$
$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}$$
$$+\frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d}$$
$$-\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d}$$
$$+\frac{9be^3\sqrt{(c+dx)^2+1}(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d}$$
$$-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (9*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(64*
d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]
])/(32*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) + (e^3*(c + d*x)^
4*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (3*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[P
i]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d) + (3*b^(3/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2048*d*E^((4*a)/b)) - (3*b^(3/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3 \text{Subst}\left(\int x^3 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3 (c+dx)^4 (a+b\text{arcsinh}(c+dx))^{3/2}}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&= -\frac{3be^3 (c+dx)^3 \sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{32d} \\
&\quad + \frac{e^3 (c+dx)^4 (a+b\text{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad + \frac{(9be^3)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} \\
&\quad + \frac{(3b^2e^3)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{64d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&\quad - \frac{(9be^3)\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d} \\
&\quad - \frac{(9b^2e^3)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{128d} \\
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{128d} \\
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{128d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d} \\
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{512d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{512d} \\
&\quad + \frac{(3be^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{256d} \\
&\quad - \frac{(3be^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{256d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad - \frac{(9be^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{3b^{3/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad + \frac{3b^{3/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} \\
&\quad + \frac{3b^{3/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad - \frac{3b^{3/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} \\
&\quad + \frac{(9be^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{256d} \\
&\quad - \frac{(9be^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{256d} \\
&= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} \\
&\quad - \frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{3b^{3/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad + \frac{3b^{3/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&\quad + \frac{3b^{3/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad - \frac{3b^{3/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.62

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \frac{be^3e^{-\frac{4a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{5}{2},-\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)+8\sqrt{2}e^{\frac{2}{b}}$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5
/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 8*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[5/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*(-8*Sqrt[2]*Gamma[5/2, (2*(a + b*ArcSinh
[c + d*x]))/b] + E^((2*a)/b)*Gamma[5/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(512*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

## Maple [F]

$$\int (dex+ce)^3\,(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = e^3\left(\int ac^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\right.$$
$$+\int ad^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx+\int bc^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 3acd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx+\int 3ac^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int bd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 3bcd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$\left.+\int 3bc^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.188 $\int (ce+dex)^2(a+b\mathbf{arcsinh}(c+dx))^{3/2}\,dx$

Optimal result . . . 1450
Rubi [A] (verified) . . . 1451
Mathematica [A] (verified) . . . 1456
Maple [F] . . . 1457
Fricas [F(-2)] . . . 1457
Sympy [F] . . . 1457
Maxima [F] . . . 1458
Giac [F] . . . 1458
Mupad [F(-1)] . . . 1458

## Optimal result

Integrand size = 25, antiderivative size = 328

$$\int (ce+dex)^2(a + b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} - \frac{3b^{3/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{b^{3/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d} - \frac{3b^{3/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{b^{3/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/288*b^(3/2)*e^2*exp(3*a/b)
*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288*b
^(3/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2
)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d-3/32*b^(3/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/d/exp(a/b)+1/3*b*e^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-1/6*b*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d} - \frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d} + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} - \frac{be^2\sqrt{(c+dx)^2+1}(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{be^2\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (b*e^2*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (e^2
*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(3*d) - (3*b^(3/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) + (b^(3/2)*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(96*d) - (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2 x^2 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int x^2 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}}{3d} \\
&\quad + \frac{(be^2)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3d} \\
&\quad + \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{12d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12d} \\
&\quad - \frac{(b^2e^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{6d} \\
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6d} \\
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6d} \\
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} - \frac{b^{3/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12d} \\
&\quad - \frac{b^{3/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{48d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{48d} \\
&\quad - \frac{(be^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{48d} \\
&\quad + \frac{(be^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{48d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{3d} \\
&\quad - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{3d} - \frac{3b^{3/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} \\
&\quad + \frac{b^{3/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d} \\
&\quad - \frac{3b^{3/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} \\
&\quad + \frac{b^{3/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73

$$
\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx =
$$

$$
-\frac{be^2e^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}}\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]

[Out] -1/216*(b*e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*A
rcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b
+ ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 27*E^((2*
a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a + b*ArcSinh[c + d*x])/b)
] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (3*(
a + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)^2/b^2)])
```

## Maple [F]

$$\int (dex + ce)^2 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{3}{2}} dx$$

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}
&\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = e^2 \left( \int ac^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \right. \\
&+ \int ad^2x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int bc^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \\
&+ \int 2acdx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\
&+ \int bd^2x^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \\
&\left. + \int 2bcdx \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(3/2),x)
[Out] e**2*(Integral(a*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*asinh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.189 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx$

Optimal result . . . 1459
Rubi [A] (verified) . . . 1459
Mathematica [A] (verified) . . . 1463
Maple [F] . . . 1464
Fricas [F(-2)] . . . 1464
Sympy [F] . . . 1464
Maxima [F] . . . 1465
Giac [F] . . . 1465
Mupad [F(-1)] . . . 1465

## Optimal result

Integrand size = 23, antiderivative size = 205

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx =$$
$$-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d}$$
$$+\frac{e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d}+\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d}$$
$$-\frac{3b^{3/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3b^{3/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/
2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(1+(d*x+c)^2)^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} + \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} - \frac{3be\sqrt{(c+dx)^2+1}(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d)
 + (e*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh
[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(3be)\text{Subst}\left(\int \frac{\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&\quad + \frac{(3b^2e)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{16d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad - \frac{(3be)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad - \frac{(3be)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad - \frac{(3be)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad - \frac{(3be)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&\quad + \frac{(3be)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad - \frac{(3be)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d} \\
&\quad + \frac{(3be)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32d} \\
&= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} - \frac{3b^{3/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&\quad + \frac{3b^{3/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$
\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(b^2\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2}, -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right) + b^2e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{5}{2}, \frac{2}{}\right.\right.}{16\sqrt{2}d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]

[Out] (e*(b^2*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b] + b^2*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (2*(
a + b*ArcSinh[c + d*x]))/b]))/(16*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[
c + d*x]])
```

## Maple [F]

$$\int (dex + ce)\left(a + b\,\operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx &= e\left(\int ac\sqrt{a + b\operatorname{asinh}\left(c + dx\right)}\,dx\right.\\ &+ \int adx\sqrt{a + b\operatorname{asinh}\left(c + dx\right)}\,dx + \int bc\sqrt{a + b\operatorname{asinh}\left(c + dx\right)}\operatorname{asinh}\left(c + dx\right)dx\\ &\left.+ \int bdx\sqrt{a + b\operatorname{asinh}\left(c + dx\right)}\operatorname{asinh}\left(c + dx\right)dx\right)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(3/2),x)
[Out] e*(Integral(a*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
asinh(c + d*x)), x) + Integral(b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^{3/2}\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.190 $\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx$

Optimal result . . . 1466
Rubi [A] (verified) . . . 1466
Mathematica [A] (verified) . . . 1469
Maple [F] . . . 1469
Fricas [F(-2)] . . . 1470
Sympy [F] . . . 1470
Maxima [F] . . . 1470
Giac [F] . . . 1470
Mupad [F(-1)] . . . 1471

## Optimal result

Integrand size = 14, antiderivative size = 150

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = -\frac{3b\sqrt{1 + (c + dx)^2}\sqrt{a + b\operatorname{arcsinh}(c + dx)}}{2d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{3/2}}{d} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\, dx = \frac{3\sqrt{\pi}b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} - \frac{3b\sqrt{(c + dx)^2 + 1}\sqrt{a + b\operatorname{arcsinh}(c + dx)}}{2d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{3/2}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d} - \frac{(3b)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d} \\
&\quad + \frac{(3b^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4d} \\
&= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8d} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8d} \\
&= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{d} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4d}
\end{aligned}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d}$$
$$+ \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$
$$+ \frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\text{arcsinh}(c+dx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a}}{8d}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

## Maple [F]

$$\int (a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \int (a + b\,\text{asinh}\,(c + dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

## Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{3/2}\,dx = \int (a + b\,\operatorname{asinh}(c + d\,x))^{3/2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

# 3.191 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$

Optimal result . . . 1472
Rubi [N/A] . . . 1472
Mathematica [N/A] . . . 1473
Maple [N/A] (verified) . . . 1473
Fricas [F(-2)] . . . 1473
Sympy [N/A] . . . 1473
Maxima [N/A] . . . 1474
Giac [N/A] . . . 1474
Mupad [N/A] . . . 1474

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \frac{\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(3/2)/(d*x+c),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{3/2}}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{3/2}}{x}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}}{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \frac{\int \frac{a\sqrt{a+b\,\text{asinh}\,(c+dx)}}{c+dx}\,dx + \int \frac{b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**(3/2)/(d*e*x+c*e),x)
[Out] (Integral(a*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

## Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}\left(dx+c\right)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

## Giac [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}\left(dx+c\right)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^{3/2}}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

# 3.192 $\int (ce+dex)^4(a+b\mathbf{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . . . 1476
Rubi [A] (verified) . . . . . 1477
Mathematica [A] (verified) . . . . . 1488
Maple [F] . . . . . 1488
Fricas [F(-2)] . . . . . 1488
Sympy [F(-1)] . . . . . 1489
Maxima [F] . . . . . 1489
Giac [F] . . . . . 1489
Mupad [F(-1)] . . . . . 1489

## Optimal result

Integrand size = 25, antiderivative size = 701

$$\int (ce+dex)^4(a + b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} + \frac{15b^{5/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} + \frac{3b^{5/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d} - \frac{15b^{5/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} - \frac{3b^{5/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(5/2)/d+3/32000*b^(5/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d-3/320
00*b^(5/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-5/2304*b^(5/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+5/2304*b^(5/2)*e^4*erfi(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+15/128*b^(5
/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/128*
b^(5/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/
15*b*e^4*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/15*b*e^4*(d*x+c
)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-1/10*b*e^4*(d*x+c)^4*(
a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/5*b^2*e^4*(d*x+c)*(a+b*ar
csinh(d*x+c))^(1/2)/d-1/15*b^2*e^4*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(1/2)/d+3
/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules

used = {5859, 12, 5777, 5812, 5798, 5772, 5819, 3389, 2211, 2236, 2235, 3393}

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{15\sqrt{\pi}b^{5/2}e^4e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}$$
$$-\frac{\sqrt{3\pi}b^{5/2}e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d}$$
$$-\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d}$$
$$+\frac{3\sqrt{\frac{\pi}{5}}b^{5/2}e^4e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d}$$
$$-\frac{15\sqrt{\pi}b^{5/2}e^4e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}$$
$$+\frac{\sqrt{3\pi}b^{5/2}e^4e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d}$$
$$+\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d}$$
$$-\frac{3\sqrt{\frac{\pi}{5}}b^{5/2}e^4e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d}$$
$$+\frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d}$$
$$-\frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d}$$
$$+\frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d}$$
$$+\frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d}$$
$$-\frac{be^4\sqrt{(c+dx)^2+1}(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d}$$
$$+\frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d}$$
$$-\frac{4be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d}$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*e^4*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) - (b^2*e^4*(c + d*
x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(15*d) + (3*b^2*e^4*(c + d*x)^5*Sqrt[a +
 b*ArcSinh[c + d*x]])/(100*d) - (4*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSi
nh[c + d*x])^(3/2))/(15*d) + (2*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^(3/2))/(15*d) - (b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(10*d) + (e^4*(c + d*x)^5*(a + b*ArcSin
h[c + d*x])^(5/2))/(5*d) + (15*b^(5/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) - (b^(5/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d) - (b^(5/2)*e^4*
E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(1280*d) + (3*b^(5/2)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(6400*d) - (15*b^(5/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d*E^(a/b)) + (b^(5/2)*e^4*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d*E^((3*a)/b)) +
 (b^(5/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(1280*d*E^((3*a)/b)) - (3*b^(5/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(6400*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int e^4 x^4 (a+b\operatorname{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \operatorname{Subst}\left(\int x^4 (a+b\operatorname{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} - \frac{(be^4)\operatorname{Subst}\left(\int \frac{x^5 (a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}\,(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad + \frac{(2be^4)\operatorname{Subst}\left(\int \frac{x^3 (a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int x^4 \sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{20d} \\
&= \frac{3b^2e^4 (c+dx)^5 \sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad + \frac{2be^4 (c+dx)^2 \sqrt{1+(c+dx)^2}\,(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}\,(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad - \frac{(4be^4)\operatorname{Subst}\left(\int \frac{x (a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{15d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int x^2 \sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{5d} \\
&\quad - \frac{(3b^3e^4)\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{200d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh^5\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad + \frac{(2b^2e^4)\operatorname{Subst}\left(\int \sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{5d} \\
&\quad + \frac{(b^3e^4)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{30d} \\
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad - \frac{(3ib^2e^4)\operatorname{Subst}\left(\int \left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{5i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{5i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{200d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{30d} \\
&\quad - \frac{(b^3e^4)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{5d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad - \frac{(ib^2e^4)\operatorname{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{30d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3200d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{640d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{320d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6400d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6400d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1280d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1280d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{640d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{640d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{120d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{40d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{10d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{10d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3200d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3200d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{240d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{240d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{640d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{640d} \\
&\quad + \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{320d} \\
&\quad - \frac{(3b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{320d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{80d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{80d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} + \frac{67b^{5/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{640d} \\
&\quad - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} \\
&\quad + \frac{3b^{5/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d} \\
&\quad - \frac{67b^{5/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{640d} \\
&\quad + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} \\
&\quad - \frac{3b^{5/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{120d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{120d} \\
&\quad + \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{40d} \\
&\quad - \frac{(b^2e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{40d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} \\
&\quad + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} \\
&\quad - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} + \frac{15b^{5/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&\quad - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} \\
&\quad - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} \\
&\quad + \frac{3b^{5/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d} \\
&\quad - \frac{15b^{5/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&\quad + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} \\
&\quad + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} \\
&\quad - \frac{3b^{5/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.46

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx =$$

$$-\frac{b^3e^4e^{-\frac{5a}{b}}\left(33750e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{7}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+27\sqrt{5}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{5(a+\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/540000*(b^3*e^4*(33750*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/
2, a/b + ArcSinh[c + d*x]] + 27*Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
*Gamma[7/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 625*Sqrt[3]*E^((2*a)/b)*Sqrt
[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-3*(a + b*ArcSinh[c + d*x]))/b]
 + 33750*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, -((a +
b*ArcSinh[c + d*x])/b)] - 625*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*
x]]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 27*Sqrt[5]*E^((10*a)/b)*Sq
rt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(d*
E^((5*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex+ce)^4\,(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(5/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.193 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . . . 1490
Rubi [A] (verified) . . . . . 1491
Mathematica [A] (verified) . . . . . 1499
Maple [F] . . . . . 1500
Fricas [F(-2)] . . . . . 1500
Sympy [F] . . . . . 1500
Maxima [F] . . . . . 1501
Giac [F] . . . . . 1501
Mupad [F(-1)] . . . . . 1501

## Optimal result

Integrand size = 25, antiderivative size = 455

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \\
&-\frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&+\frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&-\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&-\frac{15b^{5/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&+\frac{15b^{5/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} \\
&-\frac{15b^{5/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&+\frac{15b^{5/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d}
\end{aligned}
$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(5/2)/d+15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^
```

```
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5/2)*
e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/1638
4*b^(5/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a
/b)+15/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-5/
32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-225/204
8*b^2*e^3*(a+b*arcsinh(d*x+c))^(1/2)/d-45/256*b^2*e^3*(d*x+c)^2*(a+b*arcsin
h(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5819, 3393, 3388, 2211, 2236, 2235}

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = -\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} + \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} - \frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} + \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} - \frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} - \frac{5be^3\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} + \frac{15be^3\sqrt{(c+dx)^2+1}(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(2048*d) - (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcSinh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcSinh[c + d*x]])/(256*d) + (15*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*
(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*(c + d*x)^3*Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/(4*
d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(16384*d) + (15*b^(5/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(512*d) - (15*b^(5/2)*e^3*Sqrt[P
```

```
i]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16384*d*E^((4*a)/b)) +
(15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(512*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^3 x^3 (a + b\text{arcsinh}(x))^{5/2}\, dx, x, c + dx\right)}{d} \\
&= \frac{e^3 \text{Subst}\left(\int x^3 (a + b\text{arcsinh}(x))^{5/2}\, dx, x, c + dx\right)}{d} \\
&= \frac{e^3 (c + dx)^4 (a + b\text{arcsinh}(c + dx))^{5/2}}{4d} - \frac{(5be^3)\,\text{Subst}\left(\int \frac{x^4 (a + b\text{arcsinh}(x))^{3/2}}{\sqrt{1 + x^2}}\, dx, x, c + dx\right)}{8d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad + \frac{(15be^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int x^3\sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{64d} \\
&= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15be^3)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d} \\
&\quad - \frac{(45b^2e^3)\operatorname{Subst}\left(\int x\sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{128d} \\
&\quad - \frac{(15b^3e^3)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{512d} \\
&= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad + \frac{(45b^3e^3)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{512d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&= -\frac{45b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4096d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d} \\
&\quad - \frac{(45b^2e^3)\operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4096d} \\
&\quad - \frac{(15b^2e^3)\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4096d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1024d} \\
&\quad + \frac{(15b^2e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1024d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{15b^{5/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&\quad + \frac{15b^{5/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad - \frac{15b^{5/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&\quad + \frac{15b^{5/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1024d} \\
&\quad + \frac{(45b^2e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1024d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225b^2e^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{256d} \\
&\quad + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&\quad - \frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} \\
&\quad - \frac{15b^{5/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&\quad + \frac{15b^{5/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} \\
&\quad - \frac{15b^{5/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&\quad + \frac{15b^{5/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.46

$$
\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{b^3e^3e^{-\frac{4a}{b}}\left(-\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)+16\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]

[Out] (b^3*e^3*(-(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-4*(a + b*ArcSi
nh[c + d*x]))/b]) + 16*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/
b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*(-16*Sqrt[2]*Gamma[7/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E
^((2*a)/b)*Gamma[7/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(2048*d*E^((4*a)/b
)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex + ce)^3 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{5}{2}} dx$$

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}
\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx &= e^3 \left( \int a^2 c^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \right. \\
&+ \int a^2 d^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\
&+ \int b^2 c^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c + dx)\, dx \\
&+ \int 2abc^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \\
&+ \int 3a^2 cd^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 3a^2 c^2 dx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \\
&+ \int b^2 d^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c + dx)\, dx \\
&+ \int 2abd^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \\
&+ \int 3b^2 cd^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c + dx)\, dx \\
&+ \int 3b^2 c^2 dx \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c + dx)\, dx \\
&+ \int 6abcd^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \\
&+ \left. \int 6abc^2 dx \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx \right)
\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(a**2*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d**
3*x**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**3*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**3*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x), x) + Integral(3*a**2*c*d**2*x**2*sqrt(a + b*asinh(c
+ d*x)), x) + Integral(3*a**2*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Int
egral(b**2*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) + Int
egral(2*a*b*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integ
ral(3*b**2*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) + I
ntegral(3*b**2*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) +
Integral(6*a*b*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) +
Integral(6*a*b*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^3(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^3(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\operatorname{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.194 $\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . 1502
Rubi [A] (verified) . . . 1503
Mathematica [A] (verified) . . . 1510
Maple [F] . . . 1510
Fricas [F(-2)] . . . 1510
Sympy [F] . . . 1511
Maxima [F] . . . 1511
Giac [F] . . . 1512
Mupad [F(-1)] . . . 1512

## Optimal result

Integrand size = 25, antiderivative size = 394

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \\
&-\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&+\frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&-\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&+\frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d}-\frac{15b^{5/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&+\frac{5b^{5/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d} \\
&+\frac{15b^{5/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&-\frac{5b^{5/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d}
\end{aligned}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)/d+5/1728*b^(5/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/1728
*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d+15/64*b^(5/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
```

```
2))*Pi^(1/2)/d/exp(a/b)+5/9*b*e^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^
(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)
/d-5/6*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d+5/36*b^2*e^2*(d*x+c)^3*
(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5819, 3389, 2211, 2236, 2235, 3393}

$$\begin{aligned}
&\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \\
&-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d} \\
&+\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d} \\
&+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{36d} \\
&-\frac{5b^2e^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{6d}+\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}}{3d} \\
&-\frac{5be^2\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{18d} \\
&+\frac{5be^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}{9d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
 d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(36*d) + (5*b*e^2*Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (e^2*(c + d*x)^3*(a + b
*ArcSinh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) + (5*b^(5/2)*e^2*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(576*d) + (15*b
^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d*E^(a/
```

```
b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
```

```
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^2x^2(a+b\text{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\ &= \frac{e^2\text{Subst}\left(\int x^2(a+b\text{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\ &= \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}}{3d} - \frac{(5be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\text{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{6d}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad + \frac{(5be^2)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int x^2\sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{12d} \\
&= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{6d} \\
&\quad - \frac{(5b^3e^2)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{72d} \\
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{72d} \\
&\quad + \frac{(5b^3e^2)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{12d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad + \frac{(5ib^2e^2)\operatorname{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{72d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12d} \\
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{288d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{96d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{576d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{576d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{192d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{192d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{12d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{12d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} - \frac{5b^{5/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24d} \\
&\quad + \frac{5b^{5/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{288d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{288d} \\
&\quad - \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{96d} \\
&\quad + \frac{(5b^2e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{96d} \\
&= -\frac{5b^2e^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{36d} \\
&\quad + \frac{5be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{9d} \\
&\quad - \frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{3d} - \frac{15b^{5/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&\quad + \frac{5b^{5/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d} \\
&\quad + \frac{15b^{5/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} \\
&\quad - \frac{5b^{5/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.56

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx =$$

$$-\frac{b^3e^2e^{-\frac{3a}{b}}\left(-81e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{7}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{3(a+b\operatorname{ar}\right.\right.}{648d}$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/648*(b^3*e^2*(-81*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, a/
b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7
/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 81*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[7/2, -((a + b*ArcSinh[c + d*x])/b)] + Sqrt[3]*E^((6*a)
/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b]
))/(d*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex+ce)^2\,(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = e^2\Bigg(\int a^2c^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int a^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int b^2c^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+\int 2abc^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 2a^2cdx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+\int b^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+\int 2abd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+\int 2b^2cdx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+\int 4abcdx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx\Bigg)$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(a**2*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d**
2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*asinh(c + d*x)
), x) + Integral(b**2*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x),
 x) + Integral(2*b**2*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(4*a*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.195 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . 1513
Rubi [A] (verified) . . . 1514
Mathematica [A] (verified) . . . 1518
Maple [F] . . . 1519
Fricas [F(-2)] . . . 1519
Sympy [F] . . . 1519
Maxima [F] . . . 1520
Giac [F] . . . 1520
Mupad [F(-1)] . . . 1520

## Optimal result

Integrand size = 23, antiderivative size = 262

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} - \frac{15b^{5/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} - \frac{15b^{5/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/
2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arcsinh(
d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+15/64*b^2*e*(a+b*arcsinh(d*x+c))^(1/2)/
d+15/32*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5859, 12, 5777, 5812, 5783, 5819, 3393, 3388, 2211, 2236, 2235}

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = -\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}$$
$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}$$
$$+\frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d}+\frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d}$$
$$-\frac{5be(c+dx)\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d}$$
$$+\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d}+\frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*e*Sqrt[a + b*ArcSinh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqrt
[a + b*ArcSinh[c + d*x]])/(32*d) - (5*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(
a + b*ArcSinh[c + d*x])^(3/2))/(8*d) + (e*(a + b*ArcSinh[c + d*x])^(5/2))/(
4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) - (15*b^(5/2)*e
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int ex(a+b\operatorname{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\operatorname{Subst}\left(\int x(a+b\operatorname{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} - \frac{(5be)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad + \frac{(5be)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&\quad + \frac{(15b^2e)\operatorname{Subst}\left(\int x\sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{16d} \\
&= \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{(15b^3e)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{64d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&= \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad + \frac{(15b^2e)\operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{64d} \\
&= \frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{128d} \\
&= \frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{128d} \\
&\quad - \frac{(15b^2e)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{128d} \\
&= \frac{15b^2e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{32d} \\
&\quad - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad - \frac{15b^{5/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&\quad - \frac{15b^{5/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.48

$$
\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(-b^3\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2}, -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right) + b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{7}{2},\right.\right.}{32\sqrt{2}d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]

[Out] (e*(-(b^3*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh
[c + d*x]))/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (
2*(a + b*ArcSinh[c + d*x]))/b]))/(32*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

## Maple [F]

$$\int (dex + ce)\left(a + b\,\mathrm{arcsinh}\left(dx + c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)(a + b\mathrm{arcsinh}(c + dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\begin{aligned}\int (ce + dex)(a + b\mathrm{arcsinh}(c + dx))^{5/2}\,dx &= e\left(\int a^2 c\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,dx\right.\\ &+ \int a^2 dx\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,dx + \int b^2 c\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,\mathrm{asinh}^2\left(c + dx\right)dx\\ &+ \int 2abc\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,\mathrm{asinh}\left(c + dx\right)dx\\ &+ \int b^2 dx\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,\mathrm{asinh}^2\left(c + dx\right)dx\\ &+ \left.\int 2abdx\sqrt{a + b\,\mathrm{asinh}\left(c + dx\right)}\,\mathrm{asinh}\left(c + dx\right)dx\right)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(5/2),x)
[Out] e*(Integral(a**2*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*x*sqrt(
a + b*asinh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x), x) + Integral(b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(2*a*b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

## Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^{5/2}\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.196 $\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

Optimal result . . . 1521
Rubi [A] (verified) . . . 1521
Mathematica [B] (verified) . . . 1524
Maple [F] . . . 1525
Fricas [F(-2)] . . . 1525
Sympy [F] . . . 1525
Maxima [F] . . . 1526
Giac [F] . . . 1526
Mupad [F(-1)] . . . 1526

## Optimal result

Integrand size = 14, antiderivative size = 179

$$\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5819, 3389, 2211, 2236, 2235}

$$\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{15\sqrt{\pi}b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^{5/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} - \frac{(5b)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int \sqrt{a+b\operatorname{arcsinh}(x)}\,dx, x, c+dx\right)}{4d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} - \frac{(15b^3)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{8d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} \\
&\quad + \frac{(15b^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&\quad - \frac{(15b^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8d} \\
&= \frac{15b^2(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d} + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} \\
&\quad - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}
\end{aligned}
$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(179) = 358$.

Time = 1.04 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.56

$$
\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}}\right) + 4a\sqrt{b}\left(
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

## Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}\, dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^{5/2}\, dx = \int (a + b \operatorname{asinh}(c + dx))^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

## Maxima [F]

$$\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (a+b\,\operatorname{asinh}(c+d\,x))^{5/2}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

# 3.197 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$

Optimal result . . . 1527
Rubi [N/A] . . . 1527
Mathematica [N/A] . . . 1528
Maple [N/A] (verified) . . . 1528
Fricas [F(-2)] . . . 1528
Sympy [N/A] . . . 1528
Maxima [N/A] . . . 1529
Giac [N/A] . . . 1529
Mupad [N/A] . . . 1529

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \frac{\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(5/2)/(d*x+c),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{5/2}}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{5/2}}{x}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{5}{2}}}{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 22.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \frac{\int \frac{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}}{c+dx}\,dx + \int \frac{b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)}{c+dx}\,dx + \int \frac{2ab\sqrt{a+b\,\text{asinh}\,(c}}{c+}}{e}$$

```
[In] integrate((a+b*asinh(d*x+c))**(5/2)/(d*e*x+c*e),x)
[Out] (Integral(a**2*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b**2*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

## Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

## Giac [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^{5/2}}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

# 3.198 $\int (ce+dex)^4(a+b\mathbf{arcsinh}(c+dx))^{7/2}\,dx$

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Maple [F] . . . . . 1541
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Sympy [F(-1)] . . . . . 1542
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Giac [F] . . . . . 1542
Mupad [F(-1)] . . . . . 1542

## Optimal result

Integrand size = 25, antiderivative size = 835

$$
\begin{aligned}
&\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \\
&-\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&+\frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&-\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&+\frac{14b^2e^4(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&-\frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&+\frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&-\frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&+\frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&-\frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&+\frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&+\frac{105b^{7/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&-\frac{119b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} \\
&-\frac{21b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} \\
&+\frac{21b^{7/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} \\
&+\frac{105b^{7/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&-\frac{119b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} \\
&-\frac{21b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} \\
&+\frac{21b^{7/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d}
\end{aligned}
$$

```
[Out] 14/15*b^2*e^4*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d-7/45*b^2*e^4*(d*x+c)^3*(
a+b*arcsinh(d*x+c))^(3/2)/d+7/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3
/2)/d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(7/2)/d+21/320000*b^(7/2)*e^4*
exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)
/d+21/320000*b^(7/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5
^(1/2)*Pi^(1/2)/d/exp(5*a/b)-35/13824*b^(7/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-35/13824*b^(7/2)*e^4*e
rfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/
b)+105/256*b^(7/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/256*b^(7/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/
2)/d/exp(a/b)-28/75*b*e^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+
14/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/50
*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-1813/1125
*b^3*e^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+119/1125*b^3*e^4*
(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-21/1000*b^3*e^4*
(d*x+c)^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {5859, 12, 5777, 5812, 5798, 5772, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\int (ce+dex)^4(a + b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{e^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}(c+dx)^5}{5d} + \frac{7b^2e^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}(c+dx)^5}{100d} - \frac{7be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}(c+dx)^4}{50d} - \frac{21b^3e^4\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}(c+dx)^4}{1000d} - \frac{7b^2e^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}(c+dx)^3}{45d} + \frac{14be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}(c+dx)^2}{75d} + \frac{119b^3e^4\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}(c+dx)^2}{1125d} + \frac{14b^2e^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}(c+dx)}{15d} - \frac{28be^4\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} + \frac{105b^{7/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} - \frac{21b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} - \frac{119b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} + \frac{21b^{7/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} + \frac{105b^{7/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} - \frac{21b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} - \frac{119b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} + \frac{21b^{7/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} - \frac{1813b^3e^4\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d}$$

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-1813*b^3*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(1125*d)
 + (119*b^3*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*
x]])/(1125*d) - (21*b^3*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/(1000*d) + (14*b^2*e^4*(c + d*x)*(a + b*ArcSinh[c + d*x])^
(3/2))/(15*d) - (7*b^2*e^4*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(45*
d) + (7*b^2*e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(100*d) - (28*b
*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d) + (14*b*e
^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d)
 - (7*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2
))/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(7/2))/(5*d) + (105*b
^(7/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(256
*d) - (119*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(18000*d) - (21*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*
Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000*d) + (21*b^(7/2
)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(64000*d) + (105*b^(7/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(256*d*E^(a/b)) - (119*b^(7/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(18000*d*E^((3*a)/b)) - (21*b^(7/2)*
e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000
*d*E^((3*a)/b)) + (21*b^(7/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(64000*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^4 x^4 (a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int x^4 (a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 (c+dx)^5 (a+b\text{arcsinh}(c+dx))^{7/2}}{5d} - \frac{(7be^4)\,\text{Subst}\left(\int \frac{x^5 (a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{10d} \\
&= -\frac{7be^4 (c+dx)^4 \sqrt{1+(c+dx)^2}\,(a+b\text{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4 (c+dx)^5 (a+b\text{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad + \frac{(14be^4)\,\text{Subst}\left(\int \frac{x^3 (a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d} \\
&\quad + \frac{(7b^2e^4)\,\text{Subst}\left(\int x^4 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{20d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&\quad + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad - \frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad - \frac{(28be^4)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{75d} \\
&\quad - \frac{(7b^2e^4)\operatorname{Subst}\left(\int x^2(a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{15d} \\
&\quad - \frac{(21b^3e^4)\operatorname{Subst}\left(\int \frac{x^5\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{200d} \\
&= -\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&\quad - \frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&\quad + \frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&\quad - \frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad - \frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad + \frac{(14b^2e^4)\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{15d} \\
&\quad + \frac{(21b^3e^4)\operatorname{Subst}\left(\int \frac{x^3\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{250d} \\
&\quad + \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{x^3\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{30d} \\
&\quad + \frac{(21b^4e^4)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{2000d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&\quad - \frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&\quad + \frac{14b^2e^4(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&\quad + \frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&\quad - \frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad - \frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad + \frac{(21b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2000d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{125d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{45d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5d} \\
&\quad - \frac{(7b^4e^4)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{500d} \\
&\quad - \frac{(7b^4e^4)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{180d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&\quad + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&\quad - \frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&\quad + \frac{14b^2e^4(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&\quad + \frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&\quad - \frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad - \frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad + \frac{(21b^3e^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2000d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{500d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{180d} \\
&\quad + \frac{(7b^4e^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{250d} \\
&\quad + \frac{(7b^4e^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{90d} \\
&\quad + \frac{(7b^4e^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{10d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&\quad + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&\quad - \frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&\quad + \frac{14b^2e^4(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&\quad - \frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&\quad + \frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&\quad - \frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&\quad - \frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&\quad + \frac{(21b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32000d} \\
&\quad + \frac{(21b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16000d} \\
&\quad - \frac{(63b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32000d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{500d} \\
&\quad + \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{250d} \\
&\quad - \frac{(7b^3e^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{180d} \\
&\quad + \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{90d} \\
&\quad + \frac{(7b^3e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{10d} \\
&= \text{Too large to display}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.39

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^4e^{-\frac{5a}{b}}\left(-506250e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+81\sqrt{5}\sqrt{-\frac{a+b\operatorname{arcsin}}{b}}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b^4*e^4*(-506250*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, a/b +
 ArcSinh[c + d*x]] + 81*Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9
/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3125*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a +
 b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 5062
50*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, -((a + b*ArcS
inh[c + d*x])/b)] + 3125*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*G
amma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 81*Sqrt[5]*E^((10*a)/b)*Sqrt[a/
b + ArcSinh[c + d*x]]*Gamma[9/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(8100000
*d*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex+ce)^4\,(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^{7/2}\,dx$$

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.199 $\int (ce+dex)^3(a+b\mathbf{arcsinh}(c+dx))^{7/2}\,dx$

Optimal result . . . . . 1544
Rubi [A] (verified) . . . . . 1545
Mathematica [A] (verified) . . . . . 1556
Maple [F] . . . . . 1557
Fricas [F(-2)] . . . . . 1557
Sympy [F(-1)] . . . . . 1557
Maxima [F] . . . . . 1557
Giac [F] . . . . . 1558
Mupad [F(-1)] . . . . . 1558

## Optimal result

Integrand size = 25, antiderivative size = 547

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} - \frac{105b^{7/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} + \frac{105b^{7/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} + \frac{105b^{7/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} - \frac{105b^{7/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

```
[Out] -525/2048*b^2*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d-105/256*b^2*e^3*(d*x+c)^2*(a
+b*arcsinh(d*x+c))^(3/2)/d+35/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(3
/2)/d-3/32*e^3*(a+b*arcsinh(d*x+c))^(7/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(
d*x+c))^(7/2)/d+105/4096*b^(7/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-105/4096*b^(7/2)*e^3*erfi(2^(1/2)*(
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-105/131072
*b^(7/2)*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/
d+105/131072*b^(7/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2
)/d/exp(4*a/b)+21/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)
```

```
^(1/2)/d-7/32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2
)/d+1575/4096*b^3*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-105/2048*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/
2)/d
```

## Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \\
&-\frac{105\sqrt{\pi}b^{7/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&+\frac{105\sqrt{\pi}b^{7/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}e^3e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&-\frac{105b^3e^3\sqrt{(c+dx)^2+1}(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&+\frac{1575b^3e^3\sqrt{(c+dx)^2+1}(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&+\frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&-\frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&-\frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d}+\frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&-\frac{7be^3\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&+\frac{21be^3\sqrt{(c+dx)^2+1}(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d}
\end{aligned}
$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (1575*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(4096*d) - (105*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/(2048*d) - (525*b^2*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(2048
*d) - (105*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (3
5*b^2*e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (21*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(64*d) - (7*
b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(32
*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(7/2))/(32*d) + (e^3*(c + d*x)^4*(a +
 b*ArcSinh[c + d*x])^(7/2))/(4*d) - (105*b^(7/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*E
rf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(131072*d) + (105*b^(7/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(2048*d) + (105*b^(7/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(131072*d*E^((4*a)/b)) - (105*b^(7/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^3 x^3 (a + b\text{arcsinh}(x))^{7/2}\, dx, x, c + dx\right)}{d}$$

$$= \frac{e^3 \text{Subst}\left(\int x^3 (a + b\text{arcsinh}(x))^{7/2}\, dx, x, c + dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} - \frac{(7be^3)\operatorname{Subst}\left(\int \frac{x^4(a+b\operatorname{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d}$$

$$= -\frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{(21be^3)\operatorname{Subst}\left(\int \frac{x^2(a+b\operatorname{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{32d} + \frac{(35b^2e^3)\operatorname{Subst}\left(\int x^3(a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{64d}$$

$$= \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} - \frac{(21be^3)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d} - \frac{(105b^2e^3)\operatorname{Subst}\left(\int x(a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{128d} - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{x^4\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{512d}$$

$$
\begin{aligned}
&= -\frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{x^2\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2048d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{x^2\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{512d} \\
&\quad + \frac{(105b^4e^3)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{4096d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4096d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4096d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{1024d} \\
&\quad - \frac{(315b^4e^3)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{8192d} \\
&\quad - \frac{(315b^4e^3)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{2048d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4096d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32768d} \\
&\quad + \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16384d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2048d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{65536d} \\
&\quad + \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{65536d} \\
&\quad + \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32768d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32768d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16384d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4096d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32768d} \\
&\quad + \frac{(105b^3e^3)\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{32768d} \\
&\quad + \frac{(105b^3e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16384d} \\
&\quad - \frac{(105b^3e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16384d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32768d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32768d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8192d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{105b^{7/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&\quad + \frac{105b^{7/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32768d} \\
&\quad + \frac{105b^{7/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&\quad - \frac{105b^{7/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32768d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16384d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16384d} \\
&\quad + \frac{(315b^3e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4096d} \\
&\quad - \frac{(315b^3e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4096d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4096d} \\
&\quad - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2048d} \\
&\quad - \frac{525b^2e^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{35b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{256d} \\
&\quad + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{64d} \\
&\quad - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{32d} \\
&\quad - \frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} \\
&\quad - \frac{105b^{7/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&\quad + \frac{105b^{7/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} \\
&\quad + \frac{105b^{7/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} \\
&\quad - \frac{105b^{7/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.38

$$
\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2},-\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right) - 32\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2},\right.\right.}{819}
$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]

[Out] (b^4*e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-4*(a + b*ArcSinh
[c + d*x]))/b] - 32*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
*Gamma[9/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSi
nh[c + d*x]]*(-32*Sqrt[2]*Gamma[9/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((
2*a)/b)*Gamma[9/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(8192*d*E^((4*a)/b)*S
qrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex + ce)^3 \left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{7}{2}} dx$$

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \int (dex + ce)^3 (b \operatorname{arsinh}\left(dx + c\right) + a)^{\frac{7}{2}}\, dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.200 $\int (ce+dex)^2(a+b\mathbf{arcsinh}(c+dx))^{7/2}\,dx$

Optimal result . . . . . 1560
Rubi [A] (verified) . . . . . 1561
Mathematica [A] (verified) . . . . . 1571
Maple [F] . . . . . 1572
Fricas [F(-2)] . . . . . 1572
Sympy [F(-1)] . . . . . 1572
Maxima [F] . . . . . 1572
Giac [F] . . . . . 1573
Mupad [F(-1)] . . . . . 1573

## Optimal result

Integrand size = 25, antiderivative size = 481

$$
\begin{aligned}
&\int (ce+dex)^2(a \\
&+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&-\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&-\frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&+\frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&+\frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&-\frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&+\frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&-\frac{105b^{7/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&+\frac{35b^{7/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d} \\
&-\frac{105b^{7/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&+\frac{35b^{7/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d}
\end{aligned}
$$

```
[Out] -35/18*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(7/2)
/d+35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^2*exp
(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-105/128*b^(7/2)*e^
2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)+7/9*b*e^2*(a
+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/18*b*e^2*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+175/54*b^3*e^2*(1+(d*x+c)^2)^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/d-35/216*b^3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\begin{aligned}
&\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \\
&-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&+\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d} \\
&-\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&+\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d} \\
&+\frac{175b^3e^2\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&-\frac{35b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&+\frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&-\frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&+\frac{7be^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&-\frac{7be^2(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&+\frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (175*b^3*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(54*d) - (
35*b^3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/
(216*d) - (35*b^2*e^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (3
5*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(108*d) + (7*b*e^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*(c + d
*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(18*d) + (e^2*(
c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)
```

```
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) + (35*b^(7/2)*
e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(3456*d) - (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(128*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2 x^2 (a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2 \text{Subst}\left(\int x^2 (a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2 (c+dx)^3 (a+b\text{arcsinh}(c+dx))^{7/2}}{3d} - \frac{(7be^2)\,\text{Subst}\left(\int \frac{x^3 (a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{6d} \\
&= -\frac{7be^2 (c+dx)^2 \sqrt{1+(c+dx)^2} (a+b\text{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2 (c+dx)^3 (a+b\text{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad + \frac{(7be^2)\,\text{Subst}\left(\int \frac{x (a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&\quad + \frac{(35b^2e^2)\,\text{Subst}\left(\int x^2 (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{36d} \\
&= \frac{35b^2e^2 (c+dx)^3 (a+b\text{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2 \sqrt{1+(c+dx)^2} (a+b\text{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2 (c+dx)^2 \sqrt{1+(c+dx)^2} (a+b\text{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2 (c+dx)^3 (a+b\text{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad - \frac{(35b^2e^2)\,\text{Subst}\left(\int (a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{18d} \\
&\quad - \frac{(35b^3e^2)\,\text{Subst}\left(\int \frac{x^3 \sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{72d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{108d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{12d} \\
&\quad + \frac{(35b^4e^2)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{432d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{432d} \\
&\quad - \frac{(35b^4e^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{216d} \\
&\quad - \frac{(35b^4e^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{24d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{432d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{216d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1728d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1728d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{432d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{432d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{48d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3456d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3456d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3456d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3456d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{216d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{216d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{24d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} - \frac{175b^{7/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{216d} \\
&\quad - \frac{175b^{7/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{216d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1728d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1728d} \\
&\quad - \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1728d} \\
&\quad + \frac{(35b^3e^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{1728d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{54d} \\
&\quad - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{216d} \\
&\quad - \frac{35b^2e^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{18d} \\
&\quad + \frac{35b^2e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{108d} \\
&\quad + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{9d} \\
&\quad - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{18d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{3d} - \frac{105b^{7/2}e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&\quad + \frac{35b^{7/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d} \\
&\quad - \frac{105b^{7/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} \\
&\quad + \frac{35b^{7/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.46

$$
\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^2e^{-\frac{3a}{b}}\left(243e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma
$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]

[Out] (b^4*e^2*(243*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, a/b + Arc
Sinh[c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-3
*(a + b*ArcSinh[c + d*x]))/b] - 243*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d
*x])/b)]*Gamma[9/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sq
rt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(19
44*d*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int (dex + ce)^2 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{7}{2}} dx$$

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \int (dex + ce)^2 \left(b \operatorname{arsinh}(dx + c) + a\right)^{\frac{7}{2}} dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.201 $\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$

Optimal result . . . 1574
Rubi [A] (verified) . . . 1575
Mathematica [A] (verified) . . . 1580
Maple [F] . . . 1580
Fricas [F(-2)] . . . 1581
Sympy [F(-1)] . . . 1581
Maxima [F] . . . 1581
Giac [F] . . . 1581
Mupad [F(-1)] . . . 1582

## Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \\
&-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&+\frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d}+\frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&-\frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&+\frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d}+\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&-\frac{105b^{7/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d} \\
&+\frac{105b^{7/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d}
\end{aligned}$$

```
[Out] 35/64*b^2*e*(a+b*arcsinh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arcsinh
(d*x+c))^(3/2)/d+1/4*e*(a+b*arcsinh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8*b*e*
(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/128*b^3*e*(d*x
+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$\begin{aligned}
&\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \\
&-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d} \\
&+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d} \\
&-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2+1}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&+\frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d}+\frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} \\
&-\frac{7be(c+dx)\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&+\frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d}+\frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(
128*d) + (35*b^2*e*(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) + (35*b^2*e*(c +
d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (7*b*e*(c + d*x)*Sqrt[1 + (
c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(8*d) + (e*(a + b*ArcSinh[c + d
*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/(2*d) -
(105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}}{2d} - \frac{(7be)\text{Subst}\left(\int \frac{x^2(a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4d} \\
&= -\frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad + \frac{(7be)\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d} \\
&\quad + \frac{(35b^2e)\,\text{Subst}\left(\int x(a+b\text{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{16d} \\
&= \frac{35b^2e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\text{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\text{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{64d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad + \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{128d} \\
&\quad + \frac{(105b^4e)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{256d} \\
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d} \\
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{256d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{512d} \\
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d} \\
&\quad + \frac{(105b^3e)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{1024d} \\
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{(105b^3e)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{512d} \\
&\quad + \frac{(105b^3e)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{512d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{128d} \\
&\quad + \frac{35b^2e(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{64d} + \frac{35b^2e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{32d} \\
&\quad - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{8d} \\
&\quad + \frac{e(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{4d} + \frac{e(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{2d} \\
&\quad - \frac{105b^{7/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d} \\
&\quad + \frac{105b^{7/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$
\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(b^4\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2},-\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)+b^4e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{2(a}\right.\right.}{64\sqrt{2}d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e*(b^4*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b] + b^4*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (2*(
a + b*ArcSinh[c + d*x]))/b]))/(64*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[
c + d*x]])
```

## Maple [F]

$$
\int (dex+ce)\,(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}\,dx
$$

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \int (c\,e+d\,e\,x)\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.202 $\int (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx$

Optimal result . . . 1583
Rubi [A] (verified) . . . 1583
Mathematica [B] (verified) . . . 1587
Maple [F] . . . 1588
Fricas [F(-2)] . . . 1588
Sympy [F(-1)] . . . 1588
Maxima [F] . . . 1588
Giac [F] . . . 1589
Mupad [F(-1)] . . . 1589

## Optimal result

Integrand size = 14, antiderivative size = 216

$$\int (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = -\frac{105b^3\sqrt{1 + (c + dx)^2}\sqrt{a + b\operatorname{arcsinh}(c + dx)}}{8d} + \frac{35b^2(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{3/2}}{4d} - \frac{7b\sqrt{1 + (c + dx)^2}(a + b\operatorname{arcsinh}(c + dx))^{5/2}}{2d} + \frac{(c + dx)(a + b\operatorname{arcsinh}(c + dx))^{7/2}}{d} + \frac{105b^{7/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{105b^{7/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a + b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

```
[Out] 35/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^
(7/2)/d+105/32*b^(7/2)*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/32*b^(7/2)*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/
exp(a/b)-7/2*b*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/8*b^3*(
1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235\}$

$$\int (a + b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{105\sqrt{\pi}b^{7/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$
$$+ \frac{105\sqrt{\pi}b^{7/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$
$$- \frac{105b^3\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d}$$
$$+ \frac{35b^2(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{4d}$$
$$- \frac{7b\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}}{d}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d) + (35*b
^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (7*b*Sqrt[1 + (c + d*x
)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c +
d*x])^(7/2))/d + (105*b^(7/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^{7/2}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} - \frac{(7b)\operatorname{Subst}\left(\int \frac{x(a+b\operatorname{arcsinh}(x))^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} \\
&\quad + \frac{(35b^2)\operatorname{Subst}\left(\int (a+b\operatorname{arcsinh}(x))^{3/2}\,dx, x, c+dx\right)}{4d} \\
&= \frac{35b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} - \frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} \\
&\quad + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} - \frac{(105b^3)\operatorname{Subst}\left(\int \frac{x\sqrt{a+b\operatorname{arcsinh}(x)}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{8d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{35b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} \\
&\quad + \frac{(105b^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{16d} \\
&= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{35b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} \\
&\quad + \frac{(105b^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16d} \\
&= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{35b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} \\
&\quad + \frac{(105b^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32d} \\
&\quad + \frac{(105b^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{32d} \\
&= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{8d} \\
&\quad + \frac{35b^2(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{4d} \\
&\quad - \frac{7b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{d} \\
&\quad + \frac{(105b^3)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d} \\
&\quad + \frac{(105b^3)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{16d}
\end{aligned}
$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d} + \frac{35b^2(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{4d}$$
$$- \frac{7b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}}{d}$$
$$+ \frac{105b^{7/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{105b^{7/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 698 vs. $2(216) = 432$.

Time = 3.27 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.23

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{16a^3e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right) + 12a^2$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2),x]

[Out] ((16*a^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[
c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 12*a^2*Sqrt[b
]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 6*a*Sqrt[b]*(4*Sq
rt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + S
inh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*ArcS
inh[c + d*x]]*(2*b*(c + d*x)*(-10*a + 35*b*ArcSinh[c + d*x] + 4*b*ArcSinh[c
 + d*x]^3) + Sqrt[1 + (c + d*x)^2]*(-4*a^2 + 4*a*b*ArcSinh[c + d*x] - 7*b^2
*(15 + 4*ArcSinh[c + d*x]^2))) + (8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
 (-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(32*d)
```

## Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

## Maxima [F]

$$\int (a + b\operatorname{arcsinh}(c + dx))^{7/2}\, dx = \int (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^{7/2}\, dx = \int (b\,\text{arsinh}\,(dx + c) + a)^{\frac{7}{2}}\, dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^{7/2}\, dx = \int (a + b\,\text{asinh}(c + d\,x))^{7/2}\, dx$$

```
[In] int((a + b*asinh(c + d*x))^(7/2),x)
[Out] int((a + b*asinh(c + d*x))^(7/2), x)
```

# 3.203 $\int \frac{(a+b\mathbf{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$

Optimal result . . . 1590
Rubi [N/A] . . . 1590
Mathematica [N/A] . . . 1591
Maple [N/A] (verified) . . . 1591
Fricas [F(-2)] . . . 1591
Sympy [F(-1)] . . . 1591
Maxima [N/A] . . . 1592
Giac [N/A] . . . 1592
Mupad [N/A] . . . 1592

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \frac{\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(7/2)/(d*x+c),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^{7/2}}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^{7/2}}{x}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{7}{2}}}{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh(d*x+c))**(7/2)/(d*e*x+c*e),x)
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

## Giac [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^{7/2}}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

# 3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\mathbf{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 1593
Rubi [A] (verified) . . . 1594
Mathematica [A] (verified) . . . 1597
Maple [F] . . . 1598
Fricas [F(-2)] . . . 1598
Sympy [F] . . . 1598
Maxima [F] . . . 1599
Giac [F] . . . 1599
Mupad [F(-1)] . . . 1599

## Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{e^4 e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4 e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4 e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4 e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4 e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4 e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}$$

```
[Out] 1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)+1/16*e^4*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*erf(3^(
```

```
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)-1/32*e^
4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3
*a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3388, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{\sqrt{\pi}e^4e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{5}}e^4e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}$$

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b
]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) - (e^4*Sqrt[3*Pi
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((3
*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^4\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{e^4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad - \frac{(3e^4)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&= \frac{e^4\text{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32bd} \\
&\quad + \frac{e^4\text{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32bd} \\
&\quad + \frac{e^4\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{e^4\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad - \frac{(3e^4)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32bd} \\
&\quad - \frac{(3e^4)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{32bd}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^4 \text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{16bd} \\
&\quad + \frac{e^4 \text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{16bd} \\
&\quad + \frac{e^4 \text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{8bd} \\
&\quad + \frac{e^4 \text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{8bd} \\
&\quad - \frac{(3e^4) \text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{16bd} \\
&\quad - \frac{(3e^4) \text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{16bd} \\
&= \frac{e^4 e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4 e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} \\
&\quad + \frac{e^4 e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4 e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} \\
&\quad - \frac{e^4 e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4 e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx \\
&= \frac{e^4 e^{-\frac{5a}{b}}\left(-10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2}, \frac{a}{b}+\text{arcsinh}(c+dx)\right) + \sqrt{5}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2}, -\frac{5(a+b\text{ar}}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]

[Out] (e^4*(-10*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh
[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcSinh[c + d*x]))/b] - 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqr
t[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]
```

```
+ 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*A
rcSinh[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Ga
mma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Ar
cSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^4}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = e^4\left(\int \frac{c^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{d^4x^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{4cd^3x^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{6c^2d^2x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{4c^3dx}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^4}{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^4}{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{\sqrt{a+b\,\operatorname{asinh}\left(c+d\,x\right)}}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$


Optimal result . . . . . 1600
Rubi [A] (verified) . . . . . 1601
Mathematica [A] (verified) . . . . . 1604
Maple [F] . . . . . 1604
Fricas [F(-2)] . . . . . 1604
Sympy [F] . . . . . 1605
Maxima [F] . . . . . 1605
Giac [F] . . . . . 1605
Mupad [F(-1)] . . . . . 1606


## Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = -\frac{e^3 e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^3 e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^3 e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^3 e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

```
[Out] 1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/d/b^(1/2)-1/16*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = -\frac{\sqrt{\pi}e^3 e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{2}}e^3 e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{\sqrt{\pi}e^3 e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{\sqrt{\frac{\pi}{2}}e^3 e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/32*(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(Sqrt[b]*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e^3\text{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^3 \text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{e^3 \text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&= -\frac{e^3 \text{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{e^3 \text{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{16bd} \\
&\quad + \frac{e^3 \text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad - \frac{e^3 \text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&= -\frac{e^3 \text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{8bd} \\
&\quad + \frac{e^3 \text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{8bd} \\
&\quad + \frac{e^3 \text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&\quad - \frac{e^3 \text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&= -\frac{e^3 e^{\frac{4a}{b}} \sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^3 e^{\frac{2a}{b}} \sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} \\
&\quad + \frac{e^3 e^{-\frac{4a}{b}} \sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^3 e^{-\frac{2a}{b}} \sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$

$$= \frac{e^3 e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\operatorname{arcsinh}(c+dx))}{b}\right)-2\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\operatorname{arcsinh}(}{b}\right.\right.}{32d\sqrt{a+b\operatorname{arcsi}}}$$

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSinh[c
 + d*x]]*(-2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b
)*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]))/(32*d*E^((4*a)/b)*Sqrt[a +
b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^3}{\sqrt{a+b\,\operatorname{arcsinh}\left(dx+c\right)}}dx$$

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = e^3\left(\int \frac{c^3}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx + \int \frac{d^3x^3}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx + \int \frac{3cd^2x^2}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx + \int \frac{3c^2dx}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^3}{\sqrt{b\,\operatorname{arsinh}(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^3}{\sqrt{b\,\operatorname{arsinh}(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\mathbf{arcsinh}(c+dx)}}\,dx$

Optimal result . . . . . 1607
Rubi [A] (verified) . . . . . 1608
Mathematica [A] (verified) . . . . . 1611
Maple [F] . . . . . 1611
Fricas [F(-2)] . . . . . 1611
Sympy [F] . . . . . 1612
Maxima [F] . . . . . 1612
Giac [F] . . . . . 1612
Mupad [F(-1)] . . . . . 1613

## Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{e^2 e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2 e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} - \frac{e^2 e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2 e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

```
[Out] 1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)-1/8*e^2*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3388, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{3}}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} - \frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{\sqrt{\frac{\pi}{3}}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/8*(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt
[b]*d) + (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(8*Sqrt[b]*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2 x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{e^2\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^2 \text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad - \frac{e^2 \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&= -\frac{e^2 \text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad - \frac{e^2 \text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{e^2 \text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&\quad + \frac{e^2 \text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{8bd} \\
&= \frac{e^2 \text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&\quad - \frac{e^2 \text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&\quad - \frac{e^2 \text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&\quad + \frac{e^2 \text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{4bd} \\
&= -\frac{e^2 e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2 e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} \\
&\quad - \frac{e^2 e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2 e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$

$$= \frac{e^2 e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\operatorname{arcsin}}{b}\right.\right.}{24d\sqrt{}}$$

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^2}{\sqrt{a+b\,\operatorname{arcsinh}(dx+c)}}dx$$

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = e^2\left(\int \frac{c^2}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx + \int \frac{d^2x^2}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx + \int \frac{2cdx}{\sqrt{a+b\,\operatorname{asinh}(c+dx)}}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*asinh(c + d*x)),
x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^2}{\sqrt{b\,\operatorname{arsinh}(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^2}{\sqrt{b\,\operatorname{arsinh}(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.207 $\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 1614
Rubi [A] (verified) . . . 1614
Mathematica [A] (verified) . . . 1617
Maple [F] . . . 1617
Fricas [F(-2)] . . . 1617
Sympy [F] . . . 1618
Maxima [F] . . . 1618
Giac [F] . . . 1618
Mupad [F(-1)] . . . 1618

## Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

```
[Out] -1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5780, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d} - \frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/4*(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{e\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= -\frac{e\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&= -\frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{4bd} \\
&= -\frac{e\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd} \\
&\quad + \frac{e\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd} \\
&= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}bd} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}bd}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$= \frac{ee^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{2(a+b\text{arcsinh}(c+dx}{b}\right.\right.}{4\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c + d
*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*Ar
cSinh[c + d*x]))/b]))/(4*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]]
)
```

## Maple [F]

$$\int \frac{dex+ce}{\sqrt{a+b\,\text{arcsinh}\,(dx+c)}}dx$$

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$
$$= e\left(\int \frac{c}{\sqrt{a+b\,\operatorname{asinh}\,(c+dx)}}\,dx + \int \frac{dx}{\sqrt{a+b\,\operatorname{asinh}\,(c+dx)}}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x/sqrt(a + b*asin
h(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{dex+ce}{\sqrt{b\,\operatorname{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{dex+ce}{\sqrt{b\,\operatorname{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{c\,e+d\,e\,x}{\sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.208 $\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

Optimal result . . . 1619
Rubi [A] (verified) . . . 1619
Mathematica [A] (verified) . . . 1621
Maple [F] . . . 1621
Fricas [F(-2)] . . . 1622
Sympy [F] . . . 1622
Maxima [F] . . . 1622
Giac [F] . . . 1622
Mupad [F(-1)] . . . 1623

## Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5858, 5774, 3388, 2211, 2236, 2235}

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2bd} \\
&= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd} \\
&\quad + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{bd} \\
&= \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$
\begin{aligned}
&\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx \\
&= \frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2}, \frac{a}{b}+\text{arcsinh}(c+dx)\right) + \sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2}, -\frac{a+b\text{arcsinh}(c+dx)}{b}\right.\right.}{2d\sqrt{a+b\text{arcsinh}(c+dx)}}
\end{aligned}
$$

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$
\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\,(dx+c)}}dx
$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\operatorname{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

# 3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$


Optimal result . . . . . 1624
Rubi [N/A] . . . . . 1624
Mathematica [N/A] . . . . . 1625
Maple [N/A] (verified) . . . . . 1625
Fricas [F(-2)] . . . . . 1625
Sympy [N/A] . . . . . 1625
Maxima [N/A] . . . . . 1626
Giac [N/A] . . . . . 1626
Mupad [N/A] . . . . . 1626


## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcSinh[x]]), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\sqrt{a+b\,\text{arcsinh}\,(dx+c)}}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{\int \frac{1}{c\sqrt{a+b\,\text{asinh}\,(c+dx)}+dx\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(1/(c*sqrt(a + b*asinh(c + d*x)) + d*x*sqrt(a + b*asinh(c + d*x))),
 x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)), x)
```

# 3.210 $\int \frac{(ce+dex)^4}{(a+b\mathbf{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1627
Rubi [A] (verified) . . . 1628
Mathematica [A] (verified) . . . 1631
Maple [F] . . . 1632
Fricas [F(-2)] . . . 1632
Sympy [F] . . . 1632
Maxima [F] . . . 1633
Giac [F] . . . 1633
Mupad [F(-1)] . . . 1633

## Optimal result

Integrand size = 25, antiderivative size = 367

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} - \frac{e^4e^{\frac{5a}{b}}\sqrt{5\pi}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} - \frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} + \frac{e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}$$

```
[Out] -1/8*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/
d+1/8*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a
/b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(3/2)/d-3/16*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-1/16*e^4*exp(5*a/b)*erf(5^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16*e^4*e
rfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d/
exp(5*a/b)-2*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/
2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5859, 12, 5778, 3389, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\pi}e^4 e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3\sqrt{3\pi}e^4 e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} - \frac{\sqrt{5\pi}e^4 e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} + \frac{\sqrt{\pi}e^4 e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} - \frac{3\sqrt{3\pi}e^4 e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} + \frac{\sqrt{5\pi}e^4 e^{-\frac{5a}{b}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} - \frac{2e^4(c+dx)^4\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^
(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) - (e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/b)) - (3*e^4*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*
d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(2e^4)\,\text{Subst}\left(\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} + \frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} - \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d}
\end{aligned}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^4\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d}$$
$$- \frac{(5e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d}$$
$$+ \frac{(9e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d}$$
$$+ \frac{e^4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^2d}$$
$$- \frac{(5e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d}$$
$$+ \frac{(5e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d}$$
$$+ \frac{(9e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d}$$
$$- \frac{(9e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{16b^2d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^4\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4b^2d}$$
$$+ \frac{e^4\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4b^2d}$$
$$- \frac{(5e^4)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8b^2d}$$
$$+ \frac{(5e^4)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8b^2d}$$
$$+ \frac{(9e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8b^2d}$$
$$- \frac{(9e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{8b^2d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}$$
$$+ \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d} - \frac{e^4e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}$$
$$+ \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} - \frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}$$
$$+ \frac{e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}$$

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.34

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^4e^{-5\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(-e^{\frac{5a}{b}} + 3e^{\frac{5a}{b}+2\text{arcsinh}(c+dx)} - 2e^{\frac{5a}{b}+4\text{arcsinh}(c+dx)} - 2e^{\frac{5}{b}}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-E^((5*a)/b) + 3*E^((5*a)/b + 2*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 4*
ArcSinh[c + d*x]) - 2*E^((5*a)/b + 6*ArcSinh[c + d*x]) + 3*E^((5*a)/b + 8*A
rcSinh[c + d*x]) - E^((5*a)/b + 10*ArcSinh[c + d*x]) + 2*E^((6*a)/b + 5*Arc
Sinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]] + Sqrt[5]*E^(5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Ga
mma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[3]*E^((2*a)/b + 5*ArcSin
h[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSi
nh[c + d*x]))/b] + 2*E^((4*a)/b + 5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - 3*Sqrt[3]*E^((8*
a)/b + 5*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a +
b*ArcSinh[c + d*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c + d*x]))*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(16*b*d
*E^(5*(a/b + ArcSinh[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = e^4\left(\int \frac{c^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right.$$
$$+\int \frac{d^4x^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$+\int \frac{4cd^3x^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$+\int \frac{6c^2d^2x^2}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$\left.+\int \frac{4c^3dx}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt
(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.211 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1634
Rubi [A] (verified) . . . 1634
Mathematica [A] (verified) . . . 1638
Maple [F] . . . 1638
Fricas [F(-2)] . . . 1638
Sympy [F] . . . 1639
Maxima [F] . . . 1639
Giac [F] . . . 1639
Mupad [F(-1)] . . . 1640

## Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}$$

```
[Out] -1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/d-1/4*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{5859, 12, 5778, 3388, 2211, 2236, 2235\}$

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\sqrt{\pi}e^3 e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{\sqrt{\frac{\pi}{2}}e^3 e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} + \frac{\sqrt{\pi}e^3 e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{\sqrt{\frac{\pi}{2}}e^3 e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} - \frac{2e^3(c+dx)^3\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
])/(4*b^(3/2)*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(2e^3)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad + \frac{e^3\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&\quad + \frac{e^3\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^2d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^3\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad - \frac{e^3\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad + \frac{e^3\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^2d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} \\
&\quad - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} \\
&\quad - \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.97

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^3 e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{ar}}{b}}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] - E^((4*a)/b)*(-(Sqrt[2]*E^((2*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c + d
*x]))/b] - 2*Sinh[2*ArcSinh[c + d*x]] + Sinh[4*ArcSinh[c + d*x]])))/(4*b*d*
E^((4*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = e^3\left(\int \frac{c^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right.$$
$$+\int \frac{d^3x^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$+\int \frac{3cd^2x^2}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$\left.+\int \frac{3c^2dx}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(3*c*d**2*x*
*2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\operatorname{asinh}(c+d\,x))^{3/2}}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.212 $\int \frac{(ce+dex)^2}{(a+b\mathbf{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1641
Rubi [A] (verified) . . . 1641
Mathematica [A] (verified) . . . 1644
Maple [F] . . . 1645
Fricas [F(-2)] . . . 1645
Sympy [F] . . . 1645
Maxima [F] . . . 1646
Giac [F] . . . 1646
Mupad [F(-1)] . . . 1646

## Optimal result

Integrand size = 25, antiderivative size = 255

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}$$

```
[Out] 1/4*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
-1/4*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)-1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/
2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{5859, 12, 5778, 3389, 2211, 2236, 2235\}$

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{2e^2(c+dx)^2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*b^
(3/2)*d) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(4*b^(3/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2 x^2}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(2e^2)\,\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2 d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{e^2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^2 d} \\
&\quad - \frac{(3e^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{2b^2 d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad - \frac{e^2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^2d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^2d} \\
&\quad - \frac{e^2\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^2d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^2d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^2d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} \\
&\quad - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} \\
&\quad + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.28

$$
\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^2e^{-3\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}\left(-e^{\frac{3a}{b}} + e^{\frac{3a}{b}+2\operatorname{arcsinh}(c+dx)} + e^{\frac{3a}{b}+4\operatorname{arcsinh}(c+dx)} - e^{\frac{3a}{b}+6\text{ar}}\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c + d*x]) + E^((3*a)/b + 4*ArcS
inh[c + d*x]) - E^((3*a)/b + 6*ArcSinh[c + d*x]) - E^((4*a)/b + 3*ArcSinh[c
 + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] +
Sqrt[3]*E^(3*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/
2, (-3*(a + b*ArcSinh[c + d*x]))/b] - E^((2*a)/b + 3*ArcSinh[c + d*x])*Sqrt
[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(4*b*d*E^(3*(a/b + ArcSinh[c + d*x
]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = e^2\left(\int \frac{c^2}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx + \int \frac{d^2x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx + \int \frac{2cdx}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)),
x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\operatorname{asinh}(c+d\,x))^{3/2}}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.213 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1647
Rubi [A] (verified) . . . 1647
Mathematica [A] (verified) . . . 1649
Maple [F] . . . 1650
Fricas [F(-2)] . . . 1650
Sympy [F] . . . 1650
Maxima [F] . . . 1651
Giac [F] . . . 1651
Mupad [F(-1)] . . . 1651

## Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] 1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b
*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5778, 3388, 2211, 2236, 2235}

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{e\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad + \frac{(2e)\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} \\
&\quad + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{ee^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2}, -\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right) - \sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+}\right.}{2bd\sqrt{a+b\text{ar}}}$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSi
nh[c + d*x]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1
/2, (2*(a + b*ArcSinh[c + d*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c + d*x]
]))/(2*b*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

## Maple [F]

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}} dx$$

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{3/2}} dx = e\left(\int \frac{c}{a\sqrt{a + b\operatorname{asinh}(c + dx)} + b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx)} dx + \int \frac{dx}{a\sqrt{a + b\operatorname{asinh}(c + dx)} + b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx)} dx\right)$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a
 + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\,\text{asinh}\,(c+d\,x))^{3/2}}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.214 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1652
Rubi [A] (verified) . . . 1652
Mathematica [A] (verified) . . . 1654
Maple [F] . . . 1655
Fricas [F(-2)] . . . 1655
Sympy [F] . . . 1655
Maxima [F] . . . 1656
Giac [F] . . . 1656
Mupad [F(-1)] . . . 1656

## Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3389, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&\quad + \frac{2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} \\
&\quad + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(c+dx)}\right) + e^{\frac{2a}{b}+\text{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\text{arcsinh}(}\right.}{bd}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{3/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

# 3.215 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))^{3/2}}\,dx$

Optimal result . . . 1657
Rubi [N/A] . . . 1657
Mathematica [N/A] . . . 1658
Maple [N/A] (verified) . . . 1658
Fricas [F(-2)] . . . 1658
Sympy [N/A] . . . 1658
Maxima [N/A] . . . 1659
Giac [N/A] . . . 1659
Mupad [N/A] . . . 1659

