Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/197-7.3.7-Inverse-hyperbolic-tangent-functions

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 7:11am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [361]. This is test number [197].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (361)	0.00 (0)
Mathematica	99.72 (360)	0.28 (1)
Fricas	96.95 (350)	3.05 (11)
Maple	94.74 (342)	5.26 (19)
Maxima	74.24 (268)	25.76 (93)
Giac	70.36 (254)	29.64 (107)
Mupad	66.20 (239)	33.80 (122)
Sympy	27.15 (98)	72.85 (263)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

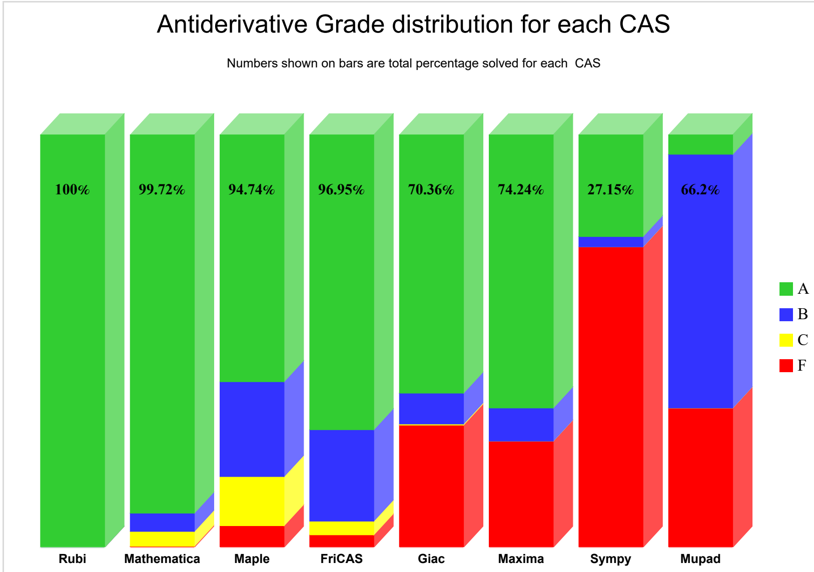
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

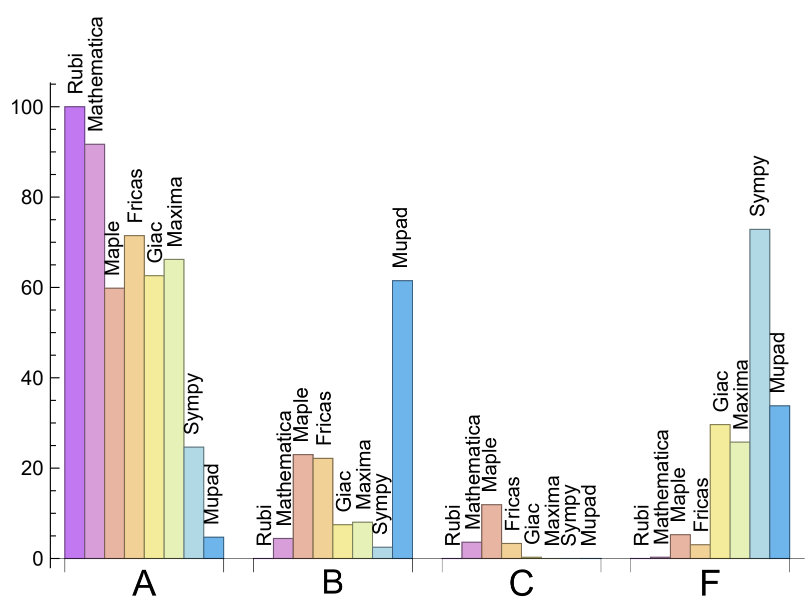
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	91.69	4.43	3.60	0.28
Fricas	71.47	22.16	3.32	3.05
Maxima	66.20	8.03	0.00	25.76
Giac	62.60	7.48	0.28	29.64
Maple	59.83	22.99	11.91	5.26
Sympy	24.65	2.49	0.00	72.85
Mupad	N/A	61.50	0.00	33.80

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	19	100.00 %	0.00 %	0.00 %
Fricas	11	100.00 %	0.00 %	0.00 %
Giac	107	79.44 %	13.08 %	7.48 %
Maxima	93	100.00 %	0.00 %	0.00 %
Sympy	263	82.89 %	7.22 %	9.89 %
Mupad	122	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

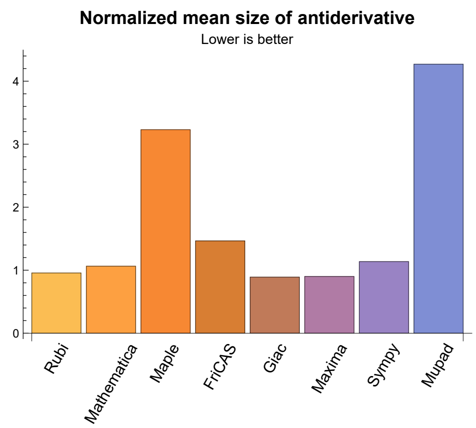
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

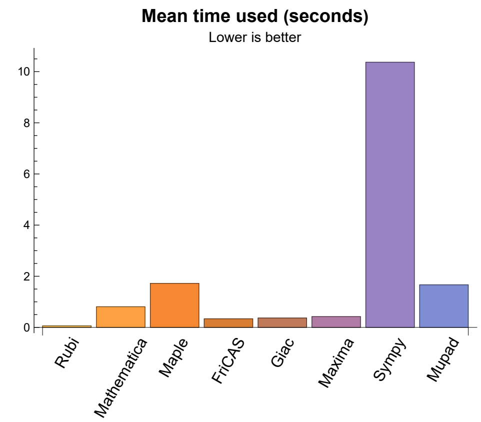
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	95.79	0.96	80.00	1.00
Mathematica	0.81	113.42	1.06	73.50	0.88
Maple	1.72	479.97	3.23	108.00	1.16
Maxima	0.42	71.51	0.90	53.00	0.83
Fricas	0.34	182.29	1.47	81.00	1.01
Sympy	10.37	43.87	1.14	41.00	1.01
Giac	0.37	58.62	0.89	45.50	0.57
Mupad	1.67	365.23	4.27	211.00	3.17

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{319, 336\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is ValueError. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima abs_integrate was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

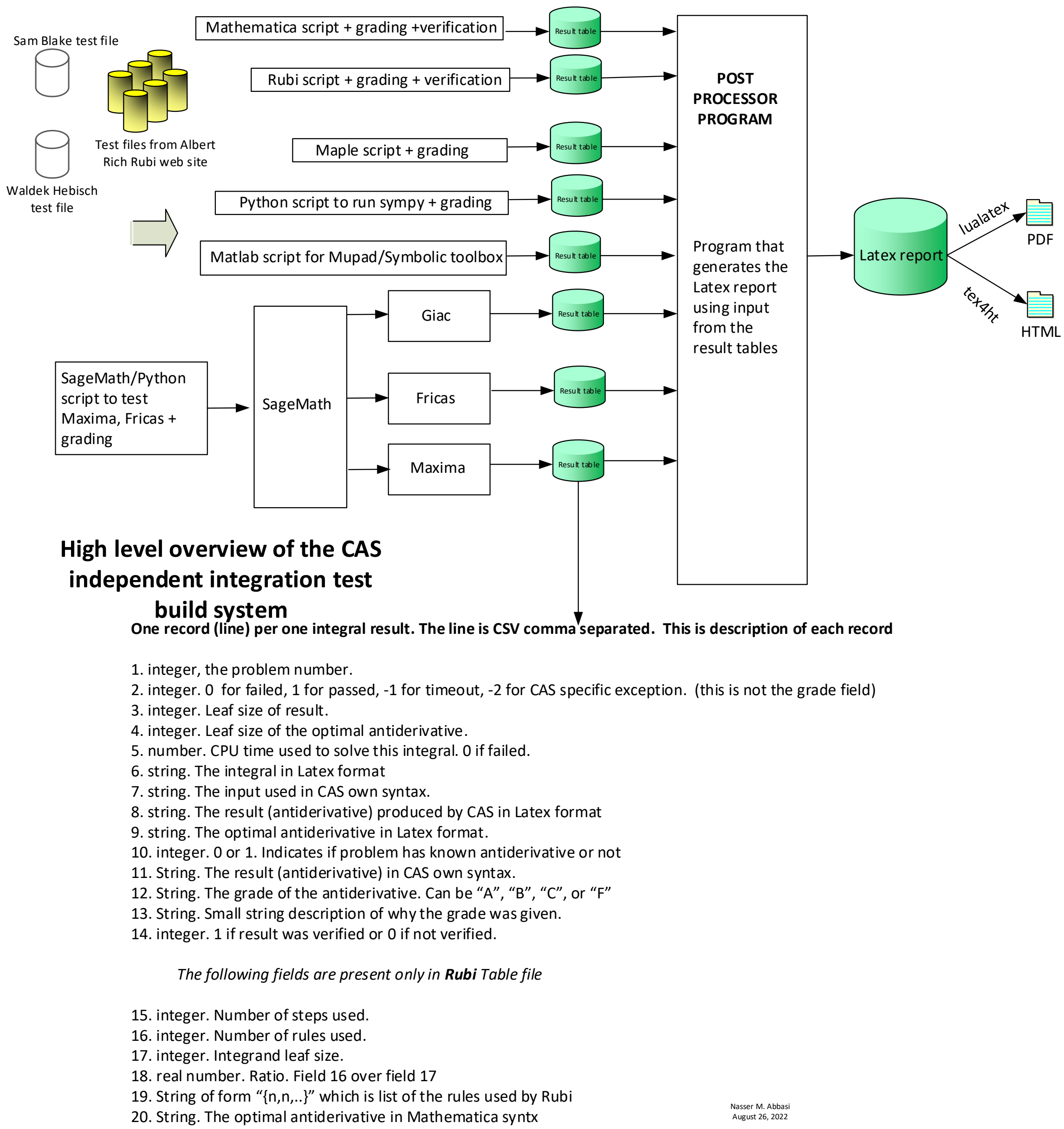
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 301, 302, 303, 304, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360 }

B grade: { 47, 57, 71, 78, 84, 286, 291, 296, 300, 305, 310, 312, 319, 329, 336, 346 }

C grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 361 }

F grade: { 4 }

2.1.3 Maple

A grade: { 4, 6, 7, 8, 13, 14, 28, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 75, 76, 77, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 217, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 261, 262, 263, 264, 270, 275, 276, 277, 278, 279, 280, 281, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 346, 347, 348, 349, 352, 357 }

B grade: { 1, 2, 3, 5, 9, 10, 11, 12, 15, 29, 31, 32, 59, 71, 73, 74, 78, 84, 86, 87, 94, 95, 96, 103, 104, 105, 134, 149, 150, 151, 158, 159, 191, 192, 193, 194, 199, 200, 201, 207, 208, 215, 216, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 249, 250, 257, 258, 260, 266, 267, 268, 269, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 340, 344, 350, 351, 353, 354 }

C grade: { 274, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 355, 356, 358, 359, 360, 361 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 85, 93, 102, 265, 271, 272, 273 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 9, 10, 28, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 254, 255, 256, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 316, 320, 324, 328, 333, 337, 341, 345, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359 }

B grade: { 11, 12, 48, 58, 71, 72, 78, 84, 315, 319, 321, 322, 323, 325, 326, 327, 332, 336, 338, 339, 340, 342, 343, 344, 346, 347, 349, 355, 360 }

C grade: { }

F grade: { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 154, 155, 156, 157, 163, 164, 165, 166, 215, 216, 217, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 253, 257, 258, 259, 260, 261, 265, 271, 272, 273, 312, 313, 314, 317, 318, 329, 330, 331, 334, 335, 361 }

2.1.5 FriCAS

A grade: { 28, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 274, 275, 276, 277, 278, 279, 280, 283, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 338, 339, 340, 341, 342, 343, 344, 345, 352, 354, 356, 357, 358, 359 }

B grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 29, 44, 54, 58, 65, 72, 84, 133, 162, 266, 267, 268, 281, 282, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 346, 347, 348, 349, 350, 351, 353, 355, 360, 361 }

C grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27 }

F grade: { 4, 31, 32, 33, 85, 93, 102, 265, 271, 272, 273 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 10, 11, 12, 28, 30, 34, 35, 37, 39, 41, 42, 43, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 62, 64, 66, 67, 68, 69, 70, 71, 72, 77, 79, 80, 81, 82, 83, 84, 89, 97, 98, 105, 106, 107, 115, 123, 124, 130, 131, 132, 133, 144, 152, 153, 159, 160, 161, 162, 168, 169, 172, 173, 174, 176, 181, 182, 184, 189, 190, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 357 }

B grade: { 38, 63, 78, 270, 275, 276, 277, 279, 280 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 32, 33, 36, 40, 44, 49, 50, 54, 59, 60, 61, 65, 73, 74, 75, 76, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 170, 171, 175, 177, 178, 179, 180, 183, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 278, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361 }

2.1.7 Giac

A grade: { 12, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 355, 357, 358, 360 }

B grade: { 28, 29, 58, 72, 84, 113, 114, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 219, 266, 274, 275, 276, 277, 279, 280, 356, 359 }

C grade: { 278 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 44, 54, 65, 85, 93, 102, 225, 226, 227, 233, 234, 235, 236, 265, 271, 272, 273, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

2.1.8 Mupad

A grade: { 30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345 }

B grade: { 12, 28, 29, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 253, 254, 255, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 355, 356, 357, 358, 359, 360 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 215, 216, 217, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 257, 258, 259, 265, 271, 272, 273, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	99	253	0	380	121	0	-1
N.S.	1	1.00	0.78	1.99	0.00	2.99	0.95	0.00	-0.01
time (sec)	N/A	0.036	0.061	0.037	0.000	0.349	1.641	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	205	0	263	95	0	-1
N.S.	1	1.00	0.87	2.03	0.00	2.60	0.94	0.00	-0.01
time (sec)	N/A	0.026	0.038	0.009	0.000	0.343	0.607	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	76	157	0	168	66	0	-1
N.S.	1	1.00	1.01	2.09	0.00	2.24	0.88	0.00	-0.01
time (sec)	N/A	0.017	0.024	0.007	0.000	0.352	0.329	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	209	0	0	0	0	-1
N.S.	1	1.00	0.00	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.104	0.030	0.113	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	111	50	129	0	0	-1
N.S.	1	1.00	0.94	2.09	0.94	2.43	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.023	0.007	0.287	0.355	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	62	61	176	0	0	-1
N.S.	1	1.00	0.80	0.78	0.77	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.030	0.007	0.277	0.353	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	74	110	102	246	0	0	-1
N.S.	1	1.00	0.70	1.05	0.97	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.033	0.007	0.260	0.356	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	158	123	340	0	0	-1
N.S.	1	1.00	0.65	1.21	0.94	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.040	0.010	0.285	0.363	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	79	224	162	442	116	0	-1
N.S.	1	1.00	0.69	1.96	1.42	3.88	1.02	0.00	-0.01
time (sec)	N/A	0.043	0.039	0.011	0.260	0.364	1.901	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	176	132	315	90	0	-1
N.S.	1	1.00	0.75	1.93	1.45	3.46	0.99	0.00	-0.01
time (sec)	N/A	0.035	0.039	0.010	0.281	0.356	0.806	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	128	102	209	65	0	-1
N.S.	1	1.00	0.82	1.88	1.50	3.07	0.96	0.00	-0.01
time (sec)	N/A	0.026	0.034	0.008	0.265	0.366	0.428	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	76	66	129	36	59	32
N.S.	1	1.00	1.00	1.90	1.65	3.22	0.90	1.48	0.80
time (sec)	N/A	0.006	0.007	0.005	0.279	0.346	0.330	0.416	1.057

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	84	0	407	0	0	-1
N.S.	1	1.00	1.11	1.53	0.00	7.40	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.030	0.006	0.000	0.371	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	123	0	571	0	0	-1
N.S.	1	1.00	1.08	1.45	0.00	6.72	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.058	0.007	0.000	0.380	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	107	171	0	727	0	0	-1
N.S.	1	1.00	0.96	1.54	0.00	6.55	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.077	0.007	0.000	0.388	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	161	0	0	402	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.332	0.231	0.000	0.106	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	147	0	0	288	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.282	0.217	0.000	0.107	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	188	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.203	0.240	0.000	0.098	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	112	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.083	0.178	0.000	0.102	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	142	0	0	187	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.246	0.162	0.000	0.090	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	154	0	0	260	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.205	0.289	0.000	0.099	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	163	0	0	355	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.282	0.293	0.000	0.100	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	124	0	0	312	0	0	-1
N.S.	1	1.00	0.42	0.00	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.086	0.307	0.000	0.109	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	109	0	0	220	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.063	0.306	0.000	0.094	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	85	0	0	128	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.55	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.072	0.296	0.000	0.098	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	118	0	0	208	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.113	0.106	0.290	0.000	0.091	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	131	0	0	281	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.068	0.289	0.000	0.091	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	39	37	59	60	223	90
N.S.	1	1.00	0.89	0.89	0.84	1.34	1.36	5.07	2.05
time (sec)	N/A	0.035	0.015	0.043	0.248	0.366	0.982	0.407	0.316

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	121	40	109	0	124	56
N.S.	1	1.00	0.89	2.57	0.85	2.32	0.00	2.64	1.19
time (sec)	N/A	0.035	0.027	0.050	0.258	0.354	0.000	0.405	1.466

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.066	0.111	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	482	1445	0	0	0	0	-1
N.S.	1	1.00	1.18	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.122	0.314	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	324	674	0	0	0	0	-1
N.S.	1	1.00	1.21	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.074	0.016	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	43	117	0	0	0	0	-1
N.S.	1	1.00	0.48	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.189	0.016	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.069	0.112	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.531	0.112	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	62	0	43	96
N.S.	1	1.00	0.92	1.11	1.03	1.68	0.00	1.16	2.59
time (sec)	N/A	0.023	0.043	0.189	0.261	0.346	0.000	0.402	1.493

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	19	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	0.83	0.57	0.83
time (sec)	N/A	0.006	0.013	0.051	0.298	0.340	0.115	0.393	1.005

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	39	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	1.70	0.57	0.83
time (sec)	N/A	0.005	0.011	0.049	0.289	0.340	0.105	0.395	0.976

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	10	19	10	16
N.S.	1	1.00	1.12	0.94	1.00	0.62	1.19	0.62	1.00
time (sec)	N/A	0.002	0.006	0.100	0.291	0.330	0.058	0.391	0.046

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	34	8	0	9	58
N.S.	1	1.00	0.90	1.10	1.62	0.38	0.00	0.43	2.76
time (sec)	N/A	0.025	0.011	0.056	0.257	0.338	0.000	0.386	1.101

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	13	14	12	17
N.S.	1	1.00	1.06	1.06	1.00	0.76	0.82	0.71	1.00
time (sec)	N/A	0.010	0.013	0.068	0.295	0.355	0.084	0.383	0.091

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	19	11	16
N.S.	1	1.00	0.78	0.87	0.83	0.48	0.83	0.48	0.70
time (sec)	N/A	0.006	0.012	0.065	0.304	0.327	0.192	0.383	0.951

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	20	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	0.87	0.57	0.83
time (sec)	N/A	0.006	0.012	0.060	0.298	0.318	0.250	0.388	0.066

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	98	73	161	0	0	203
N.S.	1	1.00	0.87	1.38	1.03	2.27	0.00	0.00	2.86
time (sec)	N/A	0.024	0.083	0.693	0.303	0.359	0.000	0.000	1.126

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	36	24	37	24	36
N.S.	1	1.00	0.88	0.90	0.86	0.57	0.88	0.57	0.86
time (sec)	N/A	0.017	0.021	35.963	0.344	0.337	0.261	0.396	0.998

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	36	24	37	24	36
N.S.	1	1.00	0.88	0.90	0.86	0.57	0.88	0.57	0.86
time (sec)	N/A	0.016	0.036	30.386	0.335	0.324	0.171	0.402	0.975

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	74	38	36	24	41	24	36
N.S.	1	1.00	2.18	1.12	1.06	0.71	1.21	0.71	1.06
time (sec)	N/A	0.035	0.041	30.592	0.328	0.326	0.148	0.408	0.938

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	20	20	20	33
N.S.	1	1.00	1.00	0.94	2.06	1.25	1.25	1.25	2.06
time (sec)	N/A	0.005	0.008	0.207	0.340	0.328	0.090	0.399	0.073

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	78	20	20	0	21	183
N.S.	1	1.00	1.08	1.59	0.41	0.41	0.00	0.43	3.73
time (sec)	N/A	0.056	0.035	0.135	0.662	0.332	0.000	0.401	0.288

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	54	24	0	21	198
N.S.	1	1.00	0.95	1.05	1.38	0.62	0.00	0.54	5.08
time (sec)	N/A	0.043	0.035	0.098	0.316	0.344	0.000	0.392	0.187

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	35	34	26	32	22	34
N.S.	1	1.00	1.17	0.97	0.94	0.72	0.89	0.61	0.94
time (sec)	N/A	0.026	0.024	0.215	0.343	0.329	0.198	0.391	0.932

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	38	36	22	37	22	32
N.S.	1	1.00	1.10	1.23	1.16	0.71	1.19	0.71	1.03
time (sec)	N/A	0.014	0.033	0.201	0.334	0.319	0.267	0.392	0.944

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	64	37	38	36	24	39	24	36
N.S.	1	1.52	0.88	0.90	0.86	0.57	0.93	0.57	0.86
time (sec)	N/A	0.038	0.022	0.214	0.347	0.348	0.385	0.384	0.955

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	177	109	300	0	0	332
N.S.	1	1.00	0.88	1.61	0.99	2.73	0.00	0.00	3.02
time (sec)	N/A	0.064	0.097	6.039	0.348	0.374	0.000	0.000	1.233

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	56	54	35	58	35	53
N.S.	1	1.00	0.89	0.92	0.89	0.57	0.95	0.57	0.87
time (sec)	N/A	0.032	0.019	0.025	0.376	0.324	0.574	0.384	1.048

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	56	54	35	56	35	53
N.S.	1	1.00	1.02	1.06	1.02	0.66	1.06	0.66	1.00
time (sec)	N/A	0.026	0.023	0.023	0.374	0.328	0.443	0.384	0.141

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	99	56	54	34	41	34	53
N.S.	1	1.00	2.91	1.65	1.59	1.00	1.21	1.00	1.56
time (sec)	N/A	0.010	0.046	29.553	0.370	0.328	0.233	0.383	0.977

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	31	20	31	47
N.S.	1	1.00	1.00	0.94	3.19	1.94	1.25	1.94	2.94
time (sec)	N/A	0.003	0.007	30.429	0.382	0.318	0.126	0.385	0.098

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	104	188	31	31	0	32	306
N.S.	1	1.00	1.35	2.44	0.40	0.40	0.00	0.42	3.97
time (sec)	N/A	0.063	0.046	0.427	0.657	0.337	0.000	0.366	0.142

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	96	65	36	0	33	415
N.S.	1	1.00	0.91	1.41	0.96	0.53	0.00	0.49	6.10
time (sec)	N/A	0.030	0.032	0.244	0.694	0.334	0.000	0.383	1.071

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	59	72	37	0	31	365
N.S.	1	1.00	1.10	0.98	1.20	0.62	0.00	0.52	6.08
time (sec)	N/A	0.028	0.033	0.272	0.348	0.324	0.000	0.400	0.205

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	52	52	37	51	35	51
N.S.	1	1.00	1.09	0.95	0.95	0.67	0.93	0.64	0.93
time (sec)	N/A	0.027	0.018	0.932	0.372	0.333	0.287	0.387	1.004

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	56	53	33	56	33	48
N.S.	1	1.00	1.61	1.81	1.71	1.06	1.81	1.06	1.55
time (sec)	N/A	0.009	0.019	0.954	0.388	0.338	0.359	0.389	0.969

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	56	54	35	60	35	53
N.S.	1	1.00	0.84	0.88	0.84	0.55	0.94	0.55	0.83
time (sec)	N/A	0.022	0.024	0.908	0.383	0.312	0.551	0.387	0.991

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	137	278	145	483	0	0	479
N.S.	1	1.00	0.89	1.81	0.94	3.14	0.00	0.00	3.11
time (sec)	N/A	0.068	0.113	33.391	0.386	0.350	0.000	0.000	1.341

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	75	46	242
N.S.	1	1.00	0.89	0.92	0.90	0.58	0.94	0.58	3.02
time (sec)	N/A	0.045	0.040	0.026	0.402	0.340	2.491	0.383	1.032

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	76	46	242
N.S.	1	1.00	0.89	0.92	0.90	0.58	0.95	0.58	3.02
time (sec)	N/A	0.043	0.024	0.026	0.414	0.323	1.571	0.377	0.151

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	78	46	77
N.S.	1	1.00	0.89	0.92	0.90	0.58	0.98	0.58	0.96
time (sec)	N/A	0.040	0.024	0.026	0.421	0.365	1.040	0.396	1.055

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	74	72	45	75	45	70
N.S.	1	1.00	0.99	1.03	1.00	0.62	1.04	0.62	0.97
time (sec)	N/A	0.037	0.018	0.028	0.415	0.378	0.683	0.390	1.020

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	71	74	72	45	75	45	70
N.S.	1	1.00	1.34	1.40	1.36	0.85	1.42	0.85	1.32
time (sec)	N/A	0.021	0.039	0.015	0.425	0.330	0.423	0.393	0.153

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	125	74	72	46	41	46	70
N.S.	1	1.00	3.68	2.18	2.12	1.35	1.21	1.35	2.06
time (sec)	N/A	0.010	0.056	31.814	0.416	0.325	0.414	0.392	1.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	42	20	42	67
N.S.	1	1.00	1.00	0.94	4.31	2.62	1.25	2.62	4.19
time (sec)	N/A	0.004	0.007	34.815	0.415	0.346	0.287	0.383	0.979

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	175	358	42	42	0	43	423
N.S.	1	1.00	1.67	3.41	0.40	0.40	0.00	0.41	4.03
time (sec)	N/A	0.055	0.073	1.200	0.667	0.335	0.000	0.377	0.135

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	206	77	47	0	44	553
N.S.	1	1.00	0.89	2.17	0.81	0.49	0.00	0.46	5.82
time (sec)	N/A	0.046	0.057	0.703	0.705	0.329	0.000	0.380	0.144

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	114	83	47	0	43	672
N.S.	1	1.00	0.93	1.31	0.95	0.54	0.00	0.49	7.72
time (sec)	N/A	0.043	0.027	0.668	0.736	0.343	0.000	0.384	1.537

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	82	77	91	48	0	42	571
N.S.	1	1.00	1.06	1.00	1.18	0.62	0.00	0.55	7.42
time (sec)	N/A	0.041	0.034	1.000	0.381	0.326	0.000	0.389	1.336

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	69	72	48	70	46	68
N.S.	1	1.00	1.05	0.93	0.97	0.65	0.95	0.62	0.92
time (sec)	N/A	0.039	0.026	3.361	0.425	0.314	0.362	0.394	1.135

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	66	74	70	44	75	44	64
N.S.	1	1.00	2.13	2.39	2.26	1.42	2.42	1.42	2.06
time (sec)	N/A	0.010	0.038	2.996	0.423	0.322	0.515	0.400	1.196

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	74	72	46	78	46	70
N.S.	1	1.00	1.11	1.16	1.12	0.72	1.22	0.72	1.09
time (sec)	N/A	0.022	0.024	3.063	0.430	0.328	0.796	0.391	1.039

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	71	74	72	46	80	46	70
N.S.	1	1.00	0.72	0.76	0.73	0.47	0.82	0.47	0.71
time (sec)	N/A	0.037	0.025	3.009	0.426	0.312	1.164	0.391	1.005

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	132	71	74	72	46	76	46	70
N.S.	1	1.65	0.89	0.92	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.053	0.025	3.069	0.423	0.316	1.556	0.395	1.072

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	76	46	70
N.S.	1	1.00	0.89	0.92	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.039	0.044	3.072	0.428	0.331	2.239	0.387	1.105

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	78	46	70
N.S.	1	1.00	0.89	0.92	0.90	0.58	0.98	0.58	0.88
time (sec)	N/A	0.037	0.025	3.096	0.420	0.333	3.534	0.379	1.039

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	177	110	110	68	41	68	104
N.S.	1	1.00	5.21	3.24	3.24	2.00	1.21	2.00	3.06
time (sec)	N/A	0.010	0.076	51.618	0.517	0.343	1.241	0.380	1.100

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.060	0.019	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	202	42	41	0	43	354
N.S.	1	1.00	0.98	2.49	0.52	0.51	0.00	0.53	4.37
time (sec)	N/A	0.044	0.032	2.387	0.547	0.421	0.000	0.406	0.129

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	111	29	29	0	30	234
N.S.	1	1.00	0.98	1.98	0.52	0.52	0.00	0.54	4.18
time (sec)	N/A	0.024	0.030	0.648	0.550	0.356	0.000	0.378	0.273

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	49	18	17	0	19	108
N.S.	1	1.00	1.00	1.58	0.58	0.55	0.00	0.61	3.48
time (sec)	N/A	0.011	0.024	0.187	0.528	0.338	0.000	0.398	0.154

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	10	17	11	12
N.S.	1	1.00	1.00	1.08	1.08	0.83	1.42	0.92	1.00
time (sec)	N/A	0.003	0.035	0.065	0.465	0.342	12.130	0.402	1.060

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	43	18	16	0	20	107
N.S.	1	1.00	0.66	0.98	0.41	0.36	0.00	0.45	2.43
time (sec)	N/A	0.020	0.019	7.723	0.520	0.336	0.000	0.387	2.870

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	64	28	26	0	30	210
N.S.	1	1.00	0.69	0.98	0.43	0.40	0.00	0.46	3.23
time (sec)	N/A	0.031	0.023	0.021	0.534	0.339	0.000	0.387	2.832

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	87	40	41	0	45	286
N.S.	1	1.00	0.72	0.95	0.43	0.45	0.00	0.49	3.11
time (sec)	N/A	0.049	0.025	0.020	0.529	0.336	0.000	0.395	2.893

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.485	0.024	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	350	70	73	0	62	669
N.S.	1	1.00	1.08	3.57	0.71	0.74	0.00	0.63	6.83
time (sec)	N/A	0.061	0.064	2.874	0.705	0.324	0.000	0.390	0.177

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	83	223	59	62	0	48	490
N.S.	1	1.00	1.11	2.97	0.79	0.83	0.00	0.64	6.53
time (sec)	N/A	0.038	0.037	0.808	0.698	0.341	0.000	0.376	1.033

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	127	44	47	0	34	302
N.S.	1	1.00	1.12	2.54	0.88	0.94	0.00	0.68	6.04
time (sec)	N/A	0.024	0.050	0.236	0.684	0.341	0.000	0.391	1.055

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	56	26	28	36	24	28
N.S.	1	1.00	0.96	2.00	0.93	1.00	1.29	0.86	1.00
time (sec)	N/A	0.011	0.035	0.086	0.667	0.342	25.242	0.389	0.078

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	13	20	12	14
N.S.	1	1.00	1.00	1.07	0.86	0.93	1.43	0.86	1.00
time (sec)	N/A	0.003	0.007	0.060	0.469	0.316	24.854	0.376	0.081

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	67	28	39	0	31	359
N.S.	1	1.00	0.76	0.96	0.40	0.56	0.00	0.44	5.13
time (sec)	N/A	0.032	0.050	0.025	0.639	0.347	0.000	0.377	3.390

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	91	45	63	0	45	432
N.S.	1	1.00	0.69	0.89	0.44	0.62	0.00	0.44	4.24
time (sec)	N/A	0.045	0.044	0.027	0.647	0.355	0.000	0.380	3.444

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	116	64	86	0	64	660
N.S.	1	1.00	0.64	0.81	0.45	0.60	0.00	0.45	4.62
time (sec)	N/A	0.064	0.038	0.027	0.652	0.337	0.000	0.381	3.716

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.394	0.040	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	371	81	95	0	61	867
N.S.	1	1.00	1.24	4.03	0.88	1.03	0.00	0.66	9.42
time (sec)	N/A	0.094	0.029	0.833	0.854	0.335	0.000	0.387	1.336

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	239	69	83	0	44	620
N.S.	1	1.00	1.21	3.37	0.97	1.17	0.00	0.62	8.73
time (sec)	N/A	0.082	0.033	0.256	0.873	0.330	0.000	0.383	1.473

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	136	48	61	54	37	46
N.S.	1	1.00	1.04	2.89	1.02	1.30	1.15	0.79	0.98
time (sec)	N/A	0.039	0.027	0.107	0.851	0.381	39.214	0.384	1.028

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	43	32	32	42	18	25
N.S.	1	1.00	0.79	1.26	0.94	0.94	1.24	0.53	0.74
time (sec)	N/A	0.019	0.032	0.084	0.845	0.325	37.239	0.385	0.082

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	24	24	12	14
N.S.	1	1.00	1.00	0.94	0.75	1.50	1.50	0.75	0.88
time (sec)	N/A	0.005	0.007	0.059	0.475	0.345	25.471	0.389	0.059

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	92	51	80	0	43	645
N.S.	1	1.00	0.76	0.95	0.53	0.82	0.00	0.44	6.65
time (sec)	N/A	0.100	0.062	0.023	0.863	0.347	0.000	0.391	3.661

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	93	117	69	109	0	60	804
N.S.	1	1.00	0.71	0.89	0.53	0.83	0.00	0.46	6.14
time (sec)	N/A	0.113	0.037	0.027	0.863	0.368	0.000	0.392	3.665

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	145	86	130	0	73	909
N.S.	1	1.00	0.63	0.85	0.51	0.76	0.00	0.43	5.35
time (sec)	N/A	0.098	0.036	0.031	0.842	0.342	0.000	0.404	2.728

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	64	0	150	811
N.S.	1	1.00	0.82	1.52	0.63	0.63	0.00	1.49	8.03
time (sec)	N/A	0.048	0.024	0.090	0.532	0.325	0.000	0.402	1.052

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	53	0	125	648
N.S.	1	1.00	0.82	1.55	0.66	0.66	0.00	1.56	8.10
time (sec)	N/A	0.035	0.025	0.080	0.524	0.341	0.000	0.388	0.990

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	42	0	102	485
N.S.	1	1.00	0.83	1.17	0.71	0.71	0.00	1.73	8.22
time (sec)	N/A	0.022	0.026	0.085	0.538	0.337	0.000	0.395	0.992

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	30	30	0	75	151
N.S.	1	1.00	0.84	1.11	0.79	0.79	0.00	1.97	3.97
time (sec)	N/A	0.011	0.033	0.082	0.523	0.349	0.000	0.394	1.038

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	12	26	18	95
N.S.	1	1.00	1.00	0.83	0.67	0.67	1.44	1.00	5.28
time (sec)	N/A	0.004	0.009	0.065	0.537	0.347	0.138	0.409	1.121

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	54	0	73	0	40	308
N.S.	1	1.00	0.97	0.86	0.00	1.16	0.00	0.63	4.89
time (sec)	N/A	0.044	0.061	0.064	0.000	0.345	0.000	0.384	2.364

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	63	0	93	0	51	341
N.S.	1	1.00	0.98	0.95	0.00	1.41	0.00	0.77	5.17
time (sec)	N/A	0.022	0.031	0.068	0.000	0.335	0.000	0.396	6.975

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	89	92	0	119	0	75	741
N.S.	1	1.00	0.71	0.74	0.00	0.95	0.00	0.60	5.93
time (sec)	N/A	0.052	0.077	0.069	0.000	0.342	0.000	0.392	5.739

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	115	185	0	145	0	93	964
N.S.	1	1.00	0.64	1.03	0.00	0.81	0.00	0.52	5.39
time (sec)	N/A	0.084	0.072	0.069	0.000	0.360	0.000	0.395	5.513

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	75	0	241	1813
N.S.	1	1.00	0.82	1.52	0.63	0.74	0.00	2.39	17.95
time (sec)	N/A	0.047	0.025	0.079	0.523	0.330	0.000	0.393	1.186

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	64	0	205	1483
N.S.	1	1.00	0.82	1.55	0.66	0.80	0.00	2.56	18.54
time (sec)	N/A	0.035	0.039	0.080	0.527	0.333	0.000	0.388	1.124

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	53	0	168	1153
N.S.	1	1.00	0.83	1.17	0.71	0.90	0.00	2.85	19.54
time (sec)	N/A	0.023	0.053	0.080	0.536	0.345	0.000	0.396	1.119

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	41	49	131	823
N.S.	1	1.00	0.84	1.11	0.82	1.08	1.29	3.45	21.66
time (sec)	N/A	0.014	0.034	0.081	0.536	0.344	2.445	0.403	1.081

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	28	26	84	97
N.S.	1	1.00	1.00	0.83	0.67	1.56	1.44	4.67	5.39
time (sec)	N/A	0.004	0.008	0.066	0.517	0.329	1.186	0.406	1.162

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	80	131	0	88	0	57	501
N.S.	1	1.00	0.88	1.44	0.00	0.97	0.00	0.63	5.51
time (sec)	N/A	0.036	0.054	0.063	0.000	0.347	0.000	0.392	5.999

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	85	0	102	0	69	459
N.S.	1	1.00	0.98	1.05	0.00	1.26	0.00	0.85	5.67
time (sec)	N/A	0.034	0.031	0.069	0.000	0.341	0.000	0.394	2.217

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	91	0	124	0	73	609
N.S.	1	1.00	0.96	0.99	0.00	1.35	0.00	0.79	6.62
time (sec)	N/A	0.035	0.044	0.069	0.000	0.356	0.000	0.395	6.032

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	117	116	0	145	0	93	1019
N.S.	1	1.00	0.80	0.79	0.00	0.99	0.00	0.64	6.98
time (sec)	N/A	0.068	0.062	0.071	0.000	0.378	0.000	0.412	5.419

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	86	0	344	2500
N.S.	1	1.00	0.82	1.52	0.63	0.85	0.00	3.41	24.75
time (sec)	N/A	0.052	0.026	0.081	0.543	0.333	0.000	0.401	1.280

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	75	94	296	2235
N.S.	1	1.00	0.82	1.55	0.66	0.94	1.18	3.70	27.94
time (sec)	N/A	0.053	0.027	0.081	0.544	0.324	113.235	0.394	1.152

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	64	71	248	1789
N.S.	1	1.00	0.83	1.17	0.71	1.08	1.20	4.20	30.32
time (sec)	N/A	0.038	0.025	0.081	0.538	0.339	62.073	0.387	1.139

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	52	49	197	773
N.S.	1	1.00	0.84	1.11	0.82	1.37	1.29	5.18	20.34
time (sec)	N/A	0.014	0.036	0.077	0.540	0.336	33.862	0.410	1.105

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	39	26	136	337
N.S.	1	1.00	1.00	0.83	0.67	2.17	1.44	7.56	18.72
time (sec)	N/A	0.004	0.009	0.065	0.509	0.338	17.679	0.405	1.120

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	99	222	0	114	0	73	789
N.S.	1	1.00	0.82	1.83	0.00	0.94	0.00	0.60	6.52
time (sec)	N/A	0.057	0.056	0.065	0.000	0.331	0.000	0.391	4.816

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	106	193	0	126	0	89	616
N.S.	1	1.00	0.96	1.75	0.00	1.15	0.00	0.81	5.60
time (sec)	N/A	0.052	0.044	0.069	0.000	0.344	0.000	0.397	4.967

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	142	0	133	0	92	614
N.S.	1	1.00	0.98	1.29	0.00	1.21	0.00	0.84	5.58
time (sec)	N/A	0.049	0.037	0.072	0.000	0.349	0.000	0.423	2.066

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	144	0	146	0	88	669
N.S.	1	1.00	0.95	1.27	0.00	1.29	0.00	0.78	5.92
time (sec)	N/A	0.050	0.051	0.072	0.000	0.344	0.000	0.396	5.675

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	169	0	167	0	108	1069
N.S.	1	1.00	0.80	1.01	0.00	1.00	0.00	0.65	6.40
time (sec)	N/A	0.087	0.080	0.072	0.000	0.360	0.000	0.406	5.848

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	150	262	0	189	0	123	1292
N.S.	1	1.00	0.68	1.19	0.00	0.86	0.00	0.56	5.85
time (sec)	N/A	0.124	0.080	0.076	0.000	0.323	0.000	0.412	6.506

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	153	64	53	0	61	496
N.S.	1	1.00	0.84	1.55	0.65	0.54	0.00	0.62	5.01
time (sec)	N/A	0.049	0.028	0.065	0.536	0.341	0.000	0.404	1.090

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	123	53	42	0	49	385
N.S.	1	1.00	0.87	1.62	0.70	0.55	0.00	0.64	5.07
time (sec)	N/A	0.037	0.027	0.063	0.537	0.349	0.000	0.402	1.066

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	68	42	31	0	37	211
N.S.	1	1.00	0.86	1.19	0.74	0.54	0.00	0.65	3.70
time (sec)	N/A	0.026	0.027	0.064	0.527	0.342	0.000	0.382	1.145

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	56	30	19	0	23	105
N.S.	1	1.00	0.89	1.56	0.83	0.53	0.00	0.64	2.92
time (sec)	N/A	0.012	0.032	0.063	0.540	0.343	0.000	0.384	1.213

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	12	24	12	52
N.S.	1	1.00	1.00	0.94	0.75	0.75	1.50	0.75	3.25
time (sec)	N/A	0.004	0.009	0.066	0.519	0.379	23.207	0.379	1.176

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	42	0	56	0	21	285
N.S.	1	1.00	0.96	0.86	0.00	1.14	0.00	0.43	5.82
time (sec)	N/A	0.011	0.038	0.065	0.000	0.366	0.000	0.389	7.189

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	95	0	93	0	47	570
N.S.	1	1.00	0.83	1.01	0.00	0.99	0.00	0.50	6.06
time (sec)	N/A	0.039	0.048	0.067	0.000	0.336	0.000	0.385	7.018

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	98	148	0	123	0	69	802
N.S.	1	1.00	0.62	0.94	0.00	0.78	0.00	0.44	5.08
time (sec)	N/A	0.073	0.061	0.066	0.000	0.396	0.000	0.393	6.009

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	117	200	0	145	0	84	1086
N.S.	1	1.00	0.55	0.94	0.00	0.68	0.00	0.40	5.12
time (sec)	N/A	0.106	0.075	0.069	0.000	0.347	0.000	0.420	5.749

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	319	64	63	0	77	1057
N.S.	1	1.00	0.87	3.36	0.67	0.66	0.00	0.81	11.13
time (sec)	N/A	0.047	0.032	0.066	0.530	0.327	0.000	0.399	1.450

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	201	52	51	0	61	660
N.S.	1	1.00	0.89	2.72	0.70	0.69	0.00	0.82	8.92
time (sec)	N/A	0.034	0.027	0.066	0.520	0.319	0.000	0.401	1.258

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	106	41	40	0	46	259
N.S.	1	1.00	0.89	1.93	0.75	0.73	0.00	0.84	4.71
time (sec)	N/A	0.024	0.028	0.066	0.544	0.344	0.000	0.393	1.293

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	40	30	29	46	29	152
N.S.	1	1.00	0.85	1.18	0.88	0.85	1.35	0.85	4.47
time (sec)	N/A	0.011	0.036	0.063	0.537	0.372	46.134	0.389	1.334

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	20	26	12	97
N.S.	1	1.00	1.00	0.94	0.75	1.25	1.62	0.75	6.06
time (sec)	N/A	0.004	0.009	0.065	0.519	0.336	45.834	0.383	1.310

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	68	0	110	0	37	614
N.S.	1	1.00	0.96	0.87	0.00	1.41	0.00	0.47	7.87
time (sec)	N/A	0.029	0.073	0.066	0.000	0.374	0.000	0.387	6.144

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	91	105	0	151	0	64	807
N.S.	1	1.00	0.73	0.85	0.00	1.22	0.00	0.52	6.51
time (sec)	N/A	0.051	0.060	0.071	0.000	0.390	0.000	0.407	5.856

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	115	131	0	189	0	80	1028
N.S.	1	1.00	0.60	0.69	0.00	0.99	0.00	0.42	5.38
time (sec)	N/A	0.087	0.083	0.075	0.000	0.353	0.000	0.403	6.072

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	133	186	0	211	0	95	1258
N.S.	1	1.00	0.54	0.76	0.00	0.86	0.00	0.39	5.13
time (sec)	N/A	0.138	0.081	0.075	0.000	0.386	0.000	0.400	5.092

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	295	64	74	0	75	817
N.S.	1	1.00	0.84	2.98	0.65	0.75	0.00	0.76	8.25
time (sec)	N/A	0.048	0.034	0.069	0.529	0.346	0.000	0.382	1.423

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	186	52	62	90	59	533
N.S.	1	1.00	0.86	2.45	0.68	0.82	1.18	0.78	7.01
time (sec)	N/A	0.034	0.030	0.067	0.540	0.341	72.695	0.396	1.296

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	91	42	52	71	39	259
N.S.	1	1.00	0.81	1.54	0.71	0.88	1.20	0.66	4.39
time (sec)	N/A	0.024	0.032	0.067	0.550	0.330	71.896	0.395	1.262

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	42	31	41	51	20	152
N.S.	1	1.00	0.82	1.11	0.82	1.08	1.34	0.53	4.00
time (sec)	N/A	0.013	0.031	0.068	0.526	0.335	72.213	0.385	1.382

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	31	27	12	103
N.S.	1	1.00	1.00	0.83	0.67	1.72	1.50	0.67	5.72
time (sec)	N/A	0.004	0.009	0.066	0.509	0.327	48.569	0.388	1.367

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	93	0	177	0	45	886
N.S.	1	1.00	0.84	0.86	0.00	1.64	0.00	0.42	8.20
time (sec)	N/A	0.042	0.104	0.068	0.000	0.348	0.000	0.385	6.263

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	113	130	0	221	0	65	1230
N.S.	1	1.00	0.73	0.84	0.00	1.43	0.00	0.42	7.94
time (sec)	N/A	0.074	0.091	0.073	0.000	0.358	0.000	0.390	7.338

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	133	157	0	255	0	93	1514
N.S.	1	1.00	0.59	0.70	0.00	1.14	0.00	0.42	6.76
time (sec)	N/A	0.106	0.079	0.075	0.000	0.354	0.000	0.394	8.471

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	150	211	0	277	0	115	2359
N.S.	1	1.00	0.54	0.76	0.00	1.00	0.00	0.41	8.49
time (sec)	N/A	0.151	0.251	0.079	0.000	0.365	0.000	0.396	6.417

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	0	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.00	0.48	2.11
time (sec)	N/A	0.006	0.027	0.109	0.265	0.327	0.000	0.394	1.242

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.006	0.020	0.104	0.260	0.325	18.981	0.373	1.087

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.006	0.020	0.103	0.266	0.321	1.666	0.384	1.100

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	16	0	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.59	0.00	0.48	2.11
time (sec)	N/A	0.006	0.019	0.102	0.267	0.323	0.000	0.399	1.092

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	19	12	0	13	56
N.S.	1	1.00	0.92	0.80	0.76	0.48	0.00	0.52	2.24
time (sec)	N/A	0.006	0.014	0.101	0.261	0.367	0.000	0.388	1.127

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	12	22	13	56
N.S.	1	1.00	0.87	0.87	0.83	0.52	0.96	0.57	2.43
time (sec)	N/A	0.007	0.015	0.099	0.270	0.354	0.388	0.377	1.116

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	20	19	11	27	11	52
N.S.	1	1.00	0.78	0.74	0.70	0.41	1.00	0.41	1.93
time (sec)	N/A	0.007	0.015	0.100	0.260	0.331	2.589	0.381	1.241

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	13	27	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.48	1.00	0.48	2.11
time (sec)	N/A	0.006	0.016	0.107	0.266	0.330	19.822	0.390	1.282

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	0	24	122
N.S.	1	1.00	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.015	0.041	0.293	0.268	0.360	0.000	0.391	1.154

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	48	24	122
N.S.	1	1.00	0.83	0.79	0.75	0.60	1.00	0.50	2.54
time (sec)	N/A	0.015	0.034	0.289	0.273	0.332	33.816	0.399	1.131

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	0	24	122
N.S.	1	1.00	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.016	0.029	0.284	0.277	0.357	0.000	0.381	1.141

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	27	0	24	122
N.S.	1	1.00	0.83	0.79	0.75	0.56	0.00	0.50	2.54
time (sec)	N/A	0.015	0.043	0.283	0.277	0.334	0.000	0.385	1.119

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	47	36	24	0	24	122
N.S.	1	1.00	0.87	1.02	0.78	0.52	0.00	0.52	2.65
time (sec)	N/A	0.014	0.023	0.273	0.271	0.350	0.000	0.393	1.154

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	37	36	23	0	24	122
N.S.	1	1.00	0.91	0.84	0.82	0.52	0.00	0.55	2.77
time (sec)	N/A	0.015	0.036	0.290	0.275	0.333	0.000	0.384	1.162

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	24	48	23	122
N.S.	1	1.00	0.83	0.79	0.75	0.50	1.00	0.48	2.54
time (sec)	N/A	0.015	0.032	0.282	0.269	0.362	2.302	0.393	1.132

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	24	49	24	122
N.S.	1	1.00	0.83	0.79	0.75	0.50	1.02	0.50	2.54
time (sec)	N/A	0.016	0.032	0.287	0.277	0.350	20.262	0.379	1.134

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	0	35	182
N.S.	1	1.00	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.025	0.021	1.199	0.282	0.341	0.000	0.390	1.165

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	70	35	182
N.S.	1	1.00	0.83	0.81	0.80	0.58	1.01	0.51	2.64
time (sec)	N/A	0.026	0.036	1.224	0.282	0.373	61.609	0.396	1.177

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	0	35	182
N.S.	1	1.00	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.026	0.022	1.211	0.289	0.340	0.000	0.399	1.170

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	38	0	35	182
N.S.	1	1.00	0.83	0.81	0.80	0.55	0.00	0.51	2.64
time (sec)	N/A	0.025	0.020	1.192	0.277	0.372	0.000	0.377	1.157

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	69	55	35	0	35	182
N.S.	1	1.00	0.88	1.06	0.85	0.54	0.00	0.54	2.80
time (sec)	N/A	0.024	0.021	1.317	0.283	0.345	0.000	0.396	1.186

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	64	55	34	0	35	182
N.S.	1	1.00	0.90	1.02	0.87	0.54	0.00	0.56	2.89
time (sec)	N/A	0.025	0.059	1.331	0.277	0.369	0.000	0.377	1.214

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	55	55	34	66	34	182
N.S.	1	1.00	0.85	0.85	0.85	0.52	1.02	0.52	2.80
time (sec)	N/A	0.025	0.061	1.222	0.283	0.343	2.052	0.407	1.216

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	35	70	34	182
N.S.	1	1.00	0.83	0.81	0.80	0.51	1.01	0.49	2.64
time (sec)	N/A	0.026	0.041	1.228	0.277	0.342	20.155	0.392	1.185

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	129	481	65	153	0	70	475
N.S.	1	1.00	0.90	3.36	0.45	1.07	0.00	0.49	3.32
time (sec)	N/A	0.095	0.091	0.125	0.467	0.363	0.000	0.395	1.721

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	330	54	132	0	59	415
N.S.	1	1.00	0.93	2.84	0.47	1.14	0.00	0.51	3.58
time (sec)	N/A	0.125	0.080	0.086	0.475	0.368	0.000	0.387	1.388

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	207	42	103	0	45	354
N.S.	1	1.00	0.97	2.33	0.47	1.16	0.00	0.51	3.98
time (sec)	N/A	0.094	0.058	0.083	0.497	0.353	0.000	0.393	1.852

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	112	31	85	0	31	296
N.S.	1	1.00	0.97	1.75	0.48	1.33	0.00	0.48	4.62
time (sec)	N/A	0.059	0.035	0.087	0.490	0.343	0.000	0.393	2.228

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	41	18	68	0	18	347
N.S.	1	1.00	0.96	0.77	0.34	1.28	0.00	0.34	6.55
time (sec)	N/A	0.036	0.027	0.081	0.460	0.380	0.000	0.392	4.005

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	76	31	93	0	31	464
N.S.	1	1.00	0.96	1.00	0.41	1.22	0.00	0.41	6.11
time (sec)	N/A	0.044	0.043	0.084	0.467	0.354	0.000	0.402	1.968

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	89	98	41	118	0	41	642
N.S.	1	1.00	0.88	0.97	0.41	1.17	0.00	0.41	6.36
time (sec)	N/A	0.114	0.123	0.085	0.468	0.380	0.000	0.393	1.832

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	107	120	52	144	0	52	822
N.S.	1	1.00	0.84	0.94	0.41	1.12	0.00	0.41	6.42
time (sec)	N/A	0.063	0.102	0.090	0.473	0.362	0.000	0.389	1.634

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	144	452	75	188	0	76	523
N.S.	1	1.00	1.07	3.35	0.56	1.39	0.00	0.56	3.87
time (sec)	N/A	0.074	0.127	0.953	0.471	0.335	0.000	0.373	1.612

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	119	294	64	161	0	65	463
N.S.	1	1.00	1.10	2.72	0.59	1.49	0.00	0.60	4.29
time (sec)	N/A	0.052	0.100	0.514	0.483	0.380	0.000	0.384	1.532

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	81	160	50	134	0	46	403
N.S.	1	1.00	0.98	1.93	0.60	1.61	0.00	0.55	4.86
time (sec)	N/A	0.037	0.055	0.423	0.478	0.398	0.000	0.396	1.771

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	61	37	115	0	36	344
N.S.	1	1.00	0.96	0.84	0.51	1.58	0.00	0.49	4.71
time (sec)	N/A	0.025	0.041	0.388	0.471	0.385	0.000	0.379	1.769

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	82	35	116	0	35	516
N.S.	1	1.00	0.82	0.85	0.36	1.20	0.00	0.36	5.32
time (sec)	N/A	0.038	0.043	0.420	0.467	0.365	0.000	0.395	2.182

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	105	51	147	0	49	705
N.S.	1	1.00	0.87	0.88	0.42	1.22	0.00	0.41	5.88
time (sec)	N/A	0.054	0.080	0.459	0.480	0.356	0.000	0.391	1.830

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	120	128	64	184	0	58	871
N.S.	1	1.00	0.83	0.88	0.44	1.27	0.00	0.40	6.01
time (sec)	N/A	0.076	0.152	0.589	0.475	0.391	0.000	0.398	2.163

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	139	151	75	210	0	70	1051
N.S.	1	1.00	0.81	0.88	0.44	1.22	0.00	0.41	6.11
time (sec)	N/A	0.097	0.174	0.945	0.477	0.347	0.000	0.398	2.138

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	147	418	86	227	0	77	571
N.S.	1	1.00	1.09	3.10	0.64	1.68	0.00	0.57	4.23
time (sec)	N/A	0.068	0.070	0.545	0.488	0.348	0.000	0.398	1.977

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	104	249	73	200	0	59	511
N.S.	1	1.00	0.95	2.26	0.66	1.82	0.00	0.54	4.65
time (sec)	N/A	0.050	0.066	0.437	0.491	0.361	0.000	0.401	1.691

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	85	61	185	0	47	667
N.S.	1	1.00	0.98	0.87	0.62	1.89	0.00	0.48	6.81
time (sec)	N/A	0.039	0.052	0.422	0.489	0.351	0.000	0.388	1.867

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	107	98	64	186	0	52	580
N.S.	1	1.00	0.86	0.78	0.51	1.49	0.00	0.42	4.64
time (sec)	N/A	0.052	0.163	0.460	0.473	0.353	0.000	0.390	1.842

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	118	112	60	186	0	47	741
N.S.	1	1.00	0.78	0.74	0.39	1.22	0.00	0.31	4.88
time (sec)	N/A	0.072	0.117	0.465	0.484	0.353	0.000	0.397	1.898

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	141	181	73	214	0	59	1077
N.S.	1	1.00	0.80	1.03	0.41	1.22	0.00	0.34	6.12
time (sec)	N/A	0.093	0.206	0.617	0.481	0.350	0.000	0.395	2.168

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	207	86	250	0	71	1362
N.S.	1	1.00	0.78	1.03	0.43	1.24	0.00	0.35	6.78
time (sec)	N/A	0.115	0.184	1.000	0.481	0.345	0.000	0.391	2.495

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	174	229	97	276	0	80	2151
N.S.	1	1.00	0.76	1.00	0.43	1.21	0.00	0.35	9.43
time (sec)	N/A	0.138	0.236	1.819	0.485	0.345	0.000	0.395	1.947

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	304	0	141	0	60	-1
N.S.	1	1.00	0.73	2.14	0.00	0.99	0.00	0.42	-0.01
time (sec)	N/A	0.054	0.059	0.127	0.000	0.360	0.000	0.391	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	84	174	0	114	0	48	-1
N.S.	1	1.00	0.81	1.67	0.00	1.10	0.00	0.46	-0.01
time (sec)	N/A	0.035	0.045	0.121	0.000	0.369	0.000	0.386	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	75	0	93	0	36	-1
N.S.	1	1.00	1.02	1.23	0.00	1.52	0.00	0.59	-0.02
time (sec)	N/A	0.020	0.026	0.122	0.000	0.351	0.000	0.362	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	149	40	89	0	57	-1
N.S.	1	1.00	1.06	3.04	0.82	1.82	0.00	1.16	-0.02
time (sec)	N/A	0.020	0.029	0.126	0.506	0.354	0.000	0.401	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	15	0	59	210
N.S.	1	1.00	0.97	0.83	0.43	0.43	0.00	1.69	6.00
time (sec)	N/A	0.010	0.024	0.121	0.488	0.359	0.000	0.412	1.714

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	34	0	112	174
N.S.	1	1.00	0.67	0.82	0.47	0.47	0.00	1.56	2.42
time (sec)	N/A	0.023	0.029	0.120	0.473	0.350	0.000	0.387	1.413

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	45	0	138	234
N.S.	1	1.00	0.60	0.95	0.41	0.41	0.00	1.25	2.13
time (sec)	N/A	0.037	0.043	0.127	0.473	0.342	0.000	0.403	1.594

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	56	0	166	294
N.S.	1	1.00	0.55	1.02	0.38	0.38	0.00	1.12	1.99
time (sec)	N/A	0.055	0.047	0.134	0.478	0.346	0.000	0.400	1.519

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	471	0	163	0	147	-1
N.S.	1	1.00	0.69	2.66	0.00	0.92	0.00	0.83	-0.01
time (sec)	N/A	0.072	0.067	0.122	0.000	0.360	0.000	0.405	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	105	304	0	140	0	122	-1
N.S.	1	1.00	0.76	2.19	0.00	1.01	0.00	0.88	-0.01
time (sec)	N/A	0.052	0.047	0.121	0.000	0.356	0.000	0.406	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	165	0	119	0	0	-1
N.S.	1	1.00	0.82	1.63	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.049	0.121	0.000	0.356	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	280	0	109	0	0	-1
N.S.	1	1.00	0.95	3.46	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.036	0.121	0.000	0.352	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	315	0	109	0	0	-1
N.S.	1	1.00	1.06	4.50	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.052	0.120	0.000	0.337	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	31	0	33	332
N.S.	1	1.00	0.97	0.83	0.43	0.89	0.00	0.94	9.49
time (sec)	N/A	0.010	0.066	0.128	0.478	0.352	0.000	0.392	1.510

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	45	0	59	228
N.S.	1	1.00	0.67	0.82	0.47	0.62	0.00	0.82	3.17
time (sec)	N/A	0.022	0.032	0.130	0.485	0.363	0.000	0.413	1.541

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	56	0	78	288
N.S.	1	1.00	0.60	0.95	0.41	0.51	0.00	0.71	2.62
time (sec)	N/A	0.039	0.040	0.141	0.475	0.338	0.000	0.408	1.635

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	67	0	97	348
N.S.	1	1.00	0.55	1.02	0.38	0.45	0.00	0.66	2.35
time (sec)	N/A	0.054	0.051	0.168	0.470	0.344	0.000	0.397	1.741

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	121	471	0	162	0	204	-1
N.S.	1	1.00	0.70	2.71	0.00	0.93	0.00	1.17	-0.01
time (sec)	N/A	0.068	0.056	0.122	0.000	0.346	0.000	0.404	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	101	286	0	141	0	0	-1
N.S.	1	1.00	0.74	2.10	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.038	0.122	0.000	0.366	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	460	0	137	0	0	-1
N.S.	1	1.00	0.83	3.80	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.050	0.124	0.000	0.359	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	97	501	0	138	0	0	-1
N.S.	1	1.00	0.92	4.73	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.048	0.122	0.000	0.330	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	532	0	137	0	0	-1
N.S.	1	1.00	1.02	5.72	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.038	0.122	0.000	0.354	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	42	0	33	396
N.S.	1	1.00	0.97	0.83	0.43	1.20	0.00	0.94	11.31
time (sec)	N/A	0.010	0.034	0.126	0.478	0.331	0.000	0.407	1.654

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	56	0	59	293
N.S.	1	1.00	0.67	0.82	0.47	0.78	0.00	0.82	4.07
time (sec)	N/A	0.024	0.035	0.140	0.471	0.348	0.000	0.437	1.625

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	67	0	78	353
N.S.	1	1.00	0.60	0.95	0.41	0.61	0.00	0.71	3.21
time (sec)	N/A	0.038	0.037	0.176	0.476	0.335	0.000	0.419	1.657

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	78	0	97	413
N.S.	1	1.00	0.55	1.02	0.38	0.53	0.00	0.66	2.79
time (sec)	N/A	0.053	0.054	0.269	0.467	0.335	0.000	0.412	1.744

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	304	0	140	0	64	-1
N.S.	1	1.00	0.72	2.10	0.00	0.97	0.00	0.44	-0.01
time (sec)	N/A	0.056	0.064	0.127	0.000	0.366	0.000	0.381	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	88	174	0	119	0	52	-1
N.S.	1	1.00	0.82	1.63	0.00	1.11	0.00	0.49	-0.01
time (sec)	N/A	0.037	0.050	0.125	0.000	0.372	0.000	0.387	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	66	80	0	91	0	38	-1
N.S.	1	1.00	1.05	1.27	0.00	1.44	0.00	0.60	-0.02
time (sec)	N/A	0.020	0.040	0.125	0.000	0.340	0.000	0.391	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	0	57	0	23	-1
N.S.	1	1.00	1.10	0.80	0.00	1.90	0.00	0.77	-0.03
time (sec)	N/A	0.009	0.024	0.125	0.000	0.363	0.000	0.411	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	15	15	0	30	101
N.S.	1	1.00	0.97	0.88	0.45	0.45	0.00	0.91	3.06
time (sec)	N/A	0.010	0.030	0.127	0.474	0.341	0.000	0.391	1.620

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	59	33	23	0	55	218
N.S.	1	1.00	0.64	0.82	0.46	0.32	0.00	0.76	3.03
time (sec)	N/A	0.022	0.042	0.126	0.464	0.339	0.000	0.387	1.573

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	34	0	77	227
N.S.	1	1.00	0.60	0.95	0.41	0.31	0.00	0.70	2.06
time (sec)	N/A	0.038	0.067	0.133	0.475	0.328	0.000	0.383	1.527

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	55	45	0	103	287
N.S.	1	1.00	0.55	1.02	0.37	0.30	0.00	0.70	1.94
time (sec)	N/A	0.053	0.072	0.129	0.472	0.344	0.000	0.378	1.636

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	122	428	0	196	0	75	-1
N.S.	1	1.00	0.73	2.58	0.00	1.18	0.00	0.45	-0.01
time (sec)	N/A	0.077	0.161	0.128	0.000	0.331	0.000	0.399	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	261	0	175	0	63	-1
N.S.	1	1.00	0.81	2.04	0.00	1.37	0.00	0.49	-0.01
time (sec)	N/A	0.051	0.066	0.126	0.000	0.360	0.000	0.395	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	130	0	145	0	48	-1
N.S.	1	1.00	0.94	1.51	0.00	1.69	0.00	0.56	-0.01
time (sec)	N/A	0.037	0.098	0.120	0.000	0.344	0.000	0.391	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	55	42	0	119	0	39	-1
N.S.	1	1.00	1.06	0.81	0.00	2.29	0.00	0.75	-0.02
time (sec)	N/A	0.018	0.091	0.120	0.000	0.355	0.000	0.400	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	0	22	0	15	163
N.S.	1	1.00	0.97	0.88	0.00	0.67	0.00	0.45	4.94
time (sec)	N/A	0.009	0.027	0.122	0.000	0.339	0.000	0.389	1.736

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	59	32	34	0	50	281
N.S.	1	1.00	0.63	0.87	0.47	0.50	0.00	0.74	4.13
time (sec)	N/A	0.023	0.034	0.122	0.474	0.319	0.000	0.389	1.490

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	105	45	49	0	107	286
N.S.	1	1.00	0.58	0.95	0.41	0.45	0.00	0.97	2.60
time (sec)	N/A	0.039	0.043	0.123	0.493	0.326	0.000	0.398	1.694

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	151	54	58	0	161	346
N.S.	1	1.00	0.54	1.02	0.36	0.39	0.00	1.09	2.34
time (sec)	N/A	0.055	0.047	0.121	0.473	0.338	0.000	0.410	1.774

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	348	0	241	0	75	-1
N.S.	1	1.00	0.79	2.27	0.00	1.58	0.00	0.49	-0.01
time (sec)	N/A	0.067	0.081	0.132	0.000	0.376	0.000	0.401	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	101	180	0	214	0	61	-1
N.S.	1	1.00	0.91	1.62	0.00	1.93	0.00	0.55	-0.01
time (sec)	N/A	0.048	0.060	0.127	0.000	0.361	0.000	0.393	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	78	59	0	186	0	49	-1
N.S.	1	1.00	1.04	0.79	0.00	2.48	0.00	0.65	-0.01
time (sec)	N/A	0.031	0.044	0.128	0.000	0.392	0.000	0.402	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	92	0	33	0	15	229
N.S.	1	1.00	0.97	2.63	0.00	0.94	0.00	0.43	6.54
time (sec)	N/A	0.011	0.036	0.121	0.000	0.336	0.000	0.398	1.704

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	58	0	43	0	25	346
N.S.	1	1.00	0.66	0.82	0.00	0.61	0.00	0.35	4.87
time (sec)	N/A	0.022	0.029	0.122	0.000	0.340	0.000	0.386	1.659

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	104	45	58	0	62	348
N.S.	1	1.00	0.62	0.98	0.42	0.55	0.00	0.58	3.28
time (sec)	N/A	0.039	0.041	0.125	0.470	0.326	0.000	0.394	1.687

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	79	150	56	71	0	119	406
N.S.	1	1.00	0.54	1.03	0.38	0.49	0.00	0.82	2.78
time (sec)	N/A	0.059	0.049	0.131	0.476	0.335	0.000	0.411	1.765

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	100	196	67	82	0	173	466
N.S.	1	1.00	0.54	1.05	0.36	0.44	0.00	0.93	2.51
time (sec)	N/A	0.075	0.049	0.127	0.466	0.448	0.000	0.426	1.861

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.111	0.049	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	654	139	374	0	332	546
N.S.	1	1.00	0.88	3.96	0.84	2.27	0.00	2.01	3.31
time (sec)	N/A	0.098	0.076	7.020	0.538	0.358	0.000	0.394	1.671

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	492	101	255	0	226	418
N.S.	1	1.00	0.88	4.07	0.83	2.11	0.00	1.87	3.45
time (sec)	N/A	0.053	0.057	2.541	0.528	0.365	0.000	0.390	1.422

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	315	68	168	0	140	304
N.S.	1	1.00	0.87	3.84	0.83	2.05	0.00	1.71	3.71
time (sec)	N/A	0.031	0.048	0.970	0.528	0.354	0.000	0.392	1.291

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	175	42	91	0	76	205
N.S.	1	1.00	0.85	3.65	0.88	1.90	0.00	1.58	4.27
time (sec)	N/A	0.014	0.040	0.399	0.535	0.363	0.000	0.395	1.231

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	21	39	51	28	121
N.S.	1	1.00	1.00	1.05	1.05	1.95	2.55	1.40	6.05
time (sec)	N/A	0.005	0.013	0.374	0.508	0.387	0.203	0.400	1.182

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.063	0.015	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.057	0.023	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.037	0.023	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	90	96
N.S.	1	1.00	0.92	18.27	1.03	0.89	0.00	2.43	2.59
time (sec)	N/A	0.015	0.038	0.124	0.264	0.352	0.000	0.387	1.596

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	76	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	3.30	3.09	0.83
time (sec)	N/A	0.015	0.017	0.122	0.291	0.405	7.947	0.391	1.105

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	160	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	6.96	3.09	0.83
time (sec)	N/A	0.006	0.012	0.075	0.301	0.601	4.155	0.378	0.061

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	10	80	69	16
N.S.	1	1.00	1.12	0.94	1.00	0.62	5.00	4.31	1.00
time (sec)	N/A	0.002	0.005	0.100	0.305	0.512	2.225	0.398	0.020

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	34	8	0	15	59
N.S.	1	1.00	0.90	1.10	1.62	0.38	0.00	0.71	2.81
time (sec)	N/A	0.022	0.013	0.079	0.256	0.351	0.000	0.410	1.204

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	13	68	70	17
N.S.	1	1.00	1.06	1.06	1.00	0.76	4.00	4.12	1.00
time (sec)	N/A	0.006	0.013	0.091	0.297	0.406	3.958	0.405	0.076

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	80	71	16
N.S.	1	1.00	0.78	0.87	0.83	0.48	3.48	3.09	0.70
time (sec)	N/A	0.006	0.011	0.089	0.300	0.354	7.727	0.397	1.066

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	33	58	0	0	-1
N.S.	1	1.00	1.74	0.78	1.22	2.15	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.014	0.224	0.291	0.446	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	479	56	88	0	0	-1
N.S.	1	1.00	1.59	9.39	1.10	1.73	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.012	0.170	0.302	0.416	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	501	78	118	0	0	-1
N.S.	1	1.00	1.42	6.51	1.01	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.024	0.171	0.305	0.425	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	271	5366	281	900	0	0	-1
N.S.	1	1.00	0.88	17.48	0.92	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.327	8.594	9.969	0.477	0.389	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	203	5062	215	746	0	0	-1
N.S.	1	1.00	0.88	21.91	0.93	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.264	6.879	1.330	0.483	0.454	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	366	361	142	552	0	0	-1
N.S.	1	1.00	2.44	2.41	0.95	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.160	4.844	0.914	0.478	0.400	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.111	12.321	0.090	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1769	149	451	0	0	-1
N.S.	1	1.00	0.93	11.41	0.96	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.198	3.598	1.285	0.733	0.408	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1710	125	382	0	0	-1
N.S.	1	1.00	0.92	13.36	0.98	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.179	4.037	1.041	0.718	0.387	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1627	101	323	0	0	-1
N.S.	1	1.00	0.90	16.11	1.00	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.156	3.612	0.880	0.723	0.452	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	265	72	239	0	0	-1
N.S.	1	1.00	2.91	3.84	1.04	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.096	3.531	0.492	0.756	0.452	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.059	3.585	0.113	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1773	146	424	0	0	-1
N.S.	1	1.00	0.86	10.55	0.87	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.212	3.169	1.282	0.718	0.372	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1716	123	360	0	0	-1
N.S.	1	1.00	0.86	12.35	0.88	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.182	4.134	1.103	0.705	0.389	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1635	100	306	0	0	-1
N.S.	1	1.00	0.85	14.86	0.91	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.156	4.081	0.941	0.700	0.352	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	200	299	73	228	0	0	-1
N.S.	1	1.00	2.63	3.93	0.96	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	3.490	0.510	0.706	0.385	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.049	3.759	0.122	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	265	5294	277	880	0	0	-1
N.S.	1	1.00	0.87	17.47	0.91	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.322	7.640	10.248	0.488	0.511	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	199	4990	213	730	0	0	-1
N.S.	1	1.00	0.87	21.79	0.93	3.19	0.00	0.00	-0.00
time (sec)	N/A	0.267	6.169	1.447	0.483	0.440	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	369	361	142	540	0	0	-1
N.S.	1	1.00	2.46	2.41	0.95	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.162	3.332	1.018	0.487	0.479	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.111	11.735	0.117	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1741	146	424	0	0	-1
N.S.	1	1.00	0.93	11.45	0.96	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.217	3.165	1.374	0.695	0.485	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1684	123	360	0	0	-1
N.S.	1	1.00	0.92	13.37	0.98	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.185	4.061	1.111	0.693	0.499	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1603	100	306	0	0	-1
N.S.	1	1.00	0.90	16.03	1.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.157	4.055	0.931	0.706	0.439	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	197	265	72	227	0	0	-1
N.S.	1	1.00	2.86	3.84	1.04	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.097	2.897	0.566	0.700	0.374	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.060	3.036	0.145	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1801	149	451	0	0	-1
N.S.	1	1.00	0.89	10.92	0.90	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.208	3.609	1.365	0.694	0.407	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1742	125	382	0	0	-1
N.S.	1	1.00	0.88	12.72	0.91	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.186	4.054	1.154	0.693	0.391	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1659	101	323	0	0	-1
N.S.	1	1.00	0.86	15.22	0.93	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.159	3.512	0.951	0.708	0.393	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	208	299	73	240	0	0	-1
N.S.	1	1.00	2.74	3.93	0.96	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.095	3.335	0.561	0.693	0.367	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.066	3.611	0.142	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	3041	0	0	-1
N.S.	1	1.00	2.17	24.60	0.00	10.07	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.744	14.719	0.000	0.484	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1788	0	0	-1
N.S.	1	1.00	1.75	23.69	0.00	7.64	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.405	11.053	0.000	0.530	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	295	2543	0	948	0	0	-1
N.S.	1	1.00	1.82	15.70	0.00	5.85	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.323	1.081	0.000	0.384	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	169	182	499	0	0	-1
N.S.	1	1.00	1.61	2.14	2.30	6.32	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.034	0.602	0.502	0.408	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.027	5.050	0.145	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6895	0	2165	0	0	-1
N.S.	1	1.00	0.88	17.46	0.00	5.48	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.649	25.551	0.000	0.446	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6521	0	1689	0	0	-1
N.S.	1	1.00	0.87	22.11	0.00	5.73	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.442	2.451	0.000	0.438	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	563	372	1185	0	0	-1
N.S.	1	1.00	23.99	2.90	1.92	6.11	0.00	0.00	-0.01
time (sec)	N/A	0.173	30.408	1.277	0.537	0.530	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.111	3.511	0.132	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2346	343	345	0	0	-1
N.S.	1	1.00	0.91	13.80	2.02	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.449	1.803	0.281	0.401	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	248	293	0	0	-1
N.S.	1	1.00	0.89	16.96	1.86	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.212	1.191	0.287	0.389	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	321	260	218	0	0	-1
N.S.	1	1.00	0.90	3.45	2.80	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.101	6.225	0.749	0.482	0.404	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.756	0.120	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2456	342	345	0	0	-1
N.S.	1	1.00	0.91	14.36	2.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.297	1.714	0.289	0.428	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2358	247	293	0	0	-1
N.S.	1	1.00	0.90	17.60	1.84	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.229	1.273	0.278	0.441	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	365	262	219	0	0	-1
N.S.	1	1.00	0.90	3.88	2.79	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.103	6.015	0.740	0.471	0.364	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	1.204	0.122	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	2799	0	0	-1
N.S.	1	1.00	2.17	24.60	0.00	9.27	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.175	14.941	0.000	0.470	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1590	0	0	-1
N.S.	1	1.00	1.75	23.69	0.00	6.79	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.115	11.232	0.000	0.460	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	295	2544	0	794	0	0	-1
N.S.	1	1.00	1.82	15.70	0.00	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.206	1.099	0.000	0.449	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	145	184	389	0	0	-1
N.S.	1	1.00	1.61	1.84	2.33	4.92	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.047	0.589	0.507	0.444	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.030	0.621	0.135	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6739	0	1799	0	0	-1
N.S.	1	1.00	0.87	17.24	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.663	28.345	0.000	0.572	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6389	0	1463	0	0	-1
N.S.	1	1.00	0.86	21.81	0.00	4.99	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.410	2.470	0.000	0.607	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	569	392	1099	0	0	-1
N.S.	1	1.00	23.01	2.93	2.02	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.170	29.488	1.359	0.533	0.593	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.110	3.882	0.168	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2456	344	180	0	0	-1
N.S.	1	1.00	0.92	14.62	2.05	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.311	1.949	0.285	0.408	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2358	249	157	0	0	-1
N.S.	1	1.00	0.90	17.86	1.89	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.202	1.342	0.282	0.410	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	321	288	122	0	0	-1
N.S.	1	1.00	0.89	3.45	3.10	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.102	23.927	0.835	0.476	0.376	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	0.760	0.163	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2346	345	180	0	0	-1
N.S.	1	1.00	0.92	13.88	2.04	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.308	1.867	0.298	0.437	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	250	157	0	0	-1
N.S.	1	1.00	0.89	16.96	1.88	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.231	1.331	0.279	0.355	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	365	286	122	0	0	-1
N.S.	1	1.00	0.89	3.88	3.04	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.105	20.188	0.823	0.489	0.360	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	1.172	0.186	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	46	31	58	65	0	0	-1
N.S.	1	1.00	2.19	1.48	2.76	3.10	0.00	0.00	-0.05
time (sec)	N/A	0.008	0.181	0.050	0.259	0.446	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	62	59	95	0	0	-1
N.S.	1	1.00	1.65	1.44	1.37	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.019	0.039	0.257	0.390	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	93	79	76	120	0	0	-1
N.S.	1	1.00	1.60	1.36	1.31	2.07	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.021	0.038	0.256	0.367	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	69	59	107	138	0	0	-1
N.S.	1	1.00	1.97	1.69	3.06	3.94	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.155	0.057	0.256	0.371	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	113	178	108	199	0	0	-1
N.S.	1	1.00	1.59	2.51	1.52	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.031	0.097	0.270	0.361	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	149	326	142	248	0	0	-1
N.S.	1	1.00	1.48	3.23	1.41	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.029	0.112	0.266	0.383	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	108	160	202	284	0	0	-1
N.S.	1	1.00	0.64	0.95	1.20	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.500	0.306	0.261	0.360	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	177	596	194	396	0	0	-1
N.S.	1	1.00	0.84	2.82	0.92	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.084	0.065	0.299	0.401	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	235	672	254	480	0	0	-1
N.S.	1	1.00	0.89	2.55	0.96	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.062	0.069	0.288	0.439	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	153	868	184	234	0	157	179
N.S.	1	1.00	1.43	8.11	1.72	2.19	0.00	1.47	1.67
time (sec)	N/A	0.126	0.096	0.388	0.485	0.423	0.000	0.534	1.594