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(3/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(3/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^{3/2}}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{ac\sqrt{a+b\,\text{asinh}\,(c+dx)}+adx\sqrt{a+b\,\text{asinh}\,(c+dx)}+bc\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+}}{e}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral(1/(a*c*sqrt(a + b*asinh(c + d*x)) + a*d*x*sqrt(a + b*asinh(c + d*x
)) + b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b*d*x*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)), x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}\,(c+d\,x)\right)^{3/2}}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)), x)
```

# 3.216 $\int \frac{(ce+dex)^4}{(a+b\mathbf{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 1660
Rubi [A] (verified) . . . 1661
Mathematica [A] (verified) . . . 1667
Maple [F] . . . 1668
Fricas [F(-2)] . . . 1668
Sympy [F] . . . 1668
Maxima [F] . . . 1669
Giac [F] . . . 1669
Mupad [F(-1)] . . . 1669

## Optimal result

Integrand size = 25, antiderivative size = 437

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} - \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} + \frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} - \frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} + \frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}$$

```
[Out] 1/12*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d+1/12*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(
a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(5/2)/d-3/8*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)+5/24*e^4*exp(5*a/b)*erf(5^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*e^4*er
fi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d/e
xp(5*a/b)-2/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3
```

```
/2)-16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-20/3*e^4*(d*x+c)^5/
b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3388, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{\sqrt{\pi}e^4e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} - \frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} + \frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d} + \frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} - \frac{3\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} + \frac{5\sqrt{5\pi}e^4e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (20*
e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^4*E^(a/b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d) - (3*e^4*E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8
*b^(5/2)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) - (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^((3*a)/b)) + (5*
e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(24*b^
(5/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{(8e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&\quad + \frac{(10e^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2 d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2 d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(16e^4)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{b^2 d} \\
&\quad + \frac{(100e^4)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2 d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(16e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(100e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{(16e^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+\right.}{b^3d} \\
&\quad + \frac{(100e^4)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} - \frac{3\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} + \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12b^3d} \\
&\quad + \frac{(4e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(4e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^3d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{24b^3d} \\
&\quad - \frac{(2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(2e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(2e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12b^3d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{8b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{12b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{12b^3d} \\
&\quad + \frac{(4e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad - \frac{(4e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad - \frac{(4e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad + \frac{(4e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6b^3d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6b^3d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4b^3d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{4b^3d}
\end{aligned}
$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}}$$
$$- \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}$$
$$- \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} + \frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}$$
$$+ \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} - \frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}$$
$$+ \frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}$$

## Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.26

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^4\left(-2e^{\text{arcsinh}(c+dx)}(2a+b+2b\text{arcsinh}(c+dx)) + e^{-\text{arcsinh}(c+dx)}(4a-2b-\right.$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-2*E^ArcSinh[c + d*x]*(2*a + b + 2*b*ArcSinh[c + d*x]) + (4*a - 2*b +
 4*b*ArcSinh[c + d*x] - 4*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c +
 d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSi
nh[c + d*x] + (-(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcSinh[c
 + d*x])) - 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
5*(a + b*ArcSinh[c + d*x]))/b])/E^((5*a)/b) + (3*E^(3*(a/b + ArcSinh[c + d*
x]))*(6*a + b + 6*b*ArcSinh[c + d*x]) + 18*Sqrt[3]*b*(-((a + b*ArcSinh[c +
 d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b])/E^((3*a)/b) -
 (4*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c +
 d*x])/b)])/E^(a/b) + (3*(-6*a + b - 6*b*ArcSinh[c + d*x] + 6*Sqrt[3]*E^(3*(
a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*
x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(3*ArcSinh[c + d*x]) + (
10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x])
)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a +
b*ArcSinh[c + d*x]))/b])/E^(5*ArcSinh[c + d*x])))/(48*b^2*d*(a + b*ArcSinh[
c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = e^4\left(\int \frac{c^4}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)}\right.$$
$$+\int \frac{d^4x^4}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+$$
$$+\int \frac{4cd^3x^3}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+$$
$$+\int \frac{6c^2d^2x^2}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+$$
$$+\int \frac{4c^3dx}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
```

```
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b
**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{5/2}}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.217 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 1670
Rubi [A] (verified) . . . 1671
Mathematica [A] (verified) . . . 1676
Maple [F] . . . 1676
Fricas [F(-2)] . . . 1677
Sympy [F] . . . 1677
Maxima [F] . . . 1678
Giac [F] . . . 1678
Mupad [F(-1)] . . . 1678

## Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

```
[Out] -2/3*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5
/2)/d+2/3*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
/exp(4*a/b)+1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d-1/3*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(1+(d*
x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4*e^3*(d*x+c)^2/b^2/d/(a+b*arc
sinh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3389, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{\pi}e^3 e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{\sqrt{2\pi}e^3 e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2\sqrt{\pi}e^3 e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{\sqrt{2\pi}e^3 e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{16e^3(c+dx)^4}{3b^2 d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e^3(c+dx)^2}{b^2 d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (16*e^3
*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e^3*E^((4*a)/b)*S
qrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (e^3
*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(3*b^(5/2)*d*E^((4*a)/b)) - (e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{(2e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{bd} \\
&\quad + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{b^2d} \\
&\quad + \frac{(64e^3)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(64e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(64e^3)\operatorname{Subst}\left(\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{8\sqrt{x}} - \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad - \frac{(4e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(16e^3)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(4e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{(4e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad - \frac{(2e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(2e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad + \frac{(8e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad - \frac{(8e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e^3)\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{(8e^3)\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad - \frac{(4e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad + \frac{(4e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad + \frac{(16e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad - \frac{(16e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d}
\end{aligned}
$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$
$$- \frac{2e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$
$$+ \frac{2e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

## Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.20

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^3e^{-4\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}\left(-8be^{4\operatorname{arcsinh}(c+dx)}\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4(a+b}\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-8*b*E^(4*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gam
ma[1/2, (-4*(a + b*ArcSinh[c + d*x]))/b] + 4*Sqrt[2]*b*E^((2*a)/b + 4*ArcSi
nh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*Ar
cSinh[c + d*x]))/b] + (E^((4*a)/b)*(-((-1 + E^(2*ArcSinh[c + d*x]))^2*(b*(-
1 + E^(4*ArcSinh[c + d*x])) + 8*a*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSin
h[c + d*x])) + 8*b*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x]))*Ar
cSinh[c + d*x])) - 8*Sqrt[2]*E^((2*a)/b + 4*ArcSinh[c + d*x])*Sqrt[a/b + Ar
cSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d
*x]))/b] + 16*E^(4*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(
a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]))/2))/(1
2*b^2*d*E^(4*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = e^3\left(\int \frac{c^3}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx}\right.$$
$$+\int \frac{d^3x^3}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$
$$+\int \frac{3cd^2x^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$
$$+\int \frac{3c^2dx}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.218 $\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 1679
Rubi [A] (verified) . . . 1680
Mathematica [A] (verified) . . . 1685
Maple [F] . . . 1685
Fricas [F(-2)] . . . 1686
Sympy [F] . . . 1686
Maxima [F] . . . 1686
Giac [F] . . . 1687
Mupad [F(-1)] . . . 1687

## Optimal result

Integrand size = 25, antiderivative size = 321

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}$$

```
[Out] -1/6*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d-1/6*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a
/b)+1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^
(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-8/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x
+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3388, 2211, 2236, 2235, 5774}

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} + \frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d} - \frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d} + \frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*e^2*
(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) + (e^2*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*
d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d
*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
```

```
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2 x^2}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{(4e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&\quad + \frac{(2e^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(8e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d} \\
&\quad + \frac{(12e^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{b^2d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(8e^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{(12e^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(4e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{(4e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{(12e^2)\operatorname{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(8e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{(8e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{b^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} \\
&\quad + \frac{4e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^3d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} \\
&\quad + \frac{4e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{b^3d}
\end{aligned}
$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}}$$
$$- \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}$$
$$+ \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}$$
$$+ \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}$$

## Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.21

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^2e^{-3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(2e^{\frac{4a}{b}+3\text{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}(a+b\text{arcsin}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] + 2*b*E^((2*a)/b + 3*ArcSinh[c + d*x])*(-((a + b*Ar
cSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - E^((3
*a)/b)*((-1 + E^(2*ArcSinh[c + d*x]))*(b*(-1 + E^(4*ArcSinh[c + d*x])) + a*
(6 + 4*E^(2*ArcSinh[c + d*x]) + 6*E^(4*ArcSinh[c + d*x])) + 2*b*(3 + 2*E^(2
*ArcSinh[c + d*x]) + 3*E^(4*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSin
h[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/(12*b^2*d*E^(3*(a
/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = e^2\left(\int \frac{c^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\right.$$
$$+\int \frac{d^2x^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+$$
$$+\int \frac{2cdx}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*asinh(c + d*x)) +
2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)**2), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{asinh}\,(c+d\,x))^{5/2}}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.219 $\int \frac{ce+dex}{(a+b\mathbf{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 1688
Rubi [A] (verified) . . . 1688
Mathematica [A] (verified) . . . 1692
Maple [F] . . . 1693
Fricas [F(-2)] . . . 1693
Sympy [F] . . . 1693
Maxima [F] . . . 1694
Giac [F] . . . 1694
Mupad [F(-1)] . . . 1694

## Optimal result

Integrand size = 23, antiderivative size = 209

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{2ee^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2ee^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

```
[Out] -2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(
a+b*arcsinh(d*x+c))^(3/2)-4/3*e/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-8/3*e*(d*x
+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {5859, 12, 5779, 5818, 5780, 5556, 3389, 2211, 2236, 2235, 5783}

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2\pi}ee^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2\sqrt{2\pi}ee^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2
)) - (4*e)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (8*e*(c + d*x)^2)/(3*b^
2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e*Sqrt[2*Pi]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/
b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&\quad + \frac{(4e)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{(16e)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{(16e)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{(16e)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(4e)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{(4e)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{(8e)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{2ee^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2ee^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$
\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{ee^{-2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}\left(-4\sqrt{2}be^{2\operatorname{arcsinh}(c+dx)}\left(-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2}, -\frac{2(a\right.\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]

[Out] (e*(-4*Sqrt[2]*b*E^(2*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/
2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*
a*E^(4*ArcSinh[c + d*x]) - b*E^(4*ArcSinh[c + d*x]) - 4*b*(1 + E^(4*ArcSinh
[c + d*x]))*ArcSinh[c + d*x] + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*Arc
Sinh[c + d*x]))/b])))/(6*b^2*d*E^(2*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSin
h[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}} dx$$

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{5/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{5/2}} dx = e\left(\int \frac{c}{a^2\sqrt{a + b\operatorname{asinh}(c + dx)} + 2ab\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx)}\right.$$

$$\left. + \int \frac{dx}{a^2\sqrt{a + b\operatorname{asinh}(c + dx)} + 2ab\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx) + b^2\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^2(c}\right.$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.220 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . 1695
Rubi [A] (verified) . . . 1695
Mathematica [A] (verified) . . . 1698
Maple [F] . . . 1699
Fricas [F(-2)] . . . 1699
Sympy [F] . . . 1699
Maxima [F] . . . 1699
Giac [F] . . . 1700
Mupad [F(-1)] . . . 1700

## Optimal result

Integrand size = 14, antiderivative size = 158

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{5858, 5773, 5818, 5774, 3388, 2211, 2236, 2235\}$

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2\sqrt{(c+dx)^2+1}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
```

```
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{3b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{3b^3d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^3d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{4\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&\quad + \frac{4\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^3d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

$$
\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\operatorname{arcsinh}(c+dx)}\right)-2b\operatorname{arcsinh}(c+dx)+\right.\right.
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]

[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\operatorname{asinh}\,(c+d\,x))^{5/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

# 3.221 $\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

Optimal result . . . . . 1701
Rubi [N/A] . . . . . 1701
Mathematica [N/A] . . . . . 1702
Maple [N/A] (verified) . . . . . 1702
Fricas [F(-2)] . . . . . 1702
Sympy [N/A] . . . . . 1702
Maxima [N/A] . . . . . 1703
Giac [N/A] . . . . . 1703
Mupad [N/A] . . . . . 1703

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(5/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(5/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^{5/2}}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.20

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{\int \frac{}{a^2c\sqrt{a+b\,\text{asinh}\,(c+dx)}+a^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}+2abc\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*asinh(c + d*x)) + a**2*d*x*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*c*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)**2 + b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2)
, x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\operatorname{asinh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)), x)
```

# 3.222 $\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 1704
Rubi [A] (verified) . . . 1705
Mathematica [A] (verified) . . . 1711
Maple [F] . . . 1712
Fricas [F(-2)] . . . 1712
Sympy [F] . . . 1712
Maxima [F] . . . 1713
Giac [F] . . . 1713
Mupad [F(-1)] . . . 1714

## Optimal result

Integrand size = 25, antiderivative size = 531

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx &= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} \\ &- \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\ &- \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\ &- \frac{e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} + \frac{9e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\ &- \frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} \\ &- \frac{9e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\ &+ \frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}\end{aligned}$$

```
[Out] -16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)-1/30*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d-9/20*e^4*erfi(3^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)
```

```
-5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(
1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/5*e^4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/
b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b^3
/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5779, 5818, 5778, 3389, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{\sqrt{\pi}e^4 e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} + \frac{9\sqrt{3\pi}e^4 e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} - \frac{5\sqrt{5\pi}e^4 e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d} + \frac{\sqrt{\pi}e^4 e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} - \frac{9\sqrt{3\pi}e^4 e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} + \frac{5\sqrt{5\pi}e^4 e^{-\frac{5a}{b}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d} - \frac{40e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{32e^4\sqrt{(c+dx)^2+1}(c+dx)^2}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e^4(c+dx)^5}{3b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (
4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e^4*(c +
d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (40*
e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b
^(7/2)*d) + (9*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(20*b^(7/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) - (9*e^
4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(20*b^(7
```

```
/2)*d*E^((3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(12*b^(7/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
```

```
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^4 x^4}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4 \text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad + \frac{(8e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&\quad + \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{bd} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{(16e^4)\,\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{5b^2d} \\
&\quad + \frac{(20e^4)\,\text{Subst}\left(\int \frac{x^4}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{3b^2d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(32e^4)\operatorname{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(40e^4)\operatorname{Subst}\left(\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{16\sqrt{x}} + \frac{9\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{16\sqrt{x}} - \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{8\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(8e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(5e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{3b^4d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{5a}{b}-\frac{5x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d} \\
&\quad - \frac{(24e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(15e^4)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{2b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(4e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(4e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(5e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d} \\
&\quad + \frac{(5e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{6b^4d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12b^4d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{5ia}{b}-\frac{5ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{12b^4d} \\
&\quad - \frac{(12e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(12e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(15e^4)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^4d} \\
&\quad - \frac{(15e^4)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{4b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(8e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad - \frac{(8e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad - \frac{(5e^4)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^4d} \\
&\quad + \frac{(5e^4)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{3b^4d} \\
&\quad - \frac{(25e^4)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6b^4d} \\
&\quad + \frac{(25e^4)\operatorname{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{6b^4d} \\
&\quad - \frac{(24e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad + \frac{(24e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad + \frac{(15e^4)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^4d} \\
&\quad - \frac{(15e^4)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{2b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^4(c+dx)^5}{3b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} \\
&\quad + \frac{9e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} - \frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d} \\
&\quad + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} - \frac{9e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\
&\quad + \frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.32

$$
\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^4\left(-6b^2e^{\operatorname{arcsinh}(c+dx)} - 3b^2e^{5\operatorname{arcsinh}(c+dx)} + e^{-\operatorname{arcsinh}(c+dx)}\left(-8a^2+4ab-6b\right.\right.}{}
$$

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-6*b^2*E^ArcSinh[c + d*x] - 3*b^2*E^(5*ArcSinh[c + d*x]) + (-8*a^2 +
4*a*b - 6*b^2 - 4*(4*a - b)*b*ArcSinh[c + d*x] - 8*b^2*ArcSinh[c + d*x]^2 +
 8*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c
 + d*x])^2*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSinh[c + d*x] - (10*(a
+ b*ArcSinh[c + d*x])*(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcS
inh[c + d*x]) + 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-5*(a + b*ArcSinh[c + d*x]))/b]))/E^((5*a)/b) + 9*(b^2*E^(3*ArcSinh[c +
 d*x]) + (2*(a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a +
 b + 6*b*ArcSinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3
/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) - (4*(a + b*
ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*
x]) + 2*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)]))/E^(a/b) + (9*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b +
6*b*ArcSinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b +
ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c +
 d*x]))/b]))/E^(3*ArcSinh[c + d*x]) - (3*b^2 + 10*(a + b*ArcSinh[c + d*x])
```

```
*(10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x
]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a
+ b*ArcSinh[c + d*x]))/b]))/E^(5*ArcSinh[c + d*x])))/(240*b^3*d*(a + b*ArcS
inh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = e^4\left(\int \frac{}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\right.$$
$$+\int \frac{d^4x^4}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2}$$
$$+\int \frac{4cd^3x^3}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2}$$
$$+\int \frac{6c^2d^2x^2}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2}$$
$$+\int \frac{4c^3dx}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2}$$

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**4*x**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
4*c*d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(6
*c**2*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral
(4*c**3*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c
+ d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\operatorname{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.223 $\int \frac{(ce+dex)^3}{(a+b\mathbf{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 1715
Rubi [A] (verified) . . . 1716
Mathematica [A] (verified) . . . 1721
Maple [F] . . . 1721
Fricas [F(-2)] . . . 1721
Sympy [F] . . . 1722
Maxima [F] . . . 1722
Giac [F] . . . 1723
Mupad [F(-1)] . . . 1723

## Optimal result

Integrand size = 25, antiderivative size = 420

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^3(c+dx)^4}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{16e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} - \frac{4e^3e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{16e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} - \frac{4e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

```
[Out] -4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)-4/15*e^3*exp(2*a/b)*erf(2^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d-4/15*e^3*e
rfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/
exp(2*a/b)-2/5*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(
5/2)-16/5*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-
128/15*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5779, 5818, 5778, 3388, 2211, 2236, 2235}

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{16\sqrt{\pi}e^3 e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$-\frac{4\sqrt{2\pi}e^3 e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3 e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$-\frac{4\sqrt{2\pi}e^3 e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{128e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}}$$
$$-\frac{16e^3\sqrt{(c+dx)^2+1}(c+dx)}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{16e^3(c+dx)^4}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{4e^3(c+dx)^2}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{2e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16
*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^3*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (128*e
^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (16*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(15*b^(7/2)*d) - (4*e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) - (4*e^3*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)
*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^3 x^3}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad + \frac{(6e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&\quad + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{(8e^3)\,\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{5b^2d} \\
&\quad + \frac{(64e^3)\,\text{Subst}\left(\int \frac{x^3}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(16e^3)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(128e^3)\,\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{15b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(8e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(8e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(64e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}-\frac{4x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad - \frac{(64e^3)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(32e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad - \frac{(32e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(32e^3)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(32e^3)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{4ia}{b}-\frac{4ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(16e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad + \frac{(16e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{4e^3e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} \\
&\quad + \frac{4e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} \\
&\quad + \frac{(64e^3)\operatorname{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad - \frac{(64e^3)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad - \frac{(64e^3)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad + \frac{(64e^3)\operatorname{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^3(c+dx)^4}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{16e^3e^{\frac{4a}{b}}\sqrt{\pi}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&\quad - \frac{4e^3e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{16e^3e^{-\frac{4a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&\quad - \frac{4e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.02

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^3\left(4(a+b\text{arcsinh}(c+dx))\left(-4ae^{-2\text{arcsinh}(c+dx)}+be^{-2\text{arcsinh}(c+dx)}-4be^{-}\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*(4*(a + b*ArcSinh[c + d*x])*((-4*a)/E^(2*ArcSinh[c + d*x]) + b/E^(2*Ar
cSinh[c + d*x]) - (4*b*ArcSinh[c + d*x])/E^(2*ArcSinh[c + d*x]) + E^(2*ArcS
inh[c + d*x])*(4*a + b + 4*b*ArcSinh[c + d*x]) + (4*Sqrt[2]*b*(-((a + b*Arc
Sinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2
*a)/b) + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[
c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) - 4*(a + b*ArcSinh[c
+ d*x])*((-8*a)/E^(4*ArcSinh[c + d*x]) + (b*(1 - 8*ArcSinh[c + d*x]))/E^(4*
ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x])*(8*a + b + 8*b*ArcSinh[c + d*x])
 + (16*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcSin
h[c + d*x]))/b])/E^((4*a)/b) + 16*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*
(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]) + 6*b^
2*Sinh[2*ArcSinh[c + d*x]] - 3*b^2*Sinh[4*ArcSinh[c + d*x]]))/(60*b^3*d*(a
+ b*ArcSinh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^3}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = e^3\left(\int \frac{}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\right.$$
$$+\int \frac{d^3x^3}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2}$$
$$+\int \frac{3cd^2x^2}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2}$$
$$+\int \frac{3c^2dx}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2}$$

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
3*c*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(3
*c**2*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^{7/2}}\,dx$$

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.224 $\int \frac{(ce+dex)^2}{(a+b\mathbf{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 1724
Rubi [A] (verified) . . . 1725
Mathematica [A] (verified) . . . 1731
Maple [F] . . . 1731
Fricas [F(-2)] . . . 1731
Sympy [F] . . . 1732
Maxima [F] . . . 1732
Giac [F] . . . 1732
Mupad [F(-1)] . . . 1733

## Optimal result

Integrand size = 25, antiderivative size = 410

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} - \frac{3e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{3e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}$$

```
[Out] -8/15*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d/
(a+b*arcsinh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(7/2)/d-1/15*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2)*(a+
b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)-2/5*
e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-16/15*e^2*
(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-24/5*e^2*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5779, 5818, 5778, 3389, 2211, 2236, 2235, 5773, 5819}

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} - \frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} - \frac{24e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}}$$
$$-\frac{16e^2\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e^2(c+dx)^3}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8e^2(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*e
^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^2*Sqrt[1 +
 (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (24*e^2*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^2*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) - (3
*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(5*b^(7/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/(15*b^(7/2)*d*E^(a/b)) + (3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
```

```
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&\quad + \frac{(4e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&\quad + \frac{(6e^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{(8e^2)\,\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d} \\
&\quad + \frac{(12e^2)\,\text{Subst}\left(\int \frac{x^2}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{5b^2d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(24e^2)\operatorname{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(16e^2)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\operatorname{arcsinh}(x)}}\,dx, x, c+dx\right)}{15b^3d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(16e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(6e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(18e^2)\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(8e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(8e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(3e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad - \frac{(9e^2)\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d} \\
&\quad + \frac{(9e^2)\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{5b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{(16e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad + \frac{(16e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad + \frac{(6e^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad - \frac{(6e^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad - \frac{(18e^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&\quad + \frac{(18e^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{5b^4d} \\
&= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{8e^2(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{4e^2(c+dx)^3}{5b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad - \frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&\quad - \frac{3e^2e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\
&\quad + \frac{3e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.16

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^2\left(3b^2e^{\text{arcsinh}(c+dx)}+e^{-\text{arcsinh}(c+dx)}\left(4a^2-2ab+3b^2+2(4a-b)b\text{arcsinh}(\right.\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(3*b^2*E^ArcSinh[c + d*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*Arc
Sinh[c + d*x] + 4*b^2*ArcSinh[c + d*x]^2 - 4*E^(a/b + ArcSinh[c + d*x])*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2, a/b + ArcSi
nh[c + d*x]])/E^ArcSinh[c + d*x] - 3*(b^2*E^(3*ArcSinh[c + d*x]) + (2*(a +
b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a + b + 6*b*ArcSinh[
c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) + (2*(a + b*ArcSinh[c + d*x])
*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a +
 b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/
E^(a/b) - (3*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b + 6*b*ArcSinh[c + d
*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]
*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(
3*ArcSinh[c + d*x])))/(60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = e^2\left(\int \frac{}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\right.$$
$$+\int \frac{d^2x^2}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2}$$
$$+\int \frac{2cdx}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2}$$

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
2*c*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
+ b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.225 $\int \frac{ce+dex}{(a+b\mathbf{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 1734
Rubi [A] (verified) . . . 1735
Mathematica [A] (verified) . . . 1738
Maple [F] . . . 1739
Fricas [F(-2)] . . . 1739
Sympy [F] . . . 1739
Maxima [F] . . . 1740
Giac [F] . . . 1740
Mupad [F(-1)] . . . 1740

## Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8e(c+dx)^2}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} + \frac{8ee^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{8ee^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

```
[Out] -4/15*e/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arcsin
h(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(1+(d*x+c)^2)^(1
/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5779, 5818, 5778, 3388, 2211, 2236, 2235, 5783}

$$\begin{aligned}\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx &= \frac{8\sqrt{2\pi}ee^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\ &+ \frac{8\sqrt{2\pi}ee^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\ &- \frac{32e\sqrt{(c+dx)^2+1}(c+dx)}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8e(c+dx)^2}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\ &- \frac{4e}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2
)) - (4*e)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*e*(c + d*x)^2)/(1
5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e*(c + d*x)*Sqrt[1 + (c + d*x
)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*e*E^((2*a)/b)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (8*e
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/
2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{ex}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&\quad + \frac{(4e)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e(c+dx)^2}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{(16e)\text{Subst}\left(\int \frac{x}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e(c+dx)^2}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(32e)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{15b^4d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e(c+dx)^2}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&\quad + \frac{(16e)\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{15b^4d} \\
&\quad + \frac{(16e)\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(c+dx)\right)}{15b^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e(c+dx)^2}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{(32e)\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&\quad + \frac{(32e)\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8e(c+dx)^2}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} \\
&\quad + \frac{8ee^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{8ee^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

$$
\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx =
$$

$$
-\frac{e\left((a+b\operatorname{arcsinh}(c+dx))\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}(4a+b+4b\operatorname{arcsinh}(c+dx))+8\sqrt{2}b\left(-\frac{a+b\operatorname{arcsinh}(c+d\right.\right.\right.\right.}{b}
$$

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]

[Out] -1/15*(e*((a + b*ArcSinh[c + d*x])*((2*E^(2*(a/b + ArcSinh[c + d*x]))*(4*a
+ b + 4*b*ArcSinh[c + d*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c + d*x])/b))^(
3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2*a)/b) + (-8*a + 2*b
 - 8*b*ArcSinh[c + d*x] + 8*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b
 + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[
c + d*x]))/b])/E^(2*ArcSinh[c + d*x])) + 3*b^2*Sinh[2*ArcSinh[c + d*x]]))/(
b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}} dx$$

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{7/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{ce + dex}{(a + b\operatorname{arcsinh}(c + dx))^{7/2}} dx = e\left(\int \frac{}{a^3\sqrt{a + b\operatorname{asinh}(c + dx)} + 3a^2b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx}\right.$$

$$\left. + \int \frac{dx}{a^3\sqrt{a + b\operatorname{asinh}(c + dx)} + 3a^2b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx) + 3ab^2\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}\right.$$

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e*(Integral(c/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(d
*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*x))*a
sinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2 + b**
3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

## Maxima [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.226 $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . . . 1741
Rubi [A] (verified) . . . . . 1741
Mathematica [A] (verified) . . . . . 1745
Maple [F] . . . . . 1745
Fricas [F(-2)] . . . . . 1745
Sympy [F] . . . . . 1746
Maxima [F] . . . . . 1746
Giac [F] . . . . . 1746
Mupad [F(-1)] . . . . . 1746

## Optimal result

Integrand size = 14, antiderivative size = 195

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}} - \frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{5858, 5773, 5818, 5819, 3389, 2211, 2236, 2235\}$

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{4\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} - \frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{2\sqrt{(c+dx)^2+1}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
```

```
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}(a+b\text{arcsinh}(x))^{5/2}}\,dx, x, c+dx\right)}{5bd} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{1}{(a+b\text{arcsinh}(x))^{3/2}}\,dx, x, c+dx\right)}{15b^2d} \\
&= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&\quad - \frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{8\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\text{arcsinh}(x)}}\,dx, x, c+dx\right)}{15b^3d}
\end{aligned}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8\operatorname{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$
$$-\frac{4\operatorname{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$
$$+\frac{4\operatorname{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(c+dx)\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{8\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d}$$
$$+\frac{8\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(c+dx)}\right)}{15b^4d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\operatorname{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\operatorname{arcsinh}(c+dx))^{3/2}}$$
$$-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$
$$+\frac{4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{-6b^2e^{\text{arcsinh}(c+dx)} - 2e^{-\text{arcsinh}(c+dx)}(4a^2+2ab(-1+4\text{arcsinh}(c+dx))+b$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

## Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

## Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

## Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{7/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

# 3.227 $\int \frac{1}{(ce+dex)(a+b\mathbf{arcsinh}(c+dx))^{7/2}}\,dx$

Optimal result . . . 1747
Rubi [N/A] . . . 1747
Mathematica [N/A] . . . 1748
Maple [N/A] (verified) . . . 1748
Fricas [F(-2)] . . . 1748
Sympy [N/A] . . . 1748
Maxima [N/A] . . . 1749
Giac [N/A] . . . 1749
Mupad [N/A] . . . 1749

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(7/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$$

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(7/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{ex(a+b\operatorname{arcsinh}(x))^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+b\operatorname{arcsinh}(x))^{7/2}}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 105.85 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.84

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{\int \frac{}{a^3c\sqrt{a+b\,\text{asinh}\,(c+dx)}+a^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2bc\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c}$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral(1/(a**3*c*sqrt(a + b*asinh(c + d*x)) + a**3*d*x*sqrt(a + b*asinh(c
 + d*x)) + 3*a**2*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a**2*b*
d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*c*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2 + 3*a*b**2*d*x*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3 + b**3*
d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x)/e
```

## Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

## Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\operatorname{asinh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)), x)
```

# 3.228 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 1750
Rubi [A] (verified) . . . 1751
Mathematica [C] (verified) . . . 1754
Maple [C] (verified) . . . 1754
Fricas [C] (verification not implemented) . . . 1755
Sympy [F(-1)] . . . 1755
Maxima [F(-2)] . . . 1756
Giac [F] . . . 1756
Mupad [F(-1)] . . . 1756

## Optimal result

Integrand size = 23, antiderivative size = 298

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} - \frac{28be^3\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{135d(1+c+dx)} + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} + \frac{28be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}} - \frac{14be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))/d/e+28/405*b*e^2*(e*(d*x+c))^(3/
2)*(1+(d*x+c)^2)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(1+(d*x+c)^2)^(1/2)/d-28/
135*b*e^3*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c+1)+28/135*b*e^(7/2
)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan
((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/
2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c)^2)^(1/2)-1
4/135*b*e^(7/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2
)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c
))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x
+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 327, 335, 311, 226, 1210}

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de}$$
$$-\frac{14be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}$$
$$+\frac{28be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}$$
$$-\frac{28be^3\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{135d(c+dx+1)}$$
$$+\frac{28be^2\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}}{405d}-\frac{4b\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}}{81d}$$

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (28*b*e^2*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(405*d) - (4*b*(e*(c +
 d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/(81*d) - (28*b*e^3*Sqrt[e*(c + d*x)]*Sq
rt[1 + (c + d*x)^2])/(135*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(9/2)*(a + b*
ArcSinh[c + d*x]))/(9*d*e) + (28*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x
)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/
(135*d*Sqrt[1 + (c + d*x)^2]) - (14*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c +
d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2
])/(135*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (ex)^{7/2}(a+b\operatorname{arcsinh}(x))\,dx, x, c+dx\right)}{d} \\
&= \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9de} \\
&= -\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} \\
&\quad + \frac{(14be)\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{81d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} \\
&\quad + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} - \frac{(14be^3)\operatorname{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{135d} \\
&= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} \\
&\quad + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} \\
&\quad - \frac{(28be^2)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{135d} \\
&= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} \\
&\quad + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} \\
&\quad - \frac{(28be^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{135d} \\
&\quad + \frac{(28be^3)\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{e}}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{135d} \\
&= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} \\
&\quad - \frac{28be^3\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{135d(1+c+dx)} + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))}{9de} \\
&\quad + \frac{28be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}} \\
&\quad - \frac{14be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.38

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{7/2}\left(45a(c+dx)^3+14b\sqrt{1+(c+dx)^2}-10b(c+dx)^2\sqrt{1+(c+dx)^2}+\right.}{405d(c+$$

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(45*a*(c + d*x)^3 + 14*b*Sqrt[1 + (c + d*x)^2] - 10*
b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 45*b*(c + d*x)^3*ArcSinh[c + d*x] - 1
4*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(405*d*(c + d*x)^2)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| derivativedivides | $\frac{2a(dex+ce)^{\frac{9}{2}}}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1-\frac{i(}{}}\right.}{de}$ |
| default | $\frac{2a(dex+ce)^{\frac{9}{2}}}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1-\frac{i(}{}}\right.}{de}$ |
| parts | $\frac{2a(dex+ce)^{\frac{9}{2}}}{9de}+\frac{2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1}}{}\right.}{de}$ |

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/9*a*(d*e*x+c*e)^(9/2)+b*(1/9*(d*e*x+c*e)^(9/2)*arcsinh(1/e*(d*e*x+
c*e))-2/9/e*(1/9*e^2*(d*e*x+c*e)^(7/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-7/45*e
^4*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+7/15*I*e^5/(I/e)^(1/2)*(
1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2+1)^
(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/
2)*(I/e)^(1/2),I)))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.03

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2\left(42\sqrt{d^3e}be^3\operatorname{weierstrassZeta}\left(-\frac{4}{d^2},0,\operatorname{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+45\,(bd^5$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/405*(42*sqrt(d^3*e)*b*e^3*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(
-4/d^2, 0, (d*x + c)/d)) + 45*(b*d^5*e^3*x^4 + 4*b*c*d^4*e^3*x^3 + 6*b*c^2*
d^3*e^3*x^2 + 4*b*c^3*d^2*e^3*x + b*c^4*d*e^3)*sqrt(d*e*x + c*e)*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(5*b*d^4*e^3*x^3 + 15*b*c*d^3*e^
3*x^2 + (15*b*c^2 - 7*b)*d^2*e^3*x + (5*b*c^3 - 7*b*c)*d*e^3)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) + 45*(a*d^5*e^3*x^4 + 4*a*c*d^4*e^3*
x^3 + 6*a*c^2*d^3*e^3*x^2 + 4*a*c^3*d^2*e^3*x + a*c^4*d*e^3)*sqrt(d*e*x + c
*e))/d^2
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \int (dex+ce)^{\frac{7}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)), x)
```

# 3.229 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 1757
Rubi [A] (verified) . . . 1757
Mathematica [C] (verified) . . . 1759
Maple [C] (verified) . . . 1760
Fricas [C] (verification not implemented) . . . 1760
Sympy [F] . . . 1761
Maxima [F(-2)] . . . 1761
Giac [F] . . . 1761
Mupad [F(-1)] . . . 1762

## Optimal result

Integrand size = 23, antiderivative size = 177

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} - \frac{10be^{5/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{147d\sqrt{1+(c+dx)^2}}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))/d/e-4/49*b*(e*(d*x+c))^(5/2)*(1+
(d*x+c)^2)^(1/2)/d+20/147*b*e^2*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d-10/
147*b*e^(5/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c
)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 327, 335, 226}

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} - \frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{147d\sqrt{(c+dx)^2+1}} + \frac{20be^2\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{147d} - \frac{4b\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}}{49d}$$

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (20*b*e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(147*d) - (4*b*(e*(c + d
*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/(49*d) + (2*(e*(c + d*x))^(7/2)*(a + b*Ar
cSinh[c + d*x]))/(7*d*e) - (10*b*e^(5/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x)^
2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(1
47*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{5/2}(a + b\text{arcsinh}(x))\,dx, x, c + dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} - \frac{(2b)\operatorname{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{7de} \\
&= -\frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} \\
&\quad + \frac{(10be)\operatorname{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{49d} \\
&= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} \\
&\quad + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} \\
&\quad - \frac{(10be^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{147d} \\
&= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} \\
&\quad + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} \\
&\quad - \frac{(20be^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{147d} \\
&= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} \\
&\quad + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))}{7de} \\
&\quad - \frac{10be^{5/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{147d\sqrt{1+(c+dx)^2}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.64

$$
\int (ce+dex)^{5/2}(a + b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{5/2}\left(21a(c+dx)^3 + 10b\sqrt{1+(c+dx)^2} - 6b(c+dx)^2\sqrt{1+(c+dx)^2} -
$$

$$
147d(c+
$$

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(21*a*(c + d*x)^3 + 10*b*Sqrt[1 + (c + d*x)^2] - 6*b
*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 21*b*(c + d*x)^3*ArcSinh[c + d*x] - 10
*b*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2]))/(147*d*(c + d*x)^2)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.20

| method | result |
| --- | --- |
| derivativedivides | $\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{2+\frac{i(dex+ce)}{e}}\sqrt{\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{21\sqrt{\frac{i}{e}}}\right)}{7e}\right)}{de}$ |
| default | $\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{2+\frac{i(dex+ce)}{e}}\sqrt{\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{21\sqrt{\frac{i}{e}}}\right)}{7e}\right)}{de}$ |
| parts | $\frac{2a(dex+ce)^{\frac{7}{2}}}{7de}+\frac{2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{2+\frac{i(dex+ce)}{e}}\sqrt{\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{21\sqrt{\frac{i}{e}}}\right)}{7e}\right)}{de}$ |

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arcsinh(1/e*(d*e*x+
c*e))-2/7/e*(1/7*e^2*(d*e*x+c*e)^(5/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-5/21*e
^4*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+5/21*e^4/(I/e)^(1/2)*(1-
I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2+1)^(1
/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.44

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,dx =$$

$$-\frac{2\left(10\sqrt{d^3}ebe^2\operatorname{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)-21\left(bd^5e^2x^3+3\,bcd^4e^2x^2+3\,bc^2d^3e^2x+bc^3d^2e^2\right)\sqrt{dex+}\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/147*(10*sqrt(d^3*e)*b*e^2*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) -
21*(b*d^5*e^2*x^3 + 3*b*c*d^4*e^2*x^2 + 3*b*c^2*d^3*e^2*x + b*c^3*d^2*e^2)*
sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(3*b
*d^4*e^2*x^2 + 6*b*c*d^3*e^2*x + (3*b*c^2 - 5*b)*d^2*e^2)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) - 21*(a*d^5*e^2*x^3 + 3*a*c*d^4*e^2*x^2
+ 3*a*c^2*d^3*e^2*x + a*c^3*d^2*e^2)*sqrt(d*e*x + c*e))/d^3
```

## Sympy [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (e(c+dx))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(c+dx))\,dx$$

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**(5/2)*(a + b*asinh(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)), x)
```

# 3.230 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 1763
Rubi [A] (verified) . . . 1764
Mathematica [C] (verified) . . . 1766
Maple [C] (verified) . . . 1766
Fricas [C] (verification not implemented) . . . 1767
Sympy [F] . . . 1768
Maxima [F(-2)] . . . 1768
Giac [F] . . . 1768
Mupad [F(-1)] . . . 1768

## Optimal result

Integrand size = 23, antiderivative size = 261

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{12be\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{25d(1+c+dx)} + \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))}{5de} - \frac{12be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}} + \frac{6be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))/d/e-4/25*b*(e*(d*x+c))^(3/2)*(1+
(d*x+c)^2)^(1/2)/d+12/25*b*e*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c
+1)-12/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^
(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d
*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+
(d*x+c)^2)^(1/2)+6/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e
^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*
arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 327, 335, 311, 226, 1210}

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))}{5de}$$
$$+\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}$$
$$-\frac{12be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}$$
$$-\frac{4b\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}}{25d}+\frac{12be\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{25d(c+dx+1)}$$

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(25*d) + (12*b*e*Sqrt[e*(c
 + d*x)]*Sqrt[1 + (c + d*x)^2])/(25*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(5/
2)*(a + b*ArcSinh[c + d*x]))/(5*d*e) - (12*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e
]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2]) + (6*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{3/2}(a+b\text{arcsinh}(x))\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5de} + \frac{(6be)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{25d}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5de} + \frac{(12b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{25d}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))}{5de}$$
$$+ \frac{(12be)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{25d} - \frac{(12be)\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{e}}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{25d}$$
$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{12be\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{25d(1+c+dx)}$$
$$+ \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))}{5de}$$
$$- \frac{12be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}}$$
$$+ \frac{6be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.33

$$\int (ce+dex)^{3/2}(a$$
$$+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{3/2}\left(5ac+5adx-2b\sqrt{1+(c+dx)^2}+5bc\operatorname{arcsinh}(c+dx)+5bdx\operatorname{arcsin}\right.}{25d}$$

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*a*c + 5*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 5*b*c
*ArcSinh[c + d*x] + 5*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/2, 3
/4, 7/4, -(c + d*x)^2]))/(25*d)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellip}\right.\right.}{5e\sqrt{\frac{i}{e}}}\right.$ $de$ |
| default | $\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellip}\right.\right.}{5e\sqrt{\frac{i}{e}}}\right.$ $de$ |
| parts | $\frac{2a(dex+ce)^{\frac{5}{2}}}{5de}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{E}\right.\right.}{5e\sqrt{}}\right.$ $de$ |

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e*x+
c*e))-2/5/e*(1/5*e^2*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-3/5*I*
e^3/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*
(d*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-Elliptic
E((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = -\frac{2\left(6\sqrt{d^3e}be\operatorname{weierstrassZeta}\left(-\frac{4}{d^2},0,\operatorname{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)-5\left(bd^3ex^2+2\,bcd^2ex+bc^2de\right)\sqrt{}\right.}{}$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/25*(6*sqrt(d^3*e)*b*e*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(-4/
d^2, 0, (d*x + c)/d)) - 5*(b*d^3*e*x^2 + 2*b*c*d^2*e*x + b*c^2*d*e)*sqrt(d*
e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(b*d^2*e*x
+ b*c*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) - 5*(a*d^3*e
*x^2 + 2*a*c*d^2*e*x + a*c^2*d*e)*sqrt(d*e*x + c*e))/d^2
```

## Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (e(c+dx))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(c+dx))\,\,dx$$

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)), x)
```

# 3.231 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx$

Optimal result . . . 1769
Rubi [A] (verified) . . . 1770
Mathematica [C] (verified) . . . 1771
Maple [C] (verified) . . . 1772
Fricas [C] (verification not implemented) . . . 1773
Sympy [F] . . . 1773
Maxima [F(-2)] . . . 1773
Giac [F] . . . 1774
Mupad [F(-1)] . . . 1774

## Optimal result

Integrand size = 23, antiderivative size = 142

$$\begin{aligned}&\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx\\&=-\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d}+\frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))}{3de}\\&\quad+\frac{2b\sqrt{e}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{9d\sqrt{1+(c+dx)^2}}\end{aligned}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))/d/e-4/9*b*(e*(d*x+c))^(1/2)*(1+(
d*x+c)^2)^(1/2)/d+2/9*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))
^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*e^(1/2)*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 327, 335, 226}

$$\begin{aligned}
&\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx \\
&= \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))}{3de} \\
&\quad + \frac{2b\sqrt{e}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{9d\sqrt{(c+dx)^2+1}} \\
&\quad - \frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{9d}
\end{aligned}$$

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(9*d) + (2*(e*(c + d*x))^(3/
2)*(a + b*ArcSinh[c + d*x]))/(3*d*e) + (2*b*Sqrt[e]*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(9*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{ex}(a+b\text{arcsinh}(x))\,dx, x, c+dx\right)}{d} \\
&= \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de} \\
&= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3de} \\
&\quad + \frac{(2be)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9d} \\
&= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3de} \\
&\quad + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{9d} \\
&= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3de} \\
&\quad + \frac{2b\sqrt{e}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{9d\sqrt{1+(c+dx)^2}}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\begin{aligned}
&\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx \\
&= \frac{2\sqrt{e(c+dx)}\left(3ac+3adx-2b\sqrt{1+(c+dx)^2}+3bc\text{arcsinh}(c+dx)+3bdx\text{arcsinh}(c+dx)+2b\,\text{Hyperg}\right.}{9d}
\end{aligned}$$

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(3*a*c + 3*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 3*b*c*A
rcSinh[c + d*x] + 3*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/4, 1/2
, 5/4, -(c + d*x)^2]))/(9*d)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| derivativedivides | $\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\ \text{EllipticF}\left(\sqrt{}\right.\right.}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{3e}\right)}{de}$ |
| default | $\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\ \text{EllipticF}\left(\sqrt{}\right.\right.}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{3e}\right)}{de}$ |
| parts | $\frac{2a(dex+ce)^{\frac{3}{2}}}{3de}+\frac{2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\ \text{Elliptic}}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{3e}\right)}{de}$ |

```
[In] int((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e*x+
c*e))-2/3/e*(1/3*e^2*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-1/3*e^
2/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d
*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx = -\frac{2\left(2\sqrt{d^2x^2+2\,cdx+c^2+1}\sqrt{dex+ce}bd^2-3\,(bd^3x+bcd^2)\sqrt{dex+ce}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+\right.\right.}}{9\,d^3}$$

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e)*b*d^2 - 3*(b*d^
3*x + b*c*d^2)*sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)) - 2*sqrt(d^3*e)*b*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) - 3*(a
*d^3*x + a*c*d^2)*sqrt(d*e*x + c*e))/d^3
```

## Sympy [F]

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\operatorname{asinh}\,(c+dx))\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx = \int \sqrt{c\,e+d\,e\,x}\,(a+b\,\text{asinh}(c+d\,x))\,\,dx$$

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)), x)
```

# 3.232 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal result . . . 1775
Rubi [A] (verified) . . . 1776
Mathematica [C] (verified) . . . 1778
Maple [C] (verified) . . . 1778
Fricas [C] (verification not implemented) . . . 1779
Sympy [F] . . . 1779
Maxima [F(-2)] . . . 1779
Giac [F] . . . 1780
Mupad [F(-1)] . . . 1780

## Optimal result

Integrand size = 23, antiderivative size = 223

$$\begin{aligned}\int \frac{a+b\operatorname{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx &= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{de(1+c+dx)} \\ &+ \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))}{de} \\ &+ \frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}} \\ &- \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}}\end{aligned}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*(e*(d*x+c))^(1/2)*(1+(d*x+
c)^2)^(1/2)/d/e/(d*x+c+1)+4*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(
1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*ar
ctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(
1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)-2*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^
(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF
(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+
c+1)^2)^(1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 5776, 335, 311, 226, 1210}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))}{de} - \frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{d\sqrt{e}\sqrt{(c+dx)^2+1}} + \frac{4b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\sqrt{(c+dx)^2+1}} - \frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{de(c+dx+1)}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(d*e*(1 + c + d*x)) + (2*Sqr
t[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/(d*e) + (4*b*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2]) - (2*b*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{\sqrt{ex}}\,dx, x, c+dx\right)}{d} \\
&= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{de} - \frac{(2b)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de} \\
&= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{de} - \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2} \\
&= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{de} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de} \\
&\quad + \frac{(4b)\text{Subst}\left(\int \frac{1-\frac{x^2}{e}}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de} \\
&= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{de(1+c+dx)} + \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{de} \\
&\quad + \frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}} \\
&\quad - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = -\frac{2\sqrt{e(c+dx)}\left(-3(a+b\operatorname{arcsinh}(c+dx))+2b(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)\right)}{3de}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-2*Sqrt[e*(c + d*x)]*(-3*(a + b*ArcSinh[c + d*x]) + 2*b*(c + d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(3*d*e)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| derivativedivides | $\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)-\text{EllipticE}\left(\sqrt{dex+\right.\right.\right.}}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{de}$ |
| default | $\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)-\text{EllipticE}\left(\sqrt{dex+\right.\right.\right.}}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{de}$ |
| parts | $\frac{2a\sqrt{dex+ce}}{de}+\frac{2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)-\text{EllipticE}\left(\sqrt{d\right.\right.\right.}}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{de}$ |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e))-2*
I/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d
*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE(
(d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{\sqrt{ce + dex}}\,dx$$
$$= \frac{2\left(\sqrt{dex + ce}bd\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) + \sqrt{dex + ce}ad + 2\sqrt{d^3e}b\text{weierstrassZeta}\left(-\frac{4}{d^2}, 0\right.\right.}{d^2e}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
 sqrt(d*e*x + c*e)*a*d + 2*sqrt(d^3*e)*b*weierstrassZeta(-4/d^2, 0, weierst
rassPInverse(-4/d^2, 0, (d*x + c)/d)))/(d^2*e)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{\sqrt{ce + dex}}\,dx = \int \frac{a + b\,\operatorname{asinh}(c + dx)}{\sqrt{e\,(c + dx)}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/sqrt(e*(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{\sqrt{ce + dex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{\sqrt{c\,e+d\,e\,x}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

# 3.233 $\int \frac{a+b\mathbf{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal result . . . 1781
Rubi [A] (verified) . . . 1781
Mathematica [C] (verified) . . . 1783
Maple [C] (verified) . . . 1783
Fricas [C] (verification not implemented) . . . 1784
Sympy [F] . . . 1784
Maxima [F(-2)] . . . 1784
Giac [F] . . . 1785
Mupad [F(-1)] . . . 1785

## Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(1/2)+2*b*(d*x+c+1)*(cos(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))
)*EllipticF(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+
c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(3/2)/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5859, 5776, 335, 226}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (2*b*(1 + c + d*x)*
Sqrt[(1 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)
]/Sqrt[e]], 1/2])/(d*e^(3/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\text{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= -\frac{2(a+b\text{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}} + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= -\frac{2(a+b\text{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = -\frac{2\left(a+b\text{arcsinh}(c+dx)-2b(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)\right)}{de\sqrt{e(c+dx)}}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] - 2*b*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/4
, -(c + d*x)^2]))/(d*e*Sqrt[e*(c + d*x)])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\text{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$ | 140 |
| default | $\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\text{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$ | 140 |
| parts | $-\frac{2a}{\sqrt{dex+ce}\,de}+\frac{2b\left(-\frac{\text{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$ | 145 |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e)
)+2/e/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^
2*(d*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{dex+ce}ad^2-2\sqrt{d^3e}(bdx+bc)\text{weierstrassPI}\right.}{d^4e^2x+cd^3e^2}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
) + sqrt(d*e*x + c*e)*a*d^2 - 2*sqrt(d^3*e)*(b*d*x + b*c)*weierstrassPInver
se(-4/d^2, 0, (d*x + c)/d))/(d^4*e^2*x + c*d^3*e^2)
```

## Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

# 3.234 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal result . . . . . 1786
Rubi [A] (verified) . . . . . 1787
Mathematica [C] (verified) . . . . . 1789
Maple [C] (verified) . . . . . 1789
Fricas [C] (verification not implemented) . . . . . 1790
Sympy [F] . . . . . 1791
Maxima [F(-2)] . . . . . 1791
Giac [F] . . . . . 1791
Mupad [F(-1)] . . . . . 1792

## Optimal result

Integrand size = 23, antiderivative size = 266

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} + \frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} - \frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*(1+(d*x+c)^2)^(1/2)/d
/e^2/(e*(d*x+c))^(1/2)+4/3*b*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/e^3/(d
*x+c+1)-4/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(5/2)/(
1+(d*x+c)^2)^(1/2)+2/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))
)^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan(
(e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/
d/e^(5/2)/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 331, 335, 311, 226, 1210}

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} + \frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}} - \frac{4b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}} + \frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{3de^3(c+dx+1)} - \frac{4b\sqrt{(c+dx)^2+1}}{3de^2\sqrt{e(c+dx)}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(3*d*e^3*(1 + c + d*x)) - (2*(a + b*ArcSinh[c
 + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2]) + (2*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\text{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2(a+b\text{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2(a+b\text{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{3de^4}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} \\
&\quad + \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{3de^3} \\
&\quad - \frac{(4b)\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{e}}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{3de^3} \\
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} + \frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}} \\
&\quad - \frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}} \\
&\quad + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.22

$$
\begin{aligned}
&\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \\
&-\frac{2\big(a+b\operatorname{arcsinh}(c+dx)+2b(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)\big)}{3de(e(c+dx))^{3/2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] + 2*b*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c + d*x)^2]))/(3*d*e*(e*(c + d*x))^(3/2))
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.76

| method | result |
| --- | --- |
| derivativedivides | $-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\text{Elli}\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right.$ $\left.\left.\vphantom{\frac{1}{1}}\right)\right)$ $\Big/ de$ |
| default | $-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\text{Elli}\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right.$ $\left.\left.\vphantom{\frac{1}{1}}\right)\right)$ $\Big/ de$ |
| parts | $-\frac{2a}{3(dex+ce)^{\frac{3}{2}}de}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},\right.\right.\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}$ $\left.\left.\left.\vphantom{\frac{1}{1}}\right)\right)\right)$ $\Big/ de$ |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e*
x+c*e))+2/3/e*(-(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(1/2)+I/e/(I/e)^(
1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)
^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*
e)^(1/2)*(I/e)^(1/2),I)))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx =$$

$$-\frac{2\left(\sqrt{dex+ce}bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{dex+ce}ad+2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{d^3e}\text{weie}\right.}{3\left(d^4e^3x^2+2\,c\right.}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
) + sqrt(d*e*x + c*e)*a*d + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(d^3*e)*w
eierstrassZeta(-4/d^2, 0, weierstrassPInverse(-4/d^2, 0, (d*x + c)/d)) + 2*
```

```
(b*d^2*x + b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e))/(d^4
*e^3*x^2 + 2*c*d^3*e^3*x + c^2*d^2*e^3)
```

## Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^{5/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(c + dx\right)}{\left(e\left(c + dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^{5/2}}\,dx = \int \frac{b\,\operatorname{arsinh}\left(dx + c\right) + a}{\left(dex + ce\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

# 3.235 $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal result . . . 1793
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## Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{15de^{7/2}\sqrt{1+(c+dx)^2}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(5/2)-4/15*b*(1+(d*x+c)^2)^(1/2)/
d/e^2/(e*(d*x+c))^(3/2)-2/15*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^
(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*a
rctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^
(1/2)/d/e^(7/2)/(1+(d*x+c)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 331, 335, 226}

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}} - \frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(15*d*e^2*(e*(c + d*x))^(3/2)) - (2*(a + b*Arc
Sinh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) - (2*b*(1 + c + d*x)*Sqrt[(1 +
(c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(15*d*e^(7/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\text{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{15de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{15de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right), \frac{1}{2}\right)}{15de^{7/2}\sqrt{1+(c+dx)^2}}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{-6(a+b\operatorname{arcsinh}(c+dx)) - 4b(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, -(c+d\right.}{15de(e(c+dx))^{5/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]

[Out] (-6*(a + b*ArcSinh[c + d*x]) - 4*b*(c + d*x)*Hypergeometric2F1[-3/4, 1/2, 1
/4, -(c + d*x)^2])/(15*d*e*(e*(c + d*x))^(5/2))
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$ |
| default | $\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$ |
| parts | $-\frac{2a}{5(dex+ce)^{\frac{5}{2}}de}+\frac{2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$ |

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e*
x+c*e))+2/5/e*(-1/3*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(3/2)-1/3/e^2
/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*
e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

## Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)+3\sqrt{dex+ce}ad^2+2\left(bd^3x^3+3bcd^2x^2+3bc^2d\right.\right.}{15\left(d^6e^4x^3+3cd^5e^4x^2+\right.}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)) + 3*sqrt(d*e*x + c*e)*a*d^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3)*sqrt(d^3*e)*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) + 2*(
b*d^3*x + b*c*d^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e))/(d^
6*e^4*x^3 + 3*c*d^5*e^4*x^2 + 3*c^2*d^4*e^4*x + c^3*d^3*e^4)
```

## Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{7/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(c + dx)}{(e\,(c + dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(7/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{7/2}}\,dx = \int \frac{b\,\text{arsinh}\,(dx + c) + a}{(dex + ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = \int \frac{a+b\,\operatorname{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

# 3.236 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1799
Rubi [A] (verified) . . . 1799
Mathematica [A] (verified) . . . 1801
Maple [F] . . . 1801
Fricas [F] . . . 1801
Sympy [F(-1)] . . . 1802
Maxima [F(-2)] . . . 1802
Giac [F] . . . 1802
Mupad [F(-1)] . . . 1802

## Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^2}{9de} - \frac{8b(e(c+dx))^{11/2}(a+b\operatorname{arcsinh}(c+dx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{4},\frac{15}{4},-(c+dx)^2\right)}{99de^2} + \frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/99*b*(e*(d*x+c))^(11/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 11/4],[15/4],-(d*x+c)^2)/d/e^2+16/1287
*b^2*(e*(d*x+c))^(13/2)*hypergeom([1, 13/4, 13/4],[15/4, 17/4],-(d*x+c)^2)/
d/e^3
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int (ce+dex)^{7/2}(a + b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3} - \frac{8b(e(c+dx))^{11/2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{4},\frac{15}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{99de^2} + \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^2}{9de}$$

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c
 + d*x)^2])/(99*d*e^2) + (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(1287*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{7/2}(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{9de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{9/2}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{9de}$$

$$= \frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{9de} - \frac{8b(e(c+dx))^{11/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{11}{4}, \frac{15}{4}, -(c+dx)^2\right)}{99de^2} + \frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1, \frac{13}{4}, \frac{13}{4}; \frac{15}{4}, \frac{17}{4}; -(c+dx)^2\right)}{1287de^3}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{9/2}\left(143(a+b\operatorname{arcsinh}(c+dx))^2-52b(c+dx)(a+b\operatorname{arcsinh}(c+dx))\right.$$

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcSinh[c + d*x])^2 - 52*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)
^2]))/(1287*d*e)
```

## Maple [F]

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\operatorname{arcsinh}\left(dx+c\right)\right)^2\,dx$$

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

## Fricas [F]

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{7}{2}}\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*c
^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2*c
^3*e^3)*arcsinh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 + 3*a
*b*c^2*d*e^3*x + a*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^{7/2}(a + b\text{arcsinh}(c + dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**2,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce + dex)^{7/2}(a + b\text{arcsinh}(c + dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce + dex)^{7/2}(a + b\text{arcsinh}(c + dx))^2\,dx = \int (dex + ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx + c) + a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^{7/2}(a + b\text{arcsinh}(c + dx))^2\,dx = \int (c\,e + d\,e\,x)^{7/2}\,(a + b\,\text{asinh}(c + d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2, x)
```

# 3.237 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1803
Rubi [A] (verified) . . . 1803
Mathematica [A] (verified) . . . 1805
Maple [F] . . . 1805
Fricas [F] . . . 1805
Sympy [F] . . . 1805
Maxima [F(-2)] . . . 1806
Giac [F] . . . 1806
Mupad [F(-1)] . . . 1806

## Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2}{7de} - \frac{8b(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},-(c+dx)^2\right)}{63de^2} + \frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/63*b*(e*(d*x+c))^(9/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 9/4],[13/4],-(d*x+c)^2)/d/e^2+16/693*b^
2*(e*(d*x+c))^(11/2)*hypergeom([1, 11/4, 11/4],[13/4, 15/4],-(d*x+c)^2)/d/e
^3
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int (ce+dex)^{5/2}(a + b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3} - \frac{8b(e(c+dx))^{9/2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{63de^2} + \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2}{7de}$$

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]

[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c +
 d*x)^2])/(63*d*e^2) + (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 1
1/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(693*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (ex)^{5/2}(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{7de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{7/2}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{7de}$$

$$= \frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{7de} - \frac{8b(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{9}{4}, \frac{13}{4}, -(c+dx)^2\right)}{63de^2} + \frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1, \frac{11}{4}, \frac{11}{4}; \frac{13}{4}, \frac{15}{4}; -(c+dx)^2\right)}{693de^3}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{7/2}\left(99(a+b\operatorname{arcsinh}(c+dx))^2-44b(c+dx)(a+b\operatorname{arcsinh}(c+dx))\,\mathrm{Hy}\right.}{}$$

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcSinh[c + d*x])^2 - 44*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2
]))/(693*d*e)
```

## Maple [F]

$$\int (dex+ce)^{\frac{5}{2}}\left(a+b\,\operatorname{arcsinh}\left(dx+c\right)\right)^2\,dx$$

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

## Fricas [F]

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{5}{2}}\left(b\,\operatorname{arsinh}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
[Out] integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [F]

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int \left(e(c+dx)\right)^{\frac{5}{2}}\left(a+b\,\operatorname{asinh}\left(c+dx\right)\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**2,x)
[Out] Integral((e*(c + d*x))**(5/2)*(a + b*asinh(c + d*x))**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2, x)
```

# 3.238 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1807
Rubi [A] (verified) . . . 1807
Mathematica [A] (verified) . . . 1809
Maple [F] . . . 1809
Fricas [F] . . . 1809
Sympy [F] . . . 1809
Maxima [F(-2)] . . . 1810
Giac [F] . . . 1810
Mupad [F(-1)] . . . 1810

## Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2}{5de} - \frac{8b(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-(c+dx)^2\right)}{35de^2} + \frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/35*b*(e*(d*x+c))^(7/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 7/4],[11/4],-(d*x+c)^2)/d/e^2+16/315*b^
2*(e*(d*x+c))^(9/2)*hypergeom([1, 9/4, 9/4],[11/4, 13/4],-(d*x+c)^2)/d/e^3
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3} - \frac{8b(e(c+dx))^{7/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{35de^2} + \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2}{5de}$$

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c +
 d*x)^2])/(35*d*e^2) + (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(315*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (ex)^{3/2}(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d} \\
&= \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{5de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{5/2}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de} \\
&= \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{5de} \\
&\quad - \frac{8b(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{4}, \frac{11}{4}, -(c+dx)^2\right)}{35de^2} \\
&\quad + \frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1, \frac{9}{4}, \frac{9}{4}; \frac{11}{4}, \frac{13}{4}; -(c+dx)^2\right)}{315de^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{5/2}\left(63(a+b\text{arcsinh}(c+dx))^2-36b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hy}}{3}$$

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcSinh[c + d*x])^2 - 36*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])
)/(315*d*e)
```

## Maple [F]

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^2\,dx$$

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

## Fricas [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2 +
2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \int \left(e(c+dx)\right)^{\frac{3}{2}}\left(a+b\,\text{asinh}\left(c+dx\right)\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2, x)
```

# 3.239 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

Optimal result . . . 1811
Rubi [A] (verified) . . . 1811
Mathematica [A] (verified) . . . 1813
Maple [F] . . . 1813
Fricas [F] . . . 1813
Sympy [F] . . . 1813
Maxima [F(-2)] . . . 1814
Giac [F] . . . 1814
Mupad [F(-1)] . . . 1814

## Optimal result

Integrand size = 25, antiderivative size = 134

$$\begin{aligned}&\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2}{3de}\\&\quad-\frac{8b(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-(c+dx)^2\right)}{15de^2}\\&\quad+\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}\end{aligned}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/15*b*(e*(d*x+c))^(5/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 5/4],[9/4],-(d*x+c)^2)/d/e^2+16/105*b^2
*(e*(d*x+c))^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],-(d*x+c)^2)/d/e^3
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\begin{aligned}&\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx\\&=\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}\\&\quad-\frac{8b(e(c+dx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{15de^2}\\&\quad+\frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2}{3de}\end{aligned}$$

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c +
d*x)^2])/(15*d*e^2) + (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, -(c + d*x)^2])/(105*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sqrt{ex}(a+b\text{arcsinh}(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{3de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{3/2}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

$$= \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{3de} - \frac{8b(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, -(c+dx)^2\right)}{15de^2} + \frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; -(c+dx)^2\right)}{105de^3}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx$$
$$= \frac{2(e(c+dx))^{3/2}\left(35(a+b\text{arcsinh}(c+dx))^2-28b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4}\right.\right.}{105de}$$

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcSinh[c + d*x])^2 - 28*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c + d*x)^2] + 8*b^2
*(c + d*x)^2*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c + d*x)^2]))/
(105*d*e)
```

## Maple [F]

$$\int (a+b\,\text{arcsinh}\,(dx+c))^2\sqrt{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

## Fricas [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e), x)
```

## Sympy [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\text{asinh}\,(c+dx))^2\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int \sqrt{dex+ce}(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \int \sqrt{c\,e+d\,e\,x}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2, x)
```

# 3.240 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$


Optimal result . . . . . 1815
Rubi [A] (verified) . . . . . 1815
Mathematica [A] (verified) . . . . . 1817
Maple [F] . . . . . 1817
Fricas [F] . . . . . 1817
Sympy [F] . . . . . 1817
Maxima [F(-2)] . . . . . 1818
Giac [F] . . . . . 1818
Mupad [F(-1)] . . . . . 1818


## Optimal result

Integrand size = 25, antiderivative size = 132

$$\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx \\
&= \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^2}{de} \\
&\quad - \frac{8b(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)}{3de^2} \\
&\quad + \frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}
\end{aligned}$$

```
[Out] -8/3*b*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))*hypergeom([1/2, 3/4],[7/4],-(
d*x+c)^2)/d/e^2+16/15*b^2*(e*(d*x+c))^(5/2)*hypergeom([1, 5/4, 5/4],[7/4, 9
/4],-(d*x+c)^2)/d/e^3+2*(a+b*arcsinh(d*x+c))^2*(e*(d*x+c))^(1/2)/d/e
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\begin{aligned}
&\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx \\
&= \frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3} \\
&\quad - \frac{8b(e(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{3de^2} \\
&\quad + \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^2}{de}
\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)
^2])/(3*d*e^2) + (16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4
}, {7/4, 9/4}, -(c + d*x)^2])/(15*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{de} - \frac{(4b)\text{Subst}\left(\int \frac{\sqrt{ex}(a+b\text{arcsinh}(x))}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{de} - \frac{8b(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -(c+dx)^2\right)}{3de^2} + \frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1, \frac{5}{4}, \frac{5}{4}; \frac{7}{4}, \frac{9}{4}; -(c+dx)^2\right)}{15de^3}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$$
$$= \frac{2\sqrt{e(c+dx)}\left(15(a+b\operatorname{arcsinh}(c+dx))^2 - 20b(c+dx)(a+b\operatorname{arcsinh}(c+dx))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{15de}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcSinh[c + d*x])^2 - 20*b*(c + d*x)*(a + b
*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2] + 8*b^2*(
c + d*x)^2*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c + d*x)^2]))/(15
*d*e)
```

## Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(dx+c))^2}{\sqrt{dex+ce}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/sqrt(d*e*x
 + c*e), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\operatorname{asinh}(c+dx))^2}{\sqrt{e(c+dx)}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

# 3.241 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$


Optimal result . . . . . 1819
Rubi [A] (verified) . . . . . 1819
Mathematica [A] (verified) . . . . . 1821
Maple [F] . . . . . 1821
Fricas [F] . . . . . 1821
Sympy [F] . . . . . 1821
Maxima [F(-2)] . . . . . 1822
Giac [F] . . . . . 1822
Mupad [F(-1)] . . . . . 1822


## Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)}{de^2} - \frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}$$

```
[Out] -16/3*b^2*(e*(d*x+c))^(3/2)*hypergeom([3/4, 3/4, 1],[5/4, 7/4],-(d*x+c)^2)/
d/e^3-2*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arcsinh(d*x+c
))*hypergeom([1/4, 1/2],[5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = -\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3} + \frac{8b\sqrt{e(c+dx)}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{de^2} - \frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{de\sqrt{e(c+dx)}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^
2])/(d*e^2) - (16*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1},
{5/4, 7/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{de\sqrt{e(c+dx)}} \\
&\quad + \frac{8b\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)}{de^2} \\
&\quad - \frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \frac{2\left(-3(a+b\text{arcsinh}(c+dx))^2 - 4b(c+dx)\left(-3(a+b\text{arcsinh}(c+dx))\,\text{Hyp}\right.\right.}{3de\sqrt{}}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcSinh[c + d*x])^2 - 4*b*(c + d*x)*(-3*(a + b*ArcSinh[c + d*
x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2] + 2*b*(c + d*x)*Hypergeo
metricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*
x)])
```

## Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^2}{(dex+ce)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

# 3.242 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx$


Optimal result . . . . . 1823
Rubi [A] (verified) . . . . . 1823
Mathematica [A] (verified) . . . . . 1825
Maple [F] . . . . . 1825
Fricas [F] . . . . . 1825
Sympy [F] . . . . . 1826
Maxima [F(-2)] . . . . . 1826
Giac [F] . . . . . 1826
Mupad [F(-1)] . . . . . 1826


## Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{3de(e(c+dx))^{3/2}} - \frac{8b(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)}{3de^2\sqrt{e(c+dx)}} + \frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arcsinh(d*x+c)
)*hypergeom([-1/4, 1/2],[3/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(1/2)+16/3*b^2*
hypergeom([1/4, 1/4, 1],[3/4, 5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^3
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3} - \frac{8b\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{3de^2\sqrt{e(c+dx)}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{3de(e(c+dx))^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(3*d*e^2*
Sqrt[e*(c + d*x)]) + (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4,
 1}, {3/4, 5/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{3de(e(c+dx))^{3/2}} \\
&\quad - \frac{8b(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -(c+dx)^2\right)}{3de^2\sqrt{e(c+dx)}} \\
&\quad + \frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4}, \frac{1}{4}, 1; \frac{3}{4}, \frac{5}{4}; -(c+dx)^2\right)}{3de^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx =$$
$$-\frac{2\left((a+b\operatorname{arcsinh}(c+dx))^2+4b(c+dx)\left((a+b\operatorname{arcsinh}(c+dx))\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)\right.\right.}{3de(e(c+dx))^{3/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])^2 + 4*b*(c + d*x)*((a + b*ArcSinh[c + d*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2] - 2*b*(c + d*x)*Hypergeometr
icPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2
))
```

## Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(dx+c))^2}{(dex+ce)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^2}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

# 3.243 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx$


Optimal result . . . . . 1827
Rubi [A] (verified) . . . . . 1827
Mathematica [A] (verified) . . . . . 1829
Maple [F] . . . . . 1829
Fricas [F] . . . . . 1829
Sympy [F] . . . . . 1830
Maxima [F(-2)] . . . . . 1830
Giac [F] . . . . . 1830
Mupad [F(-1)] . . . . . 1830


## Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{5de(e(c+dx))^{5/2}} - \frac{8b(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)}{15de^2(e(c+dx))^{3/2}} - \frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arcsinh(d*x+c
))*hypergeom([-3/4, 1/2],[1/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(3/2)-16/15*b^
2*hypergeom([-1/4, -1/4, 1],[1/4, 3/4],-(d*x+c)^2)/d/e^3/(e*(d*x+c))^(1/2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = -\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}} - \frac{8b\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{5de(e(c+dx))^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(15*d*e^2
*(e*(c + d*x))^(3/2)) - (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/
4}, -(c + d*x)^2])/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{5de(e(c+dx))^{5/2}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de} \\
&= -\frac{2(a+b\text{arcsinh}(c+dx))^2}{5de(e(c+dx))^{5/2}} \\
&\quad - \frac{8b(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, -(c+dx)^2\right)}{15de^2(e(c+dx))^{3/2}} \\
&\quad - \frac{16b^2\,{}_3F_2\left(-\frac{1}{4}, -\frac{1}{4}, 1; \frac{1}{4}, \frac{3}{4}; -(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx =$$
$$-\frac{2\big((a+b\operatorname{arcsinh}(c+dx))\left(3(a+b\operatorname{arcsinh}(c+dx))+4b(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)\right.}{15de(e(c+dx))^{5/2}}$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])*(3*(a + b*ArcSinh[c + d*x]) + 4*b*(c + d*x)*H
ypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2]) + 8*b^2*(c + d*x)^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, -(c + d*x)^2]))/(15*d*e*(e*(c + d*x
))^(5/2))
```

## Maple [F]

$$\int \frac{(a+b\,\operatorname{arcsinh}(dx+c))^2}{(dex+ce)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

## Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
+ c^4*e^4), x)
```

## Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^2}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(7/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)
```

# 3.244 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . 1831
Rubi [N/A] . . . 1831
Mathematica [N/A] . . . 1832
Maple [N/A] (verified) . . . 1832
Fricas [N/A] . . . 1832
Sympy [F(-1)] . . . 1833
Maxima [F(-2)] . . . 1833
Giac [N/A] . . . 1833
Mupad [N/A] . . . 1833

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^3}{9de} - \frac{2b\operatorname{Int}\left(\frac{(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/d/e-2/3*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/(9*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{7/2}(a+b\operatorname{arcsinh}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^3}{9de} - \frac{(2b)\operatorname{Subst}\left(\int \frac{(ex)^{9/2}(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}$$

## Mathematica [N/A]

Not integrable

Time = 74.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^3\,dx$$

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 10.24

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{7}{2}}\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral((a^3*d^3*e^3*x^3 + 3*a^3*c*d^2*e^3*x^2 + 3*a^3*c^2*d*e^3*x + a^3*c
^3*e^3 + (b^3*d^3*e^3*x^3 + 3*b^3*c*d^2*e^3*x^2 + 3*b^3*c^2*d*e^3*x + b^3*c
^3*e^3)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^2 +
 3*a*b^2*c^2*d*e^3*x + a*b^2*c^3*e^3)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^3*e^3
*x^3 + 3*a^2*b*c*d^2*e^3*x^2 + 3*a^2*b*c^2*d*e^3*x + a^2*b*c^3*e^3)*arcsinh
(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**3,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3, x)
```

# 3.245 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . 1834
Rubi [N/A] . . . 1834
Mathematica [N/A] . . . 1835
Maple [N/A] (verified) . . . 1835
Fricas [N/A] . . . 1835
Sympy [F(-1)] . . . 1836
Maxima [F(-2)] . . . 1836
Giac [N/A] . . . 1836
Mupad [N/A] . . . 1836

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3}{7de} - \frac{6b\operatorname{Int}\left(\frac{(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/(7*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{5/2}(a+b\operatorname{arcsinh}(x))^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3}{7de} - \frac{(6b)\operatorname{Subst}\left(\int \frac{(ex)^{7/2}(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{7de}$$

## Mathematica [N/A]

Not integrable

Time = 120.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^3\,dx$$

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 7.44

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral((a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^
2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^
2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^2*e^
2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c
*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**3,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3, x)
```

# 3.246 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

Optimal result . . . 1837
Rubi [N/A] . . . 1837
Mathematica [N/A] . . . 1838
Maple [N/A] (verified) . . . 1838
Fricas [N/A] . . . 1838
Sympy [N/A] . . . 1839
Maxima [F(-2)] . . . 1839
Giac [N/A] . . . 1839
Mupad [N/A] . . . 1840

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3}{5de} - \frac{6b\operatorname{Int}\left(\frac{(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{3/2}(a+b\operatorname{arcsinh}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3}{5de} - \frac{(6b)\operatorname{Subst}\left(\int \frac{(ex)^{5/2}(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}$$

## Mathematica [N/A]

Not integrable

Time = 66.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^3\,dx$$

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e
)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 20.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \left(e(c+dx)\right)^{\frac{3}{2}}\left(a+b\,\text{asinh}\left(c+dx\right)\right)^3\,dx$$

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**3,x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3, x)
```

# 3.247 $\int \sqrt{ce+dex}(a+b\mathbf{arcsinh}(c+dx))^3\,dx$

Optimal result . . . 1841
Rubi [N/A] . . . 1841
Mathematica [N/A] . . . 1842
Maple [N/A] (verified) . . . 1842
Fricas [N/A] . . . 1842
Sympy [N/A] . . . 1842
Maxima [F(-2)] . . . 1843
Giac [N/A] . . . 1843
Mupad [N/A] . . . 1843

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de} - \frac{2b\operatorname{Int}\left(\frac{(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+c
))^(3/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \sqrt{ex}(a+b\operatorname{arcsinh}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de} - \frac{(2b)\operatorname{Subst}\left(\int \frac{(ex)^{3/2}(a+b\operatorname{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 104.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx$$

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+b\,\text{arcsinh}\,(dx+c))^3\sqrt{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\text{asinh}\,(c+dx))^3\,\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx = \int \sqrt{c\,e+d\,e\,x}\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3, x)
```

# 3.248 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$

Optimal result . . . . . 1844
Rubi [N/A] . . . . . 1844
Mathematica [N/A] . . . . . 1845
Maple [N/A] (verified) . . . . . 1845
Fricas [N/A] . . . . . 1845
Sympy [N/A] . . . . . 1846
Maxima [F(-2)] . . . . . 1846
Giac [N/A] . . . . . 1846
Mupad [N/A] . . . . . 1847

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^3}{de} - \frac{6b\operatorname{Int}\left(\frac{\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arcsin
h(d*x+c))^2*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{de} - \frac{(6b)\text{Subst}\left(\int \frac{\sqrt{ex}(a+b\text{arcsinh}(x))^2}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 61.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^3}{\sqrt{dex+ce}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^3}{\sqrt{e\,(c+dx)}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^3}{\sqrt{c\,e+d\,e\,x}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

# 3.249 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$

Optimal result . . . 1848
Rubi [N/A] . . . 1848
Mathematica [N/A] . . . 1849
Maple [N/A] (verified) . . . 1849
Fricas [N/A] . . . 1849
Sympy [N/A] . . . 1850
Maxima [F(-2)] . . . 1850
Giac [N/A] . . . 1850
Mupad [N/A] . . . 1851

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{de\sqrt{e(c+dx)}} + \frac{6b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcsi
nh(d*x+c))^2/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^2/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{de\sqrt{e(c+dx)}} + \frac{(6b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^2}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 43.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^3}{(dex+ce)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2
), x)
```

## Sympy [N/A]

Not integrable

Time = 6.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^3}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

# 3.250 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$


Optimal result . . . . . 1852
Rubi [N/A] . . . . . 1852
Mathematica [N/A] . . . . . 1853
Maple [N/A] (verified) . . . . . 1853
Fricas [N/A] . . . . . 1853
Sympy [N/A] . . . . . 1854
Maxima [F(-2)] . . . . . 1854
Giac [N/A] . . . . . 1854
Mupad [N/A] . . . . . 1855


## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{3de(e(c+dx))^{3/2}} + \frac{2b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*arc
sinh(d*x+c))^2/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{3de(e(c+dx))^{3/2}} + \frac{(2b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^2}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 39.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c
^2*d*e^3*x + c^3*e^3), x)
```

## Sympy [N/A]

Not integrable

Time = 14.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^3}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

# 3.251 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$

Optimal result . . . 1856
Rubi [N/A] . . . 1856
Mathematica [N/A] . . . 1857
Maple [N/A] (verified) . . . 1857
Fricas [N/A] . . . 1857
Sympy [N/A] . . . 1858
Maxima [F(-2)] . . . 1858
Giac [N/A] . . . 1858
Mupad [N/A] . . . 1859

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{5de(e(c+dx))^{5/2}} + \frac{6b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^2/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^3}{5de(e(c+dx))^{5/2}} + \frac{(6b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^2}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

## Mathematica [N/A]

Not integrable

Time = 73.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^3}{(dex+ce)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.44

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c
^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

## Sympy [N/A]

Not integrable

Time = 61.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^3}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(7/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

# 3.252 $\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . 1860
Rubi [N/A] . . . 1860
Mathematica [N/A] . . . 1861
Maple [N/A] (verified) . . . 1861
Fricas [N/A] . . . 1861
Sympy [F(-1)] . . . 1862
Maxima [F(-2)] . . . 1862
Giac [N/A] . . . 1862
Mupad [N/A] . . . 1862

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^4}{9de} - \frac{8b\operatorname{Int}\left(\frac{(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{9e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/9*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^4)/(9*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(9*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{7/2}(a+b\operatorname{arcsinh}(x))^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}(a+b\operatorname{arcsinh}(c+dx))^4}{9de} - \frac{(8b)\operatorname{Subst}\left(\int \frac{(ex)^{9/2}(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{9de}$$

## Mathematica [N/A]

Not integrable

Time = 37.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{7}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^4\,dx$$

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 330, normalized size of antiderivative = 13.20

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral((a^4*d^3*e^3*x^3 + 3*a^4*c*d^2*e^3*x^2 + 3*a^4*c^2*d*e^3*x + a^4*c
^3*e^3 + (b^4*d^3*e^3*x^3 + 3*b^4*c*d^2*e^3*x^2 + 3*b^4*c^2*d*e^3*x + b^4*c
^3*e^3)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^3*e^3*x^3 + 3*a*b^3*c*d^2*e^3*x^2 +
 3*a*b^3*c^2*d*e^3*x + a*b^3*c^3*e^3)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^3*e
^3*x^3 + 3*a^2*b^2*c*d^2*e^3*x^2 + 3*a^2*b^2*c^2*d*e^3*x + a^2*b^2*c^3*e^3)
*arcsinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3*x^3 + 3*a^3*b*c*d^2*e^3*x^2 + 3*a^3*
b*c^2*d*e^3*x + a^3*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**4,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 172.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{7}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4, x)
```

# 3.253 $\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . 1863
Rubi [N/A] . . . 1863
Mathematica [N/A] . . . 1864
Maple [N/A] (verified) . . . 1864
Fricas [N/A] . . . 1864
Sympy [F(-1)] . . . 1865
Maxima [F(-2)] . . . 1865
Giac [N/A] . . . 1865
Mupad [N/A] . . . 1865

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4}{7de} - \frac{8b\operatorname{Int}\left(\frac{(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^4)/(7*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{5/2}(a+b\operatorname{arcsinh}(x))^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4}{7de} - \frac{(8b)\operatorname{Subst}\left(\int \frac{(ex)^{7/2}(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{7de}$$

## Mathematica [N/A]

Not integrable

Time = 74.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^4\,dx$$

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 9.60

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral((a^4*d^2*e^2*x^2 + 2*a^4*c*d*e^2*x + a^4*c^2*e^2 + (b^4*d^2*e^2*x^
2 + 2*b^4*c*d*e^2*x + b^4*c^2*e^2)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^2*e^2*x^
2 + 2*a*b^3*c*d*e^2*x + a*b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^2*
e^2*x^2 + 2*a^2*b^2*c*d*e^2*x + a^2*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 4*(a^
3*b*d^2*e^2*x^2 + 2*a^3*b*c*d*e^2*x + a^3*b*c^2*e^2)*arcsinh(d*x + c))*sqrt
(d*e*x + c*e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Timed out}$$

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**4,x)
[Out] Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{5}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\operatorname{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4, x)
```

# 3.254 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . 1866
Rubi [N/A] . . . 1866
Mathematica [N/A] . . . 1867
Maple [N/A] (verified) . . . 1867
Fricas [N/A] . . . 1867
Sympy [N/A] . . . 1868
Maxima [F(-2)] . . . 1868
Giac [N/A] . . . 1868
Mupad [N/A] . . . 1869

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4}{5de} - \frac{8b\operatorname{Int}\left(\frac{(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int (ex)^{3/2}(a+b\operatorname{arcsinh}(x))^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4}{5de} - \frac{(8b)\operatorname{Subst}\left(\int \frac{(ex)^{5/2}(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}$$

## Mathematica [N/A]

Not integrable

Time = 34.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{3}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^4\,dx$$

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.20

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4 +
4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b^2
*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c))*sq
rt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 39.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (e(c+dx))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(c+dx))^4\,dx$$

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**4,x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**4, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4, x)
```

# 3.255 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

Optimal result . . . 1870
Rubi [N/A] . . . 1870
Mathematica [N/A] . . . 1871
Maple [N/A] (verified) . . . 1871
Fricas [N/A] . . . 1871
Sympy [N/A] . . . 1871
Maxima [F(-2)] . . . 1872
Giac [N/A] . . . 1872
Mupad [N/A] . . . 1872

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4}{3de} - \frac{8b\operatorname{Int}\left(\frac{(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*x
+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \sqrt{ex}(a+b\operatorname{arcsinh}(x))^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4}{3de} - \frac{(8b)\operatorname{Subst}\left(\int \frac{(ex)^{3/2}(a+b\operatorname{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}$$

## Mathematica [N/A]

Not integrable

Time = 57.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx$$

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+b\,\text{arcsinh}\,(dx+c))^4\sqrt{dex+ce}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 6.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\text{asinh}\,(c+dx))^4\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**4, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

## Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{c\,e+d\,e\,x}\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4, x)
```

# 3.256 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

Optimal result . . . . . 1873
Rubi [N/A] . . . . . 1873
Mathematica [N/A] . . . . . 1874
Maple [N/A] (verified) . . . . . 1874
Fricas [N/A] . . . . . 1874
Sympy [N/A] . . . . . 1875
Maxima [F(-2)] . . . . . 1875
Giac [N/A] . . . . . 1875
Mupad [N/A] . . . . . 1876

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^4}{de} - \frac{8b\operatorname{Int}\left(\frac{\sqrt{e(c+dx)}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arcsin
h(d*x+c))^3*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^4}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^4}{de} - \frac{(8b)\text{Subst}\left(\int \frac{\sqrt{ex}(a+b\text{arcsinh}(x))^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 13.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^4}{\sqrt{dex+ce}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

## Sympy [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\,\operatorname{asinh}(c+dx))^4}{\sqrt{e\,(c+dx)}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^4}{\sqrt{dex+ce}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

## Mupad [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{\sqrt{c\,e+d\,e\,x}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

# 3.257 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$


Optimal result . . . . . 1877
Rubi [N/A] . . . . . 1877
Mathematica [N/A] . . . . . 1878
Maple [N/A] (verified) . . . . . 1878
Fricas [N/A] . . . . . 1878
Sympy [N/A] . . . . . 1879
Maxima [F(-2)] . . . . . 1879
Giac [N/A] . . . . . 1879
Mupad [N/A] . . . . . 1880


## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{de\sqrt{e(c+dx)}} + \frac{8b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcsi
nh(d*x+c))^3/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^3/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^4}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{de\sqrt{e(c+dx)}} + \frac{(8b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

## Mathematica [N/A]

Not integrable

Time = 16.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^2*
e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

## Sympy [N/A]

Not integrable

Time = 9.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^4}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

# 3.258 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$

Optimal result . . . 1881
Rubi [N/A] . . . 1881
Mathematica [N/A] . . . 1882
Maple [N/A] (verified) . . . 1882
Fricas [N/A] . . . 1882
Sympy [N/A] . . . 1883
Maxima [F(-2)] . . . 1883
Giac [N/A] . . . 1883
Mupad [N/A] . . . 1884

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{3de(e(c+dx))^{3/2}} + \frac{8b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(3*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^4}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{3de(e(c+dx))^{3/2}} + \frac{(8b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

## Mathematica [N/A]

Not integrable

Time = 10.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^3*
e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

## Sympy [N/A]

Not integrable

Time = 18.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^4}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

# 3.259 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$

Optimal result . . . 1885
Rubi [N/A] . . . 1885
Mathematica [N/A] . . . 1886
Maple [N/A] (verified) . . . 1886
Fricas [N/A] . . . 1886
Sympy [N/A] . . . 1887
Maxima [F(-2)] . . . 1887
Giac [N/A] . . . 1887
Mupad [N/A] . . . 1888

## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{5de(e(c+dx))^{5/2}} + \frac{8b\operatorname{Int}\left(\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

## Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$$

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^4}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2(a+b\operatorname{arcsinh}(c+dx))^4}{5de(e(c+dx))^{5/2}} + \frac{(8b)\operatorname{Subst}\left(\int \frac{(a+b\operatorname{arcsinh}(x))^3}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

## Mathematica [N/A]

Not integrable

Time = 53.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{7}{2}}}dx$$

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^4*
e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x
)
```

## Sympy [N/A]

Not integrable

Time = 68.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^4}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(7/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

## Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

# 3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathbf{arcsinh}(a+bx)^3\,dx$

Optimal result . . . . . 1889
Rubi [A] (verified) . . . . . 1889
Mathematica [A] (verified) . . . . . 1891
Maple [A] (verified) . . . . . 1892
Fricas [A] (verification not implemented) . . . . . 1892
Sympy [F] . . . . . 1893
Maxima [F] . . . . . 1893
Giac [F] . . . . . 1893
Mupad [F(-1)] . . . . . 1893

## Optimal result

Integrand size = 30, antiderivative size = 131

$$\begin{aligned}
&\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)^3\,dx \\
&= -\frac{3(a+bx)^2}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\mathrm{arcsinh}(a+bx)}{4b} \\
&\quad - \frac{3\mathrm{arcsinh}(a+bx)^2}{8b} - \frac{3(a+bx)^2\mathrm{arcsinh}(a+bx)^2}{4b} \\
&\quad + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\mathrm{arcsinh}(a+bx)^3}{2b} + \frac{\mathrm{arcsinh}(a+bx)^4}{8b}
\end{aligned}$$

```
[Out] -3/8*(b*x+a)^2/b-3/8*arcsinh(b*x+a)^2/b-3/4*(b*x+a)^2*arcsinh(b*x+a)^2/b+1/
8*arcsinh(b*x+a)^4/b+3/4*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(
b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5785, 5783, 5776, 5812, 30}

$$\begin{aligned}
&\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)^3\,dx \\
&= \frac{\mathrm{arcsinh}(a+bx)^4}{8b} + \frac{(a+bx)\sqrt{(a+bx)^2+1}\,\mathrm{arcsinh}(a+bx)^3}{2b} \\
&\quad - \frac{3(a+bx)^2\mathrm{arcsinh}(a+bx)^2}{4b} - \frac{3\mathrm{arcsinh}(a+bx)^2}{8b} \\
&\quad + \frac{3(a+bx)\sqrt{(a+bx)^2+1}\,\mathrm{arcsinh}(a+bx)}{4b} - \frac{3(a+bx)^2}{8b}
\end{aligned}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]

[Out] (-3*(a + b*x)^2)/(8*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(4*b) - (3*ArcSinh[a + b*x]^2)/(8*b) - (3*(a + b*x)^2*ArcSinh[a + b*x]^2
)/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(2*b) + ArcS
inh[a + b*x]^4/(8*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{1+x^2}\text{arcsinh}(x)^3\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^3}{2b} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \frac{3\text{Subst}\left(\int x\text{arcsinh}(x)^2\,dx, x, a+bx\right)}{2b} \\
&= -\frac{3(a+bx)^2\text{arcsinh}(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^3}{2b} \\
&\quad + \frac{\text{arcsinh}(a+bx)^4}{8b} + \frac{3\text{Subst}\left(\int \frac{x^2\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} \\
&= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{4b} - \frac{3(a+bx)^2\text{arcsinh}(a+bx)^2}{4b} \\
&\quad + \frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^3}{2b} + \frac{\text{arcsinh}(a+bx)^4}{8b} \\
&\quad - \frac{3\text{Subst}\left(\int x\,dx, x, a+bx\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{4b} \\
&= -\frac{3(a+bx)^2}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{4b} \\
&\quad - \frac{3\text{arcsinh}(a+bx)^2}{8b} - \frac{3(a+bx)^2\text{arcsinh}(a+bx)^2}{4b} \\
&\quad + \frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^3}{2b} + \frac{\text{arcsinh}(a+bx)^4}{8b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\begin{aligned}
&\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3\,dx \\
&= \frac{-3bx(2a+bx) + 6(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx) - 3(1+2a^2+4abx+2b^2x^2)\,\text{arcsinh}(a}{8b}
\end{aligned}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*b*x*(2*a + b*x) + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] - 3*(1 + 2*a^2 + 4*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 4*(a +
 b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3 + ArcSinh[a + b*
x]^4)/(8*b)
```

## Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $\frac{4\operatorname{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,bx-6\operatorname{arcsinh}(bx+a)^2b^2x^2+4\operatorname{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,a-12\operatorname{arcsinh}(bx+a)^2abx+\text{arc}$ |

```
[In] int(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a)^
2*b^2*x^2+4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x
+a)^2*a*b*x+arcsinh(b*x+a)^4-6*a^2*arcsinh(b*x+a)^2+6*arcsinh(b*x+a)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*b*x-3*b^2*x^2+6*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*a-6*a*b*x-3*arcsinh(b*x+a)^2-3*a^2-3)/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx$$

$$= \frac{4\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 - 3\,b^2x^2 + \log\left(bx+a+\sqrt{b^2x^2+\right.}$$

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/8*(4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))^3 - 3*b^2*x^2 + log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))^4 - 6*a*b*x - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

## Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^3(a+bx)\,dx$$

```
[In] integrate(asinh(b*x+a)**3*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3, x)
```

## Maxima [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^3\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

## Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^3\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{asinh}(a+b\,x)^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

# 3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathbf{arcsinh}(a+bx)^2\,dx$

Optimal result . . . . . 1894
Rubi [A] (verified) . . . . . 1895
Mathematica [A] (verified) . . . . . 1897
Maple [A] (verified) . . . . . 1897
Fricas [A] (verification not implemented) . . . . . 1897
Sympy [F] . . . . . 1898
Maxima [F] . . . . . 1898
Giac [F] . . . . . 1898
Mupad [F(-1)] . . . . . 1898

## Optimal result

Integrand size = 30, antiderivative size = 107

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)^2\,dx = \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\mathrm{arcsinh}(a+bx)}{4b} - \frac{(a+bx)^2\mathrm{arcsinh}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\mathrm{arcsinh}(a+bx)^2}{2b} + \frac{\mathrm{arcsinh}(a+bx)^3}{6b}$$

```
[Out] -1/4*arcsinh(b*x+a)/b-1/2*(b*x+a)^2*arcsinh(b*x+a)/b+1/6*arcsinh(b*x+a)^3/b
+1/4*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5785, 5783, 5776, 327, 221}

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{6b} + \frac{(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{2b} - \frac{(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b} - \frac{\operatorname{arcsinh}(a+bx)}{4b} + \frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b) - ArcSinh[a + b*x]/(4*b) - ((a + b*
x)^2*ArcSinh[a + b*x])/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x]^2)/(2*b) + ArcSinh[a + b*x]^3/(6*b)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int x\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{b} \\
&= -\frac{(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b} \\
&\quad + \frac{\operatorname{arcsinh}(a+bx)^3}{6b} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} \\
&= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b} \\
&\quad + \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b} \\
&\quad + \frac{\operatorname{arcsinh}(a+bx)^3}{6b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{4b} \\
&= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\operatorname{arcsinh}(a+bx)}{4b} - \frac{(a+bx)^2\operatorname{arcsinh}(a+bx)}{2b} \\
&\quad + \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2b} + \frac{\operatorname{arcsinh}(a+bx)^3}{6b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx$$
$$= \frac{3(a+bx)\sqrt{1+a^2+2abx+b^2x^2}-3(1+2a^2+4abx+2b^2x^2)\operatorname{arcsinh}(a+bx)+6(a+bx)\sqrt{1+a^2+2ab}}{12b}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] (3*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*(1 + 2*a^2 + 4*a*b*x + 2
*b^2*x^2)*ArcSinh[a + b*x] + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*
ArcSinh[a + b*x]^2 + 2*ArcSinh[a + b*x]^3)/(12*b)
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $\frac{6\operatorname{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,bx-6\operatorname{arcsinh}(bx+a)b^2x^2+6\operatorname{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,a-12\operatorname{arcsinh}(bx+a)abx+2a}{12b}$ |

```
[In] int(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a)
*b^2*x^2+6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x+
a)*a*b*x+2*arcsinh(b*x+a)^3-6*a^2*arcsinh(b*x+a)+3*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)*b*x+3*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*arcsinh(b*x+a))/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.50

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx$$
$$= \frac{6\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,\log\left(bx+a+\sqrt{b^2x^2+2\,ab}\right.}{}$$

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/12*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))^2 + 2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))^3 - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a))/b
```

## Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)\,dx$$

```
[In] integrate(asinh(b*x+a)**2*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2, x)
```

## Maxima [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

## Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^2\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

# 3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathbf{arcsinh}(a+bx)\,dx$

Optimal result . . . . . 1899
Rubi [A] (verified) . . . . . 1899
Mathematica [A] (verified) . . . . . 1900
Maple [A] (verified) . . . . . 1901
Fricas [A] (verification not implemented) . . . . . 1901
Sympy [F] . . . . . 1901
Maxima [B] (verification not implemented) . . . . . 1902
Giac [F] . . . . . 1902
Mupad [F(-1)] . . . . . 1903

## Optimal result

Integrand size = 28, antiderivative size = 61

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)\,dx$$
$$= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)}{2b} + \frac{\text{arcsinh}(a+bx)^2}{4b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/4*arcsinh(b*x+a)^2/b+1/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5860, 5785, 5783, 30}

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)\,dx = \frac{\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)}{2b} + \frac{\text{arcsinh}(a+bx)^2}{4b} - \frac{(a+bx)^2}{4b}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] -1/4*(a + b*x)^2/b + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*
b) + ArcSinh[a + b*x]^2/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{b} \\ &= \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b} \\ &\quad - \frac{\text{Subst}(\int x\,dx, x, a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} \\ &= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b} + \frac{\operatorname{arcsinh}(a+bx)^2}{4b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx \\ &= \frac{-bx(2a+bx)+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)+\operatorname{arcsinh}(a+bx)^2}{4b}\end{aligned}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(2*a + b*x)) + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + ArcSinh[a + b*x]^2)/(4*b)
```

## Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

| method | result | size |
|---|---|---|
| default | $\frac{2\operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,bx-b^2x^2+2\operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,a-2abx+\operatorname{arcsinh}(bx+a)^2-a^2-1}{4b}$ | 91 |

```
[In] int(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-b^2*x^2+2*arcsinh(b
*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-2*a*b*x+arcsinh(b*x+a)^2-a^2-1)/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx =$$

$$-\frac{b^2x^2+2\,abx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\log\left(bx+a+\right.}{4\,b}$$

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2)/b
```

## Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx = \int \sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)\,dx$$

```
[In] integrate(asinh(b*x+a)*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.

Time $= 0.25$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 3.90$

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx =$$

$$-\frac{1}{4}\left(x^2+\frac{2\,ax}{b}+\frac{2\,\operatorname{arsinh}\left(bx+a\right)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2}-\frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{b^2}\right)b$$

$$-\frac{1}{2}\left(\frac{a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\sqrt{b^2x^2+2\,abx+a^2+1}x-\frac{(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-$$

$$+a)$$

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(x^2 + 2*a*x/b + 2*arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))^2/b^2)*b - 1/2*(a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x - (a^2 + 1)*a
rcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b)*arcsinh(b*x + a)
```

## Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}\left(bx+a\right)\,dx$$

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\,dx = \int \operatorname{asinh}(a+b\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

# 3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)}\,dx$

Optimal result . . . 1904
Rubi [A] (verified) . . . 1904
Mathematica [A] (verified) . . . 1905
Maple [A] (verified) . . . 1906
Fricas [F] . . . 1906
Sympy [F] . . . 1906
Maxima [F] . . . 1906
Giac [F] . . . 1907
Mupad [F(-1)] . . . 1907

## Optimal result

Integrand size = 30, antiderivative size = 31

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\log(\text{arcsinh}(a+bx))}{2b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5860, 5791, 3393, 3382}

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\log(\text{arcsinh}(a+bx))}{2b}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + Log[ArcSinh[a + b*x]]/(2*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x} + \frac{\cosh(2x)}{2x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\log(\text{arcsinh}(a+bx))}{2b} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b} \\
&= \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\log(\text{arcsinh}(a+bx))}{2b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx)) + \log(\text{arcsinh}(a+bx))}{2b}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
[Out] (CoshIntegral[2*ArcSinh[a + b*x]] + Log[ArcSinh[a + b*x]])/(2*b)
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\operatorname{arcsinh}(bx+a))+\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))}{2b}$ | 23 |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a)))/b
```

## Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

## Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x), x)
```

## Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

## Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x),x)
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x), x)
```

# 3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\mathbf{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . . . 1908
Rubi [A] (verified) . . . . . 1908
Mathematica [A] (verified) . . . . . 1910
Maple [A] (verified) . . . . . 1910
Fricas [F] . . . . . 1910
Sympy [F] . . . . . 1911
Maxima [F] . . . . . 1911
Giac [F] . . . . . 1911
Mupad [F(-1)] . . . . . 1912

## Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1+(a+bx)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b}$$

```
[Out] (-1-(b*x+a)^2)/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5790, 5780, 5556, 12, 3379}

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b} - \frac{(a+bx)^2+1}{b\operatorname{arcsinh}(a+bx)}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*x]
]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{1+(a+bx)^2}{b\text{arcsinh}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= -\frac{1+(a+bx)^2}{b\text{arcsinh}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1+(a+bx)^2}{b\text{arcsinh}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}
\end{aligned}$$

$$= -\frac{1+(a+bx)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b}$$

$$= -\frac{1+(a+bx)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx$$
$$= -\frac{1+a^2+2abx+b^2x^2-\operatorname{arcsinh}(a+bx)\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b\operatorname{arcsinh}(a+bx)}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + a^2 + 2*a*b*x + b^2*x^2 - ArcSinh[a + b*x]*SinhIntegral[2*ArcSinh[a
+ b*x]])/(b*ArcSinh[a + b*x]))
```

## Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

| method | result | size |
|---|---|---|
| default | $\frac{2\,\operatorname{Shi}(2\,\operatorname{arcsinh}(bx+a))\,\operatorname{arcsinh}(bx+a)-\cosh(2\,\operatorname{arcsinh}(bx+a))-1}{2b\,\operatorname{arcsinh}(bx+a)}$ | 44 |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(2*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arc
sinh(b*x+a)
```

## Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

## Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^2(a+bx)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
 + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a^2
*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((2*b^2*x^2 + 4*a*b*x + 2*a^2
- 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a
^3 + (6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^4*x^4 + 8*a*
b^3*x^3 + 2*a^4 + 3*(4*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(4*a^3*b + 3*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3
*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^
2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
+ b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2,x)
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2, x)
```

# 3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 1913
Rubi [A] (verified) . . . 1913
Mathematica [A] (verified) . . . 1915
Maple [A] (verified) . . . 1915
Fricas [F] . . . 1915
Sympy [F] . . . 1916
Maxima [F] . . . 1916
Giac [F] . . . 1917
Mupad [F(-1)] . . . 1918

## Optimal result

Integrand size = 30, antiderivative size = 71

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx = \frac{-1-(a+bx)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b}$$

```
[Out] 1/2*(-1-(b*x+a)^2)/b/arcsinh(b*x+a)^2+Chi(2*arcsinh(b*x+a))/b-(b*x+a)*(1+(b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5860, 5790, 5778, 3382}

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b} - \frac{\sqrt{(a+bx)^2+1}(a+bx)}{b\text{arcsinh}(a+bx)} - \frac{(a+bx)^2+1}{2b\text{arcsinh}(a+bx)^2}$$

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + (a + b*x)^2)/(b*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{1+(a+bx)^2}{2b\text{arcsinh}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{1+(a+bx)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{1+(a+bx)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1+a^2+2abx+b^2x^2+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)-2\operatorname{arcsinh}(a+bx)^2\mathrm{Chi}(2\mathrm{arcs}}{2b\operatorname{arcsinh}(a+bx)^2}$$

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + a^2 + 2*a*b*x + b^2*x^2 + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2]*ArcSinh[a + b*x] - 2*ArcSinh[a + b*x]^2*CoshIntegral[2*ArcSinh[a + b
*x]])/(b*ArcSinh[a + b*x]^2)
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{4\,\mathrm{Chi}(2\,\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-2\sinh(2\,\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-\cosh(2\,\operatorname{arcsinh}(bx+a))-1}{4b\operatorname{arcsinh}(bx+a)^2}$ | 63 |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(4*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*sinh(2*arcsinh(b*x+a))*ar
csinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arcsinh(b*x+a)^2
```

## Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

## Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^3(a+bx)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**3,x)
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**3, x)
```

## Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
[Out] -1/2*((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3
*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^5*x^5 + 15*a*b^4*x^4 +
3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*
a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + ((2*b^4*x
^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(4*a^3*b + a*b)
*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (6*b^5*x^5 + 30*a*b^4*x^4 + 6*a^5
 + (60*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + 3*(20*a^3*b^2 + 7*a*b^2)*x^2 + (30*a^
4*b + 21*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*b^6*x^6
 + 36*a*b^5*x^5 + 6*a^6 + (90*a^2*b^4 + 11*b^4)*x^4 + 11*a^4 + 4*(30*a^3*b^
3 + 11*a*b^3)*x^3 + 6*(15*a^4*b^2 + 11*a^2*b^2 + b^2)*x^2 + 6*a^2 + 4*(9*a^
5*b + 11*a^3*b + 3*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^7*x^7 +
 14*a*b^6*x^6 + 2*a^7 + (42*a^2*b^5 + 5*b^5)*x^5 + 5*a^5 + 5*(14*a^3*b^4 +
5*a*b^4)*x^4 + 2*(35*a^4*b^3 + 25*a^2*b^3 + 2*b^3)*x^3 + 4*a^3 + 2*(21*a^5*
b^2 + 25*a^3*b^2 + 6*a*b^2)*x^2 + (14*a^6*b + 25*a^4*b + 12*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) + (b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^
5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3
*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a
^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^7*x^6 + 6*a*b^6*x^5
 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3
+ 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*x^3 + 3*a*b^3*x^2 +
3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^4 + 4*a
*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)
```

```
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2 + a*b^2)*x + 3*(b^6
*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b
^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*((4*b^4*x^4 + 16*a*b^3*x^3 + 4*a^4 + 2*(12*a^2*b^2 - b^2)*x
^2 - 2*a^2 + 4*(4*a^3*b - a*b)*x + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) +
 2*(8*b^5*x^5 + 40*a*b^4*x^4 + 8*a^5 + 4*(20*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4
*(20*a^3*b^2 + 3*a*b^2)*x^2 + (40*a^4*b + 12*a^2*b + b)*x + a)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 2*(12*b^6*x^6 + 72*a*b^5*x^5 + 12*a^6 + 18*(10*a^2*b^
4 + b^4)*x^4 + 18*a^4 + 24*(10*a^3*b^3 + 3*a*b^3)*x^3 + 6*(30*a^4*b^2 + 18*
a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(6*a^5*b + 6*a^3*b + a*b)*x - 1)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2) + 2*(8*b^7*x^7 + 56*a*b^6*x^6 + 8*a^7 + 4*(42*a^2
*b^5 + 5*b^5)*x^5 + 20*a^5 + 20*(14*a^3*b^4 + 5*a*b^4)*x^4 + 5*(56*a^4*b^3
+ 40*a^2*b^3 + 3*b^3)*x^3 + 15*a^3 + (168*a^5*b^2 + 200*a^3*b^2 + 45*a*b^2)
*x^2 + (56*a^6*b + 100*a^4*b + 45*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (4*b^8*x^8 + 32*a*b^7*x^7 + 4*a^8 + 14*(8*a^2*b^6 + b^6)*x^6 +
 14*a^6 + 28*(8*a^3*b^5 + 3*a*b^5)*x^5 + (280*a^4*b^4 + 210*a^2*b^4 + 17*b^
4)*x^4 + 17*a^4 + 4*(56*a^5*b^3 + 70*a^3*b^3 + 17*a*b^3)*x^3 + 2*(56*a^6*b^
2 + 105*a^4*b^2 + 51*a^2*b^2 + 4*b^2)*x^2 + 8*a^2 + 4*(8*a^7*b + 21*a^5*b +
 17*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x^8 + 8*
a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)
*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*
a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x
 + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2
*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 +
 a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b
*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)
*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2
)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6
+ a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35
*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*
a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3,x)
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3, x)
```

# 3.266 $\int (1+a^2+2abx+b^2x^2)^{3/2}\,\mathbf{arcsinh}(a+bx)^3\,dx$

Optimal result . . . 1919
Rubi [A] (verified) . . . 1920
Mathematica [A] (verified) . . . 1923
Maple [B] (verified) . . . 1924
Fricas [A] (verification not implemented) . . . 1924
Sympy [B] (verification not implemented) . . . 1925
Maxima [F] . . . 1925
Giac [F] . . . 1926
Mupad [F(-1)] . . . 1926

## Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}\int (1+a^2+2abx+b^2x^2)^{3/2}\,\operatorname{arcsinh}(a+bx)^3\,dx &= -\frac{51(a+bx)^2}{128b}\\ &-\frac{3(a+bx)^4}{128b}+\frac{45(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{64b}\\ &+\frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{32b}-\frac{27\operatorname{arcsinh}(a+bx)^2}{128b}\\ &-\frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b}-\frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b}\\ &+\frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b}\\ &+\frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b}+\frac{3\operatorname{arcsinh}(a+bx)^4}{32b}\end{aligned}$$

```
[Out] -51/128*(b*x+a)^2/b-3/128*(b*x+a)^4/b+3/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcs
inh(b*x+a)/b-27/128*arcsinh(b*x+a)^2/b-9/16*(b*x+a)^2*arcsinh(b*x+a)^2/b-3/
16*(1+(b*x+a)^2)^2*arcsinh(b*x+a)^2/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsi
nh(b*x+a)^3/b+3/32*arcsinh(b*x+a)^4/b+45/64*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5860, 5786, 5785, 5783, 5776, 5812, 30, 5798, 14}

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx =$$
$$-\frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\operatorname{arcsinh}(a+bx)^3}{4b}$$
$$+\frac{3\sqrt{(a+bx)^2+1}(a+bx)\operatorname{arcsinh}(a+bx)^3}{8b}$$
$$+\frac{3\left((a+bx)^2+1\right)^{3/2}(a+bx)\operatorname{arcsinh}(a+bx)}{32b}$$
$$+\frac{45\sqrt{(a+bx)^2+1}(a+bx)\operatorname{arcsinh}(a+bx)}{64b}$$
$$+\frac{3\operatorname{arcsinh}(a+bx)^4}{32b}-\frac{3\left((a+bx)^2+1\right)^2\operatorname{arcsinh}(a+bx)^2}{16b}$$
$$-\frac{27\operatorname{arcsinh}(a+bx)^2}{128b}-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-51*(a + b*x)^2)/(128*b) - (3*(a + b*x)^4)/(128*b) + (45*(a + b*x)*Sqrt[1
+ (a + b*x)^2]*ArcSinh[a + b*x])/(64*b) + (3*(a + b*x)*(1 + (a + b*x)^2)^(3
/2)*ArcSinh[a + b*x])/(32*b) - (27*ArcSinh[a + b*x]^2)/(128*b) - (9*(a + b*
x)^2*ArcSinh[a + b*x]^2)/(16*b) - (3*(1 + (a + b*x)^2)^2*ArcSinh[a + b*x]^2
)/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(8*b) + (
(a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/(4*b) + (3*ArcSinh[a
+ b*x]^4)/(32*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)^{3/2}\operatorname{arcsinh}(x)^3\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} \\
&\quad - \frac{3\text{Subst}\left(\int x(1+x^2)\operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{4b} \\
&\quad + \frac{3\text{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)^3\,dx, x, a+bx\right)}{4b} \\
&= -\frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} \\
&\quad + \frac{3\text{Subst}\left(\int (1+x^2)^{3/2}\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{8b} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b} - \frac{9\text{Subst}\left(\int x\operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{8b} \\
&= \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{32b} - \frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b} \\
&\quad - \frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^4}{32b} \\
&\quad - \frac{3\text{Subst}\left(\int x(1+x^2)\,dx, x, a+bx\right)}{32b} + \frac{9\text{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{32b} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{x^2\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{45(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{64b} \\
&\quad + \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{32b} - \frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b} \\
&\quad - \frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} \\
&\quad + \frac{3\operatorname{arcsinh}(a+bx)^4}{32b} - \frac{3\operatorname{Subst}\left(\int \left(x+x^3\right)\,dx, x, a+bx\right)}{32b} \\
&\quad - \frac{9\operatorname{Subst}\left(\int x\,dx, x, a+bx\right)}{64b} + \frac{9\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{64b} \\
&\quad - \frac{9\operatorname{Subst}\left(\int x\,dx, x, a+bx\right)}{16b} - \frac{9\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{16b} \\
&= -\frac{51(a+bx)^2}{128b} - \frac{3(a+bx)^4}{128b} + \frac{45(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{64b} \\
&\quad + \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{32b} - \frac{27\operatorname{arcsinh}(a+bx)^2}{128b} \\
&\quad - \frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b} - \frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b} \\
&\quad + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^4}{32b}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.13

$$
\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx =
$$

$$
-\frac{6a(17+2a^2)\,bx+3(17+6a^2)\,b^2x^2+12ab^3x^3+3b^4x^4-6\sqrt{1+a^2+2abx+b^2x^2}(17a+2a^3+17bx+6a^2
$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]

[Out] -1/128*(6*a*(17 + 2*a^2)*b*x + 3*(17 + 6*a^2)*b^2*x^2 + 12*a*b^3*x^3 + 3*b^
4*x^4 - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*
b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*(17 + 8*a^4 + 32*a^3*b*
x + 40*b^2*x^2 + 8*b^4*x^4 + 16*a*b*x*(5 + 2*b^2*x^2) + 8*a^2*(5 + 6*b^2*x^
2))*ArcSinh[a + b*x]^2 - 16*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3
+ 5*b*x + 6*a^2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^3 - 12*ArcS
inh[a + b*x]^4)/b
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(207) = 414$.

Time = 0.73 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| default | $\frac{-48-102abx-51a^2-51b^2x^2+12\,\mathrm{arcsinh}(bx+a)^4-51\,\mathrm{arcsinh}(bx+a)^2-12a\,b^3x^3-3b^4x^4+96\,\mathrm{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,a\,b^2x^2}$ |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-48-102*a*b*x-51*a^2-51*b^2*x^2+12*arcsinh(b*x+a)^4-51*arcsinh(b*x+a
)^2-12*a*b^3*x^3-3*b^4*x^4+96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*a*b^2*x^2+96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x+36*ar
csinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2+36*arcsinh(b*x+a)*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x-3*a^4+12*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)*b^3*x^3+32*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*
x^3-96*arcsinh(b*x+a)^2*a*b^3*x^3-144*arcsinh(b*x+a)^2*a^2*b^2*x^2-96*arcsi
nh(b*x+a)^2*a^3*b*x+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-2
40*arcsinh(b*x+a)^2*a*b*x+102*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
b*x-24*a^4*arcsinh(b*x+a)^2-120*a^2*arcsinh(b*x+a)^2-120*arcsinh(b*x+a)^2*b
^2*x^2+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+102*arcsinh(b*x+
a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+12*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*a^3+32*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-24*arcsin
h(b*x+a)^2*b^4*x^4-18*a^2*b^2*x^2-12*a^3*b*x)/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.41

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\,\mathrm{arcsinh}(a+bx)^3\,dx =$$

$$-\frac{3\,b^4x^4+12\,ab^3x^3+3\,(6\,a^2+17)b^2x^2-16\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a^2}}$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/128*(3*b^4*x^4 + 12*a*b^3*x^3 + 3*(6*a^2 + 17)*b^2*x^2 - 16*(2*b^3*x^3 +
 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 12*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4 + 6*(2*a^3 + 17*a)*b*x + 3*(8*b^4*x^4
+ 32*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*
a^2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^3*x^3
```

```
 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(223) = 446$.

Time $= 1.15$ (sec) , antiderivative size $= 694$, normalized size of antiderivative $= 2.95$

$$\int \left(1 + a^2 + 2abx + b^2x^2\right)^{3/2} \operatorname{arcsinh}(a$$

$$+bx)^3\,dx = \begin{cases} -\frac{3a^4 \operatorname{asinh}^2(a+bx)}{16b} - \frac{3a^3 x \operatorname{asinh}^2(a+bx)}{4} - \frac{3a^3 x}{32} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{4b} + \frac{3a^3\sqrt{a^2+2abx+b^2x^2+1}\,\text{asin}}{32b} \\ x(a^2+1)^{\frac{3}{2}} \operatorname{asinh}^3(a) \end{cases}$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-3*a**4*asinh(a + b*x)**2/(16*b) - 3*a**3*x*asinh(a + b*x)**2/4
- 3*a**3*x/32 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3
/(4*b) + 3*a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(32*b)
- 9*a**2*b*x**2*asinh(a + b*x)**2/8 - 9*a**2*b*x**2/64 + 3*a**2*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a**2*x*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a**2*asinh(a + b*x)**2/(16*b) -
 3*a*b**2*x**3*asinh(a + b*x)**2/4 - 3*a*b**2*x**3/32 + 3*a*b*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a*b*x**2*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a*x*asinh(a + b*x)**2/8 - 51
*a*x/64 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/(8*b)
+ 51*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(64*b) - 3*b**3*
x**4*asinh(a + b*x)**2/16 - 3*b**3*x**4/128 + b**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 3*b**2*x**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/32 - 15*b*x**2*asinh(a + b*x)**2/16 - 51*b*x**
2/128 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/8 + 51*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/64 + 3*asinh(a + b*x)*
*4/(32*b) - 51*asinh(a + b*x)**2/(128*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*a
sinh(a)**3, True))
```

## Maxima [F]

$$\int \left(1 + a^2 + 2abx + b^2x^2\right)^{3/2} \operatorname{arcsinh}(a + bx)^3\,dx = \int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^3\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

## Giac [F]

$$\int (1+a^2+2abx \\ +b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^3\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (1+a^2+2abx \\ +b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{asinh}(a+b\,x)^3\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.267 $\int (1+a^2+2abx+b^2x^2)^{3/2}\,\mathbf{arcsinh}(a+bx)^2\,dx$

Optimal result . . . 1927
Rubi [A] (verified) . . . 1927
Mathematica [A] (verified) . . . 1931
Maple [B] (verified) . . . 1931
Fricas [A] (verification not implemented) . . . 1932
Sympy [B] (verification not implemented) . . . 1932
Maxima [F] . . . 1933
Giac [F] . . . 1933
Mupad [F(-1)] . . . 1933

## Optimal result

Integrand size = 30, antiderivative size = 189

$$\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2\,dx = \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{9\operatorname{arcsinh}(a+bx)}{64b} - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b} - \frac{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} + \frac{\operatorname{arcsinh}(a+bx)^3}{8b}$$

```
[Out] 1/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b-9/64*arcsinh(b*x+a)/b-3/8*(b*x+a)^2*arcs
inh(b*x+a)/b-1/8*(1+(b*x+a)^2)^2*arcsinh(b*x+a)/b+1/4*(b*x+a)*(1+(b*x+a)^2)
^(3/2)*arcsinh(b*x+a)^2/b+1/8*arcsinh(b*x+a)^3/b+15/64*(b*x+a)*(1+(b*x+a)^2
)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used

$= \{5860, 5786, 5785, 5783, 5776, 327, 221, 5798, 201\}$

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{8b}$$
$$+ \frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b}$$
$$+ \frac{3(a+bx)\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2}{8b} - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b}$$
$$- \frac{\left((a+bx)^2+1\right)^2\operatorname{arcsinh}(a+bx)}{8b} - \frac{9\operatorname{arcsinh}(a+bx)}{64b}$$
$$+ \frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b} + \frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (15*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(64*b) + ((a + b*x)*(1 + (a + b*x)^2)^
(3/2))/(32*b) - (9*ArcSinh[a + b*x])/(64*b) - (3*(a + b*x)^2*ArcSinh[a + b*
x])/(8*b) - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/(8*b) + (3*(a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)
^(3/2)*ArcSinh[a + b*x]^2)/(4*b) + ArcSinh[a + b*x]^3/(8*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1+x^2)^{3/2} \operatorname{arcsinh}(x)^2 \, dx, x, a+bx\right)}{b}$$

$$
\begin{aligned}
&= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} \\
&\quad - \frac{\operatorname{Subst}\left(\int x(1+x^2)\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{2b} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)^2\,dx, x, a+bx\right)}{4b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} + \frac{\operatorname{Subst}\left(\int \left(1+x^2\right)^{3/2}\,dx, x, a+bx\right)}{8b} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b} - \frac{3\operatorname{Subst}\left(\int x\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{4b} \\
&= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b} - \frac{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}{8b} \\
&\quad + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} \\
&\quad + \frac{\operatorname{arcsinh}(a+bx)^3}{8b} + \frac{3\operatorname{Subst}\left(\int \sqrt{1+x^2}\,dx, x, a+bx\right)}{32b} + \frac{3\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b} \\
&= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} \\
&\quad - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b} - \frac{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}{8b} \\
&\quad + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} + \frac{\operatorname{arcsinh}(a+bx)^3}{8b} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{64b} - \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{16b} \\
&= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} \\
&\quad - \frac{9\operatorname{arcsinh}(a+bx)}{64b} - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b} \\
&\quad - \frac{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}{4b} + \frac{\operatorname{arcsinh}(a+bx)^3}{8b}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \frac{\sqrt{1+a^2+2abx+b^2x^2}(17a+2a^3+17bx+6a^2bx+6ab^2x^2+2b^3x^3)-(17+40a^2+8a^4)\operatorname{arcsin}$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x + 6*a
*b^2*x^2 + 2*b^3*x^3) - (17 + 40*a^2 + 8*a^4)*ArcSinh[a + b*x] - 8*b*x*(10*
a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*ArcSinh[a + b*x] + 8
*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b
^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^2 + 8*ArcSinh[a + b*x]^3)/(64*b)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(165) = 330$.

Time = 0.89 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.53

| method | result |
|---|---|
| default | $\frac{16\operatorname{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,b^3x^3-8\operatorname{arcsinh}(bx+a)b^4x^4+48\operatorname{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,a\,b^2x^2-32\operatorname{arcsinh}(bx+$ |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-8*arcsinh(b
*x+a)*b^4*x^4+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2-3
2*arcsinh(b*x+a)*a*b^3*x^3+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*a^2*b*x+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-48*arcsinh(b*x+a)*a^2*b^2
*x^2+16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3+6*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)*a*b^2*x^2-32*arcsinh(b*x+a)*a^3*b*x+40*arcsinh(b*x+a)^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x-8*ar
csinh(b*x+a)*a^4-40*arcsinh(b*x+a)*b^2*x^2+40*arcsinh(b*x+a)^2*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*a+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-80*arcsinh(b*x+a)*a
*b*x+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+8*arcsinh(b*x+a)^3-40*a^2*arcsinh
(b*x+a)+17*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-17*arcsinh(b*x+a))/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.37

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a$$

$$+bx)^2\,dx = \frac{8\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,ab}\right.$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] 1/64*(8*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 8*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - (8*b^4*x^4 + 32*a
*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*a^2 +
17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (2*b^3*x^3 + 6*a*b^2
*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(173) = 346$.

Time = 0.74 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.01

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a$$

$$+bx)^2\,dx = \begin{cases} -\frac{a^4\operatorname{asinh}(a+bx)}{8b} - \frac{a^3x\operatorname{asinh}(a+bx)}{2} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}}{32b} - \frac{3a^2bx^2\operatorname{asinh}(a+} \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}^2(a) \end{cases}$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(8*b) - a**3*x*asinh(a + b*x)/2 + a**3*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) + a**3*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)/(32*b) - 3*a**2*b*x**2*asinh(a + b*x)/4 + 3*a**2*
x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/4 + 3*a**2*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a**2*asinh(a + b*x)/(8*b) - a*b**2*x
**3*asinh(a + b*x)/2 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asin
h(a + b*x)**2/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a*
x*asinh(a + b*x)/4 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)**2/(8*b) + 17*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(64*b) - b**3*x**4*a
sinh(a + b*x)/8 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
```

```
b*x)**2/4 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*b*x**2*as
inh(a + b*x)/8 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2
/8 + 17*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/64 + asinh(a + b*x)**3/(8*b)
 - 17*asinh(a + b*x)/(64*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*asinh(a)**2, T
rue))
```

## Maxima [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^2\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

## Giac [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^2\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.268 $\int (1+a^2+2abx+b^2x^2)^{3/2}\,\mathbf{arcsinh}(a+bx)\,dx$

Optimal result . . . 1934
Rubi [A] (verified) . . . 1934
Mathematica [A] (verified) . . . 1936
Maple [B] (verified) . . . 1937
Fricas [A] (verification not implemented) . . . 1937
Sympy [B] (verification not implemented) . . . 1938
Maxima [B] (verification not implemented) . . . 1938
Giac [F] . . . 1939
Mupad [F(-1)] . . . 1939

## Optimal result

Integrand size = 28, antiderivative size = 106

$$\int (1+a^2+2abx+b^2x^2)^{3/2}\,\operatorname{arcsinh}(a+bx)\,dx = -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^2}{16b}$$

```
[Out] -5/16*(b*x+a)^2/b-1/16*(b*x+a)^4/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsinh(
b*x+a)/b+3/16*arcsinh(b*x+a)^2/b+3/8*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)/b
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5860, 5786, 5785, 5783, 30, 14}

$$\int (1+a^2+2abx + b^2x^2)^{3/2}\,\operatorname{arcsinh}(a+bx)\,dx = \frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\operatorname{arcsinh}(a+bx)}{4b} + \frac{3\sqrt{(a+bx)^2+1}(a+bx)\operatorname{arcsinh}(a+bx)}{8b} + \frac{3\operatorname{arcsinh}(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} - \frac{5(a+bx)^2}{16b}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-5*(a + b*x)^2)/(16*b) - (a + b*x)^4/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b
*x)^2]*ArcSinh[a + b*x])/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh
[a + b*x])/(4*b) + (3*ArcSinh[a + b*x]^2)/(16*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1+x^2)^{3/2}\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{4b} - \frac{\text{Subst}\left(\int x(1+x^2)\,dx, x, a+bx\right)}{4b} \\
&\quad + \frac{3\text{Subst}\left(\int \sqrt{1+x^2}\operatorname{arcsinh}(x)\,dx, x, a+bx\right)}{4b} \\
&= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{4b} - \frac{\text{Subst}\left(\int (x+x^3)\,dx, x, a+bx\right)}{4b} \\
&\quad - \frac{3\text{Subst}\left(\int x\,dx, x, a+bx\right)}{8b} + \frac{3\text{Subst}\left(\int \frac{\operatorname{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{8b} \\
&= -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b} \\
&\quad + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^2}{16b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)\,dx = \frac{-bx(10a+4a^3+5bx+6a^2bx+4ab^2x^2+b^3x^3)+2\sqrt{1+a^2+2abx+b^2x^2}(5a+2a^3+5bx+6a^2b$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]

[Out] (-(b*x*(10*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)) + 2*Sqrt
[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*x^
2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*ArcSinh[a + b*x]^2)/(16*b)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(92) = 184$.

Time = 0.72 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.47

| method | result |
|---|---|
| default | $\frac{4\,\operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,b^3x^3-b^4x^4+12\,\operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,a\,b^2x^2-4a\,b^3x^3+12\,\operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+}$ |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(4*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-b^4*x^4+12*arc
sinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2-4*a*b^3*x^3+12*arcsinh(
b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x-6*a^2*b^2*x^2+4*arcsinh(b*x+a)
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-4*a^3*b*x+10*arcsinh(b*x+a)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*b*x-a^4-5*b^2*x^2+10*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*a-10*a*b*x+3*arcsinh(b*x+a)^2-5*a^2-4)/b
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.51

$$\int \left(1 + a^2 + 2abx + b^2x^2\right)^{3/2} \operatorname{arcsinh}(a + bx)\,dx =$$

$$-\frac{b^4x^4 + 4\,ab^3x^3 + (6\,a^2 + 5)b^2x^2 + 2\,(2\,a^3 + 5\,a)bx - 2\,(2\,b^3x^3 + 6\,ab^2x^2 + 2\,a^3 + (6\,a^2 + 5)bx + 5\,a)\sqrt{b^2x^2}}{16\,b}$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] -1/16*(b^4*x^4 + 4*a*b^3*x^3 + (6*a^2 + 5)*b^2*x^2 + 2*(2*a^3 + 5*a)*b*x -
2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(95) = 190$.

Time = 0.49 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.81

$$\int \left(1 + a^2 + 2abx + b^2x^2\right)^{3/2} \operatorname{arcsinh}(a + bx)\, dx = \begin{cases} -\frac{a^3x}{4} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4b} - \frac{3a^2bx^2}{8} + \frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \frac{ab^2x^3}{4} + \frac{3abx^2\sqrt{a^2+2ab} \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}(a) \end{cases}$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**3*x/4 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)/(4*b) - 3*a**2*b*x**2/8 + 3*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)/4 - a*b**2*x**3/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*asinh(a + b*x)/4 - 5*a*x/8 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1)*asinh(a + b*x)/(8*b) - b**3*x**4/16 + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*asinh(a + b*x)/4 - 5*b*x**2/16 + 5*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/8 + 3*asinh(a + b*x)**2/(16*b), Ne(b, 0)), (x*
(a**2 + 1)**(3/2)*asinh(a), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(92) = 184$.

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.72

$$\int \left(1 + a^2 + 2abx + b^2x^2\right)^{3/2} \operatorname{arcsinh}(a + bx)\, dx = -\frac{1}{16}\left(b^2x^4 + 4\,abx^3 + 6\,a^2x^2 + \frac{4\,a^3x}{b} + 5\,x^2 + \frac{10\,ax}{b} + \frac{6\,\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2} - \frac{3\,\text{ars}}{} \right.$$

$$+\frac{1}{8}\left(2\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}x + \frac{2\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}a}{b} + \frac{3\left(a^2b^2 - (a^2+1)b^2\right)a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+}{\sqrt{-4\,a^2b^2+4}}\right.\right.}{b^3}$$

$$\left.\left.+\,a\right)\right.$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] -1/16*(b^2*x^4 + 4*a*b*x^3 + 6*a^2*x^2 + 4*a^3*x/b + 5*x^2 + 10*a*x/b + 6*a
rcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
/b^2 - 3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b^2)
```

```
*b + 1/8*(2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x + 2*(b^2*x^2 + 2*a*b*x +
a^2 + 1)^(3/2)*a/b + 3*(a^2*b^2 - (a^2 + 1)*b^2)*a^2*arcsinh(2*(b^2*x + a*b
)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1
)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)*arcsinh(b*x +
 a)
```

## Giac [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)\,dx = \int \operatorname{asinh}(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.269 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)}\,dx$

Optimal result . . . 1940
Rubi [A] (verified) . . . 1940
Mathematica [A] (verified) . . . 1942
Maple [A] (verified) . . . 1942
Fricas [F] . . . 1942
Sympy [F] . . . 1942
Maxima [F] . . . 1943
Giac [F] . . . 1943
Mupad [F(-1)] . . . 1943

## Optimal result

Integrand size = 30, antiderivative size = 47

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{8b} + \frac{3\log(\text{arcsinh}(a+bx))}{8b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x+
a))/b
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5860, 5791, 3393, 3382}

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{8b} + \frac{3\log(\text{arcsinh}(a+bx))}{8b}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + CoshIntegral[4*ArcSinh[a + b*x]]/(
8*b) + (3*Log[ArcSinh[a + b*x]])/(8*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} + \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{3\log(\text{arcsinh}(a+bx))}{8b} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b} \\
&= \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{8b} + \frac{3\log(\text{arcsinh}(a+bx))}{8b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \frac{4\operatorname{Chi}(2\operatorname{arcsinh}(a+bx)) + \operatorname{Chi}(4\operatorname{arcsinh}(a+bx)) + 3\log(\operatorname{arcsinh}(a+bx))}{8b}$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] (4*CoshIntegral[2*ArcSinh[a + b*x]] + CoshIntegral[4*ArcSinh[a + b*x]] + 3*
Log[ArcSinh[a + b*x]])/(8*b)
```

## Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| default | $\frac{3\ln(\operatorname{arcsinh}(bx+a))+4\ \operatorname{Chi}(2\ \operatorname{arcsinh}(bx+a))+\operatorname{Chi}(4\ \operatorname{arcsinh}(bx+a))}{8b}$ | 36 |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*ln(arcsinh(b*x+a))+4*Chi(2*arcsinh(b*x+a))+Chi(4*arcsinh(b*x+a)))/b
```

## Fricas [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

## Sympy [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x), x)
```

## Maxima [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\,(bx+a)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

## Giac [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\,(bx+a)}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}{\operatorname{asinh}\,(a+b\,x)}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x),x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x), x)
```

# 3.270 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 1944
Rubi [A] (verified) . . . 1944
Mathematica [A] (verified) . . . 1946
Maple [A] (verified) . . . 1946
Fricas [F] . . . 1947
Sympy [F] . . . 1947
Maxima [F] . . . 1947
Giac [F] . . . 1948
Mupad [F(-1)] . . . 1948

## Optimal result

Integrand size = 30, antiderivative size = 54

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{\left(1+(a+bx)^2\right)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(a+bx))}{2b}$$

```
[Out] -(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b+1/2*Shi(4*arcsinh
(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5860, 5790, 5819, 5556, 3379}

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(a+bx))}{2b} - \frac{\left((a+bx)^2+1\right)^2}{b\operatorname{arcsinh}(a+bx)}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*
x]]/b + SinhIntegral[4*ArcSinh[a + b*x]]/(2*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{(1+(a+bx)^2)^2}{b\,\text{arcsinh}(a+bx)} + \frac{4\text{Subst}\left(\int \frac{x(1+x^2)}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= -\frac{(1+(a+bx)^2)^2}{b\,\text{arcsinh}(a+bx)} + \frac{4\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b}
\end{aligned}$$

$$= -\frac{(1+(a+bx)^2)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{4\operatorname{Subst}\left(\int \left(\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b}$$

$$= -\frac{(1+(a+bx)^2)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b}$$
$$+ \frac{\operatorname{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b}$$

$$= -\frac{(1+(a+bx)^2)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(a+bx))}{2b}$$

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{-2(1+a^2+2abx+b^2x^2)^2 + 2\operatorname{arcsinh}(a+bx)\operatorname{Shi}(2\operatorname{arcsinh}(a+bx)) + \text{arc}}{2b\operatorname{arcsinh}(a+bx)}$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*(1 + a^2 + 2*a*b*x + b^2*x^2)^2 + 2*ArcSinh[a + b*x]*SinhIntegral[2*Arc
Sinh[a + b*x]] + ArcSinh[a + b*x]*SinhIntegral[4*ArcSinh[a + b*x]])/(2*b*Ar
cSinh[a + b*x])
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| default | $\frac{8\ \operatorname{Shi}(2\ \operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)+4\ \operatorname{Shi}(4\ \operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-4\cosh(2\ \operatorname{arcsinh}(bx+a))-\cosh(4\ \operatorname{arcsinh}(bx+a))-3}{8b\ \operatorname{arcsinh}(bx+a)}$ |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(8*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)+4*Shi(4*arcsinh(b*x+a))*arcsi
nh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/arcsinh(b*x+a)
```

## Fricas [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

## Sympy [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**2, x)
```

## Maxima [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2 + 4*(a^3*b
 + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^5*x^5 + 5*a*b^4*x^4 + a^5
 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b
 + 6*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b
^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((4*b^4*x^4 + 16*a*b
^3*x^3 + 4*a^4 + 3*(8*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(8*a^3*b + 3*a*b)*x -
1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(2*b^5*x^5 + 10*a*b^4*x^4 + 2*a^
5 + (20*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + (20*a^3*b^2 + 9*a*b^2)*x^2 + (10*a^4
*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^6*x^6 + 24*a*
b^5*x^5 + 4*a^6 + 3*(20*a^2*b^4 + 3*b^4)*x^4 + 9*a^4 + 4*(20*a^3*b^3 + 9*a*
b^3)*x^3 + 6*(10*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(2*a^5*b + 3*a
^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x
^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2, x)
```

# 3.271 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 1949
Rubi [A] (verified) . . . 1949
Mathematica [A] (verified) . . . 1952
Maple [A] (verified) . . . 1952
Fricas [F] . . . 1953
Sympy [F] . . . 1953
Maxima [F] . . . 1953
Giac [F] . . . 1955
Mupad [F(-1)] . . . 1955

## Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx = -\frac{\left(1+(a+bx)^2\right)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{b}$$

```
[Out] -1/2*(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)^2-2*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b/arc
sinh(b*x+a)+Chi(2*arcsinh(b*x+a))/b+Chi(4*arcsinh(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5860, 5790, 5814, 5791, 3393, 3382, 5819, 5556}

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{b} - \frac{\left((a+bx)^2+1\right)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\text{arcsinh}(a+bx)}$$

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x]^2) - (2*(a + b*x)*(1 + (a + b*
x)^2)^(3/2))/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b + Co
shIntegral[4*ArcSinh[a + b*x]]/b
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\text{arcsinh}(a+bx)^2} + \frac{2\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\text{arcsinh}(a+bx)} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} + \frac{8\text{Subst}\left(\int \frac{x^2\sqrt{1+x^2}}{\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\text{arcsinh}(a+bx)} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\text{arcsinh}(a+bx)} \\
&\quad + \frac{2\text{Subst}\left(\int \left(\frac{1}{2x} + \frac{\cosh(2x)}{2x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&\quad + \frac{8\text{Subst}\left(\int \left(-\frac{1}{8x} + \frac{\cosh(4x)}{8x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\operatorname{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\operatorname{arcsinh}(a+bx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= -\frac{\left(1+(a+bx)^2\right)^2}{2b\operatorname{arcsinh}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\operatorname{arcsinh}(a+bx)} \\
&\quad + \frac{\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{b} + \frac{\operatorname{Chi}(4\operatorname{arcsinh}(a+bx))}{b}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$
\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \frac{-\frac{\left(1+a^2+2abx+b^2x^2\right)\left(1+a^2+2abx+b^2x^2+4(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)\right)}{\operatorname{arcsinh}(a+bx)^2} + 2\mathrm{Ch}}{2b}
$$

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] (-(((1 + a^2 + 2*a*b*x + b^2*x^2)*(1 + a^2 + 2*a*b*x + b^2*x^2 + 4*(a + b*x
)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2)
+ 2*CoshIntegral[2*ArcSinh[a + b*x]] + 2*CoshIntegral[4*ArcSinh[a + b*x]])/
(2*b)
```

## Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| default | $\frac{16\ \mathrm{Chi}(2\ \operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2+16\ \mathrm{Chi}(4\ \operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-8\sinh(2\ \operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-4\sinh($ <br> $16b\operatorname{arcsinh}(bx+a)^2$ |

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(16*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+16*Chi(4*arcsinh(b*x+a))*
arcsinh(b*x+a)^2-8*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-4*sinh(4*arcsinh(b
*x+a))*arcsinh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/ar
csinh(b*x+a)^2
```

## Fricas [F]

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

## Sympy [F]

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}{\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**3, x)
```

## Maxima [F]

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^7*x^7 + 2
1*a*b^6*x^6 + 3*a^7 + (63*a^2*b^5 + 8*b^5)*x^5 + 8*a^5 + 5*(21*a^3*b^4 + 8*
a*b^4)*x^4 + (105*a^4*b^3 + 80*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (63*a^5*b^2 +
 80*a^3*b^2 + 21*a*b^2)*x^2 + (21*a^6*b + 40*a^4*b + 21*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^8*x^8 + 24*a*b^7*x^7 + 3*a^8
+ 2*(42*a^2*b^6 + 5*b^6)*x^6 + 10*a^6 + 12*(14*a^3*b^5 + 5*a*b^5)*x^5 + 6*(
35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 12*a^4 + 8*(21*a^5*b^3 + 25*a^3*b^3
+ 6*a*b^3)*x^3 + 6*(14*a^6*b^2 + 25*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + 6*a^2
 + 12*(2*a^7*b + 5*a^5*b + 4*a^3*b + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 +
 1) + ((4*b^6*x^6 + 24*a*b^5*x^5 + 4*a^6 + (60*a^2*b^4 + 7*b^4)*x^4 + 7*a^4
 + 4*(20*a^3*b^3 + 7*a*b^3)*x^3 + 2*(30*a^4*b^2 + 21*a^2*b^2 + b^2)*x^2 + 2
*a^2 + 4*(6*a^5*b + 7*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 +
```

```
 3*(4*b^7*x^7 + 28*a*b^6*x^6 + 4*a^7 + 3*(28*a^2*b^5 + 3*b^5)*x^5 + 9*a^5 +
 5*(28*a^3*b^4 + 9*a*b^4)*x^4 + 2*(70*a^4*b^3 + 45*a^2*b^3 + 3*b^3)*x^3 + 6
*a^3 + 6*(14*a^5*b^2 + 15*a^3*b^2 + 3*a*b^2)*x^2 + (28*a^6*b + 45*a^4*b + 1
8*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (12*b^8*x^8 + 96*
a*b^7*x^7 + 12*a^8 + 3*(112*a^2*b^6 + 11*b^6)*x^6 + 33*a^6 + 6*(112*a^3*b^5
 + 33*a*b^5)*x^5 + (840*a^4*b^4 + 495*a^2*b^4 + 31*b^4)*x^4 + 31*a^4 + 4*(1
68*a^5*b^3 + 165*a^3*b^3 + 31*a*b^3)*x^3 + (336*a^6*b^2 + 495*a^4*b^2 + 186
*a^2*b^2 + 11*b^2)*x^2 + 11*a^2 + 2*(48*a^7*b + 99*a^5*b + 62*a^3*b + 11*a*
b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^9*x^9 + 36*a*b^8*x^8 + 4*a^9
 + (144*a^2*b^7 + 13*b^7)*x^7 + 13*a^7 + 7*(48*a^3*b^6 + 13*a*b^6)*x^6 + 3*
(168*a^4*b^5 + 91*a^2*b^5 + 5*b^5)*x^5 + 15*a^5 + (504*a^5*b^4 + 455*a^3*b^
4 + 75*a*b^4)*x^4 + (336*a^6*b^3 + 455*a^4*b^3 + 150*a^2*b^3 + 7*b^3)*x^3 +
 7*a^3 + 3*(48*a^7*b^2 + 91*a^5*b^2 + 50*a^3*b^2 + 7*a*b^2)*x^2 + (36*a^8*b
 + 91*a^6*b + 75*a^4*b + 21*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (b^9*x^9 + 9*a*b^8
*x^8 + a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 4*a^7 + 28*(3*a^3*b^6 + a*b^6)*x^6 +
 6*(21*a^4*b^5 + 14*a^2*b^5 + b^5)*x^5 + 6*a^5 + 2*(63*a^5*b^4 + 70*a^3*b^4
 + 15*a*b^4)*x^4 + 4*(21*a^6*b^3 + 35*a^4*b^3 + 15*a^2*b^3 + b^3)*x^3 + 4*a
^3 + 12*(3*a^7*b^2 + 7*a^5*b^2 + 5*a^3*b^2 + a*b^2)*x^2 + (9*a^8*b + 28*a^6
*b + 30*a^4*b + 12*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b
^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3
*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*
x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2
*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2
 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b
^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)
*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2) + integrate(1/2*((16*b^6*x^6 + 96*a*b^5*x^5 + 16*a^6 +
10*(24*a^2*b^4 + b^4)*x^4 + 10*a^4 + 40*(8*a^3*b^3 + a*b^3)*x^3 + 3*(80*a^4
*b^2 + 20*a^2*b^2 - b^2)*x^2 - 3*a^2 + 2*(48*a^5*b + 20*a^3*b - 3*a*b)*x +
3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(16*b^7*x^7 + 112*a*b^6*x^6 + 16
*a^7 + (336*a^2*b^5 + 23*b^5)*x^5 + 23*a^5 + 5*(112*a^3*b^4 + 23*a*b^4)*x^4
 + (560*a^4*b^3 + 230*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (336*a^5*b^2 + 230*a^3
*b^2 + 21*a*b^2)*x^2 + (112*a^6*b + 115*a^4*b + 21*a^2*b)*x)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^2 + 12*(8*b^8*x^8 + 64*a*b^7*x^7 + 8*a^8 + 2*(112*a^2*b^6 +
 9*b^6)*x^6 + 18*a^6 + 4*(112*a^3*b^5 + 27*a*b^5)*x^5 + (560*a^4*b^4 + 270*
a^2*b^4 + 13*b^4)*x^4 + 13*a^4 + 4*(112*a^5*b^3 + 90*a^3*b^3 + 13*a*b^3)*x^
3 + (224*a^6*b^2 + 270*a^4*b^2 + 78*a^2*b^2 + 3*b^2)*x^2 + 3*a^2 + 2*(32*a^
7*b + 54*a^5*b + 26*a^3*b + 3*a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 4*(16*b^9*x^9 + 144*a*b^8*x^8 + 16*a^9 + (576*a^2*b^7 + 49*b^7)*x^7 + 49*a
^7 + 7*(192*a^3*b^6 + 49*a*b^6)*x^6 + 3*(672*a^4*b^5 + 343*a^2*b^5 + 18*b^5
)*x^5 + 54*a^5 + (2016*a^5*b^4 + 1715*a^3*b^4 + 270*a*b^4)*x^4 + (1344*a^6*
b^3 + 1715*a^4*b^3 + 540*a^2*b^3 + 25*b^3)*x^3 + 25*a^3 + 3*(192*a^7*b^2 +
343*a^5*b^2 + 180*a^3*b^2 + 25*a*b^2)*x^2 + (144*a^8*b + 343*a^6*b + 270*a^
```

```
4*b + 75*a^2*b + 4*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (16*b^10*x^1
0 + 160*a*b^9*x^9 + 16*a^10 + 2*(360*a^2*b^8 + 31*b^8)*x^8 + 62*a^8 + 16*(1
20*a^3*b^7 + 31*a*b^7)*x^7 + 7*(480*a^4*b^6 + 248*a^2*b^6 + 13*b^6)*x^6 + 9
1*a^6 + 14*(288*a^5*b^5 + 248*a^3*b^5 + 39*a*b^5)*x^5 + (3360*a^6*b^4 + 434
0*a^4*b^4 + 1365*a^2*b^4 + 61*b^4)*x^4 + 61*a^4 + 4*(480*a^7*b^3 + 868*a^5*
b^3 + 455*a^3*b^3 + 61*a*b^3)*x^3 + (720*a^8*b^2 + 1736*a^6*b^2 + 1365*a^4*
b^2 + 366*a^2*b^2 + 17*b^2)*x^2 + 17*a^2 + 2*(80*a^9*b + 248*a^7*b + 273*a^
5*b + 122*a^3*b + 17*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x
^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3
*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^
3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*
a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2
+ 9*a^2*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3
)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 +
 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^
2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b
^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^
4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b
^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
, x)
```

## Giac [F]

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3, x)
```

# 3.272 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result . . . . . 1956
Rubi [A] (verified) . . . . . 1956
Mathematica [A] (verified) . . . . . 1957
Maple [A] (verified) . . . . . 1957
Fricas [B] (verification not implemented) . . . . . 1957
Sympy [B] (verification not implemented) . . . . . 1958
Maxima [B] (verification not implemented) . . . . . 1958
Giac [F] . . . . . 1959
Mupad [B] (verification not implemented) . . . . . 1959

## Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^4}{4b}$$

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^4}{4b}$$

```
[In] Int[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\ &= \frac{\text{arcsinh}(a+bx)^4}{4b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{arcsinh}(a+bx)^4}{4b}$$

```
[In] Integrate[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
[Out] ArcSinh[a + b*x]^4/(4*b)
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\text{arcsinh}(bx+a)^4}{4b}$ | 14 |

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*arcsinh(b*x+a)^4/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4}{4\,b}$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
[Out] 1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^4(a+bx)}{4b} & \text{for } b \neq 0 \\ \frac{x\operatorname{asinh}^3(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*asinh(a)**3/sqrt(a**2 + 1
), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 11.93

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = {} & \frac{\operatorname{arsinh}\left(bx+a\right)^3 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b} \\ & - \frac{3\operatorname{arsinh}\left(bx+a\right)^2 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{2\,b} \\ & + \frac{\operatorname{arsinh}\left(bx+a\right) \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^3}{b} \\ & - \frac{\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^4}{4\,b}\end{aligned}$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - 3/2*arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2))^2/b + arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))^3/b - 1/4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))^4/b
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

## Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^4}{4\,b}$$

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^4/(4*b)
```

# 3.273 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result . . . . . 1960
Rubi [A] (verified) . . . . . 1960
Mathematica [A] (verified) . . . . . 1961
Maple [A] (verified) . . . . . 1961
Fricas [B] (verification not implemented) . . . . . 1961
Sympy [B] (verification not implemented) . . . . . 1962
Maxima [B] (verification not implemented) . . . . . 1962
Giac [F] . . . . . 1963
Mupad [B] (verification not implemented) . . . . . 1963

## Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{3b}$$

```
[Out] 1/3*arcsinh(b*x+a)^3/b
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{3b}$$

```
[In] Int[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^3/(3*b)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\ &= \frac{\text{arcsinh}(a+bx)^3}{3b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{arcsinh}(a+bx)^3}{3b}$$

```
[In] Integrate[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
[Out] ArcSinh[a + b*x]^3/(3*b)
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\text{arcsinh}(bx+a)^3}{3b}$ | 14 |

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsinh(b*x+a)^3/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{3\,b}$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
[Out] 1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^3(a+bx)}{3b} & \text{for } b \neq 0 \\ \frac{x\operatorname{asinh}^2(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*asinh(a)**2/sqrt(a**2 + 1
), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(13) = 26$.

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 8.80

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = {} & \frac{\operatorname{arsinh}\left(bx+a\right)^2 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b} \\ & - \frac{\operatorname{arsinh}\left(bx+a\right) \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{b} \\ & + \frac{\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^3}{3\,b}\end{aligned}$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2))^2/b + 1/3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2))^3/b
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

## Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^3}{3\,b}$$

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^3/(3*b)
```

# 3.274 $\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result . . . 1964
Rubi [A] (verified) . . . 1964
Mathematica [A] (verified) . . . 1965
Maple [A] (verified) . . . 1965
Fricas [B] (verification not implemented) . . . 1965
Sympy [B] (verification not implemented) . . . 1966
Maxima [B] (verification not implemented) . . . 1966
Giac [F] . . . 1966
Mupad [B] (verification not implemented) . . . 1967

## Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^2}{2b}$$

```
[Out] 1/2*arcsinh(b*x+a)^2/b
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5860, 5783}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^2}{2b}$$

```
[In] Int[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^2/(2*b)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\ &= \frac{\text{arcsinh}(a+bx)^2}{2b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{arcsinh}(a+bx)^2}{2b}$$

```
[In] Integrate[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
[Out] ArcSinh[a + b*x]^2/(2*b)
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\text{arcsinh}(bx+a)^2}{2b}$ | 14 |

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(b*x+a)^2/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,b}$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
[Out] 1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ \frac{x\,\operatorname{asinh}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*asinh(a)/sqrt(a**2 + 1),
True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b} - \frac{\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)^2}{2b}$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b - 1/2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

## Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^2}{2\,b}$$

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] asinh(a + b*x)^2/(2*b)
```

# 3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx$

Optimal result . . . 1968
Rubi [A] (verified) . . . 1968
Mathematica [A] (verified) . . . 1969
Maple [A] (verified) . . . 1969
Fricas [B] (verification not implemented) . . . 1969
Sympy [B] (verification not implemented) . . . 1970
Maxima [F] . . . 1970
Giac [F] . . . 1970
Mupad [B] (verification not implemented) . . . 1970

## Optimal result

Integrand size = 30, antiderivative size = 11

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \frac{\log(\operatorname{arcsinh}(a+bx))}{b}$$

```
[Out] ln(arcsinh(b*x+a))/b
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5782}

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \frac{\log(\operatorname{arcsinh}(a+bx))}{b}$$

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

```
[Out] Log[ArcSinh[a + b*x]]/b
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
```

```
n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx, x, a+bx\right)}{b} \\ &= \frac{\log(\text{arcsinh}(a+bx))}{b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx = \frac{\log(\text{arcsinh}(a+bx))}{b}$$

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
[Out] Log[ArcSinh[a + b*x]]/b
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\text{arcsinh}(bx+a))}{b}$ | 12 |

```
[In] int(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(arcsinh(b*x+a))/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx = \frac{\log\left(\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right)}{b}$$

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
[Out] log(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \begin{cases} \frac{\log(\operatorname{asinh}(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((log(asinh(a + b*x))/b, Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)),
True))
```

## Maxima [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

## Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \frac{\ln(\operatorname{asinh}(a+b\,x))}{b}$$

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] log(asinh(a + b*x))/b
```

# 3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . . . 1971
Rubi [A] (verified) . . . . . 1971
Mathematica [A] (verified) . . . . . 1972
Maple [A] (verified) . . . . . 1972
Fricas [B] (verification not implemented) . . . . . 1973
Sympy [B] (verification not implemented) . . . . . 1973
Maxima [B] (verification not implemented) . . . . . 1973
Giac [F] . . . . . 1974
Mupad [B] (verification not implemented) . . . . . 1974

## Optimal result

Integrand size = 30, antiderivative size = 13

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\operatorname{arcsinh}(a+bx)}$$

```
[Out] -1/b/arcsinh(b*x+a)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\operatorname{arcsinh}(a+bx)}$$

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]

[Out] -(1/(b*ArcSinh[a + b*x]))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\text{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b} \\ &= -\frac{1}{b\text{arcsinh}(a+bx)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\text{arcsinh}(a+bx)}$$

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
[Out] -(1/(b*ArcSinh[a + b*x]))
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| default | $-\frac{1}{b\ \text{arcsinh}(bx+a)}$ | 14 |

```
[In] int(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOS
E)
[Out] -1/b/arcsinh(b*x+a)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time $= 0.25$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 2.46$

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time $= 0.51$ (sec) , antiderivative size $= 26$, normalized size of antiderivative $= 2.00$

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx = \begin{cases} -\frac{1}{b\operatorname{asinh}(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^2(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*asinh(a + b*x)), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**2
), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(13) = 26$.

Time $= 0.33$ (sec) , antiderivative size $= 150$, normalized size of antiderivative $= 11.54$

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx =$$
$$-\frac{b^3x^3+3\,ab^2x^2+a^3+(3\,a^2b+b)x+(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}+a}{\left((b^2x^2+2\,abx+a^2+1)(b^2x+ab)+(b^3x^2+2\,ab^2x+a^2b+b)\sqrt{b^2x^2+2\,abx+a^2+1}\right)\log\left(bx+a+\sqrt{}\right.}$$

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/(((b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + (b^3*x^2 +
2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

## Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx$$
$$= \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2), x)
```

## Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\,\operatorname{asinh}\left(a+b\,x\right)}$$

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(b*asinh(a + b*x))
```

# 3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx$

Optimal result . . . 1975
Rubi [A] (verified) . . . 1975
Mathematica [A] (verified) . . . 1976
Maple [A] (verified) . . . 1976
Fricas [B] (verification not implemented) . . . 1977
Sympy [B] (verification not implemented) . . . 1977
Maxima [F] . . . 1977
Giac [F] . . . 1979
Mupad [B] (verification not implemented) . . . 1979

## Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2b\operatorname{arcsinh}(a+bx)^2}$$

```
[Out] -1/2/b/arcsinh(b*x+a)^2
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2b\operatorname{arcsinh}(a+bx)^2}$$

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

```
[Out] -1/2*1/(b*ArcSinh[a + b*x]^2)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\text{arcsinh}(x)^3}\,dx, x, a+bx\right)}{b} \\ &= -\frac{1}{2b\text{arcsinh}(a+bx)^2}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2b\text{arcsinh}(a+bx)^2}$$

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
[Out] -1/2*1/(b*ArcSinh[a + b*x]^2)
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $-\frac{1}{2b\,\text{arcsinh}(bx+a)^2}$ | 14 |

```
[In] int(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOS
E)
[Out] -1/2/b/arcsinh(b*x+a)^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time $= 0.25$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 2.13$

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time $= 0.77$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 1.93$

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = \begin{cases} -\frac{1}{2b\operatorname{asinh}^2(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^3(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(2*b*asinh(a + b*x)**2), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**3), True))
```

## Maxima [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
```

```
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^5*x^
5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*
a*b^2)*x^2 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) - (b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (5*a^4*b + 6*a^
2*b + b)*x + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2
 + 4*(a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6
+ (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4
*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/(((b^4*x^3 + 3*a*b^3*x^2 + 3*a^2
*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 3*(b^5*x^4 + 4*a*b^4*x^3
+ a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b +
2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 +
6*a^2*b^2 + b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^7*x^6 + 6*a*b^6*x^5
+ a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3 +
 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + 6*(a^5*b^2 + 2*a^3*b^2 + a
*b^2)*x + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2) + integrate(-1/2*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6
 + 3*(10*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4*(10*a^3*b^3 + 3*a*b^3)*x^3 - (2*b^2
*x^2 + 4*a*b*x + 2*a^2 + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(5*a^4*b^2
+ 3*a^2*b^2)*x^2 - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*(b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 12*
(a^5*b + a^3*b)*x + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^
3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(b^5*x^5 + 5*
a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2
 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(b^6*x^6 + 6*
a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3
)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b +
a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^
7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4
*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^
2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2
 + 1) + (b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*
(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4
 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9
*a^2*b^2 + b^2)*x^2 + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 1)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx$$
$$= \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3), x)
```

## Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2\,b\,\operatorname{asinh}\left(a+b\,x\right)^2}$$

```
[In] int(1/(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(2*b*asinh(a + b*x)^2)
```

# 3.278 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result . . . 1980
Rubi [A] (verified) . . . 1980
Mathematica [A] (verified) . . . 1983
Maple [A] (verified) . . . 1984
Fricas [F] . . . 1984
Sympy [F] . . . 1984
Maxima [F] . . . 1985
Giac [F] . . . 1985
Mupad [F(-1)] . . . 1985

## Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\operatorname{arcsinh}(a+bx)^2\log\left(1+e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} - \frac{3\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{2b}$$

```
[Out] arcsinh(b*x+a)^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b
-3*arcsinh(b*x+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3
,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^3/b/(1+(b*x+a)^2)
^(1/2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5860, 5787, 5797, 3799, 2221, 2611, 2320, 6724}

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{3\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{2b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b\sqrt{(a+bx)^2+1}} + \frac{\operatorname{arcsinh}(a+bx)^3}{b} - \frac{3\operatorname{arcsinh}(a+bx)^2\log\left(e^{2\operatorname{arcsinh}(a+bx)}+1\right)}{b}$$

```
[In] Int[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^3/b + ((a + b*x)*ArcSinh[a + b*x]^3)/(b*Sqrt[1 + (a + b*x)
^2]) - (3*ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x])])/b - (3*ArcSin
h[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/b + (3*PolyLog[3, -E^(2*Arc
Sinh[a + b*x])])/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^3}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\text{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\text{Subst}\left(\int \frac{x\text{arcsinh}(x)^2}{1+x^2}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\text{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{6\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{3\text{arcsinh}(a+bx)^2\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} \\
&\quad + \frac{6\text{Subst}\left(\int x\log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\operatorname{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{3\operatorname{arcsinh}(a+bx)^2 \log\left(1+e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} \\
&\quad - \frac{3\operatorname{arcsinh}(a+bx)\,\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\operatorname{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{3\operatorname{arcsinh}(a+bx)^2 \log\left(1+e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} \\
&\quad - \frac{3\operatorname{arcsinh}(a+bx)\,\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(a+bx)}\right)}{2b} \\
&= \frac{\operatorname{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\operatorname{arcsinh}(a+bx)^2 \log\left(1+e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} \\
&\quad - \frac{3\operatorname{arcsinh}(a+bx)\,\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} + \frac{3\,\operatorname{PolyLog}\left(3,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{2b}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$
\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{2\operatorname{arcsinh}(a+bx)^2\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 3\log\left(1+e^{-2a}\right.\right.}{}
$$

```
[In] Integrate[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*ArcSinh[a + b*x]^2*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSi
nh[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*Log[1 + E^(-2*ArcSinh[a
+ b*x])]) + 6*ArcSinh[a + b*x]*PolyLog[2, -E^(-2*ArcSinh[a + b*x])] + 3*Pol
yLog[3, -E^(-2*ArcSinh[a + b*x])])/(2*b)
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| default | $-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\operatorname{arcsinh}(bx+a)^3}{b(b^2x^2+2abx+a^2+1)}+\frac{2\operatorname{arcsinh}(bx+a)^3}{b}-\frac{3\operatorname{arcsinh}(bx+a)^2\ln$ |

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^3+2*arcsinh(b*x+a)
^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-3*arcsinh(b*x
+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3,-(b*x+a+(1+(b
*x+a)^2)^(1/2))^2)/b
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^3\left(a+bx\right)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}\,dx$$

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.279 $\int \frac{\text{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result . . . 1986
Rubi [A] (verified) . . . 1986
Mathematica [A] (verified) . . . 1989
Maple [A] (verified) . . . 1989
Fricas [F] . . . 1989
Sympy [F] . . . 1990
Maxima [F] . . . 1990
Giac [F] . . . 1990
Mupad [F(-1)] . . . 1990

## Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{\text{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\text{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\text{arcsinh}(a+bx)\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{2\text{arcsinh}(a+bx)}\right)}{b}$$

```
[Out] arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-p
olylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^2/b/(1+(b
*x+a)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5860, 5787, 5797, 3799, 2221, 2317, 2438}

$$\int \frac{\text{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{\text{PolyLog}\left(2,-e^{2\text{arcsinh}(a+bx)}\right)}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{(a+bx)^2+1}} + \frac{\text{arcsinh}(a+bx)^2}{b} - \frac{2\text{arcsinh}(a+bx)\log\left(e^{2\text{arcsinh}(a+bx)}+1\right)}{b}$$

```
[In] Int[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^2/b + ((a + b*x)*ArcSinh[a + b*x]^2)/(b*Sqrt[1 + (a + b*x)
^2]) - (2*ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])])/b - PolyLog[2,
-E^(2*ArcSinh[a + b*x])]/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)^2}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\text{Subst}\left(\int \frac{x\text{arcsinh}(x)}{1+x^2}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\text{Subst}(\int x\tanh(x)\,dx, x, \text{arcsinh}(a+bx))}{b} \\
&= \frac{\text{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{4\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{2\text{arcsinh}(a+bx)\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} \\
&\quad + \frac{2\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{2\text{arcsinh}(a+bx)\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{arcsinh}(a+bx)}\right)}{b} \\
&= \frac{\text{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} \\
&\quad - \frac{2\text{arcsinh}(a+bx)\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} - \frac{\text{PolyLog}\left(2, -e^{2\text{arcsinh}(a+bx)}\right)}{b}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 2\log\left(1+e^{-2\operatorname{arcsi}}\right.\right.}{b}$$

```
[In] Integrate[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (ArcSinh[a + b*x]*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSinh[a
 + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 2*Log[1 + E^(-2*ArcSinh[a + b*
x])]) + PolyLog[2, -E^(-2*ArcSinh[a + b*x])])/b
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.95

| method | result |
|---|---|
| default | $-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\operatorname{arcsinh}(bx+a)^2}{b(b^2x^2+2abx+a^2+1)}+\frac{2\operatorname{arcsinh}(bx+a)^2}{b}-\frac{2\ \operatorname{arcsinh}(bx+a)}$ |

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^2+2*arcsinh(b*x+a)
^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-polylog(2,-(b*x
+a+(1+(b*x+a)^2)^(1/2))^2)/b
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^2(a+bx)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
[Out] Integral(asinh(a + b*x)**2/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
[Out] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.280 $\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result . . . . . 1991
Rubi [A] (verified) . . . . . 1991
Mathematica [A] (verified) . . . . . 1992
Maple [B] (verified) . . . . . 1992
Fricas [B] (verification not implemented) . . . . . 1993
Sympy [F] . . . . . 1993
Maxima [B] (verification not implemented) . . . . . 1993
Giac [A] (verification not implemented) . . . . . 1994
Mupad [F(-1)] . . . . . 1994

## Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] -1/2*ln(1+(b*x+a)^2)/b+(b*x+a)*arcsinh(b*x+a)/b/(1+(b*x+a)^2)^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5860, 5787, 266}

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b\sqrt{(a+bx)^2+1}} - \frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[In] Int[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + (a + b*x)^2]) - Log[1 + (a + b*x)^
2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
```

```
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b} \\ &= \frac{(a+bx)\text{arcsinh}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b} \\ &= \frac{(a+bx)\text{arcsinh}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log\left(1+(a+bx)^2\right)}{2b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{\text{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{(a+bx)\text{arcsinh}(a+bx)}{b\sqrt{1+a^2+2abx+b^2x^2}} - \frac{\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

```
[In] Integrate[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) - Log[1
+ a^2 + 2*a*b*x + b^2*x^2]/(2*b)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(42) = 84$.

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

| method | result |
|---|---|
| default | $\frac{2\,\text{arcsinh}(bx+a)}{b} - \frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\text{arcsinh}(bx+a)}{b(b^2x^2+2abx+a^2+1)} - \frac{\ln\left(1+\left(bx+a+\sqrt{1+(b$ |

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arcsinh(b*x+a)/b-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)-
1/b*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\left(b^3x^2+2\,ab^2x+(a^2+\right.}$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) - (b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b^3*x^2 + 2*a*b^2*x + (a^2 + 1)*b)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}\left(a+bx\right)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx =$$
$$-\left(\frac{b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}+\frac{ab}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}\right)\operatorname{arsinh}\left(bx\right.$$
$$\left.+a\right)-\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -(b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*b
/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))*arcsinh(b*x
 + a) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\left(x+\frac{a}{b}\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] (x + a/b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

# 3.281 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\mathbf{arcsinh}(a+bx)}\,dx$

Optimal result . . . 1995
Rubi [N/A] . . . 1995
Mathematica [N/A] . . . 1996
Maple [N/A] (verified) . . . 1996
Fricas [N/A] . . . 1996
Sympy [N/A] . . . 1997
Maxima [N/A] . . . 1997
Giac [N/A] . . . 1997
Mupad [N/A] . . . 1998

## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \operatorname{Int}\left(\frac{1}{\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)},x\right)$$

```
[Out] Unintegrable(1/(1+(b*x+a)^2)^(3/2)/arcsinh(b*x+a),x)
```

## Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx$$

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((1 + x^2)^(3/2)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{b}$$

## Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx$$

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}\operatorname{arcsinh}\left(bx+a\right)}dx$$

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

## Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fric
as")
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}\operatorname{asinh}(a+bx)}\,dx$$

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}(bx+a)}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac
")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}(a+b\,x)\,(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}\,dx$$

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
[Out] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

# 3.282 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\mathbf{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 1999
Rubi [N/A] . . . 1999
Mathematica [N/A] . . . 2000
Maple [N/A] (verified) . . . 2000
Fricas [N/A] . . . 2000
Sympy [N/A] . . . 2001
Maxima [N/A] . . . 2001
Giac [N/A] . . . 2002
Mupad [N/A] . . . 2002

## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\left(1+(a+bx)^2\right)\operatorname{arcsinh}(a+bx)} - 2\operatorname{Int}\left(\frac{a+bx}{\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)}, x\right)$$

```
[Out] -1/b/(1+(b*x+a)^2)/arcsinh(b*x+a)-2*Unintegrable((b*x+a)/(1+(b*x+a)^2)^2/ar
csinh(b*x+a),x)
```

## Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx$$

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*(1 + (a + b*x)^2)*ArcSinh[a + b*x])) - (2*Defer[Subst][Defer[Int][x/
((1 + x^2)^2*ArcSinh[x]), x], x, a + b*x])/b
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}\operatorname{arcsinh}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{1}{b\left(1+(a+bx)^2\right)\operatorname{arcsinh}(a+bx)} - \frac{2\operatorname{Subst}\left(\int \frac{x}{(1+x^2)^2\operatorname{arcsinh}(x)}\,dx, x, a+bx\right)}{b}$$

## Mathematica [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx$$

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arcsinh}\left(bx+a\right)^2}dx$$

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fr
icas")
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)^2), x
)
```

## Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}\operatorname{asinh}^2(a+bx)}\,dx$$

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 610, normalized size of antiderivative = 20.33

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}(bx+a)^2}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="ma
xima")
```

```
[Out] -(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(((b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b^2*x + a*b) + (b^3*x^2 + 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) - integrate
((2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + (2*b^2*x^2 + 4
*a*b*x + 2*a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + a^2 + 2*(4*a^3*b + a*b)
*x + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^
2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2
*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*
b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + (b^6*x^6 + 6*a*b^5*x^5 + a^6 + 3*(5*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4
*(5*a^3*b^3 + 3*a*b^3)*x^3 + 3*(5*a^4*b^2 + 6*a^2*b^2 + b^2)*x^2 + 3*a^2 +
6*(a^5*b + 2*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^2\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

```
[Out] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

# 3.283 $\int x^3 \mathbf{arcsinh}(ax^2)\, dx$

Optimal result . . . . . 2003
Rubi [A] (verified) . . . . . 2003
Mathematica [A] (verified) . . . . . 2005
Maple [A] (verified) . . . . . 2005
Fricas [A] (verification not implemented) . . . . . 2005
Sympy [A] (verification not implemented) . . . . . 2006
Maxima [B] (verification not implemented) . . . . . 2006
Giac [A] (verification not implemented) . . . . . 2006
Mupad [B] (verification not implemented) . . . . . 2007

## Optimal result

Integrand size = 10, antiderivative size = 50

$$\int x^3 \text{arcsinh}\left(ax^2\right)\, dx = -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{\text{arcsinh}(ax^2)}{8a^2} + \frac{1}{4}x^4\text{arcsinh}\left(ax^2\right)$$

```
[Out] 1/8*arcsinh(a*x^2)/a^2+1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 281, 327, 221}

$$\int x^3 \text{arcsinh}\left(ax^2\right)\, dx = \frac{\text{arcsinh}(ax^2)}{8a^2} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{1}{4}x^4\text{arcsinh}\left(ax^2\right)$$

```
[In] Int[x^3*ArcSinh[a*x^2],x]
```

```
[Out] -1/8*(x^2*Sqrt[1 + a^2*x^4])/a + ArcSinh[a*x^2]/(8*a^2) + (x^4*ArcSinh[a*x^
2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\operatorname{arcsinh}\left(ax^2\right) - \frac{1}{4}\int \frac{2ax^5}{\sqrt{1+a^2x^4}}\,dx \\
&= \frac{1}{4}x^4\operatorname{arcsinh}\left(ax^2\right) - \frac{1}{2}a\int \frac{x^5}{\sqrt{1+a^2x^4}}\,dx \\
&= \frac{1}{4}x^4\operatorname{arcsinh}\left(ax^2\right) - \frac{1}{4}a\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx, x, x^2\right) \\
&= -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{1}{4}x^4\operatorname{arcsinh}\left(ax^2\right) + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx, x, x^2\right)}{8a} \\
&= -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{\operatorname{arcsinh}(ax^2)}{8a^2} + \frac{1}{4}x^4\operatorname{arcsinh}\left(ax^2\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int x^3 \text{arcsinh}\left(ax^2\right)\, dx = \frac{-ax^2\sqrt{1+a^2x^4} + \left(1+2a^2x^4\right)\text{arcsinh}\left(ax^2\right)}{8a^2}$$

```
[In] Integrate[x^3*ArcSinh[a*x^2],x]
```

```
[Out] (-(a*x^2*Sqrt[1 + a^2*x^4]) + (1 + 2*a^2*x^4)*ArcSinh[a*x^2])/(8*a^2)
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

| method | result | size |
|---|---|---|
| default | $\frac{x^4 \text{arcsinh}\left(a\,x^2\right)}{4} - \frac{a\left(\frac{x^2\sqrt{a^2x^4+1}}{4a^2} - \frac{\ln\left(\frac{a^2x^2}{\sqrt{a^2}}+\sqrt{a^2x^4+1}\right)}{4a^2\sqrt{a^2}}\right)}{2}$ | 71 |
| parts | $\frac{x^4 \text{arcsinh}\left(a\,x^2\right)}{4} - \frac{a\left(\frac{x^2\sqrt{a^2x^4+1}}{4a^2} - \frac{\ln\left(\frac{a^2x^2}{\sqrt{a^2}}+\sqrt{a^2x^4+1}\right)}{4a^2\sqrt{a^2}}\right)}{2}$ | 71 |

```
[In] int(x^3*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arcsinh(a*x^2)-1/2*a*(1/4*x^2/a^2*(a^2*x^4+1)^(1/2)-1/4/a^2*ln(a^2*
x^2/(a^2)^(1/2)+(a^2*x^4+1)^(1/2))/(a^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int x^3 \text{arcsinh}\left(ax^2\right)\, dx = -\frac{\sqrt{a^2x^4+1}ax^2 - \left(2\,a^2x^4+1\right)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{8\,a^2}$$

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 + 1)*log(a*x^2 + sqrt(a^2*x^4 +
1)))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{arcsinh}\left(ax^2\right)\, dx = \begin{cases} \frac{x^4 \operatorname{asinh}\left(ax^2\right)}{4} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{\operatorname{asinh}\left(ax^2\right)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**4*asinh(a*x**2)/4 - x**2*sqrt(a**2*x**4 + 1)/(8*a) + asinh(a*
x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(42) = 84$.

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.04

$$\begin{aligned} &\int x^3 \operatorname{arcsinh}\left(ax^2\right)\, dx \\ &= \frac{1}{4} x^4 \operatorname{arsinh}\left(ax^2\right) \\ &\quad + \frac{1}{16} a \left( \frac{\log\left(a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} - \frac{\log\left(-a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} + \frac{2\sqrt{a^2x^4+1}}{\left(a^4 - \frac{(a^2x^4+1)a^2}{x^4}\right)x^2} \right) \end{aligned}$$

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x^2) + 1/16*a*(log(a + sqrt(a^2*x^4 + 1)/x^2)/a^3 - log(-
a + sqrt(a^2*x^4 + 1)/x^2)/a^3 + 2*sqrt(a^2*x^4 + 1)/((a^4 - (a^2*x^4 + 1)*
a^2/x^4)*x^2))
```

## Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\begin{aligned} \int x^3 \operatorname{arcsinh}\left(ax^2\right)\, dx &= \frac{1}{4} x^4 \log\left(ax^2 + \sqrt{a^2x^4+1}\right) \\ &\quad - \frac{1}{8} a \left( \frac{\sqrt{a^2x^4+1}x^2}{a^2} + \frac{\log\left(-x^2|a| + \sqrt{a^2x^4+1}\right)}{a^2|a|} \right) \end{aligned}$$

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/8*a*(sqrt(a^2*x^4 + 1)*x^2/a^2 +
 log(-x^2*abs(a) + sqrt(a^2*x^4 + 1))/(a^2*abs(a)))
```

## Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3 \operatorname{arcsinh}\left(a x^2\right)\, dx = \frac{x^2 \operatorname{asinh}(a\,x^2)\,\left(\frac{x^2}{2}+\frac{1}{4\,a^2\,x^2}\right)}{2} - \frac{x^2\,\sqrt{a^2\,x^4+1}}{8\,a}$$

```
[In] int(x^3*asinh(a*x^2),x)
```

```
[Out] (x^2*asinh(a*x^2)*(x^2/2 + 1/(4*a^2*x^2)))/2 - (x^2*(a^2*x^4 + 1)^(1/2))/(8
*a)
```

# 3.284 $\int x^2 \mathbf{arcsinh}(ax^2)\, dx$

Optimal result . . . 2008
Rubi [A] (verified) . . . 2008
Mathematica [C] (verified) . . . 2010
Maple [C] (verified) . . . 2010
Fricas [A] (verification not implemented) . . . 2011
Sympy [F] . . . 2011
Maxima [F] . . . 2011
Giac [F] . . . 2012
Mupad [F(-1)] . . . 2012

## Optimal result

Integrand size = 10, antiderivative size = 101

$$\int x^2 \operatorname{arcsinh}(ax^2)\, dx = -\frac{2x\sqrt{1+a^2x^4}}{9a} + \frac{1}{3}x^3 \operatorname{arcsinh}(ax^2) + \frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right), \frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}$$

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/9*x*(a^2*x^4+1)^(1/2)/a+1/9*(a*x^2+1)*(cos(2*arcta
n(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^
(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(3/2)/(a^2*x^4+1)^(1
/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 327, 226}

$$\int x^2 \operatorname{arcsinh}(ax^2)\, dx = -\frac{2x\sqrt{a^2x^4+1}}{9a} + \frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right), \frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}} + \frac{1}{3}x^3 \operatorname{arcsinh}(ax^2)$$

```
[In] Int[x^2*ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(9*a) + (x^3*ArcSinh[a*x^2])/3 + ((1 + a*x^2)*Sqrt
[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(9*a^(3/
2)*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\operatorname{arcsinh}\left(ax^2\right) - \frac{1}{3}\int \frac{2ax^4}{\sqrt{1+a^2x^4}}\,dx \\
&= \frac{1}{3}x^3\operatorname{arcsinh}\left(ax^2\right) - \frac{1}{3}(2a)\int \frac{x^4}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^4}}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}\left(ax^2\right) + \frac{2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx}{9a} \\
&= -\frac{2x\sqrt{1+a^2x^4}}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}\left(ax^2\right) + \frac{\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^2 \text{arcsinh}\left(ax^2\right)\, dx = \frac{1}{9}\left(-\frac{2(x+a^2x^5)}{a\sqrt{1+a^2x^4}} + 3x^3\text{arcsinh}\left(ax^2\right) - \frac{2\sqrt{ia}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{ia}x\right),-1\right)}{a^2}\right)$$

```
[In] Integrate[x^2*ArcSinh[a*x^2],x]
```

```
[Out] ((-2*(x + a^2*x^5))/(a*Sqrt[1 + a^2*x^4]) + 3*x^3*ArcSinh[a*x^2] - (2*Sqrt[
I*a]*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/a^2)/9
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{x^3\,\text{arcsinh}\left(a\,x^2\right)}{3} - \frac{2a\left(\frac{x\sqrt{a^2x^4+1}}{3a^2} - \frac{\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ia},i\right)}{3a^2\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$ | 89 |
| parts | $\frac{x^3\,\text{arcsinh}\left(a\,x^2\right)}{3} - \frac{2a\left(\frac{x\sqrt{a^2x^4+1}}{3a^2} - \frac{\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ia},i\right)}{3a^2\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$ | 89 |

```
[In] int(x^2*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/3*a*(1/3*x/a^2*(a^2*x^4+1)^(1/2)-1/3/a^2/(I*a)^(1/
2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*EllipticF(x*(I*a)^
(1/2),I))
```

## Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int x^2 \operatorname{arcsinh}\left(ax^2\right)\, dx$$

$$= \frac{3\,ax^3 \log\left(ax^2 + \sqrt{a^2x^4+1}\right) + 2\,a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - 2\sqrt{a^2x^4+1}x}{9\,a}$$

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) + 2*a*(-1/a^2)^(3/4)*elliptic_f
(arcsin((-1/a^2)^(1/4)/x), -1) - 2*sqrt(a^2*x^4 + 1)*x)/a
```

## Sympy [F]

$$\int x^2 \operatorname{arcsinh}\left(ax^2\right)\, dx = \int x^2 \operatorname{asinh}\left(ax^2\right) dx$$

```
[In] integrate(x**2*asinh(a*x**2),x)
```

```
[Out] Integral(x**2*asinh(a*x**2), x)
```

## Maxima [F]

$$\int x^2 \operatorname{arcsinh}\left(ax^2\right)\, dx = \int x^2 \operatorname{arsinh}\left(ax^2\right)\, dx$$

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2/9*x^3 - 2*a*integrate(1/3*x^4/(a
^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(
2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqr
t(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*
a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x
+ 1)/a^(3/2) + 1/12*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)
```

## Giac [F]

$$\int x^2 \text{arcsinh}\left(ax^2\right)\, dx = \int x^2\, \text{arsinh}\left(ax^2\right)\, dx$$

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}\left(ax^2\right)\, dx = \int x^2\, \text{asinh}\left(a\, x^2\right)\, dx$$

```
[In] int(x^2*asinh(a*x^2),x)
[Out] int(x^2*asinh(a*x^2), x)
```

# 3.285 $\int x\mathbf{arcsinh}(ax^2)\,dx$

Optimal result . . . 2013
Rubi [A] (verified) . . . 2013
Mathematica [A] (verified) . . . 2014
Maple [A] (verified) . . . 2014
Fricas [A] (verification not implemented) . . . 2015
Sympy [A] (verification not implemented) . . . 2015
Maxima [A] (verification not implemented) . . . 2015
Giac [A] (verification not implemented) . . . 2016
Mupad [B] (verification not implemented) . . . 2016

## Optimal result

Integrand size = 8, antiderivative size = 34

$$\int x\text{arcsinh}(ax^2)\,dx = -\frac{\sqrt{1+a^2x^4}}{2a} + \frac{1}{2}x^2\text{arcsinh}(ax^2)$$

```
[Out] 1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6847, 5772, 267}

$$\int x\text{arcsinh}(ax^2)\,dx = \frac{1}{2}x^2\text{arcsinh}(ax^2) - \frac{\sqrt{a^2x^4+1}}{2a}$$

```
[In] Int[x*ArcSinh[a*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \text{arcsinh}(ax)\,dx, x, x^2\right) \\ &= \frac{1}{2}x^2\text{arcsinh}\left(ax^2\right) - \frac{1}{2}a\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x^2}}\,dx, x, x^2\right) \\ &= -\frac{\sqrt{1+a^2x^4}}{2a} + \frac{1}{2}x^2\text{arcsinh}\left(ax^2\right)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\text{arcsinh}\left(ax^2\right)\,dx = -\frac{\sqrt{1+a^2x^4}}{2a} + \frac{1}{2}x^2\text{arcsinh}\left(ax^2\right)$$

```
[In] Integrate[x*ArcSinh[a*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| parts | $\frac{x^2\,\text{arcsinh}\left(a\,x^2\right)}{2} - \frac{\sqrt{a^2x^4+1}}{2a}$ | 29 |
| derivativedivides | $\frac{a\,x^2\,\text{arcsinh}\left(a\,x^2\right)-\sqrt{a^2x^4+1}}{2a}$ | 31 |
| default | $\frac{a\,x^2\,\text{arcsinh}\left(a\,x^2\right)-\sqrt{a^2x^4+1}}{2a}$ | 31 |

```
[In] int(x*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

## Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int x\text{arcsinh}\left(ax^2\right)\,dx = \frac{ax^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\sqrt{a^2x^4+1}}{2\,a}$$

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - sqrt(a^2*x^4 + 1))/a
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x\text{arcsinh}\left(ax^2\right)\,dx = \begin{cases} \frac{x^2\,\text{asinh}\left(ax^2\right)}{2}-\frac{\sqrt{a^2x^4+1}}{2a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**2*asinh(a*x**2)/2 - sqrt(a**2*x**4 + 1)/(2*a), Ne(a, 0)), (0,
 True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\text{arcsinh}\left(ax^2\right)\,dx = \frac{ax^2\,\text{arsinh}\left(ax^2\right)-\sqrt{a^2x^4+1}}{2\,a}$$

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2*arcsinh(a*x^2) - sqrt(a^2*x^4 + 1))/a
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int x\operatorname{arcsinh}\left(ax^2\right)\,dx = \frac{1}{2}\,x^2\log\left(ax^2+\sqrt{a^2x^4+1}\right) - \frac{\sqrt{a^2x^4+1}}{2\,a}$$

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="giac")
[Out] 1/2*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/2*sqrt(a^2*x^4 + 1)/a
```

## Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x\operatorname{arcsinh}\left(ax^2\right)\,dx = \frac{x^2\operatorname{asinh}(a\,x^2)}{2} - \frac{\sqrt{a^2\,x^4+1}}{2\,a}$$

```
[In] int(x*asinh(a*x^2),x)
[Out] (x^2*asinh(a*x^2))/2 - (a^2*x^4 + 1)^(1/2)/(2*a)
```

# 3.286 $\int \operatorname{arcsinh}(ax^2)\,dx$

Optimal result . . . 2017
Rubi [A] (verified) . . . 2017
Mathematica [C] (verified) . . . 2019
Maple [C] (verified) . . . 2019
Fricas [A] (verification not implemented) . . . 2020
Sympy [F] . . . 2020
Maxima [F] . . . 2020
Giac [F] . . . 2021
Mupad [F(-1)] . . . 2021

## Optimal result

Integrand size = 6, antiderivative size = 162

$$\begin{aligned}\int \operatorname{arcsinh}(ax^2)\,dx &= -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2} + x\operatorname{arcsinh}(ax^2) \\ &+ \frac{2(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}} \\ &- \frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}\end{aligned}$$

```
[Out] x*arcsinh(a*x^2)-2*x*(a^2*x^4+1)^(1/2)/(a*x^2+1)+2*(a*x^2+1)*(cos(2*arctan(
x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1
/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(1/2)/(a^2*x^4+1)^(1/2
)-(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*Ell
ipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2
)/a^(1/2)/(a^2*x^4+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used

$= \{5874, 12, 311, 226, 1210\}$

$$\int \operatorname{arcsinh}\left(ax^2\right)\, dx = -\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right), \frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}} + \frac{2(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}} - \frac{2x\sqrt{a^2x^4+1}}{ax^2+1} + x\operatorname{arcsinh}\left(ax^2\right)$$

```
[In] Int[ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(1 + a*x^2) + x*ArcSinh[a*x^2] + (2*(1 + a*x^2)*Sq
rt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[
a]*Sqrt[1 + a^2*x^4]) - ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
```

```
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\operatorname{arcsinh}\left(ax^2\right) - \int \frac{2ax^2}{\sqrt{1+a^2x^4}}\,dx \\
&= x\operatorname{arcsinh}\left(ax^2\right) - (2a)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx \\
&= x\operatorname{arcsinh}\left(ax^2\right) - 2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx + 2\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2} + x\operatorname{arcsinh}\left(ax^2\right) + \frac{2(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}} \\
&\quad - \frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}
\end{aligned}$$

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.22

$$\int \operatorname{arcsinh}\left(ax^2\right)\,dx = x\operatorname{arcsinh}\left(ax^2\right) - \frac{2}{3}ax^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-a^2x^4\right)$$

```
[In] Integrate[ArcSinh[a*x^2],x]
[Out] x*ArcSinh[a*x^2] - (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(a^2*x^4)])/3
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

| method | result | size |
|---|---|---|
| default | $x\,\operatorname{arcsinh}\left(a\,x^2\right) - \frac{2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\left(\text{EllipticF}\left(x\sqrt{ia},i\right)-\text{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$ | 77 |
| parts | $x\,\operatorname{arcsinh}\left(a\,x^2\right) - \frac{2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\left(\text{EllipticF}\left(x\sqrt{ia},i\right)-\text{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$ | 77 |

```
[In] int(arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsinh(a*x^2)-2*I/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x
^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-EllipticE(x*(I*a)^(1/2),I))
```

## Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \operatorname{arcsinh}\left(ax^2\right)\, dx$$

$$= \frac{ax^2 \log\left(ax^2 + \sqrt{a^2x^4+1}\right) - 2\,ax\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + 2\,ax\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) \mid -}{ax}$$

```
[In] integrate(arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] (a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2*a*x*(-1/a^2)^(3/4)*elliptic_e(arc
sin((-1/a^2)^(1/4)/x), -1) + 2*a*x*(-1/a^2)^(3/4)*elliptic_f(arcsin((-1/a^2
)^(1/4)/x), -1) - 2*sqrt(a^2*x^4 + 1))/(a*x)
```

## Sympy [F]

$$\int \operatorname{arcsinh}\left(ax^2\right)\, dx = \int \operatorname{asinh}\left(ax^2\right) dx$$

```
[In] integrate(asinh(a*x**2),x)
```

```
[Out] Integral(asinh(a*x**2), x)
```

## Maxima [F]

$$\int \operatorname{arcsinh}\left(ax^2\right)\, dx = \int \operatorname{arsinh}\left(ax^2\right)\, dx$$

```
[In] integrate(arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x^2/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) + x*log(a*x^
2 + sqrt(a^2*x^4 + 1)) - 2*x - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sq
rt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/
sqrt(a) + 1))/sqrt(a) - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a)
 + 1))/sqrt(a) + 1/4*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - 1
/4*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)
```

## Giac [F]

$$\int \operatorname{arcsinh}\left(ax^2\right)\,dx = \int \operatorname{arsinh}\left(ax^2\right)\,dx$$

```
[In] integrate(arcsinh(a*x^2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}\left(ax^2\right)\,dx = \int \operatorname{asinh}\left(a\,x^2\right)\,dx$$

```
[In] int(asinh(a*x^2),x)
[Out] int(asinh(a*x^2), x)
```

# 3.287 $\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx$

Optimal result . . . 2022
Rubi [A] (verified) . . . 2022
Mathematica [A] (verified) . . . 2024
Maple [F] . . . 2024
Fricas [F] . . . 2024
Sympy [F] . . . 2024
Maxima [F] . . . 2025
Giac [F] . . . 2025
Mupad [F(-1)] . . . 2025

## Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = -\frac{1}{4}\operatorname{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\operatorname{arcsinh}\left(ax^2\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ax^2\right)}\right) + \frac{1}{4}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^2\right)}\right)$$

```
[Out] -1/4*arcsinh(a*x^2)^2+1/2*arcsinh(a*x^2)*ln(1-(a*x^2+(a^2*x^4+1)^(1/2))^2)+
1/4*polylog(2,(a*x^2+(a^2*x^4+1)^(1/2))^2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \frac{1}{4}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^2\right)}\right) - \frac{1}{4}\operatorname{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\operatorname{arcsinh}\left(ax^2\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ax^2\right)}\right)$$

```
[In] Int[ArcSinh[a*x^2]/x,x]
```

```
[Out] -1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 +
PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\coth(x)\,dx, x, \text{arcsinh}\left(ax^2\right)\right) \\
&= -\frac{1}{4}\text{arcsinh}\left(ax^2\right)^2 - \text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}\left(ax^2\right)\right) \\
&= -\frac{1}{4}\text{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\text{arcsinh}\left(ax^2\right)\log\left(1-e^{2\text{arcsinh}\left(ax^2\right)}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}\left(ax^2\right)\right) \\
&= -\frac{1}{4}\text{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\text{arcsinh}\left(ax^2\right)\log\left(1-e^{2\text{arcsinh}\left(ax^2\right)}\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}\left(ax^2\right)}\right) \\
&= -\frac{1}{4}\text{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\text{arcsinh}\left(ax^2\right)\log\left(1-e^{2\text{arcsinh}\left(ax^2\right)}\right) + \frac{1}{4}\,\text{PolyLog}\left(2, e^{2\text{arcsinh}\left(ax^2\right)}\right)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = -\frac{1}{4}\operatorname{arcsinh}\left(ax^2\right)^2 + \frac{1}{2}\operatorname{arcsinh}\left(ax^2\right)\log\left(1 - e^{2\operatorname{arcsinh}\left(ax^2\right)}\right) + \frac{1}{4}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^2\right)}\right)$$

```
[In] Integrate[ArcSinh[a*x^2]/x,x]
```

```
[Out] -1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 + PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a\,x^2\right)}{x}dx$$

```
[In] int(arcsinh(a*x^2)/x,x)
```

```
[Out] int(arcsinh(a*x^2)/x,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{asinh}\left(ax^2\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x**2)/x,x)
```

```
[Out] Integral(asinh(a*x**2)/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x^2)/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x}\,dx$$

```
[In] int(asinh(a*x^2)/x,x)
[Out] int(asinh(a*x^2)/x, x)
```

# 3.288 $\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx$

Optimal result . . . 2026
Rubi [A] (verified) . . . 2026
Mathematica [C] (verified) . . . 2027
Maple [C] (verified) . . . 2028
Fricas [A] (verification not implemented) . . . 2028
Sympy [F] . . . 2028
Maxima [F] . . . 2029
Giac [F] . . . 2029
Mupad [F(-1)] . . . 2029

## Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = -\frac{\operatorname{arcsinh}(ax^2)}{x} + \frac{\sqrt{a}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}$$

```
[Out] -arcsinh(a*x^2)/x+(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan
(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^2*
x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 226}

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \frac{\sqrt{a}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a^2x^4+1}} - \frac{\operatorname{arcsinh}(ax^2)}{x}$$

```
[In] Int[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{arcsinh}(ax^2)}{x} + \int \frac{2a}{\sqrt{1+a^2x^4}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^2)}{x} + (2a)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^2)}{x} + \frac{\sqrt{a}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}\end{aligned}$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = -\frac{\operatorname{arcsinh}(ax^2) + 2\sqrt{ia}x\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{ia}x\right),-1\right)}{x}$$

```
[In] Integrate[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -((ArcSinh[a*x^2] + 2*Sqrt[I*a]*x*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/x)
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arcsinh}(a\,x^2)}{x}+\frac{2a\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ia},i\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$ | 66 |
| parts | $-\frac{\operatorname{arcsinh}(a\,x^2)}{x}+\frac{2a\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ia},i\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$ | 66 |

```
[In] int(arcsinh(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(a*x^2)/x+2*a/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*
x^4+1)^(1/2)*EllipticF(x*(I*a)^(1/2),I)
```

## Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx$$

$$=\frac{2\,a^2x\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+(x-1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)+x\log\left(ax^2-\sqrt{a^2x^4+1}\right)}{x}$$

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x*(-1/a^2)^(3/4)*elliptic_f(arcsin((-1/a^2)^(1/4)/x), -1) + (x - 1)*
log(a*x^2 + sqrt(a^2*x^4 + 1)) + x*log(a*x^2 - sqrt(a^2*x^4 + 1)))/x
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx=\int \frac{\operatorname{asinh}\left(ax^2\right)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x**2)/x**2,x)
```

```
[Out] Integral(asinh(a*x**2)/x**2, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) +
1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + I
*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1
/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + sqrt(2)*log(
a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a
)*x + 1)/a^(3/2)) + 2*a*integrate(1/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2))
, x) - log(a*x^2 + sqrt(a^2*x^4 + 1))/x
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^2}\,dx$$

```
[In] int(asinh(a*x^2)/x^2,x)
```

```
[Out] int(asinh(a*x^2)/x^2, x)
```

# 3.289 $\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx$

Optimal result . . . 2030
Rubi [A] (verified) . . . 2030
Mathematica [A] (verified) . . . 2032
Maple [A] (verified) . . . 2032
Fricas [B] (verification not implemented) . . . 2032
Sympy [F] . . . 2033
Maxima [A] (verification not implemented) . . . 2033
Giac [B] (verification not implemented) . . . 2033
Mupad [F(-1)] . . . 2034

## Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} - \frac{1}{2}a\operatorname{arctanh}\left(\sqrt{1+a^2x^4}\right)$$

```
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh((a^2*x^4+1)^(1/2))
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 272, 65, 214}

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{1}{2}a\operatorname{arctanh}\left(\sqrt{a^2x^4+1}\right) - \frac{\operatorname{arcsinh}(ax^2)}{2x^2}$$

```
[In] Int[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} + \frac{1}{2}\int \frac{2a}{x\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} + a\int \frac{1}{x\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} + \frac{1}{4}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^4\right) \\
&= -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^4}\right)}{2a} \\
&= -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} - \frac{1}{2}a\operatorname{arctanh}\left(\sqrt{1+a^2x^4}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} - \frac{1}{2}a\operatorname{arctanh}\left(\sqrt{1+a^2x^4}\right)$$

```
[In] Integrate[ArcSinh[a*x^2]/x^3,x]
[Out] -1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arcsinh}(a\,x^2)}{2x^2} - \frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^4+1}}\right)}{2}$ | 28 |
| parts | $-\frac{\operatorname{arcsinh}(a\,x^2)}{2x^2} - \frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^4+1}}\right)}{2}$ | 28 |

```
[In] int(arcsinh(a*x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh(1/(a^2*x^4+1)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.21

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}+1\right) - ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}-1\right) - x^2\log\left(-ax^2+\sqrt{a^2x^4+1}\right) - (x^2}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="fricas")
[Out] -1/2*(a*x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1) + 1) - a*x^2*log(-a*x^2 + sqrt(a
^2*x^4 + 1) - 1) - x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1)) - (x^2 - 1)*log(a*x^
2 + sqrt(a^2*x^4 + 1)))/x^2
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = \int \frac{\operatorname{asinh}(ax^2)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x**2)/x**3,x)
```

```
[Out] Integral(asinh(a*x**2)/x**3, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right) - \log\left(\sqrt{a^2x^4+1}-1\right)\right) - \frac{\operatorname{arsinh}(ax^2)}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*arcs
inh(a*x^2)/x^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right) - \log\left(\sqrt{a^2x^4+1}-1\right)\right) - \frac{\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*log(
a*x^2 + sqrt(a^2*x^4 + 1))/x^2
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^3}\,dx$$

```
[In] int(asinh(a*x^2)/x^3,x)
[Out] int(asinh(a*x^2)/x^3, x)
```

# 3.290 $\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx$

Optimal result . . . 2035
Rubi [A] (verified) . . . 2035
Mathematica [C] (verified) . . . 2037
Maple [C] (verified) . . . 2038
Fricas [F] . . . 2039
Sympy [F] . . . 2039
Maxima [F] . . . 2039
Giac [F] . . . 2039
Mupad [F(-1)] . . . 2040

## Optimal result

Integrand size = 10, antiderivative size = 197

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = -\frac{2a\sqrt{1+a^2x^4}}{3x} + \frac{2a^2x\sqrt{1+a^2x^4}}{3\left(1+ax^2\right)} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3} - \frac{2a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}} + \frac{a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}}$$

```
[Out] -1/3*arcsinh(a*x^2)/x^3-2/3*a*(a^2*x^4+1)^(1/2)/x+2/3*a^2*x*(a^2*x^4+1)^(1/
2)/(a*x^2+1)-2/3*a^(3/2)*(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2
*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x
^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)+1/3*a^(3/2)*(a*x^2+1)*(cos(2*arc
tan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*
a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{5875, 12, 331, 311, 226, 1210\}$

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = -\frac{2a\sqrt{a^2x^4+1}}{3x} + \frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)} + \frac{a^{3/2}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}} - \frac{2a^{3/2}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

```
[In] Int[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 + a^2*x^4])/(3*x) + (2*a^2*x*Sqrt[1 + a^2*x^4])/(3*(1 + a*x^2)
) - ArcSinh[a*x^2]/(3*x^3) - (2*a^(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 +
 a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4]) + (a^
(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax^2)}{3x^3} + \frac{1}{3}\int \frac{2a}{x^2\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax^2)}{3x^3} + \frac{1}{3}(2a)\int \frac{1}{x^2\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3} + \frac{1}{3}\left(2a^3\right)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3} + \frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx - \frac{1}{3}\left(2a^2\right)\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x} + \frac{2a^2x\sqrt{1+a^2x^4}}{3\left(1+ax^2\right)} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3} \\
&\quad - \frac{2a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}} \\
&\quad + \frac{a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}}
\end{aligned}
$$

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx$$
$$= \frac{1}{3}\left(-\frac{2a\sqrt{1+a^2x^4}}{x} - \frac{\operatorname{arcsinh}(ax^2)}{x^3} + \frac{2a^2\left(E\left(i\operatorname{arcsinh}\left(\sqrt{ia}x\right)\middle| -1\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{ia}x\right), -1\right)\right)}{\sqrt{ia}}\right)$$

```
[In] Integrate[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] ((-2*a*Sqrt[1 + a^2*x^4])/x - ArcSinh[a*x^2]/x^3 + (2*a^2*(EllipticE[I*ArcS
inh[Sqrt[I*a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1]))/Sqrt[I*a])/
3
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arcsinh}(a\,x^2)}{3x^3} + \frac{2a\left(-\frac{\sqrt{a^2x^4+1}}{x} + \frac{ia\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$ | 101 |
| parts | $-\frac{\operatorname{arcsinh}(a\,x^2)}{3x^3} + \frac{2a\left(-\frac{\sqrt{a^2x^4+1}}{x} + \frac{ia\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$ | 101 |

```
[In] int(arcsinh(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(a*x^2)/x^3+2/3*a*(-(a^2*x^4+1)^(1/2)/x+I*a/(I*a)^(1/2)*(1-I*a*
x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-
EllipticE(x*(I*a)^(1/2),I)))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="fricas")
[Out] integral(arcsinh(a*x^2)/x^4, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(ax^2\right)}{x^4}\,dx$$

```
[In] integrate(asinh(a*x**2)/x**4,x)
[Out] Integral(asinh(a*x**2)/x**4, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="maxima")
[Out] -1/12*I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a
) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1)) - 1/12*
I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)
 - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)) + 1/12*sqrt(2
)*a^(3/2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1) - 1/12*sqrt(2)*a^(3/2)*log(a*x
^2 - sqrt(2)*sqrt(a)*x + 1) + 2*a*integrate(1/3/(a^3*x^8 + a*x^4 + (a^2*x^6
 + x^2)*sqrt(a^2*x^4 + 1)), x) - 1/3*log(a*x^2 + sqrt(a^2*x^4 + 1))/x^3
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^4}\,dx$$

```
[In] int(asinh(a*x^2)/x^4,x)
[Out] int(asinh(a*x^2)/x^4, x)
```

# 3.291 $\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx$

Optimal result . . . 2041
Rubi [A] (verified) . . . 2041
Mathematica [A] (verified) . . . 2043
Maple [F] . . . 2043
Fricas [F] . . . 2043
Sympy [F] . . . 2043
Maxima [F] . . . 2044
Giac [F] . . . 2044
Mupad [F(-1)] . . . 2044

## Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = -\frac{1}{10}\operatorname{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\operatorname{arcsinh}\left(ax^5\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ax^5\right)}\right) + \frac{1}{10}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^5\right)}\right)$$

```
[Out] -1/10*arcsinh(a*x^5)^2+1/5*arcsinh(a*x^5)*ln(1-(a*x^5+(a^2*x^10+1)^(1/2))^2
)+1/10*polylog(2,(a*x^5+(a^2*x^10+1)^(1/2))^2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \frac{1}{10}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^5\right)}\right) - \frac{1}{10}\operatorname{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\operatorname{arcsinh}\left(ax^5\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ax^5\right)}\right)$$

```
[In] Int[ArcSinh[a*x^5]/x,x]
```

```
[Out] -1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 +
 PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int x\coth(x)\,dx, x, \text{arcsinh}\left(ax^5\right)\right) \\
&= -\frac{1}{10}\text{arcsinh}\left(ax^5\right)^2 - \frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}\left(ax^5\right)\right) \\
&= -\frac{1}{10}\text{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\text{arcsinh}\left(ax^5\right)\log\left(1-e^{2\text{arcsinh}\left(ax^5\right)}\right) \\
&\quad - \frac{1}{5}\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}\left(ax^5\right)\right) \\
&= -\frac{1}{10}\text{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\text{arcsinh}\left(ax^5\right)\log\left(1-e^{2\text{arcsinh}\left(ax^5\right)}\right) \\
&\quad - \frac{1}{10}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}\left(ax^5\right)}\right) \\
&= -\frac{1}{10}\text{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\text{arcsinh}\left(ax^5\right)\log\left(1-e^{2\text{arcsinh}\left(ax^5\right)}\right) + \frac{1}{10}\,\text{PolyLog}\left(2, e^{2\text{arcsinh}\left(ax^5\right)}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = -\frac{1}{10}\operatorname{arcsinh}\left(ax^5\right)^2 + \frac{1}{5}\operatorname{arcsinh}\left(ax^5\right)\log\left(1 - e^{2\operatorname{arcsinh}\left(ax^5\right)}\right) + \frac{1}{10}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ax^5\right)}\right)$$

```
[In] Integrate[ArcSinh[a*x^5]/x,x]
```

```
[Out] -1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 +
 PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a\,x^5\right)}{x}dx$$

```
[In] int(arcsinh(a*x^5)/x,x)
```

```
[Out] int(arcsinh(a*x^5)/x,x)
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^5\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^5)/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{asinh}\left(ax^5\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x**5)/x,x)
```

```
[Out] Integral(asinh(a*x**5)/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^5\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^5\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^5)}{x}\,dx$$

```
[In] int(asinh(a*x^5)/x,x)
[Out] int(asinh(a*x^5)/x, x)
```

# 3.292 $\int x^2 \text{arcsinh}\left(\sqrt{x}\right)\, dx$

Optimal result . . . 2045
Rubi [A] (verified) . . . 2045
Mathematica [A] (verified) . . . 2047
Maple [A] (verified) . . . 2047
Fricas [A] (verification not implemented) . . . 2048
Sympy [F] . . . 2048
Maxima [A] (verification not implemented) . . . 2048
Giac [A] (verification not implemented) . . . 2049
Mupad [F(-1)] . . . 2049

## Optimal result

Integrand size = 10, antiderivative size = 72

$$\int x^2 \text{arcsinh}\left(\sqrt{x}\right)\, dx = -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{5\text{arcsinh}\left(\sqrt{x}\right)}{48} + \frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 52, 56, 221}

$$\int x^2 \text{arcsinh}\left(\sqrt{x}\right)\, dx = \frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right) + \frac{5\text{arcsinh}\left(\sqrt{x}\right)}{48} - \frac{1}{18}\sqrt{x+1}x^{5/2} + \frac{5}{72}\sqrt{x+1}x^{3/2} - \frac{5}{48}\sqrt{x+1}\sqrt{x}$$

```
[In] Int[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[1 + x])/48 + (5*x^(3/2)*Sqrt[1 + x])/72 - (x^(5/2)*Sqrt[1
+ x])/18 + (5*ArcSinh[Sqrt[x]])/48 + (x^3*ArcSinh[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1+x}}\,dx \\
&= \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{5}{36}\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{5}{48}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{5}{96}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx
\end{aligned}$$

$$= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{5}{48}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{5\operatorname{arcsinh}\left(\sqrt{x}\right)}{48} + \frac{1}{3}x^3\operatorname{arcsinh}\left(\sqrt{x}\right)$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int x^2\operatorname{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{144}\left(\sqrt{x}\sqrt{1+x}\left(-15+10x-8x^2\right) + 3\left(5+16x^3\right)\operatorname{arcsinh}\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-15 + 10*x - 8*x^2) + 3*(5 + 16*x^3)*ArcSinh[Sqrt[x]]
)/144
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{5\operatorname{arcsinh}\left(\sqrt{x}\right)}{48} + \frac{x^3\operatorname{arcsinh}\left(\sqrt{x}\right)}{3} + \frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72} - \frac{x^{\frac{5}{2}}\sqrt{1+x}}{18} - \frac{5\sqrt{x}\sqrt{1+x}}{48}$ | 47 |
| default | $\frac{5\operatorname{arcsinh}\left(\sqrt{x}\right)}{48} + \frac{x^3\operatorname{arcsinh}\left(\sqrt{x}\right)}{3} + \frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72} - \frac{x^{\frac{5}{2}}\sqrt{1+x}}{18} - \frac{5\sqrt{x}\sqrt{1+x}}{48}$ | 47 |
| parts | $\frac{x^3\operatorname{arcsinh}\left(\sqrt{x}\right)}{3} - \frac{x^{\frac{5}{2}}\sqrt{1+x}}{18} + \frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72} - \frac{5\sqrt{x}\sqrt{1+x}}{48} + \frac{5\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{96\sqrt{x}\sqrt{1+x}}$ | 69 |