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	887	64	93	0	147	111
N.S.	1	1.00	1.22	18.10	1.31	1.90	0.00	3.00	2.27
time (sec)	N/A	0.054	0.045	0.270	0.262	0.390	0.000	0.465	1.517

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	43	46	58	35	28
N.S.	1	1.00	1.02	1.51	0.96	1.02	1.29	0.78	0.62
time (sec)	N/A	0.039	0.043	0.158	0.267	0.331	0.784	0.393	0.157

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	349	43	25	0	40	28
N.S.	1	1.00	1.02	7.76	0.96	0.56	0.00	0.89	0.62
time (sec)	N/A	0.039	0.042	0.139	0.260	0.337	0.000	0.386	0.072

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	872	64	92	0	98	119
N.S.	1	1.00	1.20	17.80	1.31	1.88	0.00	2.00	2.43
time (sec)	N/A	0.047	0.046	0.231	0.277	0.337	0.000	0.488	1.711

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	150	842	184	233	0	167	187
N.S.	1	1.00	1.40	7.87	1.72	2.18	0.00	1.56	1.75
time (sec)	N/A	0.117	0.091	0.316	0.467	0.379	0.000	0.529	1.620

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	114	668	0	461	0	0	-1
N.S.	1	1.00	0.84	4.91	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.186	0.224	0.000	0.350	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [31] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	23	0.174
2	A	5	4	1.00	23	0.174
3	A	4	4	1.00	21	0.190
4	A	8	8	1.00	23	0.348
5	A	2	2	1.00	23	0.087
6	A	3	3	1.00	23	0.130
7	A	4	3	1.00	23	0.130
8	A	5	3	1.00	23	0.130
9	A	4	3	1.00	23	0.130
10	A	4	3	1.00	23	0.130
11	A	4	3	1.00	23	0.130
12	A	2	2	1.00	19	0.105
13	A	4	4	1.00	23	0.174
14	A	5	5	1.00	23	0.217
15	A	6	5	1.00	23	0.217
16	A	6	4	1.00	25	0.160
17	A	5	4	1.00	25	0.160
18	A	4	4	1.00	25	0.160
19	A	3	3	1.00	25	0.120
20	A	4	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	6	4	1.00	25	0.160
23	A	7	6	1.00	25	0.240
24	A	6	6	1.00	25	0.240
25	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	4	4	1.00	12	0.333
29	A	4	4	1.00	14	0.286
30	A	0	0	0.00	0	0.000
31	A	9	7	1.00	40	0.175
32	A	7	6	1.00	40	0.150
33	A	2	3	1.00	38	0.079
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	9	0.222
39	A	2	2	1.00	7	0.286
40	A	2	2	1.00	11	0.182
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	3	2	1.00	13	0.154
45	A	3	2	1.00	13	0.154
46	A	3	2	1.00	13	0.154
47	A	3	3	1.00	11	0.273
48	A	2	2	1.00	9	0.222
49	A	3	3	1.00	13	0.231
50	A	3	3	1.00	13	0.231
51	A	3	2	1.00	13	0.154
52	A	1	1	1.00	13	0.077
53	A	2	2	1.52	13	0.154
54	A	4	2	1.00	13	0.154
55	A	4	2	1.00	13	0.154
56	A	4	3	1.00	13	0.231
57	A	3	3	1.00	11	0.273
58	A	2	2	1.00	9	0.222
59	A	4	3	1.00	13	0.231
60	A	4	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	13	0.231
62	A	4	2	1.00	13	0.154
63	A	1	1	1.00	13	0.077
64	A	2	2	1.00	13	0.154
65	A	5	2	1.00	13	0.154
66	A	5	2	1.00	13	0.154
67	A	5	2	1.00	13	0.154
68	A	5	2	1.00	13	0.154
69	A	5	3	1.00	13	0.231
70	A	4	3	1.00	13	0.231
71	A	3	3	1.00	11	0.273
72	A	2	2	1.00	9	0.222
73	A	5	3	1.00	13	0.231
74	A	5	4	1.00	13	0.308
75	A	5	4	1.00	13	0.308
76	A	5	3	1.00	13	0.231
77	A	5	2	1.00	13	0.154
78	A	1	1	1.00	13	0.077
79	A	2	2	1.00	13	0.154
80	A	3	2	1.00	13	0.154
81	A	4	2	1.65	13	0.154
82	A	5	2	1.00	13	0.154
83	A	5	2	1.00	13	0.154
84	A	3	3	1.00	11	0.273
85	A	1	1	1.00	13	0.077
86	A	5	4	1.00	13	0.308
87	A	4	4	1.00	13	0.308
88	A	3	3	1.00	11	0.273
89	A	2	2	1.00	9	0.222
90	A	4	3	1.00	13	0.231
91	A	5	4	1.00	13	0.308
92	A	6	4	1.00	13	0.308
93	A	2	2	1.00	13	0.154
94	A	6	5	1.00	13	0.385
95	A	5	5	1.00	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	13	0.308
97	A	3	3	1.00	11	0.273
98	A	2	2	1.00	9	0.222
99	A	5	4	1.00	13	0.308
100	A	6	5	1.00	13	0.385
101	A	7	5	1.00	13	0.385
102	A	3	2	1.00	13	0.154
103	A	6	5	1.00	13	0.385
104	A	5	4	1.00	13	0.308
105	A	4	3	1.00	13	0.231
106	A	3	3	1.00	11	0.273
107	A	2	2	1.00	9	0.222
108	A	6	4	1.00	13	0.308
109	A	7	5	1.00	13	0.385
110	A	8	5	1.00	13	0.385
111	A	6	3	1.00	15	0.200
112	A	5	3	1.00	15	0.200
113	A	4	3	1.00	15	0.200
114	A	3	3	1.00	13	0.231
115	A	2	2	1.00	11	0.182
116	A	2	2	1.00	15	0.133
117	A	2	2	1.00	15	0.133
118	A	4	3	1.00	15	0.200
119	A	6	3	1.00	15	0.200
120	A	6	3	1.00	15	0.200
121	A	5	3	1.00	15	0.200
122	A	4	3	1.00	15	0.200
123	A	3	3	1.00	13	0.231
124	A	2	2	1.00	11	0.182
125	A	3	2	1.00	15	0.133
126	A	3	3	1.00	15	0.200
127	A	3	2	1.00	15	0.133
128	A	5	3	1.00	15	0.200
129	A	6	3	1.00	15	0.200
130	A	5	3	1.00	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	15	0.200
132	A	3	3	1.00	13	0.231
133	A	2	2	1.00	11	0.182
134	A	4	2	1.00	15	0.133
135	A	4	3	1.00	15	0.200
136	A	4	3	1.00	15	0.200
137	A	4	2	1.00	15	0.133
138	A	6	3	1.00	15	0.200
139	A	8	3	1.00	15	0.200
140	A	6	3	1.00	15	0.200
141	A	5	3	1.00	15	0.200
142	A	4	3	1.00	15	0.200
143	A	3	3	1.00	13	0.231
144	A	2	2	1.00	11	0.182
145	A	1	1	1.00	15	0.067
146	A	3	3	1.00	15	0.200
147	A	5	3	1.00	15	0.200
148	A	7	3	1.00	15	0.200
149	A	6	3	1.00	15	0.200
150	A	5	3	1.00	15	0.200
151	A	4	3	1.00	15	0.200
152	A	3	3	1.00	13	0.231
153	A	2	2	1.00	11	0.182
154	A	2	2	1.00	15	0.133
155	A	4	3	1.00	15	0.200
156	A	6	3	1.00	15	0.200
157	A	8	3	1.00	15	0.200
158	A	6	3	1.00	15	0.200
159	A	5	3	1.00	15	0.200
160	A	4	3	1.00	15	0.200
161	A	3	3	1.00	13	0.231
162	A	2	2	1.00	11	0.182
163	A	3	2	1.00	15	0.133
164	A	5	3	1.00	15	0.200
165	A	7	3	1.00	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	3	1.00	15	0.200
167	A	2	2	1.00	13	0.154
168	A	2	2	1.00	13	0.154
169	A	2	2	1.00	13	0.154
170	A	2	2	1.00	13	0.154
171	A	2	2	1.00	13	0.154
172	A	2	2	1.00	13	0.154
173	A	2	2	1.00	13	0.154
174	A	2	2	1.00	13	0.154
175	A	3	2	1.00	15	0.133
176	A	3	2	1.00	15	0.133
177	A	3	2	1.00	15	0.133
178	A	3	2	1.00	15	0.133
179	A	3	2	1.00	15	0.133
180	A	3	2	1.00	15	0.133
181	A	3	2	1.00	15	0.133
182	A	3	2	1.00	15	0.133
183	A	4	2	1.00	15	0.133
184	A	4	2	1.00	15	0.133
185	A	4	2	1.00	15	0.133
186	A	4	2	1.00	15	0.133
187	A	4	2	1.00	15	0.133
188	A	4	2	1.00	15	0.133
189	A	4	2	1.00	15	0.133
190	A	4	2	1.00	15	0.133
191	A	5	2	1.00	15	0.133
192	A	4	2	1.00	15	0.133
193	A	3	2	1.00	15	0.133
194	A	2	2	1.00	15	0.133
195	A	1	1	1.00	15	0.067
196	A	2	2	1.00	15	0.133
197	A	3	2	1.00	15	0.133
198	A	4	2	1.00	15	0.133
199	A	5	3	1.00	15	0.200
200	A	4	3	1.00	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.00	15	0.200
202	A	2	2	1.00	15	0.133
203	A	3	3	1.00	15	0.200
204	A	4	3	1.00	15	0.200
205	A	5	3	1.00	15	0.200
206	A	6	3	1.00	15	0.200
207	A	5	3	1.00	15	0.200
208	A	4	3	1.00	15	0.200
209	A	3	2	1.00	15	0.133
210	A	4	3	1.00	15	0.200
211	A	5	3	1.00	15	0.200
212	A	6	3	1.00	15	0.200
213	A	7	3	1.00	15	0.200
214	A	8	3	1.00	15	0.200
215	A	4	2	1.00	17	0.118
216	A	3	2	1.00	17	0.118
217	A	2	2	1.00	17	0.118
218	A	2	2	1.00	17	0.118
219	A	1	1	1.00	17	0.059
220	A	2	2	1.00	17	0.118
221	A	3	2	1.00	17	0.118
222	A	4	2	1.00	17	0.118
223	A	5	2	1.00	17	0.118
224	A	4	2	1.00	17	0.118
225	A	3	2	1.00	17	0.118
226	A	3	3	1.00	17	0.176
227	A	3	2	1.00	17	0.118
228	A	1	1	1.00	17	0.059
229	A	2	2	1.00	17	0.118
230	A	3	2	1.00	17	0.118
231	A	4	2	1.00	17	0.118
232	A	5	2	1.00	17	0.118
233	A	4	2	1.00	17	0.118
234	A	4	3	1.00	17	0.176
235	A	4	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	2	1.00	17	0.118
237	A	1	1	1.00	17	0.059
238	A	2	2	1.00	17	0.118
239	A	3	2	1.00	17	0.118
240	A	4	2	1.00	17	0.118
241	A	4	2	1.00	17	0.118
242	A	3	2	1.00	17	0.118
243	A	2	2	1.00	17	0.118
244	A	1	1	1.00	17	0.059
245	A	1	1	1.00	17	0.059
246	A	2	2	1.00	17	0.118
247	A	3	2	1.00	17	0.118
248	A	4	2	1.00	17	0.118
249	A	5	3	1.00	17	0.176
250	A	4	3	1.00	17	0.176
251	A	3	3	1.00	17	0.176
252	A	2	2	1.00	17	0.118
253	A	1	1	1.00	17	0.059
254	A	2	2	1.00	17	0.118
255	A	3	2	1.00	17	0.118
256	A	4	2	1.00	17	0.118
257	A	5	3	1.00	17	0.176
258	A	4	3	1.00	17	0.176
259	A	3	2	1.00	17	0.118
260	A	1	1	1.00	17	0.059
261	A	2	2	1.00	17	0.118
262	A	3	2	1.00	17	0.118
263	A	4	2	1.00	17	0.118
264	A	5	2	1.00	17	0.118
265	A	1	1	1.00	13	0.077
266	A	6	3	1.00	13	0.231
267	A	5	3	1.00	13	0.231
268	A	4	3	1.00	13	0.231
269	A	3	3	1.00	11	0.273
270	A	2	2	1.00	9	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	1	1	1.00	13	0.077
272	A	2	2	1.00	13	0.154
273	A	3	2	1.00	13	0.154
274	A	2	2	1.00	11	0.182
275	A	2	2	1.00	11	0.182
276	A	2	2	1.00	9	0.222
277	A	2	2	1.00	7	0.286
278	A	2	2	1.00	11	0.182
279	A	2	2	1.00	11	0.182
280	A	2	2	1.00	11	0.182
281	A	6	4	1.00	3	1.333
282	A	8	5	1.00	5	1.000
283	A	10	6	1.00	7	0.857
284	A	11	6	1.00	15	0.400
285	A	9	5	1.00	13	0.385
286	A	7	4	1.00	11	0.364
287	A	0	0	0.00	0	0.000
288	A	8	7	1.00	16	0.438
289	A	7	7	1.00	16	0.438
290	A	6	6	1.00	14	0.429
291	A	5	5	1.00	12	0.417
292	A	0	0	0.00	0	0.000
293	A	8	7	1.00	19	0.368
294	A	7	7	1.00	19	0.368
295	A	6	6	1.00	17	0.353
296	A	5	5	1.00	15	0.333
297	A	0	0	0.00	0	0.000
298	A	11	6	1.00	15	0.400
299	A	9	5	1.00	13	0.385
300	A	7	4	1.00	11	0.364
301	A	0	0	0.00	0	0.000
302	A	8	7	1.00	16	0.438
303	A	7	7	1.00	16	0.438
304	A	6	6	1.00	14	0.429
305	A	5	5	1.00	12	0.417

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	0	0	0.00	0	0.000
307	A	8	7	1.00	19	0.368
308	A	7	7	1.00	19	0.368
309	A	6	6	1.00	17	0.353
310	A	5	5	1.00	15	0.333
311	A	0	0	0.00	0	0.000
312	A	12	6	1.00	15	0.400
313	A	10	6	1.00	15	0.400
314	A	8	5	1.00	13	0.385
315	A	6	4	1.00	7	0.571
316	A	0	0	0.00	0	0.000
317	A	11	6	1.00	15	0.400
318	A	9	5	1.00	13	0.385
319	A	7	4	1.00	11	0.364
320	A	0	0	0.00	0	0.000
321	A	7	7	1.00	20	0.350
322	A	6	6	1.00	18	0.333
323	A	5	5	1.00	16	0.312
324	A	0	0	0.00	0	0.000
325	A	7	7	1.00	21	0.333
326	A	6	6	1.00	19	0.316
327	A	5	5	1.00	17	0.294
328	A	0	0	0.00	0	0.000
329	A	12	6	1.00	15	0.400
330	A	10	6	1.00	15	0.400
331	A	8	5	1.00	13	0.385
332	A	6	4	1.00	7	0.571
333	A	0	0	0.00	0	0.000
334	A	11	6	1.00	15	0.400
335	A	9	5	1.00	13	0.385
336	A	7	4	1.00	11	0.364
337	A	0	0	0.00	0	0.000
338	A	7	7	1.00	20	0.350
339	A	6	6	1.00	18	0.333
340	A	5	5	1.00	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	0	0	0.00	0	0.000
342	A	7	7	1.00	21	0.333
343	A	6	6	1.00	19	0.316
344	A	5	5	1.00	17	0.294
345	A	0	0	0.00	0	0.000
346	A	2	2	1.00	4	0.500
347	A	7	4	1.00	6	0.667
348	A	9	5	1.00	8	0.625
349	A	2	2	1.00	8	0.250
350	A	7	4	1.00	10	0.400
351	A	9	5	1.00	12	0.417
352	A	6	6	1.00	12	0.500
353	A	9	5	1.00	14	0.357
354	A	11	6	1.00	16	0.375
355	A	8	7	1.00	20	0.350
356	A	5	5	1.00	20	0.250
357	A	3	2	1.00	20	0.100
358	A	3	2	1.00	20	0.100
359	A	5	5	1.00	20	0.250
360	A	8	7	1.00	20	0.350
361	A	11	8	1.00	24	0.333

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=127

$$-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^3}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 5/96*d^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^3+1/6*x^6*arctanh(x*e^(1/2)/(
e*x^2+d)^(1/2))-5/96*d^2*x*(e*x^2+d)^(1/2)/e^(5/2)+5/144*d*x^3*(e*x^2+d)^(1
/2)/e^(3/2)-1/36*x^5*(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6356, 327, 223, 212}

$$\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^3}-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-5*d^2*x*Sqrt[d + e*x^2])/(96*e^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*e^
(3/2)) - (x^5*Sqrt[d + e*x^2])/(36*Sqrt[e]) + (5*d^3*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(96*e^3) + (x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\int \frac{x^6}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(5d)\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{36\sqrt{e}}$$

$$= \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(5d^2)\int \frac{}{\sqrt{d}}}{48e}$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}}{\sqrt{d+}}\right.$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}}{\sqrt{d+}}\right.$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}}{\sqrt{d+}}\right.}{96e^3}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 0.78

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\,(-15d^2+10dex^2-8e^2x^4)+48e^3x^6\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+15d^3\log\left(\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 48*e^3*x^6*
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + 15*d^3*Log[Sqrt[e]*x + Sqrt[d + e*x^
2]])/(288*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(97) = 194$.

time = 0.04, size = 253, normalized size = 1.99

method	result
default	$\frac{x^6 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{6} + \frac{e^{\frac{3}{2}}\left(\frac{x^7\sqrt{e x^2+d}}{8e} - \frac{7d\left(\frac{x^5\sqrt{e x^2+d}}{6e} - \frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e} - \frac{d \ln}{}\right.}{}\right.}{6e}\right.}{8e}\right.}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/6*e^(3/2)/d*(1/8*x^7/e*(e*x^2+
d)^(1/2)-7/8*d/e*(1/6*x^5/e*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1
/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(
1/2))))))-1/6*e^(1/2)/d*(1/8*x^5*(e*x^2+d)^(3/2)/e-5/8*d/e*(1/6*x^3*(e*x^2+
d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d)^(1/2)+
1/2/e^(1/2)*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*x^6*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/12*x^6*log(-x*e^(1/2) + sqrt(
x^2*e + d)) - 1/2*d*integrate(-1/3*x^6*e^(1/2*log(x^2*e + d) + 1/2)/(x^4*e^
2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(96) = 192$.
time = 0.35, size = 380, normalized size = 2.99

$$\frac{3\left(16 x^{6} \cosh\left(\frac{1}{2}\right)^{6} + 96 x^{6} \cosh\left(\frac{1}{2}\right)^{5} \sinh\left(\frac{1}{2}\right) + 240 x^{6} \cosh\left(\frac{1}{2}\right)^{4} \sinh\left(\frac{1}{2}\right)^{2} + 320 x^{6} \cosh\left(\frac{1}{2}\right)^{3} \sinh\left(\frac{1}{2}\right)^{3} + 240 x^{6} \cosh\left(\frac{1}{2}\right)^{2} \sinh\left(\frac{1}{2}\right)^{4} + 96 x^{6} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{5} + 16 x^{6} \sinh\left(\frac{1}{2}\right)^{6} + 5 d^{3}\right) \log\left(\frac{2 x^{2} \cosh\left(\frac{1}{2}\right)^{2} + 4 x^{2} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right) + 2 x^{2} \sinh\left(\frac{1}{2}\right)^{2} + 2\left(x \cosh\left(\frac{1}{2}\right) + x \sinh\left(\frac{1}{2}\right)\right) \sqrt{\frac{(x^{2} + d) \cosh\left(\frac{1}{2}\right) + (x^{2} - d) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}} + d}{d}\right) - 2\left(8 x^{5} \cosh\left(\frac{1}{2}\right)^{5} + 40 x^{5} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{4} + 8 x^{5} \sinh\left(\frac{1}{2}\right)^{5} - 10 d x^{3} \cosh\left(\frac{1}{2}\right)^{3} + 15 d^{2} x \cosh\left(\frac{1}{2}\right) + 10\left(8 x^{5} \cosh\left(\frac{1}{2}\right)^{2} - d x^{3}\right) \sinh\left(\frac{1}{2}\right)^{3} + 10\left(8 x^{5} \cosh\left(\frac{1}{2}\right)^{3} - 3 d x^{3} \cosh\left(\frac{1}{2}\right)\right) \sinh\left(\frac{1}{2}\right)^{2} + 5\left(8 x^{5} \cosh\left(\frac{1}{2}\right)^{4} - 6 d x^{3} \cosh\left(\frac{1}{2}\right)^{2} + 3 d^{2} x\right) \sinh\left(\frac{1}{2}\right)\right) \sqrt{\frac{(x^{2} + d) \cosh\left(\frac{1}{2}\right) + (x^{2} - d) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}}}{576\left(\cosh\left(\frac{1}{2}\right)^{6} + 6 \cosh\left(\frac{1}{2}\right)^{5} \sinh\left(\frac{1}{2}\right) + 15 \cosh\left(\frac{1}{2}\right)^{4} \sinh\left(\frac{1}{2}\right)^{2} + 20 \cosh\left(\frac{1}{2}\right)^{3} \sinh\left(\frac{1}{2}\right)^{3} + 15 \cosh\left(\frac{1}{2}\right)^{2} \sinh\left(\frac{1}{2}\right)^{4} + 6 \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{5} + \sinh\left(\frac{1}{2}\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/576*(3*(16*x^6*cosh(1/2)^6 + 96*x^6*cosh(1/2)^5*sinh(1/2) + 240*x^6*cosh(
1/2)^4*sinh(1/2)^2 + 320*x^6*cosh(1/2)^3*sinh(1/2)^3 + 240*x^6*cosh(1/2)^2*
sinh(1/2)^4 + 96*x^6*cosh(1/2)*sinh(1/2)^5 + 16*x^6*sinh(1/2)^6 + 5*d^3)*lo
g((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x
*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/
(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(8*x^5*cosh(1/2)^5 + 40*x^5*cosh(1/2)*
sinh(1/2)^4 + 8*x^5*sinh(1/2)^5 - 10*d*x^3*cosh(1/2)^3 + 15*d^2*x*cosh(1/2)
 + 10*(8*x^5*cosh(1/2)^2 - d*x^3)*sinh(1/2)^3 + 10*(8*x^5*cosh(1/2)^3 - 3*d
*x^3*cosh(1/2))*sinh(1/2)^2 + 5*(8*x^5*cosh(1/2)^4 - 6*d*x^3*cosh(1/2)^2 +
3*d^2*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(
1/2) - sinh(1/2))))/(cosh(1/2)^6 + 6*cosh(1/2)^5*sinh(1/2) + 15*cosh(1/2)^4
*sinh(1/2)^2 + 20*cosh(1/2)^3*sinh(1/2)^3 + 15*cosh(1/2)^2*sinh(1/2)^4 + 6*
cosh(1/2)*sinh(1/2)^5 + sinh(1/2)^6)
```

Sympy [A]
time = 1.64, size = 121, normalized size = 0.95

$$\begin{cases} \frac{5 d^{3} \operatorname{atanh}{\left(\frac{\sqrt{e} x}{\sqrt{d + e x^{2}}} \right)}}{96 e^{3}} - \frac{5 d^{2} x \sqrt{d + e x^{2}}}{96 e^{\frac{5}{2}}} + \frac{5 d x^{3} \sqrt{d + e x^{2}}}{144 e^{\frac{3}{2}}} + \frac{x^{6} \operatorname{atanh}{\left(\frac{\sqrt{e} x}{\sqrt{d + e x^{2}}} \right)}}{6} - \frac{x^{5} \sqrt{d + e x^{2}}}{36 \sqrt{e}} & \text{for}\: e \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*sq
rt(d + e*x**2)/(96*e**(5/2)) + 5*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2)) + x
**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - x**5*sqrt(d + e*x**2)/(36*sqrt(e)
), Ne(e, 0)), (0, True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \operatorname{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.2 $\int x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=101

$$\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -3/32*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^2+1/4*x^4*arctanh(x*e^(1/2)/
(e*x^2+d)^(1/2))+3/32*d*x*(e*x^2+d)^(1/2)/e^(3/2)-1/16*x^3*(e*x^2+d)^(1/2)/
e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6356, 327, 223, 212}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*e^(3/2)) - (x^3*Sqrt[d + e*x^2])/(16*Sqrt[e]) -
 (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^2) + (x^4*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\int \frac{x^4}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(3d)\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{16\sqrt{e}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\int \frac{}{\sqrt{}}}{32}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\,\mathrm{Subs}}{}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} - \frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{1}{4}x^4 \tanh^{-}$$

Mathematica [**A**]
time = 0.04, size = 88, normalized size = 0.87

$$\frac{\sqrt{e}\,x(3d-2ex^2)\sqrt{d+ex^2} + 8e^2x^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - 3d^2\log\left(\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + 8*e^2*x^4*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]] - 3*d^2*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(32*e^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(77) = 154$.
time = 0.01, size = 205, normalized size = 2.03

method	result
default	$\frac{x^{4} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{4}+\frac{e^{\frac{3}{2}} \left(\frac{x^{5} \sqrt{e\,x^{2}+d}}{6 e}-\frac{5 d \left(\frac{x^{3} \sqrt{e\,x^{2}+d}}{4 e}-\frac{3 d \left(\frac{x \sqrt{e\,x^{2}+d}}{2 e}-\frac{d \ln \left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2 e^{\frac{3}{2}}} \cdots\right)}{4 e} \cdots\right)}{6 e} \cdots\right)}{4 d} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/4*e^(3/2)/d*(1/6*x^5/e*(e*x^2+
d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2
)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))-1/4*e^(1/2)/d*(1/6*x^3*(e*
x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d)^(1
/2)+1/2/e^(1/2)*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/8*x^4*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/8*x^4*log(-x*e^(1/2) + sqrt(x^
2*e + d)) - 1/2*d*integrate(-1/2*x^4*e^(1/2*log(x^2*e + d) + 1/2)/(x^4*e^2
+ d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(76) = 152.
time = 0.34, size = 263, normalized size = 2.60

$$\frac{\left(8\,x^{4} \cosh\left(\frac{1}{2}\right)^{4}+32\,x^{4} \cosh\left(\frac{1}{2}\right)^{3} \sinh\left(\frac{1}{2}\right)+48\,x^{4} \cosh\left(\frac{1}{2}\right)^{2} \sinh\left(\frac{1}{2}\right)^{2}+32\,x^{4} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{3}+8\,x^{4} \sinh\left(\frac{1}{2}\right)^{4}-3\,d^{2}\right) \log\left(\frac{2\,x^{2} \cosh\left(\frac{1}{2}\right)^{2}+4\,x^{2} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)+2\,x^{2} \sinh\left(\frac{1}{2}\right)^{2}+2\,\left(x \cosh\left(\frac{1}{2}\right)+x \sinh\left(\frac{1}{2}\right)\right) \sqrt{\frac{\left(x^{2}+d\right) \cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\,\left(2\,x^{3} \cosh\left(\frac{1}{2}\right)^{3}+6\,x^{3} \cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{2}+2\,x^{3} \sinh\left(\frac{1}{2}\right)^{3}-3\,d x \cosh\left(\frac{1}{2}\right)+3\,\left(2\,x^{3} \cosh\left(\frac{1}{2}\right)^{2}-d x\right) \sinh\left(\frac{1}{2}\right)\right) \sqrt{\frac{\left(x^{2}+d\right) \cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right) \sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{64\,\left(\cosh\left(\frac{1}{2}\right)^{4}+4\,\cosh\left(\frac{1}{2}\right)^{3} \sinh\left(\frac{1}{2}\right)+6\,\cosh\left(\frac{1}{2}\right)^{2} \sinh\left(\frac{1}{2}\right)^{2}+4\,\cosh\left(\frac{1}{2}\right) \sinh\left(\frac{1}{2}\right)^{3}+\sinh\left(\frac{1}{2}\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/64*((8*x^4*cosh(1/2)^4 + 32*x^4*cosh(1/2)^3*sinh(1/2) + 48*x^4*cosh(1/2)^
2*sinh(1/2)^2 + 32*x^4*cosh(1/2)*sinh(1/2)^3 + 8*x^4*sinh(1/2)^4 - 3*d^2)*l
og((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(
x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))
/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(2*x^3*cosh(1/2)^3 + 6*x^3*cosh(1/2)*
sinh(1/2)^2 + 2*x^3*sinh(1/2)^3 - 3*d*x*cosh(1/2) + 3*(2*x^3*cosh(1/2)^2 -
d*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2)
 - sinh(1/2))))/(cosh(1/2)^4 + 4*cosh(1/2)^3*sinh(1/2) + 6*cosh(1/2)^2*sinh
(1/2)^2 + 4*cosh(1/2)*sinh(1/2)^3 + sinh(1/2)^4)
```

Sympy [A]
time = 0.61, size = 95, normalized size = 0.94

$$\begin{cases} -\frac{3d^2 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{3dx\sqrt{d+ex^2}}{32e^{\frac{3}{2}}} + \frac{x^4 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqrt
(d + e*x**2)/(32*e**(3/2)) + x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4 - x**
3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.3 $\int x \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=75

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 1/4*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e+1/2*x^2*arctanh(x*e^(1/2)/(e*x^2
+d)^(1/2))-1/4*x*(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6356, 327, 223, 212}

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -1/4*(x*Sqrt[d + e*x^2])/Sqrt[e] + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/(4*e) + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{e} \int \frac{x^2}{\sqrt{d+ex^2}}\,dx \\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{d\int \frac{1}{\sqrt{d+ex^2}}\,dx}{4\sqrt{e}} \\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{d\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{}{\sqrt{d+}}\right.}{4\sqrt{e}} \\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 76, normalized size = 1.01

$$
-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{d\log\left(\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{4e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -1/4*(x*Sqrt[d + e*x^2])/Sqrt[e] + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(4*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(57) = 114$.
time = 0.01, size = 157, normalized size = 2.09

method	result

default	$\frac{x^{2} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{2}+\frac{e^{\frac{3}{2}}\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{2d}-\frac{\sqrt{e}\left(\frac{x\left(e\,x^{2}\right.}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/2*e^(3/2)/d*(1/4*x^3/e*(e*x^2+
d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2
+d)^(1/2))))-1/2*e^(1/2)/d*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d
)^(1/2)+1/2/e^(1/2)*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/4*x^2*log(-x*e^(1/2) + sqrt(x^
2*e + d)) - 1/2*d*integrate(-x^2*e^(1/2*log(x^2*e + d) + 1/2)/(x^4*e^2 + d*
x^2*e - (x^2*e + d)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(56) = 112$.
time = 0.35, size = 168, normalized size = 2.24

$$\frac{\left(2x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^{2}\sinh\left(\frac{1}{2}\right)^{2}+d\right)\log\left(\frac{2x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^{2}\sinh\left(\frac{1}{2}\right)^{2}+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^{2}+d)\cosh\left(\frac{1}{2}\right)+(x^{2}-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^{2}+d)\cosh\left(\frac{1}{2}\right)+(x^{2}-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{8\left(\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + d
)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 +
2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/
2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt((
(x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(
1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2)
```

Sympy [A]
time = 0.33, size = 66, normalized size = 0.88

$$\begin{cases} \frac{d\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{x^2\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2} - \frac{x\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + x**2*atanh(sqrt(e)*x
/sqrt(d + e*x**2))/2 - x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0, True)
)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.4 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal. Leaf size=238

$$-\frac{\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{1+}}{}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2*d
^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x
*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^
(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln(x)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^
(1/2)+1/2*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x
^2/d)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6354, 2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\frac{\sqrt{d}\,\sqrt{\frac{ex^2}{d}+1}\,\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\log(x)\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d+ex^2}}+\log(x)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -1/2*(Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[d +
e*x^2] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 -
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/Sqrt[d + e*x^2] - (Sqrt[d]*Sqrt[1 + (e
*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[d + e*x^2] + ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*PolyLog[2,
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[d + e*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6354

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Sim
p[ArcTanh[c*(x/Sqrt[a + b*x^2])]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b
*x^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{(}{}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{(}{}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1\right.}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1\right.}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1\right.}{\sqrt{d+ex^2}}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

[Out] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x, x]

Maple [A]
time = 0.11, size = 209, normalized size = 0.88

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)^{2}}{2}+\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)\ln\left(1+\frac{\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}+1}{\sqrt{-\frac{x^{2}e}{e\,x^{2}+d}+1}}\right)+\operatorname{polylog}\left(2,-\frac{\sqrt{\cdot}}{\sqrt{-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x,method=_RETURNVERBOSE)

[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))^2+arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*ln(1+(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+polylog(2,-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*ln(1-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+polylog(2,(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")

[Out] integrate(arctanh(x*e^(1/2)/sqrt(x^2*e + d))/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")

[Out] integral(arctanh(x*e^(1/2)/sqrt(x^2*e + d))/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^{2}}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

3.5 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e^(1/2)*(e*x^2+d)^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6356, 270}

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2} + \frac{1}{2}\sqrt{e}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{2dx} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.94

$$-\frac{\sqrt{e}\,x\sqrt{d+ex^2}+d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[e]*x*Sqrt[d + e*x^2] + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
d*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(41) = 82$.
time = 0.01, size = 111, normalized size = 2.09

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{e\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e/d*ln(x*e^(1/2)+(e*x^2+d)^
(1/2))+1/2*e^(1/2)/d*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1/2*x*(e*x^2+d)^(1/2)+1
/2/e^(1/2)*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.94

$$-\frac{\operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)}{2\,x^2}-\frac{x^2e^{\frac{3}{2}}+de^{\frac{1}{2}}}{2\,\sqrt{x^2e+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(x*e^(1/2)/sqrt(x^2*e + d))/x^2 - 1/2*(x^2*e^(3/2) + d*e^(1/2))
/(sqrt(x^2*e + d)*d*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(41) = 82$.
time = 0.36, size = 129, normalized size = 2.43

$$-\frac{d\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2
)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*s
inh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 2*(x*cosh(1/2) + x*sinh(1/2))*
sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/
(d*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

3.6 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{12dx^3}+\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

```
[Out] -1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/6*e^(3/2)*(e*x^2+d)^(1/2)/d^2
/x-1/12*e^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 277, 270}

$$\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{12dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -1/12*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x^3) + (e^(3/2)*Sqrt[d + e*x^2])/(6*d^2*
x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{1}{4}\sqrt{e}\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx\\ &= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{e^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{6d}\\ &= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{12dx^3}+\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.80

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\,(-d+2ex^2)-3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(12*d^2*x^4)
```

Maple [A]
time = 0.01, size = 62, normalized size = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{e}\,(e\,x^2+d)^{\frac{3}{2}}}{12d^2x^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*e^(3/2)*(e*x^2+d)^(1/2)/d^2
/x-1/12*e^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.77

$$\frac{\sqrt{x^2e+d}\,e^{\frac{3}{2}}}{4\,d^2x}-\frac{(x^2e+d)^{\frac{3}{2}}e^{\frac{1}{2}}}{12\,d^2x^3}-\frac{\operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^2*e + d)*e^(3/2)/(d^2*x) - 1/12*(x^2*e + d)^(3/2)*e^(1/2)/(d^2*x
^3) - 1/4*arctanh(x*e^(1/2)/sqrt(x^2*e + d))/x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(61) = 122.
time = 0.35, size = 176, normalized size = 2.23

$$-\frac{3\,d^2\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\,\left(2\,x^3\cosh\left(\frac{1}{2}\right)^3+6\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2+2\,x^3\sinh\left(\frac{1}{2}\right)^3-dx\cosh\left(\frac{1}{2}\right)+\left(6\,x^3\cosh\left(\frac{1}{2}\right)^2-dx\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{24\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*d^2*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sin
h(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 -
 d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(2*x^3*cosh(1/2)^3 + 6*
x^3*cosh(1/2)*sinh(1/2)^2 + 2*x^3*sinh(1/2)^3 - d*x*cosh(1/2) + (6*x^3*cosh
(1/2)^2 - d*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/
(cosh(1/2) - sinh(1/2))))/(d^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**5, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

3.7 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{30dx^5}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[Out] -1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6+2/45*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x^3-4/45*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*e^(1/2)*(e*x^2+d)^(1/2)/d/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 277, 270}

$$-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{30dx^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -1/30*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x^5) + (2*e^(3/2)*Sqrt[d + e*x^2])/(45*d
^2*x^3) - (4*e^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/(6*x^6)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
```

```
x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{1}{6}\sqrt{e}\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{30dx^5} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} - \frac{\left(2e^{3/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{30dx^5} + \frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(4e^{5/2}\right)\int \frac{1}{x^2\sqrt{}}}{45d}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{30dx^5} + \frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.70

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\left(-3d^2+4dex^2-8e^2x^4\right)-15d^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

Maple [A]
time = 0.01, size = 110, normalized size = 1.05

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{e}\,\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*e^(3/2)/d*(-1/3/d/x^3*(e*x^
2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*e^(1/2)/d*(-1/5/d/x^5*(e*x^2+d)
^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

Maxima [A]
time = 0.26, size = 102, normalized size = 0.97

$$-\frac{(2\,x^4e^2+dx^2e-d^2)e^{\frac{3}{2}}}{18\,\sqrt{x^2e+d}\,d^3x^3}-\frac{\operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)}{6\,x^6}+\frac{(2\,x^4e^2-dx^2e-3\,d^2)\sqrt{x^2e+d}\,e^{\frac{1}{2}}}{90\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*x^4*e^2 + d*x^2*e - d^2)*e^(3/2)/(sqrt(x^2*e + d)*d^3*x^3) - 1/6*a
rctanh(x*e^(1/2)/sqrt(x^2*e + d))/x^6 + 1/90*(2*x^4*e^2 - d*x^2*e - 3*d^2)*
sqrt(x^2*e + d)*e^(1/2)/(d^3*x^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(81) = 162$.
time = 0.36, size = 246, normalized size = 2.34

$$-\frac{15\,d^3\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+2\left(8\,x^5\cosh\left(\frac{1}{2}\right)^5+40\,x^5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^4+8\,x^5\sinh\left(\frac{1}{2}\right)^5-4\,dx^3\cosh\left(\frac{1}{2}\right)^3+3\,d^2x\cosh\left(\frac{1}{2}\right)+4\left(20\,x^5\cosh\left(\frac{1}{2}\right)^2-dx^3\right)\sinh\left(\frac{1}{2}\right)^3+4\left(20\,x^5\cosh\left(\frac{1}{2}\right)^3-3\,dx^3\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)^2+\left(40\,x^5\cosh\left(\frac{1}{2}\right)^4-12\,dx^3\cosh\left(\frac{1}{2}\right)^2+3\,d^2x\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{180\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/180*(15*d^3*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*s
inh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2
 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 2*(8*x^5*cosh(1/2)^5 +
40*x^5*cosh(1/2)*sinh(1/2)^4 + 8*x^5*sinh(1/2)^5 - 4*d*x^3*cosh(1/2)^3 + 3*
d^2*x*cosh(1/2) + 4*(20*x^5*cosh(1/2)^2 - d*x^3)*sinh(1/2)^3 + 4*(20*x^5*co
sh(1/2)^3 - 3*d*x^3*cosh(1/2))*sinh(1/2)^2 + (40*x^5*cosh(1/2)^4 - 12*d*x^3
*cosh(1/2)^2 + 3*d^2*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*si
nh(1/2))/(cosh(1/2) - sinh(1/2))))/(d^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**7, x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)

[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)

3.8 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7}+\frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3}+\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

```
[Out] -1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8+3/140*e^(3/2)*(e*x^2+d)^(1/2)/d
^2/x^5-1/35*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3+2/35*e^(7/2)*(e*x^2+d)^(1/2)/d^
4/x-1/56*e^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 277, 270}

$$\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3}+\frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -1/56*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x^7) + (3*e^(3/2)*Sqrt[d + e*x^2])/(140*
d^2*x^5) - (e^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) + (2*e^(7/2)*Sqrt[d + e*x
^2])/(35*d^4*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
```

```
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{e}\int \frac{1}{x^8\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} - \frac{\left(3e^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3e^{5/2}\right)\int \ldots}{35 \ldots}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} \ldots$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \ldots$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 0.65

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\left(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6\right)-35d^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^6
) - 35*d^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

Maple [A]
time = 0.01, size = 158, normalized size = 1.21

method	result

default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{8x^{8}}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^{2}+d}}{5d\,x^{5}}-\frac{4e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{5d}\right)}{8d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7d\,x^{7}}-\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*e^(3/2)/d*(-1/5/d/x^5*(e*x^
2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))
)+1/8*e^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(e*x^2+d)^(
3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

Maxima [A]
time = 0.28, size = 123, normalized size = 0.94

$$-\frac{\operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^{2}e+d}}\right)}{8\,x^{8}}+\frac{(8\,x^{6}e^{3}+4\,dx^{4}e^{2}-d^{2}x^{2}e+3\,d^{3})e^{\frac{3}{2}}}{120\,\sqrt{x^{2}e+d}\,d^{4}x^{5}}-\frac{(8\,x^{6}e^{3}-4\,dx^{4}e^{2}+3\,d^{2}x^{2}e+15\,d^{3})\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}}{840\,d^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*arctanh(x*e^(1/2)/sqrt(x^2*e + d))/x^8 + 1/120*(8*x^6*e^3 + 4*d*x^4*e^
2 - d^2*x^2*e + 3*d^3)*e^(3/2)/(sqrt(x^2*e + d)*d^4*x^5) - 1/840*(8*x^6*e^3
 - 4*d*x^4*e^2 + 3*d^2*x^2*e + 15*d^3)*sqrt(x^2*e + d)*e^(1/2)/(d^4*x^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(101) = 202$.
time = 0.36, size = 340, normalized size = 2.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] -1/560*(35*d^4*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*s
inh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2
 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(16*x^7*cosh(1/2)^7 +
 112*x^7*cosh(1/2)*sinh(1/2)^6 + 16*x^7*sinh(1/2)^7 - 8*d*x^5*cosh(1/2)^5 +
 6*d^2*x^3*cosh(1/2)^3 + 8*(42*x^7*cosh(1/2)^2 - d*x^5)*sinh(1/2)^5 - 5*d^3
*x*cosh(1/2) + 40*(14*x^7*cosh(1/2)^3 - d*x^5*cosh(1/2))*sinh(1/2)^4 + 2*(2
80*x^7*cosh(1/2)^4 - 40*d*x^5*cosh(1/2)^2 + 3*d^2*x^3)*sinh(1/2)^3 + 2*(168
*x^7*cosh(1/2)^5 - 40*d*x^5*cosh(1/2)^3 + 9*d^2*x^3*cosh(1/2))*sinh(1/2)^2
```

```
+ (112*x^7*cosh(1/2)^6 - 40*d*x^5*cosh(1/2)^4 + 18*d^2*x^3*cosh(1/2)^2 - 5*
d^3*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/
2) - sinh(1/2))))/(d^4*x^8)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**9, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

3.9 $\int x^6 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=114

$$\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{d^2(d+ex^2)^{3/2}}{7e^{7/2}}+\frac{3d(d+ex^2)^{5/2}}{35e^{7/2}}-\frac{(d+ex^2)^{7/2}}{49e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/7*d^2*(e*x^2+d)^(3/2)/e^(7/2)+3/35*d*(e*x^2+d)^(5/2)/e^(7/2)-1/49*(e*x^2
+d)^(7/2)/e^(7/2)+1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*d^3*(e*x^2
+d)^(1/2)/e^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 272, 45}

$$\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{d^2(d+ex^2)^{3/2}}{7e^{7/2}}-\frac{(d+ex^2)^{7/2}}{49e^{7/2}}+\frac{3d(d+ex^2)^{5/2}}{35e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*e^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*e^(7/2)) + (
3*d*(d + e*x^2)^(5/2))/(35*e^(7/2)) - (d + e*x^2)^(7/2)/(49*e^(7/2)) + (x^7
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^6 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{e}\int \frac{x^7}{\sqrt{d+ex^2}}\,dx\\ &= \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{d+ex}}\,dx, x, x^2\right)\\ &= \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int \left(-\frac{d^3}{e^3\sqrt{d+ex}} + \frac{3d^2\sqrt{d+}}{e^3}\right.\right.\\ &= \frac{d^3\sqrt{d+ex^2}}{7e^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7e^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35e^{7/2}} - \frac{(d+ex^2)^{7/2}}{49e^{7/2}} + \frac{1}{7}x^7 \tan\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 79, normalized size = 0.69

$$\frac{\sqrt{d+ex^2}\,(16d^3 - 8d^2ex^2 + 6de^2x^4 - 5e^3x^6)}{245e^{7/2}} + \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*e^(
7/2)) + (x^7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(84) = 168.
time = 0.01, size = 224, normalized size = 1.96

method	result

default	$\frac{x^7 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{7}+\frac{e^{\frac{3}{2}}\left(\frac{x^8\sqrt{e\,x^2+d}}{9e}-\frac{8d\left(\frac{x^6\sqrt{e\,x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\right.}{5e}\right.}{7e}\right.}{9e}\right.}{7d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*e^(3/2)/d*(1/9*x^8/e*(e*x^2+
d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1
/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))-1/7*e^
(1/2)/d*(1/9*x^6*(e*x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4/7*d
/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))
```

Maxima [A]
time = 0.26, size = 162, normalized size = 1.42

$$\frac{1}{7}\,x^7 \operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)-\frac{\left(35\,(x^2e+d)^{\frac{9}{2}}-135\,(x^2e+d)^{\frac{7}{2}}d+189\,(x^2e+d)^{\frac{5}{2}}d^2-105\,(x^2e+d)^{\frac{3}{2}}d^3\right)e^{\left(-\frac{7}{2}\right)}}{2205\,d}+\frac{\left(35\,(x^2e+d)^{\frac{9}{2}}-180\,(x^2e+d)^{\frac{7}{2}}d+378\,(x^2e+d)^{\frac{5}{2}}d^2-420\,(x^2e+d)^{\frac{3}{2}}d^3+315\,\sqrt{x^2e+d}\,d^4\right)e^{\left(-\frac{7}{2}\right)}}{2205\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctanh(x*e^(1/2)/sqrt(x^2*e + d)) - 1/2205*(35*(x^2*e + d)^(9/2) -
 135*(x^2*e + d)^(7/2)*d + 189*(x^2*e + d)^(5/2)*d^2 - 105*(x^2*e + d)^(3/2
)*d^3)*e^(-7/2)/d + 1/2205*(35*(x^2*e + d)^(9/2) - 180*(x^2*e + d)^(7/2)*d
+ 378*(x^2*e + d)^(5/2)*d^2 - 420*(x^2*e + d)^(3/2)*d^3 + 315*sqrt(x^2*e +
d)*d^4)*e^(-7/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(84) = 168$.
time = 0.36, size = 442, normalized size = 3.88

$$\frac{35\left(x^7\cosh\left(\frac{1}{2}\right)^7+7\,x^7\cosh\left(\frac{1}{2}\right)^6\sinh\left(\frac{1}{2}\right)+21\,x^7\cosh\left(\frac{1}{2}\right)^5\sinh\left(\frac{1}{2}\right)^2+35\,x^7\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)^3+35\,x^7\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^4+21\,x^7\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^5+7\,x^7\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^6+x^7\sinh\left(\frac{1}{2}\right)^7\right)\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\left(5\,x^6\cosh\left(\frac{1}{2}\right)^6+30\,x^6\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^5+5\,x^6\sinh\left(\frac{1}{2}\right)^6-6\,dx^4\cosh\left(\frac{1}{2}\right)^4+8\,d^2x^2\cosh\left(\frac{1}{2}\right)^2+3\left(25\,x^6\cosh\left(\frac{1}{2}\right)^2-2\,dx^4\right)\sinh\left(\frac{1}{2}\right)^4+4\left(25\,x^6\cosh\left(\frac{1}{2}\right)^3-6\,dx^4\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)^3-16\,d^3+\left(75\,x^6\cosh\left(\frac{1}{2}\right)^4-36\,dx^4\cosh\left(\frac{1}{2}\right)^2+8\,d^2x^2\right)\sinh\left(\frac{1}{2}\right)^2+2\left(15\,x^6\cosh\left(\frac{1}{2}\right)^5-12\,dx^4\cosh\left(\frac{1}{2}\right)^3+8\,d^2x^2\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{490\left(\cosh\left(\frac{1}{2}\right)^7+7\cosh\left(\frac{1}{2}\right)^6\sinh\left(\frac{1}{2}\right)+21\cosh\left(\frac{1}{2}\right)^5\sinh\left(\frac{1}{2}\right)^2+35\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)^3+35\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^4+21\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^5+7\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^6+\sinh\left(\frac{1}{2}\right)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/490*(35*(x^7*cosh(1/2)^7 + 7*x^7*cosh(1/2)^6*sinh(1/2) + 21*x^7*cosh(1/2)
^5*sinh(1/2)^2 + 35*x^7*cosh(1/2)^4*sinh(1/2)^3 + 35*x^7*cosh(1/2)^3*sinh(1
/2)^4 + 21*x^7*cosh(1/2)^2*sinh(1/2)^5 + 7*x^7*cosh(1/2)*sinh(1/2)^6 + x^7*
sinh(1/2)^7)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sin
h(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 -
 d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(5*x^6*cosh(1/2)^6 + 30
*x^6*cosh(1/2)*sinh(1/2)^5 + 5*x^6*sinh(1/2)^6 - 6*d*x^4*cosh(1/2)^4 + 8*d^
2*x^2*cosh(1/2)^2 + 3*(25*x^6*cosh(1/2)^2 - 2*d*x^4)*sinh(1/2)^4 + 4*(25*x^
6*cosh(1/2)^3 - 6*d*x^4*cosh(1/2))*sinh(1/2)^3 - 16*d^3 + (75*x^6*cosh(1/2)
^4 - 36*d*x^4*cosh(1/2)^2 + 8*d^2*x^2)*sinh(1/2)^2 + 2*(15*x^6*cosh(1/2)^5
- 12*d*x^4*cosh(1/2)^3 + 8*d^2*x^2*cosh(1/2))*sinh(1/2))*sqrt(((x^2 + d)*co
sh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^7 + 7*c
osh(1/2)^6*sinh(1/2) + 21*cosh(1/2)^5*sinh(1/2)^2 + 35*cosh(1/2)^4*sinh(1/2
)^3 + 35*cosh(1/2)^3*sinh(1/2)^4 + 21*cosh(1/2)^2*sinh(1/2)^5 + 7*cosh(1/2)
*sinh(1/2)^6 + sinh(1/2)^7)
```

Sympy [**A**]
time = 1.90, size = 116, normalized size = 1.02

$$\begin{cases} \frac{16d^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}} - \frac{8d^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}} + \frac{6dx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}} + \frac{x^7 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*d**2*x**2*sqrt(d + e
*x**2)/(245*e**(5/2)) + 6*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + x**7*ata
nh(sqrt(e)*x/sqrt(d + e*x**2))/7 - x**6*sqrt(d + e*x**2)/(49*sqrt(e)), Ne(e
, 0)), (0, True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \operatorname{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.10 $\int x^4 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=91

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}+\frac{2d(d+ex^2)^{3/2}}{15e^{5/2}}-\frac{(d+ex^2)^{5/2}}{25e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 2/15*d*(e*x^2+d)^(3/2)/e^(5/2)-1/25*(e*x^2+d)^(5/2)/e^(5/2)+1/5*x^5*arctanh
(x*e^(1/2)/(e*x^2+d)^(1/2))-1/5*d^2*(e*x^2+d)^(1/2)/e^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 272, 45}

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}-\frac{(d+ex^2)^{5/2}}{25e^{5/2}}+\frac{2d(d+ex^2)^{3/2}}{15e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -1/5*(d^2*Sqrt[d + e*x^2])/e^(5/2) + (2*d*(d + e*x^2)^(3/2))/(15*e^(5/2)) -
 (d + e*x^2)^(5/2)/(25*e^(5/2)) + (x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{e}\int\frac{x^5}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int\left(\frac{d^2}{e^2\sqrt{d+ex}}-\frac{2d\sqrt{d+e}}{e^2}\right.\right.\\
&= -\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}+\frac{2d(d+ex^2)^{3/2}}{15e^{5/2}}-\frac{(d+ex^2)^{5/2}}{25e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.75

$$
-\frac{\sqrt{d+ex^2}\,(8d^2-4dex^2+3e^2x^4)}{75e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
[Out] -1/75*(Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/e^(5/2) + (x^5*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(67) = 134.
time = 0.01, size = 176, normalized size = 1.93

method	result
default	$\frac{x^5\,\text{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5}+\frac{e^{\frac{3}{2}}\left(\frac{x^6\sqrt{e\,x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/5*x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/5*e^(3/2)/d*(1/7*x^6/e*(e*x^2+
d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1
```

```
/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*e^(1/2)/d*(1/7*x^4*(e*x^2+d)^(3/2)/e-4
/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

Maxima [A]
time = 0.28, size = 132, normalized size = 1.45

$$\frac{1}{5}\,x^5\,\operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)-\frac{\left(15\,(x^2e+d)^{\frac{7}{2}}-42\,(x^2e+d)^{\frac{5}{2}}d+35\,(x^2e+d)^{\frac{3}{2}}d^2\right)e^{\left(-\frac{5}{2}\right)}}{525\,d}+\frac{\left(5\,(x^2e+d)^{\frac{7}{2}}-21\,(x^2e+d)^{\frac{5}{2}}d+35\,(x^2e+d)^{\frac{3}{2}}d^2-35\,\sqrt{x^2e+d}\,d^3\right)e^{\left(-\frac{5}{2}\right)}}{175\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctanh(x*e^(1/2)/sqrt(x^2*e + d)) - 1/525*(15*(x^2*e + d)^(7/2) -
42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2)*e^(-5/2)/d + 1/175*(5*(x
^2*e + d)^(7/2) - 21*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2 - 35*sq
rt(x^2*e + d)*d^3)*e^(-5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(67) = 134$.
time = 0.36, size = 315, normalized size = 3.46

$$\frac{15\left(x^5\cosh\left(\frac{1}{2}\right)^5+5\,x^5\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)+10\,x^5\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^2+10\,x^5\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^3+5\,x^5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^4+x^5\sinh\left(\frac{1}{2}\right)^5\right)\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\left(3\,x^4\cosh\left(\frac{1}{2}\right)^4+12\,x^4\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^3+3\,x^4\sinh\left(\frac{1}{2}\right)^4-4\,dx^2\cosh\left(\frac{1}{2}\right)^2+2\left(9\,x^4\cosh\left(\frac{1}{2}\right)^2-2\,dx^2\right)\sinh\left(\frac{1}{2}\right)^2+8\,d^2+4\left(3\,x^4\cosh\left(\frac{1}{2}\right)^3-2\,dx^2\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{150\left(\cosh\left(\frac{1}{2}\right)^5+5\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)+10\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^2+10\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^3+5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^4+\sinh\left(\frac{1}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/150*(15*(x^5*cosh(1/2)^5 + 5*x^5*cosh(1/2)^4*sinh(1/2) + 10*x^5*cosh(1/2)
^3*sinh(1/2)^2 + 10*x^5*cosh(1/2)^2*sinh(1/2)^3 + 5*x^5*cosh(1/2)*sinh(1/2)
^4 + x^5*sinh(1/2)^5)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) +
2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2)
 + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(3*x^4*cosh(1/
2)^4 + 12*x^4*cosh(1/2)*sinh(1/2)^3 + 3*x^4*sinh(1/2)^4 - 4*d*x^2*cosh(1/2)
^2 + 2*(9*x^4*cosh(1/2)^2 - 2*d*x^2)*sinh(1/2)^2 + 8*d^2 + 4*(3*x^4*cosh(1/
2)^3 - 2*d*x^2*cosh(1/2))*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*
sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^5 + 5*cosh(1/2)^4*sinh(1/2)
 + 10*cosh(1/2)^3*sinh(1/2)^2 + 10*cosh(1/2)^2*sinh(1/2)^3 + 5*cosh(1/2)*si
nh(1/2)^4 + sinh(1/2)^5)
```

Sympy [A]
time = 0.81, size = 90, normalized size = 0.99

$$\begin{cases} -\frac{8d^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}}+\frac{4dx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}}+\frac{x^5\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for}\ e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*d*x**2*sqrt(d + e*x**
2)/(75*e**(3/2)) + x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - x**4*sqrt(d +
 e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{atanh}\left(\frac{\sqrt{e}\,\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.11 $\int x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=68

$$\frac{d\sqrt{d+ex^2}}{3e^{3/2}} - \frac{(d+ex^2)^{3/2}}{9e^{3/2}} + \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/9*(e*x^2+d)^(3/2)/e^(3/2)+1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3
*d*(e*x^2+d)^(1/2)/e^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6356, 272, 45}

$$-\frac{(d+ex^2)^{3/2}}{9e^{3/2}} + \frac{d\sqrt{d+ex^2}}{3e^{3/2}} + \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*e^(3/2)) - (d + e*x^2)^(3/2)/(9*e^(3/2)) + (x^3*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{3}x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{3}\sqrt{e} \int \frac{x^3}{\sqrt{d+ex^2}}\, dx \\
&= \frac{1}{3}x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\, \text{Subst}\left(\int \frac{x}{\sqrt{d+ex}}\, dx, x, x^2\right) \\
&= \frac{1}{3}x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\, \text{Subst}\left(\int \left(-\frac{d}{e\sqrt{d+ex}} + \frac{\sqrt{d+ex}}{e}\right.\right. \\
&= \frac{d\sqrt{d+ex^2}}{3e^{3/2}} - \frac{(d+ex^2)^{3/2}}{9e^{3/2}} + \frac{1}{3}x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.82

$$
\frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{e^{3/2}} + 3x^3 \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 3*x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.
time = 0.01, size = 128, normalized size = 1.88

method	result
default	$\frac{x^3 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{3} + \frac{e^{\frac{3}{2}}\left(\frac{x^4\sqrt{e\,x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e} - \frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{3d} - \frac{\sqrt{e}\left(\frac{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}{5e} - \right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3*e^(3/2)/d*(1/5*x^4/e*(e*x^2+
d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))-1/3
*e^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(50) = 100$.
time = 0.27, size = 102, normalized size = 1.50

$$\frac{1}{3}\,x^3 \operatorname{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right) - \frac{\left(3\,(x^2e+d)^{\frac{5}{2}} - 5\,(x^2e+d)^{\frac{3}{2}}d\right)e^{\left(-\frac{3}{2}\right)}}{45\,d} + \frac{\left(3\,(x^2e+d)^{\frac{5}{2}} - 10\,(x^2e+d)^{\frac{3}{2}}d + 15\,\sqrt{x^2e+d}\,d^2\right)e^{\left(-\frac{3}{2}\right)}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(x*e^(1/2)/sqrt(x^2*e + d)) - 1/45*(3*(x^2*e + d)^(5/2) - 5*
(x^2*e + d)^(3/2)*d)*e^(-3/2)/d + 1/45*(3*(x^2*e + d)^(5/2) - 10*(x^2*e + d
)^(3/2)*d + 15*sqrt(x^2*e + d)*d^2)*e^(-3/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(50) = 100$.
time = 0.37, size = 209, normalized size = 3.07

$$\frac{3\left(x^3\cosh\left(\frac{1}{2}\right)^3 + 3\,x^3\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right) + 3\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2 + x^3\sinh\left(\frac{1}{2}\right)^3\right)\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2 + 4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right) + 2\,x^2\sinh\left(\frac{1}{2}\right)^2 + 2\left(x\cosh\left(\frac{1}{2}\right) + x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right) + (x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}} + d}{d}\right) - 2\left(x^2\cosh\left(\frac{1}{2}\right)^2 + 2\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right) + x^2\sinh\left(\frac{1}{2}\right)^2 - 2\,d\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right) + (x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right) - \sinh\left(\frac{1}{2}\right)}}}{18\left(\cosh\left(\frac{1}{2}\right)^3 + 3\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right) + 3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2 + \sinh\left(\frac{1}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(x^3*cosh(1/2)^3 + 3*x^3*cosh(1/2)^2*sinh(1/2) + 3*x^3*cosh(1/2)*si
nh(1/2)^2 + x^3*sinh(1/2)^3)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(
1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*co
sh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*(x^2*co
sh(1/2)^2 + 2*x^2*cosh(1/2)*sinh(1/2) + x^2*sinh(1/2)^2 - 2*d)*sqrt(((x^2 +
 d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^3
 + 3*cosh(1/2)^2*sinh(1/2) + 3*cosh(1/2)*sinh(1/2)^2 + sinh(1/2)^3)
```

Sympy [A]
time = 0.43, size = 65, normalized size = 0.96

$$\begin{cases} \frac{2d\sqrt{d+ex^2}}{9e^{\frac{3}{2}}} + \frac{x^3 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*d*sqrt(d + e*x**2)/(9*e**(3/2)) + x**3*atanh(sqrt(e)*x/sqrt(d
+ e*x**2))/3 - x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.12 $\int \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{d+ex^2}}{\sqrt{e}}+x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6352, 267}

$$x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6352

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*A
rcTanh[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /
; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx &= x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\sqrt{e}\int \frac{x}{\sqrt{d+ex^2}}\,dx\\ &= -\frac{\sqrt{d+ex^2}}{\sqrt{e}}+x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 40, normalized size = 1.00

$$-\frac{\sqrt{d+ex^2}}{\sqrt{e}}+x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.
time = 0.00, size = 76, normalized size = 1.90

method	result	size
default	$x\,\mathrm{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)+\frac{e^{\frac{3}{2}}\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{3\sqrt{e}\,d}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+e^(3/2)/d*(1/3*x^2/e*(e*x^2+d)^(1/2)-2
/3*d/e^2*(e*x^2+d)^(1/2))-1/3/e^(1/2)/d*(e*x^2+d)^(3/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.
time = 0.28, size = 66, normalized size = 1.65

$$x\,\mathrm{artanh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}\right)-\frac{\left(x^2e+d\right)^{\frac{3}{2}}e^{\left(-\frac{1}{2}\right)}}{3\,d}+\frac{\left(\left(x^2e+d\right)^{\frac{3}{2}}-3\,\sqrt{x^2e+d}\,d\right)e^{\left(-\frac{1}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctanh(x*e^(1/2)/sqrt(x^2*e + d)) - 1/3*(x^2*e + d)^(3/2)*e^(-1/2)/d + 1
/3*((x^2*e + d)^(3/2) - 3*sqrt(x^2*e + d)*d)*e^(-1/2)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(32) = 64$.
time = 0.35, size = 129, normalized size = 3.22

$$\frac{\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{\left(x^2+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^2-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-2\,\sqrt{\frac{\left(x^2+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^2-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{2\,\left(\cosh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*((x*cosh(1/2) + x*sinh(1/2))*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*s
inh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d
)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 2*sqr
t(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(co
sh(1/2) + sinh(1/2))
```

Sympy [A]
time = 0.33, size = 36, normalized size = 0.90

$$\begin{cases} x \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\sqrt{d+ex^2}}{\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - sqrt(d + e*x**2)/sqrt(e),
Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.42, size = 59, normalized size = 1.48

$$\frac{1}{2}\,x \log\left(-\frac{\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}+1}{\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}-1}\right) - \frac{\sqrt{e^2x^2+de}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(sqrt(e)*x/sqrt(e*x^2 + d) + 1)/(sqrt(e)*x/sqrt(e*x^2 + d) - 1))
 - sqrt(e^2*x^2 + d*e)/e
```

Mupad [B]
time = 1.06, size = 32, normalized size = 0.80

$$x \operatorname{atanh}\left(\frac{\sqrt{e}\;x}{\sqrt{e\,x^2+d}}\right) - \frac{\sqrt{e\,x^2+d}}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)) - (d + e*x^2)^(1/2)/e^(1/2)
```

3.13 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)}{x^2}\, dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\, \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*e^(1
/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6356, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\, \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[e]*ArcTanh[Sqrt[d + e*x^2
]/Sqrt[d]])/Sqrt[d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\sqrt{e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\frac{1}{2}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{\sqrt{e}}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 61, normalized size = 1.11

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\frac{\sqrt{e}\left(\log(x)-\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) + (Sqrt[e]*(Log[x] - Log[d + Sqrt
[d]*Sqrt[d + e*x^2]]))/Sqrt[d]
```

Maple [**A**]
time = 0.01, size = 84, normalized size = 1.53

method	result	size

default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{e}\,\sqrt{e\,x^{2}+d}}{d}+\frac{\sqrt{e}\,\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-e^(1/2)/d*(e*x^2+d)^(1/2)+e^(1/2)/d*(
(e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] d*integrate(-e^(1/2*log(x^2*e + d) + 1/2)/(x^5*e^2 + d*x^3*e - (x^3*e + d*x
)*(x^2*e + d)), x) - 1/2*(log(x*e^(1/2) + sqrt(x^2*e + d)) - log(-x*e^(1/2)
 + sqrt(x^2*e + d)))/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(43) = 86.
time = 0.37, size = 407, normalized size = 7.40

$$\left[\frac{2\,dx\log\left(-x\cosh\left(\frac{1}{2}\right)-x\sinh\left(\frac{1}{2}\right)+\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}\right)+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{d}\,\log\left(-\frac{\sqrt{d}-\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{x}\right)+(dx-d)\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)}{2\,dx},\ \frac{2\,dx\log\left(-x\cosh\left(\frac{1}{2}\right)-x\sinh\left(\frac{1}{2}\right)+\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}\right)+4\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{-d}\,\arctan\left(-\frac{\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)-\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}\right)\sqrt{-d}}{d}\right)+(dx-d)\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)}{2\,dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x*log(-x*cosh(1/2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1/2) + (x
^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))) + 2*(x*cosh(1/2) + x*sinh(1/2)
)*sqrt(d)*log(-(sqrt(d) - sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/
(cosh(1/2) - sinh(1/2))))/x) + (d*x - d)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cos
h(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt((
(x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d)
)/(d*x), 1/2*(2*d*x*log(-x*cosh(1/2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1
/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))) + 4*(x*cosh(1/2) + x*s
inh(1/2))*sqrt(-d)*arctan(-(x*cosh(1/2) + x*sinh(1/2) - sqrt(((x^2 + d)*cos
h(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))*sqrt(-d)/d) + (d*x
- d)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2
 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh
(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d))/(d*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

3.14 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}$$

```
[Out] -1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*e^(3/2)*arctanh((e*x^2+d)^(
1/2)/d^(1/2))/d^(3/2)-1/6*e^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6356, 272, 44, 65, 214}

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2
]]/(3*x^3) + (e^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{e}\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{e^{3/2}\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x\right.}{12d} \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+\right.}{6d} \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 1.08

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{e}\,x\left(\sqrt{d}\sqrt{d+ex^2}+ex^2\log(x)-ex^2\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)\right)}{d^{3/2}}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d +
 e*x^2] + e*x^2*Log[x] - e*x^2*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]))/d^(3/2))/
x^3
```

Maple [A]
time = 0.01, size = 123, normalized size = 1.45

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{e^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2}{}\right.\right.}{2d}\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/3*e^(3/2)/d^(3/2)*ln((2*d+2*d
^(1/2)*(e*x^2+d)^(1/2))/x)+1/3*e^(1/2)/d*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e/
d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] d*integrate(-1/3*e^(1/2*log(x^2*e + d) + 1/2)/(x^7*e^2 + d*x^5*e - (x^5*e +
 d*x^3)*(x^2*e + d)), x) - 1/6*(log(x*e^(1/2) + sqrt(x^2*e + d)) - log(-x*e
^(1/2) + sqrt(x^2*e + d)))/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(63) = 126$.
time = 0.38, size = 571, normalized size = 6.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(2*d^2*x^3*log(-x*cosh(1/2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1/2)
+ (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))) + (x^3*cosh(1/2)^3 + 3*x^3
```

```
*cosh(1/2)^2*sinh(1/2) + 3*x^3*cosh(1/2)*sinh(1/2)^2 + x^3*sinh(1/2)^3)*sqr
t(d)*log((sqrt(d) + sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(
1/2) - sinh(1/2))))/x) + (d^2*x^3 - d^2)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cos
h(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt((
(x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d)
 - (d*x*cosh(1/2) + d*x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*si
nh(1/2))/(cosh(1/2) - sinh(1/2))))/(d^2*x^3), 1/6*(2*d^2*x^3*log(-x*cosh(1/
2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1
/2) - sinh(1/2)))) - 2*(x^3*cosh(1/2)^3 + 3*x^3*cosh(1/2)^2*sinh(1/2) + 3*x
^3*cosh(1/2)*sinh(1/2)^2 + x^3*sinh(1/2)^3)*sqrt(-d)*arctan(-(x*cosh(1/2) +
 x*sinh(1/2) - sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2)
- sinh(1/2))))*sqrt(-d)/d) + (d^2*x^3 - d^2)*log((2*x^2*cosh(1/2)^2 + 4*x^2
*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sq
rt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d
)/d) - (d*x*cosh(1/2) + d*x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d
)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(d^2*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

3.15 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

```
[Out] -1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*e^(5/2)*arctanh((e*x^2+d)^
(1/2)/d^(1/2))/d^(5/2)+3/40*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*e^(1/2)*(e
*x^2+d)^(1/2)/d/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6356, 272, 44, 65, 214}

$$-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -1/20*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x^4) + (3*e^(3/2)*Sqrt[d + e*x^2])/(40*d
^2*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*e^(5/2)*ArcTanh
[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{1}{5}\sqrt{e}\int \frac{1}{x^5\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{d+ex}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{\left(3e^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx\right.}{40d}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4} + \frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{\left(3e^{5/2}\right)\text{Subst}}{}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4} + \frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{\left(3e^{3/2}\right)\text{Subst}}{}$$

$$= -\frac{\sqrt{e}\,\sqrt{d+ex^2}}{20dx^4} + \frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3e^{5/2}\tanh^{-1}}{40}$$

Mathematica [A]

time = 0.08, size = 107, normalized size = 0.96

$$\frac{-8\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{e}\,x\left(\sqrt{d}\,\sqrt{d+ex^2}\,(-2d+3ex^2)+3e^2x^4\log(x)-3e^2x^4\log\left(d+\sqrt{d}\,\sqrt{d+ex^2}\right)\right)}{d^{5/2}}}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] (-8*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d + e*x
^2]*(-2*d + 3*e*x^2) + 3*e^2*x^4*Log[x] - 3*e^2*x^4*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]]))/d^(5/2))/(40*x^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.
time = 0.01, size = 171, normalized size = 1.54

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{2x^2d}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{e}\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{4d\,x^4}-\frac{e\left(-\frac{(e\,x^2+}{2d}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5-1/5*e^(3/2)/d*(-1/2*(e*x^2+d)^(
1/2)/x^2/d+1/2*e/d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))+1/5*e^(1/2)
/d*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e/d*
((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] d*integrate(-1/5*e^(1/2*log(x^2*e + d) + 1/2)/(x^9*e^2 + d*x^7*e - (x^7*e +
 d*x^5)*(x^2*e + d)), x) - 1/10*(log(x*e^(1/2) + sqrt(x^2*e + d)) - log(-x*
e^(1/2) + sqrt(x^2*e + d)))/x^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(83) = 166$.
time = 0.39, size = 727, normalized size = 6.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/40*(8*d^3*x^5*log(-x*cosh(1/2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1/2)
 + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))) + 3*(x^5*cosh(1/2)^5 + 5*
x^5*cosh(1/2)^4*sinh(1/2) + 10*x^5*cosh(1/2)^3*sinh(1/2)^2 + 10*x^5*cosh(1/
2)^2*sinh(1/2)^3 + 5*x^5*cosh(1/2)*sinh(1/2)^4 + x^5*sinh(1/2)^5)*sqrt(d)*l
og(-(sqrt(d) - sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2)
- sinh(1/2))))/x) + 4*(d^3*x^5 - d^3)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1
/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^
2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) +
(3*d*x^3*cosh(1/2)^3 + 9*d*x^3*cosh(1/2)*sinh(1/2)^2 + 3*d*x^3*sinh(1/2)^3
- 2*d^2*x*cosh(1/2) + (9*d*x^3*cosh(1/2)^2 - 2*d^2*x)*sinh(1/2))*sqrt(((x^2
 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(d^3*x^5),
 1/40*(8*d^3*x^5*log(-x*cosh(1/2) - x*sinh(1/2) + sqrt(((x^2 + d)*cosh(1/2)
 + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))) + 6*(x^5*cosh(1/2)^5 + 5*
x^5*cosh(1/2)^4*sinh(1/2) + 10*x^5*cosh(1/2)^3*sinh(1/2)^2 + 10*x^5*cosh(1/
2)^2*sinh(1/2)^3 + 5*x^5*cosh(1/2)*sinh(1/2)^4 + x^5*sinh(1/2)^5)*sqrt(-d)*
arctan(-(x*cosh(1/2) + x*sinh(1/2) - sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*
sinh(1/2))/(cosh(1/2) - sinh(1/2))))*sqrt(-d)/d) + 4*(d^3*x^5 - d^3)*log((2
*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x*cos
h(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cos
h(1/2) - sinh(1/2))) + d)/d) + (3*d*x^3*cosh(1/2)^3 + 9*d*x^3*cosh(1/2)*sin
h(1/2)^2 + 3*d*x^3*sinh(1/2)^3 - 2*d^2*x*cosh(1/2) + (9*d*x^3*cosh(1/2)^2 -
 2*d^2*x)*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh
(1/2) - sinh(1/2))))/(d^3*x^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d + ex^2}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**6, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

3.16 $\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=196

$$-\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{30d^{11/4}\left(\sqrt{d}+\right.}{}$$

[Out] 2/11*x^(11/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+d)^(1/2)/e^(3/2)-4/121*x^(9/2)*(e*x^2+d)^(1/2)/e^(1/2)-60/847*d^2*x^(1/2)*(e*x^2+d)^(1/2)/e^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6356, 327, 335, 226}

$$\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}-\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]

[Out] (-60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*e^(5/2)) + (36*d*x^(5/2)*Sqrt[d + e*x^2])/(847*e^(3/2)) - (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[e]) + (2*x^(11/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(11/4)*Sqrt[d + e*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{11}\left(2\sqrt{e}\right)\int\frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}}{121\sqrt{e}}$$

$$= \frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

$$= -\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/}$$

$$= -\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/}$$

$$= -\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.33, size = 161, normalized size = 0.82

$$\frac{2}{847}\sqrt{x}\left(-\frac{2\sqrt{d+ex^2}\,(15d^2-9dex^2+7e^2x^4)}{e^{5/2}}+77x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right)+\frac{60d^{5/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{847e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/e^(5/2) +
 77*x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/847 + (60*d^(5/2)*Sqrt[(I*Sq
rt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/
Sqrt[e]]/Sqrt[x]], -1])/(847*e^2*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int x^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/11*x^(11/2)*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/11*x^(11/2)*log(-x*e^(1/
2) + sqrt(x^2*e + d)) - 2*d*integrate(-1/11*x*e^(1/2*log(x^2*e + d) + 9/2*l
og(x) + 1/2)/(x^4*e^2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 402, normalized size = 2.05

$$\frac{60\,d^{3}\operatorname{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)+77\left(x^5\cosh\left(\frac{1}{2}\right)^6+6x^5\cosh\left(\frac{1}{2}\right)^5\sinh\left(\frac{1}{2}\right)+15x^5\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)^2+20x^5\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^3+15x^5\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^4+6x^5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^5+x^5\sinh\left(\frac{1}{2}\right)^6\right)\sqrt{x}\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\left(7x^4\cosh\left(\frac{1}{2}\right)^5+35x^4\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^4+7x^4\sinh\left(\frac{1}{2}\right)^5-9dx^2\cosh\left(\frac{1}{2}\right)^3+\left(70x^4\cosh\left(\frac{1}{2}\right)^2-9dx^2\right)\sinh\left(\frac{1}{2}\right)^3+15d^2\cosh\left(\frac{1}{2}\right)+\left(70x^4\cosh\left(\frac{1}{2}\right)^3-27dx^2\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)^2+\left(35x^4\cosh\left(\frac{1}{2}\right)^4-27dx^2\cosh\left(\frac{1}{2}\right)^2+15d^2\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{847\left(\cosh\left(\frac{1}{2}\right)^6+6\cosh\left(\frac{1}{2}\right)^5\sinh\left(\frac{1}{2}\right)+15\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)^2+20\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^3+15\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^4+6\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^5+\sinh\left(\frac{1}{2}\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/847*(60*d^3*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2)
 + sinh(1/2)^2), 0, x) + 77*(x^5*cosh(1/2)^6 + 6*x^5*cosh(1/2)^5*sinh(1/2)
+ 15*x^5*cosh(1/2)^4*sinh(1/2)^2 + 20*x^5*cosh(1/2)^3*sinh(1/2)^3 + 15*x^5*
cosh(1/2)^2*sinh(1/2)^4 + 6*x^5*cosh(1/2)*sinh(1/2)^5 + x^5*sinh(1/2)^6)*sq
rt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^
2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sin
h(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 4*(7*x^4*cosh(1/2)^5 + 35*x^4*co
sh(1/2)*sinh(1/2)^4 + 7*x^4*sinh(1/2)^5 - 9*d*x^2*cosh(1/2)^3 + (70*x^4*cos
h(1/2)^2 - 9*d*x^2)*sinh(1/2)^3 + 15*d^2*cosh(1/2) + (70*x^4*cosh(1/2)^3 -
27*d*x^2*cosh(1/2))*sinh(1/2)^2 + (35*x^4*cosh(1/2)^4 - 27*d*x^2*cosh(1/2)^
2 + 15*d^2)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1
/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^6 + 6*cosh(1/2)^5*sinh(1/2) + 15*
cosh(1/2)^4*sinh(1/2)^2 + 20*cosh(1/2)^3*sinh(1/2)^3 + 15*cosh(1/2)^2*sinh(
1/2)^4 + 6*cosh(1/2)*sinh(1/2)^5 + sinh(1/2)^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8855 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{9/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.17 $\int x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=168

$$\frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)\cdots}}}{147e^{7/4}\sqrt{d+\cdots}}$$

```
[Out] 2/7*x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/49*x^(5/2)*(e*x^2+d)^(1/2)
/e^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)-10/147*d^(7/4)*(cos(2*arc
tan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)
))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x
*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6356, 327, 335, 226}

$$-\frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147e^{7/4}\sqrt{d+ex^2}}+\frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*e^(3/2)) - (4*x^(5/2)*Sqrt[d + e*x^2])/
(49*Sqrt[e]) + (2*x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7 - (10*d^(
7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellipt
icF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(7/4)*Sqrt[d + e*x^2]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\left(2\sqrt{e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}}{49\sqrt{e}}$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 147, normalized size = 0.88

$$\frac{2}{147}\sqrt{x}\left(\frac{2(5d-3ex^2)\sqrt{d+ex^2}}{e^{3/2}} + 21x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right) + \frac{20\sqrt{d}\left(\frac{i\sqrt{d}}{\sqrt{e}}\right)^{5/2}\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{147\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 21*x^3*ArcTanh[(S
qrt[e]*x)/Sqrt[d + e*x^2]]))/147 + (20*Sqrt[d]*((I*Sqrt[d])/Sqrt[e])^(5/2)*
Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]
], -1])/(147*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^(7/2)*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/7*x^(7/2)*log(-x*e^(1/2) +
 sqrt(x^2*e + d)) - 2*d*integrate(-1/7*x*e^(1/2*log(x^2*e + d) + 5/2*log(x)
 + 1/2)/(x^4*e^2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 288, normalized size = 1.71

$$-\frac{20\,d^2\text{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)-21\left(x^3\cosh\left(\frac{1}{2}\right)^4+4\,x^3\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)+6\,x^3\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^2+4\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^3+x^3\sinh\left(\frac{1}{2}\right)^4\right)\sqrt{x}\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+4\left(3\,x^2\cosh\left(\frac{1}{2}\right)^3+9\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2+3\,x^2\sinh\left(\frac{1}{2}\right)^3-5\,d\cosh\left(\frac{1}{2}\right)+\left(9\,x^2\cosh\left(\frac{1}{2}\right)^2-5\,d\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{147\left(\cosh\left(\frac{1}{2}\right)^4+4\,\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)+6\,\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^2+4\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^3+\sinh\left(\frac{1}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/147*(20*d^2*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2
) + sinh(1/2)^2), 0, x) - 21*(x^3*cosh(1/2)^4 + 4*x^3*cosh(1/2)^3*sinh(1/2)
 + 6*x^3*cosh(1/2)^2*sinh(1/2)^2 + 4*x^3*cosh(1/2)*sinh(1/2)^3 + x^3*sinh(1
/2)^4)*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*s
inh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2
 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 4*(3*x^2*cosh(1/2)^3 +
```

```
9*x^2*cosh(1/2)*sinh(1/2)^2 + 3*x^2*sinh(1/2)^3 - 5*d*cosh(1/2) + (9*x^2*co
sh(1/2)^2 - 5*d)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*s
inh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^4 + 4*cosh(1/2)^3*sinh(1/2)
+ 6*cosh(1/2)^2*sinh(1/2)^2 + 4*cosh(1/2)*sinh(1/2)^3 + sinh(1/2)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Integral(x**(5/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(5/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.18 $\int \sqrt{x} \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=142

$$-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\ldots}{\ldots}\right.\right.}{9e^{3/4}\sqrt{d+ex^2}}$$

```
[Out] 2/3*x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/9*x^(1/2)*(e*x^2+d)^(1/2)/
e^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*
arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(
1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(
1/2)/e^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6356, 327, 335, 226}

$$\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[e]) + (2*x^(3/2)*ArcTanh[(Sqrt[e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqr
t[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(
9*e^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\left(2\sqrt{e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(2d)\int\frac{1}{\sqrt{x}\sqrt{d+e\ldots}}}{9\sqrt{e}}$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(4d)\mathrm{Subst}\left(\int\frac{}{\sqrt{d+\ldots}}\right.}{9\sqrt{\ldots}}$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\ldots\right.}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 135, normalized size = 0.95

$$\frac{2}{9}\sqrt{x}\left(-\frac{2\sqrt{d+ex^2}}{\sqrt{e}}+3x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right)+\frac{4\sqrt{d}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{9\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2])/Sqrt[e] + 3*x*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]]))/9 + (4*Sqrt[d]*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(9*Sqrt[d + e*x
^2])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{x}\, \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/3*x^(3/2)*log(-x*e^(1/2) +
 sqrt(x^2*e + d)) - 2*d*integrate(-1/3*x*e^(1/2*log(x^2*e + d) + 1/2*log(x)
 + 1/2)/(x^4*e^2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 188, normalized size = 1.32

$$\frac{3 \left(x\cosh\left(\frac{1}{2}\right)^{2}+2\,x\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)^{2}\right)\sqrt{x}\,\log\left(\frac{2\,x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4\,x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^{2}\sinh\left(\frac{1}{2}\right)^{2}+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^{2}+d)\cosh\left(\frac{1}{2}\right)+(x^{2}-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\sqrt{x}\,\sqrt{\frac{(x^{2}+d)\cosh\left(\frac{1}{2}\right)+(x^{2}-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}\left(\cosh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)\right)+4\,d\,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,x\right)}{9\left(\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(x*cosh(1/2)^2 + 2*x*cosh(1/2)*sinh(1/2) + x*sinh(1/2)^2)*sqrt(x)*lo
g((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2 + 2*(x
*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/
(cosh(1/2) - sinh(1/2))) + d)/d) - 4*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x
^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2)))*(cosh(1/2) + sinh(1/2)) + 4*d*w
eierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2)
, 0, x))/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Integral(sqrt(x)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(sqrt(x)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \operatorname{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.19 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}+\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt{d+ex^2}}$$

```
[Out] -2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*e^(1/4)*(cos(2*arctan(e^(1/
4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellipt
icF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))
*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6356, 335, 226}

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt{d+ex^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*e^(1/4)*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{e}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,dx \\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{e}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right) \\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\right.}{\sqrt[4]{d}\,\sqrt{d+ex^2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 111, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{e}\,\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[e]*Sqrt[1 +
 d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/
(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(-x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(3/2)
- (x^2*e + d)*e^(log(x^2*e + d) + 3/2*log(x))), x) - log(x*e^(1/2) + sqrt(x
^2*e + d))/sqrt(x) + log(-x*e^(1/2) + sqrt(x^2*e + d))/sqrt(x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 0.99

$$\frac{4\,x\text{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,x\right)-\sqrt{x}\,\log\left(\frac{2\,x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4\,x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^{2}\sinh\left(\frac{1}{2}\right)^{2}+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^{2}+d)\cosh\left(\frac{1}{2}\right)+(x^{2}-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] (4*x*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1
/2)^2), 0, x) - sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2)
+ 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/
2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(\frac{\sqrt{e}\, x}{\sqrt{d + e x^{2}}} \right)}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

3.20 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{15d^{5/4}\sqrt{d+ex^2}}$$

```
[Out] -2/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*e^(1/2)*(e*x^2+d)^(1/2
)/d/x^(3/2)-2/15*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))
^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6356, 331, 335, 226}

$$-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(5*x^(5/2)) - (2*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)
/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/
2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{1}{5}\left(2\sqrt{e}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{\left(2e^{3/2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,dx}{15d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{\left(4e^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\right.}{15d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{}{\left(\sqrt{d}\right.}}}{15}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 142, normalized size = 0.98

$$-\frac{2\left(2\sqrt{e}\,x\sqrt{d+ex^2}+3d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}} - \frac{4\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,e^2\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{15d^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[e]*x*Sqrt[d + e*x^2] + 3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
))/(15*d*x^(5/2)) - (4*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^2*Sqrt[1 + d/(e*x^2)]*x*
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(15*d^(3/2)*Sq
rt[d + e*x^2])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(-1/5*x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(7
/2) - (x^2*e + d)*e^(log(x^2*e + d) + 7/2*log(x))), x) - 1/5*log(x*e^(1/2)
+ sqrt(x^2*e + d))/x^(5/2) + 1/5*log(-x*e^(1/2) + sqrt(x^2*e + d))/x^(5/2)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 187, normalized size = 1.29

$$-\frac{3\,d\sqrt{x}\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+4\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\,\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+4\,\left(x^3\cosh\left(\frac{1}{2}\right)^2+2\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+x^3\sinh\left(\frac{1}{2}\right)^2\right){\rm weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)}{15\,d x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*d*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x
^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) +
(x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 4*(x*cosh(1/2) + x*
sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1
/2) - sinh(1/2))) + 4*(x^3*cosh(1/2)^2 + 2*x^3*cosh(1/2)*sinh(1/2) + x^3*si
nh(1/2)^2)*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) +
sinh(1/2)^2), 0, x))/(d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

3.21 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F}{189d^{9/4}\sqrt{d+ex^2}}$$

```
[Out] -2/9*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)+20/189*e^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^(3/2)-4/63*e^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(9/4)*(cos(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^
(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1
/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(9/4)/(e*x^2+d)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6356, 331, 335, 226}

$$\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) + (20*e^(3/2)*Sqrt[d + e*x^2])/
(189*d^2*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) +
(10*e^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]
*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(9/4)*Sqrt[d +
 e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{1}{9}\left(2\sqrt{e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} - \frac{\left(10e^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10e^{5/2}\right)}{}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(20e^{5/2}\right)}{}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10e^{9/4}\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.20, size = 154, normalized size = 0.89

$$\frac{4\sqrt{e}\,x\sqrt{d+ex^2}\,(-3d+5ex^2)-42d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}}+\frac{20\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,e^3\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{189d^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (20*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^3*Sqr
t[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]],
-1])/(189*d^(5/2)*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxima"
)
```

```
[Out] 2*d*integrate(-1/9*x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(1
1/2) - (x^2*e + d)*e^(log(x^2*e + d) + 11/2*log(x))), x) - 1/9*log(x*e^(1/2
) + sqrt(x^2*e + d))/x^(9/2) + 1/9*log(-x*e^(1/2) + sqrt(x^2*e + d))/x^(9/2
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 260, normalized size = 1.50

$$-\frac{21\,d^2\sqrt{x}\,\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\left(5\,x^3\cosh\left(\frac{1}{2}\right)^3+15\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2+5\,x^3\sinh\left(\frac{1}{2}\right)^3-3\,dx\cosh\left(\frac{1}{2}\right)+3\left(5\,x^3\cosh\left(\frac{1}{2}\right)^2-dx\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\,\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}-20\left(x^5\cosh\left(\frac{1}{2}\right)^4+4\,x^5\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)+6\,x^5\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^2+4\,x^5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^3+x^5\sinh\left(\frac{1}{2}\right)^4\right)\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)}{189\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas"
)
```

```
[Out] -1/189*(21*d^2*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) +
 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2
) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 4*(5*x^3*cosh(1
/2)^3 + 15*x^3*cosh(1/2)*sinh(1/2)^2 + 5*x^3*sinh(1/2)^3 - 3*d*x*cosh(1/2)
+ 3*(5*x^3*cosh(1/2)^2 - d*x)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2)
+ (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) - 20*(x^5*cosh(1/2)^4 + 4*x
^5*cosh(1/2)^3*sinh(1/2) + 6*x^5*cosh(1/2)^2*sinh(1/2)^2 + 4*x^5*cosh(1/2)*
sinh(1/2)^3 + x^5*sinh(1/2)^4)*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*co
sh(1/2)*sinh(1/2) + sinh(1/2)^2), 0, x))/(d^2*x^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

3.22 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\cdots}}{\cdots}$$

```
[Out] -2/13*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)+36/1001*e^(3/2)*(e*x^2+d)
^(1/2)/d^2/x^(7/2)-60/1001*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/143*e^(1/2
)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(13/4)*(cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*
arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d
)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6356, 331, 335, 226}

$$-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) + (36*e^(3/2)*Sqrt[d + e*x^2]
)/(1001*d^2*x^(7/2)) - (60*e^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2)) - (2
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*e^(13/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan
[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(18e^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(90e^{5/2}\right)\cdots}{\cdots}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\cdots\right)}{13x^{13/2}}\cdots$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\cdots\right)}{13x^{13/2}}\cdots$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\cdots\right)}{13x^{13/2}}\cdots$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.28, size = 163, normalized size = 0.81

$$\frac{2\left(-\frac{2\sqrt{e}\,x\sqrt{d+ex^2}\,(7d^2-9dex^2+15e^2x^4)}{d^3}-77\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{30\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,e^4\sqrt{1+\frac{d}{ex^2}}\,x^{15/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^{7/2}\sqrt{d+ex^2}}\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[e]*x*Sqrt[d + e*x^2]*(7*d^2 - 9*d*e*x^2 + 15*e^2*x^4))/d^3 - 7
7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (30*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^4*
Sqrt[1 + d/(e*x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/
Sqrt[x]], -1])/(d^(7/2)*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxima"
)
```

```
[Out] 2*d*integrate(-1/13*x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(
15/2) - (x^2*e + d)*e^(log(x^2*e + d) + 15/2*log(x))), x) - 1/13*log(x*e^(1
/2) + sqrt(x^2*e + d))/x^(13/2) + 1/13*log(-x*e^(1/2) + sqrt(x^2*e + d))/x^
(13/2)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 355, normalized size = 1.77

$$-\frac{77\,d^3\sqrt{x}\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+4\,\left(15\,x^5\cosh\left(\frac{1}{2}\right)^5+75\,x^5\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^4+15\,x^5\sinh\left(\frac{1}{2}\right)^5-9\,dx^3\cosh\left(\frac{1}{2}\right)^3+7\,d^2x\cosh\left(\frac{1}{2}\right)+3\,\left(50\,x^5\cosh\left(\frac{1}{2}\right)^2-3\,dx^3\right)\sinh\left(\frac{1}{2}\right)^3+3\,\left(50\,x^5\cosh\left(\frac{1}{2}\right)^3-9\,dx^3\cosh\left(\frac{1}{2}\right)\right)\sinh\left(\frac{1}{2}\right)^2+\left(75\,x^5\cosh\left(\frac{1}{2}\right)^4-27\,dx^3\cosh\left(\frac{1}{2}\right)^2+7\,d^2x\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+60\,\left(x^7\cosh\left(\frac{1}{2}\right)^6+6\,x^7\cosh\left(\frac{1}{2}\right)^5\sinh\left(\frac{1}{2}\right)+15\,x^7\cosh\left(\frac{1}{2}\right)^4\sinh\left(\frac{1}{2}\right)^2+20\,x^7\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)^3+15\,x^7\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^4+6\,x^7\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^5+x^7\sinh\left(\frac{1}{2}\right)^6\right)\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)}{1001\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas"
)
```

```
[Out] -1/1001*(77*d^3*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2)
+ 2*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/
2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 4*(15*x^5*cosh
(1/2)^5 + 75*x^5*cosh(1/2)*sinh(1/2)^4 + 15*x^5*sinh(1/2)^5 - 9*d*x^3*cosh(
1/2)^3 + 7*d^2*x*cosh(1/2) + 3*(50*x^5*cosh(1/2)^2 - 3*d*x^3)*sinh(1/2)^3 +
 3*(50*x^5*cosh(1/2)^3 - 9*d*x^3*cosh(1/2))*sinh(1/2)^2 + (75*x^5*cosh(1/2)
^4 - 27*d*x^3*cosh(1/2)^2 + 7*d^2*x)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cos
h(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + 60*(x^7*cosh(1/2)^
6 + 6*x^7*cosh(1/2)^5*sinh(1/2) + 15*x^7*cosh(1/2)^4*sinh(1/2)^2 + 20*x^7*c
osh(1/2)^3*sinh(1/2)^3 + 15*x^7*cosh(1/2)^2*sinh(1/2)^4 + 6*x^7*cosh(1/2)*s
inh(1/2)^5 + x^7*sinh(1/2)^6)*weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cos
h(1/2)*sinh(1/2) + sinh(1/2)^2), 0, x))/(d^3*x^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

3.23 $\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=297

$$\frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}}-\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}}-\frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{28d^{9/4}\left(\sqrt{d}+\right.}{}$$

```
[Out] 2/9*x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)^(
1/2)/e^(3/2)-4/81*x^(7/2)*(e*x^2+d)^(1/2)/e^(1/2)-28/135*d^2*x^(1/2)*(e*x^2
+d)^(1/2)/e^2/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin
(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^
2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d
^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/(e*x^2+d)
^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6356, 327, 335, 311, 226, 1210}

$$-\frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}-\frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}}-\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}}+\frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*e^(3/2)) - (4*x^(7/2)*Sqrt[d + e*x^2])/
(81*Sqrt[e]) - (28*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^2*(Sqrt[d] + Sqrt[e]
*x)) + (2*x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/9 + (28*d^(9/4)*(Sq
rt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Ar
cTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x^2]) - (14*
d^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ell
ipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x
^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{9}(2\sqrt{e})\int\frac{x^{9/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}}{81\sqrt{e}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) -$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} - \frac{28d^2\sqrt{x}\,\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}\,x\right)} + \frac{2}{9}x^{9/2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 124, normalized size = 0.42

$$\frac{2x^{3/2}\left(14d^2+4dex^2-10e^2x^4+45e^{3/2}x^3\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-14d^2\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)\right)}{405e^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(14*d^2 + 4*d*e*x^2 - 10*e^2*x^4 + 45*e^(3/2)*x^3*Sqrt[d + e*x^2
]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[1 + (e*x^2)/d]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^(3/2)*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/9*x^(9/2)*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/9*x^(9/2)*log(-x*e^(1/2) +
 sqrt(x^2*e + d)) - 2*d*integrate(-1/9*x*e^(1/2*log(x^2*e + d) + 7/2*log(x)
 + 1/2)/(x^4*e^2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 312, normalized size = 1.05

$$\frac{84\,d^{2}\mathrm{weierstrassZeta}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,x\right)\right)+45\left(x^{4}\cosh\left(\frac{1}{2}\right)^{4}+4\,x^{4}\cosh\left(\frac{1}{2}\right)^{3}\sinh\left(\frac{1}{2}\right)+6\,x^{4}\cosh\left(\frac{1}{2}\right)^{2}\sinh\left(\frac{1}{2}\right)^{2}+4\,x^{4}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^{3}+x^{4}\sinh\left(\frac{1}{2}\right)^{4}\right)\sqrt{x}\log\left(\frac{2\,x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4\,x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^{2}\sinh\left(\frac{1}{2}\right)^{2}+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{\left(x^{2}+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\left(5\,x^{3}\cosh\left(\frac{1}{2}\right)^{3}+15\,x^{3}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^{2}+5\,x^{3}\sinh\left(\frac{1}{2}\right)^{3}-7\,dx\cosh\left(\frac{1}{2}\right)+\left(15\,x^{3}\cosh\left(\frac{1}{2}\right)^{2}-7\,dx\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{\left(x^{2}+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}}{405\left(\cosh\left(\frac{1}{2}\right)^{4}+4\,\cosh\left(\frac{1}{2}\right)^{3}\sinh\left(\frac{1}{2}\right)+6\,\cosh\left(\frac{1}{2}\right)^{2}\sinh\left(\frac{1}{2}\right)^{2}+4\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^{3}+\sinh\left(\frac{1}{2}\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/405*(84*d^2*weierstrassZeta(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + s
inh(1/2)^2), 0, weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/
2) + sinh(1/2)^2), 0, x)) + 45*(x^4*cosh(1/2)^4 + 4*x^4*cosh(1/2)^3*sinh(1/
2) + 6*x^4*cosh(1/2)^2*sinh(1/2)^2 + 4*x^4*cosh(1/2)*sinh(1/2)^3 + x^4*sinh
(1/2)^4)*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2
*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x
^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 4*(5*x^3*cosh(1/2)^3
+ 15*x^3*cosh(1/2)*sinh(1/2)^2 + 5*x^3*sinh(1/2)^3 - 7*d*x*cosh(1/2) + (15*
x^3*cosh(1/2)^2 - 7*d*x)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^
2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))))/(cosh(1/2)^4 + 4*cosh(1/2)^3*si
nh(1/2) + 6*cosh(1/2)^2*sinh(1/2)^2 + 4*cosh(1/2)*sinh(1/2)^3 + sinh(1/2)^4
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(7/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.24 $\int x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=269

$$-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{12d\sqrt{x}\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\right.}}}{25e^{5/4}\sqrt{d}}$$

```
[Out] 2/5*x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/25*x^(3/2)*(e*x^2+d)^(1/2)
/e^(1/2)+12/25*d*x^(1/2)*(e*x^2+d)^(1/2)/e/(d^(1/2)+x*e^(1/2))-12/25*d^(5/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^
2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^
2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6356, 327, 335, 311, 226, 1210}

$$\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}+\frac{12d\sqrt{x}\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[e]) + (12*d*Sqrt[x]*Sqrt[d + e*x^2])/
(25*e*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/5 - (12*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(5/4)*
Sqrt[d + e*x^2]) + (6*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[
d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25
*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\left(2\sqrt{e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx$$

$$=-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{25\sqrt{e}}$$

$$=-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(12d)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+}}\right.}{25\sqrt{e}}$$

$$=-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\left(12d^{3/2}\right)\text{Subst}\left(\int\frac{}{\sqrt{}}\right.}{25}$$

$$=-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{12d\sqrt{x}\,\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 109, normalized size = 0.41

$$\frac{2x^{3/2}\left(-2(d+ex^2)+5\sqrt{e}\,x\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+2d\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)\right)}{25\sqrt{e}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*(d + e*x^2) + 5*Sqrt[e]*x*Sqrt[d + e*x^2]*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]] + 2*d*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/
4, -((e*x^2)/d)]))/(25*Sqrt[e]*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^(5/2)*log(x*e^(1/2) + sqrt(x^2*e + d)) - 1/5*x^(5/2)*log(-x*e^(1/2) +
 sqrt(x^2*e + d)) - 2*d*integrate(-1/5*x*e^(1/2*log(x^2*e + d) + 3/2*log(x)
 + 1/2)/(x^4*e^2 + d*x^2*e - (x^2*e + d)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 220, normalized size = 0.82

$$\frac{5\left(x^{2}\cosh\left(\frac{1}{2}\right)^{2}+2\,x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+x^{2}\sinh\left(\frac{1}{2}\right)^{2}\right)\sqrt{x}\,\log\left(\frac{2\,x^{2}\cosh\left(\frac{1}{2}\right)^{2}+4\,x^{2}\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^{2}\sinh\left(\frac{1}{2}\right)^{2}+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{\left(x^{2}+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{\left(x^{2}+d\right)\cosh\left(\frac{1}{2}\right)+\left(x^{2}-d\right)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}-12\,d\,\mathrm{weierstrassZeta}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}},0,x\right)\right)}{25\left(\cosh\left(\frac{1}{2}\right)^{2}+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/25*(5*(x^2*cosh(1/2)^2 + 2*x^2*cosh(1/2)*sinh(1/2) + x^2*sinh(1/2)^2)*sqr
t(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2)^2
 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh
(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 4*(x*cosh(1/2) + x*sinh(1/2))*sqr
t(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2
))) - 12*d*weierstrassZeta(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh
(1/2)^2), 0, weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2)
+ sinh(1/2)^2), 0, x)))/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Integral(x**(3/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(3/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.25 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=232

$$-\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}\,x}+2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\right.\right.}{\sqrt[4]{e}\,\sqrt{d+ex^2}}$$

```
[Out] 2*x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4*x^(1/2)*(e*x^2+d)^(1/2)/(d^(
1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/
cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2)
)^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)-2*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*a
rctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)
/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6356, 335, 311, 226, 1210}

$$-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\,\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\,\sqrt{d+ex^2}}-\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}\,x}+2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x) + 2*Sqrt[x]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2
)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1
/2])/(e^(1/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\left(2\sqrt{e}\right)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx\\
&= 2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\left(4\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\left(4\sqrt{d}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)+\Bigg(\\
&= -\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}\,x}+2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\cdots}}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 85, normalized size = 0.37

$$2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{4\sqrt{e}\,x^{3/2}\sqrt{1+\frac{ex^2}{d}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[e]*x^(3/2)*Sqrt[1
+ (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e*
x^2])
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*d*integrate(x*e^(1/2*log(x^2*e + d) + 1/2)/((x^2*e + d)*e^(log(x^2*e + d
) + 1/2*log(x)) - (x^4*e^2 + d*x^2*e)*sqrt(x)), x) + sqrt(x)*log(x*e^(1/2)
+ sqrt(x^2*e + d)) - sqrt(x)*log(-x*e^(1/2) + sqrt(x^2*e + d))
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 128, normalized size = 0.55

$$\sqrt{x}\,\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+4\,\mathrm{weierstrassZeta}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\,\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*sinh(1/2
)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*s
inh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 4*weierstrassZeta(-4*d/(cosh(1
/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2), 0, weierstrassPInverse(-4*d/(
cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/2)^2), 0, x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/sqrt(x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

3.26 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}}+\frac{4e\sqrt{x}\,\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}}{3d^{3/4}\sqrt{d+ex}}$$

```
[Out] -2/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*e^(1/2)*(e*x^2+d)^(1/2)
/d/x^(1/2)+4/3*e*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/2))-4/3*e^(3/4)*
(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*
(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+
d)^(1/2)+2/3*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2
*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^
(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^
(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6356, 331, 335, 311, 226, 1210}

$$\frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4e\sqrt{x}\,\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*e*Sqrt[x]*Sqrt[d + e*x^2])/
(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*x
^(3/2)) - (4*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*S
qrt[d + e*x^2]) + (2*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d
] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d
^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(2e^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(4e^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\right.}{3d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{(4e)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,\right.}{3\sqrt{d}}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\,\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4e^{3/4}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 118, normalized size = 0.43

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4e^{3/2}x^{3/2}\sqrt{1+\frac{ex^2}{d}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{9d\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(3*x^(3/2)) + (4*e^(3/2)*x^(3/2)*Sqrt[1 + (e*x^2)/d]*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(9*d*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(-1/3*x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(5
/2) - (x^2*e + d)*e^(log(x^2*e + d) + 5/2*log(x))), x) - 1/3*log(x*e^(1/2)
+ sqrt(x^2*e + d))/x^(3/2) + 1/3*log(-x*e^(1/2) + sqrt(x^2*e + d))/x^(3/2)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 208, normalized size = 0.76

$$-\frac{d\sqrt{x}\log\left(\frac{2x^2\cosh\left(\frac{1}{2}\right)^2+4x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2x^2\sinh\left(\frac{1}{2}\right)^2+2\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)+4\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+4\left(x^2\cosh\left(\frac{1}{2}\right)^2+2x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+x^2\sinh\left(\frac{1}{2}\right)^2\right){\rm weierstrassZeta}\left(-\frac{4d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,{\rm weierstrassPInverse}\left(-\frac{4d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)\right)}{3dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(d*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2*x^2*
sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2) + (x^
2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) + 4*(x*cosh(1/2) + x*sin
h(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2 - d)*sinh(1/2))/(cosh(1/2)
 - sinh(1/2))) + 4*(x^2*cosh(1/2)^2 + 2*x^2*cosh(1/2)*sinh(1/2) + x^2*sinh(
1/2)^2)*weierstrassZeta(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + sinh(1/
2)^2), 0, weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sinh(1/2) + s
inh(1/2)^2), 0, x)))/(d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

3.27 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{12e^2\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)}{}$$

```
[Out] -2/7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*e^(1/2)*(e*x^2+d)^(1/2
)/d/x^(5/2)+12/35*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^2*x^(1/2)*(e*
x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(7/4)*(cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(s
in(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*
x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)-6/35*e^(7/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d
^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)
^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6356, 331, 335, 311, 226, 1210}

$$-\frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{12e^2\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) + (12*e^(3/2)*Sqrt[d + e*x^2])/
(35*d^2*Sqrt[x]) - (12*e^2*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^2*(Sqrt[d] + Sqrt
[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (12*e^(7/4
)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE
[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2]) -
(6*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*
EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e
*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 6356

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{5/2}\right)\int}{}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^{5/2}\right)\mathrm{S}}{}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^{2}\right)\mathrm{Su}}{}$$

$$= -\frac{4\sqrt{e}\,\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12e^2\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2\tanh^{-1}\left(\cdots\right)}{7x^{7}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 131, normalized size = 0.43

$$\frac{4\sqrt{e}\,x(-d^2+2dex^2+3e^2x^4) - 10d^2\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - 4e^{5/2}x^5\sqrt{1+\frac{ex^2}{d}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 4*e^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(-1/7*x*e^(1/2*log(x^2*e + d) + 1/2)/((x^4*e^2 + d*x^2*e)*x^(9
/2) - (x^2*e + d)*e^(log(x^2*e + d) + 9/2*log(x))), x) - 1/7*log(x*e^(1/2)
+ sqrt(x^2*e + d))/x^(7/2) + 1/7*log(-x*e^(1/2) + sqrt(x^2*e + d))/x^(7/2)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 281, normalized size = 0.93

$$-\frac{5\,d^2\sqrt{x}\,\log\left(\frac{2\,x^2\cosh\left(\frac{1}{2}\right)^2+4\,x^2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+2\,x^2\sinh\left(\frac{1}{2}\right)^2+2\,\left(x\cosh\left(\frac{1}{2}\right)+x\sinh\left(\frac{1}{2}\right)\right)\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}+d}{d}\right)-4\left(3\,x^3\cosh\left(\frac{1}{2}\right)^3+9\,x^3\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^2+3\,x^3\sinh\left(\frac{1}{2}\right)^3-dx\cosh\left(\frac{1}{2}\right)+\left(9\,x^3\cosh\left(\frac{1}{2}\right)^2-dx\right)\sinh\left(\frac{1}{2}\right)\right)\sqrt{x}\sqrt{\frac{(x^2+d)\cosh\left(\frac{1}{2}\right)+(x^2-d)\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)}}-12\left(x^4\cosh\left(\frac{1}{2}\right)^4+4\,x^4\cosh\left(\frac{1}{2}\right)^3\sinh\left(\frac{1}{2}\right)+6\,x^4\cosh\left(\frac{1}{2}\right)^2\sinh\left(\frac{1}{2}\right)^2+4\,x^4\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)^3+x^4\sinh\left(\frac{1}{2}\right)^4\right)\mathrm{weierstrassZeta}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{\cosh\left(\frac{1}{2}\right)^2+2\cosh\left(\frac{1}{2}\right)\sinh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)^2},0,x\right)\right)}{35\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*d^2*sqrt(x)*log((2*x^2*cosh(1/2)^2 + 4*x^2*cosh(1/2)*sinh(1/2) + 2
*x^2*sinh(1/2)^2 + 2*(x*cosh(1/2) + x*sinh(1/2))*sqrt(((x^2 + d)*cosh(1/2)
+ (x^2 - d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) + d)/d) - 4*(3*x^3*cosh(1/2
)^3 + 9*x^3*cosh(1/2)*sinh(1/2)^2 + 3*x^3*sinh(1/2)^3 - d*x*cosh(1/2) + (9*
x^3*cosh(1/2)^2 - d*x)*sinh(1/2))*sqrt(x)*sqrt(((x^2 + d)*cosh(1/2) + (x^2
- d)*sinh(1/2))/(cosh(1/2) - sinh(1/2))) - 12*(x^4*cosh(1/2)^4 + 4*x^4*cosh
(1/2)^3*sinh(1/2) + 6*x^4*cosh(1/2)^2*sinh(1/2)^2 + 4*x^4*cosh(1/2)*sinh(1/
2)^3 + x^4*sinh(1/2)^4)*weierstrassZeta(-4*d/(cosh(1/2)^2 + 2*cosh(1/2)*sin
h(1/2) + sinh(1/2)^2), 0, weierstrassPInverse(-4*d/(cosh(1/2)^2 + 2*cosh(1/
2)*sinh(1/2) + sinh(1/2)^2), 0, x)))/(d^2*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

3.28 $\int x^3 \tanh^{-1}(a+bx^4)\, dx$

Optimal. Leaf size=44

$$\frac{(a+bx^4)\tanh^{-1}(a+bx^4)}{4b}+\frac{\log\left(1-(a+bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arctanh(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 6238, 6021, 266}

$$\frac{\log\left(1-(a+bx^4)^2\right)}{8b}+\frac{(a+bx^4)\tanh^{-1}(a+bx^4)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTanh[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \tanh^{-1}\left(a+bx^4\right)\, dx &= \frac{1}{4}\text{Subst}\left(\int \tanh^{-1}(a+bx)\, dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \tanh^{-1}(x)\, dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\tanh^{-1}\left(a+bx^4\right)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\, dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\tanh^{-1}\left(a+bx^4\right)}{4b} + \frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.89

$$\frac{2(a+bx^4)\tanh^{-1}\left(a+bx^4\right)+\log\left(1-\left(a+bx^4\right)^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[a + b*x^4],x]
[Out] (2*(a + b*x^4)*ArcTanh[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

Maple [A]
time = 0.04, size = 39, normalized size = 0.89

method	result
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arctanh}(b\,x^4+a)+\frac{\ln\left(1-\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\,\mathrm{arctanh}(b\,x^4+a)+\frac{\ln\left(1-\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
risch	$\frac{x^4\ln(b\,x^4+a+1)}{8}-\frac{x^4\ln(-b\,x^4-a+1)}{8}+\frac{\ln(b\,x^4+a+1)a}{8b}-\frac{\ln(-b\,x^4-a+1)a}{8b}+\frac{\ln(b\,x^4+a+1)}{8b}+\frac{\ln(-b\,x^4-a+1)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arctanh(b*x^4+a)+1/2*ln(1-(b*x^4+a)^2))
```

Maxima [A]
time = 0.25, size = 37, normalized size = 0.84

$$\frac{2\left(bx^4+a\right)\mathrm{artanh}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="maxima")

[Out] 1/8*(2*(b*x^4 + a)*arctanh(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b

Fricas [A]
time = 0.37, size = 59, normalized size = 1.34

$$\frac{bx^4\log\left(-\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="fricas")

[Out] 1/8*(b*x^4*log(-(b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a + 1) - (a - 1)*log(b*x^4 + a - 1))/b

Sympy [A]
time = 0.98, size = 60, normalized size = 1.36

$$\begin{cases}\frac{a\,\mathrm{atanh}\,(a+bx^4)}{4b}+\frac{x^4\,\mathrm{atanh}\,(a+bx^4)}{4}+\frac{\log\,(a+bx^4+1)}{4b}-\frac{\mathrm{atanh}\,(a+bx^4)}{4b} & \text{for } b\neq 0\\ \frac{x^4\,\mathrm{atanh}\,(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atanh(b*x**4+a),x)

[Out] Piecewise((a*atanh(a + b*x**4)/(4*b) + x**4*atanh(a + b*x**4)/4 + log(a + b*x**4 + 1)/(4*b) - atanh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*atanh(a)/4, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(40) = 80.
time = 0.41, size = 223, normalized size = 5.07

$$\frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|-bx^4-a-1|}{|bx^4+a-1|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^4+a+1}{bx^4+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}+1}{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^4 - a - 1)/abs(b*x^4 + a - 1))/b^
2 - log(abs(-(b*x^4 + a + 1)/(b*x^4 + a - 1) + 1))/b^2 + log(-(a - ((b*x^4
+ a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a -
 1) - b) + 1)/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*
x^4 + a + 1)*b/(b*x^4 + a - 1) - b) - 1))/(b^2*((b*x^4 + a + 1)/(b*x^4 + a
- 1) - 1)))
```

Mupad [B]
time = 0.32, size = 90, normalized size = 2.05

$$\frac{\ln\left(b\,x^4+a-1\right)}{8\,b}-\frac{x^4\,\ln\left(-b\,x^4-a+1\right)}{8}+\frac{\ln\left(b\,x^4+a+1\right)}{8\,b}+\frac{x^4\,\ln\left(b\,x^4+a+1\right)}{8}-\frac{a\,\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{a\,\ln\left(b\,x^4+a+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a + b*x^4),x)
```

```
[Out] log(a + b*x^4 - 1)/(8*b) - (x^4*log(1 - b*x^4 - a))/8 + log(a + b*x^4 + 1)/
(8*b) + (x^4*log(a + b*x^4 + 1))/8 - (a*log(a + b*x^4 - 1))/(8*b) + (a*log(
a + b*x^4 + 1))/(8*b)
```

3.29 $\int x^{-1+n} \tanh^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=47

$$\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

```
[Out] (a+b*x^n)*arctanh(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 6238, 6021, 266}

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTanh[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6021

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTanh[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6238

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\tanh^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \tanh^{-1}(a+bx)\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \tanh^{-1}(x)\,dx, x, a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn} + \frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.89

$$\frac{2(a+bx^n)\tanh^{-1}(a+bx^n)+\log\left(1-(a+bx^n)^2\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcTanh[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(45) = 90.
time = 0.05, size = 121, normalized size = 2.57

method	result	size
risch	$\frac{x^n\ln(a+b\,x^n+1)}{2n}-\frac{x^n\ln(1-a-b\,x^n)}{2n}-\frac{\ln\left(x^n+\frac{-1+a}{b}\right)a}{2bn}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)a}{2bn}+\frac{\ln\left(x^n+\frac{-1+a}{b}\right)}{2bn}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)}{2bn}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arctanh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*x^n*ln(a+b*x^n+1)-1/2/n*x^n*ln(1-a-b*x^n)-1/2/b/n*ln(x^n+(-1+a)/b)*a+
1/2/b/n*ln(x^n+(1+a)/b)*a+1/2/b/n*ln(x^n+(-1+a)/b)+1/2/b/n*ln(x^n+(1+a)/b)
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.85

$$\frac{2\,(bx^n+a)\,\text{artanh}\,(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctanh(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.
time = 0.35, size = 109, normalized size = 2.32

$$\frac{(a+1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a+1\right)-(a-1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a-1\right)+\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)\right)\log\left(-\frac{b\cosh(n\log(x))+b\sinh(n\log(x))+a+1}{b\cosh(n\log(x))+b\sinh(n\log(x))+a-1}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*log
(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sinh(
n*log(x)))*log(-(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*log
(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atanh(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(45) = 90.
time = 0.40, size = 124, normalized size = 2.64

$$\frac{\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|-bx^n-a-1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^n+a+1}{bx^n+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^n - a - 1)/abs(b*x^n + a - 1))/b^
2 - log(abs(-(b*x^n + a + 1)/(b*x^n + a - 1) + 1))/b^2 + log(-(b*x^n + a +
1)/(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

Mupad [B]
time = 1.47, size = 56, normalized size = 1.19

$$\frac{x^n\,\mathrm{atanh}(a+b\,x^n)}{n}-\frac{\ln\left(a+b\,x^n-1\right)\,\left(a-1\right)}{2\,b\,n}+\frac{\ln\left(a+b\,x^n+1\right)\,\left(a+1\right)}{2\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*atanh(a + b*x^n),x)
```

```
[Out] (x^n*atanh(a + b*x^n))/n - (log(a + b*x^n - 1)*(a - 1))/(2*b*n) + (log(a +
b*x^n + 1)*(a + 1))/(2*b*n)
```

3.30 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^n}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.31 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=409

$$-\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}}{2c}$$

```
[Out] 2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^3/c+3/2*b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*p
olylog(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b*(a+b*arctanh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/
c-3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(1-(-c*
x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1
/2)))*polylog(3,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*b^3*polylog(4,
1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*b^3*polylog(4,-1+2/(1-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A]
time = 0.36, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 6033, 6199, 6095, 6205, 6209, 6745}

$$-\frac{3b^2\text{Li}_3\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{Li}_3\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3b\text{Li}_2\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}+\frac{3b^3\text{Li}_4\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{4c}-\frac{3b^3\text{Li}_4\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (3*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1 + 2/(1 - Sqrt[1
- c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^
2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 - Sqrt[1 - c*x
]/Sqrt[1 + c*x])])/(4*c) - (3*b^3*PolyLog[4, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt
[1 + c*x])])/(4*c)
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
```

```
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6209

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} +$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} +$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} +$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} +$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} +$$

Mathematica [A]
time = 0.12, size = 482, normalized size = 1.18

$$-\frac{8\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)+6b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)-6b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)-6b^2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)+6b^2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)+3b^3\text{PolyLog}\left(4,-\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)-3b^3\text{PolyLog}\left(4,\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(8*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - S
qrt[1 - c*x]/Sqrt[1 + c*x])] + 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1
 + c*x]))] - 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
```

```
(Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - 6*b^2*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sqrt[1 - c*x] + Sq
rt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] + 6*b^2*(a + b*ArcTanh[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1
 - c*x] - Sqrt[1 + c*x])] + 3*b^3*PolyLog[4, -((Sqrt[1 - c*x] + Sqrt[1 + c*
x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] - 3*b^3*PolyLog[4, (Sqrt[1 - c*x] + S
qrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])])/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(349) = 698$.
time = 0.31, size = 1445, normalized size = 3.53

method	result	size
default	Expression too large to display	1445

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)-b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^3*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2
))-3*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arctanh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c
*x+1)+1)^(1/2))-6*b^3/c*polylog(4,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+
1)/(c*x+1)+1)^(1/2))-b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3*b^3/c*arctanh(
(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6
*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1
/2))+b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)+1)+3/2*b^3/c*arctanh((-c*x+1)^(1/2)/(c*
x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x
+1)+1))-3/2*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+3/4*b^3/c*polylog(4,-((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-3*a*b^2/c*arctanh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+
1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylo
g(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*a*b^2/c
*polylog(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3*
a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)
+1)^(1/2))+6*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)
```

```
/(c*x+1)+1)^(1/2))+3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)+1)+3*a*b^2/c*arctanh(
(-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2
/(-(-c*x+1)/(c*x+1)+1))-3/2*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2
)+1)^2/(-(-c*x+1)/(c*x+1)+1))-3/4*a^2*b*(4*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^
(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))-dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^
4*(-(-c*x+1)/(c*x+1)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(s
qrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^
3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sqr
t(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x
 + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c*
x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
+ 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - s
qrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sqrt
(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1)*
a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-
c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x +
 1) + sqrt(-c*x + 1)))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/((c^2*x^2 - 1)*
sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctanh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{atanh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atanh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atanh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\operatorname{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\mathrm{PolyLog}\left(2\right.}{c}$$

```
[Out] 2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^2/c+b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog
(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-b*(a+b*arctanh((-c*x+1)^(1/2)/(c
*x+1)^(1/2)))*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*po
lylog(3,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2*b^2*polylog(3,-1+2/(1-(
-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A]

time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6813, 6033, 6199, 6095, 6205, 6745}

$$\frac{b\mathrm{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b\mathrm{Li}_2\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}-\frac{b^2\mathrm{Li}_3\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{2c}+\frac{b^2\mathrm{Li}_3\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcTa
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/c - (b^2*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(
2*c) + (b^2*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c)
```

Rule 6033

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6199

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
e*x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6205

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \cdots$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \cdots$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \cdots$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \cdots$$

Mathematica [A]
time = 0.07, size = 324, normalized size = 1.21

$$-\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)+b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)-b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+\sqrt{1+cx}}{\sqrt{1-cx}-\sqrt{1+cx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -((2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt
[1 - c*x]/Sqrt[1 + c*x])] + b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*
PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]
))] - b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, (Sqrt[1 - c
*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - (b^2*PolyLog[3, -((
Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))])/2 + (b^2*
PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])]
)/2)/c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(232) = 464$.
time = 0.02, size = 674, normalized size = 2.51

method	result
default	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - \frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 - \frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c} - \frac{2b^2 \operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) \mathrm{po}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2
))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3,((-c*x+1)^(
1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-b^2/c*arctanh((-c*x+1)^(
1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x
+1)+1)^(1/2))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3
,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+b^2/c*arcta
nh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-
(-c*x+1)/(c*x+1)+1)+1)+b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(
2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-1/2*b^2/c*poly
log(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-1/2*a*b*(4
*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))-dilog(1/
((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(sq
rt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^2
 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*x
 + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*log
(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c*
x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
)/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctanh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atanh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\operatorname{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*b*polylog(2,-(-c*x+1)^(1/2)/(c*x+
1)^(1/2))/c-1/2*b*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {212, 6813, 6031}

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}+\frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{b\text{Li}_2\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + (b*PolyLog[2, -(Sqrt[1 - c*x]/S
qrt[1 + c*x])])/(2*c) - (b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} + \frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} - \frac{b\text{Li}_2\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

Mathematica [A]
time = 0.19, size = 43, normalized size = 0.48

$$\frac{a\tanh^{-1}(cx)}{c} + \frac{b\left(\text{PolyLog}\left(2, -e^{-\tanh^{-1}(cx)}\right) - \text{PolyLog}\left(2, e^{-\tanh^{-1}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] (a*ArcTanh[c*x])/c + (b*(PolyLog[2, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-Ar
cTanh[c*x])]))/(2*c)
```

Maple [A]
time = 0.02, size = 117, normalized size = 1.31

method	result	size
default	$\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} - \frac{b\left(4\,\text{dilog}\left(\frac{-\frac{-cx+1}{cx+1}+1}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1\right)^2}\right) - \text{dilog}\left(\frac{\left(-\frac{-cx+1}{cx+1}+1\right)^2}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1\right)^4}\right)\right)}{4c}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-1/4*b*(4*dilog(1/((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))-dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
)^4*(-(-c*x+1)/(c*x+1)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] 1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
- (log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/c - 2
*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)
*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sqrt(c
*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*
x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*atanh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arctanh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a c^{2} x^{2} - a + b c^{2} x^{2} \operatorname{atanh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)} - b \operatorname{atanh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)\left(a+b \,\mathrm{arctanh}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x + 1
)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
 + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 - b^2
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b^2*c
^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) - sqrt(-c*x +
1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctanh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2c^2x^2 - a^2 + 2abc^2x^2\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^2c^2x^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atanh(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atanh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.36 $\int x^m \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctanh(tanh(b*x+a))/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]])/(1 +
m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\tanh^{-1}(\tanh(a+bx))\,dx&=\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.92

$$x^m\left(\frac{bx^2}{2+m}+\frac{x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(1 + m))
```

Maple [A]
time = 0.19, size = 41, normalized size = 1.11

method	result
default	$\frac{b\,x^2 \mathrm{e}^{m\ln(x)}}{2+m}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)x\,\mathrm{e}^{m\ln(x)}}{1+m}$
risch	$\frac{x\,x^m\ln\left(\mathrm{e}^{bx+a}\right)}{1+m}-\frac{x\left(4bx+i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3 m+i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3 m-2i\pi\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-4i\pi\operatorname{csgn}(i\mathrm{e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctanh(tanh(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))/(m + 1)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 1.68

$$\frac{((bm+b)x^2+(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+((bm+b)x^2+(am+2\,a)x)\sinh\left(m\log\left(x\right)\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (((b*m + b)*x^2 + (a*m + 2*a)*x)*cosh(m*log(x)) + ((b*m + b)*x^2 + (a*m + 2
*a)*x)*sinh(m*log(x)))/(m^2 + 3*m + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log(x) - \frac{\operatorname{atanh}(\tanh(a+bx))}{x} & \text{for}\ m = -2 \\ \int \frac{\operatorname{atanh}(\tanh(a+bx))}{x}\,dx & \text{for}\ m = -1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{atanh}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{atanh}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - atanh(tanh(a + b*x))/x, Eq(m, -2)), (Integral(atanh(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
atanh(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*atanh(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

Giac [A]
time = 0.40, size = 43, normalized size = 1.16

$$\frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [B]
time = 1.49, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x))/(6*m + 2*m^2 + 4)
```

3.37 $\int x^2 \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTanh[Tanh[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))\,dx&=\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))-\frac{1}{3}b\int x^3\,dx\\&=-\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^4}{12}+\frac{x^3\operatorname{arctanh}(\tanh(bx+a))}{3}$
risch	$\frac{x^3\ln\left(e^{bx+a}\right)}{3}-\frac{b\,x^4}{12}+\frac{i\pi\,x^3\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{12}+\frac{i\pi\,x^3\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)^2}{6}-\frac{i\pi\,x^3\operatorname{csgn}\left(ie^{2bx+2a}\right)}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{1}{12}\,bx^4+\frac{1}{3}\,x^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.12, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12} + \frac{x^3 \operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a)),x)
[Out] -b*x**4/12 + x**3*atanh(tanh(a + b*x))/3
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 1.00, size = 19, normalized size = 0.83

$$\frac{x^3 \operatorname{atanh}(\tanh(a + b\,x))}{3} - \frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x)),x)
[Out] (x^3*atanh(tanh(a + b*x)))/3 - (b*x^4)/12
```

3.38 $\int x \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6374, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTanh[Tanh[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))\,dx&=\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]],x]
[Out] -1/6*(x^2*(b*x - 3*ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arctanh}(\tanh(bx+a))}{2}$
risch	$\frac{x^{2}\ln\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{b\,x^{3}}{6}+\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}}{8}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}}{8}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\operatorname{csgn}\left(i\mathrm{e}^{2b\cdots}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.83

$$-\frac{1}{6}\,bx^{3}+\frac{1}{2}\,x^{2}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.
time = 0.11, size = 39, normalized size = 1.70

$$\begin{cases}\frac{x\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{2b}-\frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{6b^{2}} & \text{for}\ b\neq 0\\ \frac{x^{2}\operatorname{atanh}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**2/(2*b) - atanh(tanh(a + b*x))**3/(6*b**
2), Ne(b, 0)), (x**2*atanh(tanh(a))/2, True))
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B]
time = 0.98, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^2*atanh(tanh(a + b*x)))/2 - (b*x^3)/6
```

3.39 $\int \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

```
[Out] 1/2*arctanh(tanh(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2} + x\tanh^{-1}(\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTanh[Tanh[a + b*x]]
```

Maple [A]
time = 0.10, size = 15, normalized size = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2b}$
risch	$x \ln\left(e^{bx+a}\right) - \frac{i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right) x}{4} + \frac{i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(tanh(b*x+a))^2/b
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.00

$$-\frac{1}{2}\,bx^2 + x\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.33, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.06, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2b} & \text{for}\ b \neq 0 \\ x\,\operatorname{atanh}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*atanh(tanh(a)), Tru
e))
```

Giac [A]
time = 0.39, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.05, size = 16, normalized size = 1.00

$$x\,\mathrm{atanh}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x)),x)
```

```
[Out] x*atanh(tanh(a + b*x)) - (b*x^2)/2
```

3.40 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] b*x-(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$bx - \log(x)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))}{x} \, dx &= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right) \int \frac{1}{x} \, dx \\
&= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$bx + \left(-bx + \tanh^{-1}(\tanh(a+bx))\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x,x]
[Out] b*x + (-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

Maple [A]
time = 0.06, size = 23, normalized size = 1.10

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x) - x)$
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right) - \ln(x)\,xb + bx + \frac{i\pi\ln(x)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}}{4} - \frac{i\pi\ln(x)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}}{4} - \frac{i\pi\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x)
```

Maxima [A]
time = 0.26, size = 34, normalized size = 1.62

$$-b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{artanh}(\tanh(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="maxima")
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(tanh(b*x + a))*log(x)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="fricas")
[Out] b*x + a*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/x, x)
```

Giac [A]
time = 0.39, size = 9, normalized size = 0.43

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

Mupad [B]
time = 1.10, size = 58, normalized size = 2.76

$$b\,x-\frac{\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}+\frac{\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}-b\,x\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x,x)
```

```
[Out] b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

3.41 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2} dx$

Optimal. Leaf size=17

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

```
[Out] -arctanh(tanh(b*x+a))/x+b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 29}

$$b\log(x) - \frac{\tanh^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\int \frac{1}{x}\,dx\\ &= -\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.06

$$b - \frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] b - ArcTanh[Tanh[a + b*x]]/x + b*Log[x]
```

Maple [A]
time = 0.07, size = 18, normalized size = 1.06

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{x}+\frac{i\pi \operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+i\pi \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+i\pi \operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+i\pi}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(tanh(b*x+a))/x+b*ln(x)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 1.00

$$b\log(x)-\frac{\operatorname{artanh}(\tanh(bx+a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arctanh(tanh(b*x + a))/x
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.76

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A]
time = 0.08, size = 14, normalized size = 0.82

$$b\log(x)-\frac{\operatorname{atanh}(\tanh(a+bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**2,x)
[Out] b*log(x) - atanh(tanh(a + b*x))/x
```

Giac [A]
time = 0.38, size = 12, normalized size = 0.71

$$b\log\left(|x|\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="giac")
[Out] b*log(abs(x)) - a/x
```

Mupad [B]
time = 0.09, size = 17, normalized size = 1.00

$$b\,\ln\left(x\right)-\frac{\mathrm{atanh}(\mathrm{tanh}(a+b\,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^2,x)
[Out] b*log(x) - atanh(tanh(a + b*x))/x
```

3.42 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3} \, dx$

Optimal. Leaf size=23

$$-\frac{b}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}$$

```
[Out] -1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -1/2*b/x - ArcTanh[Tanh[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3} \, dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2} \, dx \\ &= -\frac{b}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$-\frac{bx+\tanh^{-1}(\tanh(a+bx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^3,x]
[Out] -1/2*(b*x + ArcTanh[Tanh[a + b*x]])/x^2
```

Maple [A]
time = 0.06, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}$
risch	$-\frac{\ln(e^{bx+a})}{2x^2}-\frac{4bx-i\pi \operatorname{csgn}(ie^{2bx+2a})^3+i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}(ie^{2bx+2a})\operatorname{csgn}\left(\frac{ie^{2}}{e^{2bx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{b}{2\,x}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="maxima")
[Out] -1/2*b/x - 1/2*arctanh(tanh(b*x + a))/x^2
```

Fricas [A]
time = 0.33, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A]
time = 0.19, size = 19, normalized size = 0.83

$$-\frac{b}{2x} - \frac{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**3,x)
```

```
[Out] -b/(2*x) - atanh(tanh(a + b*x))/(2*x**2)
```

Giac [A]
time = 0.38, size = 11, normalized size = 0.48

$$-\frac{2\,bx + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Mupad [B]
time = 0.95, size = 16, normalized size = 0.70

$$-\frac{\operatorname{atanh}(\tanh(a + b\,x)) + b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^3,x)
```

```
[Out] -(atanh(tanh(a + b*x)) + b*x)/(2*x^2)
```

3.43 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4} dx$

Optimal. Leaf size=23

$$-\frac{b}{6x^2} - \frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}$$

```
[Out] -1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -1/6*b/x^2 - ArcTanh[Tanh[a + b*x]]/(3*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4} dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3} + \frac{1}{3}b \int \frac{1}{x^3} dx \\
&= -\frac{b}{6x^2} - \frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{bx+2\tanh^{-1}(\tanh(a+bx))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -1/6*(b*x + 2*ArcTanh[Tanh[a + b*x]])/x^3
```

Maple [A]
time = 0.06, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}$
risch	$-\frac{\ln(e^{bx+a})}{3x^3}-\frac{2bx-i\pi\operatorname{csgn}(ie^{bx+a})^2\operatorname{csgn}(ie^{2bx+2a})-i\pi\operatorname{csgn}(ie^{2bx+2a})^3+2i\pi\operatorname{csgn}(ie^{bx+a})\operatorname{csgn}(ie^{2bx+2a})^2-i\pi\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{b}{6\,x^2}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b/x^2 - 1/3*arctanh(tanh(b*x + a))/x^3
```

Fricas [A]
time = 0.32, size = 13, normalized size = 0.57

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A]
time = 0.25, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2} - \frac{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**4,x)
```

```
[Out] -b/(6*x**2) - atanh(tanh(a + b*x))/(3*x**3)
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.57

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Mupad [B]
time = 0.07, size = 19, normalized size = 0.83

$$-\frac{\operatorname{atanh}(\tanh(a + b\,x))}{3\,x^3} - \frac{b}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^4,x)
```

```
[Out] - atanh(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

3.44 $\int x^m \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=71

$$\frac{2b^2x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m}$$

```
[Out] 2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arctanh(tanh(b*x+a))/(m^2+3*m+
2)+x^(1+m)*arctanh(tanh(b*x+a))^2/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{2bx^{m+2}\tanh^{-1}(\tanh(a+bx))}{m^2+3m+2} + \frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^2}{m+1} + \frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcTanh[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^2)/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m} - \frac{(2b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx)) \, dx}{1+m}\\ &= -\frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m} + \frac{(2b^2)\int\\ &= \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\mathrm{ta}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.87

$$\frac{x^{1+m}\left(2b^2x^2-2b(3+m)x\tanh^{-1}(\tanh(a+bx))+\left(6+5m+m^2\right)\tanh^{-1}(\tanh(a+bx))^2\right)}{(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcTanh[Tanh[a + b*x]] + (6 + 5*m + m
^2)*ArcTanh[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

Maple [A]
time = 0.69, size = 98, normalized size = 1.38

method	result
default	$\frac{b^2x^3e^{m\ln(x)}}{3+m}+\frac{\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)x\,e^{m\ln(x)}}{1+m}+\frac{2b(\operatorname{arctanh}(\tanh(bx+a))-}{2+m}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(3+m)*x^3*exp(m*ln(x))+(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(t
anh(b*x+a))-b*x-a)^2)/(1+m)*x*exp(m*ln(x))+2*b*(arctanh(tanh(b*x+a))-b*x)/(
2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.30, size = 73, normalized size = 1.03

$$\frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)}-\frac{2\,bx^2x^m\operatorname{artanh}(\tanh(bx+a))}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{artanh}(\tanh(bx+a))^2}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arctanh(tanh(b*x + a)
)/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))^2/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(71) = 142.
time = 0.36, size = 161, normalized size = 2.27

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\,\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)\cosh\left(m\log\left(x\right)\right)+\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\,\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)\sinh\left(m\log\left(x\right)\right)}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^
2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + ((b^2*m^2 + 3*b^2*m + 2*b^2)*x
^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2*m^2 + 5*a^2*m + 6*a^2)*x)*sin
h(m*log(x)))/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^2 \log(x) - \frac{b \operatorname{atanh}(\tanh(a+bx))}{x} - \frac{\operatorname{atanh}^2(\tanh(a+bx))}{2x^2} & \text{for } m = -3 \\ \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x^2}\, dx & \text{for } m = -2 \\ \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x}\, dx & \text{for } m = -1 \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m \operatorname{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m \operatorname{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m \operatorname{atanh}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m \operatorname{atanh}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{6xx^m \operatorname{atanh}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2
/(2*x**2), Eq(m, -3)), (Integral(atanh(tanh(a + b*x))**2/x**2, x), Eq(m, -2
)), (Integral(atanh(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x**m/
(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*atanh(tanh(a + b*x))/(m**3 + 6
*m**2 + 11*m + 6) - 6*b*x**2*x**m*atanh(tanh(a + b*x))/(m**3 + 6*m**2 + 11*
m + 6) + m**2*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 5
*m*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x**m*ata
nh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^2, x)
```

Mupad [B]
time = 1.13, size = 203, normalized size = 2.86

$$\frac{4\,b^2\,x^m\,x^3\,\left(m^2+3\,m+2\right)}{4\,m^3+24\,m^2+44\,m+24}+\frac{x\,x^m\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2\,\left(m^2+5\,m+6\right)}{4\,m^3+24\,m^2+44\,m+24}-\frac{4\,b\,x^m\,x^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(m^2+4\,m+3\right)}{4\,m^3+24\,m^2+44\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
```

```
b*x) + 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (4*b*
x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4*m^3 +
24)
```

3.45 $\int x^3 \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$\frac{b^2x^6}{60} - \frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 1/60*b^2*x^6-1/10*b*x^5*arctanh(tanh(b*x+a))+1/4*x^4*arctanh(tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$,
Rules used = {2199, 30}

$$-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 + \frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcTanh[Tanh[a + b*x]])/10 + (x^4*ArcTanh[Tanh[a + b*
x]]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned} \int x^3\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\int x^4\tanh^{-1}(\tanh(a+bx))\,dx \\ &= -\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{10}b^2\int x^5 \\ &= \frac{b^2x^6}{60} - \frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 \end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4\left(b^2x^2-6bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
[Out] (x^4*(b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2)
)/60
```

Maple [A]
time = 35.96, size = 38, normalized size = 0.90

method	result	size
default	$\frac{x^4\,\mathrm{arctanh}(\tanh(bx+a))^2}{4}-\frac{b\left(\frac{x^5\,\mathrm{arctanh}(\tanh(bx+a))}{5}-\frac{b\,x^6}{30}\right)}{2}$	38
risch	Expression too large to display	2083

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*b*x
^6)
```

Maxima [A]
time = 0.34, size = 36, normalized size = 0.86

$$\frac{1}{60}\,b^2x^6-\frac{1}{10}\,bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{1}{4}\,x^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 1/60*b^2*x^6 - 1/10*b*x^5*arctanh(tanh(b*x + a)) + 1/4*x^4*arctanh(tanh(b*x
 + a))^2
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.57

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.26, size = 37, normalized size = 0.88

$$\frac{b^2x^6}{60} - \frac{bx^5 \operatorname{atanh}(\tanh(a+bx))}{10} + \frac{x^4 \operatorname{atanh}^2(\tanh(a+bx))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**6/60 - b*x**5*atanh(tanh(a + b*x))/10 + x**4*atanh(tanh(a + b*x))**
2/4
```

Giac [A]
time = 0.40, size = 24, normalized size = 0.57

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B]
time = 1.00, size = 36, normalized size = 0.86

$$\frac{b^2\,x^6}{60} - \frac{b\,x^5\,\mathrm{atanh}(\tanh(a+b\,x))}{10} + \frac{x^4\,\mathrm{atanh}(\tanh(a+b\,x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^4*atanh(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*atanh(tanh(a + b*x))
)/10
```

3.46 $\int x^2 \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$\frac{b^2x^5}{30} - \frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 1/30*b^2*x^5-1/6*b*x^4*arctanh(tanh(b*x+a))+1/3*x^3*arctanh(tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2 + \frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcTanh[Tanh[a + b*x]])/6 + (x^3*ArcTanh[Tanh[a + b*x
]]^2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^2 \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{3}(2b) \int x^3 \tanh^{-1}(\tanh(a+bx)) \, dx \\
&= -\frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{6}b^2 \int x^4 \, d \\
&= \frac{b^2x^5}{30} - \frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3\left(b^2x^2-5bx\tanh^{-1}(\tanh(a+bx))+10\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^3*(b^2*x^2 - 5*b*x*ArcTanh[Tanh[a + b*x]] + 10*ArcTanh[Tanh[a + b*x]]^2)
)/30
```

Maple [A]
time = 30.39, size = 38, normalized size = 0.90

method	result	size
default	$\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{2b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))}{4}-\frac{b\,x^5}{20}\right)}{3}$	38
risch	Expression too large to display	2083

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x
^5)
```

Maxima [A]
time = 0.33, size = 36, normalized size = 0.86

$$\frac{1}{30}\,b^2x^5-\frac{1}{6}\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{1}{3}\,x^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/30*b^2*x^5 - 1/6*b*x^4*arctanh(tanh(b*x + a)) + 1/3*x^3*arctanh(tanh(b*x
+ a))^2
```

Fricas [A]
time = 0.32, size = 24, normalized size = 0.57

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.17, size = 37, normalized size = 0.88

$$\frac{b^2x^5}{30} - \frac{bx^4 \operatorname{atanh}(\tanh(a+bx))}{6} + \frac{x^3 \operatorname{atanh}^2(\tanh(a+bx))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**5/30 - b*x**4*atanh(tanh(a + b*x))/6 + x**3*atanh(tanh(a + b*x))**2
/3
```

Giac [A]
time = 0.40, size = 24, normalized size = 0.57

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.97, size = 36, normalized size = 0.86

$$\frac{b^2\,x^5}{30} - \frac{b\,x^4\,\mathrm{atanh}(\tanh(a+b\,x))}{6} + \frac{x^3\,\mathrm{atanh}(\tanh(a+b\,x))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^3*atanh(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*atanh(tanh(a + b*x))
)/6
```

3.47 $\int x \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

```
[Out] 1/3*x*arctanh(tanh(b*x+a))^3/b-1/12*arctanh(tanh(b*x+a))^4/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2199, 2188, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^3)/(3*b) - ArcTanh[Tanh[a + b*x]]^4/(12*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^3\,dx}{3b}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b} - \frac{\text{Subst}\left(\int x^3\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.04, size = 74, normalized size = 2.18

$$\frac{(a+bx)\left(-((3a-bx)(a+bx)^2)+4(2a^2+abx-b^2x^2)\tanh^{-1}(\tanh(a+bx))-6(a-bx)\tanh^{-1}(\tanh(a+bx))^2\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcTan
h[Tanh[a + b*x]] - 6*(a - b*x)*ArcTanh[Tanh[a + b*x]]^2))/(12*b^2)
```

Maple [A]
time = 30.59, size = 38, normalized size = 1.12

method	result	size
default	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^2}{2} - b\left(-\frac{b x^4}{12} + \frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}\right)$	38
risch	Expression too large to display	2083

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^2-b*(-1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a)))
```

Maxima [A]
time = 0.33, size = 36, normalized size = 1.06

$$\frac{1}{12}\,b^2x^4 - \frac{1}{3}\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^4 - 1/3*b*x^3*arctanh(tanh(b*x + a)) + 1/2*x^2*arctanh(tanh(b*x
+ a))^2
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.71

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A]
time = 0.15, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3b}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{12b^2} & \text{for } b\neq 0\\ \frac{x^2\,\mathrm{atanh}^2\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**3/(3*b) - atanh(tanh(a + b*x))**4/(12*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**2/2, True))
```

Giac [A]
time = 0.41, size = 24, normalized size = 0.71

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Mupad [B]
time = 0.94, size = 36, normalized size = 1.06

$$\frac{b^2\,x^4}{12}-\frac{b\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))}{3}+\frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*atanh(tanh(a + b*x))
)/3
```

3.48 $\int \tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

Maple [A]
time = 0.21, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3b}$	15
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3b}$	15
risch	Expression too large to display	6270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.34, size = 33, normalized size = 2.06

$$\frac{1}{3}\,b^2x^3 - bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 - b*x^2*arctanh(tanh(b*x + a)) + x*arctanh(tanh(b*x + a))^2
```