```
[In] int(x^2*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int x^2 \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = -\frac{1}{144}\left(8\,x^2 - 10\,x + 15\right)\sqrt{x+1}\sqrt{x} + \frac{1}{48}\left(16\,x^3 + 5\right)\log\left(\sqrt{x+1} + \sqrt{x}\right)$$

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 - 10*x + 15)*sqrt(x + 1)*sqrt(x) + 1/48*(16*x^3 + 5)*log(sqrt
(x + 1) + sqrt(x))
```

## Sympy [F]

$$\int x^2 \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \int x^2 \operatorname{asinh}\left(\sqrt{x}\right) dx$$

```
[In] integrate(x**2*asinh(x**(1/2)),x)
```

```
[Out] Integral(x**2*asinh(sqrt(x)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^2 \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \frac{1}{3}\,x^3 \operatorname{arsinh}\left(\sqrt{x}\right) - \frac{1}{18}\sqrt{x+1}x^{\frac{5}{2}} + \frac{5}{72}\sqrt{x+1}x^{\frac{3}{2}} - \frac{5}{48}\sqrt{x+1}\sqrt{x} + \frac{5}{48}\operatorname{arsinh}\left(\sqrt{x}\right)$$

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(sqrt(x)) - 1/18*sqrt(x + 1)*x^(5/2) + 5/72*sqrt(x + 1)*x^(3
/2) - 5/48*sqrt(x + 1)*sqrt(x) + 5/48*arcsinh(sqrt(x))
```

## Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \frac{1}{3} x^3 \log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{144}\left(2\left(4\,x-5\right)x+15\right)\sqrt{x+1}\sqrt{x} - \frac{5}{48}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(x + 1) + sqrt(x)) - 1/144*(2*(4*x - 5)*x + 15)*sqrt(x + 1)
*sqrt(x) - 5/48*log(sqrt(x + 1) - sqrt(x))
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \int x^2 \operatorname{asinh}\left(\sqrt{x}\right)\, dx$$

```
[In] int(x^2*asinh(x^(1/2)),x)
```

```
[Out] int(x^2*asinh(x^(1/2)), x)
```

# 3.293 $\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx$

Optimal result . . . 2050
Rubi [A] (verified) . . . 2050
Mathematica [A] (verified) . . . 2052
Maple [A] (verified) . . . 2052
Fricas [A] (verification not implemented) . . . 2052
Sympy [F] . . . 2053
Maxima [A] (verification not implemented) . . . 2053
Giac [A] (verification not implemented) . . . 2053
Mupad [F(-1)] . . . 2054

## Optimal result

Integrand size = 8, antiderivative size = 56

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{3}{16}\sqrt{x}\sqrt{1+x} - \frac{1}{8}x^{3/2}\sqrt{1+x} - \frac{3\text{arcsinh}\left(\sqrt{x}\right)}{16} + \frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5875, 12, 52, 56, 221}

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right) - \frac{3\text{arcsinh}\left(\sqrt{x}\right)}{16} - \frac{1}{8}\sqrt{x+1}x^{3/2} + \frac{3}{16}\sqrt{x+1}\sqrt{x}$$

```
[In] Int[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[1 + x])/16 - (x^(3/2)*Sqrt[1 + x])/8 - (3*ArcSinh[Sqrt[x]])
/16 + (x^2*ArcSinh[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{x^{3/2}}{2\sqrt{1+x}}\,dx \\
&= \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{1}{8}x^{3/2}\sqrt{1+x} + \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{3}{16}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x} - \frac{1}{8}x^{3/2}\sqrt{1+x} + \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{3}{32}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x} - \frac{1}{8}x^{3/2}\sqrt{1+x} + \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{3}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x} - \frac{1}{8}x^{3/2}\sqrt{1+x} - \frac{3\operatorname{arcsinh}\left(\sqrt{x}\right)}{16} + \frac{1}{2}x^2\operatorname{arcsinh}\left(\sqrt{x}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{16}\left((3-2x)\sqrt{x}\sqrt{1+x}+\left(-3+8x^2\right)\text{arcsinh}\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] ((3 - 2*x)*Sqrt[x]*Sqrt[1 + x] + (-3 + 8*x^2)*ArcSinh[Sqrt[x]])/16
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{3\,\text{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^2\,\text{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1+x}}{8}+\frac{3\sqrt{x}\,\sqrt{1+x}}{16}$ | 37 |
| default | $-\frac{3\,\text{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^2\,\text{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1+x}}{8}+\frac{3\sqrt{x}\,\sqrt{1+x}}{16}$ | 37 |
| parts | $\frac{x^2\,\text{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1+x}}{8}+\frac{3\sqrt{x}\,\sqrt{1+x}}{16}-\frac{3\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{32\sqrt{x}\,\sqrt{1+x}}$ | 59 |

```
[In] int(x*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = -\frac{1}{16}\left(2\,x-3\right)\sqrt{x+1}\sqrt{x}+\frac{1}{16}\left(8\,x^2-3\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x - 3)*sqrt(x + 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x + 1) + sq
rt(x))
```

## Sympy [F]

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \int x\,\text{asinh}\left(\sqrt{x}\right)\,dx$$

```
[In] integrate(x*asinh(x**(1/2)),x)
[Out] Integral(x*asinh(sqrt(x)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\,\text{arsinh}\left(\sqrt{x}\right) - \frac{1}{8}\,\sqrt{x+1}x^{\frac{3}{2}} + \frac{3}{16}\,\sqrt{x+1}\sqrt{x} - \frac{3}{16}\,\text{arsinh}\left(\sqrt{x}\right)$$

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*arcsinh(sqrt(x)) - 1/8*sqrt(x + 1)*x^(3/2) + 3/16*sqrt(x + 1)*sqrt(
x) - 3/16*arcsinh(sqrt(x))
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\begin{aligned}\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = {} & \frac{1}{2}\,x^2\log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{16}\,\sqrt{x^2+x}(2\,x-3) \\ & + \frac{3}{32}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\end{aligned}$$

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*log(sqrt(x + 1) + sqrt(x)) - 1/16*sqrt(x^2 + x)*(2*x - 3) + 3/32*lo
g(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int x\operatorname{arcsinh}\left(\sqrt{x}\right)\,dx = \int x\,\operatorname{asinh}\left(\sqrt{x}\right)\,dx$$

```
[In] int(x*asinh(x^(1/2)),x)
[Out] int(x*asinh(x^(1/2)), x)
```

# 3.294 $\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx$

Optimal result . . . 2055
Rubi [A] (verified) . . . 2055
Mathematica [A] (verified) . . . 2057
Maple [A] (verified) . . . 2057
Fricas [A] (verification not implemented) . . . 2057
Sympy [A] (verification not implemented) . . . 2058
Maxima [A] (verification not implemented) . . . 2058
Giac [A] (verification not implemented) . . . 2058
Mupad [B] (verification not implemented) . . . 2058

## Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = -\frac{1}{2}\sqrt{x}\sqrt{1+x} + \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} + x\operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5874, 12, 1978, 52, 56, 221}

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = x\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} - \frac{1}{2}\sqrt{x}\sqrt{x+1}$$

```
[In] Int[ArcSinh[Sqrt[x]],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*ArcSinh[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\operatorname{arcsinh}\left(\sqrt{x}\right) - \int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx \\
&= x\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx \\
&= x\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + x\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + x\operatorname{arcsinh}\left(\sqrt{x}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} + x\operatorname{arcsinh}\left(\sqrt{x}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \frac{1}{2}\left(-\frac{x}{\sqrt{\frac{x}{1+x}}} + 2x\operatorname{arcsinh}\left(\sqrt{x}\right) - \log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)$$

```
[In] Integrate[ArcSinh[Sqrt[x]],x]
```

```
[Out] (-(x/Sqrt[x/(1 + x)]) + 2*x*ArcSinh[Sqrt[x]] - Log[-Sqrt[x] + Sqrt[1 + x]])
/2
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} + x\ \operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\sqrt{1+x}}{2}$ | 24 |
| default | $\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} + x\ \operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\sqrt{1+x}}{2}$ | 24 |
| parts | $x\ \operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\sqrt{1+x}}{2} + \frac{\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{4\sqrt{x}\sqrt{1+x}}$ | 46 |

```
[In] int(arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \frac{1}{2}\left(2\,x+1\right)\log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{2}\sqrt{x+1}\sqrt{x}$$

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = -\frac{\sqrt{x}\sqrt{x+1}}{2} + x\operatorname{asinh}\left(\sqrt{x}\right) + \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{2}$$

```
[In] integrate(asinh(x**(1/2)),x)
[Out] -sqrt(x)*sqrt(x + 1)/2 + x*asinh(sqrt(x)) + asinh(sqrt(x))/2
```

## Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = x\operatorname{arsinh}\left(\sqrt{x}\right) - \frac{1}{2}\sqrt{x+1}\sqrt{x} + \frac{1}{2}\operatorname{arsinh}\left(\sqrt{x}\right)$$

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="maxima")
[Out] x*arcsinh(sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x) + 1/2*arcsinh(sqrt(x))
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = x\log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{2}\sqrt{x^2+x} - \frac{1}{4}\log\left(\left|-2\,x+2\sqrt{x^2+x}-1\right|\right)$$

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\, dx = \operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right) + x\operatorname{asinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\,\sqrt{x+1}}{2}$$

```
[In] int(asinh(x^(1/2)),x)
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) + x*asinh(x^(1/2)) - (x^(1/2)*(x + 1)^(1
/2))/2
```

# 3.295 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx$

Optimal result . . . 2059
Rubi [A] (verified) . . . 2059
Mathematica [A] (verified) . . . 2061
Maple [A] (verified) . . . 2061
Fricas [F] . . . 2061
Sympy [F] . . . 2062
Maxima [F] . . . 2062
Giac [F] . . . 2062
Mupad [F(-1)] . . . 2062

## Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx = -\operatorname{arcsinh}(\sqrt{x})^2 + 2\operatorname{arcsinh}(\sqrt{x})\log\left(1-e^{2\operatorname{arcsinh}(\sqrt{x})}\right) + \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(\sqrt{x})}\right)$$

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1-(x^(1/2)+(1+x)^(1/2))^2)+polylo
g(2,(x^(1/2)+(1+x)^(1/2))^2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx = \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(\sqrt{x})}\right) - \operatorname{arcsinh}(\sqrt{x})^2 + 2\operatorname{arcsinh}(\sqrt{x})\log\left(1-e^{2\operatorname{arcsinh}(\sqrt{x})}\right)$$

```
[In] Int[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\coth(x)\,dx, x, \text{arcsinh}\left(\sqrt{x}\right)\right) \\
&= -\text{arcsinh}\left(\sqrt{x}\right)^2 - 4\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}\left(\sqrt{x}\right)\right) \\
&= -\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\text{arcsinh}\left(\sqrt{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\sqrt{x}\right)}\right) \\
&\quad - 2\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}\left(\sqrt{x}\right)\right) \\
&= -\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\text{arcsinh}\left(\sqrt{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\sqrt{x}\right)}\right) \\
&\quad - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}\left(\sqrt{x}\right)}\right) \\
&= -\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\text{arcsinh}\left(\sqrt{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\sqrt{x}\right)}\right) + \text{PolyLog}\left(2, e^{2\text{arcsinh}\left(\sqrt{x}\right)}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = -\operatorname{arcsinh}\left(\sqrt{x}\right)^2 + 2\operatorname{arcsinh}\left(\sqrt{x}\right)\log\left(1 - e^{2\operatorname{arcsinh}\left(\sqrt{x}\right)}\right) + \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(\sqrt{x}\right)}\right)$$

```
[In] Integrate[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $-\operatorname{arcsinh}\left(\sqrt{x}\right)^2 + 2\operatorname{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\operatorname{polylog}\left(2, -\sqrt{x}-\sqrt{1+}\right.$ |
| default | $-\operatorname{arcsinh}\left(\sqrt{x}\right)^2 + 2\operatorname{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\operatorname{polylog}\left(2, -\sqrt{x}-\sqrt{1+}\right.$ |

```
[In] int(arcsinh(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1+x^(1/2)+(1+x)^(1/2))+2*polylog(
2,-x^(1/2)-(1+x)^(1/2))+2*arcsinh(x^(1/2))*ln(1-x^(1/2)-(1+x)^(1/2))+2*poly
log(2,x^(1/2)+(1+x)^(1/2))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(sqrt(x))/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(asinh(x**(1/2))/x,x)
[Out] Integral(asinh(sqrt(x))/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="maxima")
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] int(asinh(x^(1/2))/x,x)
[Out] int(asinh(x^(1/2))/x, x)
```

# 3.296 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx$

Optimal result . . . 2063
Rubi [A] (verified) . . . 2063
Mathematica [A] (verified) . . . 2064
Maple [A] (verified) . . . 2064
Fricas [A] (verification not implemented) . . . 2065
Sympy [F] . . . 2065
Maxima [A] (verification not implemented) . . . 2065
Giac [A] (verification not implemented) . . . 2066
Mupad [F(-1)] . . . 2066

## Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx = -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{x}$$

```
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 37}

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx = -\frac{\operatorname{arcsinh}(\sqrt{x})}{x} - \frac{\sqrt{x+1}}{\sqrt{x}}$$

```
[In] Int[ArcSinh[Sqrt[x]]/x^2,x]
```

```
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x} + \int \frac{1}{2x^{3/2}\sqrt{1+x}}\,dx \\ &= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\ &= -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}$$

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^2,x]
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1+x}}{\sqrt{x}}$ | 21 |
| default | $-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1+x}}{\sqrt{x}}$ | 21 |
| parts | $-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1+x}}{\sqrt{x}}$ | 21 |

```
[In] int(arcsinh(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{x+1}\sqrt{x}+\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}$$

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -(sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) + sqrt(x)))/x
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate(asinh(x**(1/2))/x**2,x)
[Out] Integral(asinh(sqrt(x))/x**2, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{x+1}}{\sqrt{x}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}$$

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -sqrt(x + 1)/sqrt(x) - arcsinh(sqrt(x))/x
```

## Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x} + \frac{2}{\left(\sqrt{x+1}-\sqrt{x}\right)^2-1}$$

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="giac")
[Out] -log(sqrt(x + 1) + sqrt(x))/x + 2/((sqrt(x + 1) - sqrt(x))^2 - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

```
[In] int(asinh(x^(1/2))/x^2,x)
[Out] int(asinh(x^(1/2))/x^2, x)
```

# 3.297 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^3}\,dx$

Optimal result . . . 2067
Rubi [A] (verified) . . . 2067
Mathematica [A] (verified) . . . 2068
Maple [A] (verified) . . . 2069
Fricas [A] (verification not implemented) . . . 2069
Sympy [F] . . . 2069
Maxima [A] (verification not implemented) . . . 2070
Giac [A] (verification not implemented) . . . 2070
Mupad [F(-1)] . . . 2070

## Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^3}\,dx = -\frac{\sqrt{1+x}}{6x^{3/2}} + \frac{\sqrt{1+x}}{3\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{2x^2}$$

```
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 47, 37}

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^3}\,dx = -\frac{\operatorname{arcsinh}(\sqrt{x})}{2x^2} - \frac{\sqrt{x+1}}{6x^{3/2}} + \frac{\sqrt{x+1}}{3\sqrt{x}}$$

```
[In] Int[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] -1/6*Sqrt[1 + x]/x^(3/2) + Sqrt[1 + x]/(3*Sqrt[x]) - ArcSinh[Sqrt[x]]/(2*x^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2}\int \frac{1}{2x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{6x^{3/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{6}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{6x^{3/2}} + \frac{\sqrt{1+x}}{3\sqrt{x}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\sqrt{x}\sqrt{1+x}(-1+2x) - 3\operatorname{arcsinh}\left(\sqrt{x}\right)}{6x^2}$$

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-1 + 2*x) - 3*ArcSinh[Sqrt[x]])/(6*x^2)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\operatorname{arcsinh}(\sqrt{x})}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$ | 31 |
| default | $-\frac{\operatorname{arcsinh}(\sqrt{x})}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$ | 31 |
| parts | $-\frac{\operatorname{arcsinh}(\sqrt{x})}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$ | 31 |

```
[In] int(arcsinh(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{(2\,x-1)\sqrt{x+1}\sqrt{x}-3\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{6\,x^2}$$

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*((2*x - 1)*sqrt(x + 1)*sqrt(x) - 3*log(sqrt(x + 1) + sqrt(x)))/x^2
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

```
[In] integrate(asinh(x**(1/2))/x**3,x)
[Out] Integral(asinh(sqrt(x))/x**3, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\sqrt{x+1}}{3\sqrt{x}} - \frac{\sqrt{x+1}}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x + 1)/sqrt(x) - 1/6*sqrt(x + 1)/x^(3/2) - 1/2*arcsinh(sqrt(x))/x^
2
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2} + \frac{2\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{3\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^3}$$

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x + 1) - sqrt(x))^2 - 1)
/((sqrt(x + 1) - sqrt(x))^2 - 1)^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

```
[In] int(asinh(x^(1/2))/x^3,x)
```

```
[Out] int(asinh(x^(1/2))/x^3, x)
```

# 3.298 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx$

Optimal result . . . . . 2071
Rubi [A] (verified) . . . . . 2071
Mathematica [A] (verified) . . . . . 2073
Maple [A] (verified) . . . . . 2073
Fricas [A] (verification not implemented) . . . . . 2073
Sympy [F] . . . . . 2074
Maxima [A] (verification not implemented) . . . . . 2074
Giac [A] (verification not implemented) . . . . . 2074
Mupad [F(-1)] . . . . . 2075

## Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{8\sqrt{1+x}}{45\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{3x^3}$$

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 47, 37}

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = -\frac{\operatorname{arcsinh}(\sqrt{x})}{3x^3} + \frac{4\sqrt{x+1}}{45x^{3/2}} - \frac{\sqrt{x+1}}{15x^{5/2}} - \frac{8\sqrt{x+1}}{45\sqrt{x}}$$

```
[In] Int[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] -1/15*Sqrt[1 + x]/x^(5/2) + (4*Sqrt[1 + x])/(45*x^(3/2)) - (8*Sqrt[1 + x])/
(45*Sqrt[x]) - ArcSinh[Sqrt[x]]/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3} + \frac{1}{3}\int \frac{1}{2x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3} + \frac{1}{6}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3} - \frac{2}{15}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3} + \frac{4}{45}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{8\sqrt{1+x}}{45\sqrt{x}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{\text{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = \frac{\sqrt{x}\sqrt{1+x}(-3+4x-8x^2)-15\text{arcsinh}\left(\sqrt{x}\right)}{45x^3}$$

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-3 + 4*x - 8*x^2) - 15*ArcSinh[Sqrt[x]])/(45*x^3)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$ | 41 |
| default | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$ | 41 |
| parts | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$ | 41 |

```
[In] int(arcsinh(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \frac{\text{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{(8\,x^2-4\,x+3)\sqrt{x+1}\sqrt{x}+15\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{45\,x^3}$$

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/45*((8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x) + 15*log(sqrt(x + 1) + sqrt(x)
))/x^3
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

```
[In] integrate(asinh(x**(1/2))/x**4,x)
[Out] Integral(asinh(sqrt(x))/x**4, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{8\sqrt{x+1}}{45\sqrt{x}} + \frac{4\sqrt{x+1}}{45\,x^{\frac{3}{2}}} - \frac{\sqrt{x+1}}{15\,x^{\frac{5}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{3\,x^3}$$

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="maxima")
[Out] -8/45*sqrt(x + 1)/sqrt(x) + 4/45*sqrt(x + 1)/x^(3/2) - 1/15*sqrt(x + 1)/x^(
5/2) - 1/3*arcsinh(sqrt(x))/x^3
```

## Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3} + \frac{16\left(10\left(\sqrt{x+1}-\sqrt{x}\right)^4 - 5\left(\sqrt{x+1}-\sqrt{x}\right)^2 + 1\right)}{45\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2 - 1\right)^5}$$

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="giac")
[Out] -1/3*log(sqrt(x + 1) + sqrt(x))/x^3 + 16/45*(10*(sqrt(x + 1) - sqrt(x))^4 -
 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 - 1)^5
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

```
[In] int(asinh(x^(1/2))/x^4,x)
[Out] int(asinh(x^(1/2))/x^4, x)
```

# 3.299 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx$

Optimal result . . . 2076
Rubi [A] (verified) . . . 2076
Mathematica [A] (verified) . . . 2078
Maple [A] (verified) . . . 2078
Fricas [A] (verification not implemented) . . . 2078
Sympy [F] . . . 2079
Maxima [A] (verification not implemented) . . . 2079
Giac [A] (verification not implemented) . . . 2079
Mupad [F(-1)] . . . 2080

## Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} + \frac{4\sqrt{1+x}}{35\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{4x^4}$$

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 47, 37}

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = -\frac{\operatorname{arcsinh}(\sqrt{x})}{4x^4} - \frac{2\sqrt{x+1}}{35x^{3/2}} + \frac{3\sqrt{x+1}}{70x^{5/2}} - \frac{\sqrt{x+1}}{28x^{7/2}} + \frac{4\sqrt{x+1}}{35\sqrt{x}}$$

```
[In] Int[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] -1/28*Sqrt[1 + x]/x^(7/2) + (3*Sqrt[1 + x])/(70*x^(5/2)) - (2*Sqrt[1 + x])/
(35*x^(3/2)) + (4*Sqrt[1 + x])/(35*Sqrt[x]) - ArcSinh[Sqrt[x]]/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4} + \frac{1}{4}\int \frac{1}{2x^{9/2}\sqrt{1+x}}\,dx \\
&= -\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4} + \frac{1}{8}\int \frac{1}{x^{9/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4} - \frac{3}{28}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4} + \frac{3}{35}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4} - \frac{2}{35}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} + \frac{4\sqrt{1+x}}{35\sqrt{x}} - \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{\text{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx = \frac{\sqrt{x}\sqrt{1+x}(-5+6x-8x^2+16x^3)-35\text{arcsinh}\left(\sqrt{x}\right)}{140x^4}$$

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-5 + 6*x - 8*x^2 + 16*x^3) - 35*ArcSinh[Sqrt[x]])/(14
0*x^4)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$ | 51 |
| default | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$ | 51 |
| parts | $-\frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$ | 51 |

```
[In] int(arcsinh(x^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

## Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{\text{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx = \frac{(16\,x^3-8\,x^2+6\,x-5)\sqrt{x+1}\sqrt{x}-35\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{140\,x^4}$$

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 - 8*x^2 + 6*x - 5)*sqrt(x + 1)*sqrt(x) - 35*log(sqrt(x + 1)
+ sqrt(x)))/x^4
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

```
[In] integrate(asinh(x**(1/2))/x**5,x)
[Out] Integral(asinh(sqrt(x))/x**5, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx = \frac{4\sqrt{x+1}}{35\sqrt{x}} - \frac{2\sqrt{x+1}}{35\,x^{\frac{3}{2}}} + \frac{3\sqrt{x+1}}{70\,x^{\frac{5}{2}}} - \frac{\sqrt{x+1}}{28\,x^{\frac{7}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{4\,x^4}$$

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="maxima")
[Out] 4/35*sqrt(x + 1)/sqrt(x) - 2/35*sqrt(x + 1)/x^(3/2) + 3/70*sqrt(x + 1)/x^(5
/2) - 1/28*sqrt(x + 1)/x^(7/2) - 1/4*arcsinh(sqrt(x))/x^4
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx \\
&= -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4\,x^4} \\
&\quad + \frac{8\left(35\left(\sqrt{x+1}-\sqrt{x}\right)^6 - 21\left(\sqrt{x+1}-\sqrt{x}\right)^4 + 7\left(\sqrt{x+1}-\sqrt{x}\right)^2 - 1\right)}{35\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2 - 1\right)^7}
\end{aligned}$$

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="giac")
[Out] -1/4*log(sqrt(x + 1) + sqrt(x))/x^4 + 8/35*(35*(sqrt(x + 1) - sqrt(x))^6 -
21*(sqrt(x + 1) - sqrt(x))^4 + 7*(sqrt(x + 1) - sqrt(x))^2 - 1)/((sqrt(x +
1) - sqrt(x))^2 - 1)^7
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^5}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

```
[In] int(asinh(x^(1/2))/x^5,x)
[Out] int(asinh(x^(1/2))/x^5, x)
```

# 3.300 $\int x^2 \text{arcsinh}\left(\frac{a}{x}\right)\, dx$

Optimal result . . . 2081
Rubi [A] (verified) . . . 2081
Mathematica [A] (verified) . . . 2083
Maple [A] (verified) . . . 2083
Fricas [B] (verification not implemented) . . . 2084
Sympy [F] . . . 2084
Maxima [A] (verification not implemented) . . . 2084
Giac [A] (verification not implemented) . . . 2085
Mupad [F(-1)] . . . 2085

## Optimal result

Integrand size = 10, antiderivative size = 56

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right)\, dx = \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3 \text{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

```
[Out] 1/3*x^3*arccsch(x/a)-1/6*a^3*arctanh((a^2/x^2+1)^(1/2))+1/6*a*x^2*(a^2/x^2+
1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5870, 6419, 272, 44, 65, 214}

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right)\, dx = \frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1} - \frac{1}{6}a^3 \text{arctanh}\left(\sqrt{\frac{a^2}{x^2}+1}\right) + \frac{1}{3}x^3 \text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[In] Int[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2)/6 + (x^3*ArcCsch[x/a])/3 - (a^3*ArcTanh[Sqrt[1 +
a^2/x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2 \operatorname{csch}^{-1}\left(\frac{x}{a}\right)\, dx \\
&= \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{3}a\int \frac{x}{\sqrt{1+\frac{a^2}{x^2}}}\, dx \\
&= \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\, dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{12}a^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\, dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{6}a\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\, dx, x, \sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}$$

$$= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int x^2\text{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{6}\left(a\sqrt{1+\frac{a^2}{x^2}}x^2 + 2x^3\text{arcsinh}\left(\frac{a}{x}\right) - a^3\log\left(\left(1+\sqrt{1+\frac{a^2}{x^2}}\right)x\right)\right)$$

```
[In] Integrate[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2 + 2*x^3*ArcSinh[a/x] - a^3*Log[(1 + Sqrt[1 + a^2/x
^2])*x])/6
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| derivativedivides | $-a^3\left(-\frac{x^3\,\text{arcsinh}\left(\frac{a}{x}\right)}{3a^3} - \frac{x^2\sqrt{\frac{a^2}{x^2}+1}}{6a^2} + \frac{\text{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)}{6}\right)$ | 54 |
| default | $-a^3\left(-\frac{x^3\,\text{arcsinh}\left(\frac{a}{x}\right)}{3a^3} - \frac{x^2\sqrt{\frac{a^2}{x^2}+1}}{6a^2} + \frac{\text{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)}{6}\right)$ | 54 |
| parts | $\frac{x^3\,\text{arcsinh}\left(\frac{a}{x}\right)}{3} + \frac{a\sqrt{a^2+x^2}\left(-a^2\ln\left(x+\sqrt{a^2+x^2}\right)+x\sqrt{a^2+x^2}\right)}{6\sqrt{\frac{a^2+x^2}{x^2}}\,x}$ | 70 |

```
[In] int(x^2*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-1/3/a^3*x^3*arcsinh(a/x)-1/6/a^2*x^2*(a^2/x^2+1)^(1/2)+1/6*arctanh(1
/(a^2/x^2+1)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.18

$$\int x^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{6}a^3\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+\frac{1}{6}ax^2\sqrt{\frac{a^2+x^2}{x^2}} + \frac{1}{3}\left(x^3-1\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right) + \frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt((a^2 + x^2)/x^2) - x) + 1/6*a*x^2*sqrt((a^2 + x^2)/x^2)
+ 1/3*(x^3 - 1)*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/3*log(x*sqrt((a^2
+ x^2)/x^2) + a - x) - 1/3*log(x*sqrt((a^2 + x^2)/x^2) - a - x)
```

## Sympy [F]

$$\int x^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \int x^2 \operatorname{asinh}\left(\frac{a}{x}\right)dx$$

```
[In] integrate(x**2*asinh(a/x),x)
```

```
[Out] Integral(x**2*asinh(a/x), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\begin{aligned}&\int x^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx \\ &= \frac{1}{3}x^3 \operatorname{arsinh}\left(\frac{a}{x}\right) \\ &\quad -\frac{1}{12}\left(a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)-2x^2\sqrt{\frac{a^2}{x^2}+1}\right)a\end{aligned}$$

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a/x) - 1/12*(a^2*log(sqrt(a^2/x^2 + 1) + 1) - a^2*log(sqrt(
a^2/x^2 + 1) - 1) - 2*x^2*sqrt(a^2/x^2 + 1))*a
```

## Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int x^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)\, dx = -\frac{1}{6}\, a^3 \log\left(|a|\right) \operatorname{sgn}(x) + \frac{1}{3}\, x^3 \log\left(\sqrt{\frac{a^2}{x^2}+1} + \frac{a}{x}\right) + \frac{a^3 \log\left(-x+\sqrt{a^2+x^2}\right)}{6 \operatorname{sgn}\left(x\right)} + \frac{\sqrt{a^2+x^2}ax}{6 \operatorname{sgn}\left(x\right)}$$

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] -1/6*a^3*log(abs(a))*sgn(x) + 1/3*x^3*log(sqrt(a^2/x^2 + 1) + a/x) + 1/6*a^
3*log(-x + sqrt(a^2 + x^2))/sgn(x) + 1/6*sqrt(a^2 + x^2)*a*x/sgn(x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)\, dx = \int x^2 \operatorname{asinh}\left(\frac{a}{x}\right)\, dx$$

```
[In] int(x^2*asinh(a/x),x)
```

```
[Out] int(x^2*asinh(a/x), x)
```

# 3.301 $\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx$

Optimal result . . . 2086
Rubi [A] (verified) . . . 2086
Mathematica [A] (verified) . . . 2087
Maple [A] (verified) . . . 2087
Fricas [A] (verification not implemented) . . . 2088
Sympy [F] . . . 2088
Maxima [A] (verification not implemented) . . . 2088
Giac [A] (verification not implemented) . . . 2089
Mupad [B] (verification not implemented) . . . 2089

## Optimal result

Integrand size = 8, antiderivative size = 33

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}x+\frac{1}{2}x^2\operatorname{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arccsch(x/a)+1/2*a*x*(a^2/x^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5870, 6419, 197}

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\operatorname{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[In] Int[x*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)\,dx \\ &= \frac{1}{2}x^2\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\,dx \\ &= \frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}x + \frac{1}{2}x^2\operatorname{csch}^{-1}\left(\frac{x}{a}\right)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}x\left(a\sqrt{1+\frac{a^2}{x^2}} + x\operatorname{arcsinh}\left(\frac{a}{x}\right)\right)$$

```
[In] Integrate[x*ArcSinh[a/x],x]
```

```
[Out] (x*(a*Sqrt[1 + a^2/x^2] + x*ArcSinh[a/x]))/2
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

| method | result | size |
|---|---|---|
| derivativedivides | $-a^2\left(-\frac{x^2\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2a^2} - \frac{x\sqrt{\frac{a^2}{x^2}+1}}{2a}\right)$ | 38 |
| default | $-a^2\left(-\frac{x^2\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2a^2} - \frac{x\sqrt{\frac{a^2}{x^2}+1}}{2a}\right)$ | 38 |
| parts | $\frac{x^2\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2} + \frac{a\left(a^2+x^2\right)}{2\sqrt{\frac{a^2+x^2}{x^2}}\,x}$ | 39 |

```
[In] int(x*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(-1/2/a^2*x^2*arcsinh(a/x)-1/2/a*x*(a^2/x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}\,x^2\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2+x^2}{x^2}}$$

```
[In] integrate(x*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/2*a*x*sqrt((a^2 + x^2)/x^2
)
```

## Sympy [F]

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \int x\,\operatorname{asinh}\left(\frac{a}{x}\right)dx$$

```
[In] integrate(x*asinh(a/x),x)
```

```
[Out] Integral(x*asinh(a/x), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}\,x^2\,\operatorname{arsinh}\left(\frac{a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2}{x^2}+1}$$

```
[In] integrate(x*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a/x) + 1/2*a*x*sqrt(a^2/x^2 + 1)
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}\,x^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right) - \frac{1}{2}\,a|a|\operatorname{sgn}(x) + \frac{\sqrt{a^2+x^2}a}{2\operatorname{sgn}(x)}$$

```
[In] integrate(x*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(a^2/x^2 + 1) + a/x) - 1/2*a*abs(a)*sgn(x) + 1/2*sqrt(a^2 +
 x^2)*a/sgn(x)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{x^2\operatorname{asinh}\left(\frac{a}{x}\right)}{2} + \frac{a\,x\,\sqrt{\frac{a^2}{x^2}+1}}{2}$$

```
[In] int(x*asinh(a/x),x)
```

```
[Out] (x^2*asinh(a/x))/2 + (a*x*(a^2/x^2 + 1)^(1/2))/2
```

# 3.302 $\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\, dx$

Optimal result . . . 2090
Rubi [A] (verified) . . . 2090
Mathematica [B] (verified) . . . 2091
Maple [A] (verified) . . . 2092
Fricas [B] (verification not implemented) . . . 2092
Sympy [F] . . . 2093
Maxima [A] (verification not implemented) . . . 2093
Giac [B] (verification not implemented) . . . 2093
Mupad [B] (verification not implemented) . . . 2094

## Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\, dx = x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\operatorname{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

```
[Out] x*arccsch(x/a)+a*arctanh((a^2/x^2+1)^(1/2))
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5870, 6413, 272, 65, 214}

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\, dx = a\operatorname{arctanh}\left(\sqrt{\frac{a^2}{x^2}+1}\right) + x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[In] Int[ArcSinh[a/x],x]
```

```
[Out] x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{csch}^{-1}\left(\frac{x}{a}\right)\,dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x}\,dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+\frac{a^2}{x^2}}\right)}{a} \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\operatorname{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. $2(25) = 50$.

Time $= 0.06$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 3.08$

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = x\operatorname{arcsinh}\left(\frac{a}{x}\right) + \frac{a\sqrt{a^2+x^2}\left(-\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)\right)}{2\sqrt{1+\frac{a^2}{x^2}}x}$$

```
[In] Integrate[ArcSinh[a/x],x]
```

```
[Out] x*ArcSinh[a/x] + (a*Sqrt[a^2 + x^2]*(-Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 +
x/Sqrt[a^2 + x^2]]))/(2*Sqrt[1 + a^2/x^2]*x)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| derivativedivides | $-a\left(-\frac{x \operatorname{arcsinh}\left(\frac{a}{x}\right)}{a}-\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)\right)$ | 31 |
| default | $-a\left(-\frac{x \operatorname{arcsinh}\left(\frac{a}{x}\right)}{a}-\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)\right)$ | 31 |
| parts | $x \operatorname{arcsinh}\left(\frac{a}{x}\right)+\frac{a\sqrt{a^2+x^2}\,\ln\left(x+\sqrt{a^2+x^2}\right)}{\sqrt{\frac{a^2+x^2}{x^2}}\,x}$ | 49 |

```
[In] int(arcsinh(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-1/a*x*arcsinh(a/x)-arctanh(1/(a^2/x^2+1)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.84

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = -a\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+(x-1)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)$$
$$+\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

```
[In] integrate(arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] -a*log(x*sqrt((a^2 + x^2)/x^2) - x) + (x - 1)*log((x*sqrt((a^2 + x^2)/x^2)
+ a)/x) + log(x*sqrt((a^2 + x^2)/x^2) + a - x) - log(x*sqrt((a^2 + x^2)/x^2
) - a - x)
```

## Sympy [F]

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \int \operatorname{asinh}\left(\frac{a}{x}\right)\,dx$$

```
[In] integrate(asinh(a/x),x)
```

```
[Out] Integral(asinh(a/x), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}\,a\left(\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)\right)+x\operatorname{arsinh}\left(\frac{a}{x}\right)$$

```
[In] integrate(arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(sqrt(a^2/x^2 + 1) + 1) - log(sqrt(a^2/x^2 + 1) - 1)) + x*arcsinh
(a/x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = a\log\left(|a|\right)\operatorname{sgn}(x)+x\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)-\frac{a\log\left(-x+\sqrt{a^2+x^2}\right)}{\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(arcsinh(a/x),x, algorithm="giac")
```

```
[Out] a*log(abs(a))*sgn(x) + x*log(sqrt(a^2/x^2 + 1) + a/x) - a*log(-x + sqrt(a^2
 + x^2))/sgn(x)
```

## Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = x\,\operatorname{asinh}\left(\frac{a}{x}\right) + a\,\ln\left(x + \sqrt{a^2 + x^2}\right)\,\operatorname{sign}(x)$$

```
[In] int(asinh(a/x),x)
[Out] x*asinh(a/x) + a*log(x + (a^2 + x^2)^(1/2))*sign(x)
```

# 3.303 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal result . . . 2095
Rubi [A] (verified) . . . 2095
Mathematica [A] (verified) . . . 2097
Maple [A] (verified) . . . 2097
Fricas [F] . . . 2097
Sympy [F] . . . 2098
Maxima [F] . . . 2098
Giac [F] . . . 2098
Mupad [F(-1)] . . . 2098

## Optimal result

Integrand size = 10, antiderivative size = 52

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}\operatorname{arcsinh}\left(\frac{a}{x}\right)^2 - \operatorname{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right)$$

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1-(a/x+(a^2/x^2+1)^(1/2))^2)-1/2*polylog
(2,(a/x+(a^2/x^2+1)^(1/2))^2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = -\frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\operatorname{arcsinh}\left(\frac{a}{x}\right)^2 - \operatorname{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right)$$

```
[In] Int[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x\coth(x)\,dx, x, \text{arcsinh}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 + 2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 - \text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) \\
&\quad + \text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}\left(\frac{a}{x}\right)\right) \\
&= \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 - \text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) \\
&= \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 - \text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\,\text{PolyLog}\left(2, e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}\operatorname{arcsinh}\left(\frac{a}{x}\right)^2 - \operatorname{arcsinh}\left(\frac{a}{x}\right)\log\left(1 - e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(\frac{a}{x}\right)}\right)$$

```
[In] Integrate[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.19

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^2}{2} - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{\frac{a^2}{x^2}+1}\right) - \operatorname{polylog}\left(2,-\frac{a}{x}-\sqrt{\frac{a^2}{x^2}+1}\right) - \operatorname{arcsi}$ |
| default | $\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^2}{2} - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{\frac{a^2}{x^2}+1}\right) - \operatorname{polylog}\left(2,-\frac{a}{x}-\sqrt{\frac{a^2}{x^2}+1}\right) - \operatorname{arcsi}$ |

```
[In] int(arcsinh(a/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1+a/x+(a^2/x^2+1)^(1/2))-polylog(2,-a/x-
(a^2/x^2+1)^(1/2))-arcsinh(a/x)*ln(1-a/x-(a^2/x^2+1)^(1/2))-polylog(2,a/x+(
a^2/x^2+1)^(1/2))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a/x)/x, x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(asinh(a/x)/x,x)
[Out] Integral(asinh(a/x)/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="maxima")
[Out] a*integrate(x*log(x)/(a^3 + a*x^2 + (a^2 + x^2)^(3/2)), x) + log(a + sqrt(a
^2 + x^2))*log(x) - 1/2*log(x)^2 - 1/2*log(x)*log(x^2/a^2 + 1) - 1/4*dilog(
-x^2/a^2)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a/x)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] int(asinh(a/x)/x,x)
[Out] int(asinh(a/x)/x, x)
```

# 3.304 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx$

Optimal result . . . . . 2099
Rubi [A] (verified) . . . . . 2099
Mathematica [A] (verified) . . . . . 2100
Maple [A] (verified) . . . . . 2100
Fricas [A] (verification not implemented) . . . . . 2101
Sympy [A] (verification not implemented) . . . . . 2101
Maxima [A] (verification not implemented) . . . . . 2101
Giac [A] (verification not implemented) . . . . . 2102
Mupad [B] (verification not implemented) . . . . . 2102

## Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] -arccsch(x/a)/x+(a^2/x^2+1)^(1/2)/a
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5870, 6419, 267}

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[In] Int[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx \\ &= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x} - a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^3}\,dx \\ &= \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}$$

```
[In] Integrate[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcSinh[a/x]/x
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{a\,\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{\frac{a^2}{x^2}+1}}{a}$ | 31 |
| default | $-\frac{\frac{a\,\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{\frac{a^2}{x^2}+1}}{a}$ | 31 |
| parts | $-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}+\frac{a^2+x^2}{a\sqrt{\frac{a^2+x^2}{x^2}}\,x^2}$ | 40 |

```
[In] int(arcsinh(a/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(a/x*arcsinh(a/x)-(a^2/x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{a\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right) - x\sqrt{\frac{a^2+x^2}{x^2}}}{ax}$$

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="fricas")
[Out] -(a*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - x*sqrt((a^2 + x^2)/x^2))/(a*x)
```

## Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a/x)/x**2,x)
[Out] Piecewise((-asinh(a/x)/x + sqrt(a**2/x**2 + 1)/a, Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{a\operatorname{arsinh}\left(\frac{a}{x}\right)}{x} - \sqrt{\frac{a^2}{x^2}+1}}{a}$$

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="maxima")
[Out] -(a*arcsinh(a/x)/x - sqrt(a^2/x^2 + 1))/a
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a}$$

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="giac")
[Out] -log(sqrt(a^2/x^2 + 1) + a/x)/x + sqrt(a^2/x^2 + 1)/a
```

## Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}$$

```
[In] int(asinh(a/x)/x^2,x)
[Out] (a^2/x^2 + 1)^(1/2)/a - asinh(a/x)/x
```

# 3.305 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal result . . . 2103
Rubi [A] (verified) . . . 2103
Mathematica [A] (verified) . . . 2105
Maple [A] (verified) . . . 2105
Fricas [A] (verification not implemented) . . . 2105
Sympy [F] . . . 2106
Maxima [B] (verification not implemented) . . . 2106
Giac [A] (verification not implemented) . . . 2106
Mupad [B] (verification not implemented) . . . 2107

## Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/4*arccsch(x/a)/a^2-1/2*arccsch(x/a)/x^2+1/4*(a^2/x^2+1)^(1/2)/a/x
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5870, 6419, 342, 327, 221}

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[In] Int[ArcSinh[a/x]/x^3,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/(4*a*x) - ArcCsch[x/a]/(4*a^2) - ArcCsch[x/a]/(2*x^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^4}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right)}{4a} \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{a\sqrt{1+\frac{a^2}{x^2}}x - (2a^2+x^2)\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4a^2x^2}$$

```
[In] Integrate[ArcSinh[a/x]/x^3,x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x - (2*a^2 + x^2)*ArcSinh[a/x])/(4*a^2*x^2)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{a^2\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{\frac{a^2}{x^2}+1}}{4x}+\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$ | 46 |
| default | $-\frac{\frac{a^2\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{\frac{a^2}{x^2}+1}}{4x}+\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$ | 46 |
| parts | $-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}+\frac{\sqrt{a^2+x^2}\left(-\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2+x^2}}{x}\right)x^2+\sqrt{a^2}\sqrt{a^2+x^2}\right)}{4a\sqrt{\frac{a^2+x^2}{x^2}}x^3\sqrt{a^2}}$ | 94 |

```
[In] int(arcsinh(a/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(1/2*a^2/x^2*arcsinh(a/x)-1/4*a/x*(a^2/x^2+1)^(1/2)+1/4*arcsinh(a/x)
)
```

## Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{ax\sqrt{\frac{a^2+x^2}{x^2}} - (2\,a^2+x^2)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)}{4\,a^2x^2}$$

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x*sqrt((a^2 + x^2)/x^2) - (2*a^2 + x^2)*log((x*sqrt((a^2 + x^2)/x^2)
 + a)/x))/(a^2*x^2)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$

```
[In] integrate(asinh(a/x)/x**3,x)
[Out] Integral(asinh(a/x)/x**3, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(42) = 84.

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$
$$= \frac{1}{8}\,a\left(\frac{2\,x\sqrt{\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}+1\right)-a^4} - \frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}+a\right)}{a^3} + \frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}-a\right)}{a^3}\right) - \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="maxima")
[Out] 1/8*a*(2*x*sqrt(a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 + 1) - a^4) - log(x*sqrt(a^2
/x^2 + 1) + a)/a^3 + log(x*sqrt(a^2/x^2 + 1) - a)/a^3) - 1/2*arcsinh(a/x)/x
^2
```

## Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{a\left(\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{2\sqrt{a^2+x^2}}{a^2x^2}\right)}{8\operatorname{sgn}(x)} - \frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="giac")
[Out] -1/8*a*(log(a + sqrt(a^2 + x^2))/a^3 - log(-a + sqrt(a^2 + x^2))/a^3 - 2*sq
rt(a^2 + x^2)/(a^2*x^2))/sgn(x) - 1/2*log(sqrt(a^2/x^2 + 1) + a/x)/x^2
```

## Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\sqrt{\frac{a^2}{x^2}+1}}{4\,a\,x} - \frac{\operatorname{asinh}\left(\frac{a}{x}\right)\,\left(\frac{x}{4\,a^2}+\frac{1}{2\,x}\right)}{x}$$

```
[In] int(asinh(a/x)/x^3,x)
[Out] (a^2/x^2 + 1)^(1/2)/(4*a*x) - (asinh(a/x)*(x/(4*a^2) + 1/(2*x)))/x
```

# 3.306 $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal result . . . 2108
Rubi [A] (verified) . . . 2108
Mathematica [A] (verified) . . . 2109
Maple [A] (verified) . . . 2110
Fricas [A] (verification not implemented) . . . 2110
Sympy [F] . . . 2110
Maxima [A] (verification not implemented) . . . 2111
Giac [A] (verification not implemented) . . . 2111
Mupad [F(-1)] . . . 2111

## Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] 1/9*(a^2/x^2+1)^(3/2)/a^3-1/3*arccsch(x/a)/x^3-1/3*(a^2/x^2+1)^(1/2)/a^3
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5870, 6419, 272, 45}

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3} - \frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3} - \frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[In] Int[ArcSinh[a/x]/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + a^2/x^2]/a^3 + (1 + a^2/x^2)^(3/2)/(9*a^3) - ArcCsch[x/a]/(3*
x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{3}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^5}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\operatorname{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \left(-\frac{2}{9a^3} + \frac{1}{9ax^2}\right)\sqrt{\frac{a^2+x^2}{x^2}} - \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}$$

```
[In] Integrate[ArcSinh[a/x]/x^4,x]
```

```
[Out] (-2/(9*a^3) + 1/(9*a*x^2))*Sqrt[(a^2 + x^2)/x^2] - ArcSinh[a/x]/(3*x^3)
```

## Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| parts | $-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}+\frac{(a^2+x^2)(a^2-2x^2)}{9a^3\sqrt{\frac{a^2+x^2}{x^2}}\,x^4}$ | 50 |
| derivativedivides | $-\frac{\frac{a^3\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{\frac{a^2}{x^2}+1}}{9x^2}+\frac{2\sqrt{\frac{a^2}{x^2}+1}}{9}}{a^3}$ | 53 |
| default | $-\frac{\frac{a^3\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{\frac{a^2}{x^2}+1}}{9x^2}+\frac{2\sqrt{\frac{a^2}{x^2}+1}}{9}}{a^3}$ | 53 |

```
[In] int(arcsinh(a/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(a/x)/x^3+1/9/a^3*(a^2+x^2)*(a^2-2*x^2)/((a^2+x^2)/x^2)^(1/2)/x
^4
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{3\,a^3\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-(a^2x-2\,x^3)\sqrt{\frac{a^2+x^2}{x^2}}}{9\,a^3x^3}$$

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*a^3*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - (a^2*x - 2*x^3)*sqrt((a^
2 + x^2)/x^2))/(a^3*x^3)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

```
[In] integrate(asinh(a/x)/x**4,x)
```

```
[Out] Integral(asinh(a/x)/x**4, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{1}{9}\,a\left(\frac{\left(\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4} - \frac{3\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right) - \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{3\,x^3}$$

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a*((a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(a^2/x^2 + 1)/a^4) - 1/3*arcsinh(a/x
)/x^3
```

## Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{3\,x^3} - \frac{4\left(a^2-3\left(x-\sqrt{a^2+x^2}\right)^2\right)a}{9\left(a^2-\left(x-\sqrt{a^2+x^2}\right)^2\right)^3\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(a^2/x^2 + 1) + a/x)/x^3 - 4/9*(a^2 - 3*(x - sqrt(a^2 + x^2))^
2)*a/((a^2 - (x - sqrt(a^2 + x^2))^2)^3*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

```
[In] int(asinh(a/x)/x^4,x)
```

```
[Out] int(asinh(a/x)/x^4, x)
```

# 3.307 $\int x^m \mathbf{arcsinh}(ax^n)\, dx$

Optimal result . . . 2112
Rubi [A] (verified) . . . 2112
Mathematica [A] (verified) . . . 2113
Maple [F] . . . 2114
Fricas [F(-2)] . . . 2114
Sympy [F] . . . 2114
Maxima [F] . . . 2114
Giac [F] . . . 2115
Mupad [F(-1)] . . . 2115

## Optimal result

Integrand size = 10, antiderivative size = 77

$$\int x^m \text{arcsinh}(ax^n)\, dx = \frac{x^{1+m}\text{arcsinh}(ax^n)}{1+m} - \frac{anx^{1+m+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m+n}{2n}, \frac{1+m+3n}{2n}, -a^2x^{2n}\right)}{(1+m)(1+m+n)}$$

```
[Out] x^(1+m)*arcsinh(a*x^n)/(1+m)-a*n*x^(1+m+n)*hypergeom([1/2, 1/2*(1+m+n)/n],[
1/2*(1+m+3*n)/n],-a^2*x^(2*n))/(1+m)/(1+m+n)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\int x^m \text{arcsinh}(ax^n)\, dx = \frac{x^{m+1}\text{arcsinh}(ax^n)}{m+1} - \frac{anx^{m+n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+n+1}{2n}, \frac{m+3n+1}{2n}, -a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

```
[In] Int[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[1
/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1 +
m + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\text{arcsinh}(ax^n)}{1+m} - \frac{\int \frac{anx^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m} \\ &= \frac{x^{1+m}\text{arcsinh}(ax^n)}{1+m} - \frac{(an)\int \frac{x^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m} \\ &= \frac{x^{1+m}\text{arcsinh}(ax^n)}{1+m} - \frac{anx^{1+m+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m+n}{2n}, \frac{1+m+3n}{2n}, -a^2x^{2n}\right)}{(1+m)(1+m+n)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\begin{aligned}&\int x^m\text{arcsinh}(ax^n)\,dx \\ &= \frac{x^{1+m}\left((1+m+n)\text{arcsinh}(ax^n) - anx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m+n}{2n}, \frac{1+m+3n}{2n}, -a^2x^{2n}\right)\right)}{(1+m)(1+m+n)}\end{aligned}$$

```
[In] Integrate[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*((1 + m + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, (1
+ m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))]))/((1 + m)*(1 + m + n)
)
```

## Maple [F]

$$\int x^m \operatorname{arcsinh}(a\,x^n)\,dx$$

```
[In] int(x^m*arcsinh(a*x^n),x)
```

```
[Out] int(x^m*arcsinh(a*x^n),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^m \operatorname{arcsinh}(ax^n)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x^m \operatorname{arcsinh}(ax^n)\,dx = \int x^m \operatorname{asinh}(ax^n)\,dx$$

```
[In] integrate(x**m*asinh(a*x**n),x)
```

```
[Out] Integral(x**m*asinh(a*x**n), x)
```

## Maxima [F]

$$\int x^m \operatorname{arcsinh}(ax^n)\,dx = \int x^m \operatorname{arsinh}(ax^n)\,dx$$

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(e^(m*log(x) + n*log(x))/(a^3*(m + 1)*x^(3*n) + a*(m + 1)*x^n
 + (a^2*(m + 1)*x^(2*n) + m + 1)*sqrt(a^2*x^(2*n) + 1)), x) + n*integrate(x
^m/(a^2*(m + 1)*x^(2*n) + m + 1), x) + ((m + 1)*x*x^m*log(a*x^n + sqrt(a^2*
x^(2*n) + 1)) - n*x*x^m)/(m^2 + 2*m + 1)
```

## Giac [F]

$$\int x^m \text{arcsinh}(ax^n)\ dx = \int x^m\ \text{arsinh}\left(ax^n\right)\ dx$$

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x^n), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m \text{arcsinh}(ax^n)\ dx = \int x^m\ \text{asinh}(a\,x^n)\ dx$$

```
[In] int(x^m*asinh(a*x^n),x)
[Out] int(x^m*asinh(a*x^n), x)
```

# 3.308 $\int x^2 \text{arcsinh}(ax^n)\, dx$

Optimal result . . . 2116
Rubi [A] (verified) . . . 2116
Mathematica [A] (verified) . . . 2117
Maple [F] . . . 2118
Fricas [F(-2)] . . . 2118
Sympy [F] . . . 2118
Maxima [F] . . . 2118
Giac [F] . . . 2119
Mupad [F(-1)] . . . 2119

## Optimal result

Integrand size = 10, antiderivative size = 64

$$\int x^2 \text{arcsinh}(ax^n)\, dx = \frac{1}{3}x^3 \text{arcsinh}(ax^n) - \frac{anx^{3+n}\, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2n}, \frac{3(1+n)}{2n}, -a^2x^{2n}\right)}{3(3+n)}$$

```
[Out] 1/3*x^3*arcsinh(a*x^n)-1/3*a*n*x^(3+n)*hypergeom([1/2, 1/2*(3+n)/n],[3/2*(1
+n)/n],-a^2*x^(2*n))/(3+n)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\int x^2 \text{arcsinh}(ax^n)\, dx = \frac{1}{3}x^3 \text{arcsinh}(ax^n) - \frac{anx^{n+3}\, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+3}{2n}, \frac{3(n+1)}{2n}, -a^2x^{2n}\right)}{3(n+3)}$$

```
[In] Int[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax^n) - \frac{1}{3}\int \frac{anx^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= \frac{1}{3}x^3\text{arcsinh}(ax^n) - \frac{1}{3}(an)\int \frac{x^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= \frac{1}{3}x^3\text{arcsinh}(ax^n) - \frac{anx^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2n},\frac{3(1+n)}{2n},-a^2x^{2n}\right)}{3(3+n)}\end{aligned}$$

## Mathematica [A] (verified)

Time $= 0.03$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 1.03$

$$\begin{aligned}\int x^2\text{arcsinh}(ax^n)\,dx &= \frac{1}{3}x^3\text{arcsinh}(ax^n) \\ &\quad - \frac{anx^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2n},1+\frac{3+n}{2n},-a^2x^{2n}\right)}{3(3+n)}\end{aligned}$$

```
[In] Integrate[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), 1 + (3 + n)/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

## Maple [F]

$$\int x^2 \operatorname{arcsinh}(a\,x^n)\,dx$$

```
[In] int(x^2*arcsinh(a*x^n),x)
```

```
[Out] int(x^2*arcsinh(a*x^n),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \operatorname{arcsinh}(ax^n)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x^2 \operatorname{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{asinh}(ax^n)\,dx$$

```
[In] integrate(x**2*asinh(a*x**n),x)
```

```
[Out] Integral(x**2*asinh(a*x**n), x)
```

## Maxima [F]

$$\int x^2 \operatorname{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{arsinh}(ax^n)\,dx$$

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*n*x^3 + 1/3*x^3*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) - a*n*integrate(1/3
*x^2*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) + n*integrate(
1/3*x^2/(a^2*x^(2*n) + 1), x)
```

## Giac [F]

$$\int x^2 \text{arcsinh}(ax^n)\ dx = \int x^2\, \text{arsinh}\left(ax^n\right)\ dx$$

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x^n), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax^n)\ dx = \int x^2\, \text{asinh}(a\, x^n)\ dx$$

```
[In] int(x^2*asinh(a*x^n),x)
[Out] int(x^2*asinh(a*x^n), x)
```

# 3.309 $\int x\text{arcsinh}(ax^n)\,dx$

Optimal result . . . 2120
Rubi [A] (verified) . . . 2120
Mathematica [A] (verified) . . . 2121
Maple [F] . . . 2122
Fricas [F(-2)] . . . 2122
Sympy [F] . . . 2122
Maxima [F] . . . 2122
Giac [F] . . . 2123
Mupad [F(-1)] . . . 2123

## Optimal result

Integrand size = 8, antiderivative size = 65

$$\int x\text{arcsinh}(ax^n)\,dx = \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{anx^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2+n)}$$

```
[Out] 1/2*x^2*arcsinh(a*x^n)-1/2*a*n*x^(2+n)*hypergeom([1/2, 1/2*(2+n)/n],[3/2+1/
n],-a^2*x^(2*n))/(2+n)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5875, 12, 371}

$$\int x\text{arcsinh}(ax^n)\,dx = \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{anx^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),-a^2x^{2n}\right)}{2(n+2)}$$

```
[In] Int[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*n
), (3 + 2/n)/2, -(a^2*x^(2*n))])/(2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{1}{2}\int \frac{anx^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{1}{2}(an)\int \frac{x^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{anx^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2+n)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int x\text{arcsinh}(ax^n)\,dx \\ &= \frac{x^2\left((2+n)\text{arcsinh}(ax^n) - anx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-a^2x^{2n}\right)\right)}{2(2+n)}\end{aligned}$$

```
[In] Integrate[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*((2 + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, 1/2 + n^(-1),
 3/2 + n^(-1), -(a^2*x^(2*n))]))/(2*(2 + n))
```

## Maple [F]

$$\int x \operatorname{arcsinh}(a\,x^n)\,dx$$

```
[In] int(x*arcsinh(a*x^n),x)
[Out] int(x*arcsinh(a*x^n),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax^n)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int x\text{arcsinh}(ax^n)\,dx = \int x\,\text{asinh}\,(ax^n)\,dx$$

```
[In] integrate(x*asinh(a*x**n),x)
[Out] Integral(x*asinh(a*x**n), x)
```

## Maxima [F]

$$\int x\text{arcsinh}(ax^n)\,dx = \int x\,\text{arsinh}\,(ax^n)\,dx$$

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="maxima")
[Out] -1/4*n*x^2 - a*n*integrate(1/2*x*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) +
1)^(3/2)), x) + 1/2*x^2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) + n*integrate(1/
2*x/(a^2*x^(2*n) + 1), x)
```

## Giac [F]

$$\int x\text{arcsinh}(ax^n)\ dx = \int x\,\text{arsinh}\,(ax^n)\ dx$$

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x^n), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax^n)\ dx = \int x\,\text{asinh}(a\,x^n)\ dx$$

```
[In] int(x*asinh(a*x^n),x)
[Out] int(x*asinh(a*x^n), x)
```

# 3.310 $\int \operatorname{arcsinh}(ax^n)\,dx$

Optimal result . . . 2124
Rubi [A] (verified) . . . 2124
Mathematica [A] (verified) . . . 2125
Maple [F] . . . 2126
Fricas [F(-2)] . . . 2126
Sympy [F] . . . 2126
Maxima [F] . . . 2126
Giac [F] . . . 2127
Mupad [F(-1)] . . . 2127

## Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \operatorname{arcsinh}(ax^n)\,dx = x\operatorname{arcsinh}(ax^n) - \frac{anx^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{1+n}$$

```
[Out] x*arcsinh(a*x^n)-a*n*x^(1+n)*hypergeom([1/2, 1/2*(1+n)/n],[3/2+1/2/n],-a^2*
x^(2*n))/(1+n)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5874, 12, 371}

$$\int \operatorname{arcsinh}(ax^n)\,dx = x\operatorname{arcsinh}(ax^n) - \frac{anx^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{n+1}$$

```
[In] Int[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\operatorname{arcsinh}(ax^n) - \int \frac{anx^n}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= x\operatorname{arcsinh}(ax^n) - (an)\int \frac{x^n}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= x\operatorname{arcsinh}(ax^n) - \frac{anx^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{1+n}\end{aligned}$$

## Mathematica [A] (verified)

Time $= 0.04$ (sec) , antiderivative size $= 56$, normalized size of antiderivative $= 1.00$

$$\begin{aligned}&\int \operatorname{arcsinh}(ax^n)\,dx \\ &= x\operatorname{arcsinh}(ax^n) - \frac{anx^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{1+n}\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

## Maple [F]

$$\int \operatorname{arcsinh}\left(a\,x^{n}\right)\,dx$$

```
[In] int(arcsinh(a*x^n),x)
```

```
[Out] int(arcsinh(a*x^n),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax^{n})\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \operatorname{arcsinh}(ax^{n})\,dx = \int \operatorname{asinh}\left(ax^{n}\right)\,dx$$

```
[In] integrate(asinh(a*x**n),x)
```

```
[Out] Integral(asinh(a*x**n), x)
```

## Maxima [F]

$$\int \operatorname{arcsinh}(ax^{n})\,dx = \int \operatorname{arsinh}\left(ax^{n}\right)\,dx$$

```
[In] integrate(arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) - n*
x + n*integrate(1/(a^2*x^(2*n) + 1), x) + x*log(a*x^n + sqrt(a^2*x^(2*n) +
1))
```

## Giac [F]

$$\int \operatorname{arcsinh}(ax^n)\ dx = \int \operatorname{arsinh}\left(ax^n\right)\ dx$$

```
[In] integrate(arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax^n)\ dx = \int \operatorname{asinh}(a\,x^n)\ dx$$

```
[In] int(asinh(a*x^n),x)
[Out] int(asinh(a*x^n), x)
```

# 3.311 $\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx$

Optimal result . . . 2128
Rubi [A] (verified) . . . 2128
Mathematica [A] (verified) . . . 2130
Maple [A] (verified) . . . 2130
Fricas [F(-2)] . . . 2130
Sympy [F] . . . 2131
Maxima [F] . . . 2131
Giac [F] . . . 2131
Mupad [F(-1)] . . . 2131

## Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n} + \frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n}$$

```
[Out] -1/2*arcsinh(a*x^n)^2/n+arcsinh(a*x^n)*ln(1-(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2
)/n+1/2*polylog(2,(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2)/n
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n} - \frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n}$$

```
[In] Int[ArcSinh[a*x^n]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]^2/n + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{arcsinh}(ax^n)\right)}{n} \\
&= -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}(ax^n)\right)}{n} \\
&= -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n} \\
&\quad - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax^n)\right)}{n} \\
&= -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n} \\
&= -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n} + \frac{\text{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \frac{-\operatorname{arcsinh}(ax^n)\left(\operatorname{arcsinh}(ax^n) - 2\log\left(1 - e^{2\operatorname{arcsinh}(ax^n)}\right)\right) + \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n}$$

```
[In] Integrate[ArcSinh[a*x^n]/x,x]
```

```
[Out] (-(ArcSinh[a*x^n]*(ArcSinh[a*x^n] - 2*Log[1 - E^(2*ArcSinh[a*x^n])])) + Pol
yLog[2, E^(2*ArcSinh[a*x^n])])/(2*n)
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\operatorname{arcsinh}(a\,x^n)^2}{2}+\operatorname{arcsinh}(a\,x^n)\ln\left(1+a\,x^n+\sqrt{1+a^2x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^n-\sqrt{1+a^2x^{2n}}\right)+\operatorname{arcsinh}(a\,x^n)\ln\left(1-a\,x^n-\sqrt{1}\right.}{n}$ |
| default | $\frac{-\frac{\operatorname{arcsinh}(a\,x^n)^2}{2}+\operatorname{arcsinh}(a\,x^n)\ln\left(1+a\,x^n+\sqrt{1+a^2x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^n-\sqrt{1+a^2x^{2n}}\right)+\operatorname{arcsinh}(a\,x^n)\ln\left(1-a\,x^n-\sqrt{1}\right.}{n}$ |

```
[In] int(arcsinh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/2*arcsinh(a*x^n)^2+arcsinh(a*x^n)*ln(1+a*x^n+(1+a^2*(x^n)^2)^(1/2))
+polylog(2,-a*x^n-(1+a^2*(x^n)^2)^(1/2))+arcsinh(a*x^n)*ln(1-a*x^n-(1+a^2*(
x^n)^2)^(1/2))+polylog(2,a*x^n+(1+a^2*(x^n)^2)^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{asinh}(ax^n)}{x}\,dx$$

```
[In] integrate(asinh(a*x**n)/x,x)
```

```
[Out] Integral(asinh(a*x**n)/x, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) + a*x*x^n + (a^2*x*x^(2*n) + x)*sq
rt(a^2*x^(2*n) + 1)), x) - 1/2*n*log(x)^2 + n*integrate(log(x)/(a^2*x*x^(2*
n) + x), x) + log(a*x^n + sqrt(a^2*x^(2*n) + 1))*log(x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^n)}{x}\,dx$$

```
[In] int(asinh(a*x^n)/x,x)
```

```
[Out] int(asinh(a*x^n)/x, x)
```

# 3.312 $\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx$

Optimal result . . . 2132
Rubi [A] (verified) . . . 2132
Mathematica [A] (verified) . . . 2133
Maple [F] . . . 2134
Fricas [F(-2)] . . . 2134
Sympy [F] . . . 2134
Maxima [F] . . . 2134
Giac [F] . . . 2135
Mupad [F(-1)] . . . 2135

## Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx = -\frac{\text{arcsinh}(ax^n)}{x} - \frac{anx^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n}$$

```
[Out] -arcsinh(a*x^n)/x-a*n*x^(-1+n)*hypergeom([1/2, 1/2*(-1+n)/n],[3/2-1/2/n],-a
^2*x^(2*n))/(1-n)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx = -\frac{anx^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n} - \frac{\text{arcsinh}(ax^n)}{x}$$

```
[In] Int[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -1/2*(1 - n)/n
, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{arcsinh}(ax^n)}{x} + \int \frac{anx^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^n)}{x} + (an)\int \frac{x^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^n)}{x} - \frac{anx^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = &-\frac{\operatorname{arcsinh}(ax^n)}{x} \\ &+ \frac{anx^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-1+n}{2n},1+\frac{-1+n}{2n},-a^2x^{2n}\right)}{-1+n}\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) + (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/(2*n)
, 1 + (-1 + n)/(2*n), -(a^2*x^(2*n))])/(-1 + n)
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^2}dx$$

```
[In] int(arcsinh(a*x^n)/x^2,x)
```

```
[Out] int(arcsinh(a*x^n)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{asinh}(ax^n)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x**n)/x**2,x)
```

```
[Out] Integral(asinh(a*x**n)/x**2, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n/(a^3*x^2*x^(3*n) + a*x^2*x^n + (a^2*x^2*x^(2*n) + x^2)*sq
rt(a^2*x^(2*n) + 1)), x) - n*integrate(1/(a^2*x^2*x^(2*n) + x^2), x) - (n +
 log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax^n\right)}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x^n)}{x^2}\,dx$$

```
[In] int(asinh(a*x^n)/x^2,x)
[Out] int(asinh(a*x^n)/x^2, x)
```

# 3.313 $\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx$

Optimal result . . . 2136
Rubi [A] (verified) . . . 2136
Mathematica [A] (verified) . . . 2137
Maple [F] . . . 2138
Fricas [F(-2)] . . . 2138
Sympy [F] . . . 2138
Maxima [F] . . . 2138
Giac [F] . . . 2139
Mupad [F(-1)] . . . 2139

## Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = -\frac{\text{arcsinh}(ax^n)}{2x^2} - \frac{anx^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2-n)}$$

```
[Out] -1/2*arcsinh(a*x^n)/x^2-1/2*a*n*x^(-2+n)*hypergeom([1/2, 1/2-1/n],[3/2-1/n]
,-a^2*x^(2*n))/(2-n)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = -\frac{anx^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2-n)} - \frac{\text{arcsinh}(ax^n)}{2x^2}$$