Fricas [A]
time = 0.33, size = 20, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [A]
time = 0.09, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3b} & \text{for}\ b \neq 0 \\ x\operatorname{atanh}^2\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*atanh(tanh(a))**2,
True))
```

Giac [A]
time = 0.40, size = 20, normalized size = 1.25

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Mupad [B]
time = 0.07, size = 33, normalized size = 2.06

$$\frac{b^2\,x^3}{3} - b\,x^2\,\mathrm{atanh}(\tanh(a + b\,x)) + x\,\mathrm{atanh}(\tanh(a + b\,x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2,x)
```

```
[Out] x*atanh(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*atanh(tanh(a + b*x))
```

3.49 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} \, dx$

Optimal. Leaf size=49

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log(x)$$

```
[Out] -b*x*(b*x-arctanh(tanh(b*x+a)))+1/2*arctanh(tanh(b*x+a))^2+(b*x-arctanh(tan
h(b*x+a)))^2*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2190, 2189, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])) + ArcTanh[Tanh[a + b*x]]^2/2 + (b*x -
 ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a$$
$$= -bx\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 - \left(\left(bx - \tan\right.\right.$$
$$= -bx\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 + \left(bx - \tanh\right.$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2 - (a+bx)\left(a+2bx-2\tanh^{-1}(\tanh(a+bx))\right) + \left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] (a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcTanh[Tanh[a + b*x]]) + (-(b*x)
+ ArcTanh[Tanh[a + b*x]])^2*Log[b*x]
```

Maple [A]
time = 0.14, size = 78, normalized size = 1.59

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 - 2b\left(\frac{b\,x^2\ln(x)}{2} - \frac{b\,x^2}{4} + \ln(x)\,xa - xa + \ln(x)\,x(\operatorname{arctanh}(\tanh(bx$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^2-2*b*(1/2*b*x^2*ln(x)-1/4*b*x^2+ln(x)*x*a-x*a+l
n(x)*x*(arctanh(tanh(b*x+a))-b*x-a)-x*(arctanh(tanh(b*x+a))-b*x-a))
```

Maxima [A]
time = 0.66, size = 20, normalized size = 0.41

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Fricas [A]
time = 0.33, size = 20, normalized size = 0.41

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x, x)
```

Giac [A]
time = 0.40, size = 21, normalized size = 0.43

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

Mupad [B]
time = 0.29, size = 183, normalized size = 3.73

$$\ln\left(x\right)\,\left(\frac{{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4}-a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)+a^2\right)+\frac{b^2\,x^2}{2}-b\,x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x,x)
```

```
[Out] log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x) + a^2) + (b^2*x^2)/2 - b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.50 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx$

Optimal. Leaf size=39

$$2b^2x-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-2b\big(bx-\tanh^{-1}(\tanh(a+bx))\big)\log(x)$$

```
[Out] 2*b^2*x-arctanh(tanh(b*x+a))^2/x-2*b*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2199, 2189, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] 2*b^2*x - ArcTanh[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcTanh[Tanh[a + b*x]])*L
og[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx$$
$$= 2b^2x - \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - \left(2b\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{x}\,dx$$
$$= 2b^2x - \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - 2b\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\log(x)$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.95

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - 2b^2x\log(x) + 2b\tanh^{-1}(\tanh(a+bx))(1+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcTanh[Tanh[a + b*x]]
*(1 + Log[x])
```

Maple [A]
time = 0.10, size = 41, normalized size = 1.05

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x} + 2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x) - x))$
risch	$-\frac{\ln(e^{bx+a})^2}{x} + 2b^2x + \frac{i\pi\ln(x)b\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2} - \frac{i\pi\ln(e^{bx+a})\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{2x} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(tanh(b*x+a))^2/x+2*b*(ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x))
```

Maxima [A]
time = 0.32, size = 54, normalized size = 1.38

$$2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\log\left(x\right) - 2\left(b\left(x+\frac{a}{b}\right)\log\left(x\right) - b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*arctanh(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b - arctanh(tanh(b*x + a))^2/x
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.62

$$\frac{b^2x^2 + 2\,abx\log\left(x\right) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**2, x)
```

Giac [A]
time = 0.39, size = 21, normalized size = 0.54

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B]
time = 0.19, size = 198, normalized size = 5.08

$$b\ln\left(\frac{e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) - \frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2}{4\,x} - b\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) - \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2}{4\,x} + b\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\ln\left(x\right) - 2\,b^2\,x\ln\left(x\right) - b\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\ln\left(x\right) + \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^2,x)
```

```
[Out] b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - log(
1/(exp(2*a)*exp(2*b*x) + 1))^2/(4*x) + b*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))*log(x) - 2*b^2*x*log(x) - b*log(1/(exp(2*a)*exp(2*b*x) +
 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)))/(2*x)
```

3.51 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3} \, dx$

Optimal. Leaf size=36

$$-\frac{b \tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^2 \log(x)$$

```
[Out] -b*arctanh(tanh(b*x+a))/x-1/2*arctanh(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b \tanh^{-1}(\tanh(a+bx))}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -((b*ArcTanh[Tanh[a + b*x]])/x) - ArcTanh[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3} \, dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b \int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2} \, dx \\
&= -\frac{b \tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^2 \int \frac{1}{x} \, dx \\
&= -\frac{b \tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^2 \log(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.17

$$-\frac{2bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2-b^2x^2(3+2\log(x))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -1/2*(2*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 - b^2*x^2*(3
+ 2*Log[x]))/x^2
```

Maple [A]
time = 0.22, size = 35, normalized size = 0.97

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)\right)$	35
risch	Expression too large to display	1974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^2/x^2+b*(-arctanh(tanh(b*x+a))/x+b*ln(x))
```

Maxima [A]
time = 0.34, size = 34, normalized size = 0.94

$$b^2\log(x)-\frac{b\,\operatorname{artanh}(\tanh(bx+a))}{x}-\frac{\operatorname{artanh}(\tanh(bx+a))^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

```
[Out] b^2*log(x) - b*arctanh(tanh(b*x + a))/x - 1/2*arctanh(tanh(b*x + a))^2/x^2
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.72

$$\frac{2\,b^2x^2\log(x)-4\,abx-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A]
time = 0.20, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{atanh}(\tanh(a+bx))}{x} - \frac{\operatorname{atanh}^2(\tanh(a+bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**3,x)
[Out] b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2/(2*x**2)
```

Giac [A]
time = 0.39, size = 22, normalized size = 0.61

$$b^2 \log(|x|) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

Mupad [B]
time = 0.93, size = 34, normalized size = 0.94

$$b^2\,\ln(x) - \frac{\frac{\operatorname{atanh}(\tanh(a+b\,x))^2}{2} + b\,x\,\operatorname{atanh}(\tanh(a+b\,x))}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^3,x)
[Out] b^2*log(x) - (atanh(tanh(a + b*x))^2/2 + b*x*atanh(tanh(a + b*x)))/x^2
```

3.52 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/x^3/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} \, dx = \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 1.10

$$-\frac{b^2x^2+bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
[Out] -1/3*(b^2*x^2 + b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2)/x^3
```

Maple [A]
time = 0.20, size = 38, normalized size = 1.23

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{3x^3}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}\right)}{3}$	38
risch	Expression too large to display	1978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^
2)
```

Maxima [A]
time = 0.33, size = 36, normalized size = 1.16

$$-\frac{b^2}{3\,x}-\frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^2}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/x - 1/3*b*arctanh(tanh(b*x + a))/x^2 - 1/3*arctanh(tanh(b*x + a))^
2/x^3
```

Fricas [A]
time = 0.32, size = 22, normalized size = 0.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A]
time = 0.27, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x}-\frac{b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^2}-\frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**4,x)
```

```
[Out] -b**2/(3*x) - b*atanh(tanh(a + b*x))/(3*x**2) - atanh(tanh(a + b*x))**2/(3*
x**3)
```

Giac [A]
time = 0.39, size = 22, normalized size = 0.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="giac")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Mupad [B]
time = 0.94, size = 32, normalized size = 1.03

$$-\frac{b^2\,x^2+b\,x\,\mathrm{atanh}(\tanh(a+b\,x))+\mathrm{atanh}(\tanh(a+b\,x))^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^4,x)
[Out] -(atanh(tanh(a + b*x))^2 + b^2*x^2 + b*x*atanh(tanh(a + b*x)))/(3*x^3)
```

3.53 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx$

Optimal. Leaf size=42

$$-\frac{b^2}{12x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))}{6x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^2}{4x^4}$$

```
[Out] -1/12*b^2/x^2-1/6*b*arctanh(tanh(b*x+a))/x^3-1/4*arctanh(tanh(b*x+a))^2/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2202, 2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4}\,dx}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.88

$$-\frac{b^2x^2+2bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] -1/12*(b^2*x^2 + 2*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
/x^4
```

Maple [A]
time = 0.21, size = 38, normalized size = 0.90

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{4x^4}+\frac{b\left(-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}$	38
risch	Expression too large to display	1978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/
x^3)
```

Maxima [A]
time = 0.35, size = 36, normalized size = 0.86

$$-\frac{b^2}{12\,x^2}-\frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{6\,x^3}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*b^2/x^2 - 1/6*b*arctanh(tanh(b*x + a))/x^3 - 1/4*arctanh(tanh(b*x + a
))^2/x^4
```

Fricas [A]
time = 0.35, size = 24, normalized size = 0.57

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A]
time = 0.39, size = 39, normalized size = 0.93

$$-\frac{b^2}{12x^2} - \frac{b\,\mathrm{atanh}\,(\tanh\,(a+bx))}{6x^3} - \frac{\mathrm{atanh}^2\,(\tanh\,(a+bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**5,x)
```

```
[Out] -b**2/(12*x**2) - b*atanh(tanh(a + b*x))/(6*x**3) - atanh(tanh(a + b*x))**2
/(4*x**4)
```

Giac [A]
time = 0.38, size = 24, normalized size = 0.57

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Mupad [B]
time = 0.95, size = 36, normalized size = 0.86

$$-\frac{\mathrm{atanh}(\tanh(a+b\,x))^2}{4\,x^4} - \frac{b^2}{12\,x^2} - \frac{b\,\mathrm{atanh}(\tanh(a+b\,x))}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^5,x)
```

```
[Out] - atanh(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*atanh(tanh(a + b*x)))/
(6*x^3)
```

3.54 $\int x^m \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=110

$$-\frac{6b^3x^{4+m}}{(1+m)\left(24+26m+9m^2+m^3\right)}+\frac{6b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+m}\tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}}{}$$

```
[Out] -6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arctanh(tanh(b*x+a))
/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arctanh(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m)*
arctanh(tanh(b*x+a))^3/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{6b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\tanh^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m+4}}{(m+1)\left(m^3+9m^2+26m+24\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcTanh[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^3 \, dx = \frac{x^{1+m} \tanh^{-1}(\tanh(a+bx))^3}{1+m} - \frac{(3b) \int x^{1+m} \tanh^{-1}(\tanh(a+bx))^2 \, dx}{1+m}$$
$$= -\frac{3bx^{2+m} \tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m} \tanh^{-1}(\tanh(a+bx))^3}{1+m} + \frac{(6b^2)}{}$$
$$= \frac{6b^2x^{3+m} \tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m} \tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}}{}$$
$$= -\frac{6b^3x^{4+m}}{(4+m)(6+11m+6m^2+m^3)} + \frac{6b^2x^{3+m} \tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3b}{}$$

Mathematica [A]
time = 0.10, size = 97, normalized size = 0.88

$$\frac{x^{1+m}(-6b^3x^3+6b^2(4+m)x^2\tanh^{-1}(\tanh(a+bx))-3b(12+7m+m^2)\,x\tanh^{-1}(\tanh(a+bx))^2+(24+26m+9m^2+m^3)\tanh^{-1}(\tanh(a+bx))^3)}{(1+m)(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*(12
 + 7*m + m^2)*x*ArcTanh[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcTan
h[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [A]
time = 6.04, size = 177, normalized size = 1.61

method	result
default	$\frac{b^3x^4e^{m\ln(x)}}{4+m}+\frac{\left(a^3+3a^2(\mathrm{arctanh}(\tanh(bx+a))-bx-a)+3a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)^2+(\mathrm{arctanh}(\tanh(bx+a))-bx-a)^3\right)x\,e^{m\,\mathrm{l}}}{1+m}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/(4+m)*x^4*exp(m*ln(x))+(a^3+3*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3*a*(arc
tanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^3)/(1+m)*x*exp(m*ln
(x))+3*b*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)
^2)/(2+m)*x^2*exp(m*ln(x))+3*b^2*(arctanh(tanh(b*x+a))-b*x)/(3+m)*x^3*exp(m
*ln(x))
```

Maxima [A]
time = 0.35, size = 109, normalized size = 0.99

$$-\frac{3\,bx^2x^m\,\mathrm{artanh}\,(\tanh\,(bx+a))^2}{(m+2)(m+1)}+\frac{x^{m+1}\,\mathrm{artanh}\,(\tanh\,(bx+a))^3}{m+1}-\frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)}-\frac{bx^3x^m\,\mathrm{artanh}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3*b*x^2*x^m*arctanh(tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^(m + 1)*arctanh
(tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*x^
3*x^m*arctanh(tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(110) = 220$.
time = 0.37, size = 300, normalized size = 2.73

$$\frac{((b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+3\,(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2)x^3+3\,(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b)x^2+(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3)x)\cosh(m\log(x))+((b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+3\,(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2)x^3+3\,(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b)x^2+(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3)x)\sinh(m\log(x))}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] (((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)) +
 ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log(x)))/
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^3\log(x) - \frac{b^2\operatorname{atanh}(\tanh(a+bx))}{x} - \frac{b\operatorname{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{3x^3} & \text{for}\ m=-4 \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^3}\,dx & \text{for}\ m=-3 \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^2}\,dx & \text{for}\ m=-2 \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x}\,dx & \text{for}\ m=-1 \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m\operatorname{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m\operatorname{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m\operatorname{atanh}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{21bmx^2x^m\operatorname{atanh}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{36bx^2x^m\operatorname{atanh}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{m^3xx^m\operatorname{atanh}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{9m^2xx^m\operatorname{atanh}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{26mxx^m\operatorname{atanh}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24xx^m\operatorname{atanh}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x
))**2/(2*x**2) - atanh(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(at
anh(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(atanh(tanh(a + b*x))*
*3/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a + b*x))**3/x, x), Eq(m, -1)
), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*m*x**
3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*b**
2*x**3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 3
*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2 + 50*m
+ 24) - 21*b*m*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
+ 50*m + 24) - 36*b*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + m**3*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m**3 + 3
5*m**2 + 50*m + 24) + 9*m**2*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m**3
```

```
 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24) + 24*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^3, x)
```

Mupad [B]
time = 1.23, size = 332, normalized size = 3.02

$$\frac{8\,b^3\,x^m\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}-\frac{x\,x^m\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3\,\left(m^3+9\,m^2+26\,m+24\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}-\frac{12\,b^2\,x^m\,x^3\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(m^3+7\,m^2+14\,m+8\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}+\frac{6\,b\,x^m\,x^2\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2\,\left(m^3+8\,m^2+19\,m+12\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
+ 192) - (x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(400*
m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) + (6
*b*x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))/(400*m + 2
80*m^2 + 80*m^3 + 8*m^4 + 192)
```

3.55 $\int x^3 \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=61

$$-\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -1/140*b^3*x^7+1/20*b^2*x^6*arctanh(tanh(b*x+a))-3/20*b*x^5*arctanh(tanh(b*
x+a))^2+1/4*x^4*arctanh(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/140*(b^3*x^7) + (b^2*x^6*ArcTanh[Tanh[a + b*x]])/20 - (3*b*x^5*ArcTanh[T
anh[a + b*x]]^2)/20 + (x^4*ArcTanh[Tanh[a + b*x]]^3)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{4}(3b)\int x^4\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{10}(3b^2)\int$$
$$= \frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\tanh$$
$$= -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4\left(b^3x^3 - 7b^2x^2\tanh^{-1}(\tanh(a+bx)) + 21bx\tanh^{-1}(\tanh(a+bx))^2 - 35\tanh^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
[Out] -1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 21*b*x*ArcTanh[Ta
nh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.92

$$\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^3}{4} - \frac{3b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))^2}{5} - \frac{2b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))}{6} - \frac{b\,x^7}{42}\right)}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^3,x)
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*
(1/6*x^6*arctanh(tanh(b*x+a))-1/42*b*x^7))
```

Maxima [A]
time = 0.38, size = 54, normalized size = 0.89

$$-\frac{3}{20}bx^5\operatorname{artanh}(\tanh(bx+a))^2 + \frac{1}{4}x^4\operatorname{artanh}(\tanh(bx+a))^3 - \frac{1}{140}\left(b^2x^7 - 7bx^6\operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

[Out] -3/20*b*x^5*arctanh(tanh(b*x + a))^2 + 1/4*x^4*arctanh(tanh(b*x + a))^3 - 1/140*(b^2*x^7 - 7*b*x^6*arctanh(tanh(b*x + a)))*b

Fricas [A]
time = 0.32, size = 35, normalized size = 0.57

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")

[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4

Sympy [A]
time = 0.57, size = 58, normalized size = 0.95

$$-\frac{b^3x^7}{140}+\frac{b^2x^6\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{20}-\frac{3bx^5\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{20}+\frac{x^4\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atanh(tanh(b*x+a))**3,x)

[Out] -b**3*x**7/140 + b**2*x**6*atanh(tanh(a + b*x))/20 - 3*b*x**5*atanh(tanh(a + b*x))**2/20 + x**4*atanh(tanh(a + b*x))**3/4

Giac [A]
time = 0.38, size = 35, normalized size = 0.57

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="giac")

[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4

Mupad [B]
time = 1.05, size = 53, normalized size = 0.87

$$-\frac{b^3\,x^7}{140}+\frac{b^2\,x^6\,\mathrm{atanh}(\tanh(a+b\,x))}{20}-\frac{3\,b\,x^5\,{\mathrm{atanh}(\tanh(a+b\,x))}^2}{20}+\frac{x^4\,{\mathrm{atanh}(\tanh(a+b\,x))}^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*atanh(tanh(a + b*x))^3,x)

[Out] (x^4*atanh(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*atanh(tanh(a + b*x))^2)/20 + (b^2*x^6*atanh(tanh(a + b*x)))/20

3.56 $\int x^2 \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=53

$$\frac{x^2 \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x \tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3}$$

```
[Out] 1/4*x^2*arctanh(tanh(b*x+a))^4/b-1/10*x*arctanh(tanh(b*x+a))^5/b^2+1/60*arc
tanh(tanh(b*x+a))^6/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3} - \frac{x \tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{x^2 \tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - (x*ArcTanh[Tanh[a + b*x]]^5)/(10*b^2
) + ArcTanh[Tanh[a + b*x]]^6/(60*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int x\tanh^{-1}(\tanh(a+bx))^4\,dx}{2b}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\int \tanh^{-1}(\tanh(}{10b^2}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\mathrm{Subst}\left(\int x^5\,dx,\right.}{}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\tanh^{-1}(\tanh(a}{60b^3}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3\left(b^3x^3 - 6b^2x^2\tanh^{-1}(\tanh(a+bx)) + 15bx\tanh^{-1}(\tanh(a+bx))^2 - 20\tanh^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
[Out] -1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 15*b*x*ArcTanh[Tan
h[a + b*x]]^2 - 20*ArcTanh[Tanh[a + b*x]]^3))
```

Maple [A]
time = 0.02, size = 56, normalized size = 1.06

$$\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^3}{3} - b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^2}{4} - \frac{b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))}{5} - \frac{b\,x^6}{30}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^3,x)
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5
*x^5*arctanh(tanh(b*x+a))-1/30*b*x^6))
```

Maxima [A]
time = 0.37, size = 54, normalized size = 1.02

$$-\frac{1}{4}bx^4\operatorname{artanh}(\tanh(bx+a))^2 + \frac{1}{3}x^3\operatorname{artanh}(\tanh(bx+a))^3 - \frac{1}{60}\left(b^2x^6 - 6\,bx^5\operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4*arctanh(tanh(b*x + a))^2 + 1/3*x^3*arctanh(tanh(b*x + a))^3 - 1/
60*(b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.33, size = 35, normalized size = 0.66

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A]
time = 0.44, size = 56, normalized size = 1.06

$$-\frac{b^3x^6}{60}+\frac{b^2x^5\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{10}-\frac{bx^4\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4}+\frac{x^3\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**6/60 + b**2*x**5*atanh(tanh(a + b*x))/10 - b*x**4*atanh(tanh(a + b
*x))**2/4 + x**3*atanh(tanh(a + b*x))**3/3
```

Giac [A]
time = 0.38, size = 35, normalized size = 0.66

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Mupad [B]
time = 0.14, size = 53, normalized size = 1.00

$$-\frac{b^3\,x^6}{60}+\frac{b^2\,x^5\,\mathrm{atanh}(\tanh(a+b\,x))}{10}-\frac{b\,x^4\,{\mathrm{atanh}(\tanh(a+b\,x))}^2}{4}+\frac{x^3\,{\mathrm{atanh}(\tanh(a+b\,x))}^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*atanh(tanh(a + b*x))
^2)/4 + (b^2*x^5*atanh(tanh(a + b*x)))/10
```

3.57 $\int x \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

```
[Out] 1/4*x*arctanh(tanh(b*x+a))^4/b-1/20*arctanh(tanh(b*x+a))^5/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ArcTanh[Tanh[a + b*x]]^5/(20*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^4\,dx}{4b}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{\text{Subst}\left(\int x^4\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{4b^2}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. $2(34) = 68$.
time = 0.05, size = 99, normalized size = 2.91

$$\frac{(a+bx)\left((4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\tanh^{-1}(\tanh(a+bx))+10(2a^2+abx-b^2x^2)\tanh^{-1}(\tanh(a+bx))^2-10(a-bx)\tanh^{-1}(\tanh(a+bx))^3\right)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tan
h[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^2 - 10*(a
 - b*x)*ArcTanh[Tanh[a + b*x]]^3))/(20*b^2)
```

Maple [A]
time = 29.55, size = 56, normalized size = 1.65

method	result	size
default	$\frac{x^2\operatorname{arctanh}(\tanh(bx+a))^3}{2}-\frac{3b\left(\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{2b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))}{4}-\frac{b\,x^5}{20}\right)}{3}\right)}{2}$	56
risch	Expression too large to display	8165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^3-3/2*b*(1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*
(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x^5))
```

Maxima [A]
time = 0.37, size = 54, normalized size = 1.59

$$-\frac{1}{2}\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{20}\,\left(b^2x^5-5\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^3*arctanh(tanh(b*x + a))^2 + 1/2*x^2*arctanh(tanh(b*x + a))^3 - 1/
20*(b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.33, size = 34, normalized size = 1.00

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [A]
time = 0.23, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x\,\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4b}-\frac{\mathrm{atanh}^5\left(\tanh\left(a+bx\right)\right)}{20b^2} & \text{for } b\neq 0\\ \frac{x^2\,\mathrm{atanh}^3\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**4/(4*b) - atanh(tanh(a + b*x))**5/(20*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**3/2, True))
```

Giac [A]
time = 0.38, size = 34, normalized size = 1.00

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Mupad [B]
time = 0.98, size = 53, normalized size = 1.56

$$-\frac{b^3\,x^5}{20}+\frac{b^2\,x^4\,\mathrm{atanh}(\tanh(a+b\,x))}{4}-\frac{b\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2}+\frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*atanh(tanh(a + b*x))
^2)/2 + (b^2*x^4*atanh(tanh(a + b*x)))/4
```

3.58 $\int \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Maple [A]
time = 30.43, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4b}$	15
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4b}$	15
risch	Expression too large to display	6400

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.
time = 0.38, size = 51, normalized size = 3.19

$$-\frac{3}{2}\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{4}\left(b^2x^4-4\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*x^2*arctanh(tanh(b*x + a))^2 + x*arctanh(tanh(b*x + a))^3 - 1/4*(b^2
*x^4 - 4*b*x^3*arctanh(tanh(b*x + a)))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.32, size = 31, normalized size = 1.94

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [A]
time = 0.13, size = 20, normalized size = 1.25

$$\begin{cases}\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4b} & \text{for}\ b\neq 0\\ x\operatorname{atanh}^3\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*atanh(tanh(a))**3,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.38, size = 31, normalized size = 1.94

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)a^2 + \frac{1}{4}\left(bx^2 + 2\,ax\right)^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*a^2 + 1/4*(b*x^2 + 2*a*x)^2*b
```

Mupad [B]
time = 0.10, size = 47, normalized size = 2.94

$$\frac{x\,(2\,\mathrm{atanh}(\tanh(a+b\,x)) - b\,x)\,\left(b^2\,x^2 - 2\,b\,x\,\mathrm{atanh}(\tanh(a+b\,x)) + 2\,\mathrm{atanh}(\tanh(a+b\,x))^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x*(2*atanh(tanh(a + b*x)) - b*x)*(2*atanh(tanh(a + b*x))^2 + b^2*x^2 - 2*b
*x*atanh(tanh(a + b*x))))/4
```

3.59 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} \, dx$

Optimal. Leaf size=77

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{3}\tanh^{-1}(\tanh(a-$$

```
[Out] b*x*(b*x-arctanh(tanh(b*x+a)))^2-1/2*(b*x-arctanh(tanh(b*x+a)))*arctanh(tan
h(b*x+a))^2+1/3*arctanh(tanh(b*x+a))^3-(b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2190, 2189, 29}

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3-\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] b*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcT
anh[Tanh[a + b*x]]^2)/2 + ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[
a + b*x]])^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3 - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a\ldots}{x}$$
$$= -\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{3}\tanh^{-1}(\tanh(a\ldots$$
$$= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}\ldots$$
$$= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}\ldots$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 1.35

$$\frac{1}{3}(a+bx)^3+(a+bx)\left(a^2-3a(a+bx-\tanh^{-1}(\tanh(a+bx)))+3(a+bx-\tanh^{-1}(\tanh(a+bx)))^2\right)-\frac{1}{2}(a+bx)^2\left(2a+3bx-3\tanh^{-1}(\tanh(a+bx))\right)+\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] (a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcTanh[Tanh[a + b*x]]) + 3
*(a + b*x - ArcTanh[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*ArcT
anh[Tanh[a + b*x]]))/2 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[b*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(73) = 146.
time = 0.43, size = 188, normalized size = 2.44

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3-3b\left(\frac{b^2x^3\ln(x)}{3}-\frac{b^2x^3}{9}+ab\,x^2\ln(x)-\frac{ab\,x^2}{2}+b(\operatorname{arctanh}(\tanh(bx+\ldots\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^3-3*b*(1/3*b^2*x^3*ln(x)-1/9*b^2*x^3+a*b*x^2*ln(
x)-1/2*a*b*x^2+b*(arctanh(tanh(b*x+a))-b*x-a)*x^2*ln(x)-1/2*b*(arctanh(tanh
(b*x+a))-b*x-a)*x^2+ln(x)*x*a^2-x*a^2+2*ln(x)*x*a*(arctanh(tanh(b*x+a))-b*x
-a)-2*x*a*(arctanh(tanh(b*x+a))-b*x-a)+ln(x)*x*(arctanh(tanh(b*x+a))-b*x-a)
^2-x*(arctanh(tanh(b*x+a))-b*x-a)^2)
```

Maxima [A]
time = 0.66, size = 31, normalized size = 0.40

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Fricas [A]
time = 0.34, size = 31, normalized size = 0.40

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x, x)
```

Giac [A]
time = 0.37, size = 32, normalized size = 0.42

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

Mupad [B]
time = 0.14, size = 306, normalized size = 3.97

$$\frac{b^3\,x^3}{3}-\ln\left(x\right)\,\left(\frac{{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{8}-a^3-\frac{3\,a\,{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4}+\frac{3\,a^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2}\right)-\frac{3\,b^2\,x^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4}+\frac{3\,b\,x\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x,x)
```

```
[Out] (b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2*a
```

```
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/2)
- (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/4 + (3*b*x*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
2)/4
```

3.60 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx$

Optimal. Leaf size=68

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+3b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] -3*b^2*x*(b*x-arctanh(tanh(b*x+a)))+3/2*b*arctanh(tanh(b*x+a))^2-arctanh(ta
nh(b*x+a))^3/x+3*b*(b*x-arctanh(tanh(b*x+a)))^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2190, 2189, 29}

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -3*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]]) + (3*b*ArcTanh[Tanh[a + b*x]]^2)/2
- ArcTanh[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{x} + (3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx$$

$$= \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} - \big(3b\big(bx - \tanh^{-1}(\tan$$

$$= -3b^2x\big(bx - \tanh^{-1}(\tanh(a+bx))\big) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(t}{}$$

$$= -3b^2x\big(bx - \tanh^{-1}(\tanh(a+bx))\big) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(t}{}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.91

$$-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x} - 6b^2x\tanh^{-1}(\tanh(a+bx))\log(x) + 3b\tanh^{-1}(\tanh(a+bx))^2(1+\log(x)) + \frac{3}{2}b^3x^2(-1+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcTanh[Tanh[a + b*x]]*Log[x] + 3*b
*ArcTanh[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

Maple [A]
time = 0.24, size = 96, normalized size = 1.41

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x} + 3b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 - 2b\Big(\frac{b\,x^2\ln(x)}{2} - \frac{b\,x^2}{4} + \ln(x)\,xa - xa + \ln$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(tanh(b*x+a))^3/x+3*b*(ln(x)*arctanh(tanh(b*x+a))^2-2*b*(1/2*b*x^2*
ln(x)-1/4*b*x^2+ln(x)*x*a-x*a+ln(x)*x*(arctanh(tanh(b*x+a))-b*x-a)-x*(arcta
nh(tanh(b*x+a))-b*x-a)))
```

Maxima [A]
time = 0.69, size = 65, normalized size = 0.96

$$3\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)+\frac{3}{2}\,\left(b^{2}x^{2}+4\,abx+2\,a^{2}\log\left(x\right)-2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)\right)b-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*b*arctanh(tanh(b*x + a))^2*log(x) + 3/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x)
 - 2*arctanh(tanh(b*x + a))^2*log(x))*b - arctanh(tanh(b*x + a))^3/x
```

Fricas [A]
time = 0.33, size = 36, normalized size = 0.53

$$\frac{b^{3}x^{3}+6\,ab^{2}x^{2}+6\,a^{2}bx\log\left(x\right)-2\,a^{3}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**2, x)
```

Giac [A]
time = 0.38, size = 33, normalized size = 0.49

$$\frac{1}{2}\,b^{3}x^{2}+3\,ab^{2}x+3\,a^{2}b\log\left(\left|x\right|\right)-\frac{a^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

Mupad [B]
time = 1.07, size = 415, normalized size = 6.10

$$\frac{3\,b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2}{4}-\frac{\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^3}{8\,x}+\frac{3\,b\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2}{4}+\frac{\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^3}{8\,x}-\frac{3\,b^3\,x^2}{2}+\frac{3\,b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2\,\ln\left(x\right)}{4}+\frac{3\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2}{8\,x}-\frac{3\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{8\,x}+\frac{3\,b\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)^2\,\ln\left(x\right)}{4}-\frac{3\,b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}+3\,b^3\,x^2\,\ln\left(x\right)-\frac{3\,b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}+3\,b^2\,x\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)-3\,b^2\,x\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^3/x^2,x)

[Out] (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/4 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(8*x) - (3*b^3*x^2)/2 + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/4 + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/4 - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x^2*log(x) - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + 3*b^2*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x) - 3*b^2*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x)

3.61 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3} dx$

Optimal. Leaf size=60

$$3b^3x-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-3b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] 3*b^3*x-3/2*b*arctanh(tanh(b*x+a))^2/x-1/2*arctanh(tanh(b*x+a))^3/x^2-3*b^2
*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2189, 29}

$$-3b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] 3*b^3*x - (3*b*ArcTanh[Tanh[a + b*x]]^2)/(2*x) - ArcTanh[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} + \frac{1}{2}(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx$$
$$= -\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} + (3b^2)\int \frac{\tanh^{-1}(\tan}{x}$$
$$= 3b^3x - \frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} - (3b^2(bx - \tan$$
$$= 3b^3x - \frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2(bx - \tanh$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.10

$$b^3x - \frac{3b(-bx+\tanh^{-1}(\tanh(a+bx)))^2}{x} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))^3}{2x^2} + 3b^2(-bx+\tanh^{-1}(\tanh(a+bx)))\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] b^3*x - (3*b*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcTanh[Tan
h[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

Maple [A]
time = 0.27, size = 59, normalized size = 0.98

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2} + \frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x} + 2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x)-x))\right)}{2}$	59
risch	Expression too large to display	4445

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^3/x^2+3/2*b*(-arctanh(tanh(b*x+a))^2/x+2*b*(ln(x)
*arctanh(tanh(b*x+a))-b*(x*ln(x)-x)))
```

Maxima [A]
time = 0.35, size = 72, normalized size = 1.20

$$3\left(b\operatorname{artanh}(\tanh(bx+a))\log(x) - \left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b - \frac{3b\operatorname{artanh}(\tanh(bx+a))^2}{2x} - \frac{\operatorname{artanh}(\tanh(bx+a))^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)/
b))*b)*b - 3/2*b*arctanh(tanh(b*x + a))^2/x - 1/2*arctanh(tanh(b*x + a))^3/
x^2
```

Fricas [A]
time = 0.32, size = 37, normalized size = 0.62

$$\frac{2\,b^3x^3 + 6\,ab^2x^2\log\left(x\right) - 6\,a^2bx - a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}\left(\tanh\left(a + bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**3, x)
```

Giac [A]
time = 0.40, size = 31, normalized size = 0.52

$$b^3x + 3\,ab^2\log\left(\left|x\right|\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

```
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Mupad [B]
time = 0.20, size = 365, normalized size = 6.08

$$\frac{9\,b^2\,\ln\left(\frac{\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{4}-\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^3}{16\,x^2}-\frac{9\,b^2\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{4}-\frac{3\,b^3\,x}{2}+\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^3}{16\,x^2}-\frac{3\,b\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^2}{8\,x}-\frac{3\,b^2\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}+\frac{3\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^2}{16\,x^2}-\frac{3\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{16\,x^2}-\frac{3\,b\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)^2}{8\,x}+\frac{3\,b^2\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}-3\,b^3\,x\,\ln\left(x\right)+\frac{3\,b\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^3,x)
```

```
[Out] (9*b^2*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/4 - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2) - (9*b^2*log(1/(exp(2*a)*exp(2*b
*x) + 1)))/4 - (3*b^3*x)/2 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2) -
(3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) - (3*b^2*log(1/(exp(2*a)*exp
(2*b*x) + 1))*log(x))/2 + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(16*x^2) - (3*log(1/(exp(2*a)*ex
p(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(16*
x^2) - (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) +
 (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - 3*
b^3*x*log(x) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x)
```

3.62 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4} dx$

Optimal. Leaf size=55

$$-\frac{b^2 \tanh^{-1}(\tanh(a+bx))}{x} - \frac{b \tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b^3 \log(x)$$

```
[Out] -b^2*arctanh(tanh(b*x+a))/x-1/2*b*arctanh(tanh(b*x+a))^2/x^2-1/3*arctanh(ta
nh(b*x+a))^3/x^3+b^3*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 29}

$$-\frac{b^2 \tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{b \tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] -((b^2*ArcTanh[Tanh[a + b*x]])/x) - (b*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2) -
ArcTanh[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b^2\int \frac{\tanh^{-1}(\tanh(a$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+b}{3x^3}$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+b}{3x^3}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2\tanh^{-1}(\tanh(a+bx)) - 3bx\tanh^{-1}(\tanh(a+bx))^2 - 2\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3(11+6\log(x))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] (-6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*x*ArcTanh[Tanh[a + b*x]]^2 - 2*Arc
Tanh[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

Maple [A]
time = 0.93, size = 52, normalized size = 0.95

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3} + b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2} + b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x} + b\ln(x)\right)\right)$	52
risch	Expression too large to display	7816

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^3/x^3+b*(-1/2*arctanh(tanh(b*x+a))^2/x^2+b*(-arct
anh(tanh(b*x+a))/x+b*ln(x)))
```

Maxima [A]
time = 0.37, size = 52, normalized size = 0.95

$$\left(b^2\log(x) - \frac{b\,\operatorname{artanh}(\tanh(bx+a))}{x}\right)b - \frac{b\,\operatorname{artanh}(\tanh(bx+a))^2}{2\,x^2} - \frac{\operatorname{artanh}(\tanh(bx+a))^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - 1/2*b*arctanh(tanh(b*x + a))^
2/x^2 - 1/3*arctanh(tanh(b*x + a))^3/x^3
```

Fricas [A]
time = 0.33, size = 37, normalized size = 0.67

$$\frac{6\,b^3x^3\log\left(x\right) - 18\,ab^2x^2 - 9\,a^2bx - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A]
time = 0.29, size = 51, normalized size = 0.93

$$b^3\log\left(x\right) - \frac{b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x} - \frac{b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2x^2} - \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**4,x)
```

```
[Out] b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x))**2/(2*x*
*2) - atanh(tanh(a + b*x))**3/(3*x**3)
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.64

$$b^3\log\left(|x|\right) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Mupad [B]
time = 1.00, size = 51, normalized size = 0.93

$$b^3\,\ln\left(x\right) - \frac{b^2\,x^2\,\mathrm{atanh}(\tanh(a+b\,x)) + \frac{b\,x\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2} + \frac{\mathrm{atanh}(\tanh(a+b\,x))^3}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^4,x)
```

```
[Out] b^3*log(x) - (atanh(tanh(a + b*x))^3/3 + (b*x*atanh(tanh(a + b*x))^2)/2 + b
^2*x^2*atanh(tanh(a + b*x)))/x^3
```

3.63 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5} dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.61

$$-\frac{b^3x^3+b^2x^2\tanh^{-1}(\tanh(a+bx))+bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] -1/4*(b^3*x^3 + b^2*x^2*ArcTanh[Tanh[a + b*x]] + b*x*ArcTanh[Tanh[a + b*x]]
^2 + ArcTanh[Tanh[a + b*x]]^3)/x^4
```

Maple [A]
time = 0.95, size = 56, normalized size = 1.81

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{4x^4}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{3x^3}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}\right)}{3}\right)}{4}$	56
risch	Expression too large to display	7814

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*
b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2))
```

Maxima [A]
time = 0.39, size = 53, normalized size = 1.71

$$-\frac{1}{4}\,b\left(\frac{b^2}{x}+\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^2}\right)-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^3}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) - 1/4*b*arctanh(tanh(b*x + a)
)^2/x^3 - 1/4*arctanh(tanh(b*x + a))^3/x^4
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.06

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.36, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x}-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{4x^2}-\frac{b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4x^3}-\frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**5,x)
```

```
[Out] -b**3/(4*x) - b**2*atanh(tanh(a + b*x))/(4*x**2) - b*atanh(tanh(a + b*x))**
2/(4*x**3) - atanh(tanh(a + b*x))**3/(4*x**4)
```

Giac [A]
time = 0.39, size = 33, normalized size = 1.06

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Mupad [B]
time = 0.97, size = 48, normalized size = 1.55

$$-\frac{b^3\,x^3+b^2\,x^2\,\mathrm{atanh}(\tanh(a+b\,x))+b\,x\,\mathrm{atanh}(\tanh(a+b\,x))^2+\mathrm{atanh}(\tanh(a+b\,x))^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^5,x)
```

```
[Out] -(atanh(tanh(a + b*x))^3 + b^3*x^3 + b*x*atanh(tanh(a + b*x))^2 + b^2*x^2*a
tanh(tanh(a + b*x)))/(4*x^4)
```

3.64 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6} dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/20*b*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))^2+1/5*arctanh(
tanh(b*x+a))^4/x^5/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2202, 2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.84

$$-\frac{b^3x^3+2b^2x^2\tanh^{-1}(\tanh(a+bx))+3bx\tanh^{-1}(\tanh(a+bx))^2+4\tanh^{-1}(\tanh(a+bx))^3}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] -1/20*(b^3*x^3 + 2*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 3*b*x*ArcTanh[Tanh[a + b*x]]^2 + 4*ArcTanh[Tanh[a + b*x]]^3)/x^5
```

Maple [A]
time = 0.91, size = 56, normalized size = 0.88

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{5x^5}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{4x^4}+\frac{b\left(-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}\right)}{5}$	56
risch	Expression too large to display	7813

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3))
```

Maxima [A]
time = 0.38, size = 54, normalized size = 0.84

$$-\frac{1}{20}\,b\left(\frac{b^2}{x^2}+\frac{2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^3}\right)-\frac{3\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) - 3/20*b*arctanh(tanh(b*x + a))^2/x^4 - 1/5*arctanh(tanh(b*x + a))^3/x^5
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.55

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A]
time = 0.55, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2}-\frac{b^2\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{10x^3}-\frac{3b\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{20x^4}-\frac{\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**6,x)
```

```
[Out] -b**3/(20*x**2) - b**2*atanh(tanh(a + b*x))/(10*x**3) - 3*b*atanh(tanh(a +
b*x))**2/(20*x**4) - atanh(tanh(a + b*x))**3/(5*x**5)
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.55

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Mupad [B]
time = 0.99, size = 53, normalized size = 0.83

$$-\frac{\mathrm{atanh}(\tanh(a+b\,x))^3}{5\,x^5}-\frac{b^3}{20\,x^2}-\frac{b^2\,\mathrm{atanh}(\tanh(a+b\,x))}{10\,x^3}-\frac{3\,b\,\mathrm{atanh}(\tanh(a+b\,x))^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^6,x)
```

```
[Out] - atanh(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*atanh(tanh(a + b*x))
)/(10*x^3) - (3*b*atanh(tanh(a + b*x))^2)/(20*x^4)
```

3.65 $\int x^m \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=154

$$\frac{24b^4x^{5+m}}{(1+m)(2+m)(3+m)\,(20+9m+m^2)}-\frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(1+m)\,(24+26m+9m^2+m^3)}+\frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}$$

```
[Out] 24*b^4*x^(5+m)/(1+m)/(2+m)/(3+m)/(m^2+9*m+20)-24*b^3*x^(4+m)*arctanh(tanh(b
*x+a))/(1+m)/(m^3+9*m^2+26*m+24)+12*b^2*x^(3+m)*arctanh(tanh(b*x+a))^2/(m^3
+6*m^2+11*m+6)-4*b*x^(2+m)*arctanh(tanh(b*x+a))^3/(m^2+3*m+2)+x^(1+m)*arcta
nh(tanh(b*x+a))^4/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{24b^3x^{m+4}\tanh^{-1}(\tanh(a+bx))}{(m+1)\,(m^3+9m^2+26m+24)}+\frac{12b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))^2}{m^3+6m^2+11m+6}-\frac{4bx^{m+2}\tanh^{-1}(\tanh(a+bx))^3}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^4}{m+1}+\frac{24b^4x^{m+5}}{(m+1)(m+2)(m+3)\,(m^2+9m+20)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (24*b^4*x^(5 + m))/((1 + m)*(2 + m)*(3 + m)*(20 + 9*m + m^2)) - (24*b^3*x^(
4 + m)*ArcTanh[Tanh[a + b*x]])/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (12*b^
2*x^(3 + m)*ArcTanh[Tanh[a + b*x]]^2)/(6 + 11*m + 6*m^2 + m^3) - (4*b*x^(2
+ m)*ArcTanh[Tanh[a + b*x]]^3)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a
+ b*x]]^4)/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int x^m \tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m} - \frac{(4b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))^3\,dx}{1+m} \\
&= -\frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m} + \frac{(12b \\
&= \frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3} - \frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2} + \frac{x^1 \\
&= -\frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(4+m)\,(6+11m+6m^2+m^3)} + \frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3} - \\
&= \frac{24b^4x^{5+m}}{(4+m)(5+m)\,(6+11m+6m^2+m^3)} - \frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+b}{(4+m)\,(6+11m+6m^2+n}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 137, normalized size = 0.89

$$\frac{x^{1+m}\left(24b^4x^4-24b^3(5+m)x^3\tanh^{-1}(\tanh(a+bx))+12b^2(20+9m+m^2)\,x^2\tanh^{-1}(\tanh(a+bx))^2-4b(60+47m+12m^2+m^3)\,x\tanh^{-1}(\tanh(a+bx))^3+(120+154m+71m^2+14m^3+m^4)\tanh^{-1}(\tanh(a+bx))^4\right)}{(1+m)(2+m)(3+m)(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^(1 + m)*(24*b^4*x^4 - 24*b^3*(5 + m)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2
*(20 + 9*m + m^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 4*b*(60 + 47*m + 12*m^2 +
m^3)*x*ArcTanh[Tanh[a + b*x]]^3 + (120 + 154*m + 71*m^2 + 14*m^3 + m^4)*Arc
Tanh[Tanh[a + b*x]]^4))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

Maple [A]
time = 33.39, size = 278, normalized size = 1.81

method	result
default	$\frac{b^4x^5e^{m\ln(x)}}{5+m}+\frac{\left(a^4+4a^3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+6a^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2+4a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^3+\right.}{1+m}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/(5+m)*x^5*exp(m*ln(x))+(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(a
rctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^3+(arctanh(ta
nh(b*x+a))-b*x-a)^4)/(1+m)*x*exp(m*ln(x))+4*b*(a^3+3*a^2*(arctanh(tanh(b*x+
a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^
3)/(2+m)*x^2*exp(m*ln(x))+6*b^2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arct
```

```
anh(tanh(b*x+a))-b*x-a)^2)/(3+m)*x^3*exp(m*ln(x))+4*b^3*(arctanh(tanh(b*x+a
))-b*x)/(4+m)*x^4*exp(m*ln(x))
```

Maxima [A]
time = 0.39, size = 145, normalized size = 0.94

$$-\frac{4\,bx^{2}x^{m}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{4}}{m+1}+\frac{12\left(\frac{bx^{3}x^{m}\operatorname{artanh}(\tanh(bx+a))^{2}}{(m+3)(m+2)}+\frac{2\left(\frac{b^{2}x^{5}x^{m}}{(m+5)(m+4)(m+3)}-\frac{bx^{4}x^{m}\operatorname{artanh}(\tanh(bx+a))}{(m+4)(m+3)}\right)b}{m+2}\right)b}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -4*b*x^2*x^m*arctanh(tanh(b*x + a))^3/((m + 2)*(m + 1)) + x^(m + 1)*arctanh
(tanh(b*x + a))^4/(m + 1) + 12*(b*x^3*x^m*arctanh(tanh(b*x + a))^2/((m + 3)
*(m + 2)) + 2*(b^2*x^5*x^m/((m + 5)*(m + 4)*(m + 3)) - b*x^4*x^m*arctanh(ta
nh(b*x + a))/((m + 4)*(m + 3)))*b/(m + 2))*b/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(154) = 308$.
time = 0.35, size = 483, normalized size = 3.14

$$\frac{\left((b^{4}m^{4}+10b^{4}m^{3}+35b^{4}m^{2}+50b^{4}m+24b^{4})x^{5}+4(ab^{3}m^{4}+11ab^{3}m^{3}+41ab^{3}m^{2}+61ab^{3}m+30ab^{3})x^{4}+6(a^{2}b^{2}m^{4}+12a^{2}b^{2}m^{3}+49a^{2}b^{2}m^{2}+78a^{2}b^{2}m+40a^{2}b^{2})x^{3}+4(a^{3}bm^{4}+13a^{3}bm^{3}+59a^{3}bm^{2}+107a^{3}bm+60a^{3}b)x^{2}+(a^{4}m^{4}+14a^{4}m^{3}+71a^{4}m^{2}+154a^{4}m+120a^{4})x\right)\cosh\left(m\log\left(x\right)\right)+\left((b^{4}m^{4}+10b^{4}m^{3}+35b^{4}m^{2}+50b^{4}m+24b^{4})x^{5}+4(ab^{3}m^{4}+11ab^{3}m^{3}+41ab^{3}m^{2}+61ab^{3}m+30ab^{3})x^{4}+6(a^{2}b^{2}m^{4}+12a^{2}b^{2}m^{3}+49a^{2}b^{2}m^{2}+78a^{2}b^{2}m+40a^{2}b^{2})x^{3}+4(a^{3}bm^{4}+13a^{3}bm^{3}+59a^{3}bm^{2}+107a^{3}bm+60a^{3}b)x^{2}+(a^{4}m^{4}+14a^{4}m^{3}+71a^{4}m^{2}+154a^{4}m+120a^{4})x\right)\sinh\left(m\log\left(x\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] (((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^
4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m
^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(
a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^
4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*cosh(m*log(x)) +
((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^4
 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m^
4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(a
^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^4
*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*sinh(m*log(x)))/(m
^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^{4}\log\left(x\right)-\frac{b^{3}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x}-\frac{b^{2}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{2x^{2}}-\frac{b\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{3x^{3}}-\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{4x^{4}} & \text{for}\: m=-5 \\ \int\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{4}}\,dx & \text{for}\: m=-4 \\ \int\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx & \text{for}\: m=-3 \\ \int\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx & \text{for}\: m=-2 \\ \int\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x}\,dx & \text{for}\: m=-1 \\ \frac{24b^{4}x^{5}x^{m}}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{24b^{3}mx^{4}x^{m}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{120b^{3}x^{4}x^{m}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{12b^{2}m^{2}x^{3}x^{m}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{108b^{2}mx^{3}x^{m}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{240b^{2}x^{3}x^{m}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{4bm^{3}x^{2}x^{m}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{48bm^{2}x^{2}x^{m}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{188bmx^{2}x^{m}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}-\frac{240bx^{2}x^{m}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{m^{4}xx^{m}\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{14m^{3}xx^{m}\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{71m^{2}xx^{m}\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{154mxx^{m}\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120}+\frac{120xx^{m}\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a +
b*x))**2/(2*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x
))**4/(4*x**4), Eq(m, -5)), (Integral(atanh(tanh(a + b*x))**4/x**4, x), Eq(
m, -4)), (Integral(atanh(tanh(a + b*x))**4/x**3, x), Eq(m, -3)), (Integral(
atanh(tanh(a + b*x))**4/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a + b*x)
)**4/x, x), Eq(m, -1)), (24*b**4*x**5*x**m/(m**5 + 15*m**4 + 85*m**3 + 225*
m**2 + 274*m + 120) - 24*b**3*m*x**4*x**m*atanh(tanh(a + b*x))/(m**5 + 15*m
**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 120*b**3*x**4*x**m*atanh(tanh(a +
 b*x))/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 12*b**2*m**2*x
**3*x**m*atanh(tanh(a + b*x))**2/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274
*m + 120) + 108*b**2*m*x**3*x**m*atanh(tanh(a + b*x))**2/(m**5 + 15*m**4 +
85*m**3 + 225*m**2 + 274*m + 120) + 240*b**2*x**3*x**m*atanh(tanh(a + b*x))
**2/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 4*b*m**3*x**2*x**
m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 12
0) - 48*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3
+ 225*m**2 + 274*m + 120) - 188*b*m*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5
 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 240*b*x**2*x**m*atanh(tanh
(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + m**4*x*
x**m*atanh(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m +
 120) + 14*m**3*x*x**m*atanh(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 +
225*m**2 + 274*m + 120) + 71*m**2*x*x**m*atanh(tanh(a + b*x))**4/(m**5 + 15
*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 154*m*x*x**m*atanh(tanh(a + b*x
))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 120*x*x**m*atan
h(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120), Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^4, x)
```

Mupad [B]
time = 1.34, size = 479, normalized size = 3.11

$$\frac{x\,x^m\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4\,\left(m^4+14\,m^3+71\,m^2+154\,m+120\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}+\frac{16\,b^4\,x^m\,x^5\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}+\frac{24\,b^2\,x^m\,x^3\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2\,\left(m^4+12\,m^3+49\,m^2+78\,m+40\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}-\frac{32\,b^3\,x^m\,x^4\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(m^4+11\,m^3+41\,m^2+61\,m+30\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}-\frac{8\,b\,x^m\,x^2\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3\,\left(m^4+13\,m^3+59\,m^2+107\,m+60\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4*(154*m + 71*m^2 + 14*m^3 + m^4 + 120))/(4
```

```
384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920) + (16*b^4*x^m*x^5*(5
0*m + 35*m^2 + 10*m^3 + m^4 + 24))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4
+ 16*m^5 + 1920) + (24*b^2*x^m*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(78*m + 49*m^
2 + 12*m^3 + m^4 + 40))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 +
1920) - (32*b^3*x^m*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(61*m + 41*m^2 + 11*m^3 +
m^4 + 30))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920) - (8*b*
x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(107*m + 59*m^2 + 13*m^3 + m^4 + 60))/(4
384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920)
```

3.66 $\int x^6 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=80

$$\frac{b^4x^{11}}{2310}-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\,\text{ta}$$

```
[Out] 1/2310*b^4*x^11-1/210*b^3*x^10*arctanh(tanh(b*x+a))+1/42*b^2*x^9*arctanh(ta
nh(b*x+a))^2-1/14*b*x^8*arctanh(tanh(b*x+a))^3+1/7*x^7*arctanh(tanh(b*x+a))
^4
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh(a+bx))^4+\frac{b^4x^{11}}{2310}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^11)/2310 - (b^3*x^10*ArcTanh[Tanh[a + b*x]])/210 + (b^2*x^9*ArcTanh[
Tanh[a + b*x]]^2)/42 - (b*x^8*ArcTanh[Tanh[a + b*x]]^3)/14 + (x^7*ArcTanh[T
anh[a + b*x]]^4)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^6 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{1}{7}x^7\tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{7}(4b)\int x^7\tanh^{-1}(\tanh(a+bx))^3\,dx$$
$$= -\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{7}x^7\tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{14}(3b^2)\int$$
$$= \frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{7}x^7\tanh$$
$$= -\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx)) + \frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{14}bx$$
$$= \frac{b^4x^{11}}{2310} - \frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx)) + \frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.89

$$\frac{x^7\left(b^4x^4 - 11b^3x^3\tanh^{-1}(\tanh(a+bx)) + 55b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 165bx\tanh^{-1}(\tanh(a+bx))^3 + 330\tanh^{-1}(\tanh(a+bx))^4\right)}{2310}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
[Out] (x^7*(b^4*x^4 - 11*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 165*b*x*ArcTanh[Tanh[a + b*x]]^3 + 330*ArcTanh[Tanh[a + b*x]
]^4))/2310
```

Maple [A]
time = 0.03, size = 74, normalized size = 0.92

$$\frac{x^7\operatorname{arctanh}(\tanh(bx+a))^4}{7} - \frac{4b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))^3}{8} - \frac{3b\left(\frac{x^9\operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{2b\left(\frac{x^{10}\operatorname{arctanh}(\tanh(bx+a))}{10} - \frac{x^{11}b}{110}\right)}{9}\right)}{8}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(tanh(b*x+a))^4,x)
[Out] 1/7*x^7*arctanh(tanh(b*x+a))^4-4/7*b*(1/8*x^8*arctanh(tanh(b*x+a))^3-3/8*b*
(1/9*x^9*arctanh(tanh(b*x+a))^2-2/9*b*(1/10*x^10*arctanh(tanh(b*x+a))-1/110
*x^11*b)))
```

Maxima [A]
time = 0.40, size = 72, normalized size = 0.90

$$-\frac{1}{14}bx^8\operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{7}x^7\operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{2310}\left(55bx^9\operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2x^{11} - 11bx^{10}\operatorname{artanh}(\tanh(bx+a))\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/14*b*x^8*arctanh(tanh(b*x + a))^3 + 1/7*x^7*arctanh(tanh(b*x + a))^4 + 1
/2310*(55*b*x^9*arctanh(tanh(b*x + a))^2 + (b^2*x^11 - 11*b*x^10*arctanh(ta
nh(b*x + a)))*b)*b
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.58

$$\frac{1}{11}\,b^4x^{11}+\frac{2}{5}\,ab^3x^{10}+\frac{2}{3}\,a^2b^2x^9+\frac{1}{2}\,a^3bx^8+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

Sympy [A]
time = 2.49, size = 75, normalized size = 0.94

$$\frac{b^4x^{11}}{2310}-\frac{b^3x^{10}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{210}+\frac{b^2x^9\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{42}-\frac{bx^8\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{14}+\frac{x^7\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**11/2310 - b**3*x**10*atanh(tanh(a + b*x))/210 + b**2*x**9*atanh(tan
h(a + b*x))**2/42 - b*x**8*atanh(tanh(a + b*x))**3/14 + x**7*atanh(tanh(a +
 b*x))**4/7
```

Giac [A]
time = 0.38, size = 46, normalized size = 0.58

$$\frac{1}{11}\,b^4x^{11}+\frac{2}{5}\,ab^3x^{10}+\frac{2}{3}\,a^2b^2x^9+\frac{1}{2}\,a^3bx^8+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

Mupad [B]
time = 1.03, size = 242, normalized size = 3.02

$$\frac{x^7\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}{112}+\frac{b^4\,x^{11}}{11}-\frac{b\,x^8\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{16}-\frac{b^3\,x^{10}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}+\frac{b^2\,x^9\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/112 + (b^4*x^11)/11 - (b*x^8*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^3)/16 - (b^3*x^10*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/5 + (b^2*x^9*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2)/6
```

3.67 $\int x^5 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=80

$$\frac{b^4x^{10}}{1260}-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tan$$

```
[Out] 1/1260*b^4*x^10-1/126*b^3*x^9*arctanh(tanh(b*x+a))+1/28*b^2*x^8*arctanh(tan
h(b*x+a))^2-2/21*b*x^7*arctanh(tanh(b*x+a))^3+1/6*x^6*arctanh(tanh(b*x+a))^
4
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tanh(a+bx))^4+\frac{b^4x^{10}}{1260}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^10)/1260 - (b^3*x^9*ArcTanh[Tanh[a + b*x]])/126 + (b^2*x^8*ArcTanh[T
anh[a + b*x]]^2)/28 - (2*b*x^7*ArcTanh[Tanh[a + b*x]]^3)/21 + (x^6*ArcTanh[
Tanh[a + b*x]]^4)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^5 \tanh^{-1}(\tanh(a+bx))^4 \, dx = \frac{1}{6}x^6 \tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{3}(2b) \int x^6 \tanh^{-1}(\tanh(a+bx))^3 \, dx$$
$$= -\frac{2}{21}bx^7 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{6}x^6 \tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{7}(2b^2) \int$$
$$= \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{21}bx^7 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{6}x^6 \tanh$$
$$= -\frac{1}{126}b^3x^9 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{21}bx^7$$
$$= \frac{b^4x^{10}}{1260} - \frac{1}{126}b^3x^9 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 -$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.89

$$\frac{x^6 \left(b^4x^4 - 10b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 45b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 120bx \tanh^{-1}(\tanh(a+bx))^3 + 210 \tanh^{-1}(\tanh(a+bx))^4\right)}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
[Out] (x^6*(b^4*x^4 - 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 45*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 120*b*x*ArcTanh[Tanh[a + b*x]]^3 + 210*ArcTanh[Tanh[a + b*x]
]^4))/1260
```

Maple [A]
time = 0.03, size = 74, normalized size = 0.92

$$\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))^4}{6} - \frac{2b\left(\frac{x^7 \operatorname{arctanh}(\tanh(bx+a))^3}{7} - \frac{3b\left(\frac{x^8 \operatorname{arctanh}(\tanh(bx+a))^2}{8} - \frac{b\left(\frac{x^9 \operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{x^{10}b}{90}\right)}{4}\right)}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(tanh(b*x+a))^4,x)
[Out] 1/6*x^6*arctanh(tanh(b*x+a))^4-2/3*b*(1/7*x^7*arctanh(tanh(b*x+a))^3-3/7*b*
(1/8*x^8*arctanh(tanh(b*x+a))^2-1/4*b*(1/9*x^9*arctanh(tanh(b*x+a))-1/90*x^
10*b)))
```

Maxima [A]
time = 0.41, size = 72, normalized size = 0.90

$$-\frac{2}{21}bx^7 \operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{6}x^6 \operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{1260}\left(45bx^8 \operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2x^{10} - 10bx^9 \operatorname{artanh}(\tanh(bx+a))\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/21*b*x^7*arctanh(tanh(b*x + a))^3 + 1/6*x^6*arctanh(tanh(b*x + a))^4 + 1
/1260*(45*b*x^8*arctanh(tanh(b*x + a))^2 + (b^2*x^10 - 10*b*x^9*arctanh(tan
h(b*x + a)))*b)*b
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.58

$$\frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

Sympy [A]
time = 1.57, size = 76, normalized size = 0.95

$$\frac{b^4x^{10}}{1260}-\frac{b^3x^9\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{126}+\frac{b^2x^8\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{28}-\frac{2bx^7\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{21}+\frac{x^6\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**10/1260 - b**3*x**9*atanh(tanh(a + b*x))/126 + b**2*x**8*atanh(tanh
(a + b*x))**2/28 - 2*b*x**7*atanh(tanh(a + b*x))**3/21 + x**6*atanh(tanh(a
+ b*x))**4/6
```

Giac [A]
time = 0.38, size = 46, normalized size = 0.58

$$\frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

Mupad [B]
time = 0.15, size = 242, normalized size = 3.02

$$\frac{x^6\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}{96}+\frac{b^4\,x^{10}}{10}-\frac{b\,x^7\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{14}-\frac{2\,b^3\,x^9\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{9}+\frac{3\,b^2\,x^8\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^6*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/96 + (b^4*x^10)/10 - (b*x^7*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^3)/14 - (2*b^3*x^9*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/9 + (3*b^2*x^8*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2)/16
```

3.68 $\int x^4 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=80

$$\frac{b^4x^9}{630}-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}$$

```
[Out] 1/630*b^4*x^9-1/70*b^3*x^8*arctanh(tanh(b*x+a))+2/35*b^2*x^7*arctanh(tanh(b
*x+a))^2-2/15*b*x^6*arctanh(tanh(b*x+a))^3+1/5*x^5*arctanh(tanh(b*x+a))^4
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a+bx))^4+\frac{b^4x^9}{630}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^9)/630 - (b^3*x^8*ArcTanh[Tanh[a + b*x]])/70 + (2*b^2*x^7*ArcTanh[Ta
nh[a + b*x]]^2)/35 - (2*b*x^6*ArcTanh[Tanh[a + b*x]]^3)/15 + (x^5*ArcTanh[T
anh[a + b*x]]^4)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{1}{5}x^5\tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{5}(4b)\int x^5\tanh^{-1}(\tanh(a+bx))^3\,dx$$
$$= -\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}x^5\tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{5}(2b^2)\int$$
$$= \frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}x^5\tanh$$
$$= -\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx)) + \frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{15}bx^6$$
$$= \frac{b^4x^9}{630} - \frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx)) + \frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2 -$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.89

$$\frac{1}{630}x^5\left(b^4x^4 - 9b^3x^3\tanh^{-1}(\tanh(a+bx)) + 36b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 84bx\tanh^{-1}(\tanh(a+bx))^3 + 126\tanh^{-1}(\tanh(a+bx))^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
[Out] (x^5*(b^4*x^4 - 9*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 36*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 84*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x]]^
4))/630
```

Maple [A]
time = 0.03, size = 74, normalized size = 0.92

$$\frac{x^5\operatorname{arctanh}(\tanh(bx+a))^4}{5} - \frac{4b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))^3}{6} - \frac{b\left(\frac{x^7\operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{2b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))}{8} - \frac{b\,x^9}{72}\right)}{7}\right)}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^4,x)
[Out] 1/5*x^5*arctanh(tanh(b*x+a))^4-4/5*b*(1/6*x^6*arctanh(tanh(b*x+a))^3-1/2*b*
(1/7*x^7*arctanh(tanh(b*x+a))^2-2/7*b*(1/8*x^8*arctanh(tanh(b*x+a))-1/72*b*
x^9)))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 0.90

$$-\frac{2}{15}bx^6\operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{5}x^5\operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{630}\left(36\,bx^7\operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2x^9 - 9\,bx^8\operatorname{artanh}(\tanh(bx+a))\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/15*b*x^6*arctanh(tanh(b*x + a))^3 + 1/5*x^5*arctanh(tanh(b*x + a))^4 + 1
/630*(36*b*x^7*arctanh(tanh(b*x + a))^2 + (b^2*x^9 - 9*b*x^8*arctanh(tanh(b
*x + a)))*b)*b
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.58

$$\frac{1}{9}\,b^4x^9+\frac{1}{2}\,ab^3x^8+\frac{6}{7}\,a^2b^2x^7+\frac{2}{3}\,a^3bx^6+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

Sympy [A]
time = 1.04, size = 78, normalized size = 0.98

$$\frac{b^4x^9}{630}-\frac{b^3x^8\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{70}+\frac{2b^2x^7\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{35}-\frac{2bx^6\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{15}+\frac{x^5\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**9/630 - b**3*x**8*atanh(tanh(a + b*x))/70 + 2*b**2*x**7*atanh(tanh(
a + b*x))**2/35 - 2*b*x**6*atanh(tanh(a + b*x))**3/15 + x**5*atanh(tanh(a +
 b*x))**4/5
```

Giac [A]
time = 0.40, size = 46, normalized size = 0.58

$$\frac{1}{9}\,b^4x^9+\frac{1}{2}\,ab^3x^8+\frac{6}{7}\,a^2b^2x^7+\frac{2}{3}\,a^3bx^6+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

Mupad [B]
time = 1.06, size = 77, normalized size = 0.96

$$\frac{\mathrm{atanh}(\tanh(a+b\,x))^5\,\left(126\,b^4\,x^4-84\,b^3\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))+36\,b^2\,x^2\,\mathrm{atanh}(\tanh(a+b\,x))^2-9\,b\,x\,\mathrm{atanh}(\tanh(a+b\,x))^3+\mathrm{atanh}(\tanh(a+b\,x))^4\right)}{630\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (atanh(tanh(a + b*x))^5*(atanh(tanh(a + b*x))^4 + 126*b^4*x^4 + 36*b^2*x^2*
atanh(tanh(a + b*x))^2 - 9*b*x*atanh(tanh(a + b*x))^3 - 84*b^3*x^3*atanh(ta
nh(a + b*x))))/(630*b^5)
```

3.69 $\int x^3 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=72

$$\frac{x^3 \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x^2 \tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x \tanh^{-1}(\tanh(a+bx))^7}{35b^3}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}$$

```
[Out] 1/5*x^3*arctanh(tanh(b*x+a))^5/b-1/10*x^2*arctanh(tanh(b*x+a))^6/b^2+1/35*x
*arctanh(tanh(b*x+a))^7/b^3-1/280*arctanh(tanh(b*x+a))^8/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}+\frac{x \tanh^{-1}(\tanh(a+bx))^7}{35b^3}-\frac{x^2 \tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x^3 \tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x^2*ArcTanh[Tanh[a + b*x]]^6)/(10*b
^2) + (x*ArcTanh[Tanh[a + b*x]]^7)/(35*b^3) - ArcTanh[Tanh[a + b*x]]^8/(280
*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{3\int x^2\tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{\int x\tanh^{-1}(\tan}{5\!}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(}{35b^3}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(}{35b^3}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(}{35b^3}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.99

$$\frac{1}{280}x^4\left(b^4x^4 - 8b^3x^3\tanh^{-1}(\tanh(a+bx)) + 28b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 56bx\tanh^{-1}(\tanh(a+bx))^3 + 70\tanh^{-1}(\tanh(a+bx))^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^4*(b^4*x^4 - 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 28*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]^4
))/280
```

Maple [A]
time = 0.03, size = 74, normalized size = 1.03

$$\frac{x^4 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{4} - b\left(\frac{x^5 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{5} - \frac{3b\left(\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))^2}{6} - \frac{b\left(\frac{x^7 \operatorname{arctanh}(\tanh(bx+a)}{7}\right.}{3}\right.}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6
*x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*b*x^8)
))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 1.00

$$-\frac{1}{5}\,bx^5 \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3 + \frac{1}{4}\,x^4 \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4 + \frac{1}{280}\,\left(28\,bx^6 \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^8 - 8\,bx^7 \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/5*b*x^5*arctanh(tanh(b*x + a))^3 + 1/4*x^4*arctanh(tanh(b*x + a))^4 + 1/
280*(28*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*
x + a)))*b)*b
```

Fricas [A]
time = 0.38, size = 45, normalized size = 0.62

$$\frac{1}{8}\,b^4x^8+\frac{4}{7}\,ab^3x^7+a^2b^2x^6+\frac{4}{5}\,a^3bx^5+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

Sympy [A]
time = 0.68, size = 75, normalized size = 1.04

$$\frac{b^4x^8}{280}-\frac{b^3x^7\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{35}+\frac{b^2x^6\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{10}-\frac{bx^5\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5}+\frac{x^4\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**8/280 - b**3*x**7*atanh(tanh(a + b*x))/35 + b**2*x**6*atanh(tanh(a
+ b*x))**2/10 - b*x**5*atanh(tanh(a + b*x))**3/5 + x**4*atanh(tanh(a + b*x)
)**4/4
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.62

$$\frac{1}{8}\,b^4x^8+\frac{4}{7}\,ab^3x^7+a^2b^2x^6+\frac{4}{5}\,a^3bx^5+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

Mupad [B]
time = 1.02, size = 70, normalized size = 0.97

$$\frac{b^4\,x^8}{280}-\frac{b^3\,x^7\,\mathrm{atanh}(\tanh(a+b\,x))}{35}+\frac{b^2\,x^6\,{\mathrm{atanh}(\tanh(a+b\,x))}^2}{10}-\frac{b\,x^5\,{\mathrm{atanh}(\tanh(a+b\,x))}^3}{5}+\frac{x^4\,{\mathrm{atanh}(\tanh(a+b\,x))}^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^4*atanh(tanh(a + b*x))^4)/4 + (b^4*x^8)/280 + (b^2*x^6*atanh(tanh(a + b*
x))^2)/10 - (b*x^5*atanh(tanh(a + b*x))^3)/5 - (b^3*x^7*atanh(tanh(a + b*x)
))/35
```

3.70 $\int x^2 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=53

$$\frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3}$$

```
[Out] 1/5*x^2*arctanh(tanh(b*x+a))^5/b-1/15*x*arctanh(tanh(b*x+a))^6/b^2+1/105*ar
ctanh(tanh(b*x+a))^7/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x*ArcTanh[Tanh[a + b*x]]^6)/(15*b^2
) + ArcTanh[Tanh[a + b*x]]^7/(105*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{2\int x \tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\int \tanh^{-1}(\tanh(a$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\text{Subst}\left(\int x^6\,dx, x,\right.$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\tanh^{-1}(\tanh(a+}{105b^3}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 1.34

$$\frac{1}{105}x^3\left(b^4x^4 - 7b^3x^3\tanh^{-1}(\tanh(a+bx)) + 21b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 35bx\tanh^{-1}(\tanh(a+bx))^3 + 35\tanh^{-1}(\tanh(a+bx))^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*(b^4*x^4 - 7*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]^4
))/105
```

Maple [A]
time = 0.02, size = 74, normalized size = 1.40

$$\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^4}{3} - \frac{4b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^3}{4} - \frac{3b\left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^2}{5} - \frac{2b\left(\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))}{6} - \frac{b x^7}{42}\right)}{5}\right)}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^4-4/3*b*(1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*
(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*(1/6*x^6*arctanh(tanh(b*x+a))-1/42*b*
x^7)))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 1.36

$$-\frac{1}{3}bx^4 \operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{3}x^3 \operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{105}\left(21\,bx^5 \operatorname{artanh}(\tanh(bx+a))^2 + (b^2x^7 - 7\,bx^6 \operatorname{artanh}(\tanh(bx+a)))b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/3*b*x^4*arctanh(tanh(b*x + a))^3 + 1/3*x^3*arctanh(tanh(b*x + a))^4 + 1/
105*(21*b*x^5*arctanh(tanh(b*x + a))^2 + (b^2*x^7 - 7*b*x^6*arctanh(tanh(b*
x + a)))*b)*b
```

Fricas [A]
time = 0.33, size = 45, normalized size = 0.85

$$\frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Sympy [A]
time = 0.42, size = 75, normalized size = 1.42

$$\frac{b^4x^7}{105}-\frac{b^3x^6\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{15}+\frac{b^2x^5\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5}-\frac{bx^4\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3}+\frac{x^3\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**7/105 - b**3*x**6*atanh(tanh(a + b*x))/15 + b**2*x**5*atanh(tanh(a
+ b*x))**2/5 - b*x**4*atanh(tanh(a + b*x))**3/3 + x**3*atanh(tanh(a + b*x))
**4/3
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.85

$$\frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Mupad [B]
time = 0.15, size = 70, normalized size = 1.32

$$\frac{b^4\,x^7}{105}-\frac{b^3\,x^6\,\mathrm{atanh}(\tanh(a+b\,x))}{15}+\frac{b^2\,x^5\,{\mathrm{atanh}(\tanh(a+b\,x))}^2}{5}-\frac{b\,x^4\,{\mathrm{atanh}(\tanh(a+b\,x))}^3}{3}+\frac{x^3\,{\mathrm{atanh}(\tanh(a+b\,x))}^4}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^3*atanh(tanh(a + b*x))^4)/3 + (b^4*x^7)/105 + (b^2*x^5*atanh(tanh(a + b*
x))^2)/5 - (b*x^4*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6*atanh(tanh(a + b*x))
)/15
```