```
[In] Int[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]/x^2 - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, (1 - 2/n)/
2, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{arcsinh}(ax^n)}{2x^2} + \frac{1}{2}\int \frac{anx^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^n)}{2x^2} + \frac{1}{2}(an)\int \frac{x^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\ &= -\frac{\operatorname{arcsinh}(ax^n)}{2x^2} - \frac{anx^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx \\ &= \frac{-((-2+n)\operatorname{arcsinh}(ax^n)) + anx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{1}{n},\frac{3}{2}-\frac{1}{n},-a^2x^{2n}\right)}{2(-2+n)x^2}\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] (-((-2 + n)*ArcSinh[a*x^n]) + a*n*x^n*Hypergeometric2F1[1/2, 1/2 - n^(-1),
3/2 - n^(-1), -(a^2*x^(2*n))])/(2*(-2 + n)*x^2)
```

## Maple [F]

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^3}dx$$

```
[In] int(arcsinh(a*x^n)/x^3,x)
```

```
[Out] int(arcsinh(a*x^n)/x^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\operatorname{asinh}(ax^n)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x**n)/x**3,x)
```

```
[Out] Integral(asinh(a*x**n)/x**3, x)
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] a*n*integrate(1/2*x^n/(a^3*x^3*x^(3*n) + a*x^3*x^n + (a^2*x^3*x^(2*n) + x^3
)*sqrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/2/(a^2*x^3*x^(2*n) + x^3), x)
- 1/4*(n + 2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x^2
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax^n\right)}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x^n)}{x^3}\,dx$$

```
[In] int(asinh(a*x^n)/x^3,x)
[Out] int(asinh(a*x^n)/x^3, x)
```

# 3.314 $\int (a + ib \arcsin(1 - idx^2))^4 \, dx$

Optimal result . . . 2140
Rubi [A] (verified) . . . 2140
Mathematica [A] (verified) . . . 2142
Maple [F] . . . 2142
Fricas [B] (verification not implemented) . . . 2142
Sympy [F(-2)] . . . 2143
Maxima [F] . . . 2143
Giac [F(-2)] . . . 2143
Mupad [F(-1)] . . . 2144

## Optimal result

Integrand size = 20, antiderivative size = 153

$$\begin{aligned}\int (a + ib \arcsin(1 - idx^2))^4 \, dx = {} & 384b^4x - \frac{192b^3\sqrt{2idx^2 + d^2x^4}(a + ib \arcsin(1 - idx^2))}{dx} \\ & + 48b^2x(a + ib \arcsin(1 - idx^2))^2 \\ & - \frac{8b\sqrt{2idx^2 + d^2x^4}(a + ib \arcsin(1 - idx^2))^3}{dx} \\ & + x(a + ib \arcsin(1 - idx^2))^4\end{aligned}$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^2+x*(a-I*b*arcsin(-1+I*d*x^2)
)^4-192*b^3*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a
-I*b*arcsin(-1+I*d*x^2))^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$\begin{aligned}\int (a + ib \arcsin(1 - idx^2))^4 \, dx = {} & -\frac{192b^3\sqrt{d^2x^4 + 2idx^2}(a + ib \arcsin(1 - idx^2))}{dx} \\ & + 48b^2x(a + ib \arcsin(1 - idx^2))^2 \\ & - \frac{8b\sqrt{d^2x^4 + 2idx^2}(a + ib \arcsin(1 - idx^2))^3}{dx} \\ & + x(a + ib \arcsin(1 - idx^2))^4 + 384b^4x\end{aligned}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^
2]))/(d*x) + 48*b^2*x*(a + I*b*ArcSin[1 - I*d*x^2])^2 - (8*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*(a + I*b*ArcSin[1
- I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{8b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4 + \left(48b^2\right)\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx \\
&= -\frac{192b^3\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx} \\
&\quad + 48b^2x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2 - \frac{8b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4 + \left(384b^4\right)\int 1\,dx \\
&= 384b^4x - \frac{192b^3\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx} + 48b^2x\left(a\right. \\
&\qquad \left. + ib\arcsin\left(1-idx^2\right)\right)^2 \\
&\quad - \frac{8b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx = -\frac{8b\sqrt{dx^2\left(2i+dx^2\right)}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4 + 48b^2\left(8b^2x - \frac{4b\sqrt{dx^2\left(2i+dx^2\right)}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2\right)$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*
(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(2*I +
d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^
2])^2)
```

## Maple [F]

$$\int \left(a+b\,\operatorname{arcsinh}\left(d\,x^2+i\right)\right)^4 dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^4,x)
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx$$

$$= \frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,dx+i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2+2i\,d}b^4\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,dx+i\right)^3+(a^4-$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 + 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^4 +
```

```
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 + 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 + 2*I*d))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^4 dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^4 dx = \int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^4 dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^4 + 4*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^3*b + a^4*x +
integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (-3*I*a*b^3*d + 4*I
*b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(-I*a*b^3*sqrt(d) +
I*b^4*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*
x + I)^3 + 3*(a^2*b^2*d^2*x^4 + 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*b^2*d^
(3/2)*x^3 + 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^
2 + 2*I)*sqrt(d)*x + I)^2)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d
)*x)*sqrt(d*x^2 + 2*I) - 2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^4 dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^4 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 + 1\mathrm{i}\right)\right)^4 dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^4,x)
[Out] int((a + b*asinh(d*x^2 + 1i))^4, x)
```

# 3.315 $\int (a+ib\arcsin(1-idx^2))^3\,dx$

Optimal result . . . 2145
Rubi [A] (verified) . . . 2145
Mathematica [A] (verified) . . . 2147
Maple [F] . . . 2147
Fricas [A] (verification not implemented) . . . 2147
Sympy [F(-2)] . . . 2148
Maxima [F] . . . 2148
Giac [F(-2)] . . . 2148
Mupad [F(-1)] . . . 2149

## Optimal result

Integrand size = 20, antiderivative size = 129

$$\int (a+ib\arcsin(1-idx^2))^3\,dx = 24ab^2x - \frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx} + 24ib^3x\arcsin(1-idx^2) - \frac{6b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))^2}{dx} + x(a+ib\arcsin(1-idx^2))^3$$

```
[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(-1+I*d*x^2)+x*(a-I*b*arcsin(-1+I*d*x^2))^3-48*
b^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(-1+I*d*x^2))^2*(2*I*d*x
^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4898, 4924, 12, 1602}

$$\int (a+ib\arcsin(1-idx^2))^3\,dx = -\frac{6b\sqrt{d^2x^4+2idx^2}(a+ib\arcsin(1-idx^2))^2}{dx} + x(a+ib\arcsin(1-idx^2))^3 + 24ab^2x + 24ib^3x\arcsin(1-idx^2) - \frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + (24*I)*b^3*x*ArcS
in[1 - I*d*x^2] - (6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*
x^2])^2)/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3 + \left(24b^2\right)\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx \\
&= 24ab^2x - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3 + \left(24ib^3\right)\int \arcsin\left(1-idx^2\right)\,dx \\
&= 24ab^2x + 24ib^3x\arcsin\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3 - \left(24ib^3\right)\int -\frac{2idx^2}{\sqrt{2idx^2+d^2x^4}}\,dx \\
&= 24ab^2x + 24ib^3x\arcsin\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3 - \left(48b^3d\right)\int \frac{x^2}{\sqrt{2idx^2+d^2x^4}}\,dx
\end{aligned}$$

$$= 24ab^2x - \frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx} + 24ib^3x\arcsin\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3\,dx = \frac{a(a^2+24b^2)\,dx^2 - 6b(a^2+8b^2)\sqrt{dx^2\,(2i+dx^2)} + 3ib\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2\,(2i+dx^2)}\right)\arcsin(1}{dx}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(2*I + d*x^2)] + (3*
I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 -
 I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 - I
*d*x^2]^2 - I*b^3*d*x^2*ArcSin[1 - I*d*x^2]^3)/(d*x)
```

## Maple [F]

$$\int \left(a+b\,\operatorname{arcsinh}\left(d\,x^2+i\right)\right)^3 dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^3,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.46

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3\,dx = \frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)^3 + (a^3+24\,ab^2)dx + 3\left(ab^2dx-2\sqrt{d^2x^2+2i\,d}b^3\right)\log\left(dx^2+\sqrt{d^2x^2}\right.}{}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 + 2*
I*d)*x + I)^2 - 3*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) - 6*sqrt(d^2*x^2 + 2*I*d)*(a^2*b + 8*
b^3))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^3 \, dx = \int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^3 dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^3 + 3*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^2*b + a^3*x +
integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (-3*I*a*b^2*d + 4*I*b^
3*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(-I*a*b^2*sqrt(d) + I*b
^3*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x +
 I)^2/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I
) - 2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^3 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 + 1\mathrm{i}\right)\right)^3 \, dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^3,x)
[Out] int((a + b*asinh(d*x^2 + 1i))^3, x)
```

# 3.316 $\int (a+ib\arcsin(1-idx^2))^2\,dx$

Optimal result . . . 2150
Rubi [A] (verified) . . . 2150
Mathematica [A] (verified) . . . 2151
Maple [F] . . . 2151
Fricas [A] (verification not implemented) . . . 2152
Sympy [F(-2)] . . . 2152
Maxima [F] . . . 2152
Giac [F(-2)] . . . 2153
Mupad [F(-1)] . . . 2153

## Optimal result

Integrand size = 20, antiderivative size = 76

$$\int (a+ib\arcsin(1-idx^2))^2\,dx = 8b^2x - \frac{4b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))}{dx} + x(a+ib\arcsin(1-idx^2))^2$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(-1+I*d*x^2))^2-4*b*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*
d*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$\int (a+ib\arcsin(1-idx^2))^2\,dx = -\frac{4b\sqrt{d^2x^4+2idx^2}(a+ib\arcsin(1-idx^2))}{dx} + x(a+ib\arcsin(1-idx^2))^2 + 8b^2x$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin\left(1-idx^2\right))}{dx} \\ &\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2 + \left(8b^2\right)\int 1\,dx \\ &= 8b^2x - \frac{4b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin\left(1-idx^2\right))}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2\end{aligned}$$

## Mathematica [A] (verified)

Time $= 0.01$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 1.00$

$$\begin{aligned}\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx &= 8b^2x - \frac{4b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin\left(1-idx^2\right))}{dx} \\ &\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2\end{aligned}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

## Maple [F]

$$\int \left(a+b\,\operatorname{arcsinh}\left(d\,x^2+i\right)\right)^2 dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^2,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx$$

$$=\frac{b^2dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}dx+i\right)^2+(a^2+8\,b^2)dx-4\sqrt{d^2x^2+2i\,d}dab+2\left(abdx-2\sqrt{d^2x^2+2i\,d}db^2\right)\log}{d}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 + 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx = \int \left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 - integrate(4*(
d^2*x^4 + 2*I*d*x^2 + (d^(3/2)*x^3 + I*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*
x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3
+ 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x))*b^2 + a^2*x
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^2 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 + 1\mathrm{i}\right)\right)^2 dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^2, x)
```

# 3.317 $\int (a + ib \arcsin(1 - idx^2))\, dx$

Optimal result . . . 2154
Rubi [A] (verified) . . . 2154
Mathematica [A] (verified) . . . 2155
Maple [A] (verified) . . . 2155
Fricas [A] (verification not implemented) . . . 2156
Sympy [F(-2)] . . . 2156
Maxima [A] (verification not implemented) . . . 2156
Giac [F(-2)] . . . 2157
Mupad [B] (verification not implemented) . . . 2157

## Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (a + ib \arcsin(1 - idx^2))\, dx = ax - \frac{2b\sqrt{2idx^2 + d^2x^4}}{dx} + ibx \arcsin(1 - idx^2)$$

```
[Out] a*x-I*b*x*arcsin(-1+I*d*x^2)-2*b*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4924, 12, 1602}

$$\int (a + ib \arcsin(1 - idx^2))\, dx = ax + ibx \arcsin(1 - idx^2) - \frac{2b\sqrt{d^2x^4 + 2idx^2}}{dx}$$

```
[In] Int[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + (ib)\int \arcsin\left(1 - idx^2\right)\,dx \\ &= ax + ibx\arcsin\left(1 - idx^2\right) - (ib)\int -\frac{2idx^2}{\sqrt{2idx^2 + d^2x^4}}\,dx \\ &= ax + ibx\arcsin\left(1 - idx^2\right) - (2bd)\int \frac{x^2}{\sqrt{2idx^2 + d^2x^4}}\,dx \\ &= ax - \frac{2b\sqrt{2idx^2 + d^2x^4}}{dx} + ibx\arcsin\left(1 - idx^2\right)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a + ib\arcsin\left(1 - idx^2\right)\right)\,dx = ax - \frac{2b\sqrt{dx^2\left(2i + dx^2\right)}}{dx} + ibx\arcsin\left(1 - idx^2\right)$$

```
[In] Integrate[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $ax + b\left(x\ \operatorname{arcsinh}\left(d\,x^2 + i\right) - \frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$ | 47 |
| parts | $ax + b\left(x\ \operatorname{arcsinh}\left(d\,x^2 + i\right) - \frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$ | 47 |

```
[In] int(a+b*arcsinh(I+d*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*arcsinh(I+d*x^2)-2/(2*I*d*x^2+d^2*x^4)^(1/2)*x*(d*x^2+2*I))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \left(a+ib\operatorname{arcsin}\left(1-idx^2\right)\right)\,dx=\frac{bdx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}dx+i\right)+adx-2\sqrt{d^2x^2+2i\,d}b}{d}$$

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b)/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\operatorname{arcsin}\left(1-idx^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(a+b*asinh(I+d*x**2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a+ib\operatorname{arcsin}\left(1-idx^2\right)\right)\,dx=\left(x\operatorname{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2id}}\right)b+ax$$

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*b + a*x
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)\, dx = a\,x + b\,x\,\operatorname{asinh}\left(d\,x^2 + 1\mathrm{i}\right) - \frac{2\,b\,\sqrt{\left(d\,x^2 + 1\mathrm{i}\right)^2 + 1}}{d\,x}$$

```
[In] int(a + b*asinh(d*x^2 + 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 + 1i) - (2*b*((d*x^2 + 1i)^2 + 1)^(1/2))/(d*x)
```

# 3.318 $\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx$

Optimal result . . . 2158
Rubi [A] (verified) . . . 2158
Mathematica [A] (verified) . . . 2159
Maple [F] . . . 2160
Fricas [F] . . . 2160
Sympy [F(-2)] . . . 2160
Maxima [F] . . . 2160
Giac [F(-2)] . . . 2161
Mupad [F(-1)] . . . 2161

## Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx = \frac{x\,\mathrm{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} - \frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[Out] 1/2*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b
))/b/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*x*Si(1/2
*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*a
rcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4900}

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx = \frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} - \frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)]
 - Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^
2]/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Ar
cSin[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = \frac{x\,\text{CosIntegral}\left(-\frac{i(a+ib\arcsin(1-idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} - \frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx$$
$$= \frac{x\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\arcsin\left(1-idx^2\right)\right)\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\right.\right.}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(I*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] - Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b
 - ArcSin[1 - I*d*x^2]/2]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1
 - I*d*x^2]/2]))
```

## Maple [F]

$$\int \frac{1}{a + b \operatorname{arcsinh}(d\,x^2 + i)} dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2)),x)
```

## Fricas [F]

$$\int \frac{1}{a + ib \operatorname{arcsin}(1 - idx^2)}\, dx = \int \frac{1}{b \operatorname{arsinh}(dx^2 + i) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="fricas")
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{a + ib \operatorname{arcsin}(1 - idx^2)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2)),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{a + ib \operatorname{arcsin}(1 - idx^2)}\, dx = \int \frac{1}{b \operatorname{arsinh}(dx^2 + i) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx = \int \frac{1}{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i)),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i)), x)
```

# 3.319 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx$

Optimal result . . . 2162
Rubi [A] (verified) . . . 2162
Mathematica [A] (verified) . . . 2164
Maple [F] . . . 2164
Fricas [F] . . . 2164
Sympy [F(-2)] . . . 2165
Maxima [F] . . . 2165
Giac [F(-2)] . . . 2165
Mupad [F(-1)] . . . 2166

## Optimal result

Integrand size = 20, antiderivative size = 245

$$\begin{aligned}
&\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx \\
&= -\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)} \\
&\quad + \frac{x\,\mathrm{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}
\end{aligned}$$

```
[Out] 1/4*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(cosh(1/2*a/b)-I*sinh(1/2*a/b
))/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+1/4*x*Si(1
/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^2/(cos(1
/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*(2*I*d*x^2+d^2*x^4)
^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used

$= \{4909\}$

$$\begin{aligned}
&\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx \\
&= \frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad - \frac{\sqrt{d^2x^4+2idx^2}}{2bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)}
\end{aligned}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] -1/2*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (x
*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^
2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Ar
cSin[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I
*d*x^2]/2]))
```

Rule 4909

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)} \\
&\quad + \frac{x\operatorname{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx$$
$$= \frac{-\frac{2b\sqrt{dx^2(2i+dx^2)}}{d(a+ib\arcsin(1-idx^2))} + \frac{x^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\arcsin(1-idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}}{4b^2x}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])) + (x^2*
(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)]) + (Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Arc
Sin[1 - I*d*x^2]/2]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2]))/(4*b^2*x)
```

## Maple [F]

$$\int \frac{1}{\left(a+b\,\text{arcsinh}\left(d\,x^2+i\right)\right)^2}\,dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

## Fricas [F]

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)*integral(1/
2*sqrt(d^2*x^2 + 2*I*d)*x/(a*b*d*x^2 + 2*I*a*b + (b^2*d*x^2 + 2*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)), x) - sqrt(d^2*x^2 + 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{\left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^2}\, dx = \int \frac{1}{\left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^2}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I)
 - 2)/(a*b*d^2*x^3 + 2*I*a*b*d*x + (b^2*d^2*x^3 + 2*I*b^2*d*x + (b^2*d^(3/2
)*x^2 + I*b^2*sqrt(d))*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqr
t(d)*x + I) + (a*b*d^(3/2)*x^2 + I*a*b*sqrt(d))*sqrt(d*x^2 + 2*I)) + integr
ate(1/2*(d^3*x^6 + 3*I*d^2*x^4 + (d^2*x^4 + I*d*x^2 - 2)*(d*x^2 + 2*I) + (2
*d^(5/2)*x^5 + 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 + 2*I) + 4*I)/(a*b*d
^3*x^6 + 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 + 2*I*a*b*d*x^2 - a*b
)*(d*x^2 + 2*I) + (b^2*d^3*x^6 + 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b^2*d^2*x
^4 + 2*I*b^2*d*x^2 - b^2)*(d*x^2 + 2*I) + 2*(b^2*d^(5/2)*x^5 + 3*I*b^2*d^(3
/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + 2*(a*b*d^(5/2)*x^5 + 3*I*a*b*d^(3/2)*x^3 - 2*a*b*sqrt(d)*
x)*sqrt(d*x^2 + 2*I)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^2,x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^2, x)
```

# 3.320 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx$

Optimal result . . . 2167
Rubi [A] (verified) . . . 2168
Mathematica [A] (verified) . . . 2169
Maple [F] . . . 2169
Fricas [F] . . . 2170
Sympy [F(-2)] . . . 2170
Maxima [F] . . . 2170
Giac [F(-2)] . . . 2172
Mupad [F(-1)] . . . 2172

## Optimal result

Integrand size = 20, antiderivative size = 275

$$\begin{aligned}
&\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx \\
&= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)} \\
&\quad + \frac{x\,\mathrm{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad - \frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}
\end{aligned}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))+1/16*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d
*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+
sin(1/2*arcsin(-1+I*d*x^2)))-1/16*x*Si(1/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I
*cosh(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcs
in(-1+I*d*x^2)))-1/4*(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x
^2))^2
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4912, 4900}

$$\begin{aligned}
&\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx \\
&= \frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\operatorname{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad - \frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad - \frac{x}{8b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)} - \frac{\sqrt{d^2x^4+2idx^2}}{4bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}
\end{aligned}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] -1/4*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) -
x/(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*CosIntegral[((-1/2*I)*(a + I*b
*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(16*b^3*(Cos[A
rcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)]
+ Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(16*b^3*
(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^2} \\ &\quad -\frac{x}{8b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)}+\frac{\int\frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx}{8b^2} \\ &= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}-\frac{x}{8b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)} \\ &\quad +\frac{x\,\text{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\ &\quad -\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int\frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx$$
$$=\frac{-\frac{8b^2\sqrt{dx^2(2i+dx^2)}}{d\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}-\frac{4bx^2}{a+ib\arcsin\left(1-idx^2\right)}+\frac{2ix^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\arcsin\left(1-idx^2\right)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sin\right.\right.}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}}{32b^3x}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] ((-8*b^2*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])^2) - (
4*b*x^2)/(a + I*b*ArcSin[1 - I*d*x^2]) + ((2*I)*x^2*(CosIntegral[(((-I)*a)/
b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]) - (Cosh[a/(2*
b)] - I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2]))/(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(32*b^3*x)
```

## Maple [F]

$$\int\frac{1}{\left(a+b\,\text{arcsinh}\left(d\,x^2+i\right)\right)^3}dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

## Fricas [F]

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx = \int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 +
 2*I*d)*x + I) + a*b^2), x) + 2*sqrt(d^2*x^2 + 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*
d)*x + I) + a^2*b^2*d)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}\,dx = \int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(-3*I*a*d^(9/2) - 7*I*b*d^(9/2))
*x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(I*a*d^(5/2) + 20*I*b*d^(5/2))
*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (-3*I*a
*d^3 - 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(I*a*d + I*b*d)*x)*(d
*x^2 + 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(-2*I*a*d^(7/2) -
5*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(I*a*d^(3/2) +
 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 + 2*I) + (b*d^(11/2
)*x^10 + 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 - 2*I*b*d^(5/2)*x^4 - 12*b*d^
(3/2)*x^2 + (b*d^4*x^7 + 3*I*b*d^3*x^5 - 4*b*d^2*x^3 - 4*I*b*d*x)*(d*x^2 +
2*I)^(3/2) + (3*b*d^(9/2)*x^8 + 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)*x^4 - 10*
I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2*I) + (3*b*d^5*x^9 + 15*I*b*d^4*x^
```

```
7 - 23*b*d^3*x^5 - 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2*I) - 8*I*b*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(-5*I*a*d^4 - 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^5 - (7*I*a*d^2
 + 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2*I) - 8*I*a*sqrt(d)
)/(a^2*b^2*d^(11/2)*x^9 + 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*b^2*d^(7/2)*x^5
 - 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*I*b^4*d^(9/2)*x^7 - 12*b^
4*d^(7/2)*x^5 - 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*I*b^4*d^3*x^4 - 3*b^
4*d^2*x^2 - I*b^4*d)*(d*x^2 + 2*I)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*I*b^4*d^(
7/2)*x^5 - 5*b^4*d^(5/2)*x^3 - 2*I*b^4*d^(3/2)*x)*(d*x^2 + 2*I) + 3*(b^4*d^
5*x^8 + 5*I*b^4*d^4*x^6 - 8*b^4*d^3*x^4 - 4*I*b^4*d^2*x^2)*sqrt(d*x^2 + 2*I
))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 + (a^2*b^2*d^4*x^6 + 3*I*
a^2*b^2*d^3*x^4 - 3*a^2*b^2*d^2*x^2 - I*a^2*b^2*d)*(d*x^2 + 2*I)^(3/2) + 3*
(a^2*b^2*d^(9/2)*x^7 + 4*I*a^2*b^2*d^(7/2)*x^5 - 5*a^2*b^2*d^(5/2)*x^3 - 2*
I*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2*I) + 2*(a*b^3*d^(11/2)*x^9 + 6*I*a*b^3*d^(9
/2)*x^7 - 12*a*b^3*d^(7/2)*x^5 - 8*I*a*b^3*d^(5/2)*x^3 + (a*b^3*d^4*x^6 + 3
*I*a*b^3*d^3*x^4 - 3*a*b^3*d^2*x^2 - I*a*b^3*d)*(d*x^2 + 2*I)^(3/2) + 3*(a*
b^3*d^(9/2)*x^7 + 4*I*a*b^3*d^(7/2)*x^5 - 5*a*b^3*d^(5/2)*x^3 - 2*I*a*b^3*d
^(3/2)*x)*(d*x^2 + 2*I) + 3*(a*b^3*d^5*x^8 + 5*I*a*b^3*d^4*x^6 - 8*a*b^3*d^
3*x^4 - 4*I*a*b^3*d^2*x^2)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + 3*(a^2*b^2*d^5*x^8 + 5*I*a^2*b^2*d^4*x^6 - 8*a^2*b^2*d^3*
x^4 - 4*I*a^2*b^2*d^2*x^2)*sqrt(d*x^2 + 2*I)) + integrate(1/8*(d^6*x^12 + 8
*I*d^5*x^10 - 27*d^4*x^8 - 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*I*d^3*x
^6 - 3*d^2*x^4 + 8*I*d*x^2 + 4)*(d*x^2 + 2*I)^2 + 96*I*d*x^2 + 2*(2*d^(9/2)
*x^9 + 10*I*d^(7/2)*x^7 - 15*d^(5/2)*x^5 + I*d^(3/2)*x^3 - 11*sqrt(d)*x)*(d
*x^2 + 2*I)^(3/2) + 3*(2*d^5*x^10 + 12*I*d^4*x^8 - 26*d^3*x^6 - 24*I*d^2*x^
4 + 3*d*x^2 - 10*I)*(d*x^2 + 2*I) + 2*(2*d^(11/2)*x^11 + 14*I*d^(9/2)*x^9 -
 39*d^(7/2)*x^7 - 61*I*d^(5/2)*x^5 + 61*d^(3/2)*x^3 + 30*I*sqrt(d)*x)*sqrt(
d*x^2 + 2*I) - 48)/(a*b^2*d^6*x^12 + 8*I*a*b^2*d^5*x^10 - 24*a*b^2*d^4*x^8
 - 32*I*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b^2*d^4*x^8 + 4*I*a*b^2*d^3*x^
6 - 6*a*b^2*d^2*x^4 - 4*I*a*b^2*d*x^2 + a*b^2)*(d*x^2 + 2*I)^2 + 4*(a*b^2*d
^(9/2)*x^9 + 5*I*a*b^2*d^(7/2)*x^7 - 9*a*b^2*d^(5/2)*x^5 - 7*I*a*b^2*d^(3/2
)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + 6*(a*b^2*d^5*x^10 + 6*I*a*
b^2*d^4*x^8 - 13*a*b^2*d^3*x^6 - 12*I*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2)*(d*x^2
 + 2*I) + (b^3*d^6*x^12 + 8*I*b^3*d^5*x^10 - 24*b^3*d^4*x^8 - 32*I*b^3*d^3*
x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 + 4*I*b^3*d^3*x^6 - 6*b^3*d^2*x^4 - 4*I
*b^3*d*x^2 + b^3)*(d*x^2 + 2*I)^2 + 4*(b^3*d^(9/2)*x^9 + 5*I*b^3*d^(7/2)*x^
7 - 9*b^3*d^(5/2)*x^5 - 7*I*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 + 2*I
)^(3/2) + 6*(b^3*d^5*x^10 + 6*I*b^3*d^4*x^8 - 13*b^3*d^3*x^6 - 12*I*b^3*d^2
*x^4 + 4*b^3*d*x^2)*(d*x^2 + 2*I) + 4*(b^3*d^(11/2)*x^11 + 7*I*b^3*d^(9/2)*
x^9 - 18*b^3*d^(7/2)*x^7 - 20*I*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d
*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + 4*(a*b^2*d^(11/
2)*x^11 + 7*I*a*b^2*d^(9/2)*x^9 - 18*a*b^2*d^(7/2)*x^7 - 20*I*a*b^2*d^(5/2)
*x^5 + 8*a*b^2*d^(3/2)*x^3)*sqrt(d*x^2 + 2*I)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right))^3}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^3,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^3, x)
```

# 3.321 $\int (a - ib \arcsin(1 + idx^2))^4 \, dx$

Optimal result . . . 2173
Rubi [A] (verified) . . . 2173
Mathematica [A] (verified) . . . 2175
Maple [F] . . . 2175
Fricas [B] (verification not implemented) . . . 2175
Sympy [F(-2)] . . . 2176
Maxima [F] . . . 2176
Giac [F(-2)] . . . 2176
Mupad [F(-1)] . . . 2177

## Optimal result

Integrand size = 20, antiderivative size = 153

$$\int (a - ib \arcsin(1 + idx^2))^4 \, dx = 384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))}{dx} + 48b^2x(a - ib \arcsin(1 + idx^2))^2 - \frac{8b\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))^3}{dx} + x(a - ib \arcsin(1 + idx^2))^4$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^2+x*(a-I*b*arcsin(1+I*d*x^2))^
4-192*b^3*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a-I
*b*arcsin(1+I*d*x^2))^3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$\int (a - ib \arcsin(1 + idx^2))^4 \, dx = -\frac{192b^3\sqrt{d^2x^4 - 2idx^2}(a - ib \arcsin(1 + idx^2))}{dx} + 48b^2x(a - ib \arcsin(1 + idx^2))^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}(a - ib \arcsin(1 + idx^2))^3}{dx} + x(a - ib \arcsin(1 + idx^2))^4 + 384b^4x$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x
^2]))/(d*x) + 48*b^2*x*(a - I*b*ArcSin[1 + I*d*x^2])^2 - (8*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x*(a - I*b*ArcSin[
1 + I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{8b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4 + \left(48b^2\right)\int\left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx \\
&= -\frac{192b^3\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx} \\
&\quad + 48b^2x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2 - \frac{8b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4 + \left(384b^4\right)\int 1\,dx \\
&= 384b^4x - \frac{192b^3\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx} + 48b^2x\left(a-ib\arcsin\left(1\right.\right. \\
&\qquad \left.\left. + idx^2\right)\right)^2 \\
&\quad - \frac{8b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx} + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 dx = -\frac{8b\sqrt{dx^2\left(-2i + dx^2\right)}\left(a - ib \arcsin\left(1 + idx^2\right)\right)^3}{dx} + x\left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 + 48b^2\left(8b^2 x - \frac{4b\sqrt{dx^2\left(-2i + dx^2\right)}\left(a - ib \arcsin\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib \arcsin\left(1 + idx^2\right)\right)^2\right)$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x
*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(-2*I
+ d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^2)
```

## Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^2 - i\right)\right)^4 dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 dx$$

$$= \frac{b^4 dx \log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}x - i\right)^4 + 4\left(ab^3 dx - 2\sqrt{d^2x^2 - 2i\,d}b^4\right)\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}x - i\right)^3 + (a$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 - 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^4 +
```

```
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 - 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 - 2*I*d))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 \, dx = \int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^4 dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^4 + 4*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^3*b + a^4*x +
integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (3*I*a*b^3*d - 4*I*
b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(I*a*b^3*sqrt(d) - I*
b^4*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x
- I)^3 + 3*(a^2*b^2*d^2*x^4 - 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*b^2*d^(3
/2)*x^3 - 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2
- 2*I)*sqrt(d)*x - I)^2)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*
x)*sqrt(d*x^2 - 2*I) - 2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^4 dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^4,x)
[Out] int((a + b*asinh(d*x^2 - 1i))^4, x)
```

# 3.322 $\int (a - ib \arcsin(1 + idx^2))^3 \, dx$

Optimal result . . . 2178
Rubi [A] (verified) . . . 2178
Mathematica [A] (verified) . . . 2180
Maple [F] . . . 2180
Fricas [A] (verification not implemented) . . . 2180
Sympy [F(-2)] . . . 2181
Maxima [F] . . . 2181
Giac [F(-2)] . . . 2181
Mupad [F(-1)] . . . 2182

## Optimal result

Integrand size = 20, antiderivative size = 129

$$\int (a - ib \arcsin(1 + idx^2))^3 \, dx = 24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x \arcsin(1 + idx^2) - \frac{6b\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))^2}{dx} + x(a - ib \arcsin(1 + idx^2))^3$$

```
[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(1+I*d*x^2)+x*(a-I*b*arcsin(1+I*d*x^2))^3-48*b^
3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(1+I*d*x^2))^2*(-2*I*d*x^
2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4898, 4924, 12, 1602}

$$\int (a - ib \arcsin(1 + idx^2))^3 \, dx = -\frac{6b\sqrt{d^2x^4 - 2idx^2}(a - ib \arcsin(1 + idx^2))^2}{dx} + x(a - ib \arcsin(1 + idx^2))^3 + 24ab^2x - 24ib^3x \arcsin(1 + idx^2) - \frac{48b^3\sqrt{d^2x^4 - 2idx^2}}{dx}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - (24*I)*b^3*x*Arc
Sin[1 + I*d*x^2] - (6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*
d*x^2])^2)/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^3 + \left(24b^2\right)\int \left(a-ib\arcsin\left(1+idx^2\right)\right)\,dx \\
&= 24ab^2x - \frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^3 - \left(24ib^3\right)\int \arcsin\left(1+idx^2\right)\,dx \\
&= 24ab^2x - 24ib^3x\arcsin\left(1+idx^2\right) - \frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^3 + \left(24ib^3\right)\int \frac{2idx^2}{\sqrt{-2idx^2+d^2x^4}}\,dx \\
&= 24ab^2x - 24ib^3x\arcsin\left(1+idx^2\right) - \frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}{dx} \\
&\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^3 - \left(48b^3d\right)\int \frac{x^2}{\sqrt{-2idx^2+d^2x^4}}\,dx
\end{aligned}$$

$$= 24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x \arcsin\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2}{dx} + x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^3$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx = \frac{a(a^2 + 24b^2)\,dx^2 - 6b(a^2 + 8b^2)\sqrt{dx^2\left(-2i + dx^2\right)} - 3ib\left(a^2dx^2 + 8b^2dx^2 - 4ab\sqrt{dx^2\left(-2i + dx^2\right)}\right)\arcsin}{dx}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(-2*I + d*x^2)] - (3
*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
 + I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
+ I*d*x^2]^2 + I*b^3*d*x^2*ArcSin[1 + I*d*x^2]^3)/(d*x)
```

## Maple [F]

$$\int \left(a + b\,\mathrm{arcsinh}\left(d\,x^2 - i\right)\right)^3 dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.46

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx = \frac{b^3dx\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}x - i\right)^3 + (a^3 + 24\,ab^2)dx + 3\left(ab^2dx - 2\sqrt{d^2x^2 - 2i\,d}b^3\right)\log\left(dx^2 + \sqrt{d^2x^2 -}\right.}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 - 2*
I*d)*x - I)^2 - 3*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) - 6*sqrt(d^2*x^2 - 2*I*d)*(a^2*b + 8*
b^3))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**3,x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^3 \, dx = \int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^3 dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^3 + 3*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^2*b + a^3*x +
integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (3*I*a*b^2*d - 4*I*b^3
*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(I*a*b^2*sqrt(d) - I*b^3
*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I
)^2/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I)
- 2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^3 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^3 dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^3,x)
[Out] int((a + b*asinh(d*x^2 - 1i))^3, x)
```

# 3.323 $\int (a - ib \arcsin(1 + idx^2))^2 \, dx$

Optimal result . . . 2183
Rubi [A] (verified) . . . 2183
Mathematica [A] (verified) . . . 2184
Maple [F] . . . 2184
Fricas [A] (verification not implemented) . . . 2185
Sympy [F(-2)] . . . 2185
Maxima [F] . . . 2185
Giac [F(-2)] . . . 2186
Mupad [F(-1)] . . . 2186

## Optimal result

Integrand size = 20, antiderivative size = 76

$$\int (a - ib \arcsin(1 + idx^2))^2 \, dx = 8b^2 x - \frac{4b\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))}{dx} + x(a - ib \arcsin(1 + idx^2))^2$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(1+I*d*x^2))^2-4*b*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d
*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$\int (a - ib \arcsin(1 + idx^2))^2 \, dx = -\frac{4b\sqrt{d^2x^4 - 2idx^2}(a - ib \arcsin(1 + idx^2))}{dx} + x(a - ib \arcsin(1 + idx^2))^2 + 8b^2 x$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4b\sqrt{-2idx^2+d^2x^4}(a-ib\arcsin\left(1+idx^2\right))}{dx} \\ &\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2 + \left(8b^2\right)\int 1\,dx \\ &= 8b^2x - \frac{4b\sqrt{-2idx^2+d^2x^4}(a-ib\arcsin\left(1+idx^2\right))}{dx} + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2\end{aligned}$$

## Mathematica [A] (verified)

Time $= 0.01$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 1.00$

$$\begin{aligned}\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx &= 8b^2x - \frac{4b\sqrt{-2idx^2+d^2x^4}(a-ib\arcsin\left(1+idx^2\right))}{dx} \\ &\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2\end{aligned}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

## Maple [F]

$$\int \left(a+b\,\operatorname{arcsinh}\left(d\,x^2-i\right)\right)^2 dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2 dx$$

$$= \frac{b^2 dx \log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}x - i\right)^2 + (a^2 + 8\,b^2)dx - 4\sqrt{d^2x^2 - 2i\,d}ab + 2\left(abdx - 2\sqrt{d^2x^2 - 2i\,d}b^2\right)\text{lo}}{d}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 - 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I))/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2 dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2 dx = \int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^2 dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 - integrate(4*(
d^2*x^4 - 2*I*d*x^2 + (d^(3/2)*x^3 - I*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*
x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3
- 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x))*b^2 + a^2*x
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^2 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^2 dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^2, x)
```

# 3.324 $\int (a - ib \arcsin(1 + idx^2))\, dx$

Optimal result . . . 2187
Rubi [A] (verified) . . . 2187
Mathematica [A] (verified) . . . 2188
Maple [A] (verified) . . . 2188
Fricas [A] (verification not implemented) . . . 2189
Sympy [F(-2)] . . . 2189
Maxima [A] (verification not implemented) . . . 2189
Giac [F(-2)] . . . 2190
Mupad [B] (verification not implemented) . . . 2190

## Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (a - ib \arcsin(1 + idx^2))\, dx = ax - \frac{2b\sqrt{-2idx^2 + d^2x^4}}{dx} - ibx \arcsin(1 + idx^2)$$

```
[Out] a*x-I*b*x*arcsin(1+I*d*x^2)-2*b*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4924, 12, 1602}

$$\int (a - ib \arcsin(1 + idx^2))\, dx = ax - ibx \arcsin(1 + idx^2) - \frac{2b\sqrt{d^2x^4 - 2idx^2}}{dx}$$

```
[In] Int[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax - (ib)\int \arcsin\left(1+idx^2\right)\,dx \\ &= ax - ibx\arcsin\left(1+idx^2\right) + (ib)\int \frac{2idx^2}{\sqrt{-2idx^2+d^2x^4}}\,dx \\ &= ax - ibx\arcsin\left(1+idx^2\right) - (2bd)\int \frac{x^2}{\sqrt{-2idx^2+d^2x^4}}\,dx \\ &= ax - \frac{2b\sqrt{-2idx^2+d^2x^4}}{dx} - ibx\arcsin\left(1+idx^2\right)\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a - ib\arcsin\left(1+idx^2\right)\right)\,dx = ax - \frac{2b\sqrt{dx^2\left(-2i+dx^2\right)}}{dx} - ibx\arcsin\left(1+idx^2\right)$$

```
[In] Integrate[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| default | $ax + b\left(x\ \text{arcsinh}\left(d\,x^2 - i\right) + \frac{2x\left(-d\,x^2+2i\right)}{\sqrt{d^2x^4-2id\,x^2}}\right)$ | 48 |
| parts | $ax + b\left(x\ \text{arcsinh}\left(d\,x^2 - i\right) + \frac{2x\left(-d\,x^2+2i\right)}{\sqrt{d^2x^4-2id\,x^2}}\right)$ | 48 |

```
[In] int(a+b*arcsinh(-I+d*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*arcsinh(-I+d*x^2)+2/(-2*I*d*x^2+d^2*x^4)^(1/2)*x*(-d*x^2+2*I))
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)\,dx = \frac{bdx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}x-i\right)+adx-2\sqrt{d^2x^2-2i\,d}b}{d}$$

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b)/d
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(a+b*asinh(-I+d*x**2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)\,dx = \left(x\,\mathrm{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2id}}\right)b+ax$$

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*b + a*x
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)\, dx = a\,x + b\,x\,\operatorname{asinh}\left(d\,x^2 - \mathrm{i}\right) - \frac{2\,b\,\sqrt{\left(d\,x^2 - \mathrm{i}\right)^2 + 1}}{d\,x}$$

```
[In] int(a + b*asinh(d*x^2 - 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 - 1i) - (2*b*((d*x^2 - 1i)^2 + 1)^(1/2))/(d*x)
```

# 3.325 $\int \frac{1}{a-ib\arcsin\left(1+idx^2\right)}\,dx$

Optimal result . . . 2191
Rubi [A] (verified) . . . 2191
Mathematica [A] (verified) . . . 2192
Maple [F] . . . 2193
Fricas [F] . . . 2193
Sympy [F(-2)] . . . 2193
Maxima [F] . . . 2193
Giac [F(-2)] . . . 2194
Mupad [F(-1)] . . . 2194

## Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{1}{a-ib\arcsin\left(1+idx^2\right)}\,dx = -\frac{x\,\text{CosIntegral}\left(\frac{i\left(a-ib\arcsin\left(1+idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin\left(1+idx^2\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[Out] 1/2*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/
b/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*x*Ci(1/2*I*(a
-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*arcsi
n(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4900}

$$\int \frac{1}{a-ib\arcsin\left(1+idx^2\right)}\,dx = \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin\left(idx^2+1\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} - \frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\arcsin\left(idx^2+1\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] -1/2*(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b
)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^
2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin
[1 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = -\frac{x\,\text{CosIntegral}\left(\frac{i(a-ib\arcsin(1+idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin(1+idx^2)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

$$\int \frac{1}{a-ib\arcsin\left(1+idx^2\right)}\,dx = \frac{x\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin\left(1+idx^2\right)\right)\right)\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*((-I)*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I*a)/b +
ArcSin[1 + I*d*x^2])/2]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 +
 I*d*x^2]/2]))
```

## Maple [F]

$$\int \frac{1}{a + b \operatorname{arcsinh}(d\,x^2 - i)} dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

## Fricas [F]

$$\int \frac{1}{a - ib \operatorname{arcsin}(1 + idx^2)}\, dx = \int \frac{1}{b \operatorname{arsinh}(dx^2 - i) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="fricas")
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{a - ib \operatorname{arcsin}(1 + idx^2)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2)),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{a - ib \operatorname{arcsin}(1 + idx^2)}\, dx = \int \frac{1}{b \operatorname{arsinh}(dx^2 - i) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \int \frac{1}{a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i)),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i)), x)
```

# 3.326 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}\,dx$

Optimal result . . . 2195
Rubi [A] (verified) . . . 2195
Mathematica [A] (verified) . . . 2197
Maple [F] . . . 2197
Fricas [F] . . . 2197
Sympy [F(-2)] . . . 2198
Maxima [F] . . . 2198
Giac [F(-2)] . . . 2198
Mupad [F(-1)] . . . 2199

## Optimal result

Integrand size = 20, antiderivative size = 244

$$\begin{aligned}
&\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}\,dx \\
&= -\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)} \\
&\quad + \frac{x\,\mathrm{CosIntegral}\left(\frac{i\left(a-ib\arcsin\left(1+idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\
&\quad - \frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\arcsin\left(1+idx^2\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}
\end{aligned}$$

```
[Out] 1/4*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))
/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/4*x*Shi(1/2*
(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b^2/(cos(1/2*a
rcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*(-2*I*d*x^2+d^2*x^4)^(1/2
)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used

$= \{4909\}$

$$\begin{aligned}
&\int \frac{1}{\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2}\,dx \\
&= \frac{x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{CosIntegral}\left(\frac{i\left(a - ib\arcsin\left(idx^2+1\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad - \frac{x\left(\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Shi}\left(\frac{a - ib\arcsin\left(idx^2+1\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad - \frac{\sqrt{d^2x^4 - 2idx^2}}{2bdx\left(a - ib\arcsin\left(1 + idx^2\right)\right)}
\end{aligned}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] -1/2*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) + (
x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I*S
inh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1
 + I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2]))
```

Rule 4909

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{-2idx^2 + d^2x^4}}{2bdx\left(a - ib\arcsin\left(1 + idx^2\right)\right)} \\
&\quad + \frac{x\operatorname{CosIntegral}\left(\frac{i\left(a - ib\arcsin\left(1+idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad - \frac{x\left(i\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right)\operatorname{Shi}\left(\frac{a - ib\arcsin\left(1+idx^2\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx$$
$$= \frac{-\frac{2b\sqrt{dx^2(-2i+dx^2)}}{d(a-ib\arcsin(1+idx^2))} + \frac{x^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+id\right.\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}}{4b^2x}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*(a - I*b*ArcSin[1 + I*d*x^2])) + (x^2
*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]) - (Cosh[a/(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1
 + I*d*x^2])/2]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
)/(4*b^2*x)
```

## Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(d\,x^2-i))^2}dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

## Fricas [F]

$$\int \frac{1}{(a-ib\,\text{arcsin}\,(1+idx^2))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx^2-i)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)*integral(1/
2*sqrt(d^2*x^2 - 2*I*d)*x/(a*b*d*x^2 - 2*I*a*b + (b^2*d*x^2 - 2*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)), x) - sqrt(d^2*x^2 - 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{\left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2}\, dx = \int \frac{1}{\left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^2}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I)
 - 2)/(a*b*d^2*x^3 - 2*I*a*b*d*x + (b^2*d^2*x^3 - 2*I*b^2*d*x + (b^2*d^(3/2
)*x^2 - I*b^2*sqrt(d))*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqr
t(d)*x - I) + (a*b*d^(3/2)*x^2 - I*a*b*sqrt(d))*sqrt(d*x^2 - 2*I)) + integr
ate(1/2*(d^3*x^6 - 3*I*d^2*x^4 + (d^2*x^4 - I*d*x^2 - 2)*(d*x^2 - 2*I) + (2
*d^(5/2)*x^5 - 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 4*I)/(a*b*d
^3*x^6 - 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 - 2*I*a*b*d*x^2 - a*b
)*(d*x^2 - 2*I) + (b^2*d^3*x^6 - 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b^2*d^2*x
^4 - 2*I*b^2*d*x^2 - b^2)*(d*x^2 - 2*I) + 2*(b^2*d^(5/2)*x^5 - 3*I*b^2*d^(3
/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + 2*(a*b*d^(5/2)*x^5 - 3*I*a*b*d^(3/2)*x^3 - 2*a*b*sqrt(d)*
x)*sqrt(d*x^2 - 2*I)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a - ib \arcsin\left(1 + idx^2\right)\right)^2}\, dx = \int \frac{1}{\left(a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)\right)^2}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^2,x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^2, x)
```

# 3.327 $\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx$

Optimal result . . . 2200
Rubi [A] (verified) . . . 2200
Mathematica [A] (verified) . . . 2202
Maple [F] . . . 2202
Fricas [F] . . . 2202
Sympy [F(-2)] . . . 2203
Maxima [F] . . . 2203
Giac [F(-2)] . . . 2204
Mupad [F(-1)] . . . 2205

## Optimal result

Integrand size = 20, antiderivative size = 272

$$\begin{aligned}
&\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx \\
&= -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\,(a-ib\arcsin(1+idx^2))^2} - \frac{x}{8b^2\,(a-ib\arcsin(1+idx^2))} \\
&\quad - \frac{x\,\mathrm{CosIntegral}\left(\frac{i(a-ib\arcsin(1+idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} \\
&\quad + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\arcsin(1+idx^2)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}
\end{aligned}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(1+I*d*x^2))+1/16*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2
))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1
/2*arcsin(1+I*d*x^2)))-1/16*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh
(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I
*d*x^2)))-1/4*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^2
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

$= \{4912, 4900\}$

$$\int \frac{1}{\left(a - ib\arcsin\left(1 + idx^2\right)\right)^3}\,dx$$
$$= -\frac{x\left(\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\operatorname{CosIntegral}\left(\frac{i\left(a - ib\arcsin\left(idx^2+1\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$
$$+ \frac{x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Shi}\left(\frac{a - ib\arcsin\left(idx^2+1\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$
$$- \frac{x}{8b^2\left(a - ib\arcsin\left(1 + idx^2\right)\right)} - \frac{\sqrt{d^2x^4 - 2idx^2}}{4bdx\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] -1/4*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2) -
 x/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) - (x*CosIntegral[((I/2)*(a - I*b*A
rcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(16*b^3*(Cos[Arc
Sin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*
Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(16*b^3*(
Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{-2idx^2 + d^2x^4}}{4bdx\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2}$$
$$- \frac{x}{8b^2\left(a - ib\arcsin\left(1 + idx^2\right)\right)} + \frac{\int \frac{1}{a - ib\arcsin\left(1 + idx^2\right)}\,dx}{8b^2}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^2} - \frac{x}{8b^2\left(a-ib\arcsin\left(1+idx^2\right)\right)} - \frac{x\,\mathrm{CosIntegral}\left(\frac{i\left(a-ib\arcsin\left(1+idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\arcsin\left(1+idx^2\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}\,dx$$

$$= \frac{-\frac{4bx^2}{a-ib\arcsin(1+idx^2)} + \frac{8b^2\sqrt{dx^2(-2i+dx^2)}}{d(ia+b\arcsin(1+idx^2))^2} - \frac{2ix^2\left(\mathrm{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}}{32b^3x}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] ((-4*b*x^2)/(a - I*b*ArcSin[1 + I*d*x^2]) + (8*b^2*Sqrt[d*x^2*(-2*I + d*x^2
)])/(d*(I*a + b*ArcSin[1 + I*d*x^2])^2) - ((2*I)*x^2*(CosIntegral[((I*a)/b
+ ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + (Cosh[a/(2*b)
] + I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]))/(Cos[
ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(32*b^3*x)
```

## Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^3}\,dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

## Fricas [F]

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}\,dx = \int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2-i\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
```

```
2 - 2*I*d)*x - I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 -
 2*I*d)*x - I) + a*b^2), x) + 2*sqrt(d^2*x^2 - 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*
d)*x - I) + a^2*b^2*d)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a - ib \arcsin\left(1 + idx^{2}\right)\right)^{3}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{\left(a - ib \arcsin\left(1 + idx^{2}\right)\right)^{3}}\, dx = \int \frac{1}{\left(b \operatorname{arsinh}\left(dx^{2} - i\right) + a\right)^{3}}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(3*I*a*d^(9/2) + 7*I*b*d^(9/2))*
x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(-I*a*d^(5/2) - 20*I*b*d^(5/2))
*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*I*a*
d^3 + 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(-I*a*d - I*b*d)*x)*(d
*x^2 - 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(2*I*a*d^(7/2) + 5
*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(-I*a*d^(3/2) -
 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 - 2*I) + (b*d^(11/2
)*x^10 - 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 + 2*I*b*d^(5/2)*x^4 - 12*b*d^
(3/2)*x^2 + (b*d^4*x^7 - 3*I*b*d^3*x^5 - 4*b*d^2*x^3 + 4*I*b*d*x)*(d*x^2 -
2*I)^(3/2) + (3*b*d^(9/2)*x^8 - 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)*x^4 + 10*
I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2*I) + (3*b*d^5*x^9 - 15*I*b*d^4*x^
7 - 23*b*d^3*x^5 + 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2*I) + 8*I*b*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(5*I*a*d^4 + 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^5 - (-7*I*a*d^2
 - 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2*I) + 8*I*a*sqrt(d)
)/(a^2*b^2*d^(11/2)*x^9 - 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*b^2*d^(7/2)*x^5
+ 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*I*b^4*d^(9/2)*x^7 - 12*b^
4*d^(7/2)*x^5 + 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*I*b^4*d^3*x^4 - 3*b^
4*d^2*x^2 + I*b^4*d)*(d*x^2 - 2*I)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*I*b^4*d^(
7/2)*x^5 - 5*b^4*d^(5/2)*x^3 + 2*I*b^4*d^(3/2)*x)*(d*x^2 - 2*I) + 3*(b^4*d^
5*x^8 - 5*I*b^4*d^4*x^6 - 8*b^4*d^3*x^4 + 4*I*b^4*d^2*x^2)*sqrt(d*x^2 - 2*I
```

```
))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 + (a^2*b^2*d^4*x^6 - 3*I*
a^2*b^2*d^3*x^4 - 3*a^2*b^2*d^2*x^2 + I*a^2*b^2*d)*(d*x^2 - 2*I)^(3/2) + 3*
(a^2*b^2*d^(9/2)*x^7 - 4*I*a^2*b^2*d^(7/2)*x^5 - 5*a^2*b^2*d^(5/2)*x^3 + 2*
I*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2*I) + 2*(a*b^3*d^(11/2)*x^9 - 6*I*a*b^3*d^(9
/2)*x^7 - 12*a*b^3*d^(7/2)*x^5 + 8*I*a*b^3*d^(5/2)*x^3 + (a*b^3*d^4*x^6 - 3
*I*a*b^3*d^3*x^4 - 3*a*b^3*d^2*x^2 + I*a*b^3*d)*(d*x^2 - 2*I)^(3/2) + 3*(a*
b^3*d^(9/2)*x^7 - 4*I*a*b^3*d^(7/2)*x^5 - 5*a*b^3*d^(5/2)*x^3 + 2*I*a*b^3*d
^(3/2)*x)*(d*x^2 - 2*I) + 3*(a*b^3*d^5*x^8 - 5*I*a*b^3*d^4*x^6 - 8*a*b^3*d^
3*x^4 + 4*I*a*b^3*d^2*x^2)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + 3*(a^2*b^2*d^5*x^8 - 5*I*a^2*b^2*d^4*x^6 - 8*a^2*b^2*d^3*
x^4 + 4*I*a^2*b^2*d^2*x^2)*sqrt(d*x^2 - 2*I)) + integrate(1/8*(d^6*x^12 - 8
*I*d^5*x^10 - 27*d^4*x^8 + 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*I*d^3*x
^6 - 3*d^2*x^4 - 8*I*d*x^2 + 4)*(d*x^2 - 2*I)^2 - 96*I*d*x^2 + 2*(2*d^(9/2)
*x^9 - 10*I*d^(7/2)*x^7 - 15*d^(5/2)*x^5 - I*d^(3/2)*x^3 - 11*sqrt(d)*x)*(d
*x^2 - 2*I)^(3/2) + 3*(2*d^5*x^10 - 12*I*d^4*x^8 - 26*d^3*x^6 + 24*I*d^2*x^
4 + 3*d*x^2 + 10*I)*(d*x^2 - 2*I) + 2*(2*d^(11/2)*x^11 - 14*I*d^(9/2)*x^9 -
 39*d^(7/2)*x^7 + 61*I*d^(5/2)*x^5 + 61*d^(3/2)*x^3 - 30*I*sqrt(d)*x)*sqrt(
d*x^2 - 2*I) - 48)/(a*b^2*d^6*x^12 - 8*I*a*b^2*d^5*x^10 - 24*a*b^2*d^4*x^8
+ 32*I*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b^2*d^4*x^8 - 4*I*a*b^2*d^3*x^
6 - 6*a*b^2*d^2*x^4 + 4*I*a*b^2*d*x^2 + a*b^2)*(d*x^2 - 2*I)^2 + 4*(a*b^2*d
^(9/2)*x^9 - 5*I*a*b^2*d^(7/2)*x^7 - 9*a*b^2*d^(5/2)*x^5 + 7*I*a*b^2*d^(3/2
)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + 6*(a*b^2*d^5*x^10 - 6*I*a*
b^2*d^4*x^8 - 13*a*b^2*d^3*x^6 + 12*I*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2)*(d*x^2
 - 2*I) + (b^3*d^6*x^12 - 8*I*b^3*d^5*x^10 - 24*b^3*d^4*x^8 + 32*I*b^3*d^3*
x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 - 4*I*b^3*d^3*x^6 - 6*b^3*d^2*x^4 + 4*I
*b^3*d*x^2 + b^3)*(d*x^2 - 2*I)^2 + 4*(b^3*d^(9/2)*x^9 - 5*I*b^3*d^(7/2)*x^
7 - 9*b^3*d^(5/2)*x^5 + 7*I*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 - 2*I
)^(3/2) + 6*(b^3*d^5*x^10 - 6*I*b^3*d^4*x^8 - 13*b^3*d^3*x^6 + 12*I*b^3*d^2
*x^4 + 4*b^3*d*x^2)*(d*x^2 - 2*I) + 4*(b^3*d^(11/2)*x^11 - 7*I*b^3*d^(9/2)*
x^9 - 18*b^3*d^(7/2)*x^7 + 20*I*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d
*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + 4*(a*b^2*d^(11/
2)*x^11 - 7*I*a*b^2*d^(9/2)*x^9 - 18*a*b^2*d^(7/2)*x^7 + 20*I*a*b^2*d^(5/2)
*x^5 + 8*a*b^2*d^(3/2)*x^3)*sqrt(d*x^2 - 2*I)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a - ib\arcsin\left(1 + idx^2\right)\right)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a - ib \arcsin\left(1 + idx^2\right)\right)^3}\, dx = \int \frac{1}{\left(a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)\right)^3}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^3,x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^3, x)
```

# 3.328 $\int (a + ib \arcsin(1 - idx^2))^{5/2}\, dx$

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## Optimal result

Integrand size = 22, antiderivative size = 348

$$\int (a + ib \arcsin(1 - idx^2))^{5/2}\, dx = 15b^2 x \sqrt{a + ib \arcsin(1 - idx^2)} - \frac{5b\sqrt{2idx^2 + d^2x^4}(a + ib \arcsin(1 - idx^2))^{3/2}}{dx} + x(a + ib \arcsin(1 - idx^2))^{5/2} + \frac{15b^2\sqrt{\pi}x \operatorname{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1 - idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 - idx^2)\right)\right)} - \frac{15\sqrt{-\frac{i}{b}}b^3\sqrt{\pi}x \operatorname{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1 - idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 - idx^2)\right)}$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((-I/b)^(1/2)*(a-I*b*ar
csin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/
(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(-I/b)^(1/2)-15*b
^3*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*co
sh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2
))+sin(1/2*arcsin(-1+I*d*x^2)))-5*b*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)*(2*I*d
*x^2+d^2*x^4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4895}

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{5/2}\, dx =$$
$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}b^3 x\left(\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$
$$+\frac{15\sqrt{\pi}b^2 x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$+ 15b^2 x\sqrt{a + ib\arcsin\left(1-idx^2\right)}$$
$$-\frac{5b\sqrt{d^2x^4 + 2idx^2}\left(a + ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx} + x\left(a + ib\arcsin\left(1-idx^2\right)\right)^{5/2}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] - (5*b*Sqrt[(2*I)*d*x^2 + d^2*x^
4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x
^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[
1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (15*Sqrt[(-I)/b
]*b^3*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/
Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] -
Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
```

```
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx} \\ &\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2} + \left(15b^2\right)\int\sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx \\ &= 15b^2x\sqrt{a+ib\arcsin\left(1-idx^2\right)} - \frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx} \\ &\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2} \\ &\quad + \frac{15b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\ &\quad - \frac{15\sqrt{-\frac{i}{b}}b^3\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx = \\ &-\frac{5b\sqrt{dx^2\left(2i+dx^2\right)}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2} \\ &+\frac{15b^2x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)-\sqrt{\pi}\,\text{FresnelC}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-\right.\right.}\end{aligned}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]

[Out] (-5*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[(-I)/b]*Sqrt[a +
I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]
)/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + Sqrt[Pi]*FresnelS[(Sqrt[(-I
)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a
/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

## Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^2 + i\right)\right)^{\frac{5}{2}} dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**(5/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{5/2}\, dx = \int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{5/2}\, dx = \int \left(a + b \operatorname{asinh}\left(d\, x^2 + 1\mathrm{i}\right)\right)^{5/2} dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^(5/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

# 3.329 $\int (a+ib\arcsin(1-idx^2))^{3/2}\,dx$

Optimal result . . . 2211
Rubi [A] (verified) . . . 2212
Mathematica [A] (verified) . . . 2213
Maple [F] . . . 2214
Fricas [F(-2)] . . . 2214
Sympy [F(-2)] . . . 2214
Maxima [F] . . . 2214
Giac [F(-2)] . . . 2215
Mupad [F(-1)] . . . 2215

## Optimal result

Integrand size = 22, antiderivative size = 312

$$\int (a+ib\arcsin(1-idx^2))^{3/2}\,dx =$$
$$-\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\arcsin(1-idx^2)}}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}$$
$$+\frac{3\sqrt{ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$
$$-\frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(-1+I*d*x^
2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(c
os(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)+3*b*x*F
resnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*
a/b)-sinh(1/2*a/b))*(I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))-3*b*(2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(-1+I*d*
x^2))^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4903}

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{3/2}\, dx =$$
$$-\frac{3\sqrt{\pi} b^2 x \left(\cosh\left(\frac{a}{2b}\right) - i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{3b\sqrt{d^2x^4 + 2idx^2}\sqrt{a + ib\arcsin\left(1-idx^2\right)}}{dx}$$
$$+\frac{3\sqrt{\pi}\sqrt{ib} bx\left(-\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$
$$+ x\left(a + ib\arcsin\left(1-idx^2\right)\right)^{3/2}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*Sqrt[I*b]*b*Sqrt[Pi]*x*Fresnel
C[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)]
 - Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
  - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*
Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx} \\ &\quad + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2} + \left(3b^2\right)\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx \\ &= -\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2} \\ &\quad + \frac{3\sqrt{ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)} \\ &\quad - \frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}\,dx = \\ &-\frac{3b\sqrt{dx^2\left(2i+dx^2\right)}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx} + x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2} \\ &+\frac{3b^2\sqrt{\pi}x\left(-\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\right.\right.\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(2*I + d*x^2)]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x) +
x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*b^2*Sqrt[Pi]*x*(-(FresnelS[Sqrt[
a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi]
)]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

## Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^2 + i\right)\right)^{\frac{3}{2}} dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \left(a + ib \operatorname{arcsin}\left(1 - idx^2\right)\right)^{3/2}\, dx = \int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a + ib \arcsin\left(1 - idx^2\right)\right)^{3/2}\, dx = \int \left(a + b \operatorname{asinh}\left(d\, x^2 + 1\mathrm{i}\right)\right)^{3/2} dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^(3/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

# 3.330 $\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx$

Optimal result . . . 2216
Rubi [A] (verified) . . . 2217
Mathematica [A] (verified) . . . 2218
Maple [F] . . . 2218
Fricas [F(-2)] . . . 2218
Sympy [F(-2)] . . . 2219
Maxima [F] . . . 2219
Giac [F(-2)] . . . 2219
Mupad [F(-1)] . . . 2219

## Optimal result

Integrand size = 22, antiderivative size = 263

$$\begin{aligned}
&\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx \\
&= x\sqrt{a+ib\arcsin\left(1-idx^2\right)} \\
&\quad + \frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad - \frac{\sqrt{-\frac{i}{b}}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}
\end{aligned}$$

```
[Out] x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(-I/b)^(1/2)-b*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d
*x^2))^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2
)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+x*(a-I*b*arcsin
(-1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4895}

$$\begin{aligned}
&\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx \\
&= -\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)} \\
&\quad+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad+x\sqrt{a+ib\arcsin\left(1-idx^2\right)}
\end{aligned}$$

```
[In] Int[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sq
rt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
 - I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{a+ib\arcsin\left(1-idx^2\right)} \\
&\quad+\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\
&\quad-\frac{\sqrt{-\frac{i}{b}}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.98

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx$$

$$= \frac{x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{\pi}\,\text{FresnelS}\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-i\right.\right.\right.}$$

```
[In] Integrate[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a
+ I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + S
qrt[Pi]*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]
*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

## Maple [F]

$$\int \sqrt{a+b\,\text{arcsinh}\left(d\,x^2+i\right)}dx$$

```
[In] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(I+d*x**2))**(1/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \int \sqrt{b\,\mathrm{arsinh}\left(dx^2+i\right)+a}\,dx$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

```
[In] int((a + b*asinh(d*x^2 + 1i))^(1/2),x)
[Out] int((a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

# 3.331 $\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx$

Optimal result . . . 2220
Rubi [A] (verified) . . . 2220
Mathematica [A] (verified) . . . 2222
Maple [F] . . . 2222
Fricas [F(-2)] . . . 2222
Sympy [F(-2)] . . . 2223
Maxima [F] . . . 2223
Giac [F(-2)] . . . 2223
Mupad [F(-1)] . . . 2223

## Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}&\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx \\ &= -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\ &\quad -\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

```
[Out] -x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(I*b)^(1/2)-x*FresnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*
b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin
(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used

$= \{4903\}$

$$\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx$$
$$= -\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[In] Int[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(
Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx$$
$$= \frac{\sqrt{\pi}x\left(-\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\right.\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[In] Integrate[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[P
i])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 -
I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I
*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\left(d\,x^2+i\right)}}dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\operatorname{arcsin}\left(1-idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a+ib\operatorname{arcsin}\left(1-idx^2\right)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{arsinh}\left(dx^2+i\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\operatorname{arcsin}\left(1-idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+ib\operatorname{arcsin}\left(1-idx^2\right)}}\,dx = \int \frac{1}{\sqrt{a+b\operatorname{asinh}\left(d\,x^2+1\mathrm{i}\right)}}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

# 3.332 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx$

Optimal result . . . 2224
Rubi [A] (verified) . . . 2224
Mathematica [A] (verified) . . . 2226
Maple [F] . . . 2226
Fricas [F(-2)] . . . 2226
Sympy [F(-2)] . . . 2227
Maxima [F] . . . 2227
Giac [F(-2)] . . . 2227
Mupad [F(-1)] . . . 2227

## Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\arcsin\left(1-idx^2\right)}} - \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)} + \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$

```
[Out] -(-I/b)^(3/2)*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(
1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))
+sin(1/2*arcsin(-1+I*d*x^2)))+(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*a
rcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-(2*I*d*x^2+d^2*x
^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used

$= \{4906\}$

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\arcsin\left(1-idx^2\right)}}$$
$$-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$
$$+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
 + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\arcsin\left(1-idx^2\right)}}$$
$$-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$
$$+\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\arcsin(1-idx^2)}} - \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)} + \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

## Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(d\,x^2+i))^{\frac{3}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{3/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\left(dx^2+i\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right))^{3/2}}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2),x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

# 3.333 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}\,dx$

Optimal result . . . 2228
Rubi [A] (verified) . . . 2229
Mathematica [A] (verified) . . . 2230
Maple [F] . . . 2231
Fricas [F(-2)] . . . 2231
Sympy [F(-2)] . . . 2231
Maxima [F] . . . 2231
Giac [F(-2)] . . . 2232
Mupad [F(-1)] . . . 2232

## Optimal result

Integrand size = 22, antiderivative size = 326

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+ib\arcsin\left(1-idx^2\right)}}$$
$$-\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[Out] -1/3*x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cos
h(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(-1+I*d*x^2
))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*
(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/3*x/b^2/
(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4903}

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{x}{3b^2\sqrt{a+ib\arcsin\left(1-idx^2\right)}}-\frac{\sqrt{d^2x^4+2idx^2}}{3bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)
) - x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt
[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh
[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I
*d*x^2]/2])) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt
[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[A
rcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}} \\ &\quad -\frac{x}{3b^2\sqrt{a+ib\arcsin\left(1-idx^2\right)}}+\frac{\int\frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx}{3b^2} \\ &= -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+ib\arcsin\left(1-idx^2\right)}} \\ &\quad -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\ &\quad -\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.94

$$\int\frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{b\sqrt{dx^2(2i+dx^2)}}{dx(a+ib\arcsin(1-idx^2))^{3/2}}+\frac{x}{\sqrt{a+ib\arcsin(1-idx^2)}}+\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}+\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a}}{\sqrt{ib}}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arc\right.\right.}}{3b^2}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]

[Out] -1/3*((b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2
)) + x/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[Sqrt[a + I*
b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b
)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 + (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2])))/b^2
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(d\,x^2+i))^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx^2+i)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right))^{5/2}}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

# 3.334 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx$

Optimal result . . . 2233
Rubi [A] (verified) . . . 2234
Mathematica [A] (verified) . . . 2235
Maple [F] . . . 2236
Fricas [F(-2)] . . . 2236
Sympy [F(-2)] . . . 2236
Maxima [F] . . . 2236
Giac [F(-2)] . . . 2237
Mupad [F(-1)] . . . 2237