3.71 $\int x \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

```
[Out] 1/5*x*arctanh(tanh(b*x+a))^5/b-1/30*arctanh(tanh(b*x+a))^6/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - ArcTanh[Tanh[a + b*x]]^6/(30*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\text{Subst}\left(\int x^5\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 125 vs. $2(34) = 68$.
time = 0.06, size = 125, normalized size = 3.68

$$-\frac{(a+bx)\left((5a-bx)(a+bx)^4-6(4a-bx)(a+bx)^3\tanh^{-1}(\tanh(a+bx))+15(3a-bx)(a+bx)^2\tanh^{-1}(\tanh(a+bx))^2-20(2a^2+abx-b^2x^2)\tanh^{-1}(\tanh(a+bx))^3+15(a-bx)\tanh^{-1}(\tanh(a+bx))^4\right)}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] -1/30*((a + b*x)*((5*a - b*x)*(a + b*x)^4 - 6*(4*a - b*x)*(a + b*x)^3*ArcTa
nh[Tanh[a + b*x]] + 15*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tanh[a + b*x]]^2 - 2
0*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^3 + 15*(a - b*x)*ArcTanh
[Tanh[a + b*x]]^4))/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.
time = 31.81, size = 74, normalized size = 2.18

method	result
default	$\frac{x^2\,\mathrm{arctanh}(\tanh(bx+a))^4}{2}-2b\left(\frac{x^3\,\mathrm{arctanh}(\tanh(bx+a))^3}{3}-b\left(\frac{x^4\,\mathrm{arctanh}(\tanh(bx+a))^2}{4}-\frac{b\left(\frac{x^5\,\mathrm{arctanh}(\tanh(bx+a))}{5}-\frac{b\,x^6}{30}\right.}{2}\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^4-2*b*(1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x
^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*b*x^6)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.
time = 0.42, size = 72, normalized size = 2.12

$$-\frac{2}{3}\,bx^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{2}\,x^2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{30}\left(15\,bx^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^6-6\,bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*x^3*arctanh(tanh(b*x + a))^3 + 1/2*x^2*arctanh(tanh(b*x + a))^4 + 1/
30*(15*b*x^4*arctanh(tanh(b*x + a))^2 + (b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x
 + a)))*b)*b
```

Fricas [A]
time = 0.33, size = 46, normalized size = 1.35

$$\frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

Sympy [A]
time = 0.41, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x\,\operatorname{atanh}^{5}\left(\tanh\left(a+bx\right)\right)}{5b}-\frac{\operatorname{atanh}^{6}\left(\tanh\left(a+bx\right)\right)}{30b^{2}} & \text{for}\ b\neq 0\\ \frac{x^{2}\operatorname{atanh}^{4}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**5/(5*b) - atanh(tanh(a + b*x))**6/(30*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**4/2, True))
```

Giac [A]
time = 0.39, size = 46, normalized size = 1.35

$$\frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

Mupad [B]
time = 1.00, size = 70, normalized size = 2.06

$$\frac{b^4\,x^6}{30}-\frac{b^3\,x^5\,\mathrm{atanh}(\tanh(a+b\,x))}{5}+\frac{b^2\,x^4\,{\mathrm{atanh}(\tanh(a+b\,x))}^2}{2}-\frac{2\,b\,x^3\,{\mathrm{atanh}(\tanh(a+b\,x))}^3}{3}+\frac{x^2\,{\mathrm{atanh}(\tanh(a+b\,x))}^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^4)/2 + (b^4*x^6)/30 + (b^2*x^4*atanh(tanh(a + b*x
))^2)/2 - (2*b*x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^5*atanh(tanh(a + b*x)
))/5
```

3.72 $\int \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \tanh^{-1}(\tanh(a+bx))^4 \, dx &= \frac{\text{Subst}\left(\int x^4 \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{5b} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Maple [A]
time = 34.82, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^5}{5b}$	15
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^5}{5b}$	15
risch	Expression too large to display	21473

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.
time = 0.41, size = 69, normalized size = 4.31

$$-2\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{5}\left(10\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^5-5\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2*b*x^2*arctanh(tanh(b*x + a))^3 + x*arctanh(tanh(b*x + a))^4 + 1/5*(10*b*
x^3*arctanh(tanh(b*x + a))^2 + (b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.35, size = 42, normalized size = 2.62

$$\frac{1}{5}\,b^4x^5+ab^3x^4+2\,a^2b^2x^3+2\,a^3bx^2+a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Sympy [A]
time = 0.29, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{5}\left(\tanh\left(a+bx\right)\right)}{5b} & \text{for}\ b\neq 0\\ x\operatorname{atanh}^{4}\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**5/(5*b), Ne(b, 0)), (x*atanh(tanh(a))**4,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.38, size = 42, normalized size = 2.62

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Mupad [B]
time = 0.98, size = 67, normalized size = 4.19

$$\frac{b^4\,x^5}{5} - b^3\,x^4\,\mathrm{atanh}(\tanh(a+b\,x)) + 2\,b^2\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))^2 - 2\,b\,x^2\,\mathrm{atanh}(\tanh(a+b\,x))^3 + x\,\mathrm{atanh}(\tanh(a+b\,x))^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4,x)
```

```
[Out] x*atanh(tanh(a + b*x))^4 + (b^4*x^5)/5 + 2*b^2*x^3*atanh(tanh(a + b*x))^2 -
 2*b*x^2*atanh(tanh(a + b*x))^3 - b^3*x^4*atanh(tanh(a + b*x))
```

3.73 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x}\,dx$

Optimal. Leaf size=105

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{3}\left(bx-\tanh^{-1}\right.$$

```
[Out] -b*x*(b*x-arctanh(tanh(b*x+a)))^3+1/2*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(
tanh(b*x+a))^2-1/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^3+1/4*ar
ctanh(tanh(b*x+a))^4+(b*x-arctanh(tanh(b*x+a)))^4*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2190, 2189, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{4}\tanh^{-1}(\tanh(a+bx))^4+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])^3) + ((b*x - ArcTanh[Tanh[a + b*x]])^2
*ArcTanh[Tanh[a + b*x]]^2)/2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh
[a + b*x]]^3)/3 + ArcTanh[Tanh[a + b*x]]^4/4 + (b*x - ArcTanh[Tanh[a + b*x]
])^4*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x}\,dx &= \frac{1}{4}\tanh^{-1}(\tanh(a+bx))^4-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a
\\
&= -\frac{1}{3}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{4}\tanh^{-1}(\tanh(a
\\
&= \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{3}\left(bx-\tanh^{-1}(\tan
\\
&= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}
\\
&= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 175, normalized size = 1.67

$$\frac{1}{4}(a+bx)^4+\frac{1}{2}(a+bx)^2\left(a^2-4a\left(a+bx-\tanh^{-1}(\tanh(a+bx))\right)+6\left(a+bx-\tanh^{-1}(\tanh(a+bx))\right)^2\right)+(a+bx)\left(a^3-4a^2\left(a+bx-\tanh^{-1}(\tanh(a+bx))\right)+6a\left(a+bx-\tanh^{-1}(\tanh(a+bx))\right)^2-4\left(a+bx-\tanh^{-1}(\tanh(a+bx))\right)^3\right)-\frac{1}{3}(a+bx)^3\left(3a+4bx-4\tanh^{-1}(\tanh(a+bx))\right)+\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] (a + b*x)^4/4 + ((a + b*x)^2*(a^2 - 4*a*(a + b*x - ArcTanh[Tanh[a + b*x]])
+ 6*(a + b*x - ArcTanh[Tanh[a + b*x]])^2))/2 + (a + b*x)*(a^3 - 4*a^2*(a +
b*x - ArcTanh[Tanh[a + b*x]]) + 6*a*(a + b*x - ArcTanh[Tanh[a + b*x]])^2 -
4*(a + b*x - ArcTanh[Tanh[a + b*x]])^3) - ((a + b*x)^3*(3*a + 4*b*x - 4*Arc
Tanh[Tanh[a + b*x]]))/3 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(99) = 198$.
time = 1.20, size = 358, normalized size = 3.41

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^4-4b\Big(\frac{b^3x^4\ln(x)}{4}-\frac{b^3x^4}{16}+a\,b^2x^3\ln(x)-\frac{a\,b^2x^3}{3}+b^2(\operatorname{arctanh}(\tanh(bx$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^4-4*b*(1/4*b^3*x^4*ln(x)-1/16*b^3*x^4+a*b^2*x^3*
ln(x)-1/3*a*b^2*x^3+b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^3*ln(x)-1/3*b^2*(arc
tanh(tanh(b*x+a))-b*x-a)*x^3+3/2*a^2*b*x^2*ln(x)-3/4*a^2*b*x^2+3*a*b*(arcta
nh(tanh(b*x+a))-b*x-a)*x^2*ln(x)-3/2*a*b*(arctanh(tanh(b*x+a))-b*x-a)*x^2+3
```

```
/2*b*(arctanh(tanh(b*x+a))-b*x-a)^2*x^2*ln(x)-3/4*b*(arctanh(tanh(b*x+a))-b
*x-a)^2*x^2+ln(x)*x*a^3-x*a^3+3*ln(x)*x*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*
x*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3*ln(x)*x*a*(arctanh(tanh(b*x+a))-b*x-a)
^2-3*x*a*(arctanh(tanh(b*x+a))-b*x-a)^2+ln(x)*x*(arctanh(tanh(b*x+a))-b*x-a
)^3-x*(arctanh(tanh(b*x+a))-b*x-a)^3)
```

Maxima [A]
time = 0.67, size = 42, normalized size = 0.40

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="maxima")
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Fricas [A]
time = 0.34, size = 42, normalized size = 0.40

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="fricas")
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^4\left(\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x,x)
[Out] Integral(atanh(tanh(a + b*x))**4/x, x)
```

Giac [A]
time = 0.38, size = 43, normalized size = 0.41

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(abs(x))
```

Mupad [B]
time = 0.14, size = 423, normalized size = 4.03

$$\ln\left(x\right)\,\left(\frac{{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}{16}+\frac{3\,a^2\,{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2}+a^4-\frac{a\,{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{2}-2\,a^3\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)+\frac{b^4\,x^4}{4}-\frac{2\,b^3\,x^3\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3}+\frac{3\,b^2\,x^2\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4}-\frac{b\,x\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x,x)
```

```
[Out] log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4/16 + (3*a^2*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2)/2 + a^4 - (a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 - 2*a^3*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)) + (b^4*x^4)/4 - (2*b^3*x^3*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/3 + (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/4 - (b*x*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3)/2
```

3.74 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2}\,dx$

Optimal. Leaf size=95

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2+\frac{4}{3}b\tanh^{-1}(\tanh$$

```
[Out] 4*b^2*x*(b*x-arctanh(tanh(b*x+a)))^2-2*b*(b*x-arctanh(tanh(b*x+a)))*arctanh
(tanh(b*x+a))^2+4/3*b*arctanh(tanh(b*x+a))^3-arctanh(tanh(b*x+a))^4/x-4*b*(
b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2190, 2189, 29}

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+\frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2-4b\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

```
[Out] 4*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - 2*b*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^2 + (4*b*ArcTanh[Tanh[a + b*x]]^3)/3 - ArcTanh[Ta
nh[a + b*x]]^4/x - 4*b*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{x} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx$$
$$= \frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3 - \frac{\tanh^{-1}(\tanh(a+bx))^4}{x} - \Big(4b\big(bx-\tanh^{-1}(\tanh$$
$$= -2b\big(bx-\tanh^{-1}(\tanh(a+bx))\big)\tanh^{-1}(\tanh(a+bx))^2 + \frac{4}{3}b\tanh^{-1}(\tanh($$
$$= 4b^2x\big(bx-\tanh^{-1}(\tanh(a+bx))\big)^2 - 2b\big(bx-\tanh^{-1}(\tanh(a+bx))\big)\tanh^{-1}$$
$$= 4b^2x\big(bx-\tanh^{-1}(\tanh(a+bx))\big)^2 - 2b\big(bx-\tanh^{-1}(\tanh(a+bx))\big)\tanh^{-1}$$

Mathematica [A]
time = 0.06, size = 85, normalized size = 0.89

$$-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x} + \frac{2}{3}b^4x^3(5-6\log(x)) - 12b^2x\tanh^{-1}(\tanh(a+bx))^2\log(x) + 4b\tanh^{-1}(\tanh(a+bx))^3(1+\log(x)) + 6b^3x^2\tanh^{-1}(\tanh(a+bx))(-1+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
[Out] -(ArcTanh[Tanh[a + b*x]]^4/x) + (2*b^4*x^3*(5 - 6*Log[x]))/3 - 12*b^2*x*Arc
Tanh[Tanh[a + b*x]]^2*Log[x] + 4*b*ArcTanh[Tanh[a + b*x]]^3*(1 + Log[x]) +
6*b^3*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(93) = 186.
time = 0.70, size = 206, normalized size = 2.17

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{x} + 4b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3 - 3b\Big(\frac{b^2x^3\ln(x)}{3} - \frac{b^2x^3}{9} + ab\,x^2\ln(x) - \frac{ab\,x^2}{2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh(tanh(b*x+a))^4/x+4*b*(ln(x)*arctanh(tanh(b*x+a))^3-3*b*(1/3*b^2*x^
3*ln(x)-1/9*b^2*x^3+a*b*x^2*ln(x)-1/2*a*b*x^2+b*(arctanh(tanh(b*x+a))-b*x-a
)*x^2*ln(x)-1/2*b*(arctanh(tanh(b*x+a))-b*x-a)*x^2+ln(x)*x*a^2-x*a^2+2*ln(x
)*x*a*(arctanh(tanh(b*x+a))-b*x-a)-2*x*a*(arctanh(tanh(b*x+a))-b*x-a)+ln(x)
*x*(arctanh(tanh(b*x+a))-b*x-a)^2-x*(arctanh(tanh(b*x+a))-b*x-a)^2))
```

Maxima [A]
time = 0.70, size = 77, normalized size = 0.81

$$4\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}\log\left(x\right)-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{4}}{x}+\frac{2}{3}\left(2\,b^{3}x^{3}+9\,ab^{2}x^{2}+18\,a^{2}bx+6\,a^{3}\log\left(x\right)-6\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}\log\left(x\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="maxima")
```

```
[Out] 4*b*arctanh(tanh(b*x + a))^3*log(x) - arctanh(tanh(b*x + a))^4/x + 2/3*(2*b
^3*x^3 + 9*a*b^2*x^2 + 18*a^2*b*x + 6*a^3*log(x) - 6*arctanh(tanh(b*x + a))
^3*log(x))*b
```

Fricas [A]
time = 0.33, size = 47, normalized size = 0.49

$$\frac{b^{4}x^{4}+6\,ab^{3}x^{3}+18\,a^{2}b^{2}x^{2}+12\,a^{3}bx\log\left(x\right)-3\,a^{4}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 + 6*a*b^3*x^3 + 18*a^2*b^2*x^2 + 12*a^3*b*x*log(x) - 3*a^4)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**2, x)
```

Giac [A]
time = 0.38, size = 44, normalized size = 0.46

$$\frac{1}{3}\,b^{4}x^{3}+2\,ab^{3}x^{2}+6\,a^{2}b^{2}x+4\,a^{3}b\log\left(|x|\right)-\frac{a^{4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(abs(x)) - a^4/x
```

Mupad [B]
time = 0.14, size = 553, normalized size = 5.82

$$\ln(x)\left(4a^3b-\frac{b\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{2}+3ab\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-6a^2b\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)-\frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+24a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+16a^4-8a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{16x}+\frac{b^4x^3}{3}+\frac{3b^2x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2}-b^3x^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^2,x)
```

```
[Out] log(x)*(4*a^3*b - (b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 + 3*a*b*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2 - 6*a^2*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^
4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))/(16*x) + (b^4*x^3)/3 + (3*b^2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - b^3*
x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.75 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx$

Optimal. Leaf size=87

$$-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+3b^2\tanh^{-1}(\tanh(a+bx))^2-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x}-\frac{\tanh^{-1}(\tanh(a}{2x^2}$$

```
[Out] -6*b^3*x*(b*x-arctanh(tanh(b*x+a)))+3*b^2*arctanh(tanh(b*x+a))^2-2*b*arctan
h(tanh(b*x+a))^3/x-1/2*arctanh(tanh(b*x+a))^4/x^2+6*b^2*(b*x-arctanh(tanh(b
*x+a)))^2*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2190, 2189, 29}

$$-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+3b^2\tanh^{-1}(\tanh(a+bx))^2+6b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

```
[Out] -6*b^3*x*(b*x - ArcTanh[Tanh[a + b*x]]) + 3*b^2*ArcTanh[Tanh[a + b*x]]^2 -
(2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^2
*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + (6b^2)\int \frac{\tanh^{-1}(\tan$$
$$= 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+b}{2x^2}$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-}}{}$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-}}{}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 0.93

$$-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + 6b^4x^2\log(x) - 6b^3x\tanh^{-1}(\tanh(a+bx))(1+2\log(x)) + 3b^2\tanh^{-1}(\tanh(a+bx))^2(3+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

```
[Out] (-2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^
4*x^2*Log[x] - 6*b^3*x*ArcTanh[Tanh[a + b*x]]*(1 + 2*Log[x]) + 3*b^2*ArcTan
h[Tanh[a + b*x]]^2*(3 + 2*Log[x])
```

Maple [A]
time = 0.67, size = 114, normalized size = 1.31

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{2x^2} + 2b\Big(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x} + 3b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 - 2b\Big(\frac{bx^2\ln(x)}{2} -$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^4/x^2+2*b*(-arctanh(tanh(b*x+a))^3/x+3*b*(ln(x)*a
rctanh(tanh(b*x+a))^2-2*b*(1/2*b*x^2*ln(x)-1/4*b*x^2+ln(x)*x*a-x*a+ln(x)*x*
(arctanh(tanh(b*x+a))-b*x-a)-x*(arctanh(tanh(b*x+a))-b*x-a))))
```

Maxima [A]
time = 0.74, size = 83, normalized size = 0.95

$$-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{x}+3\,\left(2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)+\left(b^{2}x^{2}+4\,abx+2\,a^{2}\log\left(x\right)-2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)\right)b\right)b-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{4}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="maxima")
```

```
[Out] -2*b*arctanh(tanh(b*x + a))^3/x + 3*(2*b*arctanh(tanh(b*x + a))^2*log(x) +
(b^2*x^2 + 4*a*b*x + 2*a^2*log(x) - 2*arctanh(tanh(b*x + a))^2*log(x))*b)*b
 - 1/2*arctanh(tanh(b*x + a))^4/x^2
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.54

$$\frac{b^{4}x^{4}+8\,ab^{3}x^{3}+12\,a^{2}b^{2}x^{2}\log\left(x\right)-8\,a^{3}bx-a^{4}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 + 8*a*b^3*x^3 + 12*a^2*b^2*x^2*log(x) - 8*a^3*b*x - a^4)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**3, x)
```

Giac [A]
time = 0.38, size = 43, normalized size = 0.49

$$\frac{1}{2}\,b^{4}x^{2}+4\,ab^{3}x+6\,a^{2}b^{2}\log\left(|x|\right)-\frac{8\,a^{3}bx+a^{4}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(abs(x)) - 1/2*(8*a^3*b*x + a^4)/x^2
```

Mupad [B]
time = 1.54, size = 672, normalized size = 7.72

$$\frac{9b^2\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2}{4}-\frac{\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^4}{32x^2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^4}{32x^2}+\frac{9b^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2}{4}-3b^3x\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+\frac{b\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^3}{4x}+\frac{\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^3}{8x^2}+\frac{\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^3\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{8x^2}-\frac{b\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^3}{4x}+\frac{3b^2\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2\ln(x)}{2}-\frac{3\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2}{16x^2}+\frac{3b^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2\ln(x)}{2}+6b^4x^2\ln(x)-\frac{9b^2\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}+3b^3x\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+\frac{3b\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2}{4x}-\frac{3b\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{4x}-3b^2\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln(x)+6b^3x\ln\left(\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln(x)-6b^3x\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^3,x)
```

```
[Out] (9*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))^4/(32*x^2) - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/(3
2*x^2) + (9*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/4 -
 3*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + (b*log(1/(e
xp(2*a)*exp(2*b*x) + 1))^3)/(4*x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(8*x^2) + (log(1/(exp(2*
a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))
/(8*x^2) - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(4*x)
 + (3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/2 - (3*log(1/(exp(2*a)
*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)
/(16*x^2) + (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*l
og(x))/2 + 6*b^4*x^2*log(x) - (9*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x*log(1/(exp(2*a
)*exp(2*b*x) + 1)) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x) - (3*b*log(1/(exp(2*a)*exp(2*
b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x) -
3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))*log(x) + 6*b^3*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x) -
 6*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x)
```

3.76 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4}\,dx$

Optimal. Leaf size=77

$$4b^4x-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3}-4b^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] 4*b^4*x-2*b^2*arctanh(tanh(b*x+a))^2/x-2/3*b*arctanh(tanh(b*x+a))^3/x^2-1/3
*arctanh(tanh(b*x+a))^4/x^3-4*b^3*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2189, 29}

$$-4b^3\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}+4b^4x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] 4*b^4*x - (2*b^2*ArcTanh[Tanh[a + b*x]]^2)/x - (2*b*ArcTanh[Tanh[a + b*x]]^
3)/(3*x^2) - ArcTanh[Tanh[a + b*x]]^4/(3*x^3) - 4*b^3*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3} + \frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3} + (2b^2)\int \frac{\tanh^{-1}(\tan$$
$$= -\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a}{3x^3}$$
$$= 4b^4x - \frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tan}{3}$$
$$= 4b^4x - \frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tan}{3}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.06

$$-\frac{6b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 2bx\tanh^{-1}(\tanh(a+bx))^3 + \tanh^{-1}(\tanh(a+bx))^4 + 2b^4x^4(5+6\log(x)) - 2b^3x^3\tanh^{-1}(\tanh(a+bx))(11+6\log(x))}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] -1/3*(6*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]^3 +
 ArcTanh[Tanh[a + b*x]]^4 + 2*b^4*x^4*(5 + 6*Log[x]) - 2*b^3*x^3*ArcTanh[Ta
nh[a + b*x]]*(11 + 6*Log[x]))/x^3
```

Maple [A]
time = 1.00, size = 77, normalized size = 1.00

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{3x^3} + \frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2} + \frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x} + 2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x)-x))\right)}{2}\right)}{3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^4/x^3+4/3*b*(-1/2*arctanh(tanh(b*x+a))^3/x^2+3/2*
b*(-arctanh(tanh(b*x+a))^2/x+2*b*(ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x))
))
```

Maxima [A]
time = 0.38, size = 91, normalized size = 1.18

$$2\left(2\left(b\operatorname{artanh}(\tanh(bx+a))\log(x) - \left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b - \frac{b\operatorname{artanh}(\tanh(bx+a))^2}{x}\right)b - \frac{2b\operatorname{artanh}(\tanh(bx+a))^3}{3x^2} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="maxima")
```

```
[Out] 2*(2*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(
x)/b))*b)*b - b*arctanh(tanh(b*x + a))^2/x)*b - 2/3*b*arctanh(tanh(b*x + a)
)^3/x^2 - 1/3*arctanh(tanh(b*x + a))^4/x^3
```

Fricas [A]
time = 0.33, size = 48, normalized size = 0.62

$$\frac{3\,b^4x^4 + 12\,ab^3x^3\log\left(x\right) - 18\,a^2b^2x^2 - 6\,a^3bx - a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 12*a*b^3*x^3*log(x) - 18*a^2*b^2*x^2 - 6*a^3*b*x - a^4)/x^
3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{4}\left(\tanh\left(a + bx\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**4, x)
```

Giac [A]
time = 0.39, size = 42, normalized size = 0.55

$$b^4x + 4\,ab^3\log\left(|x|\right) - \frac{18\,a^2b^2x^2 + 6\,a^3bx + a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="giac")
```

```
[Out] b^4*x + 4*a*b^3*log(abs(x)) - 1/3*(18*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/x^3
```

Mupad [B]
time = 1.34, size = 571, normalized size = 7.42

$$\frac{11\,b^3\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3}-\frac{{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^4}{48\,x^3}-\frac{11\,b^3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3}-\frac{10\,b^4\,x}{3}-\frac{{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^4}{48\,x^3}+\frac{b\,{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^3}{12\,x^2}-2\,b^3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right)+\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^3}{12\,x^3}+\frac{{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^3\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{12\,x^3}-\frac{b\,{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^3}{12\,x^2}+2\,b^3\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right)-4\,b^4\,x\,\ln\left(x\right)-\frac{b^2\,{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2}{2\,x}-\frac{{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2\,{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2}{8\,x^3}-\frac{b^2\,{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2}{2\,x}+\frac{b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{x}+\frac{b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2}{4\,x^2}-\frac{b\,{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^4,x)

[Out] (11*b^3*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/3 - log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x^3) - (11*b^3*log(1/(exp(2*a)*exp(2
*b*x) + 1)))/3 - (10*b^4*x)/3 - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x^3)
 + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^2) - 2*b^3*log(1/(exp(2*a)*
exp(2*b*x) + 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^3) + (log(1/(exp(2*a)*exp(2*
b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(12*x^3)
 - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^2) + 2*
b^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x) - 4*b^4*x*l
og(x) - (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(2*x) - (log(1/(exp(2*a)*e
xp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(
8*x^3) - (b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(2*x)
 + (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)))/x + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x^2) - (b*log(1/(exp(2*a)*exp(2
*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x^2)
```

3.77 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5} \, dx$

Optimal. Leaf size=74

$$-\frac{b^3 \tanh^{-1}(\tanh(a+bx))}{x}-\frac{b^2 \tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b \tanh^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}+$$

```
[Out] -b^3*arctanh(tanh(b*x+a))/x-1/2*b^2*arctanh(tanh(b*x+a))^2/x^2-1/3*b*arctan
h(tanh(b*x+a))^3/x^3-1/4*arctanh(tanh(b*x+a))^4/x^4+b^4*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 29}

$$-\frac{b^3 \tanh^{-1}(\tanh(a+bx))}{x}-\frac{b^2 \tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}-\frac{b \tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^4 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -((b^3*ArcTanh[Tanh[a + b*x]])/x) - (b^2*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2)
- (b*ArcTanh[Tanh[a + b*x]]^3)/(3*x^3) - ArcTanh[Tanh[a + b*x]]^4/(4*x^4) +
 b^4*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b^2\int \frac{\tanh^{-1}(\tanh(a}{x^3}$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+}{4x^4}$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+}{3x^3}$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+}{3x^3}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 1.05

$$-\frac{12b^3x^3\tanh^{-1}(\tanh(a+bx)) + 6b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 4bx\tanh^{-1}(\tanh(a+bx))^3 + 3\tanh^{-1}(\tanh(a+bx))^4 - b^4x^4(25+12\log(x))}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -1/12*(12*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]]
^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^4 - b^4*x^4*
(25 + 12*Log[x]))/x^4
```

Maple [A]
time = 3.36, size = 69, normalized size = 0.93

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4x^4} + b\Big(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3} + b\Big(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2} + b\Big(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x} + b\ln$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^4/x^4+b*(-1/3*arctanh(tanh(b*x+a))^3/x^3+b*(-1/2*
arctanh(tanh(b*x+a))^2/x^2+b*(-arctanh(tanh(b*x+a))/x+b*ln(x))))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 0.97

$$\frac{1}{2}\left(2\left(b^2\log(x) - \frac{b\,\operatorname{artanh}(\tanh(bx+a))}{x}\right)b - \frac{b\,\operatorname{artanh}(\tanh(bx+a))^2}{x^2}\right)b - \frac{b\,\operatorname{artanh}(\tanh(bx+a))^3}{3\,x^3} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - b*arctanh(tanh(b*x + a
))^2/x^2)*b - 1/3*b*arctanh(tanh(b*x + a))^3/x^3 - 1/4*arctanh(tanh(b*x + a
))^4/x^4
```

Fricas [A]
time = 0.31, size = 48, normalized size = 0.65

$$\frac{12\,b^4x^4\log\left(x\right)-48\,ab^3x^3-36\,a^2b^2x^2-16\,a^3bx-3\,a^4}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^4*x^4*log(x) - 48*a*b^3*x^3 - 36*a^2*b^2*x^2 - 16*a^3*b*x - 3*a^
4)/x^4
```

Sympy [A]
time = 0.36, size = 70, normalized size = 0.95

$$b^4\log\left(x\right)-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x}-\frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2x^2}-\frac{b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^3}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**5,x)
```

```
[Out] b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a + b*x))**2/(2
*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x))**4/(4*x*
*4)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.62

$$b^4\log\left(|x|\right)-\frac{48\,ab^3x^3+36\,a^2b^2x^2+16\,a^3bx+3\,a^4}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="giac")
```

```
[Out] b^4*log(abs(x)) - 1/12*(48*a*b^3*x^3 + 36*a^2*b^2*x^2 + 16*a^3*b*x + 3*a^4)
/x^4
```

Mupad [B]
time = 1.14, size = 68, normalized size = 0.92

$$b^4\,\ln\left(x\right)-\frac{{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^4}{4\,x^4}-\frac{b^2\,{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^2}{2\,x^2}-\frac{b^3\,\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}{x}-\frac{b\,{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^5,x)
```

```
[Out] b^4*log(x) - atanh(tanh(a + b*x))^4/(4*x^4) - (b^2*atanh(tanh(a + b*x))^2)/
(2*x^2) - (b^3*atanh(tanh(a + b*x)))/x - (b*atanh(tanh(a + b*x))^3)/(3*x^3)
```

3.78 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6} dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.
time = 0.04, size = 66, normalized size = 2.13

$$-\frac{b^4x^4+b^3x^3\tanh^{-1}(\tanh(a+bx))+b^2x^2\tanh^{-1}(\tanh(a+bx))^2+bx\tanh^{-1}(\tanh(a+bx))^3+\tanh^{-1}(\tanh(a+bx))^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] -1/5*(b^4*x^4 + b^3*x^3*ArcTanh[Tanh[a + b*x]] + b^2*x^2*ArcTanh[Tanh[a + b
*x]]^2 + b*x*ArcTanh[Tanh[a + b*x]]^3 + ArcTanh[Tanh[a + b*x]]^4)/x^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 3.00, size = 74, normalized size = 2.39

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{4}}{5x^{5}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{4x^{4}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3x^{3}}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^{2}}\right)}{3}\right)}{4}\right)}{5}$	74
risch	Expression too large to display	22619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^4/x^5+4/5*b*(-1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*
b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))
/x^2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(29) = 58$.
time = 0.42, size = 70, normalized size = 2.26

$$-\frac{1}{5}\left(b\left(\frac{b^{2}}{x}+\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^{2}}\right)+\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{x^{3}}\right)b-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{5\,x^{4}}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) + b*arctanh(tanh(b*x + a))^2
/x^3)*b - 1/5*b*arctanh(tanh(b*x + a))^3/x^4 - 1/5*arctanh(tanh(b*x + a))^4
/x^5
```

Fricas [A]
time = 0.32, size = 44, normalized size = 1.42

$$-\frac{5\,b^{4}x^{4}+10\,ab^{3}x^{3}+10\,a^{2}b^{2}x^{2}+5\,a^{3}bx+a^{4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

time = 0.52, size = 75, normalized size = 2.42

$$-\frac{b^4}{5x} - \frac{b^3 \operatorname{atanh}(\tanh(a+bx))}{5x^2} - \frac{b^2 \operatorname{atanh}^2(\tanh(a+bx))}{5x^3} - \frac{b \operatorname{atanh}^3(\tanh(a+bx))}{5x^4} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**6,x)
```

```
[Out] -b**4/(5*x) - b**3*atanh(tanh(a + b*x))/(5*x**2) - b**2*atanh(tanh(a + b*x)
)**2/(5*x**3) - b*atanh(tanh(a + b*x))**3/(5*x**4) - atanh(tanh(a + b*x))**
4/(5*x**5)
```

Giac [A]

time = 0.40, size = 44, normalized size = 1.42

$$-\frac{5\,b^4x^4 + 10\,ab^3x^3 + 10\,a^2b^2x^2 + 5\,a^3bx + a^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="giac")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

Mupad [B]

time = 1.20, size = 64, normalized size = 2.06

$$-\frac{b^4\,x^4 + b^3\,x^3\,\mathrm{atanh}(\tanh(a+b\,x)) + b^2\,x^2\,\mathrm{atanh}(\tanh(a+b\,x))^2 + b\,x\,\mathrm{atanh}(\tanh(a+b\,x))^3 + \mathrm{atanh}(\tanh(a+b\,x))^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^6,x)
```

```
[Out] -(atanh(tanh(a + b*x))^4 + b^4*x^4 + b^2*x^2*atanh(tanh(a + b*x))^2 + b*x*a
tanh(tanh(a + b*x))^3 + b^3*x^3*atanh(tanh(a + b*x)))/(5*x^5)
```

3.79 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7} dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/30*b*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^2+1/6*arctanh(
tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2202, 2198}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx}{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.11

$$-\frac{b^4x^4+2b^3x^3\tanh^{-1}(\tanh(a+bx))+3b^2x^2\tanh^{-1}(\tanh(a+bx))^2+4bx\tanh^{-1}(\tanh(a+bx))^3+5\tanh^{-1}(\tanh(a+bx))^4}{30x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] -1/30*(b^4*x^4 + 2*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 3*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 5*ArcTanh[Tanh[a + b*x]]^4)/
x^6
```

Maple [A]
time = 3.06, size = 74, normalized size = 1.16

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{6x^6}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{5x^5}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{4x^4}+\frac{b\left(-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}\right)}{5}\right)}{3}$	74
risch	Expression too large to display	2262

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctanh(tanh(b*x+a))^4/x^6+2/3*b*(-1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*
b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a
))/x^3)))
```

Maxima [A]
time = 0.43, size = 72, normalized size = 1.12

$$-\frac{1}{30}\left(b\left(\frac{b^2}{x^2}+\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^3}\right)+\frac{3\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^4}\right)b-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{15\,x^5}-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="maxima")
```

```
[Out] -1/30*(b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) + 3*b*arctanh(tanh(b*x
+ a))^2/x^4)*b - 2/15*b*arctanh(tanh(b*x + a))^3/x^5 - 1/6*arctanh(tanh(b*x
 + a))^4/x^6
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.72

$$-\frac{15\,b^4x^4+40\,ab^3x^3+45\,a^2b^2x^2+24\,a^3bx+5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

Sympy [A]
time = 0.80, size = 78, normalized size = 1.22

$$-\frac{b^4}{30x^2}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{15x^3}-\frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{10x^4}-\frac{2b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{15x^5}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**7,x)
```

```
[Out] -b**4/(30*x**2) - b**3*atanh(tanh(a + b*x))/(15*x**3) - b**2*atanh(tanh(a +
 b*x))**2/(10*x**4) - 2*b*atanh(tanh(a + b*x))**3/(15*x**5) - atanh(tanh(a
+ b*x))**4/(6*x**6)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.72

$$-\frac{15\,b^4x^4+40\,ab^3x^3+45\,a^2b^2x^2+24\,a^3bx+5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

Mupad [B]
time = 1.04, size = 70, normalized size = 1.09

$$-\frac{\mathrm{atanh}(\tanh(a+b\,x))^4}{6\,x^6}-\frac{b^4}{30\,x^2}-\frac{b^2\,\mathrm{atanh}(\tanh(a+b\,x))^2}{10\,x^4}-\frac{b^3\,\mathrm{atanh}(\tanh(a+b\,x))}{15\,x^3}-\frac{2\,b\,\mathrm{atanh}(\tanh(a+b\,x))^3}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^7,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(6*x^6) - b^4/(30*x^2) - (b^2*atanh(tanh(a + b*x))
^2)/(10*x^4) - (b^3*atanh(tanh(a + b*x)))/(15*x^3) - (2*b*atanh(tanh(a + b*
x))^3)/(15*x^5)
```

3.80 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8} dx$

Optimal. Leaf size=98

$$\frac{b^2 \tanh^{-1}(\tanh(a+bx))^5}{105x^5 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{b \tanh^{-1}(\tanh(a+bx))^5}{21x^6 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/105*b^2*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^3+1/21*b*ar
ctanh(tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))^2+1/7*arctanh(tanh(b*x+
a))^5/x^7/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2202, 2198}

$$\frac{b^2 \tanh^{-1}(\tanh(a+bx))^5}{105x^5 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{b \tanh^{-1}(\tanh(a+bx))^5}{21x^6 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] (b^2*ArcTanh[Tanh[a + b*x]]^5)/(105*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (b*ArcTanh[Tanh[a + b*x]]^5)/(21*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \cdots$$
$$= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{105x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} \cdots$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.72

$$-\frac{b^4x^4+3b^3x^3\tanh^{-1}(\tanh(a+bx))+6b^2x^2\tanh^{-1}(\tanh(a+bx))^2+10bx\tanh^{-1}(\tanh(a+bx))^3+15\tanh^{-1}(\tanh(a+bx))^4}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] -1/105*(b^4*x^4 + 3*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 + 10*b*x*ArcTanh[Tanh[a + b*x]]^3 + 15*ArcTanh[Tanh[a + b*x]]^
4)/x^7
```

Maple [A]
time = 3.01, size = 74, normalized size = 0.76

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{7x^7}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{6x^6}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{5x^5}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{4x^4}-\frac{b}{12x^3}\right)}{5}\right)}{2}\right)}{7}$	74
risch	Expression too large to display	22625

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*arctanh(tanh(b*x+a))^4/x^7+4/7*b*(-1/6*arctanh(tanh(b*x+a))^3/x^6+1/2*
b*(-1/5*arctanh(tanh(b*x+a))^2/x^5+2/5*b*(-1/4*arctanh(tanh(b*x+a))/x^4-1/1
2/x^3*b)))
```

Maxima [A]
time = 0.43, size = 72, normalized size = 0.73

$$-\frac{1}{105}\left(b\left(\frac{b^2}{x^3}+\frac{3\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^4}\right)+\frac{6\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^5}\right)b-\frac{2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{21\,x^6}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(b*(b^2/x^3 + 3*b*arctanh(tanh(b*x + a))/x^4) + 6*b*arctanh(tanh(b*x
 + a))^2/x^5)*b - 2/21*b*arctanh(tanh(b*x + a))^3/x^6 - 1/7*arctanh(tanh(b*
x + a))^4/x^7
```

Fricas [A]
time = 0.31, size = 46, normalized size = 0.47

$$-\frac{35\,b^4x^4+105\,ab^3x^3+126\,a^2b^2x^2+70\,a^3bx+15\,a^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

Sympy [A]
time = 1.16, size = 80, normalized size = 0.82

$$-\frac{b^4}{105x^3}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{35x^4}-\frac{2b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{35x^5}-\frac{2b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{21x^6}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**8,x)
```

```
[Out] -b**4/(105*x**3) - b**3*atanh(tanh(a + b*x))/(35*x**4) - 2*b**2*atanh(tanh(
a + b*x))**2/(35*x**5) - 2*b*atanh(tanh(a + b*x))**3/(21*x**6) - atanh(tanh
(a + b*x))**4/(7*x**7)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.47

$$-\frac{35\,b^4x^4+105\,ab^3x^3+126\,a^2b^2x^2+70\,a^3bx+15\,a^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

Mupad [B]
time = 1.01, size = 70, normalized size = 0.71

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{7\,x^7}-\frac{b^4}{105\,x^3}-\frac{2\,b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{35\,x^5}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{35\,x^4}-\frac{2\,b\operatorname{atanh}(\tanh(a+b\,x))^3}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^8,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(7*x^7) - b^4/(105*x^3) - (2*b^2*atanh(tanh(a + b*
x))^2)/(35*x^5) - (b^3*atanh(tanh(a + b*x)))/(35*x^4) - (2*b*atanh(tanh(a +
 b*x))^3)/(21*x^6)
```

3.81 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9} dx$

Optimal. Leaf size=80

$$-\frac{b^4}{280x^4}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{70x^5}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{28x^6}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{14x^7}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{8x^8}$$

```
[Out] -1/280*b^4/x^4-1/70*b^3*arctanh(tanh(b*x+a))/x^5-1/28*b^2*arctanh(tanh(b*x+
a))^2/x^6-1/14*b*arctanh(tanh(b*x+a))^3/x^7-1/8*arctanh(tanh(b*x+a))^4/x^8
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2202, 2198}

$$\frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] (b^3*ArcTanh[Tanh[a + b*x]]^5)/(280*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^4) +
 (b^2*ArcTanh[Tanh[a + b*x]]^5)/(56*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (3*b*ArcTanh[Tanh[a + b*x]]^5)/(56*x^7*(b*x - ArcTanh[Tanh[a + b*x]])^2) +
 ArcTanh[Tanh[a + b*x]]^5/(8*x^8*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rule 2202

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{2}$$
$$= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} +$$
$$= \frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.89

$$-\frac{b^4x^4+4b^3x^3\tanh^{-1}(\tanh(a+bx))+10b^2x^2\tanh^{-1}(\tanh(a+bx))^2+20bx\tanh^{-1}(\tanh(a+bx))^3+35\tanh^{-1}(\tanh(a+bx))^4}{280x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] -1/280*(b^4*x^4 + 4*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 10*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 + 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]
^4)/x^8
```

Maple [A]
time = 3.07, size = 74, normalized size = 0.92

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{8x^8}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{7x^7}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{6x^6}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{5x^5}-\frac{b}{20x^4}\right)}{3}\right)}{7}\right)}{2}$	74
risch	Expression too large to display	22625

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*arctanh(tanh(b*x+a))^4/x^8+1/2*b*(-1/7/x^7*arctanh(tanh(b*x+a))^3+3/7*
b*(-1/6/x^6*arctanh(tanh(b*x+a))^2+1/3*b*(-1/5/x^5*arctanh(tanh(b*x+a))-1/2
0*b/x^4)))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 0.90

$$-\frac{1}{280}\left(b\left(\frac{b^2}{x^4}+\frac{4\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^5}\right)+\frac{10\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^6}\right)b-\frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{14\,x^7}-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="maxima")
```

```
[Out] -1/280*(b*(b^2/x^4 + 4*b*arctanh(tanh(b*x + a))/x^5) + 10*b*arctanh(tanh(b*
x + a))^2/x^6)*b - 1/14*b*arctanh(tanh(b*x + a))^3/x^7 - 1/8*arctanh(tanh(b
*x + a))^4/x^8
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.58

$$-\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

Sympy [A]
time = 1.56, size = 76, normalized size = 0.95

$$-\frac{b^4}{280x^4}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{70x^5}-\frac{b^2\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{28x^6}-\frac{b\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{14x^7}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**9,x)
```

```
[Out] -b**4/(280*x**4) - b**3*atanh(tanh(a + b*x))/(70*x**5) - b**2*atanh(tanh(a
+ b*x))**2/(28*x**6) - b*atanh(tanh(a + b*x))**3/(14*x**7) - atanh(tanh(a +
 b*x))**4/(8*x**8)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.58

$$-\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="giac")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

Mupad [B]
time = 1.07, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{8\,x^8}-\frac{b^4}{280\,x^4}-\frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{28\,x^6}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{70\,x^5}-\frac{b\operatorname{atanh}(\tanh(a+b\,x))^3}{14\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^9,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(8*x^8) - b^4/(280*x^4) - (b^2*atanh(tanh(a + b*x)
)^2)/(28*x^6) - (b^3*atanh(tanh(a + b*x)))/(70*x^5) - (b*atanh(tanh(a + b*x
))^3)/(14*x^7)
```

3.82 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx$

Optimal. Leaf size=80

$$-\frac{b^4}{630x^5}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+}{9x^9}$$

```
[Out] -1/630*b^4/x^5-1/126*b^3*arctanh(tanh(b*x+a))/x^6-1/42*b^2*arctanh(tanh(b*x
+a))^2/x^7-1/18*b*arctanh(tanh(b*x+a))^3/x^8-1/9*arctanh(tanh(b*x+a))^4/x^9
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{b^4}{630x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -1/630*b^4/x^5 - (b^3*ArcTanh[Tanh[a + b*x]])/(126*x^6) - (b^2*ArcTanh[Tanh
[a + b*x]]^2)/(42*x^7) - (b*ArcTanh[Tanh[a + b*x]]^3)/(18*x^8) - ArcTanh[Ta
nh[a + b*x]]^4/(9*x^9)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9} + \frac{1}{9}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^9}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9} + \frac{1}{6}b^2\int \frac{\tanh^{-1}(\tanh(}{x^8}$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8} - \frac{\tanh^{-1}(\tanh(a+}{9x^9}$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(\tanh(a+}{18x^8}$$
$$= -\frac{b^4}{630x^5} - \frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(}{}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.89

$$-\frac{b^4x^4 + 5b^3x^3\tanh^{-1}(\tanh(a+bx)) + 15b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 35bx\tanh^{-1}(\tanh(a+bx))^3 + 70\tanh^{-1}(\tanh(a+bx))^4}{630x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -1/630*(b^4*x^4 + 5*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 15*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 + 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]
^4)/x^9
```

Maple [A]
time = 3.07, size = 74, normalized size = 0.92

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{9x^9} + \frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{8x^8} + \frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{7x^7} + \frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{6x^6} - \frac{b}{30x^5}\right)}{7}\right)}{8}\right)}{9}$	74
risch	Expression too large to display	2262

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*arctanh(tanh(b*x+a))^4/x^9+4/9*b*(-1/8/x^8*arctanh(tanh(b*x+a))^3+3/8*
b*(-1/7/x^7*arctanh(tanh(b*x+a))^2+2/7*b*(-1/6/x^6*arctanh(tanh(b*x+a))-1/3
0*b/x^5)))
```

Maxima [A]
time = 0.43, size = 72, normalized size = 0.90

$$-\frac{1}{630}\left(b\left(\frac{b^2}{x^5}+\frac{5\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^6}\right)+\frac{15\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^7}\right)b-\frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{18\,x^8}-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(b*(b^2/x^5 + 5*b*arctanh(tanh(b*x + a))/x^6) + 15*b*arctanh(tanh(b*
x + a))^2/x^7)*b - 1/18*b*arctanh(tanh(b*x + a))^3/x^8 - 1/9*arctanh(tanh(b
*x + a))^4/x^9
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.58

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Sympy [A]
time = 2.24, size = 76, normalized size = 0.95

$$-\frac{b^4}{630x^5}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{126x^6}-\frac{b^2\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{42x^7}-\frac{b\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{18x^8}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**10,x)
```

```
[Out] -b**4/(630*x**5) - b**3*atanh(tanh(a + b*x))/(126*x**6) - b**2*atanh(tanh(a
 + b*x))**2/(42*x**7) - b*atanh(tanh(a + b*x))**3/(18*x**8) - atanh(tanh(a
+ b*x))**4/(9*x**9)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.58

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Mupad [B]
time = 1.10, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{9\,x^9}-\frac{b^4}{630\,x^5}-\frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{42\,x^7}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{126\,x^6}-\frac{b\operatorname{atanh}(\tanh(a+b\,x))^3}{18\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^10,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(9*x^9) - b^4/(630*x^5) - (b^2*atanh(tanh(a + b*x)
)^2)/(42*x^7) - (b^3*atanh(tanh(a + b*x)))/(126*x^6) - (b*atanh(tanh(a + b*
x))^3)/(18*x^8)
```

3.83 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\, dx$

Optimal. Leaf size=80

$$-\frac{b^4}{1260x^6}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}$$

```
[Out] -1/1260*b^4/x^6-1/210*b^3*arctanh(tanh(b*x+a))/x^7-1/60*b^2*arctanh(tanh(b*
x+a))^2/x^8-2/45*b*arctanh(tanh(b*x+a))^3/x^9-1/10*arctanh(tanh(b*x+a))^4/x
^10
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{b^4}{1260x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -1/1260*b^4/x^6 - (b^3*ArcTanh[Tanh[a + b*x]])/(210*x^7) - (b^2*ArcTanh[Tan
h[a + b*x]]^2)/(60*x^8) - (2*b*ArcTanh[Tanh[a + b*x]]^3)/(45*x^9) - ArcTanh
[Tanh[a + b*x]]^4/(10*x^10)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}} + \frac{1}{5}(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{10}}\,dx \\
&= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}} + \frac{1}{15}(2b^2)\int \tanh^{-1}(\\
&= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9} - \frac{\tanh^{-1}(\tanh(a+}{10x^{10}} \\
&= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh^{-1}(\tanh(a}{45x^9} \\
&= -\frac{b^4}{1260x^6} - \frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh}{}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.89

$$
-\frac{b^4x^4+6b^3x^3\tanh^{-1}(\tanh(a+bx))+21b^2x^2\tanh^{-1}(\tanh(a+bx))^2+56bx\tanh^{-1}(\tanh(a+bx))^3+126\tanh^{-1}(\tanh(a+bx))^4}{1260x^{10}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -1/1260*(b^4*x^4 + 6*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x
]]^4)/x^10
```

Maple [A]
time = 3.10, size = 74, normalized size = 0.92

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{10x^{10}}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{9x^9}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{8x^8}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{7x^7}-\frac{b}{42x^6}\right)}{4}\right)}{3}\right)}{5}$	74
risch	Expression too large to display	22625

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*arctanh(tanh(b*x+a))^4/x^10+2/5*b*(-1/9/x^9*arctanh(tanh(b*x+a))^3+1/
3*b*(-1/8/x^8*arctanh(tanh(b*x+a))^2+1/4*b*(-1/7/x^7*arctanh(tanh(b*x+a))-1
/42*b/x^6)))
```

Maxima [A]
time = 0.42, size = 72, normalized size = 0.90

$$-\frac{1}{1260}\left(b\left(\frac{b^2}{x^6}+\frac{6\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^7}\right)+\frac{21\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^8}\right)b-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{45\,x^9}-\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(b*(b^2/x^6 + 6*b*arctanh(tanh(b*x + a))/x^7) + 21*b*arctanh(tanh(b
*x + a))^2/x^8)*b - 2/45*b*arctanh(tanh(b*x + a))^3/x^9 - 1/10*arctanh(tanh
(b*x + a))^4/x^10
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.58

$$-\frac{210\,b^4x^4+720\,ab^3x^3+945\,a^2b^2x^2+560\,a^3bx+126\,a^4}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

Sympy [A]
time = 3.53, size = 78, normalized size = 0.98

$$-\frac{b^4}{1260x^6}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{210x^7}-\frac{b^2\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{60x^8}-\frac{2b\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{45x^9}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**11,x)
```

```
[Out] -b**4/(1260*x**6) - b**3*atanh(tanh(a + b*x))/(210*x**7) - b**2*atanh(tanh(
a + b*x))**2/(60*x**8) - 2*b*atanh(tanh(a + b*x))**3/(45*x**9) - atanh(tanh
(a + b*x))**4/(10*x**10)
```

Giac [A]
time = 0.38, size = 46, normalized size = 0.58

$$-\frac{210\,b^4x^4+720\,ab^3x^3+945\,a^2b^2x^2+560\,a^3bx+126\,a^4}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

Mupad [B]
time = 1.04, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{10\,x^{10}}-\frac{b^4}{1260\,x^6}-\frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{60\,x^8}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{210\,x^7}-\frac{2\,b\operatorname{atanh}(\tanh(a+b\,x))^3}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^11,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(10*x^10) - b^4/(1260*x^6) - (b^2*atanh(tanh(a + b
*x))^2)/(60*x^8) - (b^3*atanh(tanh(a + b*x)))/(210*x^7) - (2*b*atanh(tanh(a
 + b*x))^3)/(45*x^9)
```

3.84 $\int x \tanh^{-1}(\tanh(a+bx))^6 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

```
[Out] 1/7*x*arctanh(tanh(b*x+a))^7/b-1/56*arctanh(tanh(b*x+a))^8/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x \tanh^{-1}(\tanh(a+bx))^6 \, dx = \frac{x \tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^7 \, dx}{7b}$$
$$= \frac{x \tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\text{Subst}\left(\int x^7 \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{7b^2}$$
$$= \frac{x \tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(34) = 68.
time = 0.08, size = 177, normalized size = 5.21

$$-\frac{(a+bx)\left((7a-bx)(a+bx)^6-8(6a-bx)(a+bx)^5\tanh^{-1}(\tanh(a+bx))+28(5a-bx)(a+bx)^4\tanh^{-1}(\tanh(a+bx))^2-56(4a-bx)(a+bx)^3\tanh^{-1}(\tanh(a+bx))^3+70(3a-bx)(a+bx)^2\tanh^{-1}(\tanh(a+bx))^4-56(2a^2+abx-b^2x^2)\tanh^{-1}(\tanh(a+bx))^5+28(a-bx)\tanh^{-1}(\tanh(a+bx))^6\right)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] -1/56*((a + b*x)*((7*a - b*x)*(a + b*x)^6 - 8*(6*a - b*x)*(a + b*x)^5*ArcTa
nh[Tanh[a + b*x]] + 28*(5*a - b*x)*(a + b*x)^4*ArcTanh[Tanh[a + b*x]]^2 - 5
6*(4*a - b*x)*(a + b*x)^3*ArcTanh[Tanh[a + b*x]]^3 + 70*(3*a - b*x)*(a + b*
x)^2*ArcTanh[Tanh[a + b*x]]^4 - 56*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a
 + b*x]]^5 + 28*(a - b*x)*ArcTanh[Tanh[a + b*x]]^6))/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(30) = 60.
time = 51.62, size = 110, normalized size = 3.24

method	result
default	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^6}{2} - 3b\left(\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^5}{3} - \frac{5b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^4}{4} - b\left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^3}{5} - \frac{3b\left(x^6 \operatorname{arc}\right.}{}\right.\right.}{3}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^6-3*b*(1/3*x^3*arctanh(tanh(b*x+a))^5-5/3*b*(1
/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6*
x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*b*x^8))
)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(30) = 60$.
time = 0.52, size = 110, normalized size = 3.24

$$-bx^3 \operatorname{artanh}(\tanh(bx+a))^5 + \frac{1}{2}x^2 \operatorname{artanh}(\tanh(bx+a))^6 + \frac{1}{56}(70\,bx^4 \operatorname{artanh}(\tanh(bx+a))^4 - (56\,bx^5 \operatorname{artanh}(\tanh(bx+a))^3 - (28\,bx^6 \operatorname{artanh}(\tanh(bx+a))^2 + (b^2x^8 - 8\,bx^7 \operatorname{artanh}(\tanh(bx+a)))b)b)b)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="maxima")
```

```
[Out] -b*x^3*arctanh(tanh(b*x + a))^5 + 1/2*x^2*arctanh(tanh(b*x + a))^6 + 1/56*(
70*b*x^4*arctanh(tanh(b*x + a))^4 - (56*b*x^5*arctanh(tanh(b*x + a))^3 - (2
8*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*x + a)
))*b)*b)*b)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.
time = 0.34, size = 68, normalized size = 2.00

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="fricas")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Sympy [A]
time = 1.24, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{atanh}^{7}(\tanh(a+bx))}{7b} - \frac{\operatorname{atanh}^{8}(\tanh(a+bx))}{56b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atanh}^{6}(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**6,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**7/(7*b) - atanh(tanh(a + b*x))**8/(56*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**6/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.
time = 0.38, size = 68, normalized size = 2.00

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Mupad [B]
time = 1.10, size = 104, normalized size = 3.06

$$\frac{b^6\,x^8}{56}-\frac{b^5\,x^7\,\mathrm{atanh}(\tanh(a+b\,x))}{7}+\frac{b^4\,x^6\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2}-b^3\,x^5\,\mathrm{atanh}(\tanh(a+b\,x))^3+\frac{5\,b^2\,x^4\,\mathrm{atanh}(\tanh(a+b\,x))^4}{4}-b\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))^5+\frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^6,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^6)/2 + (b^6*x^8)/56 + (5*b^2*x^4*atanh(tanh(a + b
*x))^4)/4 - b^3*x^5*atanh(tanh(a + b*x))^3 + (b^4*x^6*atanh(tanh(a + b*x))^
2)/2 - b*x^3*atanh(tanh(a + b*x))^5 - (b^5*x^7*atanh(tanh(a + b*x)))/7
```

3.85 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=53

$$-\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arctanh(tanh(b*x+a))))/(1+m)/(b*
x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2195}

$$-\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcTanh[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcTanh[Tanh[a + b*x]])))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))}\,dx = -\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.96

$$\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{-bx+\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\operatorname{arctanh}\left(\tanh \left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a)),x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\operatorname{atanh}\left(\tanh \left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x)),x)
[Out] int(x^m/atanh(tanh(a + b*x)), x)
```

3.86 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=81

$$\frac{x^3}{3b}+\frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

```
[Out] 1/3*x^3/b+1/2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^2+x*(b*x-arctanh(tanh(b*x+a)
))^2/b^3+(b*x-arctanh(tanh(b*x+a)))^3*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2190, 2189, 2188, 29}

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcTan
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Ta
nh[a + b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{x^3}{3b} + \frac{x^2(bx-\tanh^{-1}(\tanh(a+bx)))}{2b^2} + \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))^2 \int}{b^2}$$
$$= \frac{x^3}{3b} + \frac{x^2(bx-\tanh^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx-\tanh^{-1}(\tanh(a+bx)))^2}{b^3} -$$
$$= \frac{x^3}{3b} + \frac{x^2(bx-\tanh^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx-\tanh^{-1}(\tanh(a+bx)))^2}{b^3} -$$
$$= \frac{x^3}{3b} + \frac{x^2(bx-\tanh^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx-\tanh^{-1}(\tanh(a+bx)))^2}{b^3} +$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.98

$$\frac{x^3}{3b} - \frac{x^2(-bx+\tanh^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(-bx+\tanh^{-1}(\tanh(a+bx)))^2}{b^3} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))^3 \log(\tanh^{-1}(\tanh(a+bx)))}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[A
rcTanh[Tanh[a + b*x]]])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(77) = 154.
time = 2.39, size = 202, normalized size = 2.49

method	result
default	$\frac{x^3}{3b} - \frac{a x^2}{2b^2} - \frac{x^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{2b^2} + \frac{x a^2}{b^3} + \frac{2xa(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^3} + \frac{x(\operatorname{arctanh}(\tanh(bx+a))-bx-}{b^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/b-1/2/b^2*a*x^2-1/2/b^2*x^2*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*x*a^
2+2/b^3*x*a*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*x*(arctanh(tanh(b*x+a))-b*x-
a)^2-1/b^4*ln(arctanh(tanh(b*x+a)))*a^3-3/b^4*ln(arctanh(tanh(b*x+a)))*a^2*
```

```
(arctanh(tanh(b*x+a))-b*x-a)-3/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh
(b*x+a))-b*x-a)^2-1/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-
a)^3
```

Maxima [A]
time = 0.55, size = 42, normalized size = 0.52

$$-\frac{a^3 \log (bx+a)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A]
time = 0.42, size = 41, normalized size = 0.51

$$\frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log (bx+a)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}(\tanh (a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a)),x)
[Out] Integral(x**3/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.41, size = 43, normalized size = 0.53

$$-\frac{a^3 \log (|bx+a|)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B]
time = 0.13, size = 354, normalized size = 4.37

$$\frac{x^3}{3\,b}+\frac{x^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4\,b^2}+\frac{x\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^3}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\left({\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3-8\,a^3-6\,a\,{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2+12\,a^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x)),x)
```

```
[Out] x^3/(3*b) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^2) + (x*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 -
 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)))/(8*b^4)
```

3.87 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=56

$$\frac{x^2}{2b}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

```
[Out] 1/2*x^2/b+x*(b*x-arctanh(tanh(b*x+a)))/b^2+(b*x-arctanh(tanh(b*x+a)))^2*ln(
arctanh(tanh(b*x+a)))/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2190, 2189, 2188, 29}

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) + (x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 + ((b*x - ArcTanh[Tanh[a
 + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\mathrm{ta}}}{b^2}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \mathrm{Sub}}{b}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\mathrm{t}\right.}{b^3}$$

Mathematica [**A**]
time = 0.03, size = 55, normalized size = 0.98

$$\frac{x^2}{2b} - \frac{x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(54) = 108.
time = 0.65, size = 111, normalized size = 1.98

method	result
default	$\frac{x^2}{2b} - \frac{xa}{b^2} - \frac{x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^2} + \frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))a^2}{b^3} + \frac{2\ln(\operatorname{arctanh}(\tanh(bx+a)))a(\operatorname{arctanh}(\tanh(bx+a}{b^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b-1/b^2*x*a-1/b^2*x*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*ln(arctanh(t
anh(b*x+a)))*a^2+2/b^3*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b*x
-a)+1/b^3*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [**A**]
time = 0.55, size = 29, normalized size = 0.52

$$\frac{a^2\log\left(bx+a\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.52

$$\frac{b^2x^2 - 2\,abx + 2\,a^2\log\left(bx + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.38, size = 30, normalized size = 0.54

$$\frac{a^2\log\left(|bx + a|\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B]
time = 0.27, size = 234, normalized size = 4.18

$$\frac{x^2}{2\,b}+\frac{x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^2}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\left({\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)+4\,a^2\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x)),x)
```

```
[Out] x^2/(2*b) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*a^2
))/(4*b^3)
```

3.88 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=31

$$\frac{x}{b}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

```
[Out] x/b+(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2189, 2188, 29}

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]],x]
[Out] x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{x}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}\\
&= \frac{x}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= \frac{x}{b}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{x}{b} - \frac{\left(-bx + \tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]],x]
[Out] x/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Maple [A]
time = 0.19, size = 49, normalized size = 1.58

method	result	size
default	$\frac{x}{b}-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))a}{b^2}-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^2}$	49
risch	Expression too large to display	2837

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] x/b-1/b^2*ln(arctanh(tanh(b*x+a)))*a-1/b^2*ln(arctanh(tanh(b*x+a)))*(arctan
h(tanh(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.53, size = 18, normalized size = 0.58

$$\frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.55

$$\frac{bx - a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.61

$$\frac{x}{b} - \frac{a\log\left(|bx+a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B]
time = 0.15, size = 108, normalized size = 3.48

$$\frac{x}{b}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x)),x)
```

```
[Out] x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2)
```

3.89 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=12

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

```
[Out] ln(arctanh(tanh(b*x+a)))/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 12, normalized size = 1.00

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-1),x]

[Out] Log[ArcTanh[Tanh[a + b*x]]]/b

Maple [A]
time = 0.06, size = 13, normalized size = 1.08

method	result
derivativedivides	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b}$
default	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b}$
risch	$\ln\left(\ln\left(\mathrm{e}^{bx+a}\right)+\frac{i\pi\left(-\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-\operatorname{csgn}\left(i\mathrm{e}^{bx+}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)

[Out] ln(arctanh(tanh(b*x+a)))/b

Maxima [A]
time = 0.47, size = 13, normalized size = 1.08

$$\frac{\log\left(-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="maxima")

[Out] log(-b*x - a)/b

Fricas [A]
time = 0.34, size = 10, normalized size = 0.83

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="fricas")

[Out] log(b*x + a)/b

Sympy [A]
time = 12.13, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for}\: b \neq 0 \\ \frac{x}{\operatorname{atanh}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((log(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/atanh(tanh(a)), True)
)
```

Giac [A]
time = 0.40, size = 11, normalized size = 0.92

$$\frac{\log(|bx + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 1.06, size = 12, normalized size = 1.00

$$\frac{\ln(\operatorname{atanh}(\tanh(a + b\,x)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x)),x)
```

```
[Out] log(atanh(tanh(a + b*x)))/b
```

3.90 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=44

$$-\frac{\log(x)}{bx - \tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx - \tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -ln(x)/(b*x-arctanh(tanh(b*x+a)))+ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tan
h(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2191, 2188, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx - \tanh^{-1}(\tanh(a+bx))} - \frac{\log(x)}{bx - \tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] -(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x
 - ArcTanh[Tanh[a + b*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))}\,dx &= -\frac{\int \frac{1}{x}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{b\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))} \\
&= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))} \\
&= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.66

$$\frac{-\log(x)+\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-Log[x] + Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])
```

Maple [A]
time = 7.72, size = 43, normalized size = 0.98

method	result
default	$-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{\operatorname{arctanh}(\tanh(bx+a))-bx}+\frac{\ln(x)}{\operatorname{arctanh}(\tanh(bx+a))-bx}$
risch	$\frac{4i\ln\left(-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-\pi\text{csgn}\left(ie^{bx+a}\right)^2\text{csgn}\left(ie^{2bx\cdots\right.\right.}{-\pi\text{csgn}\left(ie^{2bx+2a}\right)^3+2\pi\,\text{csgn}\left(ie^{bx+a}\right)\text{csgn}\left(ie^{2bx+2a}\right)^2+\pi\,\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*x+
a))-b*x)*ln(x)
```

Maxima [A]
time = 0.52, size = 18, normalized size = 0.41

$$-\frac{\log(bx+a)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.36

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a)),x)
[Out] Integral(1/(x*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 20, normalized size = 0.45

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B]
time = 2.87, size = 107, normalized size = 2.43

$$-\frac{4\operatorname{atanh}\left(\frac{4\,b\,x}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}-1\right)}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))),x)
[Out] -(4*atanh((4*b*x)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 1))/(log(1/(exp(2*a)*exp(2*b*x)
+ 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.91 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=65

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/x/(b*x-arctanh(tanh(b*x+a)))-b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^2+b*ln(ar
ctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] 1/(x*(b*x - ArcTanh[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcTanh[Tanh[a + b
*x]])^2 + (b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x}\,dx}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b^2}{(bx-}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\text{Subst}}{(}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log(}{(bx-}$$

Mathematica [**A**]
time = 0.02, size = 45, normalized size = 0.69

$$\frac{-\tanh^{-1}(\tanh(a+bx)) + bx\left(1-\log(x)+\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-ArcTanh[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcTanh[Tanh[a + b*x]]]))/
(x*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [**A**]
time = 0.02, size = 64, normalized size = 0.98

$$\frac{b\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2} - \frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x} - \frac{b\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(arctanh(tanh(b*x+a)))-1/(arctanh(tanh(b
*x+a))-b*x)/x-1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(x)
```

Maxima [**A**]
time = 0.53, size = 28, normalized size = 0.43

$$\frac{b\log\left(bx+a\right)}{a^2} - \frac{b\log\left(x\right)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 0.34, size = 26, normalized size = 0.40

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 30, normalized size = 0.46

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 2.83, size = 210, normalized size = 3.23

$$\frac{2\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+4\,b\,x+b\,x\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,1{}\mathrm{i}+\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,1{}\mathrm{i}+b\,x\,2{}\mathrm{i}}{\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8{}\mathrm{i}}{x\,{\left(\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))),x)
```

```
[Out] (2*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 4*b*x + b*x*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(
exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))*8i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

3.92 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=92

$$\frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b^2\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

```
[Out] b/x/(b*x-arctanh(tanh(b*x+a)))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a)))-b^2*ln(
x)/(b*x-arctanh(tanh(b*x+a)))^3+b^2*ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(t
anh(b*x+a)))^3
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$-\frac{b^2\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] b/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcTanh[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcTanh[Tanh[a + b*x]])^3 + (b^2*Log[ArcTanh[T
anh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{(b}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{(b}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{(b}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{(b}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.72

$$\frac{-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2+b^2x^2\left(3-2\log(x)+2\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2*
Log[x] + 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

Maple [A]
time = 0.02, size = 87, normalized size = 0.95

$$-\frac{b^2\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3}-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^2}+\frac{b^2\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3}+\frac{}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a)),x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(arctanh(tanh(b*x+a)))-1/2/(arctanh(t
anh(b*x+a))-b*x)/x^2+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(x)+1/(arctanh(ta
nh(b*x+a))-b*x)^2*b/x
```

Maxima [A]
time = 0.53, size = 40, normalized size = 0.43

$$-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A]
time = 0.34, size = 41, normalized size = 0.45

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a)),x)
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.40, size = 45, normalized size = 0.49

$$-\frac{b^2\log\left(\left|bx+a\right|\right)}{a^3}+\frac{b^2\log\left(\left|x\right|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B]
time = 2.89, size = 286, normalized size = 3.11

$$\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\left(2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+8\,b\,x\right)+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2+12\,b^2\,x^2+8\,b\,x\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+b^2\,x^2\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1i+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1i+b\,x\,2i}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,16i}{x^2\,\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*atanh(tanh(a + b*x))),x)

[Out] (log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(2*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 8*b*x) + log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 12*b^2*x^2 + 8*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1)) + b^2*x^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i)/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)

3.93 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=65

$$-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}-\frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -x^m/b/arctanh(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arctanh(tan
h(b*x+a))))/b/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$-\frac{x^m\,{}_2F_1\left(1,m;m+1;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x^m/(b*ArcTanh[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcTanh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))} + \frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))} - \frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.49, size = 51, normalized size = 0.78

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{-bx+\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^2,x)
[Out] int(x^m/arctanh(tanh(b*x+a))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a))^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x))^2,x)
```

```
[Out] int(x^m/atanh(tanh(a + b*x))^2, x)
```

3.94 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=98

$$\frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{x^4}{b\tanh^{-1}(\tanh(a+bx))}+\frac{4(bx-$$

```
[Out] 4/3*x^3/b^2+2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^3+4*x*(b*x-arctanh(tanh(b*x+
a)))^2/b^4-x^4/b/arctanh(tanh(b*x+a))+4*(b*x-arctanh(tanh(b*x+a)))^3*ln(arc
tanh(tanh(b*x+a)))/b^5
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^4}{b\tanh^{-1}(\tanh(a+bx))}+\frac{4x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcTanh[Tanh[a + b*x]]) + (4*(b*x -
 ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{4\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{4x^3}{3b^2} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\tanh^{-1}}}{b^2}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(4(-\right.}{}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 1.08

$$\frac{x^3}{3b^2} - \frac{x^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{3x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}{b^5\tanh^{-1}(\tanh(a+bx))} - \frac{4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^3/(3*b^2) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcTanh[Tanh[a + b*x]])^4/(b^5*A
```

```
rcTanh[Tanh[a + b*x]]) - (4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh
[Tanh[a + b*x]]])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(96) = 192$.
time = 2.87, size = 350, normalized size = 3.57

method	result
default	$\frac{x^3}{3b^2} - \frac{a\,x^2}{b^3} - \frac{x^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^3} + \frac{3x\,a^2}{b^4} + \frac{6a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)x}{b^4} + \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx-}{b^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/b^2-1/b^3*a*x^2-1/b^3*x^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4*x*a^2+
6/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x+3/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x-1/b^5/arctanh(tanh(b*x+a))*a^4-4/b^5/arctanh(tanh(b*x+a))*a^3*(arctanh(t
anh(b*x+a))-b*x-a)-6/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x
-a)^2-4/b^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/b^5/arc
tanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^4-4/b^5*ln(arctanh(tanh(b*x+
a)))*a^3-12/b^5*ln(arctanh(tanh(b*x+a)))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-1
2/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4/b^5*ln(ar
ctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [A]
time = 0.70, size = 70, normalized size = 0.71

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4}{3\,(b^6x + ab^5)} - \frac{4\,a^3\log\left(bx + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4)/(b^6*x + a*
b^5) - 4*a^3*log(b*x + a)/b^5
```

Fricas [A]
time = 0.32, size = 73, normalized size = 0.74

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\,(a^3bx + a^4)\log\left(bx + a\right)}{3\,(b^6x + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.39, size = 62, normalized size = 0.63

$$-\frac{4\,a^3\log\left(\left|bx+a\right|\right)}{b^5}-\frac{a^4}{(bx+a)b^5}+\frac{b^4x^3-3\,ab^3x^2+9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

Mupad [B]
time = 0.18, size = 669, normalized size = 6.83

$$\frac{x^3}{3\,b^2}-\frac{{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4+24\,a^2\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2+16\,a^4-8\,a\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3-32\,a^3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b\,\left(8\,a\,b^4+8\,b^5\,x-4\,b^4\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}+\frac{x^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^3}+\frac{3\,x\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^4}+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left({\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3-8\,a^3-6\,a\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2+12\,a^2\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x))) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^3) + (3*x*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
```

```
p(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)))/(2*b^5)
```

3.95 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=75

$$\frac{3x^2}{2b^2}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh\right.}{b^4}$$

```
[Out] 3/2*x^2/b^2+3*x*(b*x-arctanh(tanh(b*x+a)))/b^3-x^3/b/arctanh(tanh(b*x+a))+3
*(b*x-arctanh(tanh(b*x+a)))^2*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcTanh
[Tanh[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a +
 b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{3x^2}{2b^2} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\tanh^{-1}}}{b^2}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx\right.\right.}{}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx\right.\right.}{}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\right.}{}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 1.11

$$\frac{x^2}{2b^2} - \frac{2x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^2/(2*b^2) - (2*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3/(b^4*ArcTanh[Tanh[a + b*x]]) + (3*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(73) = 146.
time = 0.81, size = 223, normalized size = 2.97

method	result
default	$\frac{x^2}{2b^2}-\frac{2xa}{b^3}-\frac{2(\text{arctanh}(\tanh(bx+a))-bx-a)x}{b^3}+\frac{3\ln(\text{arctanh}(\tanh(bx+a)))a^2}{b^4}+\frac{6\ln(\text{arctanh}(\tanh(bx+a)))a(\text{arctanh}(\tanh($
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b^2-2/b^3*x*a-2/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x+3/b^4*ln(arctanh
(tanh(b*x+a)))*a^2+6/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b
*x-a)+3/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4/a
rctanh(tanh(b*x+a))*a^3+3/b^4/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a)
)-b*x-a)+3/b^4/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4/
arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [A]
time = 0.70, size = 59, normalized size = 0.79

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3}{2\,(b^5x+ab^4)}+\frac{3\,a^2\log{(bx+a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3)/(b^5*x + a*b^4) + 3*a^2*log
(b*x + a)/b^4
```

Fricas [A]
time = 0.34, size = 62, normalized size = 0.83

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log{(bx+a)}}{2\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\operatorname{atanh}^2{(\tanh{(a+bx)})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.38, size = 48, normalized size = 0.64

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

Mupad [B]
time = 1.03, size = 490, normalized size = 6.53

$$\frac{x^2}{2\,b^2}+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(3\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2-12\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+12\,a^2\right)}{4\,b^4}-\frac{{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3-8\,a^3-6\,a\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2+12\,a^2\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4\,b\,\left(2\,a\,b^3+2\,b^4\,x-b^3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}+\frac{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
 log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12
*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x))) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

3.96 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=50

$$\frac{2x}{b^2} - \frac{x^2}{b \tanh^{-1}(\tanh(a+bx))} + \frac{2(bx - \tanh^{-1}(\tanh(a+bx))) \log(\tanh^{-1}(\tanh(a+bx)))}{b^3}$$

```
[Out] 2*x/b^2-x^2/b/arctanh(tanh(b*x+a))+2*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(
tanh(b*x+a)))/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2189, 2188, 29}

$$\frac{2(bx - \tanh^{-1}(\tanh(a+bx))) \log(\tanh^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{b \tanh^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x)/b^2 - x^2/(b*ArcTanh[Tanh[a + b*x]]) + (2*(b*x - ArcTanh[Tanh[a + b*x
]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= \frac{2x}{b^2}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}-\frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\tanh^{-1}(\text{t}}}{b^2}\\ &= \frac{2x}{b^2}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}-\frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\text{Subst}\left(\int\right.}{b^3}\\ &= \frac{2x}{b^2}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\right.}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 1.12

$$\frac{bx-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}{\tanh^{-1}(\tanh(a+bx))}+2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b*x - (-(b*x) + ArcTanh[Tanh[a + b*x]])^2/ArcTanh[Tanh[a + b*x]] + 2*(b*x
- ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(50) = 100.
time = 0.24, size = 127, normalized size = 2.54

method	result
default	$\frac{x}{b^2}-\frac{2\ln(\text{arctanh}(\tanh(bx+a)))a}{b^3}-\frac{2\ln(\text{arctanh}(\tanh(bx+a)))(\text{arctanh}(\tanh(bx+a))-bx-a)}{b^3}-\frac{a^2}{b^3\,\text{arctanh}(\tanh(bx+a))}-\frac{2a}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2-2/b^3*ln(arctanh(tanh(b*x+a)))*a-2/b^3*ln(arctanh(tanh(b*x+a)))*(arct
anh(tanh(b*x+a))-b*x-a)-1/b^3/arctanh(tanh(b*x+a))*a^2-2/b^3/arctanh(tanh(b
```

```
*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)-1/b^3/arctanh(tanh(b*x+a))*(arctanh(t
anh(b*x+a))-b*x-a)^2
```

Maxima [A]
time = 0.68, size = 44, normalized size = 0.88

$$\frac{b^2x^2+abx-a^2}{b^4x+ab^3}-\frac{2\,a\log{(bx+a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] (b^2*x^2 + a*b*x - a^2)/(b^4*x + a*b^3) - 2*a*log(b*x + a)/b^3
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.94

$$\frac{b^2x^2+abx-a^2-2\,(abx+a^2)\log{(bx+a)}}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\operatorname{atanh}^2{(\tanh{(a+bx)})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**2,x)
[Out] Integral(x**2/atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.39, size = 34, normalized size = 0.68

$$\frac{x}{b^2}-\frac{2\,a\log{(|bx+a|)}}{b^3}-\frac{a^2}{(bx+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B]
time = 1.06, size = 302, normalized size = 6.04

$$\frac{x}{b^2}-\frac{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)+4\,a^2}{2\,b\,\left(2\,a\,b^2+2\,b^3\,x-b^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))
 + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp
(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

3.97 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=28

$$-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

```
[Out] -x/b/arctanh(tanh(b*x+a))+ln(arctanh(tanh(b*x+a)))/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x/(b*ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}\\
&= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.96

$$
\frac{1-\frac{bx}{\tanh^{-1}(\tanh(a+bx))}+\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (1 - (b*x)/ArcTanh[Tanh[a + b*x]] + Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Maple [A]
time = 0.09, size = 56, normalized size = 2.00

method	result
default	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^2}+\frac{a}{b^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{\operatorname{arctanh}(\tanh(bx+a))-bx-a}{b^2\operatorname{arctanh}(\tanh(bx+a))}$
risch	$\frac{4ix}{b\left(-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-\pi\operatorname{csgn}\left(ie^{bx+a}\right)^2\operatorname{csgn}\left(ie^{2bx+2a}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(arctanh(tanh(b*x+a)))/b^2+1/b^2/arctanh(tanh(b*x+a))*a+1/b^2/arctanh(tan
h(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.67, size = 26, normalized size = 0.93

$$
\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A]
time = 0.34, size = 28, normalized size = 1.00

$$\frac{(bx + a)\log(bx + a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 25.24, size = 36, normalized size = 1.29

$$\begin{cases} -\frac{x}{b\,\mathrm{atanh}\,(\tanh(a+bx))} + \frac{\log(\mathrm{atanh}\,(\tanh(a+bx)))}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2\,\mathrm{atanh}^2\,(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-x/(b*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**2, Ne
(b, 0)), (x**2/(2*atanh(tanh(a))**2), True))
```

Giac [A]
time = 0.39, size = 24, normalized size = 0.86

$$\frac{\log(|bx + a|)}{b^2} + \frac{a}{(bx + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B]
time = 0.08, size = 28, normalized size = 1.00

$$\frac{\ln(\mathrm{atanh}(\tanh(a + b\,x)))}{b^2} - \frac{x}{b\,\mathrm{atanh}\,(\tanh(a + b\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^2,x)
```

```
[Out] log(atanh(tanh(a + b*x)))/b^2 - x/(b*atanh(tanh(a + b*x)))
```

3.98 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -1/b/arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{b\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.06, size = 15, normalized size = 1.07

method	result
derivativedivides	$-\frac{1}{b \operatorname{arctanh}(\tanh(bx+a))}$
default	$-\frac{1}{b \operatorname{arctanh}(\tanh(bx+a))}$
risch	$\frac{}{b\left(-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}-\pi \operatorname{csgn}\left(i e^{bx+a}\right)^{2} \mathrm{c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.47, size = 12, normalized size = 0.86

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.32, size = 13, normalized size = 0.93

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 24.85, size = 20, normalized size = 1.43

$$\begin{cases} -\frac{1}{b \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)} & \text{for}\ b \neq 0 \\ \frac{x}{\operatorname{atanh}^{2}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-1/(b*atanh(tanh(a + b*x))), Ne(b, 0)), (x/atanh(tanh(a))**2, Tr
ue))
```

Giac [A]
time = 0.38, size = 12, normalized size = 0.86

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.08, size = 14, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^2,x)
```

```
[Out] -1/(b*atanh(tanh(a + b*x)))
```

3.99 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=70

$$-\frac{1}{(bx-\tanh^{-1}(\tanh(a+bx)))\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{(bx-\tanh^{-1}(\tanh(a+bx)))^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{(bx-\tanh^{-1}(\tanh(a+bx)))^2}$$

```
[Out] -1/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))+ln(x)/(b*x-arctanh(tanh(
b*x+a)))^2-ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$-\frac{1}{(bx-\tanh^{-1}(\tanh(a+bx)))\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{(bx-\tanh^{-1}(\tanh(a+bx)))^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{(bx-\tanh^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -(1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcTanh[Tanh[a + b*x]])^2 - Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Ta
nh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a}}{-bx+\tanh^{-1}(\tan}$$
$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{}{\left(bx-\tanh^{-1}(\tan\right.}$$
$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x}{\left(bx-\tanh^{-1}(\tan\right.}$$
$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x}{\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.76

$$\frac{-bx+\tanh^{-1}(\tanh(a+bx))\left(1+\log(bx)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b*x) + ArcTanh[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcTanh[Tanh[a + b*x]]
]))/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.02, size = 67, normalized size = 0.96

$$-\frac{\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2}+\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\ln (}{\left(\operatorname{arctanh}\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*
x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x+a))-b*x)^2*ln(x)
```

Maxima [A]
time = 0.64, size = 28, normalized size = 0.40

$$\frac{1}{abx+a^2}-\frac{\log\left(bx+a\right)}{a^2}+\frac{\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Fricas [A]
time = 0.35, size = 39, normalized size = 0.56

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.38, size = 31, normalized size = 0.44

$$-\frac{\log(|bx+a|)}{a^2}+\frac{\log(|x|)}{a^2}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B]
time = 3.39, size = 359, normalized size = 5.13

$$\frac{8\,b\,x-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\left(-4+\operatorname{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8\mathrm{i}\right)+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\left(-4+\operatorname{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8\mathrm{i}\right)}{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (8*b*x - log(1/(exp(2*a)*exp(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2
i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))*8i - 4) + log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*
1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b
*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*
8i - 4))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

3.100 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=102

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b$$

```
[Out] -2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/x/(b*x-arctanh(tan
h(b*x+a)))/arctanh(tanh(b*x+a))+2*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^3-2*b*
ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{2b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{2b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-2*b)/((b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^3 - (2*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh
[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{(2b)\int \frac{1}{x\tanh^{-1}(\tan}}{-bx+\tanh^{-1}(\text{ta}}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A]
time = 0.04, size = 70, normalized size = 0.69

$$\frac{-b^2x^2+\tanh^{-1}(\tanh(a+bx))^2+2bx\tanh^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b^2*x^2) + ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(x*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcT
anh[Tanh[a + b*x]])
```

Maple [A]
time = 0.03, size = 91, normalized size = 0.89

$$-\frac{b}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{2b\ln\left(\operatorname{arctanh}(\tanh(bx+a))\right)}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)^3}-\frac{}{\left(\operatorname{arctanh}(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))+2/(arctanh(tanh(b*x+
a))-b*x)^3*b*ln(arctanh(tanh(b*x+a)))-1/(arctanh(tanh(b*x+a))-b*x)^2/x-2/(a
rctanh(tanh(b*x+a))-b*x)^3*b*ln(x)
```

Maxima [A]
time = 0.65, size = 45, normalized size = 0.44

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A]
time = 0.35, size = 63, normalized size = 0.62

$$-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log\left(bx+a\right)+2\,(b^2x^2+abx)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.38, size = 45, normalized size = 0.44

$$\frac{2\,b\log\left(|bx+a|\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx+a}{(bx^2+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

Mupad [B]
time = 3.44, size = 432, normalized size = 4.24

$$-\frac{4\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\left(8\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)+b\,x\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)\,32\mathrm{i}\right)+4\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2-16\,b^2\,x^2+b\,x\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)\,32\mathrm{i}}{x\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^2),x)
```

```
[Out] -(4*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))*(8*log(1/(exp(2*a)*exp(2*b*x) + 1)) + b*x*atan((log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x
) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i) + 4*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 16*b^2*x^2 + b*x*log(1/(exp(2*a)*exp(2*
b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i -
log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) +
 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i)/
(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.101 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=143

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b}{2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+$$

```
[Out] -3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))+3/2*b/x/(b*x-arcta
nh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/2/x^2/(b*x-arctanh(tanh(b*x+a)))/
arctanh(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4-3*b^2*ln(arct
anh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

Rubi [A]

time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b^2\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{3b}{2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcTanh[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - Ar
cTanh[Tanh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{(3b)\int \frac{1}{x^2 \tanh^{-}}}{2\left(bx-\tanh^{-1}\right.}$$

$$= \frac{3b}{2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x^2\left(bx-\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-}\right.}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 0.64

$$-\frac{2b^3x^3-6bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3-3b^2x^2\tanh^{-1}(\tanh(a+bx))\left(-1+2\log(x)-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2x^2\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -1/2*(2*b^3*x^3 - 6*b*x*ArcTanh[Tanh[a + b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3
 - 3*b^2*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a +
 b*x]]]))/(x^2*ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.03, size = 116, normalized size = 0.81

$$-\frac{3b^2\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4}+\frac{b^2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}-\frac{}{2\left(\operatorname{arctanh}\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tan
h(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))-1/2/(arctanh(tanh(b*x+a))-b*x)^2/
x^2+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(x)+2/(arctanh(tanh(b*x+a))-b*x)^3
*b/x
```

Maxima [A]
time = 0.65, size = 64, normalized size = 0.45

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3bx^3+a^4x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Fricas [A]
time = 0.34, size = 86, normalized size = 0.60

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\left(b^3x^3+ab^2x^2\right)\log\left(bx+a\right)+6\left(b^3x^3+ab^2x^2\right)\log\left(x\right)}{2\left(a^4bx^3+a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**2,x)
```

[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**2), x)

Giac [A]
time = 0.38, size = 64, normalized size = 0.45

$$-\frac{3\,b^{2}\log\left(|bx+a|\right)}{a^{4}}+\frac{3\,b^{2}\log\left(|x|\right)}{a^{4}}+\frac{6\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}}{2\,(bx+a)a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

Mupad [B]
time = 3.72, size = 660, normalized size = 4.62

$$-\frac{6\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^{2}-6\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^{2}\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^{3}-2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^{3}-32b^{3}x^{3}+24bx\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^{2}+24b^{2}x^{2}\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-24b^{2}x^{2}\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+24bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^{2}-48bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+b^{2}x^{2}\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\right)96\mathrm{i}-b^{2}x^{2}\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\right)96\mathrm{i}}{x^{2}\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\right)\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^2),x)
```

```
[Out] -(6*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))^2 - 6*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*log(1/(exp(2*a)*exp(2*b*x) + 1))^3 -
2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 - 32*b^3*x^3 + 24*
b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 24*b^2*x^2*log
(1/(exp(2*a)*exp(2*b*x) + 1)) - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 + b^2*x^2*
log(1/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/
(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x))*96i - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - lo
g(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1
)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - 4
8*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)))/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)
```

3.102 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=94

$$-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{mx^{-1+m}\,{}_2F_1\left(1,-1+m;m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -1/2*x^m/b/arctanh(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arctanh(tanh(b*x+a))-1
/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arctanh(tanh(b*x+a))))/b^2/(
b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2199, 2195}

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^m/(b*ArcTanh[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcTanh[Tanh[
a + b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - Arc
Tanh[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{((1-m)m)\int \frac{\cdots}{\tan\cdots}}{2b\cdots}$$
$$= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,{}_2F_1\left(1,\cdots\right.}{2b^2\left(bx-\cdots\right.}$$

Mathematica [A]
time = 0.39, size = 51, normalized size = 0.54

$$\frac{x^{1+m}\,{}_2F_1\left(3,1+m;2+m;-\frac{bx}{-bx+\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] integral(x^m/arctanh(tanh(b*x + a))^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**3,x)
[Out] Integral(x**m/atanh(tanh(a + b*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x))^3,x)
[Out] int(x^m/atanh(tanh(a + b*x))^3, x)
```

3.103 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{3x^2}{b^3}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{6\left(bx-\tanh^{-1}\right.}{}$$

```
[Out] 3*x^2/b^3+6*x*(b*x-arctanh(tanh(b*x+a)))/b^4-1/2*x^4/b/arctanh(tanh(b*x+a))
^2-2*x^3/b^2/arctanh(tanh(b*x+a))+6*(b*x-arctanh(tanh(b*x+a)))^2*ln(arctanh
(tanh(b*x+a)))/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{3x^2}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcTanh[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcTanh[Tanh[a + b*x]]) + (6*(b*x - ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{2\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{6\int \frac{x^2}{\tanh^{-1}(\tanh(a+$$

$$= \frac{3x^2}{b^3}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}-\frac{(6(-bx+\mathrm{t}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\,\mathrm{ta}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\,\mathrm{ta}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\,\mathrm{ta}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.24

$$\frac{x^2}{2b^3}-\frac{3x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}{b^5\tanh^{-1}(\tanh(a+bx))}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}{2b^5\tanh^{-1}(\tanh(a+bx))^2}+\frac{6\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] x^2/(2*b^3) - (3*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + Ar
cTanh[Tanh[a + b*x]])^3)/(b^5*ArcTanh[Tanh[a + b*x]]) - (-(b*x) + ArcTanh[T
```

```
anh[a + b*x]])^4/(2*b^5*ArcTanh[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(90) = 180$.
time = 0.83, size = 371, normalized size = 4.03

method	result
default	$\frac{x^2}{2b^3} - \frac{3xa}{b^4} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)x}{b^4} + \frac{6\ln(\operatorname{arctanh}(\tanh(bx+a)))a^2}{b^5} + \frac{12\ln(\operatorname{arctanh}(\tanh(bx+a)))a(\operatorname{arctanh}(\tanh(b}{b^5}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b^3-3/b^4*x*a-3/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x+6/b^5*ln(arctanh
(tanh(b*x+a)))*a^2+12/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-
b*x-a)+6/b^5*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2-1/2/b^
5/arctanh(tanh(b*x+a))^2*a^4-2/b^5/arctanh(tanh(b*x+a))^2*a^3*(arctanh(tanh
(b*x+a))-b*x-a)-3/b^5/arctanh(tanh(b*x+a))^2*a^2*(arctanh(tanh(b*x+a))-b*x-
a)^2-2/b^5/arctanh(tanh(b*x+a))^2*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/2/b^5/
arctanh(tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b*x-a)^4+4/b^5/arctanh(tanh(b*
x+a))*a^3+12/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+12/b
^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+4/b^5/arctanh(tanh
(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [A]
time = 0.85, size = 81, normalized size = 0.88

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)} + \frac{6\,a^2\log\left(bx + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4)/(b^7*x^2 +
 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a)/b^5
```

Fricas [A]
time = 0.34, size = 95, normalized size = 1.03

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\,(a^2b^2x^2 + 2\,a^3bx + a^4)\log\left(bx + a\right)}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.39, size = 61, normalized size = 0.66

$$\frac{6\,a^2\log\left(|bx+a|\right)}{b^5}+\frac{b^3x^2-6\,ab^2x}{2\,b^6}+\frac{8\,a^3bx+7\,a^4}{2\,\left(bx+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

Mupad [B]
time = 1.34, size = 867, normalized size = 9.42

$$\frac{\frac{7\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+24a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+16a^4-8a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{4b}-x\left(4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3-24a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+48a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{2b^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+x\left(16ab^5-8b^5\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)+8a^2b^4+8b^6x^2-8ab^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{x^2}{2b^3}+\frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-12a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+12a^2\right)}{2b^5}+\frac{3x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^3,x)
```

```
[Out] ((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3
- 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 48*a^2*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
```

```
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(16*a*b^5 - 8*b^5*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 - 8*a*b^4*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 12*a^2))/(2*b^5) + (
3*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^4)
```

3.104 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=71

$$\frac{3x}{b^3}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}$$

```
[Out] 3*x/b^3-1/2*x^3/b/arctanh(tanh(b*x+a))^2-3/2*x^2/b^2/arctanh(tanh(b*x+a))+3
*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2199, 2189, 2188, 29}

$$\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcTanh[Tan
h[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]
])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x}{\tanh^{-1}(\tanh(a+b}}{b^2}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{(3(-bx+\text{ta}}{}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{(3(-bx+\text{ta}}{}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3(bx-\tanh}{}$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 1.21

$$-\frac{b^3x^3 + 3b^2x^2\tanh^{-1}(\tanh(a+bx)) + \tanh^{-1}(\tanh(a+bx))^3\left(5+6\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right) - bx\tanh^{-1}(\tanh(a+bx))^2\left(11+6\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2b^4\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^3
*(5 + 6*Log[ArcTanh[Tanh[a + b*x]]]) - b*x*ArcTanh[Tanh[a + b*x]]^2*(11 + 6
*Log[ArcTanh[Tanh[a + b*x]]]))/(b^4*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(67) = 134$.
time = 0.26, size = 239, normalized size = 3.37

method	result
default	$\frac{x}{b^3} - \frac{3a^2}{b^4\operatorname{arctanh}(\tanh(bx+a))} - \frac{6a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^4\operatorname{arctanh}(\tanh(bx+a))} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}{b^4\operatorname{arctanh}(\tanh(bx+a))} + \frac{a^3}{2b^4\operatorname{arctanh}(\tanh(bx+}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^3-3/b^4/arctanh(tanh(b*x+a))*a^2-6/b^4/arctanh(tanh(b*x+a))*a*(arctanh(
tanh(b*x+a))-b*x-a)-3/b^4/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
^2+1/2/b^4/arctanh(tanh(b*x+a))^2*a^3+3/2/b^4/arctanh(tanh(b*x+a))^2*a^2*(a
rctanh(tanh(b*x+a))-b*x-a)+3/2/b^4/arctanh(tanh(b*x+a))^2*a*(arctanh(tanh(b
*x+a))-b*x-a)^2+1/2/b^4/arctanh(tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b*x-a)
^3-3/b^4*ln(arctanh(tanh(b*x+a)))*a-3/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh
(tanh(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.87, size = 69, normalized size = 0.97

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}-\frac{3\,a\log{(bx+a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^
2*b^4) - 3*a*log(b*x + a)/b^4
```

Fricas [A]
time = 0.33, size = 83, normalized size = 1.17

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\,(ab^2x^2+2\,a^2bx+a^3)\log{(bx+a)}}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\operatorname{atanh}^3{(\tanh{(a+bx)})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.38, size = 44, normalized size = 0.62

$$\frac{x}{b^3}-\frac{3\,a\log{(|bx+a|)}}{b^4}-\frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

Mupad [B]
time = 1.47, size = 620, normalized size = 8.73

$$\frac{x}{b^3}-\frac{x\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-12a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+12a^2\right)-\frac{5\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{4b}}{b^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+x\left(8ab^4-4b^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)+4a^2b^3+4b^5x^2-4ab^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(3\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+6bx\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^3,x)
```

```
[Out] x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(4
*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4*b^4*(2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
 log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))/(2*b^4)
```

3.105 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=47

$$-\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

```
[Out] -1/2*x^2/b/arctanh(tanh(b*x+a))^2-x/b^2/arctanh(tanh(b*x+a))+ln(arctanh(tan
h(b*x+a)))/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^2/(b*ArcTanh[Tanh[a + b*x]]^2) - x/(b^2*ArcTanh[Tanh[a + b*x]]) + Lo
g[ArcTanh[Tanh[a + b*x]]]/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}}{b^2}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \text{t}\right.}{}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(\right.}{b^3}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.04

$$\frac{3 - \frac{b^2x^2}{\tanh^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\tanh^{-1}(\tanh(a+bx))} + 2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3 - (b^2*x^2)/ArcTanh[Tanh[a + b*x]]^2 - (2*b*x)/ArcTanh[Tanh[a + b*x]] +
2*Log[ArcTanh[Tanh[a + b*x]]])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(45) = 90$.
time = 0.11, size = 136, normalized size = 2.89

method	result
default	$\frac{2a}{b^3 \operatorname{arctanh}(\tanh(bx+a))} + \frac{2\operatorname{arctanh}(\tanh(bx+a)) - 2bx - 2a}{b^3 \operatorname{arctanh}(\tanh(bx+a))} - \frac{a^2}{2b^3 \operatorname{arctanh}(\tanh(bx+a))^2} - \frac{a(\operatorname{arctanh}(\tanh(bx+a)) - bx - a)}{b^3 \operatorname{arctanh}(\tanh(bx+a))^2} -$
risch	$-\frac{4i\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)x - \pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 x + \pi \operatorname{csgn}\left(ie^{bx+a}\right)^2 \operatorname{csgn}\left(ie^{2bx+}\right.\right.}{b^2\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right) - \pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 + \pi \operatorname{csgn}\left(ie^{bx+a}\right)^2 \operatorname{csgn}\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3/arctanh(tanh(b*x+a))*a+2/b^3/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a
))-b*x-a)-1/2/b^3/arctanh(tanh(b*x+a))^2*a^2-1/b^3/arctanh(tanh(b*x+a))^2*a
*(arctanh(tanh(b*x+a))-b*x-a)-1/2/b^3/arctanh(tanh(b*x+a))^2*(arctanh(tanh(
b*x+a))-b*x-a)^2+ln(arctanh(tanh(b*x+a)))/b^3
```

Maxima [A]
time = 0.85, size = 48, normalized size = 1.02

$$\frac{4\,abx + 3\,a^2}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)} + \frac{\log(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Fricas [A]
time = 0.38, size = 61, normalized size = 1.30

$$\frac{4\,abx + 3\,a^2 + 2\,(b^2x^2 + 2\,abx + a^2)\log(bx + a)}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A]
time = 39.21, size = 54, normalized size = 1.15

$$\begin{cases} -\frac{x^2}{2b\,\operatorname{atanh}^2(\tanh(a+bx))} - \frac{x}{b^2\operatorname{atanh}(\tanh(a+bx))} + \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3\operatorname{atanh}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x**2/(2*b*atanh(tanh(a + b*x))**2) - x/(b**2*atanh(tanh(a + b*x
))) + log(atanh(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*atanh(tanh(a))**3
), True))
```

Giac [A]
time = 0.38, size = 37, normalized size = 0.79

$$\frac{\log(|bx + a|)}{b^3} + \frac{4\,ax + \frac{3\,a^2}{b}}{2\,(bx + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 1.03, size = 46, normalized size = 0.98

$$\frac{\ln\left(\operatorname{atanh}(\tanh(a+b\,x))\right)}{b^3}-\frac{\frac{b^2\,x^2}{2}+b\,x\operatorname{atanh}(\tanh(a+b\,x))}{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^3,x)
```

```
[Out] log(atanh(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*atanh(tanh(a + b*x)))/(b
^3*atanh(tanh(a + b*x))^2)
```

3.106 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=34

$$-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -1/2*x/b/arctanh(tanh(b*x+a))^2-1/2/b^2/arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2199, 2188, 30}

$$-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x/(b*ArcTanh[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{2b^2}$$
$$= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.79

$$-\frac{bx+\tanh^{-1}(\tanh(a+bx))}{2b^2\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*(b*x + ArcTanh[Tanh[a + b*x]])/(b^2*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [A]
time = 0.08, size = 43, normalized size = 1.26

method	result
default	$-\frac{1}{b^2\,\mathrm{arctanh}(\tanh(bx+a))} - \frac{bx-\mathrm{arctanh}(\tanh(bx+a))}{2b^2\,\mathrm{arctanh}(\tanh(bx+a))^2}$
risch	$-\frac{2i\left(\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(ie^{2bx+2a}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\mathrm{csgn}\left(ie^{bx+a}\right)^2\mathrm{csgn}\left(ie^{2bx+2a}\right.\right.}{b^2\left(\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(ie^{2bx+2a}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\mathrm{csgn}\left(ie^{bx+a}\right)^2\mathrm{csgn}\left(ie^{2bx+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/arctanh(tanh(b*x+a))-1/2*(b*x-arctanh(tanh(b*x+a)))/b^2/arctanh(tanh
(b*x+a))^2
```

Maxima [A]
time = 0.84, size = 32, normalized size = 0.94

$$-\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [A]
time = 0.32, size = 32, normalized size = 0.94

$$-\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A]
time = 37.24, size = 42, normalized size = 1.24

$$\begin{cases} -\frac{x}{2b\operatorname{atanh}^2(\tanh(a+bx))} - \frac{1}{2b^2\operatorname{atanh}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x^2}{2\operatorname{atanh}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**3,x)
[Out] Piecewise((-x/(2*b*atanh(tanh(a + b*x))**2) - 1/(2*b**2*atanh(tanh(a + b*x)
)), Ne(b, 0)), (x**2/(2*atanh(tanh(a))**3), True))
```

Giac [A]
time = 0.39, size = 18, normalized size = 0.53

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.08, size = 25, normalized size = 0.74

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))+b\,x}{2\,b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^3,x)
[Out] -(atanh(tanh(a + b*x)) + b*x)/(2*b^2*atanh(tanh(a + b*x))^2)
```

3.107 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2/b/arctanh(tanh(b*x+a))^2
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/2*1/(b*ArcTanh[Tanh[a + b*x]]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3),x]

[Out] -1/2*1/(b*ArcTanh[Tanh[a + b*x]]^2)

Maple [A]
time = 0.06, size = 15, normalized size = 0.94

method	result
derivativedivides	$-\frac{1}{2b\,\mathrm{arctanh}(\tanh(bx+a))^2}$
default	$-\frac{1}{2b\,\mathrm{arctanh}(\tanh(bx+a))^2}$
risch	$\frac{}{b\left(-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(i e^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-\pi\mathrm{csgn}\left(i e^{bx+a}\right)^2 \mathrm{c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)

[Out] -1/2/b/arctanh(tanh(b*x+a))^2

Maxima [A]
time = 0.47, size = 12, normalized size = 0.75

$$-\frac{1}{2\,(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")

[Out] -1/2/((b*x + a)^2*b)

Fricas [A]
time = 0.35, size = 24, normalized size = 1.50

$$-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")

[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)

Sympy [A]
time = 25.47, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{2b\,\mathrm{atanh}^2\,(\tanh\,(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\mathrm{atanh}^3\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-1/(2*b*atanh(tanh(a + b*x))**2), Ne(b, 0)), (x/atanh(tanh(a))**
3, True))
```

Giac [A]
time = 0.39, size = 12, normalized size = 0.75

$$-\frac{1}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B]
time = 0.06, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^3,x)
```

```
[Out] -1/(2*b*atanh(tanh(a + b*x))^2)
```

3.108 $\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}+\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -1/2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+1/(b*x-arctanh(tanh(
b*x+a)))^2/arctanh(tanh(b*x+a))-ln(x)/(b*x-arctanh(tanh(b*x+a)))^3+ln(arcta
nh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}-\frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/2*1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + 1/((b*x
- ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) - Log[x]/(b*x - ArcTanh
[Tanh[a + b*x]])^3 + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*
x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x\tanh^{-1}(\tanh}}{bx-\tanh^{-1}(\tanh}$$
$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh\right.}$$
$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh\right.}$$
$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh\right.}$$
$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh\right.}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 0.76

$$\frac{b^2x^2-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2\left(3+2\log(bx)-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
[Out] (b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2*(3 + 2*L
og[b*x] - 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*
x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A]
time = 0.02, size = 92, normalized size = 0.95

$$-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3}+\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{2\,(\operatorname{arctanh}(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^3,x)
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*
x+a))-b*x)^2/arctanh(tanh(b*x+a))+1/2/(arctanh(tanh(b*x+a))-b*x)/arctanh(ta
nh(b*x+a))^2+1/(arctanh(tanh(b*x+a))-b*x)^3*ln(x)
```

Maxima [A]
time = 0.86, size = 51, normalized size = 0.53

$$\frac{2\,bx + 3\,a}{2\,(a^2b^2x^2 + 2\,a^3bx + a^4)} - \frac{\log(bx + a)}{a^3} + \frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Fricas [A]
time = 0.35, size = 80, normalized size = 0.82

$$\frac{2\,abx + 3\,a^2 - 2\,(b^2x^2 + 2\,abx + a^2)\log(bx + a) + 2\,(b^2x^2 + 2\,abx + a^2)\log(x)}{2\,(a^3b^2x^2 + 2\,a^4bx + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{atanh}^3(\tanh(a + bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**3), x)
```

Giac [A]
time = 0.39, size = 43, normalized size = 0.44

$$-\frac{\log(|bx + a|)}{a^3} + \frac{\log(|x|)}{a^3} + \frac{2\,abx + 3\,a^2}{2\,(bx + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

Mupad [B]
time = 3.66, size = 645, normalized size = 6.65

$$-\frac{12\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2-24\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+12\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2+16\,b^2\,x^2+b\,x\left(32\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-32\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)16\mathrm{i}-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)16\mathrm{i}+\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)32\mathrm{i}}{\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)^2\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^3),x)
```

```
[Out] -(12*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - 24*log(1/(exp(2*a)*exp(2*b*x) + 1
))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(l
og(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))*16i - log(1/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b
*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i + 12*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 16*b^2*x^2 + log(1/(exp(2*a)*exp(2*b*
x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*
x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i + b*x*(32*log(1/(exp(2*a)
*exp(2*b*x) + 1)) - 32*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.109 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=131

$$-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+$$

```
[Out] -3/2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/x/(b*x-arctanh
(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+3*b/(b*x-arctanh(tanh(b*x+a)))^3/arct
anh(tanh(b*x+a))-3*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4+3*b*ln(arctanh(tanh
(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

Rubi [A]

time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}-\frac{3b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{3b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b)/(2*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcT
anh[Tanh[a + b*x]])^4 + (3*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Ta
nh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{(3b)\int \frac{1}{x\tanh^{-1}(\text{ta}}}{-bx+\tanh^{-1}(\text{t}}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh\right.}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 0.71

$$-\frac{b^3x^3-6b^2x^2\tanh^{-1}(\tanh(a+bx))+2\tanh^{-1}(\tanh(a+bx))^3+3bx\tanh^{-1}(\tanh(a+bx))^2\left(1+2\log(x)-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2x\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/2*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]
^3 + 3*b*x*ArcTanh[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a +
b*x]]]))/(x*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.03, size = 117, normalized size = 0.89

$$-\frac{b}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}+\frac{3b\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}}-\frac{}{\left(\operatorname{arctanh}\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))^2+3/(arctanh(tanh(
b*x+a))-b*x)^4*b*ln(arctanh(tanh(b*x+a)))-2/(arctanh(tanh(b*x+a))-b*x)^3*b/
arctanh(tanh(b*x+a))-1/(arctanh(tanh(b*x+a))-b*x)^3/x-3/(arctanh(tanh(b*x+a
))-b*x)^4*b*ln(x)
```

Maxima [A]
time = 0.86, size = 69, normalized size = 0.53

$$-\frac{6\,b^{2}x^{2}+9\,abx+2\,a^{2}}{2\,\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^{4}}-\frac{3\,b\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A]
time = 0.37, size = 109, normalized size = 0.83

$$-\frac{6\,ab^{2}x^{2}+9\,a^{2}bx+2\,a^{3}-6\,\left(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx\right)\log\left(bx+a\right)+6\,\left(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx\right)\log\left(x\right)}{2\,\left(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**3), x)
```

Giac [A]
time = 0.39, size = 60, normalized size = 0.46

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

Mupad [B]
time = 3.67, size = 804, normalized size = 6.14

$$-\frac{24\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-24\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2-8\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3+8\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3+32\,b^3x^3+24\,bx\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2+96\,b^2x^2\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-96\,b^2x^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+24\,bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2-bx\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,bx}\right)96\mathrm{i}-bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,bx}\right)96\mathrm{i}-48\,bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,bx}\right)192\mathrm{i}}{x\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\right)^2\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,bx\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^3),x)
```

```
[Out] -(24*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - 24*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 8*log(1/(exp(2*a)*exp(2*b*x) + 1))^3
+ 8*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 + 32*b^3*x^3 + 2
4*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 96*b^2*x^2*l
og(1/(exp(2*a)*exp(2*b*x) + 1)) - 96*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - b*x*lo
g((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*
1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - b*x*log(1/(exp(2*a)*exp(2*b*x) +
 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(
1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1))
 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - 48*
b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(
exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))*192i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1))
 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)
```

3.110 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=170

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2}+\frac{2b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+$$

```
[Out] -3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^2+2*b/x/(b*x-arcta
nh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a))
)/arctanh(tanh(b*x+a))^2+6*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*
x+a))-6*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^5+6*b^2*ln(arctanh(tanh(b*x+a)
))/(b*x-arctanh(tanh(b*x+a)))^5
```

Rubi [A]

time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$\frac{6b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))}-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2}-\frac{6b^2\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5}+\frac{6b^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}+\frac{2b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTa
nh[Tanh[a + b*x]])^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{(2b)\int \frac{}{x^2 \tanh^{-1}}}{bx-\tanh^{-1}(}$$

$$= \frac{2b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{2x^2\left(bx-\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}\right.}$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 0.63

$$\frac{-b^4x^4+8b^3x^3\tanh^{-1}(\tanh(a+bx))-8bx\tanh^{-1}(\tanh(a+bx))^3+\tanh^{-1}(\tanh(a+bx))^4-12b^2x^2\tanh^{-1}(\tanh(a+bx))^2\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

[Out] (-(b^4*x^4) + 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 8*b*x*ArcTanh[Tanh[a + b*x
]]^3 + ArcTanh[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])^5*
ArcTanh[Tanh[a + b*x]]^2)

Maple [A]
time = 0.03, size = 145, normalized size = 0.85

$$-\frac{6b^2\ln(\operatorname{arctanh}(\tanh(bx+a)))}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^5}+\frac{3b^2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{2(\operatorname{arctanh}(\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/arctanh(tanh(b*x+a))^3,x)

[Out] -6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*ln(arctanh(tanh(b*x+a)))+3/(arctanh(tan
h(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))+1/2/(arctanh(tanh(b*x+a))-b*x)^3*
b^2/arctanh(tanh(b*x+a))^2-1/2/(arctanh(tanh(b*x+a))-b*x)^3/x^2+6/(arctanh(
tanh(b*x+a))-b*x)^5*b^2*ln(x)+3/(arctanh(tanh(b*x+a))-b*x)^4*b/x

Maxima [A]
time = 0.84, size = 86, normalized size = 0.51

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}-\frac{6\,b^2\log(bx+a)}{a^5}+\frac{6\,b^2\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")

[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5

Fricas [A]
time = 0.34, size = 130, normalized size = 0.76

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\log(bx+a)+12\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\log(x)}{2\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")

[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**3), x)
```

Giac [A]
time = 0.40, size = 73, normalized size = 0.43

$$-\frac{6\,b^2\log\left(|bx+a|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(bx^2+ax\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B]
time = 2.73, size = 909, normalized size = 5.35

$$\frac{4\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^4-16\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-16\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3+4\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^4+24\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2-64b^4x^4-64bx\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3-256b^3x^3\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)+256b^3x^3\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+64bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3+192bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2-192bx\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+b^2x^2\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\right)384\mathrm{i}+b^2x^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\right)384\mathrm{i}-b^2x^2\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\right)768\mathrm{i}}{x^2\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\right)^2\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (4*log(1/(exp(2*a)*exp(2*b*x) + 1))^4 - 16*log(1/(exp(2*a)*exp(2*b*x) + 1))
^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - 16*log(1/(exp(2*a
)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 +
 4*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^4 + 24*log(1/(exp(2
*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
^2 - 64*b^4*x^4 - 64*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))^3 - 256*b^3*x^3*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 256*b^3*x^3*log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 64*b*x*log(1/(exp(2*a)*exp(2*
b*x) + 1))^3 + 192*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 192*b*x*log(1/(exp(2*a)*exp(2*b*x) +
1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b^2*x^2*log(1/
(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp
(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))*384i + b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log
(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)
) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*384i - b
^2*x^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
```

```
)*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2
*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)*768i)/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)
```

3.111 $\int x^4 \sqrt{\tanh^{-1}(\tanh(a+bx))}\, dx$

Optimal. Leaf size=101

$$\frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{128x\tanh^{-1}(\text{t}}{31}$$

```
[Out] 2/3*x^4*arctanh(tanh(b*x+a))^(3/2)/b-16/15*x^3*arctanh(tanh(b*x+a))^(5/2)/b
^2+32/35*x^2*arctanh(tanh(b*x+a))^(7/2)/b^3-128/315*x*arctanh(tanh(b*x+a))^
(9/2)/b^4+256/3465*arctanh(tanh(b*x+a))^(11/2)/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{11/2}}{3465b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(5/2))/(15*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (128*x*
ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(11/2
))/(3465*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \sqrt{\tanh^{-1}(\tanh(a+bx))}\, dx = \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8 \int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}\, dx}{3b}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16 \int x$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2 \text{t}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2 \text{t}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2 \text{t}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2 \text{t}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.82

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{3/2} \left(1155b^4x^4 - 1848b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 1584b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 704bx \tanh^{-1}(\tanh(a+bx))^3 + 128 \tanh^{-1}(\tanh(a+bx))^4\right)}{3465b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(1155*b^4*x^4 - 1848*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 1584*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 704*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(3465*b^5)
```

Maple [A]
time = 0.09, size = 154, normalized size = 1.52

method	result
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11} + \frac{2(-4 \operatorname{arctanh}(\tanh(bx+a))+4bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9} + \frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2+(-2 \operatorname{arctanh}(\tanh(bx+a))+\right.}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-4*arctanh(tanh(b*x+a))+4*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh
(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(7/2)+2/5*(b*x-arctanh(tanh(b*
x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*
x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A]
time = 0.53, size = 64, normalized size = 0.63

$$\frac{2 \left(315 \, b^{5} x^{5} + 35 \, a b^{4} x^{4} - 40 \, a^{2} b^{3} x^{3} + 48 \, a^{3} b^{2} x^{2} - 64 \, a^{4} b x + 128 \, a^{5}\right) \sqrt{b x + a}}{3465 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

Fricas [A]
time = 0.32, size = 64, normalized size = 0.63

$$\frac{2 \left(315 \, b^{5} x^{5} + 35 \, a b^{4} x^{4} - 40 \, a^{2} b^{3} x^{3} + 48 \, a^{3} b^{2} x^{2} - 64 \, a^{4} b x + 128 \, a^{5}\right) \sqrt{b x + a}}{3465 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \sqrt{\operatorname{atanh}\left(\tanh\left(a + b x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.40, size = 150, normalized size = 1.49

$$\frac{\sqrt{2} \left(\frac{11 \, \sqrt{2} \left(35 \, (b x + a)^{\frac{9}{2}} - 180 \, (b x + a)^{\frac{7}{2}} a + 378 \, (b x + a)^{\frac{5}{2}} a^{2} - 420 \, (b x + a)^{\frac{3}{2}} a^{3} + 315 \, \sqrt{b x + a} \, a^{4}\right) a}{b^{4}} + \frac{5 \, \sqrt{2} \left(63 \, (b x + a)^{\frac{11}{2}} - 385 \, (b x + a)^{\frac{9}{2}} a + 990 \, (b x + a)^{\frac{7}{2}} a^{2} - 1386 \, (b x + a)^{\frac{5}{2}} a^{3} + 1155 \, (b x + a)^{\frac{3}{2}} a^{4} - 693 \, \sqrt{b x + a} \, a^{5}\right)}{b^{4}}\right)}{3465 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(11*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 37
8*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/
b^4 + 5*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
```

```
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)/b^4)/b
```

Mupad [B]
time = 1.05, size = 811, normalized size = 8.03

$$\frac{2x^5\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{11}-\frac{x^4\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}+\frac{2bx}{11}\right)}{9b}-\frac{128\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)^4\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}+\frac{2bx}{11}\right)}{315b^5}-\frac{8x^3\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}+\frac{2bx}{11}\right)}{63b^2}-\frac{64x\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)^3\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}+\frac{2bx}{11}\right)}{315b^4}-\frac{16x^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)^2\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}+\frac{2bx}{11}\right)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 - (x^4*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/11 + (2*b*x)/11))/(9*b) - (128*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x)^4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(315*b^5)
- (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(
2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/11 + (2*b*x)/11))/(63*b^2) - (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(315*b^4) - (16
*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/11 + (2*b*x)/11))/(105*b^3)
```

3.112 $\int x^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=80

$$\frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{32\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}$$

```
[Out] 2/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b-4/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b^2
+16/35*x*arctanh(tanh(b*x+a))^(7/2)/b^3-32/315*arctanh(tanh(b*x+a))^(9/2)/b
^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2}+\frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(5/2))/(5*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (32*ArcTanh
[Tanh[a + b*x]]^(9/2))/(315*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}\, dx = \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\, dx}{b}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{8\int x \operatorname{ta}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x \operatorname{tan}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x \operatorname{tan}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x \operatorname{tan}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.82

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{3/2} \left(105b^3x^3 - 126b^2x^2 \tanh^{-1}(\tanh(a+bx)) + 72bx \tanh^{-1}(\tanh(a+bx))^2 - 16 \tanh^{-1}(\tanh(a+bx))^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 126*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 72*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(3
15*b^4)
```

Maple [A]
time = 0.08, size = 124, normalized size = 1.55

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9} + \frac{2(-3 \operatorname{arctanh}(\tanh(bx+a))+3bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7} + \frac{2\left((bx-\operatorname{arctanh}(\tanh(bx+a)))(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx)\right.}{5}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(5/2)+1/3
*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A]
time = 0.52, size = 53, normalized size = 0.66

$$\frac{2\,(35\,b^4x^4 + 5\,ab^3x^3 - 6\,a^2b^2x^2 + 8\,a^3bx - 16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Fricas [A]
time = 0.34, size = 53, normalized size = 0.66

$$\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 125, normalized size = 1.56

$$\frac{\sqrt{2}\left(\frac{9\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a}{b^3}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)}{b^3}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(9*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*
x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^3 + sqrt(2)*(35*(b*x + a)^(9/2
) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a
^3 + 315*sqrt(b*x + a)*a^4)/b^3)/b
```

Mupad [B]
time = 0.99, size = 648, normalized size = 8.10

$$\frac{2\,x^4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{9}-\frac{x^3\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{7\,b}-\frac{16\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^3\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{35\,b^4}-\frac{6\,x^2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{35\,b^2}-\frac{8\,x\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^2\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{35\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 - (x^3*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/9 + (2*b*x)/9))/(7*b) - (16*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^4) - (6*x^2*
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 +
 (2*b*x)/9))/(35*b^2) - (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^3)
```

3.113 $\int x^2 \sqrt{\tanh^{-1}(\tanh(a+bx))}\, dx$

Optimal. Leaf size=59

$$\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}$$

```
[Out] 2/3*x^2*arctanh(tanh(b*x+a))^(3/2)/b-8/15*x*arctanh(tanh(b*x+a))^(5/2)/b^2+
16/105*arctanh(tanh(b*x+a))^(7/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(5
/2))/(15*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(7/2))/(105*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{8\int \tanh$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{8\text{Subst}(}{}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16\tanh$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(35b^2x^2-28bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(105*b^3)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.17

method	result	siz
default	$\frac{\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(-2\,\text{arctanh}(\tanh(bx+a))+2bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\text{arctanh}(\tanh(bx+a)))^2\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
+a))^(3/2))
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Fricas [A]
time = 0.34, size = 42, normalized size = 0.71

$$\frac{2\,(15\,b^3x^3 + 3\,ab^2x^2 - 4\,a^2bx + 8\,a^3)\,\sqrt{bx + a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\sqrt{\operatorname{atanh}\,(\tanh\,(a + bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(47) = 94.
time = 0.39, size = 102, normalized size = 1.73

$$\frac{\sqrt{2}\left(\frac{7\,\sqrt{2}\,\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\,\sqrt{2}\,\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*a/b^2 + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

Mupad [B]
time = 0.99, size = 485, normalized size = 8.22

$$\frac{2\,x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{7}-\frac{x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}+\frac{2\,b\,x}{7}\right)}{5\,b}-\frac{8\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^2\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}+\frac{2\,b\,x}{7}\right)}{15\,b^3}-\frac{4\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}+\frac{2\,b\,x}{7}\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(1/2),x)

[Out] (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 - (x^2*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/7 + (2*b*x)/7))/(5*b) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 + (2*b*x)/7))/(15*b^3) - (4*x*(lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(2*a)*exp(2*b
*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 + (2
*b*x)/7))/(15*b^2)
```

3.114 $\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

```
[Out] 2/3*x*arctanh(tanh(b*x+a))^(3/2)/b-4/15*arctanh(tanh(b*x+a))^(5/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2199, 2188, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))
/(15*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\text{Subst}\left(\int x^{3/2}\,dx, x, \tanh^{-1}(\tanh(a+bx\right.}{3b^2}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.84

$$\frac{2\left(5bx - 2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*(5*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*b^2
)
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.11

method	result	size
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\mathrm{arctanh}(\tanh(bx+a)))\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(3/2))
```

Maxima [A]
time = 0.52, size = 30, normalized size = 0.79

$$\frac{2\left(3\,b^2x^2 + abx - 2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.79

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(1/2),x)
[Out] Integral(x*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(30) = 60.
time = 0.39, size = 75, normalized size = 1.97

$$\frac{\sqrt{2}\left(\frac{5\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a}{b}+\frac{\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/15*sqrt(2)*(5*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + sqrt(2)
*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

Mupad [B]
time = 1.04, size = 151, normalized size = 3.97

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+5\,b\,x\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^(1/2),x)
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 5*b*x))/(15*
b^2)
```

3.115 $\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}$	15
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

Maxima [A]
time = 0.54, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A]
time = 0.35, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A]
time = 0.14, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{3b} & \text{for } b \neq 0 \\ x\sqrt{\mathrm{atanh}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(atanh(ta
nh(a))), True))
```

Giac [A]
time = 0.41, size = 18, normalized size = 1.00

$$\frac{\sqrt{2}\,(2\,bx+2\,a)^{\frac{3}{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(2*b*x + 2*a)^(3/2)/b
```

Mupad [B]
time = 1.12, size = 95, normalized size = 5.28

$$-\frac{\left(\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right)\sqrt{\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] -((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b)
```

3.116 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} \, dx$

Optimal. Leaf size=63

$$-2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+2\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x
-arctanh(tanh(b*x+a)))^(1/2)+2*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2192}

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\,\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx = 2\sqrt{\tanh^{-1}(\tanh(a+bx))} - (bx - \tanh^{-1}(\tanh(a+bx))) \int \frac{}{x\sqrt{\tanh^{-1}}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx$$

Mathematica [**A**]
time = 0.06, size = 61, normalized size = 0.97

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))} - 2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] 2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqr
t[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [**A**]
time = 0.06, size = 54, normalized size = 0.86

method	result
default	$2\sqrt{\operatorname{arctanh}(\tanh(bx+a))} - 2\sqrt{\operatorname{arctanh}(\tanh(bx+a)) - bx}\ \operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(}}{\sqrt{\operatorname{arctanh}(\tanh(bx}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)-2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arc
tanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="maxima")
```

[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x, x)

Fricas [A]
time = 0.34, size = 73, normalized size = 1.16

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x, x)
```

Giac [A]
time = 0.38, size = 40, normalized size = 0.63

$$\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{2}\sqrt{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2)*sqrt(b
*x + a))
```

Mupad [B]
time = 2.36, size = 308, normalized size = 4.89

$$2\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}+\ln\left(-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\sqrt{\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-b\,x}+b\,x}{x\,\sqrt{\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-b\,x}}\right)\,\sqrt{\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x,x)
```

```
[Out] 2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2) + log(-(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log
((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*(log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2 - b*x)^(1/2) + b*x)/(x*(log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2 - b*x)^(1/2)
))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2 - b*x)^(1/2)
```

3.117 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

```
[Out] b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(1/2)-arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 2192}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.98

$$-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] -(Sqrt[ArcTanh[Tanh[a + b*x]]]/x) - (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]
/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]
```

Maple [A]
time = 0.07, size = 63, normalized size = 0.95

method	result	size
default	$2b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2bx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-1/2/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^2, x)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.41

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2\,ax}, \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**2,x)
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**2, x)
```

Giac [A]
time = 0.40, size = 51, normalized size = 0.77

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\,\sqrt{bx+a}\,b}{x}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2*sqrt(2)*(sqrt(2)*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2)*sqrt(b*x + a)*b/x)/b

Mupad [B]
time = 6.97, size = 341, normalized size = 5.17

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}}{x}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}\,2{}\mathrm{i}\right)\,1{}\mathrm{i}}{x}\right)\,1{}\mathrm{i}}{2\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(1/2)/x^2,x)

[Out] (2^(1/2)*b*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*1i)/x)*1i)/(2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)/x

3.118 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3} dx$

Optimal. Leaf size=125

$$\frac{b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/4*b/x/arctanh(tanh(b*x+a))^(1/2)+1/4*b^
2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-1/2*arctanh(tanh(b*
x+a))^(1/2)/x^2
```

Rubi [A]

time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2}-\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b/(4*x*Sqrt[ArcTanh[Tanh[a +
b*x]]]) + b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
]) - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2} + \frac{1}{4}b\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2} - \frac{1}{8}b^2\int \frac{}{x\,\mathrm{ta}}$$

$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-}}}$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx}}$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 0.71

$$\frac{1}{4}\left(\frac{\left(-2+\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (((-2 + (b*x)/(b*x - ArcTanh[Tanh[a + b*x]]))*Sqrt[ArcTanh[Tanh[a + b*x]]])
/x^2 + (b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/4
```

Maple [A]
time = 0.07, size = 92, normalized size = 0.74

method	result
default	$2b^{2}\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8}}{b^{2}x^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{8(\operatorname{arctanh}(\tanh(bx+a))-bx)^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((-1/8/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(3/2)-1/8*arct
anh(tanh(b*x+a))^(1/2))/b^2/x^2+1/8/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctan
h(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^3, x)
```

Fricas [A]
time = 0.34, size = 119, normalized size = 0.95

$$\left[\frac{\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(abx+2\,a^{2})\sqrt{bx+a}}{8\,a^{2}x^{2}},-\frac{\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(abx+2\,a^{2})\sqrt{bx+a}}{4\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**3, x)
```

Giac [A]
time = 0.39, size = 75, normalized size = 0.60

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\, b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a}+\frac{\sqrt{2}\left((bx+a)^{\frac{3}{2}} b^3+\sqrt{bx+a}\, a b^3\right)}{a b^2 x^2}\right)}{8\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqr
t(2)*((b*x + a)^(3/2)*b^3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 5.74, size = 741, normalized size = 5.93

$$\frac{b\sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}}}{2x\left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+2bx\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+2bx\right)}{x^2\left(2\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-2\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+4bx\right)}+\frac{\sqrt{2}\, b^2 \ln\left(\frac{\left(2\sqrt{2}\, a-\sqrt{2}\left(2a-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)+2bx\right)+\sqrt{2}\, bx+\sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+2bx}\; 2i\right)\left(\left(2a-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)+2bx\right)\right)4i}{x\sqrt{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+2bx}}\right)1i}{4\left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)-\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)+2bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^3,x)
```

```
[Out] (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x
) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(x^2*(2*log(2/(exp(2*a)*exp(2*b*
x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x
)) + (2^(1/2)*b^2*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*
b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*4i)/(x*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^(1/2)))*1i)/(4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))
```

3.119 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4}\,dx$

Optimal. Leaf size=179

$$\frac{b^3\operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

```
[Out] 1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/24*b^2/x/arctanh(tanh(b*x+a))^(3/2)-1/2
4*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/12*b/x^2/arct
anh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*
x+a))^(1/2)-1/3*arctanh(tanh(b*x+a))^(1/2)/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{b^3\operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3}-\frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^2/(24*x*ArcTanh[Tanh[a + b*
x]]^(3/2)) - b^3/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(3/2)) - b/(12*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + b^3/(8*(b*x - ArcTanh[Ta
nh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - Sqrt[ArcTanh[Tanh[a + b*x]]
]/(3*x^3)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} + \frac{1}{6}b\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= -\frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} - \frac{1}{24}b^2\int$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh}}{}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx)}$$

Mathematica [A]

time = 0.07, size = 115, normalized size = 0.64

$$\frac{1}{24}\left(-\frac{3b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-3b^2x^2+14bx\tanh^{-1}(\tanh(a+bx))-8\tanh^{-1}(\tanh(a+bx))^2\right)}{x^3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] ((-3*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (Sqrt[ArcTanh[Tanh[a +
 b*x]]]*(-3*b^2*x^2 + 14*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*
x]]^2))/(x^3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/24
```

Maple [A]
time = 0.07, size = 185, normalized size = 1.03

method	result
default	$2b^3\left(\frac{\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16a^2+32a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+16(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx}}{16}}{b^3x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*((1/16/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*
x-a)^2)*arctanh(tanh(b*x+a))^(5/2)-1/6/(arctanh(tanh(b*x+a))-b*x)*arctanh(t
anh(b*x+a))^(3/2)-1/16*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-1/16/(a^2+2*a*(a
rctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*
x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b
*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^4, x)
```

Fricas [A]
time = 0.36, size = 145, normalized size = 0.81

$$\left[\frac{3 \sqrt{a} b^{3} x^{3} \log\left(\frac{b x - 2 \sqrt{b x + a} \sqrt{a} + 2 a}{x}\right) + 2 \left(3 a b^{2} x^{2} - 2 a^{2} b x - 8 a^{3}\right) \sqrt{b x + a}}{48 a^{3} x^{3}}, \frac{3 \sqrt{-a} b^{3} x^{3} \arctan\left(\frac{\sqrt{b x + a} \sqrt{-a}}{a}\right) + \left(3 a b^{2} x^{2} - 2 a^{2} b x - 8 a^{3}\right) \sqrt{b x + a}}{24 a^{3} x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}{\left(\tanh{\left(a + b x \right)} \right)}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**4, x)
```

Giac [A]
time = 0.39, size = 93, normalized size = 0.52

$$\frac{\sqrt{2} \left(\frac{3 \sqrt{2} b^{4} \arctan\left(\frac{\sqrt{b x + a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{2}} + \frac{\sqrt{2} \left(3 (b x + a)^{\frac{5}{2}} b^{4} - 8 (b x + a)^{\frac{3}{2}} a b^{4} - 3 \sqrt{b x + a} a^{2} b^{4}\right)}{a^{2} b^{3} x^{3}}\right)}{48 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) +
 sqrt(2)*(3*(b*x + a)^(5/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)
*a^2*b^4)/(a^2*b^3*x^3))/b
```

Mupad [B]
time = 5.51, size = 964, normalized size = 5.39

$$\frac{b \sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}}}{3 x^2 \left(2 \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - 2 \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 4 b x\right)} + \frac{b^2 \sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}}}{2 x \left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^2} - \frac{\sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}} \left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)}{x^3 \left(3 \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - 3 \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 6 b x\right)} + \frac{\sqrt{2} b^3 \ln\left(\frac{\left(\sqrt{2} b x - \sqrt{2} \left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x\right) + \sqrt{\frac{\ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right)}{2}} \sqrt{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x}\, 2 i\right) \left(\left(2a - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^5 + 40 a^2 \left(2a - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^3 - 80 a^3 \left(2a - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^2 - 32 a^5 - 10 a \left(2a - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^4 + 80 a^4 \left(2a - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + \ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)\right) 4 i}{x \sqrt{\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x}}\right) 1 i}{4 \left(\ln\left(\frac{2}{e^{2a} e^{2bx}+1}\right) - \ln\left(\frac{2 e^{2a} e^{2bx}}{e^{2a} e^{2bx}+1}\right) + 2 b x\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(1/2)/x^4,x)

[Out] (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*4i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))

3.120 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2} \, dx$

Optimal. Leaf size=101

$$\frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3} - \frac{128x \tanh^{-1}$$

```
[Out] 2/5*x^4*arctanh(tanh(b*x+a))^(5/2)/b-16/35*x^3*arctanh(tanh(b*x+a))^(7/2)/b
^2+32/105*x^2*arctanh(tanh(b*x+a))^(9/2)/b^3-128/1155*x*arctanh(tanh(b*x+a)
)^(11/2)/b^4+256/15015*arctanh(tanh(b*x+a))^(13/2)/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{256 \tanh^{-1}(\tanh(a+bx))^{13/2}}{15015b^5} - \frac{128x \tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4} + \frac{32x^2 \tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (128*x
*ArcTanh[Tanh[a + b*x]]^(11/2))/(1155*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(1
3/2))/(15015*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{48\int x^2 \cdots}{\cdots}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tan\cdots}{\cdots}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tan\cdots}{\cdots}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tan\cdots}{\cdots}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tan\cdots}{\cdots}$$

Mathematica [A]

time = 0.02, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(3003b^4x^4 - 3432b^3x^3\tanh^{-1}(\tanh(a+bx)) + 2288b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 832bx\tanh^{-1}(\tanh(a+bx))^3 + 128\tanh^{-1}(\tanh(a+bx))^4\right)}{15015b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(3003*b^4*x^4 - 3432*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 2288*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 832*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(15015*b^5)
```

Maple [A]

time = 0.08, size = 154, normalized size = 1.52

method	result
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13} + \frac{2(-4\,\mathrm{arctanh}(\tanh(bx+a))+4bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11} + \frac{2\left(2(bx-\mathrm{arctanh}(\tanh(bx+a)))^2+(-2\,\mathrm{arctanh}(\tanh(bx+a))+2\cdots\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arcta
nh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(9/2)+2/7*(b*x-arctanh(tanh(
b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(7/2)+1/5*(
b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(5/2))
```

Maxima [A]
time = 0.52, size = 64, normalized size = 0.63

$$\frac{2\,(1155\,b^5x^5+315\,ab^4x^4-280\,a^2b^3x^3+240\,a^3b^2x^2-192\,a^4bx+128\,a^5)(bx+a)^{\frac{3}{2}}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^5*x^5 + 315*a*b^4*x^4 - 280*a^2*b^3*x^3 + 240*a^3*b^2*x^2 -
 192*a^4*b*x + 128*a^5)*(b*x + a)^(3/2)/b^5
```

Fricas [A]
time = 0.33, size = 75, normalized size = 0.74

$$\frac{2\,(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**4*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.
time = 0.39, size = 241, normalized size = 2.39

$$\frac{\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^2}{b^4}+\frac{130\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a}{b^4}+\frac{15\sqrt{2}\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)}{b^4}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
```

```
a^2/b^4 + 130*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b
*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 6
93*sqrt(b*x + a)*a^5)*a/b^4 + 15*sqrt(2)*(231*(b*x + a)^(13/2) - 1638*(b*x
+ a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*
(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^
4)/b
```

Mupad [B]
time = 1.19, size = 1813, normalized size = 17.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^5*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
(24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(11*b
) + (x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (10*((2
4*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(
exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 + b*x))/(11*b)))/(9*b) + (128*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*
a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))/2 + b*x)^4)/(315*b^5) + (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(
```

```
2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
 b*x))/(63*b^2) + (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(3
15*b^4) + (16*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2
+ (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(105*b^3
)
```

3.121 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2} \, dx$

Optimal. Leaf size=80

$$\frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x \tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3} - \frac{32 \tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}$$

```
[Out] 2/5*x^3*arctanh(tanh(b*x+a))^(5/2)/b-12/35*x^2*arctanh(tanh(b*x+a))^(7/2)/b
^2+16/105*x*arctanh(tanh(b*x+a))^(9/2)/b^3-32/1155*arctanh(tanh(b*x+a))^(11
/2)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$-\frac{32 \tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4} + \frac{16x \tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (12*x^2*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(11/2))/(1155*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{6\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{24\int x$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x \text{ ta}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x \text{ ta}$$
$$= \frac{2x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x \text{ ta}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(231b^3x^3 - 198b^2x^2\tanh^{-1}(\tanh(a+bx)) + 88bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a+bx))^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 198*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 88*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(1
155*b^4)
```

Maple [A]
time = 0.08, size = 124, normalized size = 1.55

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-3\,\mathrm{arctanh}(\tanh(bx+a))+3bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2\left((bx-\mathrm{arctanh}(\tanh(bx+a)))(-2\,\mathrm{arctanh}(\tanh(bx+a))+2bx\right.}{7}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-3*arctanh(tanh(b*x+a))+3*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tan
h(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(7/2)+1
/5*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(5/2))
```

Maxima [A]
time = 0.53, size = 53, normalized size = 0.66

$$\frac{2\,(105\,b^4x^4 + 35\,ab^3x^3 - 30\,a^2b^2x^2 + 24\,a^3bx - 16\,a^4)(bx+a)^{\frac{3}{2}}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^4*x^4 + 35*a*b^3*x^3 - 30*a^2*b^2*x^2 + 24*a^3*b*x - 16*a^4)*
(b*x + a)^(3/2)/b^4
```

Fricas [A]
time = 0.33, size = 64, normalized size = 0.80

$$\frac{2\left(105\,b^{5}x^{5}+140\,ab^{4}x^{4}+5\,a^{2}b^{3}x^{3}-6\,a^{3}b^{2}x^{2}+8\,a^{4}bx-16\,a^{5}\right)\sqrt{bx+a}}{1155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(64) = 128.
time = 0.39, size = 205, normalized size = 2.56

$$\frac{\sqrt{2}\left(\frac{99\sqrt{2}\left(5\left(bx+a\right)^{\frac{7}{2}}-21\left(bx+a\right)^{\frac{5}{2}}a+35\left(bx+a\right)^{\frac{3}{2}}a^{2}-35\sqrt{bx+a}\,a^{3}\right)a^{2}}{b^{3}}+\frac{22\sqrt{2}\left(35\left(bx+a\right)^{\frac{9}{2}}-180\left(bx+a\right)^{\frac{7}{2}}a+378\left(bx+a\right)^{\frac{5}{2}}a^{2}-420\left(bx+a\right)^{\frac{3}{2}}a^{3}+315\sqrt{bx+a}\,a^{4}\right)a}{b^{3}}+\frac{5\sqrt{2}\left(63\left(bx+a\right)^{\frac{11}{2}}-385\left(bx+a\right)^{\frac{9}{2}}a+990\left(bx+a\right)^{\frac{7}{2}}a^{2}-1386\left(bx+a\right)^{\frac{5}{2}}a^{3}+1155\left(bx+a\right)^{\frac{3}{2}}a^{4}-693\sqrt{bx+a}\,a^{5}\right)}{b^{3}}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(99*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(
b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*sqrt(2)*(35*(b*x +
a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^
(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^3 + 5*sqrt(2)*(63*(b*x + a)^(11/2) -
 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^3)/b
```

Mupad [B]
time = 1.12, size = 1483, normalized size = 18.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^4*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
(20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*b)
 + (x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (8*((20*
b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))/2 + b*x))/(9*b)))/(7*b) + (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2/2 + (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp
(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
+ b*x)^3)/(35*b^4) + (6*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2/2 + (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) +
1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3
5*b^2) + (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (8
*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log
```

```
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

3.122 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}$$

```
[Out] 2/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b-8/35*x*arctanh(tanh(b*x+a))^(7/2)/b^2+
16/315*arctanh(tanh(b*x+a))^(9/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(7
/2))/(35*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{8\int \tanh^{-}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{8\text{Subst}\big(\int$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16\tanh^{-1}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(63b^2x^2 - 36bx\tanh^{-1}(\tanh(a+bx)) + 8\tanh^{-1}(\tanh(a+bx))^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(315*b^3)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.17

method	result	size
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-2\,\mathrm{arctanh}(\tanh(bx+a))+2bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\mathrm{arctanh}(\tanh(bx+a)))^2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
+a))^(5/2))
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\,(35\,b^3x^3 + 15\,ab^2x^2 - 12\,a^2bx + 8\,a^3)(bx+a)^{\frac{3}{2}}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^3*x^3 + 15*a*b^2*x^2 - 12*a^2*b*x + 8*a^3)*(b*x + a)^(3/2)/b^3
```

Fricas [**A**]
time = 0.34, size = 53, normalized size = 0.90

$$\frac{2\left(35\,b^{4}x^{4}+50\,ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}-4\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{315\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**2*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(47) = 94.
time = 0.40, size = 168, normalized size = 2.85

$$\frac{\sqrt{2}\left(\frac{21\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^{2}\right)a^{2}}{b^{2}}+\frac{18\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\sqrt{bx+a}\,a^{3}\right)a}{b^{2}}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\sqrt{bx+a}\,a^{4}\right)}{b^{2}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(21*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*a^2/b^2 + 18*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + sqrt(2)*(35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^2)/b
```

Mupad [**B**]
time = 1.12, size = 1153, normalized size = 19.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 + (x^3*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((
16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(7*b) +
 (x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (6*((16*b*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2
*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 + b*x))/(7*b)))/(5*b) + (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^2/2 + (6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
)^2)/(15*b^3) + (4*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
/2 + (6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(15*b^2)
```

3.123 $\int x \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

```
[Out] 2/5*x*arctanh(tanh(b*x+a))^(5/2)/b-4/35*arctanh(tanh(b*x+a))^(7/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2199, 2188, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))
/(35*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{2\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{2\text{Subst}\left(\int x^{5/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{5b^2}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.84

$$\frac{2\left(7bx - 2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
[Out] (2*(7*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*b^2
)
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.11

method	result	size
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\mathrm{arctanh}(\tanh(bx+a)))\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(5/2))
```

Maxima [A]
time = 0.54, size = 31, normalized size = 0.82

$$\frac{2\,(5\,b^2x^2 + 3\,abx - 2\,a^2)(bx+a)^{\frac{3}{2}}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/35*(5*b^2*x^2 + 3*a*b*x - 2*a^2)*(b*x + a)^(3/2)/b^2
```

Fricas [A]
time = 0.34, size = 41, normalized size = 1.08

$$\frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [A]
time = 2.44, size = 49, normalized size = 1.29

$$\begin{cases} \frac{2x\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{5b} - \frac{4\operatorname{atanh}^{\frac{7}{2}}(\tanh(a+bx))}{35b^2} & \text{for } b \neq 0 \\ \frac{x^2\operatorname{atanh}^{\frac{3}{2}}(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*x*atanh(tanh(a + b*x))**(5/2)/(5*b) - 4*atanh(tanh(a + b*x))**
(7/2)/(35*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(3/2)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.
time = 0.40, size = 131, normalized size = 3.45

$$\frac{\sqrt{2}\left(\frac{35\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^2}{b}+\frac{14\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a}{b}+\frac{3\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(35*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*
sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a
/b + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)/b)/b
```

Mupad [B]
time = 1.08, size = 823, normalized size = 21.66

$$\frac{2bx^3\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{7}+\frac{x^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{12b\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{7}-2b\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{5b}+\frac{x\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2}+\frac{4\left(\frac{12b\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{7}-2b\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5b}\right)}{3b}+\frac{2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2}+\frac{4\left(\frac{12b\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{7}-2b\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5b}\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 + (x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((
12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(5*b) +
 (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (4*((12*b*(l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a
)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 + b*x))/(5*b)))/(3*b) + (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^2/2 + (4*((12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b))*(log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))
/(3*b^2)
```

3.124 $\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2),x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5b}$	15
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

Maxima [A]
time = 0.52, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b
```

Fricas [A]
time = 0.33, size = 28, normalized size = 1.56

$$\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A]
time = 1.19, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{5b} & \text{for } b \neq 0 \\ x \operatorname{atanh}^{\frac{3}{2}}(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(5/2)/(5*b), Ne(b, 0)), (x*atanh(tanh(a)
)**(3/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(14) = 28$.
time = 0.41, size = 84, normalized size = 4.67

$$\frac{\sqrt{2}\left(15\sqrt{2}\sqrt{bx+a}\,a^2+10\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a+\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*sqrt(b*x + a)*a^2 + 10*sqrt(2)*((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*a + sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a +
15*sqrt(b*x + a)*a^2))/b
```

Mupad [B]
time = 1.16, size = 97, normalized size = 5.39

$$\frac{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(10*b)
```

3.125 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx$

Optimal. Leaf size=91

$$2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}-2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-
arctanh(tanh(b*x+a)))^(3/2)+2/3*arctanh(tanh(b*x+a))^(3/2)-2*(b*x-arctanh(t
anh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2192}

$$2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] 2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(
b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt
[ArcTanh[Tanh[a + b*x]]] + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx = \frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2} - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{\tanh^{-1}(t}}{}$$

$$= -2\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{2}{3}\tanh^{-1}(\mathrm{ta}$$

$$= 2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx - \tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^{3}$$

Mathematica [**A**]
time = 0.05, size = 80, normalized size = 0.88

$$-\frac{2}{3}\left(3bx\sqrt{\tanh^{-1}(\tanh(a+bx))} - 4\tanh^{-1}(\tanh(a+bx))^{3/2} + 3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] - 4*ArcTanh[Tanh[a + b*x]]^(3/2) +
3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)))/3
```

Maple [**A**]
time = 0.06, size = 131, normalized size = 1.44

method	result
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} + 2\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}\;a + 2(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right) - bx - a)\sqrt{\mathrm{arc}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)+2*arctanh(tanh(b*x+a))^(1/2)*a+2*(arctanh(ta
nh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-2*(a^2+2*a*(arctanh(tanh(b*x+a
))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x, x)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 0.97

$$\left[a^{\frac{3}{2}} \log\left(\frac{bx - 2\sqrt{bx+a}\,\sqrt{a}\, + 2a}{x}\right) + \frac{2}{3}\,(bx+4a)\sqrt{bx+a}\,, 2\sqrt{-a}\,a \arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right) + \frac{2}{3}\,(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x, x)
```

Giac [A]
time = 0.39, size = 57, normalized size = 0.63

$$\frac{1}{3}\,\sqrt{2}\left(\frac{3\,\sqrt{2}\,a^{2} \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{2}\,(bx+a)^{\frac{3}{2}} + 3\,\sqrt{2}\,\sqrt{bx+a}\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2
)*(b*x + a)^(3/2) + 3*sqrt(2)*sqrt(b*x + a)*a)
```

Mupad [B]
time = 6.00, size = 501, normalized size = 5.51

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4\,b\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3}-2\,b\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)}{b}+\frac{2\,b\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,2{}\mathrm{i}\right)\,4{}\mathrm{i}\right)}{x\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x,x)

[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((4*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/3 - 2*b*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)))/b + (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x
)*4i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)*1i
)/4 + (2*b*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3
```

3.126 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} dx$

Optimal. Leaf size=81

$$-3b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-$$

```
[Out] -arctanh(tanh(b*x+a))^(3/2)/x-3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-ar
ctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)+3*b*arctanh(tan
h(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2192}

$$-3b\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\,\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] -3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - A
rcTanh[Tanh[a + b*x]]^(3/2)/x
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
```

```
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} + \frac{1}{2}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx$$
$$= 3b\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} - \frac{1}{2}(3b(bx - \tan$$
$$= -3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+b$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.98

$$3b\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} - 3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - ArcTanh[Tanh[a + b*x]]^(3/2)/x - 3*b*Arc
Tanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sq
rt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [A]
time = 0.07, size = 85, normalized size = 1.05

method	result
default	$2b\left(\sqrt{\operatorname{arctanh}(\tanh(bx+a))} + \frac{\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2}+\frac{bx}{2}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{bx} - \frac{3\sqrt{\operatorname{arctan}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(arctanh(tanh(b*x+a))^(1/2)+(-1/2*arctanh(tanh(b*x+a))+1/2*b*x)*arctanh
(tanh(b*x+a))^(1/2)/b/x-3/2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctan
h(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^2, x)
```

Fricas [A]
time = 0.34, size = 102, normalized size = 1.26

$$\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(2\,bx-a)\sqrt{bx+a}}{2\,x},\ \frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(2\,bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**2, x)
```

Giac [A]
time = 0.39, size = 69, normalized size = 0.85

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{2}\,\sqrt{bx+a}\,b^2-\frac{\sqrt{2}\,\sqrt{bx+a}\,ab}{x}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="giac")

[Out] 1/2*sqrt(2)*(3*sqrt(2)*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(2)*sqrt(b*x + a)*b^2 - sqrt(2)*sqrt(b*x + a)*a*b/x)/b

Mupad [B]
time = 2.22, size = 459, normalized size = 5.67

$$3b\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}+\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{2x}-\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{2x}+b\ln\left(-\frac{4\sqrt{2}\,\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-4\sqrt{2}\,\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+8\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-2bx}+4\sqrt{2}\,bx}{x\sqrt{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-2bx}}\right)\sqrt{\frac{9\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{8}-\frac{9\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{8}-\frac{9bx}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(3/2)/x^2,x)

[Out] 3*b*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x) - (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x) + b*log(-(4*2^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 4*2^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 8*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) + 4*2^(1/2)*b*x)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((9*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/8 - (9*log(1/(exp(2*a)*exp(2*b*x) + 1)))/8 - (9*b*x)/4)^(1/2)

3.127 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\, dx$

Optimal. Leaf size=92

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}$$