## Optimal result

Integrand size = 22, antiderivative size = 389

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}} - \frac{\sqrt{2idx^2+d^2x^4}}{15b^3dx\sqrt{a+ib\arcsin\left(1-idx^2\right)}} - \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} + \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/15*(-I/b)^(3/2)*x*FresnelC((
-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sin
h(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d
*x^2)))+1/15*(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2)
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arc
sin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/5*(2*I*d*x^2+d^2*x^4)^(1/2)
/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(5/2)-1/15*(2*I*d*x^2+d^2*x^4)^(1/2)/b^3/
d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4906}

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\arcsin\left(1-idx^2\right)}}$$
$$-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$
$$-\frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}-\frac{\sqrt{d^2x^4+2idx^2}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}$$

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] -1/5*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2)
) - x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) - Sqrt[(2*I)*d*x^2 + d^2
*x^4]/(15*b^3*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (((-I)/b)^(3/2)*Sqrt
[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*
(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Si
n[ArcSin[1 - I*d*x^2]/2])) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)
/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
```

```
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}} \\ &\quad -\frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}+\frac{\int\frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx}{15b^2} \\ &= -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}} \\ &\quad -\frac{\sqrt{2idx^2+d^2x^4}}{15b^3dx\sqrt{a+ib\arcsin\left(1-idx^2\right)}} \\ &\quad -\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)} \\ &\quad +\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94

$$\int\frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx=\frac{-\frac{3b\sqrt{dx^2\left(2i+dx^2\right)}}{d}-x^2\left(a+ib\arcsin\left(1-idx^2\right)\right)+\frac{\sqrt{dx^2\left(2i+dx^2\right)}\left(-ia+b\arcsin\left(1-idx^2\right)\right)^2}{bd}}{x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}-\frac{\left(-\frac{i}{b}\right)}{}$$

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]

[Out] (((-3*b*Sqrt[d*x^2*(2*I + d*x^2)])/d - x^2*(a + I*b*ArcSin[1 - I*d*x^2]) +
(Sqrt[d*x^2*(2*I + d*x^2)]*((-I)*a + b*ArcSin[1 - I*d*x^2])^2)/(b*d))/(x*(a
 + I*b*ArcSin[1 - I*d*x^2])^(5/2)) - (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(S
qrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
+ (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(d\,x^2+i))^{\frac{7}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(7/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

## Maxima [F]

$$\int \frac{1}{(a+ib\,\mathrm{arcsin}\,(1-idx^2))^{7/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx^2+i)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-7/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin\left(1-idx^2\right))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right))^{7/2}}\,dx$$

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2), x)
```

# 3.335 $\int (a - ib \arcsin(1 + idx^2))^{5/2}\, dx$

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Rubi [A] (verified) . . . 2239
Mathematica [A] (verified) . . . 2240
Maple [F] . . . 2241
Fricas [F(-2)] . . . 2241
Sympy [F(-2)] . . . 2241
Maxima [F] . . . 2241
Giac [F(-2)] . . . 2242
Mupad [F(-1)] . . . 2242

## Optimal result

Integrand size = 22, antiderivative size = 348

$$\int (a - ib \arcsin(1 + idx^2))^{5/2}\, dx = 15b^2 x \sqrt{a - ib \arcsin(1 + idx^2)} - \frac{5b\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))^{3/2}}{dx} + x(a - ib \arcsin(1 + idx^2))^{5/2} + \frac{15b^2\sqrt{\pi}x \operatorname{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)\right)} - \frac{15b^2\sqrt{\pi}x \operatorname{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)\right)}$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((I/b)^(1/2)*(a-I*b*arcs
in(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)-15*b^2*x*F
resnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b
)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*
d*x^2)))/(I/b)^(1/2)-5*b*(a-I*b*arcsin(1+I*d*x^2))^(3/2)*(-2*I*d*x^2+d^2*x^
4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4895}

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{5/2}\, dx =$$
$$-\frac{15\sqrt{\pi} b^2 x \left(\cosh\left(\frac{a}{2b}\right) + i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$
$$+\frac{15\sqrt{\pi} b^2 x \left(\cosh\left(\frac{a}{2b}\right) - i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$
$$+ 15b^2 x\sqrt{a - ib\arcsin\left(1+idx^2\right)}$$
$$-\frac{5b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx} + x\left(a - ib\arcsin\left(1+idx^2\right)\right)^{5/2}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] - (5*b*Sqrt[(-2*I)*d*x^2 + d^2*x
^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (15*b^2*Sqrt[Pi]*x*F
resnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*
b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
```

```
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx} \\ &\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} + \left(15b^2\right)\int\sqrt{a-ib\arcsin\left(1+idx^2\right)}\,dx \\ &= 15b^2x\sqrt{a-ib\arcsin\left(1+idx^2\right)} - \frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx} \\ &\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} \\ &\quad + \frac{15b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\ &\quad - \frac{15b^2\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}\,dx = \\ &-\frac{5b\sqrt{dx^2\left(-2i+dx^2\right)}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx} + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} \\ &+\frac{15b^2x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(1+idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)+\sqrt{\pi}\,\text{FresnelS}\left(\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx\right.\right.}\end{aligned}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]

[Out] (-5*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[I/b]*Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqr
t[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi]*FresnelC[(Sqrt[I/b]*Sqr
t[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])
))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

## Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^2 - i\right)\right)^{\frac{5}{2}} dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
[Out] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**(5/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{5/2} dx = \int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{5/2}\, dx = \int \left(a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)\right)^{5/2} dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

# 3.336 $\int (a - ib \arcsin(1 + idx^2))^{3/2}\, dx$

Optimal result . . . 2243
Rubi [A] (verified) . . . 2244
Mathematica [A] (verified) . . . 2245
Maple [F] . . . 2246
Fricas [F(-2)] . . . 2246
Sympy [F(-2)] . . . 2246
Maxima [F] . . . 2246
Giac [F(-2)] . . . 2247
Mupad [F(-1)] . . . 2247

## Optimal result

Integrand size = 22, antiderivative size = 310

$$\int (a - ib \arcsin(1 + idx^2))^{3/2}\, dx =$$
$$-\frac{3b\sqrt{-2idx^2 + d^2x^4}\sqrt{a - ib\arcsin(1 + idx^2)}}{dx} + x(a - ib\arcsin(1 + idx^2))^{3/2}$$
$$-\frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)\right)}$$
$$-\frac{3\sqrt{-ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)}$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(1+I*d*x^2)
)^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-3*b*x*Fre
snelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/
b)+sinh(1/2*a/b))*(-I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2
*arcsin(1+I*d*x^2)))-3*b*(-2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(1+I*d*x^2
))^(1/2)/d/x
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4903}

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{3/2}\, dx =$$
$$-\frac{3\sqrt{\pi} b^2 x \left(\cosh\left(\frac{a}{2b}\right) + i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelS}\left(\frac{\sqrt{a - ib \arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$
$$-\frac{3b\sqrt{d^2x^4 - 2idx^2}\sqrt{a - ib\arcsin\left(1 + idx^2\right)}}{dx}$$
$$-\frac{3\sqrt{\pi}\sqrt{-ib}bx\left(\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a - ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)}$$
$$+ x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^{3/2}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2])) - (3*Sqrt[(-I)*b]*b*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 +
 I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3b\sqrt{-2idx^2+d^2x^4}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx} \\ &\quad + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} + \left(3b^2\right)\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx \\ &= -\frac{3b\sqrt{-2idx^2+d^2x^4}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx} + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} \\ &\quad - \frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\ &\quad - \frac{3\sqrt{-ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.82

$$\begin{aligned}&\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx = \\ &-\frac{3b\sqrt{dx^2\left(-2i+dx^2\right)}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx} + x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} \\ &-\frac{3(-ib)^{3/2}\sqrt{\pi}x\Big(-\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)(}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}\end{aligned}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]

[Out] (-3*b*Sqrt[d*x^2*(-2*I + d*x^2)]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x) +
 x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*((-I)*b)^(3/2)*Sqrt[Pi]*x*(-(Fr
esnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(
2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqr
t[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Cos[ArcSin[1 + I*
d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

## Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^2 - i\right)\right)^{\frac{3}{2}} dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
[Out] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \left(a - ib \operatorname{arcsin}\left(1 + idx^2\right)\right)^{3/2} dx = \int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{3/2}\, dx = \int \left(a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)\right)^{3/2} dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

# 3.337 $\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx$

Optimal result . . . 2248
Rubi [A] (verified) . . . 2249
Mathematica [A] (verified) . . . 2250
Maple [F] . . . 2250
Fricas [F(-2)] . . . 2251
Sympy [F(-2)] . . . 2251
Maxima [F] . . . 2251
Giac [F(-2)] . . . 2251
Mupad [F(-1)] . . . 2252

## Optimal result

Integrand size = 22, antiderivative size = 262

$$\begin{aligned}
&\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx \\
&= x\sqrt{a - ib \arcsin\left(1 + idx^2\right)} \\
&\quad + \frac{\sqrt{\pi} x \operatorname{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i \sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad - \frac{\sqrt{\pi} x \operatorname{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i \sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}
\end{aligned}$$

```
[Out] x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*
a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1
+I*d*x^2)))/(I/b)^(1/2)-x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1
/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*
d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)+x*(a-I*b*arcsin(1+I*d*x^2))
^(1/2)
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4895}

$$\begin{aligned}
&\int \sqrt{a - ib\arcsin\left(1 + idx^2\right)}\, dx \\
&= -\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\arcsin\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad + \frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\arcsin\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)} \\
&\quad + x\sqrt{a - ib\arcsin\left(1 + idx^2\right)}
\end{aligned}$$

```
[In] Int[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/
(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sq
rt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] -
Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\sqrt{a-ib\arcsin\left(1+idx^2\right)} \\ &+ \frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\ &- \frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.99

$$\int \sqrt{a-ib\arcsin\left(1+idx^2\right)}\,dx$$

$$= \frac{x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(1+idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)+\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}}{}\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right.\right.}$$

```
[In] Integrate[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2]
 - Sin[ArcSin[1 + I*d*x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*
ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi
]*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/
(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2]))
```

## Maple [F]

$$\int \sqrt{a+b\,\text{arcsinh}\left(d\,x^2-i\right)}dx$$

```
[In] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*asinh(-I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \int \sqrt{b \operatorname{arsinh}\left(dx^2 - i\right) + a}\, dx$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \int \sqrt{a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)}\, dx$$

```
[In] int((a + b*asinh(d*x^2 - 1i))^(1/2),x)
[Out] int((a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

# 3.338 $\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx$

Optimal result . . . 2253
Rubi [A] (verified) . . . 2253
Mathematica [A] (verified) . . . 2255
Maple [F] . . . 2255
Fricas [F(-2)] . . . 2255
Sympy [F(-2)] . . . 2256
Maxima [F] . . . 2256
Giac [F(-2)] . . . 2256
Mupad [F(-1)] . . . 2256

## Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}&\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx \\ &= -\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\ &\quad -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}\end{aligned}$$

```
[Out] -x*FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))/(-I*b)^(1/2)-x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(
1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used

$= \{4903\}$

$$\int \frac{1}{\sqrt{a - ib \arcsin\left(1 + idx^2\right)}}\, dx$$
$$= -\frac{\sqrt{\pi} x \left(\cosh\left(\frac{a}{2b}\right) - i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelC}\left(\frac{\sqrt{a - ib \arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$
$$- \frac{\sqrt{\pi} x \left(\cosh\left(\frac{a}{2b}\right) + i \sinh\left(\frac{a}{2b}\right)\right) \text{FresnelS}\left(\frac{\sqrt{a - ib \arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$

```
[In] Int[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d
*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{\pi} x \,\text{FresnelC}\left(\frac{\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right) \left(\cosh\left(\frac{a}{2b}\right) - i \sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$
$$- \frac{\sqrt{\pi} x \,\text{FresnelS}\left(\frac{\sqrt{a - ib \arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right) \left(\cosh\left(\frac{a}{2b}\right) + i \sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1 + idx^2\right)\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx$$
$$= \frac{\sqrt{\pi}x\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)\right.\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[In] Integrate[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqr
t[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(
Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

## Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\left(d\,x^2-i\right)}}dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - ib \arcsin\left(1 + idx^2\right)}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a - ib \arcsin\left(1 + idx^2\right)}}\, dx = \int \frac{1}{\sqrt{b \operatorname{arsinh}\left(dx^2 - i\right) + a}}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - ib \arcsin\left(1 + idx^2\right)}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - ib \arcsin\left(1 + idx^2\right)}}\, dx = \int \frac{1}{\sqrt{a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)}}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

# 3.339 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx$

Optimal result . . . 2257
Rubi [A] (verified) . . . 2257
Mathematica [A] (verified) . . . 2259
Maple [F] . . . 2259
Fricas [F(-2)] . . . 2259
Sympy [F(-2)] . . . 2260
Maxima [F] . . . 2260
Giac [F(-2)] . . . 2260
Mupad [F(-1)] . . . 2260

## Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\arcsin\left(1+idx^2\right)}} + \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)} - \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

```
[Out] (I/b)^(3/2)*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2)
)*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(
1/2*arcsin(1+I*d*x^2)))-(I/b)^(3/2)*x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+
I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2
*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-(-2*I*d*x^2+d^2*x^4)^(1/2)/
b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used

$= \{4906\}$

$$\int \frac{1}{\left(a - ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{d^2x^4 - 2idx^2}}{bdx\sqrt{a - ib\arcsin\left(1+idx^2\right)}}$$
$$- \frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2} x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$
$$+ \frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2} x\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{-2idx^2 + d^2x^4}}{bdx\sqrt{a - ib\arcsin\left(1+idx^2\right)}}$$
$$+ \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$
$$- \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{3/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\arcsin(1+idx^2)}} + \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)} - \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

## Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(d\,x^2-i))^{\frac{3}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}}\, dx = \int \frac{1}{(b \operatorname{arsinh}(dx^2 - i) + a)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}}\, dx = \int \frac{1}{(a + b \operatorname{asinh}(d\, x^2 - \mathrm{i}))^{3/2}}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2),x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

# 3.340 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}\,dx$

Optimal result . . . 2261
Rubi [A] (verified) . . . 2262
Mathematica [A] (verified) . . . 2263
Maple [F] . . . 2264
Fricas [F(-2)] . . . 2264
Sympy [F(-2)] . . . 2264
Maxima [F] . . . 2264
Giac [F(-2)] . . . 2265
Mupad [F(-1)] . . . 2265

## Optimal result

Integrand size = 22, antiderivative size = 326

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a-ib\arcsin\left(1+idx^2\right)}}$$
$$-\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{-ib}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{3b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[Out] -1/3*x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cos
h(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/
2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(1+I*d*x^2))
^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I*b)^(1/2)*
Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/3*(-
2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/3*x/b^2/(a
-I*b*arcsin(1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4903}

$$\int \frac{1}{\left(a - ib\arcsin\left(1+idx^2\right)\right)^{5/2}}\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-ib}x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$
$$-\frac{x}{3b^2\sqrt{a-ib\arcsin\left(1+idx^2\right)}}-\frac{\sqrt{d^2x^4-2idx^2}}{3bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2
)) - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqr
t[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*
Sinh[a/(2*b)]))/(3*Sqrt[(-I)*b]*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSi
n[1 + I*d*x^2]/2])) - (Sqrt[(-I)*b]*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin
[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/
(3*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}} \\ &\quad -\frac{x}{3b^2\sqrt{a-ib\arcsin\left(1+idx^2\right)}}+\frac{\int\frac{1}{\sqrt{a-ib\arcsin(1+idx^2)}}\,dx}{3b^2} \\ &= -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a-ib\arcsin\left(1+idx^2\right)}} \\ &\quad -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\ &\quad -\frac{\sqrt{-ib}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{3b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.94

$$\int\frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{b\sqrt{dx^2(-2i+dx^2)}}{dx(a-ib\arcsin(1+idx^2))^{3/2}}+\frac{x}{\sqrt{a-ib\arcsin(1+idx^2)}}+\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}+\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\right.}{\sqrt{-ib}(\cos(\frac{1}{2}}}{3b^2}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*((b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/
2)) + x/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelC[Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/
(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2])) + (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*
b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[
1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/b^2
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(d\,x^2-i))^{\frac{5}{2}}}\,dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx^2-i)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{5/2}}\, dx = \int \frac{1}{(a + b \operatorname{asinh}(d\, x^2 - \mathrm{i}))^{5/2}}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

# 3.341 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx$

Optimal result . . . 2266
Rubi [A] (verified) . . . 2267
Mathematica [A] (verified) . . . 2268
Maple [F] . . . 2269
Fricas [F(-2)] . . . 2269
Sympy [F(-2)] . . . 2269
Maxima [F] . . . 2269
Giac [F(-2)] . . . 2270
Mupad [F(-1)] . . . 2270

## Optimal result

Integrand size = 22, antiderivative size = 389

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}} - \frac{\sqrt{-2idx^2+d^2x^4}}{15b^3dx\sqrt{a-ib\arcsin\left(1+idx^2\right)}} - \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} + \frac{\sqrt{\frac{i}{b}}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{15b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/15*(I/b)^(3/2)*x*FresnelC((I/
b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/
2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2))
)+1/15*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*
cosh(1/2*a/b)+sinh(1/2*a/b))*(I/b)^(1/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d
*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/5*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-
I*b*arcsin(1+I*d*x^2))^(5/2)-1/15*(-2*I*d*x^2+d^2*x^4)^(1/2)/b^3/d/x/(a-I*b
*arcsin(1+I*d*x^2))^(1/2)
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4906}

$$\int \frac{1}{\left(a - ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{d^2x^4 - 2idx^2}}{15b^3dx\sqrt{a - ib\arcsin\left(1+idx^2\right)}} + \frac{\sqrt{\pi}\sqrt{\frac{i}{b}}x\left(\sinh\left(\frac{a}{2b}\right) + i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{15b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} - \frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} - \frac{x}{15b^2\left(a - ib\arcsin\left(1+idx^2\right)\right)^{3/2}} - \frac{\sqrt{d^2x^4 - 2idx^2}}{5bdx\left(a - ib\arcsin\left(1+idx^2\right)\right)^{5/2}}$$

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] -1/5*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2
)) - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - Sqrt[(-2*I)*d*x^2 + d
^2*x^4]/(15*b^3*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - ((I/b)^(3/2)*Sqrt[
Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cos
h[a/(2*b)] + I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2])) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a -
 I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15
*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
```

```
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}} \\
&\quad -\frac{x}{15b^2\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}+\frac{\int\frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx}{15b^2} \\
&= -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}} \\
&\quad -\frac{\sqrt{-2idx^2+d^2x^4}}{15b^3dx\sqrt{a-ib\arcsin\left(1+idx^2\right)}} \\
&\quad -\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} \\
&\quad +\frac{\sqrt{\frac{i}{b}}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{15b^3\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.95

$$\int\frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx=\frac{-\frac{3b\sqrt{dx^2\left(-2i+dx^2\right)}}{d}-x^2\left(a-ib\arcsin\left(1+idx^2\right)\right)+\frac{\sqrt{dx^2\left(-2i+dx^2\right)}\left(ia+b\arcsin\left(1+idx^2\right)\right)^2}{bd}}{x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}+\frac{\sqrt{\frac{i}{b}}\sqrt{}}{}$$

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(-2*I + d*x^2)])/d - x^2*(a - I*b*ArcSin[1 + I*d*x^2]) +
 (Sqrt[d*x^2*(-2*I + d*x^2)]*(I*a + b*ArcSin[1 + I*d*x^2])^2)/(b*d))/(x*(a
- I*b*ArcSin[1 + I*d*x^2])^(5/2)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelC[(Sqrt[I/
b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*((-I)*Cosh[a/(2*b)] + Sinh[
a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) +
(Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]
)/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(15*b^2)
```

## Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(d\,x^2-i))^{\frac{7}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(7/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

## Maxima [F]

$$\int \frac{1}{(a-ib\,\mathrm{arcsin}\,(1+idx^2))^{7/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(dx^2-i)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-7/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{7/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{7/2}}\, dx = \int \frac{1}{(a + b \operatorname{asinh}(d\, x^2 - \mathrm{i}))^{7/2}}\, dx$$

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2), x)
```

# 3.342 $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$


Optimal result . . . . . 2271
Rubi [N/A] . . . . . 2271
Mathematica [N/A] . . . . . 2272
Maple [N/A] (verified) . . . . . 2272
Fricas [N/A] . . . . . 2272
Sympy [F(-1)] . . . . . 2273
Maxima [N/A] . . . . . 2273
Giac [N/A] . . . . . 2273
Mupad [N/A] . . . . . 2274


## Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \text{Int}\left(\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}, x\right)$$

```
[Out] Unintegrable((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

## Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

## Giac [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

## Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\int \frac{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
[Out] -int((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

# 3.343 $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$


Optimal result . . . 2275
Rubi [A] (verified) . . . 2276
Mathematica [A] (verified) . . . 2279
Maple [B] (verified) . . . 2280
Fricas [F] . . . 2281
Sympy [F] . . . 2281
Maxima [F] . . . 2281
Giac [F] . . . 2282
Mupad [F(-1)] . . . 2282


## Optimal result

Integrand size = 40, antiderivative size = 261

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$
$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{3b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{3b^2\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \operatorname{PolyLog}\left(3, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{3b^3 \operatorname{PolyLog}\left(4, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}$$

```
[Out] -1/4*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+3/2*b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*po
lylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/
2*b^2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/4*b^3*polylog(4,1/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6813, 5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$= \frac{3b^2\,\text{PolyLog}\left(3, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3b\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc} - \frac{\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c} + \frac{3b^3\,\text{PolyLog}\left(4, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{4c}$$

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (3*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog
[2, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, E^(-2*ArcSinh[Sqrt[1 - c*x]/S
qrt[1 + c*x]])])/(2*c) + (3*b^3*PolyLog[4, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])])/(4*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{\text{Subst}\left(\int x^3 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^3}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad + \frac{3\text{Subst}\left(\int x^2 \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad + \frac{3b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad - \frac{(3b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad + \frac{3b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad + \frac{3b^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{PolyLog}\left(3, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad - \frac{\left(3b^2\right)\text{Subst}\left(\int \text{PolyLog}\left(3, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{2c}
\end{aligned}$$

$$= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{3b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{3b^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{PolyLog}\left(3, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{(3b^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{4c}$$

$$= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{3b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{3b^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{PolyLog}\left(3, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$
$$+ \frac{3b^3\,\text{PolyLog}\left(4, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{4c}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$
$$= \frac{\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{b} - 4\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) - 6b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)$$

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]

[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/b - 4*(a + b*ArcSinh[Sqrt[1
 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]
)] - 6*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, E^(2*Arc
```

```
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] - 3*b^
3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(4*c)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(285) = 570$.

Time = 1.40 (sec) , antiderivative size = 1113, normalized size of antiderivative = 4.26

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1113 |
| parts | Expression too large to display | 1113 |

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)-b^3*(-1/4/c*arcsinh((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^4+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+3/c*arcsinh((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^2*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1
))^(1/2))-6/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(-c*x+1)^(1/
2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+6/c*polylog(4,-(-c*x+1)^(1/2)/
(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^3*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+3/
c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^
(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-6/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+6/c*pol
ylog(4,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))-3*a*b^2*(-
1/3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3+1/c*arcsinh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1
)+2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+
1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^
(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+2/c*arcsin
h((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(
-c*x+1)/(c*x+1))^(1/2))-2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x
+1)/(c*x+1))^(1/2)))-3*a^2*b*(-1/2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^
2+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)
+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+1/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-
(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-
(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*polylog(2,(-c*
x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))
```

## Fricas [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arcsinh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

## Sympy [F]

$$\begin{aligned}\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = &-\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{asinh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx \\ &- \int \frac{3ab^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx \\ &- \int \frac{3a^2b\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx\end{aligned}$$

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*asinh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*asinh(sqrt(-c*x
 + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

## Maxima [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^3/c + integrate(1/8*((sqrt(2)*b^3
```

```
 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^3 - 6*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a
*b^2)*log(c*x + 1)^2 - 6*(4*sqrt(2)*a*b^2 - 2*(sqrt(2)*b^3 + sqrt(-c*x + 1)
*b^3)*log(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b
^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2 + 12*(sq
rt(2)*a^2*b + sqrt(-c*x + 1)*a^2*b)*log(c*x + 1) - 6*(4*sqrt(2)*a^2*b + 4*s
qrt(-c*x + 1)*a^2*b + (sqrt(2)*b^3 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^2 - 4
*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a*b^2)*log(c*x + 1))*log(sqrt(2) + sqrt(-c
*x + 1)))/(sqrt(2)*c^2*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

## Giac [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\operatorname{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

# 3.344 $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result . . . 2283
Rubi [A] (verified) . . . 2284
Mathematica [A] (verified) . . . 2287
Maple [B] (verified) . . . 2287
Fricas [F] . . . 2288
Sympy [F] . . . 2288
Maxima [F] . . . 2288
Giac [F] . . . 2289
Mupad [F(-1)] . . . 2289

## Optimal result

Integrand size = 40, antiderivative size = 194

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

```
[Out] -1/3*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(
2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+1/2*b^2*
polylog(3,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 5775, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{b\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc} - \frac{\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c} + \frac{b^2\,\text{PolyLog}\left(3, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2,
E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, E^(-2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b\text{arcsinh}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{\text{Subst}\left(\int x^2 \coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x^2}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}
\end{aligned}$$

$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{2\operatorname{Subst}\left(\int x\log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$- \frac{b\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{b^2\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{2c}$$

$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}$$
$$+ \frac{b^2\operatorname{PolyLog}\left(3, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{2c}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$
$$=\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-3b\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-6b^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)+3b^3\text{PolyLog}\left(3,e^{2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{6bc}$$

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*(a + b*ArcSinh[Sqrt[1 - c
*x]/Sqrt[1 + c*x]] - 3*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
]) - 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 3*b^3*PolyLog[3, E^(2*ArcSinh[Sqrt[1
- c*x]/Sqrt[1 + c*x]])])/(6*b*c)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(214) = 428.

Time = 0.40 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.22

| method | result |
|---|---|
| default | $\frac{a^2\ln(cx+1)}{2c}-\frac{a^2\ln(cx-1)}{2c}-b^2\left(-\frac{\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}+\frac{\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c}+\frac{2\,\text{arcsinh}\left(\right.}\right.$ |
| parts | $\frac{a^2\ln(cx+1)}{2c}-\frac{a^2\ln(cx-1)}{2c}-b^2\left(-\frac{\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}+\frac{\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c}+\frac{2\,\text{arcsinh}\left(\right.}\right.$ |

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2*(-1/3/c*arcsinh((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^3+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+2/c*arcsinh((-c*x+1)^(1/2)/
(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))
^(1/2))-2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1
/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/(c*x+1
)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-2/c*p
olylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))-2*a*b*(-
1/2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2+1/c*arcsinh((-c*x+1)^(1/2)/(c
```

```
*x+1)^(1/2))*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+
1/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c
*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1
+(-c*x+1)/(c*x+1))^(1/2))+1/c*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c
*x+1)/(c*x+1))^(1/2)))
```

## Fricas [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arcsinh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

## Sympy [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*asinh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

## Maxima [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2/c + integrate(-1/4*((sqrt(2)*b^
2 + sqrt(-c*x + 1)*b^2)*log(c*x + 1)^2 - 4*(sqrt(2)*a*b + sqrt(-c*x + 1)*a*
b)*log(c*x + 1) + 2*(4*sqrt(2)*a*b - 2*(sqrt(2)*b^2 + sqrt(-c*x + 1)*b^2)*l
og(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-
c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/(sqrt(2)*c^2*x^2 +
 (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

## Giac [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\operatorname{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

# 3.345 $\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result . . . 2290
Rubi [A] (verified) . . . 2290
Mathematica [A] (verified) . . . 2293
Maple [A] (verified) . . . 2293
Fricas [F] . . . 2294
Sympy [F] . . . 2294
Maxima [F] . . . 2294
Giac [F] . . . 2295
Mupad [F(-1)] . . . 2295

## Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\operatorname{PolyLog}\left(2, e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

```
[Out] -1/2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c
*x+1))^(1/2))^2)/c+1/2*b*polylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x
+1)/(c*x+1))^(1/2))^2)/c
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used

$= \{212, 6813, 5775, 3797, 2221, 2317, 2438\}$

$$\int \frac{a + b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\left(a + b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc} - \frac{\log\left(1 - e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a + b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (b*PolyLog[2, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(
2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad + \frac{\text{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{2c}
\end{aligned}$$

$$= -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{b}\right)}\right)}{2c}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$
$$= \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-2b\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-b^2\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsin}}\right.}{2bc}$$

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*(a + b*ArcSinh[Sqrt[1 - c*x]/
Sqrt[1 + c*x]] - 2*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])]) -
 b^2*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*b*c)
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

| method | result |
|---|---|
| default | $\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} - b\left(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c} + \frac{\operatorname{polylog}\left(2,-\frac{\sqrt{-}\right.}{\sqrt{}}\right.$ |
| parts | $\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} - b\left(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c} + \frac{\operatorname{polylog}\left(2,-\frac{\sqrt{-}\right.}{\sqrt{}}\right.$ |

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-b*(-1/2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+1/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+
1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/
```

```
2))*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*polyl
og(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))
```

## Fricas [F]

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

## Sympy [F]

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\,\text{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*asinh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

## Maxima [F]

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(2) + sqrt(-c*x + 1)))/c + 8*integrate(-1/4*(sqrt(2)*log(c*x
+ 1) - sqrt(2)*log(-c*x + 1))/(sqrt(2)*c*x + (c*x - 1)*sqrt(-c*x + 1) - sqr
t(2)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

## Giac [F]

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{a+b\,\operatorname{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

# 3.346 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result . . . 2296
Rubi [N/A] . . . 2296
Mathematica [N/A] . . . 2297
Maple [N/A] (verified) . . . 2297
Fricas [N/A] . . . 2297
Sympy [N/A] . . . 2298
Maxima [N/A] . . . 2298
Giac [N/A] . . . 2298
Mupad [N/A] . . . 2299

## Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}, x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

## Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

## Sympy [N/A]

Not integrable

Time = 163.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\begin{aligned}&\int \frac{1}{\left(1-c^2x^2\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx\\&=-\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx\end{aligned}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*asinh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*asinh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

## Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(1-c^2x^2\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

## Giac [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(1-c^2x^2\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

## Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\operatorname{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

# 3.347 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$


Optimal result . . . . . 2300
Rubi [N/A] . . . . . 2300
Mathematica [N/A] . . . . . 2301
Maple [N/A] (verified) . . . . . 2301
Fricas [N/A] . . . . . 2301
Sympy [F(-1)] . . . . . 2302
Maxima [N/A] . . . . . 2302
Giac [N/A] . . . . . 2303
Mupad [N/A] . . . . . 2303


## Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsinh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 474, normalized size of antiderivative = 11.85

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] -4*(sqrt(2) + sqrt(-c*x + 1))/(2*sqrt(2)*a*b*c^2*x - 2*sqrt(2)*a*b*c - 4*sq
rt(-c*x + 1)*a*b*c - (sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x + 1)*
b^2*c)*log(c*x + 1) + 2*(sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x +
1)*b^2*c)*log(sqrt(2) + sqrt(-c*x + 1))) - integrate((4*c*x + (sqrt(2)*c*x
- 3*sqrt(2))*sqrt(-c*x + 1) - 4)/(2*a*b*c^3*x^3 - 6*a*b*c^2*x^2 + 6*a*b*c*x
 - 4*(a*b*c*x - a*b)*(c*x - 1) - 2*a*b - (b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*b
^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)*b^2*c^2*x^2 - 2*sqr
t(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(c*x + 1) + 2*(b^2*c^3*x^3 -
 3*b^2*c^2*x^2 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)
*b^2*c^2*x^2 - 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(sqrt(2)
 + sqrt(-c*x + 1)) - 4*(sqrt(2)*a*b*c^2*x^2 - 2*sqrt(2)*a*b*c*x + sqrt(2)*a
*b)*sqrt(-c*x + 1)), x)
```

## Giac [N/A]

Not integrable

Time = 20.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

## Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

# 3.348 $\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx$

Optimal result . . . 2304
Rubi [A] (verified) . . . 2304
Mathematica [A] (verified) . . . 2306
Maple [A] (verified) . . . 2306
Fricas [F(-2)] . . . 2307
Sympy [F] . . . 2307
Maxima [F] . . . 2307
Giac [F] . . . 2308
Mupad [F(-1)] . . . 2308

## Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] -1/2*arcsinh(c*exp(b*x+a))^2/b+arcsinh(c*exp(b*x+a))*ln(1-(c*exp(b*x+a)+(1+
c^2*exp(b*x+a)^2)^(1/2))^2)/b+1/2*polylog(2,(c*exp(b*x+a)+(1+c^2*exp(b*x+a)
^2)^(1/2))^2)/b
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2320, 5775, 3797, 2221, 2317, 2438}

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \frac{\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b} - \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[In] Int[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -1/2*ArcSinh[c*E^(a + b*x)]^2/b + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \text{arcsinh}\left(ce^{a+bx}\right)\right)}{b}$$

$$= -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} - \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}\left(ce^{a+bx}\right)\right)}{b}$$

$$= -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}\left(ce^{a+bx}\right)\right)}{b}$$

$$= -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

$$= -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

## Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \frac{-\operatorname{arcsinh}\left(ce^{a+bx}\right)\left(\operatorname{arcsinh}\left(ce^{a+bx}\right) - 2\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)\right) + \operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[In] Integrate[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] (-(ArcSinh[c*E^(a + b*x)]*(ArcSinh[c*E^(a + b*x)] - 2*Log[1 - E^(2*ArcSinh[
c*E^(a + b*x)])])) + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])])/(2*b)
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.01

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{\operatorname{arcsinh}\left(c\,e^{bx+a}\right)^2}{2}+\operatorname{arcsinh}\left(c\,e^{bx+a}\right)\ln\left(1+c\,e^{bx+a}+\sqrt{1+c^2e^{2bx+2a}}\right)+\operatorname{polylog}\left(2,-c\,e^{bx+a}-\sqrt{1+c^2e^{2bx+2a}}\right)+\operatorname{arcsinh}}{b}$ |
| default | $\frac{-\frac{\operatorname{arcsinh}\left(c\,e^{bx+a}\right)^2}{2}+\operatorname{arcsinh}\left(c\,e^{bx+a}\right)\ln\left(1+c\,e^{bx+a}+\sqrt{1+c^2e^{2bx+2a}}\right)+\operatorname{polylog}\left(2,-c\,e^{bx+a}-\sqrt{1+c^2e^{2bx+2a}}\right)+\operatorname{arcsinh}}{b}$ |

```
[In] int(arcsinh(c*exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*arcsinh(c*exp(b*x+a))^2+arcsinh(c*exp(b*x+a))*ln(1+c*exp(b*x+a)+(
1+c^2*exp(b*x+a)^2)^(1/2))+polylog(2,-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(1/
2))+arcsinh(c*exp(b*x+a))*ln(1-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(1/2))+pol
ylog(2,c*exp(b*x+a)+(1+c^2*exp(b*x+a)^2)^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{asinh}\left(ce^{a+bx}\right)\,dx$$

```
[In] integrate(asinh(c*exp(b*x+a)),x)
```

```
[Out] Integral(asinh(c*exp(a + b*x)), x)
```

## Maxima [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(ce^{(bx+a)}\right)\,dx$$

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) + c*e^(b*x + a) + (c^2*e^
(2*b*x + 2*a) + 1)^(3/2)), x) + x*log(c*e^(b*x + a) + sqrt(c^2*e^(2*b*x + 2
*a) + 1)) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*e^(2*b*x +
 2*a)))/b
```

## Giac [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(ce^{(bx+a)}\right)\,dx$$

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="giac")
[Out] integrate(arcsinh(c*e^(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{asinh}\left(c\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\right)\,dx$$

```
[In] int(asinh(c*exp(a + b*x)),x)
[Out] int(asinh(c*exp(b*x)*exp(a)), x)
```

# 3.349 $\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx$

Optimal result . . . 2309
Rubi [A] (verified) . . . 2309
Mathematica [A] (verified) . . . 2311
Maple [B] (verified) . . . 2311
Fricas [A] (verification not implemented) . . . 2312
Sympy [A] (verification not implemented) . . . 2312
Maxima [B] (verification not implemented) . . . 2313
Giac [A] (verification not implemented) . . . 2314
Mupad [F(-1)] . . . 2314

## Optimal result

Integrand size = 12, antiderivative size = 165

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx = \frac{e^{-3\operatorname{arcsinh}(a+bx)}}{48b^4} + \frac{3ae^{-2\operatorname{arcsinh}(a+bx)}}{16b^4} - \frac{(1-6a^2)\,e^{-\operatorname{arcsinh}(a+bx)}}{8b^4} + \frac{a(3-4a^2)\,e^{2\operatorname{arcsinh}(a+bx)}}{16b^4} - \frac{(1-6a^2)\,e^{3\operatorname{arcsinh}(a+bx)}}{24b^4} - \frac{3ae^{4\operatorname{arcsinh}(a+bx)}}{32b^4} + \frac{e^{5\operatorname{arcsinh}(a+bx)}}{80b^4} + \frac{a(3-4a^2)\operatorname{arcsinh}(a+bx)}{8b^4}$$

```
[Out] 1/48/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))^3+3/16*a/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2
))^2+1/8*(6*a^2-1)/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))+1/16*a*(-4*a^2+3)*(b*x+a
+(1+(b*x+a)^2)^(1/2))^2/b^4-1/24*(-6*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b
^4-3/32*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^4+1/80*(b*x+a+(1+(b*x+a)^2)^(1/2)
)^5/b^4+1/8*a*(-4*a^2+3)*arcsinh(b*x+a)/b^4
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5873, 2320, 12, 1642}

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx = \frac{(3-4a^2)\,ae^{2\operatorname{arcsinh}(a+bx)}}{16b^4} + \frac{(3-4a^2)\,a\operatorname{arcsinh}(a+bx)}{8b^4} - \frac{(1-6a^2)\,e^{-\operatorname{arcsinh}(a+bx)}}{8b^4} - \frac{(1-6a^2)\,e^{3\operatorname{arcsinh}(a+bx)}}{24b^4} + \frac{3ae^{-2\operatorname{arcsinh}(a+bx)}}{16b^4} - \frac{3ae^{4\operatorname{arcsinh}(a+bx)}}{32b^4} + \frac{e^{-3\operatorname{arcsinh}(a+bx)}}{48b^4} + \frac{e^{5\operatorname{arcsinh}(a+bx)}}{80b^4}$$

```
[In] Int[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcSinh[a + b*x])) + (3*a)/(16*b^4*E^(2*ArcSinh[a + b*x])) -
 (1 - 6*a^2)/(8*b^4*E^ArcSinh[a + b*x]) + (a*(3 - 4*a^2)*E^(2*ArcSinh[a + b
*x]))/(16*b^4) - ((1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcSinh[a + b*x]))/(32*b^4) + E^(5*ArcSinh[a + b*x])/(80*b^4) + (a*(3 - 4*a
^2)*ArcSinh[a + b*x])/(8*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \cosh(x) \left(-\frac{a}{b} + \frac{\sinh(x)}{b}\right)^3 dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{16b^3x^4} dx, x, e^{\text{arcsinh}(a+bx)}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{x^4} dx, x, e^{\text{arcsinh}(a+bx)}\right)}{16b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4} - \frac{6a}{x^3} - \frac{2(-1+6a^2)}{x^2} + \frac{2a(3-4a^2)}{x} + 2a(3-4a^2)\,x + 2(-1+6a^2)\,x^2 - 6ax^3 + x^4\right) dx, x,\right.}{16b^4}$$

$$= \frac{e^{-3\operatorname{arcsinh}(a+bx)}}{48b^4} + \frac{3ae^{-2\operatorname{arcsinh}(a+bx)}}{16b^4} - \frac{(1-6a^2)\,e^{-\operatorname{arcsinh}(a+bx)}}{8b^4} + \frac{a(3-4a^2)\,e^{2\operatorname{arcsinh}(a+bx)}}{16b^4}$$
$$- \frac{(1-6a^2)\,e^{3\operatorname{arcsinh}(a+bx)}}{24b^4} - \frac{3ae^{4\operatorname{arcsinh}(a+bx)}}{32b^4} + \frac{e^{5\operatorname{arcsinh}(a+bx)}}{80b^4} + \frac{a(3-4a^2)\operatorname{arcsinh}(a+bx)}{8b^4}$$

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx$$
$$= \frac{30ab^4x^4 + 24b^5x^5 - \sqrt{1+a^2+2abx+b^2x^2}(16 - 83a^2 + 6a^4 + a(29-6a^2)\,bx + 2(-4+3a^2)\,b^2x^2 - 6ab^3}{120b^4}$$

```
[In] Integrate[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(16 - 83*a^2
 + 6*a^4 + a*(29 - 6*a^2)*b*x + 2*(-4 + 3*a^2)*b^2*x^2 - 6*a*b^3*x^3 - 24*b
^4*x^4) + 15*a*(3 - 4*a^2)*ArcSinh[a + b*x])/(120*b^4)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(205) = 410$.

Time = 0.75 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.81

| method | result |
|---|---|
| default | $\frac{x^2\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{5b^2} - \frac{7a\left(\frac{x\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{\left(4b^2\left(a^2+1\right)\right.}{b}\right.}{4b}\right.}{4b^2}\right.}{5b^2}$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-7/5*a/b*(1/4*x*(b^2*x^2+2*a*b*x+a
^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(2*
b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2
)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1
/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
```

```
x+a^2+1)^(1/2))/(b^2)^(1/2)))-2/5*(a^2+1)/b^2*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4
*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)))+1/5*b*x^5+1/4*a*x^4
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3 \, dx$$

$$= \frac{24\,b^5x^5 + 30\,ab^4x^4 + 15\,(4\,a^3 - 3\,a)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + (24\,b^4x^4 + 6\,ab^3x^3 - 2\,(3\,a^2$$

$$120\,b^4$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + 15*(4*a^3 - 3*a)*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)) + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 - 4)*b^2*
x^2 - 6*a^4 + (6*a^3 - 29*a)*b*x + 83*a^2 - 16)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))/b^4
```

## Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.76

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3 \, dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

$$+ \begin{cases} \left( -\frac{a\left( -\frac{3a\left( -\frac{5a\left(\frac{1}{5} - \frac{3a^2}{20}\right)}{3b} - \frac{a\left(3a^2+3\right)}{20b} \right)}{2b} - \frac{\left(\frac{1}{5} - \frac{3a^2}{20}\right)\left(2a^2+2\right)}{3b^2} \right)}{b} - \frac{\left(a^2+1\right)\left( -\frac{5a\left(\frac{1}{5} - \frac{3a^2}{20}\right)}{3b} - \frac{a\left(3a^2+3\right)}{20b} \right)}{2b^2} \right) \log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+} \right) \\ \hline \sqrt{b^2} \\ \\ \frac{\left(-3a^2-3\right)\left(a^2+2abx+1\right)^{\frac{7}{2}}}{7} + \frac{\left(a^2+2abx+1\right)^{\frac{9}{2}}}{9} + \frac{\left(a^2+2abx+1\right)^{\frac{5}{2}}\cdot\left(3a^4+6a^2+3\right)}{5} + \frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}\left(-a^6-3a^4-3a^2-1\right)}{3} \\ \hline 8a^4b^4 \\ \\ \frac{x^4\sqrt{a^2+1}}{4} \end{cases}$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**3,x)
```

```
[Out] a*x**4/4 + b*x**5/5 + Piecewise(((-a*(-3*a*(-5*a*(1/5 - 3*a**2/20)/(3*b) -
a*(3*a**2 + 3)/(20*b))/(2*b) - (1/5 - 3*a**2/20)*(2*a**2 + 2)/(3*b**2))/b -
 (a**2 + 1)*(-5*a*(1/5 - 3*a**2/20)/(3*b) - a*(3*a**2 + 3)/(20*b))/(2*b**2)
)*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))
/sqrt(b**2) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*(a*x**3/(20*b) + x**4/5
+ x**2*(1/5 - 3*a**2/20)/(3*b**2) + x*(-5*a*(1/5 - 3*a**2/20)/(3*b) - a*(3*
a**2 + 3)/(20*b))/(2*b**2) + (-3*a*(-5*a*(1/5 - 3*a**2/20)/(3*b) - a*(3*a**
2 + 3)/(20*b))/(2*b) - (1/5 - 3*a**2/20)*(2*a**2 + 2)/(3*b**2))/b**2), Ne(b
**2, 0)), (((-3*a**2 - 3)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x +
 1)**(9/2)/9 + (a**2 + 2*a*b*x + 1)**(5/2)*(3*a**4 + 6*a**2 + 3)/5 + (a**2
+ 2*a*b*x + 1)**(3/2)*(-a**6 - 3*a**4 - 3*a**2 - 1)/3)/(8*a**4*b**4), Ne(a*
b, 0)), (x**4*sqrt(a**2 + 1)/4, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(205) = 410$.

Time $= 0.30$ (sec) , antiderivative size $= 491$, normalized size of antiderivative $= 2.98$

$$
\begin{aligned}
\int e^{\operatorname{arcsinh}(a+bx)} x^3 \, dx = {} & \frac{1}{5} bx^5 + \frac{1}{4} ax^4 + \frac{(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}} x^2}{5\,b^2} \\
& - \frac{(a^2+1)a^3 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{5\,b^4} \\
& - \frac{7\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}} ax}{20\,b^3} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}(a^2+1)ax}{5\,b^3} \\
& + \frac{(a^2+1)^2 a \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{5\,b^4} \\
& + \frac{7\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}} a^2}{12\,b^4} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}(a^2+1)a^2}{5\,b^4} \\
& + \frac{7\,(5\,a^2b^2 - (a^2+1)b^2)a^3 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{40\,b^6} \\
& - \frac{2\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}(a^2+1)}{15\,b^4} \\
& - \frac{7\,(5\,a^2b^2 - (a^2+1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}ax}{40\,b^5} \\
& - \frac{7\,(5\,a^2b^2 - (a^2+1)b^2)(a^2+1)a \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{40\,b^6} \\
& - \frac{7\,(5\,a^2b^2 - (a^2+1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}a^2}{40\,b^6}
\end{aligned}
$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x^2/b^2 - 1
/5*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^3 + 1/5*(a^2 + 1)^2*a*arcsinh(2*(b^2*
x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 7/12*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*
a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2)*(a^2 + 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*a*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 7/40*(5*a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^6
```

## Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx = \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$
$$+ \frac{1}{120}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\left(4\,x+\frac{a}{b}\right)x - \frac{3\,a^2b^5-4\,b^5}{b^7}\right)x + \frac{6\,a^3b^4-29\,ab^4}{b^7}\right)x - \frac{6\,a^4b^3-83\,a^2}{b^7}\right.$$
$$\left. + \frac{(4\,a^3-3\,a)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{8\,b^3|b|}\right.$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/120*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*(4*x
 + a/b)*x - (3*a^2*b^5 - 4*b^5)/b^7)*x + (6*a^3*b^4 - 29*a*b^4)/b^7)*x - (6
*a^4*b^3 - 83*a^2*b^3 + 16*b^3)/b^7) + 1/8*(4*a^3 - 3*a)*log(-a*b - (x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*abs(b))
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)} x^3\,dx = \int x^3\left(a+\sqrt{(a+b\,x)^2+1}+b\,x\right)\,dx$$

```
[In] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

# 3.350 $\int e^{\operatorname{arcsinh}(a+bx)} x^2\,dx$

Optimal result . . . 2315
Rubi [A] (verified) . . . 2315
Mathematica [A] (verified) . . . 2317
Maple [B] (verified) . . . 2317
Fricas [A] (verification not implemented) . . . 2318
Sympy [A] (verification not implemented) . . . 2318
Maxima [A] (verification not implemented) . . . 2319
Giac [A] (verification not implemented) . . . 2319
Mupad [F(-1)] . . . 2320

## Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2\,dx = -\frac{e^{-2\operatorname{arcsinh}(a+bx)}}{16b^3} - \frac{ae^{-\operatorname{arcsinh}(a+bx)}}{2b^3} - \frac{(1-4a^2)\,e^{2\operatorname{arcsinh}(a+bx)}}{16b^3} - \frac{ae^{3\operatorname{arcsinh}(a+bx)}}{6b^3} + \frac{e^{4\operatorname{arcsinh}(a+bx)}}{32b^3} - \frac{(1-4a^2)\operatorname{arcsinh}(a+bx)}{8b^3}$$

```
[Out] -1/16/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2))^2-1/2*a/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2
))-1/16*(-4*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^3-1/6*a*(b*x+a+(1+(b*x+a
)^2)^(1/2))^3/b^3+1/32*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^3-1/8*(-4*a^2+1)*arc
sinh(b*x+a)/b^3
```

## Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5873, 2320, 12, 1642}

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2\,dx = -\frac{(1-4a^2)\,e^{2\operatorname{arcsinh}(a+bx)}}{16b^3} - \frac{(1-4a^2)\operatorname{arcsinh}(a+bx)}{8b^3} - \frac{ae^{-\operatorname{arcsinh}(a+bx)}}{2b^3} - \frac{ae^{3\operatorname{arcsinh}(a+bx)}}{6b^3} - \frac{e^{-2\operatorname{arcsinh}(a+bx)}}{16b^3} + \frac{e^{4\operatorname{arcsinh}(a+bx)}}{32b^3}$$

```
[In] Int[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] -1/16*1/(b^3*E^(2*ArcSinh[a + b*x])) - a/(2*b^3*E^ArcSinh[a + b*x]) - ((1 -
 4*a^2)*E^(2*ArcSinh[a + b*x]))/(16*b^3) - (a*E^(3*ArcSinh[a + b*x]))/(6*b^
3) + E^(4*ArcSinh[a + b*x])/(32*b^3) - ((1 - 4*a^2)*ArcSinh[a + b*x])/(8*b^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cosh(x) \left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2 dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\left(1+2ax-x^2\right)^2\left(1+x^2\right)}{8b^2x^3} dx, x, e^{\text{arcsinh}(a+bx)}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\left(1+2ax-x^2\right)^2\left(1+x^2\right)}{x^3} dx, x, e^{\text{arcsinh}(a+bx)}\right)}{8b^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1+4a^2}{x}+\left(-1+4a^2\right)x-4ax^2+x^3\right) dx, x, e^{\text{arcsinh}(a+bx)}\right)}{8b^3} \\
&= -\frac{e^{-2\text{arcsinh}(a+bx)}}{16b^3} - \frac{ae^{-\text{arcsinh}(a+bx)}}{2b^3} - \frac{\left(1-4a^2\right)e^{2\text{arcsinh}(a+bx)}}{16b^3} \\
&\quad - \frac{ae^{3\text{arcsinh}(a+bx)}}{6b^3} + \frac{e^{4\text{arcsinh}(a+bx)}}{32b^3} - \frac{\left(1-4a^2\right)\text{arcsinh}(a+bx)}{8b^3}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2 \, dx$$

$$= \frac{8ab^3x^3 + 6b^4x^4 + \sqrt{1 + a^2 + 2abx + b^2x^2}(2a^3 + 3bx - 2a^2bx + 6b^3x^3 + a(-13 + 2b^2x^2)) + 3(-1 + 2a)(1$$
$$24b^3$$

```
[In] Integrate[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2*a^3 + 3*b*x
 - 2*a^2*b*x + 6*b^3*x^3 + a*(-13 + 2*b^2*x^2)) + 3*(-1 + 2*a)*(1 + 2*a)*Ar
cSinh[a + b*x])/(24*b^3)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(143) = 286.

Time = 0.63 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.50

| method | result |
|---|---|
| default | $\frac{x\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2}\right.\right.\right.}{8b^2\sqrt{b^2}}\right.}{b}\right.}{4b}$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+1
)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*
(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/4*b*x^4+1/3*a*x^3
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2\,dx$$

$$= \frac{6\,b^4x^4 + 8\,ab^3x^3 - 3\,(4\,a^2-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + (6\,b^3x^3+2\,ab^2x^2+2\,a^3-(2\,a^2-$$

$$24\,b^3$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 - 3*(4*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 - 3)*b*x -
 13*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

## Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.54

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2\,dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

$$+ \begin{cases} \dfrac{\left(-\dfrac{a\left(-\frac{3a\left(\frac{1}{4}-\frac{a^2}{6}\right)}{2b}-\frac{a\left(2a^2+2\right)}{12b}\right)}{b}-\dfrac{\left(\frac{1}{4}-\frac{a^2}{6}\right)\left(a^2+1\right)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} + \sqrt{a^2+2abx+b^2x^2+1}\left(\frac{ax}{12} \right. \\ \dfrac{\frac{\left(-2a^2-2\right)\left(a^2+2abx+1\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2abx+1\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}\left(a^4+2a^2+1\right)}{3}}{4a^3b^3} \\ \dfrac{x^3\sqrt{a^2+1}}{3} \end{cases}$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + Piecewise(((-a*(-3*a*(1/4 - a**2/6)/(2*b) - a*(2*a**2
 + 2)/(12*b))/b - (1/4 - a**2/6)*(a**2 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*(a*x**2/(12*b) + x**3/4 + x*(1/4 - a**2/6)/(2*b
**2) + (-3*a*(1/4 - a**2/6)/(2*b) - a*(2*a**2 + 2)/(12*b))/b**2), Ne(b**2,
0)), (((-2*a**2 - 2)*(a**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x + 1)**
(7/2)/7 + (a**2 + 2*a*b*x + 1)**(3/2)*(a**4 + 2*a**2 + 1)/3)/(4*a**3*b**3),
 Ne(a*b, 0)), (x**3*sqrt(a**2 + 1)/3, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.37

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx = {} & \frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x}{4\,b^2}\\ & -\frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{12\,b^3}\\ & -\frac{(5\,a^2b^2-(a^2+1)b^2)a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}\\ & +\frac{(5\,a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^4}\\ & +\frac{(5\,a^2b^2-(a^2+1)b^2)(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}\\ & +\frac{(5\,a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^5}\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^2 - 5/1
2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 + 1)*b^
2)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/
8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 1/8
*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a/b^5
```

## Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\begin{aligned}&\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx\\ &= \frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3\\ &\quad+\frac{1}{24}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\,x+\frac{a}{b}\right)x-\frac{2\,a^2b^3-3\,b^3}{b^5}\right)x+\frac{2\,a^3b^2-13\,ab^2}{b^5}\right)\\ &\quad-\frac{(4\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{8\,b^2|b|}\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*x + a
/b)*x - (2*a^2*b^3 - 3*b^3)/b^5)*x + (2*a^3*b^2 - 13*a*b^2)/b^5) - 1/8*(4*a
^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(
b^2*abs(b))
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2 \, dx = \int x^2 \left( a + \sqrt{(a+b\,x)^2 + 1} + b\,x \right) \, dx$$

```
[In] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

# 3.351 $\int e^{\mathbf{arcsinh}(a+bx)} x\,dx$

Optimal result . . . . . 2321
Rubi [A] (verified) . . . . . 2321
Mathematica [A] (verified) . . . . . 2322
Maple [A] (verified) . . . . . 2323
Fricas [A] (verification not implemented) . . . . . 2323
Sympy [A] (verification not implemented) . . . . . 2323
Maxima [B] (verification not implemented) . . . . . 2324
Giac [A] (verification not implemented) . . . . . 2324
Mupad [F(-1)] . . . . . 2325

## Optimal result

Integrand size = 10, antiderivative size = 67

$$\int e^{\operatorname{arcsinh}(a+bx)} x\,dx = \frac{e^{-\operatorname{arcsinh}(a+bx)}}{4b^2} - \frac{a e^{2\operatorname{arcsinh}(a+bx)}}{4b^2} + \frac{e^{3\operatorname{arcsinh}(a+bx)}}{12b^2} - \frac{a\operatorname{arcsinh}(a+bx)}{2b^2}$$

```
[Out] 1/4/b^2/(b*x+a+(1+(b*x+a)^2)^(1/2))-1/4*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^2
+1/12*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b^2-1/2*a*arcsinh(b*x+a)/b^2
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5873, 2320, 12, 1642}

$$\int e^{\operatorname{arcsinh}(a+bx)} x\,dx = -\frac{a e^{2\operatorname{arcsinh}(a+bx)}}{4b^2} - \frac{a\operatorname{arcsinh}(a+bx)}{2b^2} + \frac{e^{-\operatorname{arcsinh}(a+bx)}}{4b^2} + \frac{e^{3\operatorname{arcsinh}(a+bx)}}{12b^2}$$

```
[In] Int[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcSinh[a + b*x]) - (a*E^(2*ArcSinh[a + b*x]))/(4*b^2) + E^(3*Ar
cSinh[a + b*x])/(12*b^2) - (a*ArcSinh[a + b*x])/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{4bx^2}\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{x^2}\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{4b^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2a}{x}-2ax+x^2\right)\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{4b^2} \\
&= \frac{e^{-\text{arcsinh}(a+bx)}}{4b^2}-\frac{ae^{2\text{arcsinh}(a+bx)}}{4b^2}+\frac{e^{3\text{arcsinh}(a+bx)}}{12b^2}-\frac{a\text{arcsinh}(a+bx)}{2b^2}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int e^{\text{arcsinh}(a+bx)}x\,dx = \frac{1}{6}\left(3ax^2+2bx^3+\frac{\sqrt{1+a^2+2abx+b^2x^2}(2-a^2+abx+2b^2x^2)}{b^2}-\frac{3a\text{arcsinh}(a+bx)}{b^2}\right)$$

```
[In] Integrate[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 - a^2 + a*b*x +
2*b^2*x^2))/b^2 - (3*a*ArcSinh[a + b*x])/b^2)/6
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.16

| method | result |
|---|---|
| default | $\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2}-\frac{a\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)}{b}+\frac{bx^3}{3}+\frac{a}$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2
)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/3*b*x^3+1/2*a*x^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx$$
$$=\frac{2\,b^3x^3+3\,ab^2x^2+3\,a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+(2\,b^2x^2+abx-a^2+2)\sqrt{b^2x^2+2\,abx+}}{6\,b^2}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + 3*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)) + (2*b^2*x^2 + a*b*x - a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.94

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}$$
$$+\begin{cases}\frac{\left(-\frac{a\left(\frac{1}{3}-\frac{a^2}{6}\right)}{b}-\frac{a\left(a^2+1\right)}{6b}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}}+\left(\frac{ax}{6b}+\frac{x^2}{3}+\frac{\frac{1}{3}-\frac{a^2}{6}}{b^2}\right)\sqrt{a^2+2abx+b^2x^2+1} & \text{f}\\ \frac{\left(-a^2-1\right)\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx+1\right)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{f}\\ \frac{x^2\sqrt{a^2+1}}{2} & \text{c}\end{cases}$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x,x)
```

```
[Out] a*x**2/2 + b*x**3/3 + Piecewise(((-a*(1/3 - a**2/6)/b - a*(a**2 + 1)/(6*b))
*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/
sqrt(b**2) + (a*x/(6*b) + x**2/3 + (1/3 - a**2/6)/b**2)*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1), Ne(b**2, 0)), (((-a**2 - 1)*(a**2 + 2*a*b*x + 1)**(3/2)/
3 + (a**2 + 2*a*b*x + 1)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a
**2 + 1)/2, True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(83) = 166$.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.61

$$\int e^{\operatorname{arcsinh}(a+bx)} x\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2 + \frac{a^3 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b} - \frac{(a^2+1)a \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^2} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^2}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b - 1/2*(a
^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 -
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a
^2 + 1)^(3/2)/b^2
```

## Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int e^{\operatorname{arcsinh}(a+bx)} x\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2 + \frac{1}{6}\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x+\frac{a}{b}\right)x - \frac{a^2b-2\,b}{b^3}\right) + \frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b|b|}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x + a/b)*
x - (a^2*b - 2*b)/b^3) + 1/2*a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*abs(b))/(b*abs(b))
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)} x\,dx = \int x\left(a+\sqrt{(a+b\,x)^2+1}+b\,x\right)\,dx$$

```
[In] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
[Out] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

# 3.352 $\int e^{\mathbf{arcsinh}(a+bx)}\,dx$

Optimal result . . . 2326
Rubi [A] (verified) . . . 2326
Mathematica [A] (verified) . . . 2327
Maple [B] (verified) . . . 2328
Fricas [B] (verification not implemented) . . . 2328
Sympy [A] (verification not implemented) . . . 2328
Maxima [B] (verification not implemented) . . . 2329
Giac [B] (verification not implemented) . . . 2329
Mupad [F(-1)] . . . 2330

## Optimal result

Integrand size = 8, antiderivative size = 31

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \frac{e^{2\operatorname{arcsinh}(a+bx)}}{4b} + \frac{\operatorname{arcsinh}(a+bx)}{2b}$$

```
[Out] 1/4*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b+1/2*arcsinh(b*x+a)/b
```

## Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5872, 2320, 12, 14}

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \frac{\operatorname{arcsinh}(a+bx)}{2b} + \frac{e^{2\operatorname{arcsinh}(a+bx)}}{4b}$$

```
[In] Int[E^ArcSinh[a + b*x],x]
```

```
[Out] E^(2*ArcSinh[a + b*x])/(4*b) + ArcSinh[a + b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5872

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^x \cosh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{2b} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{\text{arcsinh}(a+bx)}\right)}{2b} \\ &= \frac{e^{2\text{arcsinh}(a+bx)}}{4b} + \frac{\text{arcsinh}(a+bx)}{2b}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int e^{\text{arcsinh}(a+bx)}\,dx = \frac{(a+bx)\left(a+bx+\sqrt{1+a^2+2abx+b^2x^2}\right)+\text{arcsinh}(a+bx)}{2b}$$

```
[In] Integrate[E^ArcSinh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ArcSinh[a + b*x]
)/(2*b)
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.87

| method | result | size |
|---|---|---|
| default | $ax + \frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}} + \frac{bx^2}{2}$ | 89 |
| parts | $ax + \frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}} + \frac{bx^2}{2}$ | 89 |

```
[In] int(b*x+a+(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*b*x^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx$$
$$= \frac{b^2x^2 + 2\,abx + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}(bx + a) - \log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{2\,b}$$

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a) - log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

## Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.00

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx$$
$$= ax + \frac{bx^2}{2}$$
$$+ \begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right)\sqrt{a^2 + 2abx + b^2x^2 + 1} + \frac{\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{2\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ \frac{(a^2+2abx+1)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2+1} & \text{otherwise} \end{cases}$$

```
[In] integrate(b*x+a+(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**2/2 + Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(
b**2))/(2*sqrt(b**2)), Ne(b**2, 0)), ((a**2 + 2*a*b*x + 1)**(3/2)/(3*a*b),
Ne(a*b, 0)), (x*sqrt(a**2 + 1), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(35) = 70$.

Time $= 0.26$ (sec) , antiderivative size $= 141$, normalized size of antiderivative $= 4.55$

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \frac{1}{2}\,bx^2 + ax - \frac{a^2 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b} + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}x + \frac{(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b}$$

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x - 1/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2))/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x + 1/2*(a^2 + 1)*arcs
inh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + 1/2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*a/b
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time $= 0.29$ (sec) , antiderivative size $= 80$, normalized size of antiderivative $= 2.58$

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \frac{1}{2}\,bx^2 + ax + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(x+\frac{a}{b}\right) - \frac{\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,|b|}$$

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x + a/b) - 1/2*log
(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/abs(b)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \int a + \sqrt{(a+b\,x)^2+1} + b\,x\,dx$$

```
[In] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x,x)
[Out] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x, x)
```

# 3.353 $\int \frac{e^{\mathbf{arcsinh}(a+bx)}}{x}\,dx$

Optimal result . . . 2331
Rubi [A] (verified) . . . 2331
Mathematica [A] (verified) . . . 2333
Maple [A] (verified) . . . 2334
Fricas [A] (verification not implemented) . . . 2334
Sympy [F] . . . 2334
Maxima [A] (verification not implemented) . . . 2335
Giac [A] (verification not implemented) . . . 2335
Mupad [B] (verification not implemented) . . . 2336

## Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx + \sqrt{1+a^2+2abx+b^2x^2} + a\operatorname{arcsinh}(a+bx) - \sqrt{1+a^2}\operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right) + a\log(x)$$

```
[Out] b*x+a*arcsinh(b*x+a)+a*ln(x)-arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))*(a^2+1)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

## Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5878, 14, 748, 857, 633, 221, 738, 212}

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = -\sqrt{a^2+1}\operatorname{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right) + \sqrt{a^2+2abx+b^2x^2+1} + a\operatorname{arcsinh}(a+bx) + a\log(x) + bx$$

```
[In] Int[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^2
]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
])] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x}\,dx \\
&= \int \left(b+\frac{a}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\right)dx \\
&= bx+a\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\,dx \\
&= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x)-\frac{1}{2}\int \frac{-2(1+a^2)-2abx}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx \\
&= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x) \\
&\quad -\left(-1-a^2\right)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+(ab)\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx \\
&= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x) \\
&\quad -\left(2\left(1+a^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx,x,\frac{2(1+a^2)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right) \\
&\quad +\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{2b} \\
&= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\text{arcsinh}(a+bx) \\
&\quad -\sqrt{1+a^2}\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)+a\log(x)
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\begin{aligned}
\int \frac{e^{\text{arcsinh}(a+bx)}}{x}\,dx &= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\text{arcsinh}(a+bx)+\left(a+\sqrt{1+a^2}\right)\log(x) \\
&\quad -\sqrt{1+a^2}\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)
\end{aligned}$$

```
[In] Integrate[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] + (a + Sqrt[1
+ a^2])*Log[x] - Sqrt[1 + a^2]*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]]
```

## Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| default | $\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx-}}{x}\right.$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b*x+a*ln(x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.53

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx - a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+a\log(x)$$
$$+\sqrt{a^2+1}\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right.$$
$$\left.+\sqrt{b^2x^2+2\,abx+a^2+1}\right)$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] b*x - a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*log(x) + sqrt
(a^2 + 1)*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 - sq
rt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)
```

## Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x}\,dx$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.80

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx + a\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right) + a\log\left(x\right) - \sqrt{a^2+1}\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}|x|} + \frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}|x|} + \frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}|x|}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] b*x + a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)) + a*log
(x) - sqrt(a^2 + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs
(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

## Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.78

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx - \frac{ab\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{|b|} + a\log\left(|x|\right) + \sqrt{a^2+1}\log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*b*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))
/abs(b) + a*log(abs(x)) + sqrt(a^2 + 1)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)
```

## Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = \sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+b\,x+a\,\ln\left(x\right) - \frac{\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} - \frac{a^2\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} + \frac{a\,b\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x,x)
```

```
[Out] (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + b*x + a*log(x) - log(a*b + (a^2 + 1)/
x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2
) - (a^2*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))/x))/(a^2 + 1)^(1/2) + (a*b*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1
/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

# 3.354 $\int \frac{e^{\mathbf{arcsinh}(a+bx)}}{x^2}\,dx$

Optimal result . . . 2337
Rubi [A] (verified) . . . 2337
Mathematica [A] (verified) . . . 2339
Maple [B] (verified) . . . 2340
Fricas [B] (verification not implemented) . . . 2340
Sympy [F] . . . 2341
Maxima [A] (verification not implemented) . . . 2341
Giac [B] (verification not implemented) . . . 2341
Mupad [B] (verification not implemented) . . . 2342

## Optimal result

Integrand size = 12, antiderivative size = 99

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = -\frac{a}{x} - \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x} + b\operatorname{arcsinh}(a+bx) - \frac{ab\operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}} + b\log(x)$$

```
[Out] -a/x+b*arcsinh(b*x+a)+b*ln(x)-a*b*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(1/2)-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/x
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5878, 14, 746, 857, 633, 221, 738, 212}

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = -\frac{ab\operatorname{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\sqrt{a^2+2abx+b^2x^2+1}}{x} + b\operatorname{arcsinh}(a+bx) - \frac{a}{x} + b\log(x)$$

```
[In] Int[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*Ar
cTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])
/Sqrt[1 + a^2] + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^2}\,dx \\
&= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\right)dx \\
&= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\,dx \\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\int \frac{2ab+2b^2x}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx \\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x) \\
&\quad +(ab)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+b^2\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx \\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right) \\
&\quad -(2ab)\text{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx,x,\frac{2(1+a^2)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right) \\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\text{arcsinh}(a+bx) \\
&\quad -\frac{ab\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}}+b\log(x)
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$
\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}\,dx = b\text{arcsinh}(a+bx) - \frac{a+\sqrt{1+a^2+2abx+b^2x^2}+\left(-1-\frac{a}{\sqrt{1+a^2}}\right)bx\log(x)+\frac{abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\sqrt{1+a^2}}}{x}
$$

```
[In] Integrate[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] b*ArcSinh[a + b*x] - (a + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 - a/Sqrt[
1 + a^2])*b*x*Log[x] + (a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
a^2 + 2*a*b*x + b^2*x^2]])/Sqrt[1 + a^2])/x
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(91) = 182.