```
[Out] -1/2*arctanh(tanh(b*x+a))^(3/2)/x^2+3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh(b*x+a)))^(1/2)-3/4*b
*arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A]

time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2199, 2192}

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (3*b*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(4*x) - ArcTanh[Tanh[a + b*x]]^(3/2)/(2*x^2)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{4}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$$

$$= -\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{8}(3b^2)\int \frac{}{x\sqrt{}}$$

$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx}}{4x}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] ((-3*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/x
^2 - (3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/4
```

Maple [A]
time = 0.07, size = 91, normalized size = 0.99

method	result
default	$2b^2\left(\frac{-\frac{5\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(\frac{3\,\mathrm{arctanh}(\tanh(bx+a))}{8}-\frac{3bx}{8}\right)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^2x^2} - \frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctan}}}{\sqrt{\mathrm{arctanh}\,(}}\right)}{8\sqrt{\mathrm{arctanh}\,(\mathrm{tan}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((-5/8*arctanh(tanh(b*x+a))^(3/2)+(3/8*arctanh(tanh(b*x+a))-3/8*b*x)*
arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-3/8/(arctanh(tanh(b*x+a))-b*x)^(1/2)*ar
ctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.36, size = 124, normalized size = 1.35

$$\left[\frac{3\,\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(5\,abx+2\,a^2)\sqrt{bx+a}}{8\,ax^2},\frac{3\,\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(5\,abx+2\,a^2)\sqrt{bx+a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**3, x)
```

Giac [A]
time = 0.40, size = 73, normalized size = 0.79

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\left(5\,(bx+a)^{\frac{3}{2}}b^3-3\sqrt{bx+a}\,ab^3\right)}{b^2x^2}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2
)*(5*(b*x + a)^(3/2)*b^3 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 6.03, size = 609, normalized size = 6.62

$$\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2x^2\left(2\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+4bx\right)}-\frac{b\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{5\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{4}-\frac{5\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{4}+\frac{5bx}{2}\right)}{x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\sqrt{2}\,b^2\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\,bx-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2i\right)16i}{x}\right)3i}{8\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^3,x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^2*(2*log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 4*b*x)) + (2^(1/2)*b^2*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(
2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/x)*3i)/(8*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2
)) - (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((5*log(2/(exp(2*a)*exp(2*b*x) + 1)))/4
- (5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/4 + (5*b*x)/2)
)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.128 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx$

Optimal. Leaf size=146

$$\frac{b^3\operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/3*arctanh(tanh(b*x+a))^(3/2)/x^3-1/8*b^
2/x/arctanh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(t
anh(b*x+a))^(1/2)-1/4*b*arctanh(tanh(b*x+a))^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{b^3\operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3}-\frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b^2/(8*x*Sqrt[ArcTanh[Tanh[a
+ b*x]]]) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x
]]]) - (b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*x^2) - ArcTanh[Tanh[a + b*x]]^(3
/2)/(3*x^3)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{2}b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx$$

$$= -\frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{8}b^2\int \frac{}{x^2\sqrt{\mathrm{ta}}}$$

$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\mathrm{ta}}{}$$

$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-}}}$$

$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx)}}$$

Mathematica [A]
time = 0.06, size = 117, normalized size = 0.80

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-\frac{7b}{12x^2} - \frac{b^2}{8x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)} - \frac{-bx+\tanh^{-1}(\tanh(a+bx))}{3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[ArcTanh[Tanh[a +
```

b*x]]]*((-7*b)/(12*x^2) - b^2/(8*x*(-(b*x) + ArcTanh[Tanh[a + b*x]])) - (-(b*x) + ArcTanh[Tanh[a + b*x]])/(3*x^3))

Maple [A]
time = 0.07, size = 116, normalized size = 0.79

method	result
default	$2b^3\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6}+\left(\frac{\operatorname{arctanh}(\tanh(bx+a))}{16}-\frac{bx}{16}\right)\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^3x^3}+\frac{\operatorname{arcta}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(tanh(b*x+a))^(3/2)/x^4,x,method=_RETURNVERBOSE)

[Out] 2*b^3*((-1/16/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(5/2)-1/6*arctanh(tanh(b*x+a))^(3/2)+(1/16*arctanh(tanh(b*x+a))-1/16*b*x)*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3+1/16/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^4, x)

Fricas [A]
time = 0.38, size = 145, normalized size = 0.99

$$\left[\frac{3\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,a^2x^3},-\frac{3\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="fricas")

[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**4, x)
```

Giac [A]
time = 0.41, size = 93, normalized size = 0.64

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}b^4+8\,(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}\,a^2b^4\right)}{ab^3x^3}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) +
sqrt(2)*(3*(b*x + a)^(5/2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*
a^2*b^4)/(a*b^3*x^3))/b
```

Mupad [B]
time = 5.42, size = 1019, normalized size = 6.98

$$\frac{11\,b^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{12\,x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2x^3\left(3\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+6bx\right)}-\frac{2b^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{x\left(3\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+6bx\right)}-\frac{3b\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{7\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{18}-\frac{7\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{18}+\frac{7bx}{9}\right)}{x^2\left(2\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+4bx\right)}+\frac{\sqrt{2}\,b^3\ln\left(\frac{\left(2\sqrt{2}\,a-\sqrt{2}\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{2}\,bx+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2i\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)16i}{x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)1i}{8\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^4,x)
```

```
[Out] (11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(12*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + ((log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp
(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^3*(3*log(2/(exp(2*a
)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 6*b*x)) - (2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(3*log(2/(exp(2*a)*exp
(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
```

```
6*b*x)) + (2^(1/2)*b^3*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(
1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i)/(x*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2)))*1i)/(8*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)) - (3*b*(log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2
*b*x) + 1))/2)^(1/2)*((7*log(2/(exp(2*a)*exp(2*b*x) + 1)))/18 - (7*log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/18 + (7*b*x)/9))/(x^2*(2*lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 4*b*x))
```

3.129 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2} \, dx$

Optimal. Leaf size=101

$$\frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{128x\tanh^{-1}}{}$$

```
[Out] 2/7*x^4*arctanh(tanh(b*x+a))^(7/2)/b-16/63*x^3*arctanh(tanh(b*x+a))^(9/2)/b
^2+32/231*x^2*arctanh(tanh(b*x+a))^(11/2)/b^3-128/3003*x*arctanh(tanh(b*x+a
))^(13/2)/b^4+256/45045*arctanh(tanh(b*x+a))^(15/2)/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{15/2}}{45045b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(9/2))/(63*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (128*
x*ArcTanh[Tanh[a + b*x]]^(13/2))/(3003*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(
15/2))/(45045*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\int x^2}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tan}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tan}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tan}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tan}{}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(6435b^4x^4-5720b^3x^3\tanh^{-1}(\tanh(a+bx))+3120b^2x^2\tanh^{-1}(\tanh(a+bx))^2-960bx\tanh^{-1}(\tanh(a+bx))^3+128\tanh^{-1}(\tanh(a+bx))^4\right)}{45045b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(6435*b^4*x^4 - 5720*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 3120*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 960*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(45045*b^5)
```

Maple [A]
time = 0.08, size = 154, normalized size = 1.52

method	result
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{15}{2}}}{15}+\frac{2(-4\,\mathrm{arctanh}(\tanh(bx+a))+4bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2\left(2(bx-\mathrm{arctanh}(\tanh(bx+a)))^2+(-2\,\mathrm{arctanh}(\tanh(bx+a))+2\right.}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/15*arctanh(tanh(b*x+a))^(15/2)+1/13*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(13/2)+1/11*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arct
anh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(11/2)+2/9*(b*x-arctanh(tan
h(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(9/2)+1/7
*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(7/2))
```

Maxima [A]
time = 0.54, size = 64, normalized size = 0.63

$$\frac{2\,(3003\,b^5x^5+1155\,ab^4x^4-840\,a^2b^3x^3+560\,a^3b^2x^2-320\,a^4bx+128\,a^5)(bx+a)^{\frac{5}{2}}}{45045\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^5*x^5 + 1155*a*b^4*x^4 - 840*a^2*b^3*x^3 + 560*a^3*b^2*x^2
- 320*a^4*b*x + 128*a^5)*(b*x + a)^(5/2)/b^5
```

Fricas [A]
time = 0.33, size = 86, normalized size = 0.85

$$\frac{2\,(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a^6bx+128\,a^7)\sqrt{bx+a}}{45045\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
- 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)/b^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(81) = 162.
time = 0.40, size = 344, normalized size = 3.41

$$\frac{\sqrt{2}\left(\frac{143\,\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^3}{b^4}+\frac{195\,\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a^2}{b^4}+\frac{45\,\sqrt{2}\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)a}{b^4}+\frac{7\,\sqrt{2}\left(429\,(bx+a)^{\frac{15}{2}}-3465\,(bx+a)^{\frac{13}{2}}a+12285\,(bx+a)^{\frac{11}{2}}a^2-25025\,(bx+a)^{\frac{9}{2}}a^3+32175\,(bx+a)^{\frac{7}{2}}a^4-27027\,(bx+a)^{\frac{5}{2}}a^5+15015\,(bx+a)^{\frac{3}{2}}a^6-6435\,\sqrt{bx+a}\,a^7\right)}{b^4}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
a^3/b^4 + 195*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b
```

```
*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 6
93*sqrt(b*x + a)*a^5)*a^2/b^4 + 45*sqrt(2)*(231*(b*x + a)^(13/2) - 1638*(b*
x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 900
9*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*
a/b^4 + 7*sqrt(2)*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(
b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4
 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x +
a)*a^7)/b^4)/b
```

Mupad [B]
time = 1.28, size = 2500, normalized size = 24.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^7*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/15 + (x^5*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
- (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(13*b
)))/(11*b) - (x^6*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*b^2*(log(2/(exp(2*a
)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 + b*x))/15)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(13*b) - (x^4*(log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)
)/(11*b)))/(9*b) - (128*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
```

```
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2
*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
)^4)/(315*b^5) - (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2
*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/(63*b^2) - (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/
4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)
)/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) +
 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)
/(315*b^4) - (16*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) +...
```

3.130 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$\frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{32\tanh^{-1}(\tanh$$

```
[Out] 2/7*x^3*arctanh(tanh(b*x+a))^(7/2)/b-4/21*x^2*arctanh(tanh(b*x+a))^(9/2)/b^
2+16/231*x*arctanh(tanh(b*x+a))^(11/2)/b^3-32/3003*arctanh(tanh(b*x+a))^(13
/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2199, 2188, 30}

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(9/2))/(21*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(13/2))/(3003*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{6\int x^2\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{8\int x \,\mathrm{ta}}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\,\mathrm{tan}}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\,\mathrm{tan}}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\,\mathrm{tan}}{}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(429b^3x^3 - 286b^2x^2\tanh^{-1}(\tanh(a+bx)) + 104bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a+bx))^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 286*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 104*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(
3003*b^4)
```

Maple [A]
time = 0.08, size = 124, normalized size = 1.55

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13} + \frac{2(-3\,\mathrm{arctanh}(\tanh(bx+a))+3bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11} + \frac{2\left((bx-\mathrm{arctanh}(\tanh(bx+a)))(-2\,\mathrm{arctanh}(\tanh(bx+a))+2b\right.}{9}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-3*arctanh(tanh(b*x+a))+3*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(t
anh(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(9/2)
+1/7*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(7/2))
```

Maxima [A]
time = 0.54, size = 53, normalized size = 0.66

$$\frac{2\,(231\,b^4x^4 + 105\,ab^3x^3 - 70\,a^2b^2x^2 + 40\,a^3bx - 16\,a^4)(bx+a)^{\frac{5}{2}}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*b^4*x^4 + 105*a*b^3*x^3 - 70*a^2*b^2*x^2 + 40*a^3*b*x - 16*a^4)
*(b*x + a)^(5/2)/b^4
```

Fricas [A]
time = 0.32, size = 75, normalized size = 0.94

$$\frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [A]
time = 113.24, size = 94, normalized size = 1.18

$$\begin{cases} \frac{2x^3\operatorname{atanh}^{\frac{7}{2}}(\tanh(a+bx))}{7b}-\frac{4x^2\operatorname{atanh}^{\frac{9}{2}}(\tanh(a+bx))}{21b^2}+\frac{16x\operatorname{atanh}^{\frac{11}{2}}(\tanh(a+bx))}{231b^3}-\frac{32\operatorname{atanh}^{\frac{13}{2}}(\tanh(a+bx))}{3003b^4} & \text{for}\ b\neq 0\\ \frac{x^4\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*x**3*atanh(tanh(a + b*x))**(7/2)/(7*b) - 4*x**2*atanh(tanh(a +
 b*x))**(9/2)/(21*b**2) + 16*x*atanh(tanh(a + b*x))**(11/2)/(231*b**3) - 32
*atanh(tanh(a + b*x))**(13/2)/(3003*b**4), Ne(b, 0)), (x**4*atanh(tanh(a))*
*(5/2)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(64) = 128$.
time = 0.39, size = 296, normalized size = 3.70

$$\frac{\sqrt{2}\left(\frac{429\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^3}{b^3}+\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^2}{b^3}+\frac{65\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a}{b^3}+\frac{5\sqrt{2}\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)}{b^3}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/15015*sqrt(2)*(429*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*sqrt(2)*(35*(b*x
 + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b^3 + 65*sqrt(2)*(63*(b*x + a)^(1
```

```
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*sqrt(2
)*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^
2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3
/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

Mupad [B]
time = 1.15, size = 2235, normalized size = 27.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^4*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
- (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b
)))/(9*b) - (x^5*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^2*(log(2/(exp(2*a)
*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 + b*x))/13)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(11*b) - (x^3*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (8*((3*b*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^2*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
 b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x)
+ 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/
(9*b)))/(7*b) - (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/
4 - (8*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x) - (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2
```

```
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))
/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1
))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(
35*b^4) - (6*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 -
 (8*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x) - (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) +
1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(1
1*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(35*b^
2) - (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (8*((3
*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (2
4*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

3.131 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=59

$$\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}$$

```
[Out] 2/7*x^2*arctanh(tanh(b*x+a))^(7/2)/b-8/63*x*arctanh(tanh(b*x+a))^(9/2)/b^2+
16/693*arctanh(tanh(b*x+a))^(11/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(9
/2))/(63*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(11/2))/(693*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{8\int \tanh^{-}}{}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{8\text{Subst}(\int}{}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\tanh^{-1}}{}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(99b^2x^2 - 44bx\tanh^{-1}(\tanh(a+bx)) + 8\tanh^{-1}(\tanh(a+bx))^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(693*b^3)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.17

method	result	size
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-2*arctanh(tanh(b*x+a))+2*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b
*x+a))^(7/2))
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\,(63\,b^3x^3 + 35\,ab^2x^2 - 20\,a^2bx + 8\,a^3)(bx+a)^{\frac{5}{2}}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*b^3*x^3 + 35*a*b^2*x^2 - 20*a^2*b*x + 8*a^3)*(b*x + a)^(5/2)/b^3
```

Fricas [A]
time = 0.34, size = 64, normalized size = 1.08

$$\frac{2\left(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5\right)\sqrt{bx+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [A]
time = 62.07, size = 71, normalized size = 1.20

$$\begin{cases} \frac{2x^2\operatorname{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{7b}-\frac{8x\operatorname{atanh}^{\frac{9}{2}}\left(\tanh\left(a+bx\right)\right)}{63b^2}+\frac{16\operatorname{atanh}^{\frac{11}{2}}\left(\tanh\left(a+bx\right)\right)}{693b^3} & \text{for } b\neq 0\\ \frac{x^3\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*x**2*atanh(tanh(a + b*x))**(7/2)/(7*b) - 8*x*atanh(tanh(a + b*
x))**(9/2)/(63*b**2) + 16*atanh(tanh(a + b*x))**(11/2)/(693*b**3), Ne(b, 0)
), (x**3*atanh(tanh(a))**(5/2)/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(47) = 94.
time = 0.39, size = 248, normalized size = 4.20

$$\frac{\sqrt{2}\left(\frac{231\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^3}{b^2}+\frac{297\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^2}{b^2}+\frac{33\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a}{b^2}+\frac{5\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)}{b^2}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(231*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*
sqrt(b*x + a)*a^2)*a^3/b^2 + 297*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*sqrt(
2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 -
420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^2 + 5*sqrt(2)*(63*(b*x
 + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x
+ a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

Mupad [B]
time = 1.14, size = 1789, normalized size = 30.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^3*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 (20*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))
)/(7*b) - (x^4*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(log(2/(exp(2*a)*e
xp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 + b*x))/11)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) - (x^2*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6*((3*b*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b)
))/(5*b) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 (20*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))
*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(15*b^3)
- (4*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
```

```
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6*((3*b*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^
2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(15*b^2)
```

3.132 $\int x \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

```
[Out] 2/7*x*arctanh(tanh(b*x+a))^(7/2)/b-4/63*arctanh(tanh(b*x+a))^(9/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))
/(63*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{2\int \tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{2\text{Subst}\left(\int x^{7/2}\,dx, x, \tanh^{-1}(\tanh(a+bx\right.}{7b^2}$$
$$= \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.84

$$\frac{2\left(9bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
[Out] (2*(9*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*b^2
)
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.11

method	result	size
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\mathrm{arctanh}(\tanh(bx+a)))\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(7/2))
```

Maxima [A]
time = 0.54, size = 31, normalized size = 0.82

$$\frac{2\,(7\,b^2x^2+5\,abx-2\,a^2)(bx+a)^{\frac{5}{2}}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] 2/63*(7*b^2*x^2 + 5*a*b*x - 2*a^2)*(b*x + a)^(5/2)/b^2
```

Fricas [A]
time = 0.34, size = 52, normalized size = 1.37

$$\frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

Sympy [A]
time = 33.86, size = 49, normalized size = 1.29

$$\begin{cases} \frac{2x\,\mathrm{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{7b}-\frac{4\,\mathrm{atanh}^{\frac{9}{2}}\left(\tanh\left(a+bx\right)\right)}{63b^2} & \text{for}\ b\neq 0 \\ \frac{x^2\,\mathrm{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*x*atanh(tanh(a + b*x))**(7/2)/(7*b) - 4*atanh(tanh(a + b*x))**
(9/2)/(63*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(5/2)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(30) = 60.
time = 0.41, size = 197, normalized size = 5.18

$$\frac{\sqrt{2}\left(\frac{105\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^3}{b}+\frac{63\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^2}{b}+\frac{27\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a}{b}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(105*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63
*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a^2/b + 27*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)
^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b + sqrt(2)*(35*(b*x + a)^(9/2) - 180*
(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315
*sqrt(b*x + a)*a^4)/b)/b
```

Mupad [B]
time = 1.11, size = 773, normalized size = 20.34

$$\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{63\,b^2}-\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^4\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{252\,b^2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^4\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{252\,b^2}+\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{63\,b^2}-\frac{x\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}+\frac{x\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{42\,b^2}-\frac{3\,x\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}+\frac{3\,x\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*atanh(tanh(a + b*x))^(5/2),x)

[Out] (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (log((exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) - (log(
1/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) + (log(1
/(exp(2*a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (x*log(1/(exp(2*a)*exp(2*b*x)
+ 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (x*log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) - (log(1/(exp(2*
a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^
2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2))/(42*b^2) - (3*x*log(1/(exp(2*a)*exp(2*b*x) + 1)
)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2))/(28*b) + (3*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b)

3.133 $\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int x^{5/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2),x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7b}$	15
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

Maxima [A]
time = 0.51, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] 2/7*(b*x + a)^(7/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.
time = 0.34, size = 39, normalized size = 2.17

$$\frac{2\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [A]
time = 17.68, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2\,\mathrm{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{7b} & \text{for}\ b \neq 0 \\ x\,\mathrm{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(7/2)/(7*b), Ne(b, 0)), (x*atanh(tanh(a)
)**(5/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(14) = 28.
time = 0.41, size = 136, normalized size = 7.56

$$\frac{\sqrt{2}\left(35\sqrt{2}\sqrt{bx+a}\,a^3+35\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^2+7\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a+\sqrt{2}\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)\right)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/35*sqrt(2)*(35*sqrt(2)*sqrt(b*x + a)*a^3 + 35*sqrt(2)*((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*a^2 + 7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*
a + 15*sqrt(b*x + a)*a^2)*a + sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3))/b
```

Mupad [B]
time = 1.12, size = 337, normalized size = 18.72

$$\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28b}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28b}-\frac{3\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28b}+\frac{3\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2))/(28*b) - (log(1/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
))/(28*b) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (3*log(1/(ex
p(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b)
```

3.134 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}\,dx$

Optimal. Leaf size=121

$$-2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}+2\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x
-arctanh(tanh(b*x+a)))^(5/2)-2/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*
x+a))^(3/2)+2/5*arctanh(tanh(b*x+a))^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))^2*a
rctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2192}

$$-2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)+2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{5}\tanh^{-1}(\tanh(a+bx))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + 2*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]] - (2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tan
h[a + b*x]]^(3/2))/3 + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/5
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}\,dx = \frac{2}{5}\tanh^{-1}(\tanh(a+bx))^{5/2} - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh}{x}$$

$$= -\frac{2}{3}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{2}{5}\tanh^{-1}(\tanh$$

$$= 2\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{2}{3}\left(bx - \tanh^{-1}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx - \tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 0.82

$$\frac{2}{15}\left(15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - 35bx\tanh^{-1}(\tanh(a+bx))^{3/2} + 23\tanh^{-1}(\tanh(a+bx))^{5/2} - 15\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

```
[Out] (2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 35*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 23*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*ArcTanh[Sqrt[ArcTanh[Tanh[a +
 b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^(5/2)))/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(105) = 210$.
time = 0.06, size = 222, normalized size = 1.83

method	result
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5} + \frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}\,a}{3} + \frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}\,(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{3} + 2\sqrt{\mathrm{arctan}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)+2/3*arctanh(tanh(b*x+a))^(3/2)*a+2/3*arctanh
(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*arctanh(tanh(b*x+a))^(1/
2)*a^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+2*(arcta
```

```
nh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-2*(a^3+3*a^2*(arctanh(t
anh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))
-b*x-a)^3)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1
/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x, x)
```

Fricas [A]
time = 0.33, size = 114, normalized size = 0.94

$$\left[a^{\frac{5}{2}} \log\left(\frac{bx - 2\sqrt{bx + a}\sqrt{a} + 2a}{x}\right) + \frac{2}{15}\left(3b^2x^2 + 11abx + 23a^2\right)\sqrt{bx + a}, 2\sqrt{-a}a^2 \arctan\left(\frac{\sqrt{bx + a}\sqrt{-a}}{a}\right) + \frac{2}{15}\left(3b^2x^2 + 11abx + 23a^2\right)\sqrt{bx + a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a + bx))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x, x)
```

Giac [A]
time = 0.39, size = 73, normalized size = 0.60

$$\frac{1}{15}\sqrt{2}\left(\frac{15\sqrt{2}a^3 \arctan\left(\frac{\sqrt{bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 3\sqrt{2}(bx + a)^{\frac{5}{2}} + 5\sqrt{2}(bx + a)^{\frac{3}{2}}a + 15\sqrt{2}\sqrt{bx + a}\,a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 3*sq
rt(2)*(b*x + a)^(5/2) + 5*sqrt(2)*(b*x + a)^(3/2)*a + 15*sqrt(2)*sqrt(b*x +
 a)*a^2)
```

Mupad [B]
time = 4.82, size = 789, normalized size = 6.52

$$\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{3b\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2}-\frac{2\left(3b^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-\frac{8b^2\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5}\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{3b}\right)}{b}+\frac{2b^2x^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{5}-\frac{x\left(3b^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-\frac{8b^2\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5}\right)\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{3b}+\frac{\sqrt{2}\ln\left(\frac{\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-\sqrt{2}bx+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2\mathrm{i}\right)16\mathrm{i}}{x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x,x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (2*(3*b^2
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/5)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 + b*x))/(3*b)))/b + (2*b^2*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/5 + (
2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
+ 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*16i)/(x*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)*1i)/8 - (x*(3*b^2*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/5)*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2))/(3*b)
```

3.135 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2}\,dx$

Optimal. Leaf size=110

$$5b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+b\right.$$

```
[Out] 5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(3/2)+5/3*b*arctanh(tanh(b*x+a))^(3/2)-arctanh(tanh
(b*x+a))^(5/2)/x-5*b*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2192}

$$5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}+\frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

```
[Out] 5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 5*b*(b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]] + (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/3 - ArcTa
nh[Tanh[a + b*x]]^(5/2)/x
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} + \frac{1}{2}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx$$

$$= \frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} - \frac{1}{2}\big(5b\big(bx - \tanh$$

$$= -5b\big(bx - \tanh^{-1}(\tanh(a+bx))\big)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{3}b\tanh^{-1}(\mathrm{ta}$$

$$= 5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\big(bx - \tanh^{-1}(\tanh(a+bx))\big)^{3/}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 0.96

$$-5b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2} + \sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\frac{2b^2x}{3} + \frac{14}{3}b\left(-bx+\tanh^{-1}(\tanh(a+bx))\right) - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

```
[Out] -5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + Sqrt[ArcTanh[Tanh[a + b*x]]
]*((2*b^2*x)/3 + (14*b*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/3 - (-(b*x) + Arc
Tanh[Tanh[a + b*x]])^2/x)
```

Maple [A]
time = 0.07, size = 193, normalized size = 1.75

method	result

default	$2b\left(\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\,a+2(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a)\,\sqrt{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(tanh(b*x+a))^(5/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 2*b*(1/3*arctanh(tanh(b*x+a))^(3/2)+2*arctanh(tanh(b*x+a))^(1/2)*a+2*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+(-1/2*a^2-a*(arctanh(tanh(b*x+a))-b*x-a)-1/2*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2)/b/x-5/2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="maxima")

[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^2, x)

Fricas [A]
time = 0.34, size = 126, normalized size = 1.15

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^{2}x^{2}+14\,abx-3\,a^{2}\right)\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(2\,b^{2}x^{2}+14\,abx-3\,a^{2}\right)\sqrt{bx+a}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="fricas")

[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**2,x)

[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**2, x)

Giac [A]
time = 0.40, size = 89, normalized size = 0.81

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}\,a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{2}\,(bx+a)^{\frac{3}{2}}b^2+12\,\sqrt{2}\,\sqrt{bx+a}\,ab^2-\frac{3\,\sqrt{2}\,\sqrt{bx+a}\,a^2b}{x}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="giac")

[Out] 1/6*sqrt(2)*(15*sqrt(2)*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(2)*(b*x + a)^(3/2)*b^2 + 12*sqrt(2)*sqrt(b*x + a)*a*b^2 - 3*sqrt(2)*sqrt(b*x + a)*a^2*b/x)/b

Mupad [B]
time = 4.97, size = 616, normalized size = 5.60

$$\frac{2\,b^2\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3}-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,x}-\frac{\left(3\,b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-\frac{4\,b^2\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{b}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,2{}\mathrm{i}\right)\,16{}\mathrm{i}}{x\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}\,5{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(5/2)/x^2,x)

[Out] (2*b^2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3 - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*x) - ((3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (4*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/3)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/b + (2^(1/2)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)*5i)/8

3.136 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3}\,dx$

Optimal. Leaf size=110

$$-\frac{15}{4}b^2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+$$

```
[Out] -5/4*b*arctanh(tanh(b*x+a))^(3/2)/x-1/2*arctanh(tanh(b*x+a))^(5/2)/x^2-15/4
*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b
*x-arctanh(tanh(b*x+a)))^(1/2)+15/4*b^2*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2192}

$$-\frac{15}{4}b^2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\,\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)+\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/4 + (15*b^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/4 - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(4*x) - ArcTanh[Tanh[a + b*
x]]^(5/2)/(2*x^2)
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{4}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{8}(15b^2)\int \frac{\sqrt{\text{t}}}{}$$

$$= \frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tan}{2}$$

$$= -\frac{15}{4}b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+b}$$

Mathematica [A]
time = 0.04, size = 108, normalized size = 0.98

$$-\frac{-15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} + 5bx\tanh^{-1}(\tanh(a+bx))^{3/2} + 2\tanh^{-1}(\tanh(a+bx))^{5/2} + 15b^2x^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

```
[Out] -1/4*(-15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x
]]^(3/2) + 2*ArcTanh[Tanh[a + b*x]]^(5/2) + 15*b^2*x^2*ArcTanh[Sqrt[ArcTanh
[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTa
nh[Tanh[a + b*x]]])/x^2
```

Maple [A]
time = 0.07, size = 142, normalized size = 1.29

method	result

default	$2b^2\left(\sqrt{\operatorname{arctanh}(\tanh(bx+a))}+\frac{\left(-\frac{9\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{7a^2}{8}+\frac{7a(\operatorname{arctanh}(\tanh(bx+}{4}\right.}{b^2x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)+((-9/8*arctanh(tanh(b*x+a))+9/8*b*x)*arct
anh(tanh(b*x+a))^(3/2)+(7/8*a^2+7/4*a*(arctanh(tanh(b*x+a))-b*x-a)+7/8*(arc
tanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8*(arcta
nh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh
(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^3, x)
```

Fricas [A]
time = 0.35, size = 133, normalized size = 1.21

$$\left[\frac{15\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(8\,b^2x^2-9\,abx-2\,a^2)\sqrt{bx+a}}{8\,x^2},\ \frac{15\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(8\,b^2x^2-9\,abx-2\,a^2)\sqrt{bx+a}}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**3, x)
```

Giac [A]
time = 0.42, size = 92, normalized size = 0.84

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{2}\sqrt{bx+a}\,b^3-\frac{\sqrt{2}\left(9\,(bx+a)^{\frac{3}{2}}ab^3-7\sqrt{bx+a}\,a^2b^3\right)}{b^2x^2}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(15*sqrt(2)*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*s
qrt(2)*sqrt(b*x + a)*b^3 - sqrt(2)*(9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x +
a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 2.07, size = 614, normalized size = 5.58

$$2b^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}+b^2\ln\left(\frac{64\left(2\sqrt{2}\,a-2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2bx}-\sqrt{2}\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{2}\,bx\right)}{x\sqrt{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2bx}}\right)\sqrt{\frac{225\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{128}-\frac{225\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{128}-\frac{225bx}{64}}-\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{4x^2\left(2\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+4bx\right)}+\frac{9b\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2) + b^2*log((64*(2*2^(1/2)*a - 2*(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) - 2^(1/2)*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x) + 2^(1/2)*b*x))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((225*log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/128 - (225*log(2/(exp(2
*a)*exp(2*b*x) + 1)))/128 - (225*b*x)/64)^(1/2) - ((log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (9*b*
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(8*x)
```

3.137 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4}\,dx$

Optimal. Leaf size=113

$$\frac{5b^3\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2}-$$

```
[Out] -5/12*b*arctanh(tanh(b*x+a))^(3/2)/x^2-1/3*arctanh(tanh(b*x+a))^(5/2)/x^3+5
/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/
(b*x-arctanh(tanh(b*x+a)))^(1/2)-5/8*b^2*arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 2192}

$$\frac{5b^3\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])/(8*x) - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(12*x^2) - ArcTanh[Tanh[
a + b*x]]^(5/2)/(3*x^3)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{6}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{8}(5b^2)\int \frac{\sqrt{\mathrm{ta}}}{}$$

$$= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\mathrm{ta}}{}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx)}}{8x}$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 0.95

$$\frac{1}{24}\left(-\frac{15b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} - \frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} - \frac{8\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3} - \frac{15b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

```
[Out] ((-15*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (10*b*ArcTanh[Tanh[a + b*x]]^(3
/2))/x^2 - (8*ArcTanh[Tanh[a + b*x]]^(5/2))/x^3 - (15*b^3*ArcTanh[Sqrt[ArcT
anh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + A
rcTanh[Tanh[a + b*x]]])/24
```

Maple [A]
time = 0.07, size = 144, normalized size = 1.27

method	result

default	$2b^3\left(\frac{-\frac{11\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(\frac{5\,\mathrm{arctanh}(\tanh(bx+a))}{6}-\frac{5bx}{6}\right)\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(-\frac{5a^2}{16}-\frac{5a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{8}-5\right.}{b^3x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*((-11/16*arctanh(tanh(b*x+a))^(5/2)+(5/6*arctanh(tanh(b*x+a))-5/6*b*x
)*arctanh(tanh(b*x+a))^(3/2)+(-5/16*a^2-5/8*a*(arctanh(tanh(b*x+a))-b*x-a)-
5/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-5/
16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^4, x)
```

Fricas [A]
time = 0.34, size = 146, normalized size = 1.29

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,ax^3},\ \frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**4, x)
```

Giac [A]
time = 0.40, size = 88, normalized size = 0.78

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\left(33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\sqrt{bx+a}\,a^2b^4\right)}{b^3x^3}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(15*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt
(2)*(33*(b*x + a)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a
^2*b^4)/(b^3*x^3))/b
```

Mupad [B]
time = 5.68, size = 669, normalized size = 5.92

$$\frac{13\,b\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x\right)^2}{12\,x^2\left(2\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-2\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+4\,b\,x\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x\right)^3}{4\,x^3\left(3\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-3\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+6\,b\,x\right)}-\frac{11\,b^2\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}}}{8\,x}+\frac{\sqrt{2}\,b^3\ln\left(\frac{\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x}\left(\sqrt{2}\,b\,x-\sqrt{2}\left(\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x}\,2\mathrm{i}\right)64\mathrm{i}}{x}\right)5\mathrm{i}}{16\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}+1}\right)+2\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^4,x)
```

```
[Out] (2^(1/2)*b^3*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^
(1/2)*b*x)*64i)/x)*5i)/(16*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - ((log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^3*(3*log(2/(exp(2*a)*exp(2*b
*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*
x)) - (11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(8*x) + (13*b*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(12*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) +
 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x))
```

3.138 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5}\,dx$

Optimal. Leaf size=167

$$\frac{5b^4\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{}{64\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

```
[Out] 5/64*b^4*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(3/2)-5/24*b*arctanh(tanh(b*x+a))^(3/2)/x^3-1/
4*arctanh(tanh(b*x+a))^(5/2)/x^4-5/64*b^3/x/arctanh(tanh(b*x+a))^(1/2)+5/64
*b^4/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-5/32*b^2*arctanh
(tanh(b*x+a))^(1/2)/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{5b^4\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

```
[Out] (5*b^4*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(64*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - (5*b^3)/(64*x*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (5*b^4)/(64*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh
[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(32*x^2) - (5*b*Ar
cTanh[Tanh[a + b*x]]^(3/2))/(24*x^3) - ArcTanh[Tanh[a + b*x]]^(5/2)/(4*x^4)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4} + \frac{1}{8}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4} + \frac{1}{16}(5b^2)\int \frac{\sqrt{}}{}$$

$$= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3} - \frac{\tanh^{-1}(\tan}{}$$

$$= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2} - \frac{5b\tanh^{-}}{}$$

$$= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh}}$$

$$= \frac{5b^4\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx}}$$

Mathematica [A]
time = 0.08, size = 134, normalized size = 0.80

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^3x^3+10b^2x^2\tanh^{-1}(\tanh(a+bx))+8bx\tanh^{-1}(\tanh(a+bx))^2-48\tanh^{-1}(\tanh(a+bx))^3\right)}{192x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

```
[Out] (5*b^4*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(64*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) - (Sqrt[ArcTanh[Tanh[
a + b*x]]]*(15*b^3*x^3 + 10*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 8*b*x*ArcTanh[
Tanh[a + b*x]]^2 - 48*ArcTanh[Tanh[a + b*x]]^3))/(192*x^4*(b*x - ArcTanh[Ta
nh[a + b*x]]))
```

Maple [A]
time = 0.07, size = 169, normalized size = 1.01

method	result
default	$2b^4 \left(\frac{-\frac{5 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\operatorname{arctanh}(\tanh(bx+a))-bx)} - \frac{73 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{384} + \left(\frac{55 \operatorname{arctanh}(\tanh(bx+a))}{384} - \frac{55bx}{384}\right) \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}} + \left(-\frac{5a^2}{128} - 5\right.}{b^4x^4} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4*((-5/128/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(7/2)-73/384
*arctanh(tanh(b*x+a))^(5/2)+(55/384*arctanh(tanh(b*x+a))-55/384*b*x)*arctan
h(tanh(b*x+a))^(3/2)+(-5/128*a^2-5/64*a*(arctanh(tanh(b*x+a))-b*x-a)-5/128*
(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^4/x^4+5/128/(
arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh
(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^5, x)
```

Fricas [A]
time = 0.36, size = 167, normalized size = 1.00

$$\left[\frac{15\,\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{384\,a^2x^4},\ -\frac{15\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{192\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
```

```
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**5,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**5, x)
```

Giac [A]
time = 0.41, size = 108, normalized size = 0.65

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\sqrt{bx+a}\,a^3b^5\right)}{ab^4x^4}\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(15*sqrt(2)*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a)
+ sqrt(2)*(15*(b*x + a)^(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)
^(3/2)*a^2*b^5 + 15*sqrt(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

Mupad [B]
time = 5.85, size = 1069, normalized size = 6.40

$$\frac{5b^3\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{32x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{4x^4\left(4\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-4\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+8bx\right)}-\frac{59b^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{48x^2\left(2\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+4bx\right)}+\frac{17b\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{16x^3\left(3\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+6bx\right)}+\frac{\sqrt{2}\,b^4\ln\left(\frac{\left(2\sqrt{2}\,a-\sqrt{2}\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{2}\,bx+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2i\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)1024i}{x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)5i}{64\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^5,x)
```

```
[Out] (5*b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(
2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^4*(4*log(2/(exp(2*a)
*exp(2*b*x) + 1)) - 4*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
```

```
) + 8*b*x)) + (2^(1/2)*b^4*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1024i)/(x*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^(1/2)))*5i)/(64*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)) - (59*
b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(48*x^2*(2*log(2/(
exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 4*b*x)) + (17*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^2)/(16*x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))
```

3.139 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6}\,dx$

Optimal. Leaf size=221

$$\frac{3b^5\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{}{128\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

```
[Out] 3/128*b^5*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/128*b^4/x/arctanh(tanh(b*x+a))^(3/2)-
1/128*b^5/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/8*b*arcta
nh(tanh(b*x+a))^(3/2)/x^4-1/5*arctanh(tanh(b*x+a))^(5/2)/x^5-1/64*b^3/x^2/a
rctanh(tanh(b*x+a))^(1/2)+3/128*b^5/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(ta
nh(b*x+a))^(1/2)-1/16*b^2*arctanh(tanh(b*x+a))^(1/2)/x^3
```

Rubi [A]

time = 0.12, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{3b^5\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{3b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5}-\frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] (3*b^5*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(128*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^4/(128*x*ArcTanh[Tanh[a
 + b*x]]^(3/2)) - b^5/(128*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
b*x]]^(3/2)) - b^3/(64*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^5)/(128*(b*
x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]])/(16*x^3) - (b*ArcTanh[Tanh[a + b*x]]^(3/2))/(8*x^4) -
 ArcTanh[Tanh[a + b*x]]^(5/2)/(5*x^5)
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{2}b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^5}\,dx$$

$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{16}\left(3b^2\right)\int \frac{\sqrt{\ }}{}$$

$$= -\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tan}{5}$$

$$= -\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-}}{}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\mathrm{ta}}}{}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-}}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-}}$$

$$= \frac{3b^5\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx}$$

Mathematica [A]
time = 0.08, size = 150, normalized size = 0.68

$$\frac{1}{640}\left(-\frac{15b^5\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(15b^4x^4+10b^3x^3\tanh^{-1}(\tanh(a+bx))+8b^2x^2\tanh^{-1}(\tanh(a+bx))^2-176bx\tanh^{-1}(\tanh(a+bx))^3+128\tanh^{-1}(\tanh(a+bx))^4\right)}{x^5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] ((-15*b^5*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (Sqrt[ArcTanh[Tanh[a
+ b*x]]]*(15*b^4*x^4 + 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 8*b^2*x^2*ArcTan
h[Tanh[a + b*x]]^2 - 176*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a
+ b*x]]^4))/(x^5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/640
```

Maple [A]
time = 0.08, size = 262, normalized size = 1.19

method	result
default	$2b^5\left(\frac{\frac{3\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{256\left(a^2+2a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)+(\mathrm{arctanh}(\tanh(bx+a))-bx-a)^2\right)}-\frac{7\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\mathrm{arctanh}(\tanh(bx+a))-bx)}-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{10}+\left(\frac{7\,a}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^5*((3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b
*x-a)^2)*arctanh(tanh(b*x+a))^(9/2)-7/128/(arctanh(tanh(b*x+a))-b*x)*arctan
h(tanh(b*x+a))^(7/2)-1/10*arctanh(tanh(b*x+a))^(5/2)+(7/128*arctanh(tanh(b*
x+a))-7/128*b*x)*arctanh(tanh(b*x+a))^(3/2)+(-3/256*a^2-3/128*a*(arctanh(ta
nh(b*x+a))-b*x-a)-3/256*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a)
)^(1/2))/b^5/x^5-3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(
b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x
+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^6, x)
```

Fricas [A]
time = 0.32, size = 189, normalized size = 0.86

$$\left[\frac{15 \sqrt{a} b^{5} x^{5} \log\left(\frac{bx - 2 \sqrt{bx + a} \sqrt{a} + 2 a}{x}\right) + 2 \left(15 a b^{4} x^{4} - 10 a^{2} b^{3} x^{3} - 248 a^{3} b^{2} x^{2} - 336 a^{4} b x - 128 a^{5}\right) \sqrt{bx + a}}{1280 a^{3} x^{5}}, \frac{15 \sqrt{-a} b^{5} x^{5} \arctan\left(\frac{\sqrt{bx + a} \sqrt{-a}}{a}\right) + \left(15 a b^{4} x^{4} - 10 a^{2} b^{3} x^{3} - 248 a^{3} b^{2} x^{2} - 336 a^{4} b x - 128 a^{5}\right) \sqrt{bx + a}}{640 a^{3} x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^5)
*sqrt(b*x + a))/(a^3*x^5), 1/640*(15*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*
x - 128*a^5)*sqrt(b*x + a))/(a^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a + bx\right)\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**6,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**6, x)
```

Giac [A]
time = 0.41, size = 123, normalized size = 0.56

$$\frac{\sqrt{2}\left(\frac{15 \sqrt{2} b^{6} \arctan\left(\frac{\sqrt{bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a^{2}} + \frac{\sqrt{2}\left(15 (bx+a)^{\frac{9}{2}} b^{6} - 70 (bx+a)^{\frac{7}{2}} a b^{6} - 128 (bx+a)^{\frac{5}{2}} a^{2} b^{6} + 70 (bx+a)^{\frac{3}{2}} a^{3} b^{6} - 15 \sqrt{bx + a}\, a^{4} b^{6}\right)}{a^{2} b^{5} x^{5}}\right)}{1280 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(2)*(15*sqrt(2)*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2
) + sqrt(2)*(15*(b*x + a)^(9/2)*b^6 - 70*(b*x + a)^(7/2)*a*b^6 - 128*(b*x +
 a)^(5/2)*a^2*b^6 + 70*(b*x + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x + a)*a^4*b^6)/
(a^2*b^5*x^5))/b
```

Mupad [B]
time = 6.51, size = 1292, normalized size = 5.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(5/2)/x^6,x)

[Out] (3*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^5*(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x)) + (b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(16*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (2^(1/2)*b^5*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1024i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*3i)/(64*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)) - (93*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(80*x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x)) + (21*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(20*x^4*(4*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 4*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 8*b*x))

3.140 $\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=99

$$\frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}-\frac{128x\tanh^{-1}$$

```
[Out] -16/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b^2+32/5*x^2*arctanh(tanh(b*x+a))^(5/2
)/b^3-128/35*x*arctanh(tanh(b*x+a))^(7/2)/b^4+256/315*arctanh(tanh(b*x+a))^
(9/2)/b^5+2*x^4*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^4*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (16*x^3*ArcTanh[Tanh[a + b*x]]^(3/
2))/(3*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (128*x*ArcTan
h[Tanh[a + b*x]]^(7/2))/(35*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*
b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\int x^2\,\mathrm{ta}}{}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\,\mathrm{tan}}{}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\,\mathrm{tan}}{}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\,\mathrm{tan}}{}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\,\mathrm{tan}}{}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.84

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(315b^4x^4-840b^3x^3\tanh^{-1}(\tanh(a+bx))+1008b^2x^2\tanh^{-1}(\tanh(a+bx))^2-576bx\tanh^{-1}(\tanh(a+bx))^3+128\tanh^{-1}(\tanh(a+bx))^4\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(315*b^4*x^4 - 840*b^3*x^3*ArcTanh[Tanh[a +
 b*x]] + 1008*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 576*b*x*ArcTanh[Tanh[a + b
*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(315*b^5)
```

Maple [A]
time = 0.06, size = 153, normalized size = 1.55

method	result
default	$\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-4\,\mathrm{arctanh}(\tanh(bx+a))+4bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2\left(2(bx-\mathrm{arctanh}(\tanh(bx+a)))^2+(-2\,\mathrm{arctanh}(\tanh(bx+a))+2bx\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-4*arctanh(tanh(b*x+a))+4*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh(t
anh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(5/2)+2/3*(b*x-arctanh(tanh(b*x+
a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(3/2)+(b*x-arct
anh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.54, size = 64, normalized size = 0.65

$$\frac{2\,(35\,b^5x^5-5\,ab^4x^4+8\,a^2b^3x^3-16\,a^3b^2x^2+64\,a^4bx+128\,a^5)}{315\,\sqrt{bx+a}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 64*a^4*b
*x + 128*a^5)/(sqrt(b*x + a)*b^5)
```

Fricas [A]
time = 0.34, size = 53, normalized size = 0.54

$$\frac{2\,(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.40, size = 61, normalized size = 0.62

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

Mupad [B]
time = 1.09, size = 496, normalized size = 5.01

$$\frac{2\,x^4\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{9\,b}+\frac{256\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^4}{315\,b^5}+\frac{16\,x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{63\,b^2}+\frac{128\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^3}{315\,b^4}+\frac{32\,x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) + (256*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x)^4)/(315*b^5) + (16*x^3*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/(63*b^2) + (128*x*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x)^3)/(315*b^4) + (32*x^2*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x)^2)/(105*b^3)
```

3.141 $\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=76

$$\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}-\frac{32\tanh^{-1}(\tan$$

```
[Out] -4*x^2*arctanh(tanh(b*x+a))^(3/2)/b^2+16/5*x*arctanh(tanh(b*x+a))^(5/2)/b^3
-32/35*arctanh(tanh(b*x+a))^(7/2)/b^4+2*x^3*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2199, 2188, 30}

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}+\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*x^2*ArcTanh[Tanh[a + b*x]]^(3/2
))/b^2 + (16*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (32*ArcTanh[Tanh[a +
 b*x]]^(7/2))/(35*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{6\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{8\int x \tanh$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.87

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(35b^3x^3 - 70b^2x^2\tanh^{-1}(\tanh(a+bx)) + 56bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a+bx))^3\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 70*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 56*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(35*
b^4)
```

Maple [A]
time = 0.06, size = 123, normalized size = 1.62

method	result
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(-3\operatorname{arctanh}(\tanh(bx+a))+3bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2\left((bx-\operatorname{arctanh}(\tanh(bx+a)))(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)+(\right.}{3}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
```

```
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(3/2)+(b*
x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [**A**]
time = 0.54, size = 53, normalized size = 0.70

$$\frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)}{35\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

Fricas [**A**]
time = 0.35, size = 42, normalized size = 0.55

$$\frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [**A**]
time = 0.40, size = 49, normalized size = 0.64

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

Mupad [B]
time = 1.07, size = 385, normalized size = 5.07

$$\frac{2x^3\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{7b}+\frac{32\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)^3}{35b^4}+\frac{12x^2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{35b^2}+\frac{16x\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)^2}{35b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(7*b) + (32*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x)^3)/(35*b^4) + (12*x^2*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x))/(35*b^2) + (16*x*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

3.142 $\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=57

$$\frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3}$$

```
[Out] -8/3*x*arctanh(tanh(b*x+a))^(3/2)/b^2+16/15*arctanh(tanh(b*x+a))^(5/2)/b^3+
2*x^2*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (8*x*ArcTanh[Tanh[a + b*x]]^(3/2))
/(3*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\int \tanh^{-1}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\mathrm{Subst}\left(\int x\right.}{}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\tanh^{-1}(}{}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.86

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^2x^2-20bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(15*b^3)
```

Maple [A]
time = 0.06, size = 68, normalized size = 1.19

method	result
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx\right.\right.}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(3/2)+(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))
^(1/2))
```

Maxima [A]
time = 0.53, size = 42, normalized size = 0.74

$$\frac{2\left(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3\right)}{15\,\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Fricas [A]
time = 0.34, size = 31, normalized size = 0.54

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.38, size = 37, normalized size = 0.65

$$\frac{2\left(3\left(bx+a\right)^{\frac{5}{2}}-10\left(bx+a\right)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B]
time = 1.14, size = 211, normalized size = 3.70

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(15\,b^2\,x^2-10\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+10\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-4\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 15
*b^2*x^2 - 10*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
10*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(15*b^3)
```

3.143 $\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

```
[Out] -4/3*arctanh(tanh(b*x+a))^(3/2)/b^2+2*x*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2199, 2188, 30}

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*
b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\text{Subst}\left(\int \sqrt{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$
$$= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.89

$$\frac{2\left(3bx - 2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*(3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2)
```

Maple [A]
time = 0.06, size = 56, normalized size = 1.56

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} - 2\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}\,a - 2(\mathrm{arctanh}(\tanh(bx+a)) - bx - a)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/3*arctanh(tanh(b*x+a))^(3/2)-arctanh(tanh(b*x+a))^(1/2)*a-(arctanh
(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.54, size = 30, normalized size = 0.83

$$\frac{2\left(b^2x^2 - abx - 2\,a^2\right)}{3\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)

Fricas [A]
time = 0.34, size = 19, normalized size = 0.53

$$\frac{2\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/atanh(tanh(b*x+a))**(1/2),x)

[Out] Integral(x/sqrt(atanh(tanh(a + b*x))), x)

Giac [A]
time = 0.38, size = 23, normalized size = 0.64

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")

[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2

Mupad [B]
time = 1.21, size = 105, normalized size = 2.92

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+3\,b\,x\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/atanh(tanh(a + b*x))^(1/2),x)

[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 3*b*x))/(3*b^2)

3.144 $\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b}$	15
default	$\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/b
```

Maxima [A]
time = 0.52, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A]
time = 0.38, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A]
time = 23.21, size = 24, normalized size = 1.50

$$\begin{cases} \frac{2\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{b} & \text{for}\ b\neq 0 \\ \frac{x}{\sqrt{\operatorname{atanh}\left(\tanh\left(a\right)\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/sqrt(atanh(tanh(a)
)), True))
```

Giac [A]
time = 0.38, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Mupad [B]
time = 1.18, size = 52, normalized size = 3.25

$$\frac{2\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/b
```

3.145 $\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=49

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2192}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.86

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)}{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A]
time = 0.37, size = 56, normalized size = 1.14

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.39, size = 21, normalized size = 0.43

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 7.19, size = 285, normalized size = 5.82

$$\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,\left(\frac{\sqrt{2}\,b\,x}{2}-\frac{\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{2}+\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{x}\right)\,1{}\mathrm{i}}{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2^(1/2)*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^(1/2)*1i - (2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/2 + (2
^(1/2)*b*x)/2)*1i)/x)*1i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
```

3.146 $\int \frac{1}{x^2 \sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=94

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(3/2)-1/x/arctanh(tanh(b*x+a))^(1/2)+b/(b*x-arctanh(t
anh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]])
+ b/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{2}b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-}}}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx}}$$

Mathematica [A]
time = 0.05, size = 78, normalized size = 0.83

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - Sqrt[ArcTanh[Tanh[a + b*x]]]
/(x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.07, size = 95, normalized size = 1.01

method	result	si
default	$2b\left(\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\operatorname{arctanh}(\tanh(bx+a))+4bx\right)bx}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)}{\left(-4\operatorname{arctanh}(\tanh(bx+a))+4bx\right)\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4
*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arcta
nh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A]
time = 0.34, size = 93, normalized size = 0.99

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2a^2x},\,-\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.38, size = 47, normalized size = 0.50

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx+a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

Mupad [B]
time = 7.02, size = 570, normalized size = 6.06

$$\frac{2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\sqrt{2}\,b\ln\left(\frac{\left(\sqrt{2}\,bx-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2i\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)1i}{2x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)1i}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2)*b*lo
g((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 -
 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^(3/2)
```

3.147 $\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=158

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/4*b/x/arctanh(tanh(b*x+a))^(3/2)-1/4*b^
2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/2/x^2/arctanh(tan
h(b*x+a))^(1/2)+3/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(
1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b/(4*x*ArcTanh[Tanh[a + b*x
]]^(3/2)) - b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) - 1/(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^2)/(4*(b*x - ArcTanh[T
anh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{4}b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}\left(3b^2\right)$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b}{4x\tanh^{-1}(\tanh(a+b}$$

Mathematica [A]
time = 0.06, size = 98, normalized size = 0.62

$$\frac{1}{4}\left(-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{\left(5bx-2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] ((-3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + ((5*b*x - 2*ArcTanh[Ta
nh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(x^2*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^2))/4
```

Maple [A]
time = 0.07, size = 148, normalized size = 0.94

method	result
default	$2b^{2}\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\operatorname{arctanh}(\tanh(bx+a))+4bx\right)b^{2}x^{2}}+\frac{\frac{6\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\operatorname{arctanh}(\tanh(bx+a))+4bx\right)bx}-\frac{6\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\right.}}{\sqrt{\operatorname{arctanh}\left(\tanh\left(b\right.\right.}}\right.}{\left(-4\operatorname{arctanh}(\tanh(bx+a))+4bx\right)\sqrt{\operatorname{arctanh}\left(\right.}}}{-4\operatorname{arctanh}(\tanh(bx+a))+4bx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3
/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(
tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x
+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*
x)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A]
time = 0.40, size = 123, normalized size = 0.78

$$\left[\frac{3\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\left(3\,abx-2\,a^{2}\right)\sqrt{bx+a}}{8\,a^{3}x^{2}},\ \frac{3\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,abx-2\,a^{2}\right)\sqrt{bx+a}}{4\,a^{3}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.39, size = 69, normalized size = 0.44

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}\,ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 6.01, size = 802, normalized size = 5.08

$$\frac{2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{x^2\left(2\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+4bx\right)}+\frac{3b\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}+\frac{\sqrt{2}\,b^2\ln\left(\frac{\left(\sqrt{2}\,bx-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\;2\mathrm{i}\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^5+40a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-80a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-32a^5-10a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+80a^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)1\mathrm{i}}{x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)3\mathrm{i}}{2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2
*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (3*b*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*e
xp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^2*log
((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(
```

```
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 +
40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))*1i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*3i)/(2*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(5/2))
```

3.148 $\int \frac{1}{x^4 \sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=212

$$\frac{5b^3 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

```
[Out] 5/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(7/2)-1/8*b^2/x/arctanh(tanh(b*x+a))^(5/2)+1/8*
b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)+1/12*b/x^2/arctan
h(tanh(b*x+a))^(3/2)-5/24*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x
+a))^(3/2)-1/3/x^3/arctanh(tanh(b*x+a))^(1/2)+5/8*b^3/(b*x-arctanh(tanh(b*x
+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{5b^3 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) - b^2/(8*x*ArcTanh[Tanh[a + b
*x]]^(5/2)) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(5/2)) + b/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (5*b^3)/(24*(b*x - ArcTa
nh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - 1/(3*x^3*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh
[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{6}b\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}b^2\int$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{3x^3\sqrt{}}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{b^2}{8x\tanh^{-1}(\tanh(a+bx)}$$

Mathematica [A]

time = 0.07, size = 117, normalized size = 0.55

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(33b^2x^2-26bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2\right)}{24x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (5*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) + (Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(33*b^2*x^2 - 26*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a +
b*x]]^2))/(24*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A]
time = 0.07, size = 200, normalized size = 0.94

method	result
default	$2b^3\left(\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)b^3x^3}+\frac{\frac{10\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)b^2x^2}+\frac{10\left(\frac{6\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)bx}\right.}{-4\operatorname{arctanh}(\tanh}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(2/3*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^3/x
^3+10/3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(arctanh(tanh(b*x+a))^(1/2)/(-4*arc
tanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arcta
nh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tan
h(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+
a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x^4*sqrt(arctanh(tanh(b*x + a)))), x)

Fricas [A]
time = 0.35, size = 145, normalized size = 0.68

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^4x^3},-\frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")

[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqrt(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/atanh(tanh(b*x+a))**(1/2),x)

[Out] Integral(1/(x**4*sqrt(atanh(tanh(a + b*x)))), x)

Giac [A]
time = 0.42, size = 84, normalized size = 0.40

$$-\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}\,a^2b^4}{a^3b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")

[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*x^3))/b

Mupad [B]
time = 5.75, size = 1086, normalized size = 5.12

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{x^3\left(3\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-3\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+6\,b\,x\right)}+\frac{5\,b^2\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{x\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}+\frac{10\,b\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{3\,x^2\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)\left(2\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-2\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+4\,b\,x\right)}+\frac{\sqrt{2}\,b^3\ln\left(\frac{\left(\left(\sqrt{2}\,b\,x-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,21i\right)\left(\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^7+84\,a^2\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^5-280\,a^3\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^4+560\,a^4\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3-672\,a^5\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2-128\,a^7-14\,a\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^6+448\,a^6\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)\right)1i}{x\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}\right)5i}{2\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*atanh(tanh(a + b*x))^(1/2)),x)

[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x)) + (5*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (2^(1/2)*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*5i)/(2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2)) + (10*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x))

3.149 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{128x\tanh^{-1}(t}{5}$$

```
[Out] -32*x^2*arctanh(tanh(b*x+a))^(3/2)/b^3+128/5*x*arctanh(tanh(b*x+a))^(5/2)/b
^4-256/35*arctanh(tanh(b*x+a))^(7/2)/b^5-2*x^4/b/arctanh(tanh(b*x+a))^(1/2)
+16*x^3*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$-\frac{256\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^5}+\frac{128x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (16*x^3*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (32*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (128*x*ArcTanh[Tan
h[a + b*x]]^(5/2))/(5*b^4) - (256*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{48\int x^2\sqrt{}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tan}{}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tan}{}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tan}{}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tan}{}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.87

$$-\frac{2\left(35b^4x^4-280b^3x^3\tanh^{-1}(\tanh(a+bx))+560b^2x^2\tanh^{-1}(\tanh(a+bx))^2-448bx\tanh^{-1}(\tanh(a+bx))^3+128\tanh^{-1}(\tanh(a+bx))^4\right)}{35b^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(35*b^4*x^4 - 280*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 560*b^2*x^2*ArcTanh[
Tanh[a + b*x]]^2 - 448*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a +
b*x]]^4))/(35*b^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(81) = 162.
time = 0.07, size = 319, normalized size = 3.36

method	result
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}a}{5}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{5}+4\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}a^2+8\operatorname{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/7*arctanh(tanh(b*x+a))^(7/2)-4/5*arctanh(tanh(b*x+a))^(5/2)*a-4/5*
arctanh(tanh(b*x+a))^(5/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*arctanh(tanh(b*x+
a))^(3/2)*a^2+4*arctanh(tanh(b*x+a))^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)+2
*arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-4*a^3*arctanh(ta
nh(b*x+a))^(1/2)-12*a^2*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(
1/2)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-4*(arct
anh(tanh(b*x+a))-b*x-a)^3*arctanh(tanh(b*x+a))^(1/2)-(a^4+4*a^3*(arctanh(ta
nh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*
x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.53, size = 64, normalized size = 0.67

$$\frac{2\left(5\,b^5x^5-3\,ab^4x^4+8\,a^2b^3x^3-48\,a^3b^2x^2-192\,a^4bx-128\,a^5\right)}{35\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^5*x^5 - 3*a*b^4*x^4 + 8*a^2*b^3*x^3 - 48*a^3*b^2*x^2 - 192*a^4*b*
x - 128*a^5)/((b*x + a)^(3/2)*b^5)
```

Fricas [A]
time = 0.33, size = 63, normalized size = 0.66

$$\frac{2\left(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4\right)\sqrt{bx+a}}{35\,(b^6x+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.40, size = 77, normalized size = 0.81

$$-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30}-28\,(bx+a)^{\frac{5}{2}}ab^{30}+70\,(bx+a)^{\frac{3}{2}}a^2b^{30}-140\,\sqrt{bx+a}\,a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

Mupad [B]
time = 1.45, size = 1057, normalized size = 11.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^4) + (2*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2/(2*b^3) + (4*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (12*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 + b*x))/(7*b^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b))*(log(2/
(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(7*b^2)
 + (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (4*(
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)/b^2 + (12*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b^2))*(lo
```

```
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x))/(5*b)))/(3*b) - ((log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^4)/(4*b^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) + (x^2*((log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)/b^2 + (12*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b^2))*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2))/(5*b)
```

3.150 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{32\tanh^{-1}(\tan$$

```
[Out] -16*x*arctanh(tanh(b*x+a))^(3/2)/b^3+32/5*arctanh(tanh(b*x+a))^(5/2)/b^4-2*
x^3/b/arctanh(tanh(b*x+a))^(1/2)+12*x^2*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]

time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{32\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (12*x^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (16*x*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (32*ArcTanh[Tanh[a +
 b*x]]^(5/2))/(5*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{6\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{24\int x\sqrt{\text{t}}}{}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{}}{}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{}}{}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{}}{}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.89

$$\frac{2\left(-5b^3x^3+30b^2x^2\tanh^{-1}(\tanh(a+bx))-40bx\tanh^{-1}(\tanh(a+bx))^2+16\tanh^{-1}(\tanh(a+bx))^3\right)}{5b^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
[Out] (2*(-5*b^3*x^3 + 30*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 40*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(5*b^4*Sqrt[ArcTanh[Tanh[a + b*x]
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(64) = 128.
time = 0.07, size = 201, normalized size = 2.72

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}-2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}a-2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}(\mathrm{arctanh}(\tanh(bx+a))-bx-a)+6\sqrt{\mathrm{arctanh}\,(\tanh}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/5*arctanh(tanh(b*x+a))^(5/2)-arctanh(tanh(b*x+a))^(3/2)*a-arctanh(
tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+3*arctanh(tanh(b*x+a))^(1/2
)*a^2+6*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+3*(arctan
h(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-a^3-3*a^2*(arctanh(tan
h(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+a))-b
*x-a)^3)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.52, size = 52, normalized size = 0.70

$$\frac{2\,(b^4x^4 - ab^3x^3 + 6\,a^2b^2x^2 + 24\,a^3bx + 16\,a^4)}{5\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^4*x^4 - a*b^3*x^3 + 6*a^2*b^2*x^2 + 24*a^3*b*x + 16*a^4)/((b*x + a)^
(3/2)*b^4)
```

Fricas [A]
time = 0.32, size = 51, normalized size = 0.69

$$\frac{2\,(b^3x^3 - 2\,ab^2x^2 + 8\,a^2bx + 16\,a^3)\sqrt{bx+a}}{5\,(b^5x + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.40, size = 61, normalized size = 0.82

$$\frac{2\,a^{3}}{\sqrt{bx+a}\,b^{4}}+\frac{2\,\left(\left(bx+a\right)^{\frac{5}{2}}b^{16}-5\,\left(bx+a\right)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}\,a^{2}b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

Mupad [B]
time = 1.26, size = 660, normalized size = 8.92

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}{2\,b^{3}}+\frac{2\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}{b^{2}}+\frac{8\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{5\,b^{2}}\right)\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3\,b}\right)}{b}+\frac{2\,x^{2}\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{5\,b^{2}}+\frac{x\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}{b^{2}}+\frac{8\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{5\,b^{2}}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b}-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{3}}{2\,b^{4}\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (2*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^2))*(log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(5*b^2) +
 (x*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^2))
*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^3)/(2*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.151 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}} dx$

Optimal. Leaf size=55

$$-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}$$

```
[Out] -16/3*arctanh(tanh(b*x+a))^(3/2)/b^3-2*x^2/b/arctanh(tanh(b*x+a))^(1/2)+8*x
*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}+\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^2)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (8*x*Sqrt[ArcTanh[Tanh[a + b*x]
]])/b^2 - (16*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{4\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\int \sqrt{\tanh^{-}}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\mathrm{Subst}(\int \sqrt{}}{}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16\tanh^{-1}(\mathrm{t}}{}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.89

$$-\frac{2\left(3b^2x^2-12bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2\right)}{3b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(3*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a + b*x]]^2
))/(3*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.
time = 0.07, size = 106, normalized size = 1.93

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-4\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}\,a-4(\mathrm{arctanh}(\tanh(bx+a))-bx-a)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/3*arctanh(tanh(b*x+a))^(3/2)-2*arctanh(tanh(b*x+a))^(1/2)*a-2*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(a^2+2*a*(arctanh(tanh(
b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.54, size = 41, normalized size = 0.75

$$\frac{2\left(b^{3}x^{3}-3\,ab^{2}x^{2}-12\,a^{2}bx-8\,a^{3}\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)/((b*x + a)^(3/2)*b^3)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.73

$$\frac{2\left(b^{2}x^{2}-4\,abx-8\,a^{2}\right)\sqrt{bx+a}}{3\left(b^{4}x+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(3/2),x)
[Out] Integral(x**2/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.39, size = 46, normalized size = 0.84

$$-\frac{2\,a^{2}}{\sqrt{bx+a}\,b^{3}}+\frac{2\left(\left(bx+a\right)^{\frac{3}{2}}b^{6}-6\,\sqrt{bx+a}\,ab^{6}\right)}{3\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

Mupad [B]
time = 1.29, size = 259, normalized size = 4.71

$$-\frac{4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(3\,b^2\,x^2-6\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-4\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\right)}{3\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] -(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 3
*b^2*x^2 - 6*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6
*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.152 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}} dx$

Optimal. Leaf size=34

$$-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}$$

```
[Out] -2*x/b/arctanh(tanh(b*x+a))^(1/2)+4*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x]]])/
b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tanh^{-1}(\tanh(a+bx)\right)}{b^2}$$

$$= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.85

$$\frac{-2bx + 4\tanh^{-1}(\tanh(a+bx))}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
[Out] (-2*b*x + 4*ArcTanh[Tanh[a + b*x]])/(b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A]
time = 0.06, size = 40, normalized size = 1.18

method	result	size
default	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}}{b^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(arctanh(tanh(b*x+a))^(1/2)-(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [A]
time = 0.54, size = 30, normalized size = 0.88

$$\frac{2\left(b^2x^2 + 3\,abx + 2\,a^2\right)}{(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b^2*x^2 + 3*a*b*x + 2*a^2)/((b*x + a)^(3/2)*b^2)
```

Fricas [A]
time = 0.37, size = 29, normalized size = 0.85

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 46.13, size = 46, normalized size = 1.35

$$\begin{cases} -\frac{2x}{b\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}+\frac{4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{b^2} & \text{for } b\neq 0 \\ \frac{x^2}{2\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((-2*x/(b*sqrt(atanh(tanh(a + b*x)))) + 4*sqrt(atanh(tanh(a + b*x)
))/b**2, Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(3/2)), True))
```

Giac [A]
time = 0.39, size = 29, normalized size = 0.85

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+a}\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

Mupad [B]
time = 1.33, size = 152, normalized size = 4.47

$$-\frac{4\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+b\,x\right)}{b^2\,\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] -(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(b^2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))
))
```

3.153 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/b/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{2}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}$	15
default	$-\frac{2}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A]
time = 0.52, size = 12, normalized size = 0.75

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A]
time = 0.34, size = 20, normalized size = 1.25

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A]
time = 45.83, size = 26, normalized size = 1.62

$$\begin{cases} -\frac{2}{b\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}} & \text{for}\: b \neq 0 \\ \frac{x}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((-2/(b*sqrt(atanh(tanh(a + b*x)))), Ne(b, 0)), (x/atanh(tanh(a))*
*(3/2), True))
```

Giac [A]
time = 0.38, size = 12, normalized size = 0.75

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Mupad [B]
time = 1.31, size = 97, normalized size = 6.06

$$\frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{b\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(b*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))))
```

3.154 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=78

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(3/2)-2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+
a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2194, 2192}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{}{x\sqrt{\tanh^{-1}}}}{-bx+\tan}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \frac{2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[ArcTanh[Tanh[a + b*
x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.07, size = 68, normalized size = 0.87

method	result
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)}{\left(\mathrm{arctanh}(\tanh(bx+a))-bx\right)^{\frac{3}{2}}}+\frac{2}{\left(\mathrm{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2))+2/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x
+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 110, normalized size = 1.41

$$\left[\frac{(bx+a)\sqrt{a}\,\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3},\ \frac{2\left((bx+a)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.39, size = 37, normalized size = 0.47

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

Mupad [B]
time = 6.14, size = 614, normalized size = 7.87

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-\sqrt{2}\,bx+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2\mathrm{i}\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)1\mathrm{i}}{2x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)2\mathrm{i}}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
 + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*2i)/(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(3/2) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))
```

3.155 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=124

$$-\frac{3b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b
*x-arctanh(tanh(b*x+a)))^(5/2)-1/x/arctanh(tanh(b*x+a))^(3/2)+b/(b*x-arctan
h(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-3*b/(b*x-arctanh(tanh(b*x+a)))^2
/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$-\frac{3b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]
]])/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2
)) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)) - (3*b
)/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}(3b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{t}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{t}$$

$$= -\frac{3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{1}{x\tanh^{-1}(\tanh(a+b$$

Mathematica [A]
time = 0.06, size = 91, normalized size = 0.73

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{2bx+\tanh^{-1}(\tanh(a+bx))}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (3*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (2*b*x + ArcTanh[Tanh[a +
b*x]])/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.07, size = 105, normalized size = 0.85

method	result
default	$2b\left(-\frac{\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2bx}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}-\frac{}{(\operatorname{arctanh}(\tanh(bx+a))-}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^2*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-3/
2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arct
anh(tanh(b*x+a))-b*x)^(1/2)))-1/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A]
time = 0.39, size = 151, normalized size = 1.22

$$\left[\frac{3\,(b^2x^2+abx)\sqrt{a}\,\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\,(3\,abx+a^2)\sqrt{bx+a}}{2\,(a^3bx^2+a^4x)},-\frac{3\,(b^2x^2+abx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+a^2)\sqrt{bx+a}}{a^3bx^2+a^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.41, size = 64, normalized size = 0.52

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

Mupad [B]
time = 5.86, size = 807, normalized size = 6.51

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x}-\frac{24\,b\,x}{{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{2\,b\,x^2-x\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\left(\sqrt{2}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)-\sqrt{2}\,b\,x+\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x}\,2{}\mathrm{i}\right)\,\left({\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^5+40\,a^2\,{\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3-80\,a^3\,{\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2-32\,a^5-10\,a\,{\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4+80\,a^4\,\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)\,1{}\mathrm{i}}{2\,x\,\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,6{}\mathrm{i}}{{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2)*b*lo
g((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i + 2^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 +
 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
```

```
2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*6i)/(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(5/2)
```

3.156 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=191

$$-\frac{15b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}}-\frac{}{4\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

```
[Out] -15/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(7/2)+3/4*b/x/arctanh(tanh(b*x+a))^(5/2)-3/4*
b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-1/2/x^2/arctanh(t
anh(b*x+a))^(3/2)+5/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))
^(3/2)-15/4*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$-\frac{15b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{15b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}-\frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + (3*b)/(4*x*ArcTanh[Tanh[a
 + b*x]]^(5/2)) - (3*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(5/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b^2)/(4*(b*x -
 ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (15*b^2)/(4*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{4}(3b)\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,d$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}(15b^2)$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}}$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}}$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}}$$

$$= -\frac{15b^2 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{3b}{4x \tanh^{-1}(\tanh(a+}$$

Mathematica [A]
time = 0.08, size = 115, normalized size = 0.60

$$\frac{1}{4}\left(-\frac{15b^2 \tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{8b^2x^2 + 9bx\tanh^{-1}(\tanh(a+bx)) - 2\tanh^{-1}(\tanh(a+bx))^2}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] ((-15*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (8*b^2*x^2 + 9*b*x*Ar
cTanh[Tanh[a + b*x]] - 2*ArcTanh[Tanh[a + b*x]]^2)/(x^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/4
```

Maple [A]
time = 0.08, size = 131, normalized size = 0.69

method	result
default	$2b^2\left(\frac{\frac{\frac{7\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(-\frac{9\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^2x^2}-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}}}{\sqrt{\operatorname{arctanh}\left(\mathrm{ta}\right.}}\right.}{8\sqrt{\operatorname{arctanh}\left(\tanh\right.}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^3*((7/8*arctanh(tanh(b*x+a))^(3/2)+(-9/
8*arctanh(tanh(b*x+a))+9/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8/(a
rctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(
tanh(b*x+a))-b*x)^(1/2)))+1/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a
))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 189, normalized size = 0.99

$$\left[\frac{15\,(b^3x^3+ab^2x^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{bx+a}}{8\,(a^4bx^3+a^5x^2)},\ \frac{15\,(b^3x^3+ab^2x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{bx+a}}{4\,(a^4bx^3+a^5x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.40, size = 80, normalized size = 0.42

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^3}+\frac{2\,b^2}{\sqrt{bx+a}\,a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\sqrt{bx+a}\,ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 6.07, size = 1028, normalized size = 5.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2
)*2i + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
```

```
)) + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^
5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2)))*15i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1
))/2)^(1/2))/(x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + ((log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((14*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (60*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(
2*a)*exp(2*b*x) + 1))))
```

3.157 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=245

$$-\frac{35b^3 \operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{5b^2}{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}+\frac{5}{8\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

```
[Out] -35/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(9/2)-5/8*b^2/x/arctanh(tanh(b*x+a))^(7/2)+5/
8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)+1/4*b/x^2/arcta
nh(tanh(b*x+a))^(5/2)-7/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x
+a))^(5/2)-1/3/x^3/arctanh(tanh(b*x+a))^(3/2)+35/24*b^3/(b*x-arctanh(tanh(b
*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)-35/8*b^3/(b*x-arctanh(tanh(b*x+a)))^4/
arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$-\frac{35b^3 \operatorname{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{7b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}-\frac{5b^2}{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}-\frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{b}{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-35*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) - (5*b^2)/(8*x*ArcTanh[Tanh
[a + b*x]]^(7/2)) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[
a + b*x]]^(7/2)) + b/(4*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (7*b^3)/(8*(b*x
 - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - 1/(3*x^3*ArcTa
nh[Tanh[a + b*x]]^(3/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])^3*Ar
cTanh[Tanh[a + b*x]]^(3/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^4*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}b \int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$

$$= \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}(5b^2)$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{}{3x^3 \tan}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-}}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-}}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-}}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-}}$$

$$= -\frac{35b^3 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx - \tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{5b^2}{8x \tanh^{-1}(\tanh(a+}$$