Time = 0.76 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.87

| method | result |
|---|---|
| default | $-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{(a^2+1)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+}}{x}\right.\right.}{a^2+1}$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+b*ln(x)-a/x
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(91) = 182.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.85

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx$$

$$= \frac{\sqrt{a^2+1}abx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)bx\log\left(-bx-a+\right.}{(a^2+1}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*a*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) - (
a^2 + 1)*b*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (a^2 + 1)*
b*x*log(x) - a^3 - (a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
 1) - a)/((a^2 + 1)*x)
```

## Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^2}\,dx$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**2,x)
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2, x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.72

$$\begin{aligned}&\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx \\ &= -\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} \\ &\quad + b\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)+b\log(x)-\frac{a}{x}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{x}\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="maxima")
[Out] -a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) + b*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)) + b*log(x) - a/x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(91) = 182.

Time = 0.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx &= \frac{ab\log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} \\ &\quad -\frac{b^2\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{|b|}+b\log(|x|) \\ &\quad -\frac{a}{x}+\frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab^5+a^2b^4|b|+b^4|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)b^4}\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2
+ 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 +
1)))/sqrt(a^2 + 1) - b^2*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))*abs(b))/abs(b) + b*log(abs(x)) - a/x + 2*((x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))*a*b^5 + a^2*b^4*abs(b) + b^4*abs(b))/(((x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)*b^4)
```

## Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.72

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = b\,\ln(x) - \frac{a}{x} + \ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1} + \frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\sqrt{b^2} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)} + \frac{a^3\,b\,\operatorname{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)} + \frac{a\,b\,\operatorname{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{2\,a\,b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}$$

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x + log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b
^2)^(1/2))*(b^2)^(1/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^2 + 1))
+ (a^3*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))))/(a^2 + 1)^(3/2) - (a^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/(
x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2
 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2) - (2*a*b*log(a*b + (a^2 + 1)/x + (
(a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x))/(a^2 + 1)^(1/2)
```

# 3.355 $\int \frac{e^{\mathbf{arcsinh}(a+bx)}}{x^3}\,dx$

Optimal result . . . 2343
Rubi [A] (verified) . . . 2343
Mathematica [A] (verified) . . . 2345
Maple [B] (verified) . . . 2345
Fricas [A] (verification not implemented) . . . 2346
Sympy [F] . . . 2346
Maxima [B] (verification not implemented) . . . 2346
Giac [B] (verification not implemented) . . . 2347
Mupad [F(-1)] . . . 2348

## Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b}{x} - \frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2} - \frac{b^2 \operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2(1+a^2)^{3/2}}$$

```
[Out] -1/2*a/x^2-b/x-1/2*b^2*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(a^2+1)^(3/2)-1/2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)/(a^2+1)/x^2
```

## Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5878, 14, 734, 738, 212}

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = -\frac{b^2 \operatorname{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2(a^2+1)^{3/2}} - \frac{(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{2(a^2+1)x^2} - \frac{a}{2x^2} - \frac{b}{x}$$

```
[In] Int[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*a/x^2 - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2
*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^3}\,dx \\
&= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\right)dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2}+\frac{b^2\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx}{2(1+a^2)}
\end{aligned}$$

$$= -\frac{a}{2x^2} - \frac{b}{x} - \frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2} - \frac{b^2\text{Subst}\left(\int \frac{1}{4(1+a^2)-x^2}\,dx, x, \frac{2(1+a^2)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right)}{1+a^2}$$

$$= -\frac{a}{2x^2} - \frac{b}{x} - \frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2} - \frac{b^2\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2(1+a^2)^{3/2}}$$

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx = \frac{1}{2}\left(-\frac{a}{x^2} - \frac{2b}{x} - \frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{(1+a^2)x^2} + \frac{b^2\log(x)}{(1+a^2)^{3/2}} - \frac{b^2\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{3/2}}\right)$$

```
[In] Integrate[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
/((1 + a^2)*x^2) + (b^2*Log[x])/(1 + a^2)^(3/2) - (b^2*Log[1 + a^2 + a*b*x
+ Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(3/2))/2
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(102) = 204.

Time = 0.75 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.96

| method | result |
|---|---|
| default | $-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2(a^2+1)x^2} - ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x} + \frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1} + \frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} - \sqrt{a^2+1}\ln\left(\frac{2a}{}\right.\right.\right.$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/
x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(
a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/
```

```
2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x))-b/x-1/2*a/x^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.56

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx$$

$$= \frac{\sqrt{a^2+1}b^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-a^5-(a^3+a)b^2x^2-2\,a^3-}{2\,(a^4+2\,a^2+1)x^2}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/
x) - a^5 - (a^3 + a)*b^2*x^2 - 2*a^3 - 2*(a^4 + 2*a^2 + 1)*b*x - (a^4 + (a^
3 + a)*b*x + 2*a^2 + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - a)/((a^4 + 2*a^
2 + 1)*x^2)
```

## Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^3}\,dx$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**3,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**3, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(102) = 204.

Time = 0.38 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.70

$$\begin{aligned}
&\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx \\
&= \frac{a^2b^2 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} \\
&\quad - \frac{b^2 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\sqrt{a^2+1}} \\
&\quad + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}b^2}{2\,(a^2+1)} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab}{2\,(a^2+1)x} \\
&\quad - \frac{b}{x} - \frac{a}{2\,x^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2\,(a^2+1)x^2}
\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 1/2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) +
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/(a^2 + 1) + 1/2*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*a*b/((a^2 + 1)*x) - b/x - 1/2*a/x^2 - 1/2*(b^2*x^2 + 2*a*
b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(102) = 204.

Time = 0.34 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.31

$$\begin{aligned}
\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx &= \frac{b^2 \log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{2\,bx+a}{2\,x^2} \\
&\quad + \frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^2b^2+2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^4b^2+4\left(x|b|-\sqrt{b^2x^2+2}\right.}{}
\end{aligned}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(
a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^
2 + 1)))/(a^2 + 1)^(3/2) - 1/2*(2*b*x + a)/x^2 + (2*(x*abs(b) - sqrt(b^2*x^
```

```
2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 + 2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*a^4*b^2 + 4*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*
b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*b^2 + 3*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^2 + 4*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^2*a*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))*b^2)/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 -
1)^2*(a^2 + 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^3}\,dx$$

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3, x)
```

# 3.356 $\int \frac{e^{\mathbf{arcsinh}(a+bx)}}{x^4}\,dx$

Optimal result . . . 2349
Rubi [A] (verified) . . . 2349
Mathematica [A] (verified) . . . 2351
Maple [B] (verified) . . . 2352
Fricas [A] (verification not implemented) . . . 2353
Sympy [F] . . . 2353
Maxima [B] (verification not implemented) . . . 2353
Giac [B] (verification not implemented) . . . 2354
Mupad [F(-1)] . . . 2355

## Optimal result

Integrand size = 12, antiderivative size = 156

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = -\frac{a}{3x^3} - \frac{b}{2x^2} + \frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2} - \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3} + \frac{ab^3\operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2\left(1+a^2\right)^{5/2}}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2-1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b^
3*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1
)^(5/2)+1/2*a*b*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^2/x^2
```

## Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5878, 14, 744, 734, 738, 212}

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = \frac{ab^3\operatorname{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}} + \frac{ab(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2} - \frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3} - \frac{a}{3x^3} - \frac{b}{2x^2}$$

```
[In] Int[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 + a
^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^4}\,dx \\
&= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\right)dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{(ab)\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx}{1+a^2} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2} \\
&\quad -\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{(ab^3)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx}{2\left(1+a^2\right)^2} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2} \\
&\quad -\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{(ab^3)\,\text{Subst}\left(\int \frac{1}{4(1+a^2)-x^2}\,dx,x,\frac{2\left(1+a^2\right)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\left(1+a^2\right)^2} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2} \\
&\quad -\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{ab^3\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2\left(1+a^2\right)^{5/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4}\,dx \\
&= \frac{1}{6}\left(-\frac{2a}{x^3}-\frac{3b}{x^2}-\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(2+2a^4+abx+a^3bx+2b^2x^2+a^2\left(4-b^2x^2\right)\right)}{\left(1+a^2\right)^2x^3}\right. \\
&\qquad \left.-\frac{3ab^3\log(x)}{\left(1+a^2\right)^{5/2}}+\frac{3ab^3\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\left(1+a^2\right)^{5/2}}\right)
\end{aligned}
$$

```
[In] Integrate[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 + a
*b*x + a^3*b*x + 2*b^2*x^2 + a^2*(4 - b^2*x^2)))/((1 + a^2)^2*x^3) - (3*a*b
^3*Log[x])/(1 + a^2)^(5/2) + (3*a*b^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*S
qrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(5/2))/6
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(136) = 272.

Time = 0.95 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.22

| method | result |
|---|---|
| default | $-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3(a^2+1)x^3}-ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2}-ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+\right.\right.\right.\right.$ |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3-a*b/(a^2+1)*(-1/2/(a^2+1)/x^
2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*
b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/
(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(
a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1
/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)))
-1/3*a/x^3-1/2*b/x^2
```

## Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.47

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx$$

$$= \frac{3\sqrt{a^2+1}ab^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-2\,a^7+(a^4-a^2-2)b^3$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + 1)*a*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) +
 a)/x) - 2*a^7 + (a^4 - a^2 - 2)*b^3*x^3 - 6*a^5 - 6*a^3 - 3*(a^6 + 3*a^4 +
 3*a^2 + 1)*b*x - (2*a^6 - (a^4 - a^2 - 2)*b^2*x^2 + 6*a^4 + (a^5 + 2*a^3 +
 a)*b*x + 6*a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a)/((a^6 + 3*a^4
 + 3*a^2 + 1)*x^3)
```

## Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^4}\,dx$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**4,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**4, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(136) = 272$.

Time = 0.36 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.26

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx$$
$$= -\frac{a^3b^3 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}}$$
$$+\frac{ab^3 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}}$$
$$-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab^3}{2\,(a^2+1)^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^2b^2}{2\,(a^2+1)^2x}$$
$$+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{2\,(a^2+1)^2x^2}-\frac{b}{2\,x^2}-\frac{a}{3\,x^3}-\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,(a^2+1)x^3}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*a*b^3*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3
/2) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/(a^2 + 1)^2 - 1/2*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^2*x) + 1/2*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)*a*b/((a^2 + 1)^2*x^2) - 1/2*b/x^2 - 1/3*a/x^3 - 1/3*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(136) = 272.

Time = 0.33 (sec) , antiderivative size = 715, normalized size of antiderivative = 4.58

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = -\frac{ab^3 \log\left(\frac{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\sqrt{b^2x^2+2\,abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{2\,(a^4+2\,a^2+1)\sqrt{a^2+1}}-\frac{3\,bx+2\,a}{6\,x^3}$$
$$+\frac{20\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^5b^3+12\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^7b^3+6\left(x|b|-\sqrt{b^2x^2+2}\right.}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/2*a*b^3*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sq
rt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt
```

```
(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 1/6*(3*b*x + 2*a)/x^3 + 1/3
*(20*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 12*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 6*(x*abs(b) - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^4*a^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2*a^6*b^2*abs(b) + 2*a^8*b^2*abs(b) + 3*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^3*a^3*b^3 + 33*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
*a^5*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^2*b^2*abs(
b) + 48*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b) + 8
*a^6*b^2*abs(b) + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a*b^3
 + 30*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 6*(x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 12*a^4*b^2*abs(b) + 9*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 8*a^2*b^2*abs(b) + 2*b^2*ab
s(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
- a^2 - 1)^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^4}\,dx$$

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4, x)
```

# 3.357 $\int \frac{e^{\mathbf{arcsinh}(a+bx)}}{x^5}\,dx$

Optimal result . . . 2356
Rubi [A] (verified) . . . 2357
Mathematica [A] (verified) . . . 2359
Maple [B] (verified) . . . 2360
Fricas [A] (verification not implemented) . . . 2361
Sympy [F] . . . 2361
Maxima [B] (verification not implemented) . . . 2361
Giac [B] (verification not implemented) . . . 2363
Mupad [F(-1)] . . . 2364

## Optimal result

Integrand size = 12, antiderivative size = 207

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = -\frac{a}{4x^4} - \frac{b}{3x^3} + \frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2} - \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{4\,(1+a^2)\,x^4} + \frac{5ab(1+a^2+2abx+b^2x^2)^{3/2}}{12\,(1+a^2)^2\,x^3} + \frac{(1-4a^2)\,b^4\operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{8\,(1+a^2)^{7/2}}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3-1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*b
*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)^2/x^3+1/8*(-4*a^2+1)*b^4*arctanh((a*
b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(7/2)+1/8*(
-4*a^2+1)*b^2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^3/x^2
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5878, 14, 758, 820, 734, 738, 212}

$$\begin{aligned}\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = &\frac{\left(1-4a^2\right)b^4 \operatorname{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{8\left(a^2+1\right)^{7/2}}\\ &+\frac{\left(1-4a^2\right)b^2(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{8\left(a^2+1\right)^3x^2}\\ &-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{4\left(a^2+1\right)x^4}\\ &+\frac{5ab\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{12\left(a^2+1\right)^2x^3}-\frac{a}{4x^4}-\frac{b}{3x^3}\end{aligned}$$

```
[In] Int[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/
2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(1 +
 a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sq
rt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^5}\,dx \\
&= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\right)dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\,dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}-\frac{\int \frac{(5ab+b^2x)\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx}{4\left(1+a^2\right)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4} \\
&\quad + \frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3} - \frac{\left(\left(1-4a^2\right)b^2\right)\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx}{4\left(1+a^2\right)^2} \\
&= -\frac{a}{4x^4} - \frac{b}{3x^3} + \frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2} \\
&\quad - \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4} + \frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3} \\
&\quad - \frac{\left(\left(1-4a^2\right)b^4\right)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx}{8\left(1+a^2\right)^3} \\
&= -\frac{a}{4x^4} - \frac{b}{3x^3} + \frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2} \\
&\quad - \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4} + \frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3} \\
&\quad + \frac{\left(\left(1-4a^2\right)b^4\right)\operatorname{Subst}\left(\int \frac{1}{4(1+a^2)-x^2}\,dx, x, \frac{2(1+a^2)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right)}{4\left(1+a^2\right)^3} \\
&= -\frac{a}{4x^4} - \frac{b}{3x^3} + \frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2} \\
&\quad - \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4} + \frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3} \\
&\quad + \frac{\left(1-4a^2\right)b^4\operatorname{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{8\left(1+a^2\right)^{7/2}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$
\begin{aligned}
&\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx \\
&= \frac{1}{24}\left(-\frac{6a}{x^4} - \frac{8b}{x^3} - \frac{\sqrt{1+a^2+2abx+b^2x^2}\left(6 + \frac{2abx}{1+a^2} - \frac{\left(-3+2a^2\right)b^2x^2}{\left(1+a^2\right)^2} + \frac{a\left(-13+2a^2\right)b^3x^3}{\left(1+a^2\right)^3}\right)}{x^4}\right. \\
&\qquad + \frac{3(-1+2a)(1+2a)b^4\log(x)}{\left(1+a^2\right)^{7/2}} \\
&\qquad \left. - \frac{3(-1+2a)(1+2a)b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\left(1+a^2\right)^{7/2}}\right)
\end{aligned}
$$

```
[In] Integrate[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(6 + (2*a*b*x)
/(1 + a^2) - ((-3 + 2*a^2)*b^2*x^2)/(1 + a^2)^2 + (a*(-13 + 2*a^2)*b^3*x^3)
/(1 + a^2)^3))/x^4 + (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[x])/(1 + a^2)^(7/2) -
(3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(7/2))/24
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(183) = 366$.

Time = 0.91 (sec) , antiderivative size = 1003, normalized size of antiderivative = 4.85

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1003 |

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4-5/4*a*b/(a^2+1)*(-1/3*(b^2*x
^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3-a*b/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+
2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2
*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/
4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^
2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(
b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a
^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))))-1/4*b^2/(
a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/
(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)
^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)))-1/3*b/x^3-1/4*a/x^4
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.43

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx$$

$$= \frac{3\,(4\,a^2-1)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-6\,a^9-(2\,a^5-}{}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 - 1)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt
(a^2 + 1) + a)/x) - 6*a^9 - (2*a^5 - 11*a^3 - 13*a)*b^4*x^4 - 24*a^7 - 36*a
^5 - 24*a^3 - 8*(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 - 1
1*a^3 - 13*a)*b^3*x^3 + 24*a^6 - (2*a^6 + a^4 - 4*a^2 - 3)*b^2*x^2 + 36*a^4
 + 2*(a^7 + 3*a^5 + 3*a^3 + a)*b*x + 24*a^2 + 6)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 6*a)/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

## Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^5}\,dx$$

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**5,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**5, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(183) = 366.

Time = 0.38 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.87

$$
\begin{aligned}
&\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx \\
&= \frac{5\,a^4b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{8\,(a^2+1)^{\frac{7}{2}}} \\
&\quad - \frac{3\,a^2b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{4\,(a^2+1)^{\frac{5}{2}}} \\
&\quad + \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a^2b^4}{8\,(a^2+1)^3} \\
&\quad + \frac{b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{8\,(a^2+1)^{\frac{3}{2}}} \\
&\quad - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}b^4}{8\,(a^2+1)^2} + \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a^3b^3}{8\,(a^2+1)^3x} \\
&\quad - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab^3}{8\,(a^2+1)^2x} - \frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2b^2}{8\,(a^2+1)^3x^2} \\
&\quad + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}b^2}{8\,(a^2+1)^2x^2} + \frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{12\,(a^2+1)^2x^3} \\
&\quad - \frac{b}{3\,x^3} - \frac{a}{4\,x^4} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{4\,(a^2+1)x^4}
\end{aligned}
$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 5/8*a^4*b^4*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/4*a^2*b^4*arcsinh(2*a*b*x/(sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(
5/2) + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^4/(a^2 + 1)^3 + 1/8*b^4*
arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
)*abs(x)))/(a^2 + 1)^(3/2) - 1/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4/(a^2
 + 1)^2 + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*b^3/((a^2 + 1)^3*x) - 1
/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/((a^2 + 1)^2*x) - 5/8*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2*b^2/((a^2 + 1)^3*x^2) + 1/8*(b^2*x^2 + 2*a*b*
x + a^2 + 1)^(3/2)*b^2/((a^2 + 1)^2*x^2) + 5/12*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2)*a*b/((a^2 + 1)^2*x^3) - 1/3*b/x^3 - 1/4*a/x^4 - 1/4*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(183) = 366$.

Time = 0.35 (sec) , antiderivative size = 1173, normalized size of antiderivative = 5.67

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2*b^4 - b^4)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1)) - 1/12*(4*
b*x + 3*a)/x^4 + 1/12*(32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*
a^6*b^4 + 256*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^8*b^4 + 96
*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^10*b^4 + 144*(x*abs(b) -
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^7*b^3*abs(b) + 224*(x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^9*b^3*abs(b) + 16*a^11*b^3*abs(b) - 12*(x
*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^7*a^2*b^4 + 140*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^4*b^4 + 716*(x*abs(b) - sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))^3*a^6*b^4 + 372*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))*a^8*b^4 + 432*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^5
*b^3*abs(b) + 704*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^7*b^3*
abs(b) + 80*a^9*b^3*abs(b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
))^7*b^4 + 129*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^2*b^4 + 6
85*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^4*b^4 + 543*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^6*b^4 + 432*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^4*a^3*b^3*abs(b) + 768*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))^2*a^5*b^3*abs(b) + 160*a^7*b^3*abs(b) + 21*(x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*b^4 + 246*(x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^3*a^2*b^4 + 357*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*a^4*b^4 + 144*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a*b^3*
abs(b) + 320*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*b^3*abs(b
) + 160*a^5*b^3*abs(b) + 21*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^
3*b^4 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^4 + 32*(x*a
bs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a*b^3*abs(b) + 80*a^3*b^3*abs(
b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*b^4 + 16*a*b^3*abs(b)
)/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)^2 - a^2 - 1)^4)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^5}\,dx$$

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5,x)
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5, x)
```

# 3.358 $\int e^{\operatorname{arcsinh}(a+bx)^2} x^3\,dx$

Optimal result . . . 2365
Rubi [A] (verified) . . . 2366
Mathematica [A] (verified) . . . 2371
Maple [F] . . . 2371
Fricas [F] . . . 2371
Sympy [F] . . . 2371
Maxima [F] . . . 2372
Giac [F] . . . 2372
Mupad [F(-1)] . . . 2372

## Optimal result

Integrand size = 14, antiderivative size = 359

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)^2} x^3\,dx = &-\frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{16b^4e}+\frac{3a^2\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8b^4e}\\&+\frac{\sqrt{\pi}\operatorname{erfi}(2-\operatorname{arcsinh}(a+bx))}{32b^4e^4}-\frac{\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{16b^4e}\\&+\frac{3a^2\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{8b^4e}+\frac{\sqrt{\pi}\operatorname{erfi}(2+\operatorname{arcsinh}(a+bx))}{32b^4e^4}\\&-\frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}}\\&+\frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}}\\&-\frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}}\\&+\frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}}\\&-\frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}}\\&-\frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}}\end{aligned}$$

```
[Out] -1/32*erfi(-2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)+1/16*erfi(-1+arcsinh(b*x+
a))*Pi^(1/2)/b^4/exp(1)-3/8*a^2*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1)
-1/16*erfi(1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1)+3/8*a^2*erfi(1+arcsinh(b*x
+a))*Pi^(1/2)/b^4/exp(1)+1/32*erfi(2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)-3/
16*a*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+arcsi
nh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)
/b^4/exp(1/4)+3/16*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3
```

```
*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arcsinh(b*x
+a))*Pi^(1/2)/b^4/exp(9/4)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx = &-\frac{\sqrt{\pi}a^3\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}b^4}\\ &-\frac{\sqrt{\pi}a^3\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}b^4}\\ &+\frac{3\sqrt{\pi}a^2\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8eb^4}+\frac{3\sqrt{\pi}a^2\operatorname{erfi}(\operatorname{arcsinh}(a+bx)+1)}{8eb^4}\\ &-\frac{3\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-3)\right)}{16e^{9/4}b^4}\\ &+\frac{3\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{16\sqrt[4]{e}b^4}\\ &+\frac{3\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{16\sqrt[4]{e}b^4}\\ &-\frac{3\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+3)\right)}{16e^{9/4}b^4}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{16eb^4}+\frac{\sqrt{\pi}\operatorname{erfi}(2-\operatorname{arcsinh}(a+bx))}{32e^4b^4}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}(\operatorname{arcsinh}(a+bx)+1)}{16eb^4}+\frac{\sqrt{\pi}\operatorname{erfi}(\operatorname{arcsinh}(a+bx)+2)}{32e^4b^4}\end{aligned}$$

```
[In] Int[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] -1/16*(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[
1 - ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(3
2*b^4*E^4) - (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt
[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcSinh[a +
 b*x]])/(32*b^4*E^4) - (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16
*b^4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^
(1/4)) - (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) +
 (3*a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sq
rt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Er
fi[(3 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3 dx, x, \text{arcsinh}(a+bx)\right)}{b}$$

$$
\begin{aligned}
&= \frac{\operatorname{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))^3}{b^3}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\operatorname{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))^3\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&= \frac{\operatorname{Subst}\left(\int \left(-a^3e^{x^2}\cosh(x) + 3a^2e^{x^2}\cosh(x)\sinh(x) - 3ae^{x^2}\cosh(x)\sinh^2(x) + e^{x^2}\cosh(x)\sinh^3(x)\right.\right.}{b^4} \\
&= \frac{\operatorname{Subst}\left(\int e^{x^2}\cosh(x)\sinh^3(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad + \frac{(3a^2)\operatorname{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2} + \frac{1}{8}e^{-2x+x^2} - \frac{1}{8}e^{2x+x^2} + \frac{1}{16}e^{4x+x^2}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int \left(\frac{1}{8}e^{-3x+x^2} - \frac{1}{8}e^{-x+x^2} - \frac{e^{x+x^2}}{8} + \frac{1}{8}e^{3x+x^2}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad + \frac{(3a^2)\operatorname{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2} + \frac{1}{4}e^{2x+x^2}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2} + \frac{e^{x+x^2}}{2}\right)\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{Subst}\left(\int e^{-4x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{16b^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{4x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{16b^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{-2x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{2x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int e^{-3x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad + \frac{(3a)\operatorname{Subst}\left(\int e^{-x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad + \frac{(3a)\operatorname{Subst}\left(\int e^{x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int e^{3x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4} \\
&\quad - \frac{(3a^2)\operatorname{Subst}\left(\int e^{-2x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^4} \\
&\quad + \frac{(3a^2)\operatorname{Subst}\left(\int e^{2x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^4} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int e^{-x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^4} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int e^{x+x^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{16b^4e^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{16b^4e^4} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4e^{9/4}} \\
&\quad - \frac{(3a)\operatorname{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4e^{9/4}} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4e} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4e} \\
&\quad - \frac{(3a^2)\operatorname{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^4e} \\
&\quad + \frac{(3a^2)\operatorname{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{4b^4e} \\
&\quad + \frac{(3a)\operatorname{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4\sqrt[4]{e}} \\
&\quad + \frac{(3a)\operatorname{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{8b^4\sqrt[4]{e}} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^4\sqrt[4]{e}} \\
&\quad - \frac{a^3\operatorname{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \operatorname{arcsinh}(a+bx)\right)}{2b^4\sqrt[4]{e}} \\
&= -\frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{16b^4e} + \frac{3a^2\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8b^4e} \\
&\quad + \frac{\sqrt{\pi}\operatorname{erfi}(2-\operatorname{arcsinh}(a+bx))}{32b^4e^4} - \frac{\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{16b^4e} \\
&\quad + \frac{3a^2\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{8b^4e} + \frac{\sqrt{\pi}\operatorname{erfi}(2+\operatorname{arcsinh}(a+bx))}{32b^4e^4} \\
&\quad - \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}} + \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}} \\
&\quad - \frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}} + \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}} \\
&\quad - \frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}} - \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3\,dx$$

$$= \frac{\sqrt{\pi}\left(2a(-3+4a^2)\,e^{15/4}\operatorname{erfi}\left(\frac{1}{2}-\operatorname{arcsinh}(a+bx)\right)+2(-1+6a^2)\,e^3\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))+6ae^{7/4}\operatorname{erfi}(\right.}{}$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(2*a*(-3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcSinh[a + b*x]] + 2*(-1 +
 6*a^2)*E^3*Erfi[1 - ArcSinh[a + b*x]] + 6*a*E^(7/4)*Erfi[3/2 - ArcSinh[a +
 b*x]] + Erfi[2 - ArcSinh[a + b*x]] + 6*a*E^(15/4)*Erfi[1/2 + ArcSinh[a + b
*x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcSinh[a + b*x]] - 2*E^3*Erfi[1 + ArcSinh
[a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcSinh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
 ArcSinh[a + b*x]] + Erfi[2 + ArcSinh[a + b*x]]))/(32*b^4*E^4)
```

## Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} x^3 dx$$

```
[In] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

## Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3\,dx = \int x^3 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arcsinh(b*x + a)^2), x)
```

## Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3\,dx = \int x^3 e^{\operatorname{asinh}^2(a+bx)}\,dx$$

```
[In] integrate(exp(asinh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(asinh(a + b*x)**2), x)
```

## Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx = \int x^3 e^{\left(\operatorname{arsinh}(bx+a)^2\right)} \, dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="maxima")
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

## Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx = \int x^3 e^{\left(\operatorname{arsinh}(bx+a)^2\right)} \, dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="giac")
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx = \int x^3 \, \mathrm{e}^{\operatorname{asinh}(a+b\,x)^2} \, dx$$

```
[In] int(x^3*exp(asinh(a + b*x)^2),x)
[Out] int(x^3*exp(asinh(a + b*x)^2), x)
```

# 3.359 $\int e^{\operatorname{arcsinh}(a+bx)^2} x^2\,dx$

Optimal result . . . 2373
Rubi [A] (verified) . . . 2374
Mathematica [A] (verified) . . . 2377
Maple [F] . . . 2378
Fricas [F] . . . 2378
Sympy [F] . . . 2378
Maxima [F] . . . 2378
Giac [F] . . . 2379
Mupad [F(-1)] . . . 2379

## Optimal result

Integrand size = 14, antiderivative size = 251

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)^2} x^2\,dx = &-\frac{a\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{4b^3e} - \frac{a\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{4b^3e}\\ &+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-3+2\operatorname{arcsinh}(a+bx))\right)}{16b^3e^{9/4}}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}}\\ &+\frac{a^2\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}}\\ &+\frac{a^2\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}}\\ &+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(3+2\operatorname{arcsinh}(a+bx))\right)}{16b^3e^{9/4}}\end{aligned}$$

```
[Out] 1/4*a*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1)-1/4*a*erfi(1+arcsinh(b*x+
a))*Pi^(1/2)/b^3/exp(1)+1/16*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4
)-1/16*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/4*a^2*erfi(-1/2+ar
csinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)-1/16*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/
b^3/exp(1/4)+1/4*a^2*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*er
fi(3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)^2} x^2\,dx = &\frac{\sqrt{\pi}a^2\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}b^3}\\ &+\frac{\sqrt{\pi}a^2\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}b^3}\\ &-\frac{\sqrt{\pi}a\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{4eb^3}-\frac{\sqrt{\pi}a\operatorname{erfi}(\operatorname{arcsinh}(a+bx)+1)}{4eb^3}\\ &+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-3)\right)}{16e^{9/4}b^3}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{16\sqrt[4]{e}b^3}\\ &-\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{16\sqrt[4]{e}b^3}\\ &+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+3)\right)}{16e^{9/4}b^3}\end{aligned}$$

```
[In] Int[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -1/4*(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(b^3*E) - (a*Sqrt[Pi]*Erfi[1 +
 ArcSinh[a + b*x]])/(4*b^3*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2]
)/(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E
^(1/4)) + (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4))
- (Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[
Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 +
 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2 dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(a-\sinh(x))^2}{b^2} dx, x, \operatorname{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(a-\sinh(x))^2 dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \left(a^2 e^{x^2}\cosh(x) - 2ae^{x^2}\cosh(x)\sinh(x) + e^{x^2}\cosh(x)\sinh^2(x)\right) dx, x, \operatorname{arcsinh}(a+bx)\right)}{b^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(2a)\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{8}e^{-3x+x^2} - \frac{1}{8}e^{-x+x^2} - \frac{e^{x+x^2}}{8} + \frac{1}{8}e^{3x+x^2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2} + \frac{1}{4}e^{2x+x^2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2} + \frac{e^{x+x^2}}{2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int e^{-3x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3} + \frac{\text{Subst}\left(\int e^{3x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3} \\
&\quad + \frac{a\text{Subst}\left(\int e^{-2x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3} - \frac{a\text{Subst}\left(\int e^{2x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3} \\
&\quad + \frac{a^2\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3} + \frac{a^2\text{Subst}\left(\int e^{x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3e^{9/4}} \\
&\quad + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3e^{9/4}} \\
&\quad + \frac{a\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3e} \\
&\quad - \frac{a\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3e} \\
&\quad - \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3\sqrt[4]{e}} \\
&\quad - \frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{8b^3\sqrt[4]{e}} \\
&\quad + \frac{a^2\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3\sqrt[4]{e}} \\
&\quad + \frac{a^2\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^3\sqrt[4]{e}} \\
&= -\frac{a\sqrt{\pi}\text{erfi}(1-\text{arcsinh}(a+bx))}{4b^3e} - \frac{a\sqrt{\pi}\text{erfi}(1+\text{arcsinh}(a+bx))}{4b^3e} \\
&\quad + \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-3+2\text{arcsinh}(a+bx))\right)}{16b^3e^{9/4}} - \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}} \\
&\quad + \frac{a^2\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}} - \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(1+2\text{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}} \\
&\quad + \frac{a^2\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(1+2\text{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}} + \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(3+2\text{arcsinh}(a+bx))\right)}{16b^3e^{9/4}}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

$$
\int e^{\text{arcsinh}(a+bx)^2}x^2\,dx =
$$

$$
-\frac{\sqrt{\pi}\left(\left(-1+4a^2\right)e^2\text{erfi}\left(\frac{1}{2}-\text{arcsinh}(a+bx)\right)+4ae^{5/4}\text{erfi}(1-\text{arcsinh}(a+bx))+\text{erfi}\left(\frac{3}{2}-\text{arcsinh}(a+b\right.\right.}{}
$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -1/16*(Sqrt[Pi]*((-1 + 4*a^2)*E^2*Erfi[1/2 - ArcSinh[a + b*x]] + 4*a*E^(5/4
)*Erfi[1 - ArcSinh[a + b*x]] + Erfi[3/2 - ArcSinh[a + b*x]] + E^2*Erfi[1/2
+ ArcSinh[a + b*x]] - 4*a^2*E^2*Erfi[1/2 + ArcSinh[a + b*x]] + 4*a*E^(5/4)*
Erfi[1 + ArcSinh[a + b*x]] - Erfi[3/2 + ArcSinh[a + b*x]]))/(b^3*E^(9/4))
```

## Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} x^2 dx$$

```
[In] int(exp(arcsinh(b*x+a)^2)*x^2,x)
[Out] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

## Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)} \, dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="fricas")
[Out] integral(x^2*e^(arcsinh(b*x + a)^2), x)
```

## Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = \int x^2 e^{\operatorname{asinh}^2(a+bx)} \, dx$$

```
[In] integrate(exp(asinh(b*x+a)**2)*x**2,x)
[Out] Integral(x**2*exp(asinh(a + b*x)**2), x)
```

## Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)} \, dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

## Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2\,dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="giac")
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2\,dx = \int x^2\,\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

```
[In] int(x^2*exp(asinh(a + b*x)^2),x)
[Out] int(x^2*exp(asinh(a + b*x)^2), x)
```

# 3.360 $\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx$

Optimal result . . . 2380
Rubi [A] (verified) . . . 2380
Mathematica [A] (verified) . . . 2383
Maple [F] . . . 2383
Fricas [F] . . . 2383
Sympy [F] . . . 2384
Maxima [F] . . . 2384
Giac [F] . . . 2384
Mupad [F(-1)] . . . 2384

## Optimal result

Integrand size = 12, antiderivative size = 117

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8b^2e} + \frac{\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{8b^2e} - \frac{a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}} - \frac{a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}}$$

```
[Out] -1/8*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1)+1/8*erfi(1+arcsinh(b*x+a))
*Pi^(1/2)/b^2/exp(1)-1/4*a*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)-
1/4*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8eb^2} + \frac{\sqrt{\pi}\operatorname{erfi}(\operatorname{arcsinh}(a+bx)+1)}{8eb^2} - \frac{\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}eb^2} - \frac{\sqrt{\pi}a\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}eb^2}$$

```
[In] Int[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcSin
h[a + b*x]])/(8*b^2*E) - (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))}{b}\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\cosh(x)+e^{x^2}\cosh(x)\sinh(x)\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&\quad - \frac{a\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&\quad - \frac{a\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b^2} \\
&= -\frac{\text{Subst}\left(\int e^{-2x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^2} \\
&\quad - \frac{a\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2} - \frac{a\text{Subst}\left(\int e^{x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^2e} \\
&\quad + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{4b^2e} \\
&\quad - \frac{a\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2\sqrt[4]{e}} \\
&\quad - \frac{a\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b^2\sqrt[4]{e}}
\end{aligned}
$$

$$= \frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8b^2e} + \frac{\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{8b^2e} - \frac{a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}} - \frac{a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}}$$

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx$$

$$= \frac{\sqrt{\pi}\left(2ae^{3/4}\operatorname{erfi}\left(\frac{1}{2}-\operatorname{arcsinh}(a+bx)\right)+\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))-2ae^{3/4}\operatorname{erfi}\left(\frac{1}{2}+\operatorname{arcsinh}(a+bx)\right)+\operatorname{erfi}\right.}{8b^2e}$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(2*a*E^(3/4)*Erfi[1/2 - ArcSinh[a + b*x]] + Erfi[1 - ArcSinh[a +
b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[1 + ArcSinh[a + b*x
]]))/(8*b^2*E)
```

## Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} x dx$$

```
[In] int(exp(arcsinh(b*x+a)^2)*x,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x,x)
```

## Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \int x e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arcsinh(b*x + a)^2), x)
```

## Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \int x e^{\operatorname{asinh}^2(a+bx)}\,dx$$

```
[In] integrate(exp(asinh(b*x+a)**2)*x,x)
[Out] Integral(x*exp(asinh(a + b*x)**2), x)
```

## Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \int x e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="maxima")
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

## Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \int x e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="giac")
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x\,dx = \int x\,\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

```
[In] int(x*exp(asinh(a + b*x)^2),x)
[Out] int(x*exp(asinh(a + b*x)^2), x)
```

# 3.361 $\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx$

Optimal result . . . 2385
Rubi [A] (verified) . . . 2385
Mathematica [A] (verified) . . . 2386
Maple [F] . . . 2387
Fricas [F] . . . 2387
Sympy [F] . . . 2387
Maxima [F] . . . 2387
Giac [F] . . . 2388
Mupad [F(-1)] . . . 2388

## Optimal result

Integrand size = 10, antiderivative size = 65

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}}$$

```
[Out] 1/4*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arcsinh(b*x+
a))*Pi^(1/2)/b/exp(1/4)
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5872, 5624, 2266, 2235}

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}b} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(2\operatorname{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}b}$$

```
[In] Int[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5872

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2} + \frac{e^{x+x^2}}{2}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b\sqrt[4]{e}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \text{arcsinh}(a+bx)\right)}{2b\sqrt[4]{e}} \\
&= \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}} + \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(1+2\text{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}}
\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int e^{\text{arcsinh}(a+bx)^2}\,dx = \frac{\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}+\text{arcsinh}(a+bx)\right) + \text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)\right)}{4b\sqrt[4]{e}}$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*(Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[(-1 + 2*ArcSinh[a + b*x])/2]
))/(4*b*E^(1/4))
```

## Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}\,dx$$

```
[In] int(exp(arcsinh(b*x+a)^2),x)
[Out] int(exp(arcsinh(b*x+a)^2),x)
```

## Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="fricas")
[Out] integral(e^(arcsinh(b*x + a)^2), x)
```

## Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\operatorname{asinh}^2(a+bx)}\,dx$$

```
[In] integrate(exp(asinh(b*x+a)**2),x)
[Out] Integral(exp(asinh(a + b*x)**2), x)
```

## Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="maxima")
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

## Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="giac")
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

```
[In] int(exp(asinh(a + b*x)^2),x)
[Out] int(exp(asinh(a + b*x)^2), x)
```

# 3.362 $\int \frac{e^{\mathbf{arcsinh}(a+bx)^2}}{x}\,dx$

Optimal result . . . 2389
Rubi [N/A] . . . 2389
Mathematica [N/A] . . . 2390
Maple [N/A] (verified) . . . 2390
Fricas [N/A] . . . 2390
Sympy [N/A] . . . 2390
Maxima [N/A] . . . 2391
Giac [N/A] . . . 2391
Mupad [N/A] . . . 2391

## Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \operatorname{Int}\left(\frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}, x\right)$$

```
[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx$$

```
[In] Int[E^ArcSinh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x,x]
[Out] Integrate[E^ArcSinh[a + b*x]^2/x, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}}{x}\,dx$$

```
[In] int(exp(arcsinh(b*x+a)^2)/x,x)
[Out] int(exp(arcsinh(b*x+a)^2)/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(e^(arcsinh(b*x + a)^2)/x, x)
```

## Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\operatorname{asinh}^2(a+bx)}}{x}\,dx$$

```
[In] integrate(exp(asinh(b*x+a)**2)/x,x)
[Out] Integral(exp(asinh(a + b*x)**2)/x, x)
```

## Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}}{x}\,dx$$

```
[In] int(exp(asinh(a + b*x)^2)/x,x)
[Out] int(exp(asinh(a + b*x)^2)/x, x)
```

# 3.363 $\int \frac{e^{\mathbf{arcsinh}(a+bx)^2}}{x^2}\,dx$

Optimal result . . . 2392
Rubi [N/A] . . . 2392
Mathematica [N/A] . . . 2393
Maple [N/A] (verified) . . . 2393
Fricas [N/A] . . . 2393
Sympy [N/A] . . . 2393
Maxima [N/A] . . . 2394
Giac [N/A] . . . 2394
Mupad [N/A] . . . 2394

## Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}\,dx = \text{Int}\left(\frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}, x\right)$$

```
[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x^2,x)
```

## Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}\,dx$$

```
[In] Int[E^ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}\,dx$$

## Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx$$

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x^2,x]
[Out] Integrate[E^ArcSinh[a + b*x]^2/x^2, x]
```

## Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}}{x^2}dx$$

```
[In] int(exp(arcsinh(b*x+a)^2)/x^2,x)
[Out] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(e^(arcsinh(b*x + a)^2)/x^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\operatorname{asinh}^2(a+bx)}}{x^2}\,dx$$

```
[In] integrate(exp(asinh(b*x+a)**2)/x**2,x)
[Out] Integral(exp(asinh(a + b*x)**2)/x**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}}{x^2}\,dx$$

```
[In] int(exp(asinh(a + b*x)^2)/x^2,x)
[Out] int(exp(asinh(a + b*x)^2)/x^2, x)
```

# 3.364 $\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal result . . . 2395
Rubi [A] (verified) . . . 2395
Mathematica [A] (verified) . . . 2397
Maple [A] (verified) . . . 2397
Fricas [F] . . . 2398
Sympy [F] . . . 2398
Maxima [F] . . . 2398
Giac [F] . . . 2399
Mupad [F(-1)] . . . 2399

## Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{\text{arcsinh}(a+bx)^2}{2d} + \frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d} + \frac{\text{PolyLog}\left(2,e^{2\text{arcsinh}(a+bx)}\right)}{2d}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/d+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/
d+1/2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/d
```

## Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5859, 12, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{PolyLog}\left(2,e^{2\text{arcsinh}(a+bx)}\right)}{2d} - \frac{\text{arcsinh}(a+bx)^2}{2d} + \frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d}$$

```
[In] Int[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2/d + (ArcSinh[a + b*x]*Log[1 - E^(2*ArcSinh[a + b*x]
)])/d + PolyLog[2, E^(2*ArcSinh[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b \text{arcsinh}(x)}{dx}\, dx, x, a + bx\right)}{b}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\text{arcsinh}(x)}{x}\,dx, x, a+bx\right)}{d} \\
&= \frac{\text{Subst}(\int x\coth(x)\,dx, x, \text{arcsinh}(a+bx))}{d} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2d} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{arcsinh}(a+bx)\right)}{d} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2d} + \frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d} \\
&\quad - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(a+bx)\right)}{d} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2d} + \frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{arcsinh}(a+bx)}\right)}{2d} \\
&= -\frac{\text{arcsinh}(a+bx)^2}{2d} + \frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d} + \frac{\text{PolyLog}\left(2, e^{2\text{arcsinh}(a+bx)}\right)}{2d}
\end{aligned}
$$

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$
\begin{aligned}
&\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx \\
&= \frac{-\text{arcsinh}(a+bx)\left(\text{arcsinh}(a+bx) - 2\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)\right) + \text{PolyLog}\left(2, e^{2\text{arcsinh}(a+bx)}\right)}{2d}
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a + b*x]/((a*d)/b + d*x),x]

[Out] (-(ArcSinh[a + b*x]*(ArcSinh[a + b*x] - 2*Log[1 - E^(2*ArcSinh[a + b*x])]))
 + PolyLog[2, E^(2*ArcSinh[a + b*x])])/(2*d)
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.23

| method | result |
| --- | --- |
| derivativedivides | $\frac{-\frac{b \operatorname{arcsinh}(bx+a)^2}{2d}+\frac{b \operatorname{arcsinh}(bx+a) \ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{arcsinh}(bx+a) \ln\left(1-bx-\right.}{d}}{b}$ |
| default | $\frac{-\frac{b \operatorname{arcsinh}(bx+a)^2}{2d}+\frac{b \operatorname{arcsinh}(bx+a) \ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{arcsinh}(bx+a) \ln\left(1-bx-\right.}{d}}{b}$ |

```
[In] int(arcsinh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*b/d*arcsinh(b*x+a)^2+b/d*arcsinh(b*x+a)*ln(1+b*x+a+(1+(b*x+a)^2)^
(1/2))+b/d*polylog(2,-b*x-a-(1+(b*x+a)^2)^(1/2))+b/d*arcsinh(b*x+a)*ln(1-b*
x-a-(1+(b*x+a)^2)^(1/2))+b/d*polylog(2,b*x+a+(1+(b*x+a)^2)^(1/2)))
```

## Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsinh(b*x + a)/(b*d*x + a*d), x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{asinh}\left(a+bx\right)}{a+bx}\,dx}{d}$$

```
[In] integrate(asinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asinh(a + b*x)/(a + b*x), x)/d
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

```
[In] int(asinh(a + b*x)/(d*x + (a*d)/b),x)
[Out] int(asinh(a + b*x)/(d*x + (a*d)/b), x)
```

# 3.365 $\int \frac{x}{\sqrt{1+x^2}\mathbf{arcsinh}(x)}\,dx$

Optimal result . . . 2400
Rubi [A] (verified) . . . 2400
Mathematica [A] (verified) . . . 2401
Maple [A] (verified) . . . 2401
Fricas [F] . . . 2401
Sympy [F] . . . 2402
Maxima [F] . . . 2402
Giac [F] . . . 2402
Mupad [F(-1)] . . . 2402

## Optimal result

Integrand size = 15, antiderivative size = 3

$$\int \frac{x}{\sqrt{1+x^2}\operatorname{arcsinh}(x)}\,dx = \operatorname{Shi}(\operatorname{arcsinh}(x))$$

```
[Out] Shi(arcsinh(x))
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5819, 3379}

$$\int \frac{x}{\sqrt{1+x^2}\operatorname{arcsinh}(x)}\,dx = \operatorname{Shi}(\operatorname{arcsinh}(x))$$

```
[In] Int[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
```

```
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(x)\right)\\ &= \text{Shi}(\text{arcsinh}(x))\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \text{Shi}(\text{arcsinh}(x))$$

```
[In] Integrate[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
[Out] SinhIntegral[ArcSinh[x]]
```

## Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| default | $\text{Shi}\left(\text{arcsinh}\left(x\right)\right)$ | 4 |

```
[In] int(x/arcsinh(x)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] Shi(arcsinh(x))
```

## Fricas [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\left(x\right)}\,dx$$

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

## Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{asinh}\,(x)}\,dx$$

```
[In] integrate(x/asinh(x)/(x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(x**2 + 1)*asinh(x)), x)
```

## Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\,(x)}\,dx$$

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

## Giac [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\,(x)}\,dx$$

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\text{asinh}\,(x)\,\sqrt{x^2+1}}\,dx$$

```
[In] int(x/(asinh(x)*(x^2 + 1)^(1/2)),x)
[Out] int(x/(asinh(x)*(x^2 + 1)^(1/2)), x)
```

# 3.366 $\int x^3 \mathbf{arcsinh}(a+bx^4)\,dx$

Optimal result . . . 2403
Rubi [A] (verified) . . . 2403
Mathematica [A] (verified) . . . 2404
Maple [A] (verified) . . . 2405
Fricas [A] (verification not implemented) . . . 2405
Sympy [A] (verification not implemented) . . . 2405
Maxima [A] (verification not implemented) . . . 2406
Giac [B] (verification not implemented) . . . 2406
Mupad [B] (verification not implemented) . . . 2406

## Optimal result

Integrand size = 12, antiderivative size = 45

$$\int x^3 \operatorname{arcsinh}(a+bx^4)\,dx = -\frac{\sqrt{1+(a+bx^4)^2}}{4b} + \frac{(a+bx^4)\operatorname{arcsinh}(a+bx^4)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arcsinh(b*x^4+a)/b-1/4*(1+(b*x^4+a)^2)^(1/2)/b
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 5858, 5772, 267}

$$\int x^3 \operatorname{arcsinh}(a+bx^4)\,dx = \frac{(a+bx^4)\operatorname{arcsinh}(a+bx^4)}{4b} - \frac{\sqrt{(a+bx^4)^2+1}}{4b}$$

```
[In] Int[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] -1/4*Sqrt[1 + (a + b*x^4)^2]/b + ((a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \operatorname{arcsinh}(a+bx)\,dx, x, x^4\right) \\ &= \frac{\text{Subst}\left(\int \operatorname{arcsinh}(x)\,dx, x, a+bx^4\right)}{4b} \\ &= \frac{(a+bx^4)\operatorname{arcsinh}(a+bx^4)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx^4\right)}{4b} \\ &= -\frac{\sqrt{1+(a+bx^4)^2}}{4b} + \frac{(a+bx^4)\operatorname{arcsinh}(a+bx^4)}{4b}\end{aligned}$$

## Mathematica [A] (verified)

Time $= 0.02$ (sec) , antiderivative size $= 41$, normalized size of antiderivative $= 0.91$

$$\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx = \frac{-\sqrt{1+(a+bx^4)^2} + (a+bx^4)\operatorname{arcsinh}(a+bx^4)}{4b}$$

```
[In] Integrate[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] (-Sqrt[1 + (a + b*x^4)^2] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(b x^4+a)\operatorname{arcsinh}(b x^4+a)-\sqrt{1+(b x^4+a)^2}}{4b}$ | 38 |
| default | $\frac{(b x^4+a)\operatorname{arcsinh}(b x^4+a)-\sqrt{1+(b x^4+a)^2}}{4b}$ | 38 |

```
[In] int(x^3*arcsinh(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*((b*x^4+a)*arcsinh(b*x^4+a)-(1+(b*x^4+a)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx$$
$$= \frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2+1}}{4\,b}$$

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/b
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx = \begin{cases} \frac{a \operatorname{asinh}\left(a+bx^4\right)}{4b} + \frac{x^4 \operatorname{asinh}\left(a+bx^4\right)}{4} - \frac{\sqrt{a^2+2abx^4+b^2x^8+1}}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{asinh}\left(a\right)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*asinh(b*x**4+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x**4)/(4*b) + x**4*asinh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx = \frac{\left(bx^4+a\right)\operatorname{arsinh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2+1}}{4\,b}$$

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arcsinh(b*x^4 + a) - sqrt((b*x^4 + a)^2 + 1))/b
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(39) = 78.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.33

$$\begin{aligned}
&\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx \\
&= \frac{1}{4}\,x^4 \log\left(bx^4+a+\sqrt{\left(bx^4+a\right)^2+1}\right) \\
&\quad -\frac{1}{4}\,b\left(\frac{a\log\left(-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,abx^4+a^2+1}}{b^2}\right)
\end{aligned}$$

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 + 1)) - 1/4*b*(a*log(-a*b - (x^4
*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))*abs(b))/(b*abs(b)) + sqrt(b^
2*x^8 + 2*a*b*x^4 + a^2 + 1)/b^2)
```

## Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\begin{aligned}
\int x^3 \operatorname{arcsinh}\left(a+bx^4\right)\,dx = {} & \frac{x^4\,\operatorname{asinh}(b\,x^4+a)}{4}-\frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}}{4\,b} \\
& +\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}+\frac{b^2\,x^4+a\,b}{\sqrt{b^2}}\right)}{4\,\sqrt{b^2}}
\end{aligned}$$

```
[In] int(x^3*asinh(a + b*x^4),x)
```

```
[Out] (x^4*asinh(a + b*x^4))/4 - (a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2)/(4*b) + (a
*log((a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2) + (a*b + b^2*x^4)/(b^2)^(1/2)))/
(4*(b^2)^(1/2))
```

# 3.367 $\int x^{-1+n}\mathbf{arcsinh}(a+bx^n)\,dx$

Optimal result . . . 2407
Rubi [A] (verified) . . . 2407
Mathematica [A] (verified) . . . 2408
Maple [F] . . . 2409
Fricas [B] (verification not implemented) . . . 2409
Sympy [B] (verification not implemented) . . . 2409
Maxima [A] (verification not implemented) . . . 2410
Giac [B] (verification not implemented) . . . 2410
Mupad [B] (verification not implemented) . . . 2410

## Optimal result

Integrand size = 14, antiderivative size = 46

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = -\frac{\sqrt{1+(a+bx^n)^2}}{bn}+\frac{(a+bx^n)\operatorname{arcsinh}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arcsinh(a+b*x^n)/b/n-(1+(a+b*x^n)^2)^(1/2)/b/n
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5858, 5772, 267}

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = \frac{(a+bx^n)\operatorname{arcsinh}(a+bx^n)}{bn}-\frac{\sqrt{(a+bx^n)^2+1}}{bn}$$

```
[In] Int[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 + (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
```

```
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{arcsinh}(a+bx)\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \operatorname{arcsinh}(x)\,dx, x, a+bx^n\right)}{bn} \\ &= \frac{\left(a+bx^n\right)\operatorname{arcsinh}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx^n\right)}{bn} \\ &= -\frac{\sqrt{1+\left(a+bx^n\right)^2}}{bn} + \frac{\left(a+bx^n\right)\operatorname{arcsinh}(a+bx^n)}{bn}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = \frac{-\sqrt{1+\left(a+bx^n\right)^2} + \left(a+bx^n\right)\operatorname{arcsinh}(a+bx^n)}{bn}$$

```
[In] Integrate[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] (-Sqrt[1 + (a + b*x^n)^2] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

## Maple [F]

$$\int x^{n-1} \operatorname{arcsinh}(a + b\,x^n)\,dx$$

```
[In] int(x^(n-1)*arcsinh(a+b*x^n),x)
[Out] int(x^(n-1)*arcsinh(a+b*x^n),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(44) = 88$.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.30

$$\int x^{-1+n}\operatorname{arcsinh}(a + bx^n)\,dx$$
$$= \frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\log\left(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a + \sqrt{\frac{2\,ab+(a^2+b^2+1)}{\cos}}\right.}{bn}$$

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="fricas")
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) - sqrt((2*a*b + (a^2
+ b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 9.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int x^{-1+n}\operatorname{arcsinh}(a + bx^n)\,dx$$
$$= \begin{cases} \log(x)\operatorname{asinh}(a) & \text{for } b = 0 \wedge n = 0 \\ \frac{x x^{n-1}\operatorname{asinh}(a)}{n} & \text{for } b = 0 \\ \log(x)\operatorname{asinh}(a+b) & \text{for } n = 0 \\ \frac{a\operatorname{asinh}(a+bx^n)}{bn} + \frac{x^n\operatorname{asinh}(a+bx^n)}{n} - \frac{\sqrt{a^2+2abx^n+b^2x^{2n}+1}}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+n)*asinh(a+b*x**n),x)
[Out] Piecewise((log(x)*asinh(a), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)*asinh(a)/n,
 Eq(b, 0)), (log(x)*asinh(a + b), Eq(n, 0)), (a*asinh(a + b*x**n)/(b*n) + x
**n*asinh(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) + 1)/(b*n)
, True))
```

## Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = \frac{(bx^n+a)\operatorname{arsinh}(bx^n+a)-\sqrt{(bx^n+a)^2+1}}{bn}$$

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arcsinh(b*x^n + a) - sqrt((b*x^n + a)^2 + 1))/(b*n)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = -\frac{b\left(\frac{a\log\left(-ab-\left(x^n|b|-\sqrt{b^2x^{2\,n}+2\,abx^n+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2+1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{(bx^n+a)^2+1}\right)}{n}$$

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1))*ab
s(b))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1)/b^2) - x^n*log(b
*x^n + a + sqrt((b*x^n + a)^2 + 1)))/n
```

## Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx = \frac{x^n\operatorname{asinh}(a+b\,x^n)}{n}-\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}}{b\,n}+\frac{a\,\ln\left(\frac{a\,b+b^2\,x^n}{\sqrt{b^2}}+\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}\right)}{n\,\sqrt{b^2}}$$

```
[In] int(x^(n - 1)*asinh(a + b*x^n),x)
```

```
[Out] (x^n*asinh(a + b*x^n))/n - (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^(1/2)/(b*n)
+ (a*log((a*b + b^2*x^n)/(b^2)^(1/2) + (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^
(1/2)))/(n*(b^2)^(1/2))
```

# 3.368 $\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx$

Optimal result . . . 2411
Rubi [A] (verified) . . . 2411
Mathematica [B] (verified) . . . 2413
Maple [A] (verified) . . . 2414
Fricas [B] (verification not implemented) . . . 2414
Sympy [F] . . . 2415
Maxima [F] . . . 2415
Giac [F] . . . 2415
Mupad [B] (verification not implemented) . . . 2415

## Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\operatorname{arctanh}\left(\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arccsch(a/c+b*x/c)/b+c*arctanh((1+1/(a/c+b*x/c)^2)^(1/2))/b
```

## Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5870, 6449, 379, 272, 65, 213}

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \frac{c\operatorname{arctanh}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b} + \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

```
[In] Int[ArcSinh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)^
(-2)]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6449

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsch[c + d*
x]/d), x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)\,dx \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}\,dx \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x}\,dx, x, \frac{a}{c}+\frac{bx}{c}\right)}{b} \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b}
\end{aligned}$$

$$= \frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}$$

$$= \frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\text{arctanh}\left(\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}$$

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. $2(49) = 98$.

Time $= 0.89$ (sec) , antiderivative size $= 357$, normalized size of antiderivative $= 7.29$

$$\int \text{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = x\text{arcsinh}\left(\frac{c}{a+bx}\right) + \frac{(a+bx)\sqrt{\frac{a^2+c^2+2abx+b^2x^2}{(a+bx)^2}}\left(a\text{arctanh}\left(\frac{bcx\left(\sqrt{a^2+c^2}-\sqrt{a^2+c^2+2abx+b^2x^2}\right)}{a\left(a^2+c^2+abx-\sqrt{a^2+c^2}\sqrt{a^2+c^2+2abx+b^2x^2}\right)}\right)+a\text{arctanh}\left(\frac{-a^4-a^3bx+b^2c^2x^2}{}\right.\right.}{}$$

```
[In] Integrate[ArcSinh[c/(a + b*x)],x]
```

```
[Out] x*ArcSinh[c/(a + b*x)] + ((a + b*x)*Sqrt[(a^2 + c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*(a*ArcTanh[(b*c*x*(Sqrt[a^2 + c^2] - Sqrt[a^2 + c^2 + 2*a*b*x +
b^2*x^2]))/(a*(a^2 + c^2 + a*b*x - Sqrt[a^2 + c^2]*Sqrt[a^2 + c^2 + 2*a*b*x
 + b^2*x^2]))] + a*ArcTanh[(-a^4 - a^3*b*x + b^2*c^2*x^2 + a^2*(-c^2 + Sqrt
[a^2 + c^2]*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]))/(b^2*c*Sqrt[a^2 + c^2]*x^
2)] + c*(Log[Sqrt[a^2 + c^2] - b*x - Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]] -
 Log[b^2*(Sqrt[a^2 + c^2] + b*x - Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])])))/
(b*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| derivativedivides | $-\frac{c\left(-\frac{(bx+a)\operatorname{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}+1}}\right)\right)}{b}$ |
| default | $-\frac{c\left(-\frac{(bx+a)\operatorname{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}+1}}\right)\right)}{b}$ |
| parts | $x\operatorname{arcsinh}\left(\frac{c}{bx+a}\right)+\frac{c\sqrt{b^2x^2+2abx+a^2+c^2}\left(\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2+c^2}\sqrt{b^2}+ab}{\sqrt{b^2}}\right)b\sqrt{c^2}+a\ln\left(\frac{2\left(\sqrt{c^2}\sqrt{b^2x^2+2abx}\right.\right.\right.}{bx+a}$ |

```
[In] int(arcsinh(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*c*(-1/c*(b*x+a)*arcsinh(c/(b*x+a))-arctanh(1/(c^2/(b*x+a)^2+1)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.94

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx$$

$$=\frac{bx\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)+a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a+c\right)-a\log\left(-bx+(bx+\right.}{b}$$

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x
 + a^2)) + c)/(b*x + a)) + a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
 a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a + c) - a*log(-b*x + (b*x + a)*sq
rt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a - c) - c*
log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - a))/b
```

## Sympy [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \int \operatorname{asinh}\left(\frac{c}{a+bx}\right)\,dx$$

```
[In] integrate(asinh(c/(b*x+a)),x)
[Out] Integral(asinh(c/(a + b*x)), x)
```

## Maxima [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(\frac{c}{bx+a}\right)\,dx$$

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="maxima")
[Out] -1/2*I*c*(log(I*(b^2*x + a*b)/(b*c) + 1) - log(-I*(b^2*x + a*b)/(b*c) + 1))
/b + 1/2*(2*b*x*log(c + sqrt(b^2*x^2 + 2*a*b*x + a^2 + c^2)) + a*log(b^2*x^
2 + 2*a*b*x + a^2 + c^2) - 2*(b*x + a)*log(b*x + a))/b + integrate((b^2*c*x
^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c + c^3 + (b^2*x^2 + 2*a*b*x + a
^2 + c^2)^(3/2)), x)
```

## Giac [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(\frac{c}{bx+a}\right)\,dx$$

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="giac")
[Out] integrate(arcsinh(c/(b*x + a)), x)
```

## Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \frac{c\,\operatorname{atanh}\left(\sqrt{\frac{c^2}{(a+b\,x)^2}+1}\right)}{b} + \frac{\operatorname{asinh}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b}$$

```
[In] int(asinh(c/(a + b*x)),x)
[Out] (c*atanh((c^2/(a + b*x)^2 + 1)^(1/2)))/b + (asinh(c/(a + b*x))*(a + b*x))/b
```

# 3.369 $\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx$

Optimal result . . . 2416
Rubi [F] . . . 2416
Mathematica [B] (warning: unable to verify) . . . 2417
Maple [F] . . . 2417
Fricas [A] (verification not implemented) . . . 2417
Sympy [F] . . . 2418
Maxima [A] (verification not implemented) . . . 2418
Giac [A] (verification not implemented) . . . 2418
Mupad [B] (verification not implemented) . . . 2418

## Optimal result

Integrand size = 7, antiderivative size = 27

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = \operatorname{arcsinh}(\sinh(x)) + \log(\operatorname{arcsinh}(\sinh(x)))\left(-\operatorname{arcsinh}(\sinh(x)) + x\sqrt{\cosh^2(x)}\operatorname{sech}(x)\right)$$

```
[Out] arcsinh(sinh(x))+ln(arcsinh(sinh(x)))*(-arcsinh(sinh(x))+x*sech(x)*(cosh(x)
^2)^(1/2))
```

## Rubi [F]

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = \int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx$$

```
[In] Int[x/ArcSinh[Sinh[x]],x]
```

```
[Out] Defer[Int][x/ArcSinh[Sinh[x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx$$

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 148 vs. $2(27) = 54$.

Time $= 0.22$ (sec) , antiderivative size $= 148$, normalized size of antiderivative $= 5.48$

$$\begin{aligned}
&\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx \\
&= -\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^{x}\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\left(-1\right. \\
&\qquad\left. +\log\left(\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^{x}\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\right)\right) \\
&\quad+\frac{e^{x}\sqrt{2+e^{-2x}+e^{2x}}\,x\log\left(\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^{x}\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\right)}{1+e^{2x}}
\end{aligned}$$

```
[In] Integrate[x/ArcSinh[Sinh[x]],x]
```

```
[Out] -(Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*x) + E^(2*x)])/(2*E^x)]*(-1 + Log[
Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*x) + E^(2*x)])/(2*E^x)]])) + (E^x*Sq
rt[2 + E^(-2*x) + E^(2*x)]*x*Log[Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*x)
+ E^(2*x)])/(2*E^x)]])/(1 + E^(2*x))
```

## Maple [F]

$$\int \frac{x}{\operatorname{arcsinh}\left(\sinh\left(x\right)\right)}dx$$

```
[In] int(x/arcsinh(sinh(x)),x)
```

```
[Out] int(x/arcsinh(sinh(x)),x)
```

## Fricas [A] (verification not implemented)

none

Time $= 0.25$ (sec) , antiderivative size $= 1$, normalized size of antiderivative $= 0.04$

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

## Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = \int \frac{x}{\operatorname{asinh}\left(\sinh\left(x\right)\right)}\,dx$$

```
[In] integrate(x/asinh(sinh(x)),x)
[Out] Integral(x/asinh(sinh(x)), x)
```

## Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="maxima")
[Out] x
```

## Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="giac")
[Out] x
```

## Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

```
[In] int(x/asinh(sinh(x)),x)
[Out] x
```

# 3.370 $\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$

Optimal result . . . 2419
Rubi [A] (verified) . . . 2419
Mathematica [A] (verified) . . . 2420
Maple [F] . . . 2420
Fricas [B] (verification not implemented) . . . 2420
Sympy [F] . . . 2421
Maxima [F] . . . 2421
Giac [F] . . . 2421
Mupad [F(-1)] . . . 2422

## Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \frac{\sqrt{bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

```
[Out] arcsinh((b*x^2-1)^(1/2))^(1+n)*(b*x^2)^(1/2)/b/(1+n)/x
```

## Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5871, 5783}

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \frac{\sqrt{bx^2}\operatorname{arcsinh}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

```
[In] Int[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5871

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{\text{arcsinh}(x)^n}{\sqrt{1+x^2}}\,dx, x, \sqrt{-1+bx^2}\right)}{bx} \\ &= \frac{\sqrt{bx^2}\,\text{arcsinh}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}\end{aligned}$$

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \frac{\sqrt{bx^2}\,\text{arcsinh}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

```
[In] Integrate[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

## Maple [F]

$$\int \frac{\text{arcsinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

```
[In] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
[Out] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

$$\int \frac{\text{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$$
$$= \frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\sinh}{(bn+b)x}$$

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="fricas")
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 - 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 - 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

## Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \begin{cases} -\frac{2x}{\pi} & \text{for } b = 0 \wedge n = -1 \\ -ix\left(\frac{i\pi}{2}\right)^n & \text{for } b = 0 \\ \int \frac{1}{\sqrt{bx^2-1}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx & \text{for } n = -1 \\ \frac{\sqrt{bx^2}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)\operatorname{asinh}^n\left(\sqrt{bx^2-1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh((b*x**2-1)**(1/2))**n/(b*x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-2*x/pi, Eq(b, 0) & Eq(n, -1)), (-I*x*(I*pi/2)**n, Eq(b, 0)), (I
ntegral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x), Eq(n, -1)), (sqrt
(b*x**2)*asinh(sqrt(b*x**2 - 1))*asinh(sqrt(b*x**2 - 1))**n/(b*n*x + b*x),
True))
```

## Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

## Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

```
[In] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2),x)
[Out] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2), x)
```

# 3.371 $\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx$

Optimal result . . . 2423
Rubi [A] (verified) . . . 2423
Mathematica [A] (verified) . . . 2424
Maple [F] . . . 2424
Fricas [A] (verification not implemented) . . . 2424
Sympy [F] . . . 2425
Maxima [F] . . . 2425
Giac [F] . . . 2425
Mupad [B] (verification not implemented) . . . 2425

## Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\sqrt{bx^2}\log\left(\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)\right)}{bx}$$

```
[Out] ln(arcsinh((b*x^2-1)^(1/2)))*(b*x^2)^(1/2)/b/x
```

## Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5871, 5782}

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\sqrt{bx^2}\log\left(\operatorname{arcsinh}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

```
[In] Int[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5871

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
```

```
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{bx^2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx, x, \sqrt{-1+bx^2}\right)}{bx}$$
$$= \frac{\sqrt{bx^2}\log\left(\text{arcsinh}\left(\sqrt{-1+bx^2}\right)\right)}{bx}$$

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{x\log\left(\text{arcsinh}\left(\sqrt{-1+bx^2}\right)\right)}{\sqrt{bx^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
[Out] (x*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/Sqrt[b*x^2]
```

## Maple [F]

$$\int \frac{1}{\text{arcsinh}\left(\sqrt{b\,x^2-1}\right)\sqrt{b\,x^2-1}}dx$$

```
[In] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
[Out] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

## Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)}{bx}$$

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))/(b*x)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\,\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

```
[In] integrate(1/asinh((b*x**2-1)**(1/2))/(b*x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\,\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\,\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

## Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\ln\left(\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)\right)\,\sqrt{x^2}}{\sqrt{b}\,x}$$

```
[In] int(1/(asinh((b*x^2 - 1)^(1/2))*(b*x^2 - 1)^(1/2)),x)
[Out] (log(asinh((b*x^2 - 1)^(1/2)))*(x^2)^(1/2))/(b^(1/2)*x)
```

# CHAPTER 4

# APPENDIX

4.1 Listing of Grading functions . . . . . 2427

## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])   #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
         #is checked before calling the grading function that is passed.
         #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```