Mathematica [A]

time = 0.08, size = 133, normalized size = 0.54

$$\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{48b^3x^3+87b^2x^2\tanh^{-1}(\tanh(a+bx))-38bx\tanh^{-1}(\tanh(a+bx))^2+8\tanh^{-1}(\tanh(a+bx))^3}{24x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (35*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) - (48*b^3*x^3 + 87*b^
2*x^2*ArcTanh[Tanh[a + b*x]] - 38*b*x*ArcTanh[Tanh[a + b*x]]^2 + 8*ArcTanh[
Tanh[a + b*x]]^3)/(24*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^4)
```

Maple [A]
time = 0.08, size = 186, normalized size = 0.76

method	result
default	$2b^3\left(-\frac{\frac{\frac{19\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(-\frac{17\,\mathrm{arctanh}(\tanh(bx+a))}{6}+\frac{17bx}{6}\right)\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{29a^2}{16}+\frac{29a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{8}+29(\right.}{b^3x^3}}{(\mathrm{arctanh}(\tan}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^4*((19/16*arctanh(tanh(b*x+a))^(5/2)+(
-17/6*arctanh(tanh(b*x+a))+17/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(29/16*a^2+
29/8*a*(arctanh(tanh(b*x+a))-b*x-a)+29/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*a
rctanh(tanh(b*x+a))^(1/2))/b^3/x^3-35/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*a
rctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-1/(arc
tanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A]
time = 0.39, size = 211, normalized size = 0.86

$$\left[\frac{105\,\left(b^{4}x^{4}+ab^{3}x^{3}\right)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,\left(105\,ab^{3}x^{3}+35\,a^{2}b^{2}x^{2}-14\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{48\,\left(a^{5}bx^{4}+a^{6}x^{3}\right)},\,-\frac{105\,\left(b^{4}x^{4}+ab^{3}x^{3}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(105\,ab^{3}x^{3}+35\,a^{2}b^{2}x^{2}-14\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{24\,\left(a^{5}bx^{4}+a^{6}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
 + 2*a)/x) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b
*x + a))/(a^5*b*x^4 + a^6*x^3), -1/24*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(-a)*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*
b*x + 8*a^4)*sqrt(b*x + a))/(a^5*b*x^4 + a^6*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**4*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.40, size = 95, normalized size = 0.39

$$-\frac{35\,b^{3}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^{4}}-\frac{2\,b^{3}}{\sqrt{bx+a}\,a^{4}}-\frac{57\,\left(bx+a\right)^{\frac{5}{2}}b^{3}-136\,\left(bx+a\right)^{\frac{3}{2}}ab^{3}+87\,\sqrt{bx+a}\,a^{2}b^{3}}{24\,a^{4}b^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) - 2*b^3/(sqrt(b*x +
 a)*a^4) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)*a*b^3 + 87*sq
rt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3)
```

Mupad [B]
time = 5.09, size = 1258, normalized size = 5.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atanh(tanh(a + b*x))^(3/2)),x)

[Out] (((38*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (140*b^3*x)/(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^4)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (4*(log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^3*log((((log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*
b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i + 2^(1/2)*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^9 + 144*a^2*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 672*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 +
 2016*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 4032*a^5*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^4 + 5376*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 4608*a^7*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2 - 512*a^9 - 18*a*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8
+ 2304*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^(1/2)))*35i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(9/2) - (22*b*(log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2))/(3*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.158 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{128x\tanh^{-1}(}{}$$

```
[Out] -2/3*x^4/b/arctanh(tanh(b*x+a))^(3/2)-128/3*x*arctanh(tanh(b*x+a))^(3/2)/b^
4+256/15*arctanh(tanh(b*x+a))^(5/2)/b^5-16/3*x^3/b^2/arctanh(tanh(b*x+a))^(
1/2)+32*x^2*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A]

time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^4)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (16*x^3)/(3*b^2*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (32*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (128*x*ArcTa
nh[Tanh[a + b*x]]^(3/2))/(3*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{8\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\int \frac{}{\sqrt{\tan}}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tan}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tan}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tan}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tan}}{}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.84

$$-\frac{2\left(5b^4x^4+40b^3x^3\tanh^{-1}(\tanh(a+bx))-240b^2x^2\tanh^{-1}(\tanh(a+bx))^2+320bx\tanh^{-1}(\tanh(a+bx))^3-128\tanh^{-1}(\tanh(a+bx))^4\right)}{15b^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(5*b^4*x^4 + 40*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 240*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 320*b*x*ArcTanh[Tanh[a + b*x]]^3 - 128*ArcTanh[Tanh[a + b*
x]]^4))/(15*b^5*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(81) = 162$.
time = 0.07, size = 295, normalized size = 2.98

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}-\frac{8 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}} a}{3}-\frac{8 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}} (\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+12 \sqrt{\operatorname{arctanh}\left(\tanh\left(bx\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/5*arctanh(tanh(b*x+a))^(5/2)-4/3*arctanh(tanh(b*x+a))^(3/2)*a-4/3*
arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+6*arctanh(tanh(b*x+
a))^(1/2)*a^2+12*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+
6*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-4*a^3-12*a^2*
(arctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4*(arctanh
(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(1/2)-1/3*(a^4+4*a^3*(arctanh(
tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(
b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(3/2)
)
```

Maxima [A]
time = 0.53, size = 64, normalized size = 0.65

$$\frac{2\left(3 b^{5} x^{5}-5 a b^{4} x^{4}+40 a^{2} b^{3} x^{3}+240 a^{3} b^{2} x^{2}+320 a^{4} b x+128 a^{5}\right)}{15\left(bx+a\right)^{\frac{5}{2}} b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^5*x^5 - 5*a*b^4*x^4 + 40*a^2*b^3*x^3 + 240*a^3*b^2*x^2 + 320*a^4*
b*x + 128*a^5)/((b*x + a)^(5/2)*b^5)
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.75

$$\frac{2\left(3 b^{4} x^{4}-8 a b^{3} x^{3}+48 a^{2} b^{2} x^{2}+192 a^{3} b x+128 a^{4}\right)\sqrt{bx+a}}{15\left(b^{7} x^{2}+2 a b^{6} x+a^{2} b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.38, size = 75, normalized size = 0.76

$$\frac{2\left(12\left(bx+a\right)a^{3}-a^{4}\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^{5}}+\frac{2\left(3\left(bx+a\right)^{\frac{5}{2}}b^{20}-20\left(bx+a\right)^{\frac{3}{2}}ab^{20}+90\sqrt{bx+a}\,a^{2}b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

Mupad [B]
time = 1.42, size = 817, normalized size = 8.25

$$\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{3\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{2}}{2b^{4}}+\frac{2\left(\frac{2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{b^{3}}+\frac{8\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5b^{3}}\right)\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{3b}\right)}{b}+\frac{2x^{2}\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{5b^{3}}+\frac{x\left(\frac{2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{b^{3}}+\frac{8\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}+bx\right)}{5b^{3}}\right)\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{3b}-\frac{2\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{3}}{b^{5}\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{4}}{6b^{5}\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (2*((
2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^3))*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(
5*b^3) + (x*((2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2*a)*exp(2*b
*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*
x))/(5*b^3))*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - (2*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(b^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - ((log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
```

```
/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/(6*b^5*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.159 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{32\tanh^{-1}(\tanh(}{3b^4}$$

```
[Out] -2/3*x^3/b/arctanh(tanh(b*x+a))^(3/2)-32/3*arctanh(tanh(b*x+a))^(3/2)/b^4-4
*x^2/b^2/arctanh(tanh(b*x+a))^(1/2)+16*x*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^3)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (4*x^2)/(b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (32*ArcTanh[Tanh[
a + b*x]]^(3/2))/(3*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{}{\sqrt{\tanh}}}{}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh}}{}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh}}{}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh}}{}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.86

$$-\frac{2\left(b^3x^3+6b^2x^2\tanh^{-1}(\tanh(a+bx))-24bx\tanh^{-1}(\tanh(a+bx))^2+16\tanh^{-1}(\tanh(a+bx))^3\right)}{3b^4\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 24*b*x*ArcTanh[Tanh[a + b
*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(3*b^4*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(64) = 128.
time = 0.07, size = 186, normalized size = 2.45

method	result
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-6\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\,a-6(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/3*arctanh(tanh(b*x+a))^(3/2)-3*arctanh(tanh(b*x+a))^(1/2)*a-3*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(3*a^2+6*a*(arctanh(tan
h(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/
2)-1/3*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b
*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A]
time = 0.54, size = 52, normalized size = 0.68

$$\frac{2\left(b^{4}x^{4}-5\,ab^{3}x^{3}-30\,a^{2}b^{2}x^{2}-40\,a^{3}bx-16\,a^{4}\right)}{3\left(bx+a\right)^{\frac{5}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^4*x^4 - 5*a*b^3*x^3 - 30*a^2*b^2*x^2 - 40*a^3*b*x - 16*a^4)/((b*x +
a)^(5/2)*b^4)
```

Fricas [A]
time = 0.34, size = 62, normalized size = 0.82

$$\frac{2\left(b^{3}x^{3}-6\,ab^{2}x^{2}-24\,a^{2}bx-16\,a^{3}\right)\sqrt{bx+a}}{3\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

Sympy [A]
time = 72.69, size = 90, normalized size = 1.18

$$\begin{cases} -\frac{2x^{3}}{3b\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}-\frac{4x^{2}}{b^{2}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}+\frac{16x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{b^{3}}-\frac{32\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{3b^{4}} & \text{for}\: b\neq 0\\ \frac{x^{4}}{4\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*x**3/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4*x**2/(b**2*sqrt(at
anh(tanh(a + b*x)))) + 16*x*sqrt(atanh(tanh(a + b*x)))/b**3 - 32*atanh(tanh
(a + b*x))**(3/2)/(3*b**4), Ne(b, 0)), (x**4/(4*atanh(tanh(a))**(5/2)), Tru
e))
```

Giac [A]
time = 0.40, size = 59, normalized size = 0.78

$$-\frac{2\left(9\left(bx+a\right)a^{2}-a^{3}\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^{4}}+\frac{2\left(\left(bx+a\right)^{\frac{3}{2}}b^{8}-9\sqrt{bx+a}\,ab^{8}\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

Mupad [B]
time = 1.30, size = 533, normalized size = 7.01

$$\frac{2\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b^3}+\frac{\left(\frac{2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}+\frac{4\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3\,b^3}\right)\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{b}-\frac{3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{b^4\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{3\,b^4\,{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^3) + (((2*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b
^3 + (4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3*b^3))*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/b
- (3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(b^4*(log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
+ 1)))) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(3*b^4*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))^2)
```

3.160 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3}$$

```
[Out] -2/3*x^2/b/arctanh(tanh(b*x+a))^(3/2)-8/3*x/b^2/arctanh(tanh(b*x+a))^(1/2)+
16/3*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2188, 30}

$$\frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3}-\frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^2)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (8*x)/(3*b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{4\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{}{\sqrt{\tan}}}{}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\text{Subst}\Big(}{}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\sqrt{\tanh}}{}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.81

$$-\frac{2\left(b^2x^2+4bx\tanh^{-1}(\tanh(a+bx))-8\tanh^{-1}(\tanh(a+bx))^2\right)}{3b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
[Out] (-2*(b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*x]]^2))/
(3*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A]
time = 0.07, size = 91, normalized size = 1.54

method	result
default	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}-\frac{2\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{2(-2\operatorname{arctanh}(\tanh(bx}{\sqrt{\operatorname{arctanh}(\tanh}}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(arctanh(tanh(b*x+a))^(1/2)-1/3*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)
+(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(3/2)-(-2*arctanh(tan
h(b*x+a))+2*b*x)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.55, size = 42, normalized size = 0.71

$$\frac{2\,(3\,b^3x^3+15\,ab^2x^2+20\,a^2bx+8\,a^3)}{3\,(bx+a)^{\frac{5}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b^3*x^3 + 15*a*b^2*x^2 + 20*a^2*b*x + 8*a^3)/((b*x + a)^(5/2)*b^3)
```

Fricas [A]
time = 0.33, size = 52, normalized size = 0.88

$$\frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)\sqrt{bx+a}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

Sympy [A]
time = 71.90, size = 71, normalized size = 1.20

$$\begin{cases} -\frac{2x^2}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}-\frac{8x}{3b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}}+\frac{16\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{3b^3} & \text{for } b\neq 0 \\ \frac{x^3}{3\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*x**2/(3*b*atanh(tanh(a + b*x))**(3/2)) - 8*x/(3*b**2*sqrt(ata
nh(tanh(a + b*x)))) + 16*sqrt(atanh(tanh(a + b*x)))/(3*b**3), Ne(b, 0)), (x
**3/(3*atanh(tanh(a))**(5/2)), True))
```

Giac [A]
time = 0.40, size = 39, normalized size = 0.66

$$\frac{2\,\sqrt{bx+a}}{b^3}+\frac{2\,(6\,(bx+a)a-a^2)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

Mupad [B]
time = 1.26, size = 259, normalized size = 4.39

$$\frac{8\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(-b^2\,x^2-2\,b\,x\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}^2-4\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}^2\right)}{3\,b^3\,{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - b^
2*x^2 - 2*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.161 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/3*x/b/arctanh(tanh(b*x+a))^(3/2)-4/3/b^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - 4/(3*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$
$$= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \tanh^{-1}(\tanh(a+bx\right.}{3b^2}$$
$$= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.82

$$-\frac{2(bx+2\tanh^{-1}(\tanh(a+bx)))}{3b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b*x + 2*ArcTanh[Tanh[a + b*x]]))/(3*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A]
time = 0.07, size = 42, normalized size = 1.11

method	result	size
default	$\frac{-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{2}{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-1/3*(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/arctanh
(tanh(b*x+a))^(1/2))
```

Maxima [A]
time = 0.53, size = 31, normalized size = 0.82

$$-\frac{2\left(3\,b^2x^2+5\,abx+2\,a^2\right)}{3\,(bx+a)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

[Out] -2/3*(3*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x + a)^(5/2)*b^2)

Fricas [A]
time = 0.34, size = 41, normalized size = 1.08

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")

[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)

Sympy [A]
time = 72.21, size = 51, normalized size = 1.34

$$\begin{cases} -\frac{2x}{3b\,\mathrm{atanh}^{\frac{3}{2}}\,(\tanh{(a+bx)})}-\frac{4}{3b^2\sqrt{\mathrm{atanh}\,(\tanh{(a+bx)})}} & \text{for}\ b\neq 0\\ \frac{x^2}{2\,\mathrm{atanh}^{\frac{5}{2}}\,(\tanh{(a)})} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/atanh(tanh(b*x+a))**(5/2),x)

[Out] Piecewise((-2*x/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4/(3*b**2*sqrt(atanh(ta nh(a + b*x)))), Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(5/2)), True))

Giac [A]
time = 0.39, size = 20, normalized size = 0.53

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")

[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)

Mupad [B]
time = 1.38, size = 152, normalized size = 4.00

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+b\,x\right)}{3\,b^2\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/atanh(tanh(a + b*x))^(5/2),x)

[Out] -(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(3*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)

3.162 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}} dx$

Optimal. Leaf size=18

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2188, 30}

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}$	15
default	$-\frac{2}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A]
time = 0.51, size = 12, normalized size = 0.67

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.33, size = 31, normalized size = 1.72

$$-\frac{2\,\sqrt{bx+a}}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A]
time = 48.57, size = 27, normalized size = 1.50

$$\begin{cases} -\frac{2}{3b\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)} & \text{for}\ b\neq 0\\ \frac{x}{\mathrm{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2/(3*b*atanh(tanh(a + b*x))**(3/2)), Ne(b, 0)), (x/atanh(tanh(a
))**(5/2), True))
```

Giac [A]
time = 0.39, size = 12, normalized size = 0.67

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Mupad [B]
time = 1.37, size = 103, normalized size = 5.72

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] -(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.163 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=108

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{}{(b}$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(5/2)-2/3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x
+a))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2194, 2192}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[A
rcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{\int \frac{1}{x\tanh^{-1}(\mathrm{t}}}{bx-\tanh^{-}}$$

$$= -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\left(bx-\tanh\right.}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{}{3\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 0.84

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{2\left(-bx+4\tanh^{-1}(\tanh(a+bx))\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(-(b*x) + 4*ArcTanh[Tanh
[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x
]])^2)
```

Maple [A]
time = 0.07, size = 93, normalized size = 0.86

method	result
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)}{\left(\mathrm{arctanh}(\tanh(bx+a))-bx\right)^{\frac{5}{2}}} + \frac{2}{\left(\mathrm{arctanh}(\tanh(bx+a))-bx\right)^{2}\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^(5/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2))+2/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b
*x+a))^(1/2)+2/3/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A]
time = 0.35, size = 177, normalized size = 1.64

$$\left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx+4\,a^2)\sqrt{bx+a}}{3\,(a^3b^2x^2+2\,a^4bx+a^5)},\ \frac{2\left(3\,(b^2x^2+2\,abx+a^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+4\,a^2)\sqrt{bx+a}\right)}{3\,(a^3b^2x^2+2\,a^4bx+a^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(5/2),x)
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.42

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x + a)^(3/2)*a^2)

Mupad [B]
time = 6.26, size = 886, normalized size = 8.20

$$\frac{16\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}-\frac{16\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{3\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{2}\,bx-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,2i\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^5+40a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-80a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-32a^5-10a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+80a^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)1i}{2x\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)4i}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*atanh(tanh(a + b*x))^(5/2)),x)

[Out] (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*4i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2) + (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))

3.164 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=155

$$\frac{5b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\text{ ta}}$$

[Out] 5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh(b*x+a)))^(7/2)-1/x/arctanh(tanh(b*x+a))^(5/2)+b/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-5/3*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+5*b/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{5b\text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{5b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}-\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]

[Out] (5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)) - (5*b)/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b)/((b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n + 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}-\frac{1}{2}(5b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan$$

$$= \frac{5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5}$$

Mathematica [A]
time = 0.09, size = 113, normalized size = 0.73

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{-2b^2x^2+14bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2}{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (-2*b^2*x^2 + 14*b*x*ArcTa
```

```
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)/(3*x*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A]
time = 0.07, size = 130, normalized size = 0.84

method	result
default	$2b\left(-\frac{\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2bx}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3}-\frac{}{3(\operatorname{arctanh}(\tanh(bx+a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^3*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-5/
2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arct
anh(tanh(b*x+a))-b*x)^(1/2)))-1/3/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh
(b*x+a))^(3/2)-2/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 221, normalized size = 1.43

$$\left[\frac{15\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^2x^2+20\,a^2bx+3\,a^3)\sqrt{bx+a}}{6\,(a^4b^2x^3+2\,a^5bx^2+a^6x)},-\frac{15\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2+20\,a^2bx+3\,a^3)\sqrt{bx+a}}{3\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A]
time = 0.39, size = 65, normalized size = 0.42

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

Mupad [B]
time = 7.34, size = 1230, normalized size = 7.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] (2^(1/2)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*
2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^5*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*ex
```

```
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^(1/2)))*20i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (32*b*(log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
1))/2)^(1/2))/(3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
log(2/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + ((log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b
*x) + 1))/2)^(1/2)*(x*((32*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (6*b*(8*log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 16*b*x))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) - (8*log(2/(exp(2*a)*exp(2*
b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 16*
b*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.165 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=224

$$\frac{35b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}}-\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

```
[Out] 35/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(9/2)+5/4*b/x/arctanh(tanh(b*x+a))^(7/2)-5/4*b
^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)-1/2/x^2/arctanh(ta
nh(b*x+a))^(5/2)+7/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^
(5/2)-35/12*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)+35/
4*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{35b^2 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{35b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^2}{12\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{7b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{5/2}}-\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}-\frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}+\frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (35*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x
]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (5*b)/(4*x*ArcTanh[Tanh[a
+ b*x]]^(7/2)) - (5*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
 b*x]]^(7/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) + (7*b^2)/(4*(b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^2)/(12*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (35*b^2)/(4*(b
*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{4}(5b)\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}$$

$$= \frac{5b}{4x \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{8}(35b^2)$$

$$= \frac{5b}{4x \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{5b}{4x \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{5b}{4x \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{5b}{4x \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{35b^2 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx - \tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{5b}{4x \tanh^{-1}(\tanh(a+b}$$

Mathematica [A]
time = 0.08, size = 133, normalized size = 0.59

$$-\frac{35b^2 \tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{-8b^3x^3 + 80b^2x^2\tanh^{-1}(\tanh(a+bx)) + 39bx\tanh^{-1}(\tanh(a+bx))^2 - 6\tanh^{-1}(\tanh(a+bx))^3}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-35*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) + (-8*b^3*x^3 + 80*b
^2*x^2*ArcTanh[Tanh[a + b*x]] + 39*b*x*ArcTanh[Tanh[a + b*x]]^2 - 6*ArcTanh
[Tanh[a + b*x]]^3)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[T
anh[a + b*x]])^4)
```

Maple [A]
time = 0.08, size = 157, normalized size = 0.70

method	result
default	$2b^2\left(\frac{\frac{\frac{11\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(-\frac{13\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{13bx}{8}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b^2x^2}-\frac{35\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}}}{\sqrt{\operatorname{arctanh}(\mathrm{ta}}}\right.}{8\sqrt{\operatorname{arctanh}(\tanh}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^4*((11/8*arctanh(tanh(b*x+a))^(3/2)+(-1
3/8*arctanh(tanh(b*x+a))+13/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-35/8
/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arcta
nh(tanh(b*x+a))-b*x)^(1/2)))+3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*
x+a))^(1/2)+1/3/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A]
time = 0.35, size = 255, normalized size = 1.14

$$\left[\frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{24\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)},\ \frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{12\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A]
time = 0.39, size = 93, normalized size = 0.42

$$\frac{35\,b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^4} + \frac{2\,(9\,(bx+a)b^2 + ab^2)}{3\,(bx+a)^{\frac{3}{2}}a^4} + \frac{11\,(bx+a)^{\frac{3}{2}}b^2 - 13\,\sqrt{bx+a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B]
time = 8.47, size = 1514, normalized size = 6.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((4*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
```

```
2*b*x) + 1)) + 2*b*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^2 -
 ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((140*b)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (280
*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
 + (2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(
1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^9 + 144*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 672*a^3*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^6 + 2016*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 4
032*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 5376*a^6*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 4608*a^7*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 512*a^9 - 18*a*(2*a
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^8 + 2304*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)
/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*70i)/(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(9/2)
```

3.166 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=278

$$\frac{105b^3 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} - \frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{}{24\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

```
[Out] 105/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(11/2)-35/24*b^2/x/arctanh(tanh(b*x+a))^(9/2)
+35/24*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(9/2)+5/12*b/x^2
/arctanh(tanh(b*x+a))^(7/2)-15/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(t
anh(b*x+a))^(7/2)-1/3/x^3/arctanh(tanh(b*x+a))^(5/2)+21/8*b^3/(b*x-arctanh(
tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(5/2)-35/8*b^3/(b*x-arctanh(tanh(b*x+a
)))^4/arctanh(tanh(b*x+a))^(3/2)+105/8*b^3/(b*x-arctanh(tanh(b*x+a)))^5/arc
tanh(tanh(b*x+a))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2192}

$$\frac{105b^3 \text{ArcTan}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} + \frac{105b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{21b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{15b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{9/2}} - \frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} - \frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{5b}{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (105*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) - (35*b^2)/(24*x*ArcTanh[T
anh[a + b*x]]^(9/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[
Tanh[a + b*x]]^(9/2)) + (5*b)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - (15*b
^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - 1/(
3*x^3*ArcTanh[Tanh[a + b*x]]^(5/2)) + (21*b^3)/(8*(b*x - ArcTanh[Tanh[a + b
*x]])^3*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a +
 b*x]])^4*ArcTanh[Tanh[a + b*x]]^(3/2)) + (105*b^3)/(8*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2192

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx &= -\frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{6}(5b)\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}} \\
&= \frac{5b}{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{24}(35 \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{5b}{12x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{}{3x^3} \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh} \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh} \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh} \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh} \\
&= -\frac{35b^2}{24x \tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh} \\
&= \frac{105b^3 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} - \frac{35b^2}{24x \tanh^{-1}(\tanh(a}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 150, normalized size = 0.54

$$
\frac{1}{24}\left(\frac{315b^3 \tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} + \frac{-16b^4x^4 + 208b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 165b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 50bx \tanh^{-1}(\tanh(a+bx))^3 + 8\tanh^{-1}(\tanh(a+bx))^4}{x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5 \tanh^{-1}(\tanh(a+bx))^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] ((315*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) + (-16*b^4*x^4 + 208*b
^3*x^3*ArcTanh[Tanh[a + b*x]] + 165*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 50*b
*x*ArcTanh[Tanh[a + b*x]]^3 + 8*ArcTanh[Tanh[a + b*x]]^4)/(x^3*(b*x - ArcTa
nh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/2)))/24
```

Maple [A]
time = 0.08, size = 211, normalized size = 0.76

method	result
default	$2b^3\left(-\dfrac{\dfrac{\frac{41\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(-\frac{35\,\mathrm{arctanh}(\tanh(bx+a))}{6}+\frac{35bx}{6}\right)\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{55a^2}{16}+\frac{55a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{8}+\frac{55(}{}\right.}{b^3x^3}}{(\mathrm{arctanh}(\tan}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^5*((41/16*arctanh(tanh(b*x+a))^(5/2)+(
-35/6*arctanh(tanh(b*x+a))+35/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(55/16*a^2+
55/8*a*(arctanh(tanh(b*x+a))-b*x-a)+55/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*a
rctanh(tanh(b*x+a))^(1/2))/b^3/x^3-105/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-1/3/(
arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(3/2)-4/(arctanh(tanh(b*x+
a))-b*x)^5/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 277, normalized size = 1.00

$$\left[\frac{315\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4bx+8\,a^5)\sqrt{bx+a}}{48\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)},\ -\frac{315\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4bx+8\,a^5)\sqrt{bx+a}}{24\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) - 2*(315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^
2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8
*x^3), -1/24*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(-a)*arctan(sqr
```

```
t(b*x + a)*sqrt(-a)/a) + (315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^2*x^2
- 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(5/2),x)
[Out] Integral(1/(x**4*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A]
time = 0.40, size = 115, normalized size = 0.41

$$-\frac{105\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^5}-\frac{315\,(bx+a)^4b^3-840\,(bx+a)^3ab^3+693\,(bx+a)^2a^2b^3-144\,(bx+a)a^3b^3-16\,a^4b^3}{24\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
[Out] -105/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/24*(315*(b*x +
 a)^4*b^3 - 840*(b*x + a)^3*a*b^3 + 693*(b*x + a)^2*a^2*b^3 - 144*(b*x + a)
*a^3*b^3 - 16*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5)
```

Mupad [B]
time = 6.42, size = 2359, normalized size = 8.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atanh(tanh(a + b*x))^(5/2)),x)
[Out] (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (((140*
b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - (840*b^3*x)/(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)*(
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*
exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - ((log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
```

2)*(x*((((1232*b^4)/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (80*b^3*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b) - (952*b^3)/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^2*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)) - (68*b*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) + (2^(1/2)*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^11 + 220*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^9 - 1320*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8 + 5280*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 14784*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 29568*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 42240*a^7*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 42240*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 28160*a^9*(2*a - log((2*exp

```
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^2 - 2048*a^11 - 22*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^10 + 1126
4*a^10*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1
/2)))*210i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(11/2)
```

3.167 $\int x^{7/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$-\frac{4}{99}bx^{11/2}+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2199, 30}

$$\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{9}(2b)\int x^{9/2}\,dx\\ &= -\frac{4}{99}bx^{11/2}+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.85

$$\frac{2}{99}x^{9/2}\left(-2bx+11\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(9/2)*(-2*b*x + 11*ArcTanh[Tanh[a + b*x]]))/99
```

Maple [A]
time = 0.11, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{11}{2}}}{99}+\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}$
default	$-\frac{4b\,x^{\frac{11}{2}}}{99}+\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}$
risch	$\frac{2x^{\frac{9}{2}}\ln(\mathrm{e}^{bx+a})}{9}-\frac{\left(-11i\pi\,x^4\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-11i\pi\,x^4\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2+11i\pi\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.70

$$-\frac{4}{99}\,bx^{\frac{11}{2}}+\frac{2}{9}\,x^{\frac{9}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -4/99*b*x^(11/2) + 2/9*x^(9/2)*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.67

$$\frac{2}{99}\left(9\,bx^5+11\,ax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/99*(9*b*x^5 + 11*a*x^4)*sqrt(x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.48

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 2/11*b*x^(11/2) + 2/9*a*x^(9/2)
```

Mupad [B]
time = 1.24, size = 57, normalized size = 2.11

$$\frac{x^{9/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{4\,b\,x^{11/2}}{99}-\frac{x^{9/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x)),x)
[Out] (x^(9/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/9 - (4*b*x^(
11/2))/99 - (x^(9/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/9
```

3.168 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$-\frac{4}{63}bx^{9/2}+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{7}(2b)\int x^{7/2}\,dx\\ &= -\frac{4}{63}bx^{9/2}+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.85

$$\frac{2}{63}x^{7/2}\left(-2bx+9\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(7/2)*(-2*b*x + 9*ArcTanh[Tanh[a + b*x]]))/63
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{9}{2}}}{63}+\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}$
default	$-\frac{4b\,x^{\frac{9}{2}}}{63}+\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}$
risch	$\frac{2x^{\frac{7}{2}}\ln\left(\mathrm{e}^{bx+a}\right)}{7}-\frac{\left(-9i\pi\,x^{3}\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+9i\pi\,x^{3}\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-9i\pi\,x^{3}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.70

$$-\frac{4}{63}\,bx^{\frac{9}{2}}+\frac{2}{7}\,x^{\frac{7}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -4/63*b*x^(9/2) + 2/7*x^(7/2)*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.67

$$\frac{2}{63}\left(7\,bx^{4}+9\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)

Sympy [A]
time = 18.98, size = 26, normalized size = 0.96

$$-\frac{4bx^{\frac{9}{2}}}{63} + \frac{2x^{\frac{7}{2}}\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] -4*b*x**(9/2)/63 + 2*x**(7/2)*atanh(tanh(a + b*x))/7
```

Giac [A]
time = 0.37, size = 13, normalized size = 0.48

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Mupad [B]
time = 1.09, size = 57, normalized size = 2.11

$$\frac{x^{7/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7} - \frac{4\,b\,x^{9/2}}{63} - \frac{x^{7/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(7/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/7 - (4*b*x^(
9/2))/63 - (x^(7/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/7
```

3.169 $\int x^{3/2}\tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$-\frac{4}{35}bx^{7/2}+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2199, 30}

$$\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{35}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}(\tanh(a+bx))\,dx&=\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{5}(2b)\int x^{5/2}\,dx\\&=-\frac{4}{35}bx^{7/2}+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.85

$$\frac{2}{35}x^{5/2}\left(-2bx+7\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(5/2)*(-2*b*x + 7*ArcTanh[Tanh[a + b*x]]))/35
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}$
default	$-\frac{4b\,x^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}$
risch	$\frac{2x^{\frac{5}{2}}\ln\left(e^{bx+a}\right)}{5}-\frac{\left(7i\pi\,x^{2}\operatorname{csgn}\left(ie^{2bx+2a}\right)^{3}-7i\pi\,x^{2}\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}-14i\pi\,x^{2}\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2b\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.70

$$-\frac{4}{35}\,bx^{\frac{7}{2}}+\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -4/35*b*x^(7/2) + 2/5*x^(5/2)*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.32, size = 18, normalized size = 0.67

$$\frac{2}{35}\left(5\,bx^{3}+7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A]
time = 1.67, size = 26, normalized size = 0.96

$$-\frac{4bx^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a)),x)
[Out] -4*b*x**(7/2)/35 + 2*x**(5/2)*atanh(tanh(a + b*x))/5
```

Giac [A]
time = 0.38, size = 13, normalized size = 0.48

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Mupad [B]
time = 1.10, size = 57, normalized size = 2.11

$$\frac{x^{5/2}\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{5}-\frac{4\,b\,x^{7/2}}{35}-\frac{x^{5/2}\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x)),x)
[Out] (x^(5/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/5 - (4*b*x^(
7/2))/35 - (x^(5/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/5
```

3.170 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$-\frac{4}{15}bx^{5/2}+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{3}(2b)\int x^{3/2}\,dx\\ &= -\frac{4}{15}bx^{5/2}+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.85

$$\frac{2}{15}x^{3/2}\left(-2bx+5\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(3/2)*(-2*b*x + 5*ArcTanh[Tanh[a + b*x]]))/15
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{5}{2}}}{15}+\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))}{3}$
default	$-\frac{4b\,x^{\frac{5}{2}}}{15}+\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))}{3}$
risch	$\frac{2x^{\frac{3}{2}}\ln\left(\mathrm{e}^{bx+a}\right)}{3}-\frac{\left(-10i\pi x\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}+5i\pi x\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+5\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))*x^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.70

$$-\frac{4}{15}\,bx^{\frac{5}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="maxima")
[Out] -4/15*b*x^(5/2) + 2/3*x^(3/2)*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.32, size = 16, normalized size = 0.59

$$\frac{2}{15}\,\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.48

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Mupad [B]
time = 1.09, size = 57, normalized size = 2.11

$$\frac{x^{3/2}\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}-\frac{4\,b\,x^{5/2}}{15}-\frac{x^{3/2}\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(3/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*b*x^(
5/2))/15 - (x^(3/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/3
```

3.171 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$-\frac{4}{3}bx^{3/2}+2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-\frac{4}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
```

```
[Out] (-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-(2b)\int \sqrt{x}\,dx\\ &= -\frac{4}{3}bx^{3/2}+2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.92

$$\frac{2}{3}\sqrt{x}\left(-2bx+3\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
[Out] (2*Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/3
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.80

method	result
derivativedivides	$-\frac{4b\,x^{\frac{3}{2}}}{3}+2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$
default	$-\frac{4b\,x^{\frac{3}{2}}}{3}+2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$
risch	$2\sqrt{x}\ \ln\left(\mathrm{e}^{bx+a}\right)-\frac{\left(-3i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-6i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}+3i\pi\mathrm{csgn}\left(i\mathrm{e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.76

$$-\frac{4}{3}\,bx^{\frac{3}{2}}+2\,\sqrt{x}\ \mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
[Out] -4/3*b*x^(3/2) + 2*sqrt(x)*arctanh(tanh(b*x + a))
```

Fricas [A]
time = 0.37, size = 12, normalized size = 0.48

$$\frac{2}{3}\left(bx+3\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/sqrt(x), x)
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.52

$$\frac{2}{3}\, bx^{\frac{3}{2}} + 2\, a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Mupad [B]
time = 1.13, size = 56, normalized size = 2.24

$$\sqrt{x}\ \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\frac{4\,b\,x^{3/2}}{3}-\sqrt{x}\ \ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(1/2),x)
```

```
[Out] x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - (4*b*x^(3/2)
)/3 - x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1))
```

3.172 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx$

Optimal. Leaf size=23

$$4b\sqrt{x}-\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

```
[Out] -2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2199, 30}

$$4b\sqrt{x}-\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

```
[Out] 4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}+(2b)\int \frac{1}{\sqrt{x}}\,dx\\ &= 4b\sqrt{x}-\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.87

$$\frac{4bx - 2\tanh^{-1}(\tanh(a + bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
[Out] (4*b*x - 2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.87

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 4b\sqrt{x}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 4b\sqrt{x}$
risch	$-\frac{2\ln(\mathrm{e}^{bx+a})}{\sqrt{x}} + \frac{-i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2 - 2i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2 + i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\mathrm{csgn}(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.83

$$4\,b\sqrt{x} - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx + a\right)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="maxima")
[Out] 4*b*sqrt(x) - 2*arctanh(tanh(b*x + a))/sqrt(x)
```

Fricas [A]
time = 0.35, size = 12, normalized size = 0.52

$$\frac{2\left(bx - a\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

Sympy [A]
time = 0.39, size = 22, normalized size = 0.96

$$4b\sqrt{x} - \frac{2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(3/2),x)
```

```
[Out] 4*b*sqrt(x) - 2*atanh(tanh(a + b*x))/sqrt(x)
```

Giac [A]
time = 0.38, size = 13, normalized size = 0.57

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Mupad [B]
time = 1.12, size = 56, normalized size = 2.43

$$\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{\sqrt{x}} + 4\,b\,\sqrt{x} - \frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(3/2),x)
```

```
[Out] log(1/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2) + 4*b*x^(1/2) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2)
```

3.173 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx$

Optimal. Leaf size=27

$$-\frac{4b}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}$$

```
[Out] -2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-4*b)/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}}\,dx\\ &= -\frac{4b}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.78

$$-\frac{2\left(2bx+\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-2*(2*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2))
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{4b}{3\sqrt{x}}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{4b}{3\sqrt{x}}$
risch	$-\frac{2\ln\left(\mathrm{e}^{bx+a}\right)}{3x^{\frac{3}{2}}}-\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+2i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}-i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.70

$$-\frac{4\,b}{3\,\sqrt{x}}-\frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*b/sqrt(x) - 2/3*arctanh(tanh(b*x + a))/x^(3/2)
```

Fricas [A]
time = 0.33, size = 11, normalized size = 0.41

$$-\frac{2\left(3\,bx+a\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A]
time = 2.59, size = 27, normalized size = 1.00

$$-\frac{4b}{3\sqrt{x}} - \frac{2\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(5/2),x)
[Out] -4*b/(3*sqrt(x)) - 2*atanh(tanh(a + b*x))/(3*x**(3/2))
```

Giac [A]
time = 0.38, size = 11, normalized size = 0.41

$$-\frac{2\left(3\,bx+a\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="giac")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Mupad [B]
time = 1.24, size = 52, normalized size = 1.93

$$-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(5/2),x)
[Out] -(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*ex
p(2*b*x) + 1)) + 4*b*x)/(3*x^(3/2))
```

3.174 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}}\,dx$

Optimal. Leaf size=27

$$-\frac{4b}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}$$

```
[Out] -4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}-\frac{4b}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

```
[Out] (-4*b)/(15*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]])/(5*x^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}+\frac{1}{5}(2b)\int \frac{1}{x^{5/2}}\,dx\\ &= -\frac{4b}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.85

$$-\frac{2\left(2bx+3\tanh^{-1}(\tanh(a+bx))\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
[Out] (-2*(2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(15*x^(5/2))
```

Maple [A]
time = 0.11, size = 20, normalized size = 0.74

method	result
derivativedivides	$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{5x^{\frac{5}{2}}}$
default	$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{5x^{\frac{5}{2}}}$
risch	$-\frac{2\ln\left(\mathrm{e}^{bx+a}\right)}{5x^{\frac{5}{2}}}-\frac{3i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+6i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}-3i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.70

$$-\frac{4\,b}{15\,x^{\frac{3}{2}}}-\frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="maxima")
[Out] -4/15*b/x^(3/2) - 2/5*arctanh(tanh(b*x + a))/x^(5/2)
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.48

$$-\frac{2\left(5\,bx+3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

Sympy [A]
time = 19.82, size = 27, normalized size = 1.00

$$-\frac{4b}{15x^{\frac{3}{2}}} - \frac{2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(7/2),x)
[Out] -4*b/(15*x**(3/2)) - 2*atanh(tanh(a + b*x))/(5*x**(5/2))
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.48

$$-\frac{2\left(5\,bx+3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="giac")
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

Mupad [B]
time = 1.28, size = 57, normalized size = 2.11

$$\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}} - \frac{4\,b}{15\,x^{3/2}} - \frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(7/2),x)
[Out] log(1/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2)) - (4*b)/(15*x^(3/2)) - log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2))
```

3.175 $\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=48

$$\frac{16b^2x^{13/2}}{1287}-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 16/1287*b^2*x^(13/2)-8/99*b*x^(11/2)*arctanh(tanh(b*x+a))+2/9*x^(9/2)*arcta
nh(tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2199, 30}

$$-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16b^2x^{13/2}}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(13/2))/1287 - (8*b*x^(11/2)*ArcTanh[Tanh[a + b*x]])/99 + (2*x^(9
/2)*ArcTanh[Tanh[a + b*x]]^2)/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{9}(4b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{99}(8\\ &= \frac{16b^2x^{13/2}}{1287}-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.83

$$\frac{2x^{9/2}\left(8b^2x^2-52bx\tanh^{-1}(\tanh(a+bx))+143\tanh^{-1}(\tanh(a+bx))^2\right)}{1287}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(9/2)*(8*b^2*x^2 - 52*b*x*ArcTanh[Tanh[a + b*x]] + 143*ArcTanh[Tanh[a
+ b*x]]^2))/1287
```

Maple [A]
time = 0.29, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{8b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11}-\frac{2x^{\frac{13}{2}}b}{143}\right)}{9}$	38
default	$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{8b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11}-\frac{2x^{\frac{13}{2}}b}{143}\right)}{9}$	38
risch	Expression too large to display	2093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^2-8/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)-2/143*x^(13/2)*b)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.75

$$\frac{16}{1287}\,b^2x^{\frac{13}{2}}-\frac{8}{99}\,bx^{\frac{11}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{9}\,x^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/1287*b^2*x^(13/2) - 8/99*b*x^(11/2)*arctanh(tanh(b*x + a)) + 2/9*x^(9/2)
*arctanh(tanh(b*x + a))^2
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.60

$$\frac{2}{1287}\left(99\,b^2x^6+234\,abx^5+143\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/1287*(99*b^2*x^6 + 234*a*b*x^5 + 143*a^2*x^4)*sqrt(x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [A]
time = 0.39, size = 24, normalized size = 0.50

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/11*a*b*x^(11/2) + 2/9*a^2*x^(9/2)
```

Mupad [B]
time = 1.15, size = 122, normalized size = 2.54

$$\frac{x^{9/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{18}+\frac{2\,b^2\,x^{13/2}}{13}-\frac{2\,b\,x^{11/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/18 + (2*b^2*x^(13/2))/13 - (2*b*x^(11/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x))/11
```

3.176 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$\frac{16}{693} b^2 x^{11/2} - \frac{8}{63} b x^{9/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 16/693*b^2*x^(11/2)-8/63*b*x^(9/2)*arctanh(tanh(b*x+a))+2/7*x^(7/2)*arctanh
(tanh(b*x+a))^2
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-\frac{8}{63} b x^{9/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{693} b^2 x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(11/2))/693 - (8*b*x^(9/2)*ArcTanh[Tanh[a + b*x]])/63 + (2*x^(7/2
)*ArcTanh[Tanh[a + b*x]]^2)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{7}(4b) \int x^{7/2} \tanh^{-1}(\tanh(a+bx)) \, dx \\ &= -\frac{8}{63} b x^{9/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{63}(8 \\ &= \frac{16}{693} b^2 x^{11/2} - \frac{8}{63} b x^{9/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+b\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.83

$$\frac{2}{693}x^{7/2}\left(8b^2x^2-44bx\tanh^{-1}(\tanh(a+bx))+99\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(7/2)*(8*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]] + 99*ArcTanh[Tanh[a +
 b*x]]^2))/693
```

Maple [A]
time = 0.29, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{7}-\frac{8b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}$	38
default	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{7}-\frac{8b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}$	38
risch	Expression too large to display	2093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^2-8/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-
2/99*b*x^(11/2))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.75

$$\frac{16}{693}\,b^2x^{\frac{11}{2}}-\frac{8}{63}\,bx^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{7}\,x^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/693*b^2*x^(11/2) - 8/63*b*x^(9/2)*arctanh(tanh(b*x + a)) + 2/7*x^(7/2)*a
rctanh(tanh(b*x + a))^2
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.60

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 33.82, size = 48, normalized size = 1.00

$$\frac{16b^2x^{\frac{11}{2}}}{693} - \frac{8bx^{\frac{9}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{63} + \frac{2x^{\frac{7}{2}}\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] 16*b**2*x**(11/2)/693 - 8*b*x**(9/2)*atanh(tanh(a + b*x))/63 + 2*x**(7/2)*a
tanh(tanh(a + b*x))**2/7
```

Giac [A]
time = 0.40, size = 24, normalized size = 0.50

$$\frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Mupad [B]
time = 1.13, size = 122, normalized size = 2.54

$$\frac{x^{7/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{14}+\frac{2\,b^2\,x^{11/2}}{11}-\frac{2\,b\,x^{9/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/14 + (2*b^2*x^(11/2))/11 - (2*b*x^(9/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/9
```

3.177 $\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=48

$$\frac{16}{315}b^2x^{9/2}-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 16/315*b^2*x^(9/2)-8/35*b*x^(7/2)*arctanh(tanh(b*x+a))+2/5*x^(5/2)*arctanh(
tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2199, 30}

$$-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{315}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(9/2))/315 - (8*b*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 + (2*x^(5/2)
*ArcTanh[Tanh[a + b*x]]^2)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx&=\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{5}(4b)\int x^{5/2}\tanh^{-1}(\tanh(a+bx))\,dx\\&=-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{35}(8b\\&=\frac{16}{315}b^2x^{9/2}-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.83

$$\frac{2}{315}x^{5/2}\left(8b^2x^2-36bx\tanh^{-1}(\tanh(a+bx))+63\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2)*(8*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]] + 63*ArcTanh[Tanh[a +
 b*x]]^2))/315
```

Maple [A]
time = 0.28, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{8b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2bx^{\frac{9}{2}}}{63}\right)}{5}$	38
default	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{8b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2bx^{\frac{9}{2}}}{63}\right)}{5}$	38
risch	Expression too large to display	2093

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^2-8/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-
2/63*b*x^(9/2))
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.75

$$\frac{16}{315}\,b^2x^{\frac{9}{2}}-\frac{8}{35}\,bx^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/315*b^2*x^(9/2) - 8/35*b*x^(7/2)*arctanh(tanh(b*x + a)) + 2/5*x^(5/2)*ar
ctanh(tanh(b*x + a))^2
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.60

$$\frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \operatorname{atanh}^2(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.38, size = 24, normalized size = 0.50

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Mupad [B]
time = 1.14, size = 122, normalized size = 2.54

$$\frac{x^{5/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10}+\frac{2\,b^2\,x^{9/2}}{9}-\frac{2\,b\,x^{7/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/10 + (2*b^2*x^(9/2))/9 - (2*b*x^(7/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x))/7
```

3.178 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$\frac{16}{105}b^2x^{7/2}-\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 16/105*b^2*x^(7/2)-8/15*b*x^(5/2)*arctanh(tanh(b*x+a))+2/3*x^(3/2)*arctanh(
tanh(b*x+a))^2
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{105}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(7/2))/105 - (8*b*x^(5/2)*ArcTanh[Tanh[a + b*x]])/15 + (2*x^(3/2)
*ArcTanh[Tanh[a + b*x]]^2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{3}(4b)\int x^{3/2}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{15}(\\ &= \frac{16}{105}b^2x^{7/2}-\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+b\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.83

$$\frac{2}{105}x^{3/2}\left(8b^2x^2-28bx\tanh^{-1}(\tanh(a+bx))+35\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2)*(8*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]] + 35*ArcTanh[Tanh[a +
 b*x]]^2))/105
```

Maple [A]
time = 0.28, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{8b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{2bx^{\frac{7}{2}}}{35}\right)}{3}$	38
default	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{8b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{2bx^{\frac{7}{2}}}{35}\right)}{3}$	38
risch	Expression too large to display	2027

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^2-8/3*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))-
2/35*b*x^(7/2))
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.75

$$\frac{16}{105}b^2x^{\frac{7}{2}}-\frac{8}{15}bx^{\frac{5}{2}}\operatorname{artanh}(\tanh(bx+a))+\frac{2}{3}x^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/105*b^2*x^(7/2) - 8/15*b*x^(5/2)*arctanh(tanh(b*x + a)) + 2/3*x^(3/2)*ar
ctanh(tanh(b*x + a))^2
```

Fricas [A]
time = 0.33, size = 27, normalized size = 0.56

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.39, size = 24, normalized size = 0.50

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Mupad [B]
time = 1.12, size = 122, normalized size = 2.54

$$\frac{x^{3/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6}+\frac{2\,b^2\,x^{7/2}}{7}-\frac{2\,b\,x^{5/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/6 + (2*b^2*x^(7/2))/7 - (2*b*x^(5/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))/5
```

3.179 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=46

$$\frac{16}{15}b^2x^{5/2}-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2$$

```
[Out] 16/15*b^2*x^(5/2)-8/3*b*x^(3/2)*arctanh(tanh(b*x+a))+2*arctanh(tanh(b*x+a))
^2*x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2199, 30}

$$-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{15}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (16*b^2*x^(5/2))/15 - (8*b*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3 + 2*Sqrt[x]*Ar
cTanh[Tanh[a + b*x]]^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-(4b)\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{3}(8b^2)\int x\\ &= \frac{16}{15}b^2x^{5/2}-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.87

$$\frac{2}{15}\sqrt{x}\left(8b^2x^2-20bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(8*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a +
 b*x]]^2))/15
```

Maple [A]
time = 0.27, size = 47, normalized size = 1.02

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4(\operatorname{arctanh}(\tanh(bx+a))-bx)bx^{\frac{3}{2}}}{3}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}$	47
default	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4(\operatorname{arctanh}(\tanh(bx+a))-bx)bx^{\frac{3}{2}}}{3}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}$	47
risch	Expression too large to display	1978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*b^2*x^(5/2)+4/3*(arctanh(tanh(b*x+a))-b*x)*b*x^(3/2)+2*(arctanh(tanh(b*
x+a))-b*x)^2*x^(1/2)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.78

$$\frac{16}{15}b^2x^{\frac{5}{2}}-\frac{8}{3}bx^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a))+2\sqrt{x}\operatorname{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/15*b^2*x^(5/2) - 8/3*b*x^(3/2)*arctanh(tanh(b*x + a)) + 2*sqrt(x)*arctan
h(tanh(b*x + a))^2
```

Fricas [A]
time = 0.35, size = 24, normalized size = 0.52

$$\frac{2}{15}\left(3b^2x^2+10abx+15a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/sqrt(x), x)
```

Giac [A]
time = 0.39, size = 24, normalized size = 0.52

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Mupad [B]
time = 1.15, size = 122, normalized size = 2.65

$$\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2}+\frac{2\,b^2\,x^{5/2}}{5}-\frac{2\,b\,x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(1/2),x)
```

```
[Out] (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 + (2*b^2*x^(5/2))/5 - (2*b*x^(3/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))/3
```

3.180 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{16}{3}b^2x^{3/2}+8b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}$$

```
[Out] -16/3*b^2*x^(3/2)-2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*arctanh(tanh(b*x+a))
*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$8b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{16}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-16*b^2*x^(3/2))/3 + 8*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (2*ArcTanh[Tanh[
a + b*x]]^2)/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx$$
$$= 8b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - (8b^2)\int \sqrt{x}\,dx$$
$$= -\frac{16}{3}b^2x^{3/2} + 8b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.91

$$-\frac{2\left(8b^2x^2 - 12bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-2*(8*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
))/(3*Sqrt[x])
```

Maple [A]
time = 0.29, size = 37, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^2}{\sqrt{x}} + 8b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x} - \frac{2bx^{\frac{3}{2}}}{3}\right)$	37
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^2}{\sqrt{x}} + 8b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x} - \frac{2bx^{\frac{3}{2}}}{3}\right)$	37
risch	Expression too large to display	1977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x
^(3/2))
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.82

$$-\frac{16}{3}\,b^2x^{\frac{3}{2}} + 8\,b\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*b^2*x^(3/2) + 8*b*sqrt(x)*arctanh(tanh(b*x + a)) - 2*arctanh(tanh(b*x
 + a))^2/sqrt(x)
```

Fricas [A]
time = 0.33, size = 23, normalized size = 0.52

$$\frac{2\left(b^{2} x^{2} + 6\, a b x - 3\, a^{2}\right)}{3\, \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{2}\left(\tanh\left(a + bx\right)\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**(3/2), x)
```

Giac [A]
time = 0.38, size = 24, normalized size = 0.55

$$\frac{2}{3}\, b^{2} x^{\frac{3}{2}} + 4\, a b \sqrt{x} - \frac{2\, a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Mupad [B]
time = 1.16, size = 122, normalized size = 2.77

$$\frac{2\, b^{2}\, x^{3/2}}{3} - \frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}+1}\right) - \ln\left(\frac{2\, \mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}}{\mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}+1}\right) + 2\, b\, x\right)^{2}}{2\, \sqrt{x}} - 2\, b\, \sqrt{x}\, \left(\ln\left(\frac{2}{\mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}+1}\right) - \ln\left(\frac{2\, \mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}}{\mathrm{e}^{2\, a}\, \mathrm{e}^{2\, b\, x}+1}\right) + 2\, b\, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(3/2),x)
```

```
[Out] (2*b^2*x^(3/2))/3 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*x^(1/2)) - 2*b*x^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)
```

3.181 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}} \, dx$

Optimal. Leaf size=48

$$\frac{16b^2\sqrt{x}}{3} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}$$

```
[Out] -2/3*arctanh(tanh(b*x+a))^2/x^(3/2)-8/3*b*arctanh(tanh(b*x+a))/x^(1/2)+16/3
*b^2*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} + \frac{16b^2\sqrt{x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (16*b^2*Sqrt[x])/3 - (8*b*ArcTanh[Tanh[a + b*x]])/(3*Sqrt[x]) - (2*ArcTanh[
Tanh[a + b*x]]^2)/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}} + \frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx$$
$$= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}} + \frac{1}{3}(8b^2)\int \frac{1}{\sqrt{x}}\,dx$$
$$= \frac{16b^2\sqrt{x}}{3} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.83

$$\frac{2\left(8b^2x^2 - 4bx\tanh^{-1}(\tanh(a+bx)) - \tanh^{-1}(\tanh(a+bx))^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (2*(8*b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] - ArcTanh[Tanh[a + b*x]]^2))/(
3*x^(3/2))
```

Maple [A]
time = 0.28, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}} + \frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 2b\sqrt{x}\right)}{3}$	38
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}} + \frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 2b\sqrt{x}\right)}{3}$	38
risch	Expression too large to display	1972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^2/x^(3/2)+8/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*
b*x^(1/2))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.75

$$\frac{16}{3}\,b^2\sqrt{x} - \frac{8\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,\sqrt{x}} - \frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*sqrt(x) - 8/3*b*arctanh(tanh(b*x + a))/sqrt(x) - 2/3*arctanh(tanh(
b*x + a))^2/x^(3/2)
```

Fricas [A]
time = 0.36, size = 24, normalized size = 0.50

$$\frac{2\,(3\,b^2x^2-6\,abx-a^2)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A]
time = 2.30, size = 48, normalized size = 1.00

$$\frac{16b^2\sqrt{x}}{3}-\frac{8b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3\sqrt{x}}-\frac{2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(5/2),x)
```

```
[Out] 16*b**2*sqrt(x)/3 - 8*b*atanh(tanh(a + b*x))/(3*sqrt(x)) - 2*atanh(tanh(a +
 b*x))**2/(3*x**(3/2))
```

Giac [A]
time = 0.39, size = 23, normalized size = 0.48

$$2\,b^2\sqrt{x}-\frac{2\,(6\,abx+a^2)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Mupad [B]
time = 1.13, size = 122, normalized size = 2.54

$$2\,b^2\,\sqrt{x}-\frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6\,x^{3/2}}+\frac{2\,b\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(5/2),x)
```

```
[Out] 2*b^2*x^(1/2) - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(6*x^(3/2)) + (2*b*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))/x^(1/2)
```

3.182 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}} dx$

Optimal. Leaf size=48

$$-\frac{16b^2}{15\sqrt{x}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}$$

```
[Out] -8/15*b*arctanh(tanh(b*x+a))/x^(3/2)-2/5*arctanh(tanh(b*x+a))^2/x^(5/2)-16/
15*b^2/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}-\frac{16b^2}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-16*b^2)/(15*Sqrt[x]) - (8*b*ArcTanh[Tanh[a + b*x]])/(15*x^(3/2)) - (2*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}} + \frac{1}{5}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx$$
$$= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}} + \frac{1}{15}(8b^2)\int \frac{1}{x^{3/2}}\,d$$
$$= -\frac{16b^2}{15\sqrt{x}} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.83

$$-\frac{2(8b^2x^2 + 4bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-2*(8*b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(15*x^(5/2))
```

Maple [A]
time = 0.29, size = 38, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^2}{5x^{\frac{5}{2}}} + \frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}} - \frac{2b}{3\sqrt{x}}\right)}{5}$	38
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^2}{5x^{\frac{5}{2}}} + \frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}} - \frac{2b}{3\sqrt{x}}\right)}{5}$	38
risch	Expression too large to display	1978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*arctanh(tanh(b*x+a))^2/x^(5/2)+8/5*b*(-1/3*arctanh(tanh(b*x+a))/x^(3/2
)-2/3*b/x^(1/2))
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.75

$$-\frac{16\,b^2}{15\,\sqrt{x}} - \frac{8\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{15\,x^{\frac{3}{2}}} - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] -16/15*b^2/sqrt(x) - 8/15*b*arctanh(tanh(b*x + a))/x^(3/2) - 2/5*arctanh(ta
nh(b*x + a))^2/x^(5/2)
```

Fricas [A]
time = 0.35, size = 24, normalized size = 0.50

$$-\frac{2\,(15\,b^2x^2+10\,abx+3\,a^2)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

Sympy [A]
time = 20.26, size = 49, normalized size = 1.02

$$-\frac{16b^2}{15\sqrt{x}}-\frac{8b\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{15x^{\frac{3}{2}}}-\frac{2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(7/2),x)
```

```
[Out] -16*b**2/(15*sqrt(x)) - 8*b*atanh(tanh(a + b*x))/(15*x**(3/2)) - 2*atanh(ta
nh(a + b*x))**2/(5*x**(5/2))
```

Giac [A]
time = 0.38, size = 24, normalized size = 0.50

$$-\frac{2\,(15\,b^2x^2+10\,abx+3\,a^2)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

Mupad [B]
time = 1.13, size = 122, normalized size = 2.54

$$\frac{2\,b\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,x^{3/2}}-\frac{2\,b^2}{\sqrt{x}}-\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{10\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(7/2),x)
```

```
[Out] (2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*x^(3/2)) - (2*b^2)/x^(1/2) - (log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2/(10*x^(5/2))
```

3.183 $\int x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$-\frac{32b^3x^{15/2}}{6435}+\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -32/6435*b^3*x^(15/2)+16/429*b^2*x^(13/2)*arctanh(tanh(b*x+a))-4/33*b*x^(11
/2)*arctanh(tanh(b*x+a))^2+2/9*x^(9/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A]

time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{15/2}}{6435}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(15/2))/6435 + (16*b^2*x^(13/2)*ArcTanh[Tanh[a + b*x]])/429 - (4
*b*x^(11/2)*ArcTanh[Tanh[a + b*x]]^2)/33 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]
]^3)/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{3}(2b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{33}$$
$$= \frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 +$$
$$= -\frac{32b^3x^{15/2}}{6435} + \frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tan$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.83

$$-\frac{2x^{9/2}\left(16b^3x^3 - 120b^2x^2\tanh^{-1}(\tanh(a+bx)) + 390bx\tanh^{-1}(\tanh(a+bx))^2 - 715\tanh^{-1}(\tanh(a+bx))^3\right)}{6435}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(9/2)*(16*b^3*x^3 - 120*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 390*b*x*ArcT
anh[Tanh[a + b*x]]^2 - 715*ArcTanh[Tanh[a + b*x]]^3))/6435
```

Maple [A]
time = 1.20, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{11} - \frac{4b\left(\frac{x^{\frac{13}{2}}\operatorname{arctanh}(\tanh(bx+a))}{13} - \frac{2x^{\frac{15}{2}}b}{195}\right)}{11}\right)}{3}$	56
default	$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{11} - \frac{4b\left(\frac{x^{\frac{13}{2}}\operatorname{arctanh}(\tanh(bx+a))}{13} - \frac{2x^{\frac{15}{2}}b}{195}\right)}{11}\right)}{3}$	56
risch	Expression too large to display	8179

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^3-4/3*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)^2-4/11*b*(1/13*x^(13/2)*arctanh(tanh(b*x+a))-2/195*x^(15/2)*b))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.80

$$-\frac{4}{33}\,bx^{\frac{11}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{2}{9}\,x^{\frac{9}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{16}{6435}\left(2\,b^2x^{\frac{15}{2}}-15\,bx^{\frac{13}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/33*b*x^(11/2)*arctanh(tanh(b*x + a))^2 + 2/9*x^(9/2)*arctanh(tanh(b*x +
a))^3 - 16/6435*(2*b^2*x^(15/2) - 15*b*x^(13/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.58

$$\frac{2}{6435}\left(429\,b^3x^7+1485\,ab^2x^6+1755\,a^2bx^5+715\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/6435*(429*b^3*x^7 + 1485*a*b^2*x^6 + 1755*a^2*b*x^5 + 715*a^3*x^4)*sqrt(x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.51

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/13*a*b^2*x^(13/2) + 6/11*a^2*b*x^(11/2) + 2/9*a^3*x^(
9/2)
```

Mupad [B]
time = 1.17, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{15/2}}{15}-\frac{x^{9/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{36}+\frac{3\,b\,x^{11/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{22}-\frac{3\,b^2\,x^{13/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(15/2))/15 - (x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/36 + (3*b*x^(11/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/22 - (3*b^2*x^(13/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
))/13
```

3.184 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$-\frac{32b^3x^{13/2}}{3003}+\frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -32/3003*b^3*x^(13/2)+16/231*b^2*x^(11/2)*arctanh(tanh(b*x+a))-4/21*b*x^(9/
2)*arctanh(tanh(b*x+a))^2+2/7*x^(7/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$\frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{13/2}}{3003}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(13/2))/3003 + (16*b^2*x^(11/2)*ArcTanh[Tanh[a + b*x]])/231 - (4
*b*x^(9/2)*ArcTanh[Tanh[a + b*x]]^2)/21 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]]
^3)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{7}(6b)\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{21}($$
$$= \frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}$$
$$= -\frac{32b^3x^{13/2}}{3003} + \frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.83

$$\frac{2x^{7/2}\left(-16b^3x^3 + 104b^2x^2\tanh^{-1}(\tanh(a+bx)) - 286bx\tanh^{-1}(\tanh(a+bx))^2 + 429\tanh^{-1}(\tanh(a+bx))^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (2*x^(7/2)*(-16*b^3*x^3 + 104*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 286*b*x*ArcT
anh[Tanh[a + b*x]]^2 + 429*ArcTanh[Tanh[a + b*x]]^3))/3003
```

Maple [A]
time = 1.22, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{7} - \frac{12b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11} - \frac{2x^{\frac{13}{2}}b}{143}\right)}{9}\right)}{7}$	56
default	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{7} - \frac{12b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11} - \frac{2x^{\frac{13}{2}}b}{143}\right)}{9}\right)}{7}$	56
risch	Expression too large to display	8179

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^3-12/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))
^2-4/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a))-2/143*x^(13/2)*b))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.80

$$-\frac{4}{21}\,bx^{\frac{9}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}+\frac{2}{7}\,x^{\frac{7}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}-\frac{16}{3003}\left(2\,b^{2}x^{\frac{13}{2}}-13\,bx^{\frac{11}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/21*b*x^(9/2)*arctanh(tanh(b*x + a))^2 + 2/7*x^(7/2)*arctanh(tanh(b*x + a
))^3 - 16/3003*(2*b^2*x^(13/2) - 13*b*x^(11/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.37, size = 40, normalized size = 0.58

$$\frac{2}{3003}\left(231\,b^{3}x^{6}+819\,ab^{2}x^{5}+1001\,a^{2}bx^{4}+429\,a^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

Sympy [A]
time = 61.61, size = 70, normalized size = 1.01

$$-\frac{32b^{3}x^{\frac{13}{2}}}{3003}+\frac{16b^{2}x^{\frac{11}{2}}\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{231}-\frac{4bx^{\frac{9}{2}}\,\mathrm{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{21}+\frac{2x^{\frac{7}{2}}\,\mathrm{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] -32*b**3*x**(13/2)/3003 + 16*b**2*x**(11/2)*atanh(tanh(a + b*x))/231 - 4*b*
x**(9/2)*atanh(tanh(a + b*x))**2/21 + 2*x**(7/2)*atanh(tanh(a + b*x))**3/7
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.51

$$\frac{2}{13}\,b^{3}x^{\frac{13}{2}}+\frac{6}{11}\,ab^{2}x^{\frac{11}{2}}+\frac{2}{3}\,a^{2}bx^{\frac{9}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

Mupad [B]
time = 1.18, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{13/2}}{13}-\frac{x^{7/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{28}+\frac{b\,x^{9/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{6}-\frac{3\,b^2\,x^{11/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(13/2))/13 - (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/28 + (b*x^(9/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/6 - (3*b^2*x^(11/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/1
1
```

3.185 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$-\frac{32b^3x^{11/2}}{1155}+\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -32/1155*b^3*x^(11/2)+16/105*b^2*x^(9/2)*arctanh(tanh(b*x+a))-12/35*b*x^(7/
2)*arctanh(tanh(b*x+a))^2+2/5*x^(5/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{11/2}}{1155}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(11/2))/1155 + (16*b^2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/105 - (12
*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2)/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]]
^3)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{5}(6b)\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{35}($$
$$= \frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{5}$$
$$= -\frac{32b^3x^{11/2}}{1155} + \frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh($$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.83

$$-\frac{2x^{5/2}\left(16b^3x^3 - 88b^2x^2\tanh^{-1}(\tanh(a+bx)) + 198bx\tanh^{-1}(\tanh(a+bx))^2 - 231\tanh^{-1}(\tanh(a+bx))^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(5/2)*(16*b^3*x^3 - 88*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 198*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))/1155
```

Maple [A]
time = 1.21, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{5} - \frac{12b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{4b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{2bx^{\frac{11}{2}}}{99}\right)}{7}\right)}{5}$	56
default	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{5} - \frac{12b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{4b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{2bx^{\frac{11}{2}}}{99}\right)}{7}\right)}{5}$	56
risch	Expression too large to display	8179

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^3-12/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))
^2-4/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-2/99*b*x^(11/2)))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.80

$$-\frac{12}{35}\,bx^{\frac{7}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}+\frac{2}{5}\,x^{\frac{5}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}-\frac{16}{1155}\,\left(2\,b^{2}x^{\frac{11}{2}}-11\,bx^{\frac{9}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -12/35*b*x^(7/2)*arctanh(tanh(b*x + a))^2 + 2/5*x^(5/2)*arctanh(tanh(b*x +
a))^3 - 16/1155*(2*b^2*x^(11/2) - 11*b*x^(9/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.58

$$\frac{2}{1155}\,\left(105\,b^{3}x^{5}+385\,ab^{2}x^{4}+495\,a^{2}bx^{3}+231\,a^{3}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\,\mathrm{atanh}^{3}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.51

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{2}{3}\,ab^{2}x^{\frac{9}{2}}+\frac{6}{7}\,a^{2}bx^{\frac{7}{2}}+\frac{2}{5}\,a^{3}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Mupad [B]
time = 1.17, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{11/2}}{11}-\frac{x^{5/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{20}+\frac{3\,b\,x^{7/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{14}-\frac{b^2\,x^{9/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(11/2))/11 - (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/20 + (3*b*x^(7/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/14 - (b^2*x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/3
```

3.186 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$-\frac{32}{315}b^3x^{9/2}+\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -32/315*b^3*x^(9/2)+16/35*b^2*x^(7/2)*arctanh(tanh(b*x+a))-4/5*b*x^(5/2)*ar
ctanh(tanh(b*x+a))^2+2/3*x^(3/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{315}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(9/2))/315 + (16*b^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 - (4*b*x
^(5/2)*ArcTanh[Tanh[a + b*x]]^2)/5 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \sqrt{x}\ \tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3 - (2b)\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}(8b$$
$$= \frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{3}x^3$$
$$= -\frac{32}{315}b^3x^{9/2} + \frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.83

$$-\frac{2}{315}x^{3/2}\left(16b^3x^3 - 72b^2x^2\tanh^{-1}(\tanh(a+bx)) + 126bx\tanh^{-1}(\tanh(a+bx))^2 - 105\tanh^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(3/2)*(16*b^3*x^3 - 72*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 126*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/315
```

Maple [A]
time = 1.19, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{3} - 4b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{5} - \frac{4b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7} - \frac{2bx^{\frac{9}{2}}}{63}\right)}{5}\right)$	56
default	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{3} - 4b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{5} - \frac{4b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7} - \frac{2bx^{\frac{9}{2}}}{63}\right)}{5}\right)$	56
risch	Expression too large to display	7981

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^3-4*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))^2-
4/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-2/63*b*x^(9/2)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.80

$$-\frac{4}{5}bx^{\frac{5}{2}}\operatorname{artanh}(\tanh(bx+a))^2 + \frac{2}{3}x^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a))^3 - \frac{16}{315}\left(2b^2x^{\frac{9}{2}} - 9bx^{\frac{7}{2}}\operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/5*b*x^(5/2)*arctanh(tanh(b*x + a))^2 + 2/3*x^(3/2)*arctanh(tanh(b*x + a)
)^3 - 16/315*(2*b^2*x^(9/2) - 9*b*x^(7/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.37, size = 38, normalized size = 0.55

$$\frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\ \operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.38, size = 35, normalized size = 0.51

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Mupad [B]
time = 1.16, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{9/2}}{9}-\frac{x^{3/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{12}+\frac{3\,b\,x^{5/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{10}-\frac{3\,b^2\,x^{7/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(9/2))/9 - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/12 + (3*b*x^(5/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/10 - (3*b^2*x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/7
```

3.187 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=65

$$-\frac{32}{35}b^3x^{7/2}+\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3$$

```
[Out] -32/35*b^3*x^(7/2)+16/5*b^2*x^(5/2)*arctanh(tanh(b*x+a))-4*b*x^(3/2)*arctan
h(tanh(b*x+a))^2+2*arctanh(tanh(b*x+a))^3*x^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{35}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (-32*b^3*x^(7/2))/35 + (16*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5 - 4*b*x^(3
/2)*ArcTanh[Tanh[a + b*x]]^2 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = 2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^3 - (6b)\int \sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^2\,dx$$

$$= -4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^3 + \left(8b^2\right)\int x^3$$

$$= \frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\,\tanh$$

$$= -\frac{32}{35}b^3x^{7/2} + \frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.88

$$\frac{2}{35}\sqrt{x}\left(-16b^3x^3 + 56b^2x^2\tanh^{-1}(\tanh(a+bx)) - 70bx\tanh^{-1}(\tanh(a+bx))^2 + 35\tanh^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(-16*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 70*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 35*ArcTanh[Tanh[a + b*x]]^3))/35
```

Maple [A]
time = 1.32, size = 69, normalized size = 1.06

method	result
derivativedivides	$\frac{2b^3x^{\frac{7}{2}}}{7} + \frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2x^{\frac{5}{2}}}{5} + 2(\operatorname{arctanh}(\tanh(bx+a)) - bx)^2\,b\,x^{\frac{3}{2}} + 2(\operatorname{arctanh}($
default	$\frac{2b^3x^{\frac{7}{2}}}{7} + \frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2x^{\frac{5}{2}}}{5} + 2(\operatorname{arctanh}(\tanh(bx+a)) - bx)^2\,b\,x^{\frac{3}{2}} + 2(\operatorname{arctanh}($
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*b^3*x^(7/2)+6/5*(arctanh(tanh(b*x+a))-b*x)*b^2*x^(5/2)+2*(arctanh(tanh(
b*x+a))-b*x)^2*b*x^(3/2)+2*(arctanh(tanh(b*x+a))-b*x)^3*x^(1/2)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$-4\,bx^{\frac{3}{2}}\,\operatorname{artanh}(\tanh(bx+a))^2 + 2\sqrt{x}\,\operatorname{artanh}(\tanh(bx+a))^3 - \frac{16}{35}\left(2\,b^2x^{\frac{7}{2}} - 7\,bx^{\frac{5}{2}}\,\operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4*b*x^(3/2)*arctanh(tanh(b*x + a))^2 + 2*sqrt(x)*arctanh(tanh(b*x + a))^3
- 16/35*(2*b^2*x^(7/2) - 7*b*x^(5/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.54

$$\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/sqrt(x), x)
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.54

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Mupad [B]
time = 1.19, size = 182, normalized size = 2.80

$$\frac{2\,b^3\,x^{7/2}}{7}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{4}+\frac{b\,x^{3/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2}-\frac{3\,b^2\,x^{5/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(1/2),x)
```

```
[Out] (2*b^3*x^(7/2))/7 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/4 + (b*x^(3/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^2)/2 - (3*b^2*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/5
```

3.188 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{32}{5}b^3x^{5/2}-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}$$

```
[Out] 32/5*b^3*x^(5/2)-16*b^2*x^(3/2)*arctanh(tanh(b*x+a))-2*arctanh(tanh(b*x+a))
^3/x^(1/2)+12*b*arctanh(tanh(b*x+a))^2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+\frac{32}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (32*b^3*x^(5/2))/5 - 16*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 12*b*Sqrt[x]*A
rcTanh[Tanh[a + b*x]]^2 - (2*ArcTanh[Tanh[a + b*x]]^3)/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}} + (6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx$$
$$= 12b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^2 - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}} - (24b^2)\int \sqrt{x}$$
$$= -16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 12b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^2 - \frac{2\tanh}{}$$
$$= \frac{32}{5}b^3x^{5/2} - 16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 12b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.90

$$\frac{2\left(16b^3x^3 - 40b^2x^2\tanh^{-1}(\tanh(a+bx)) + 30bx\tanh^{-1}(\tanh(a+bx))^2 - 5\tanh^{-1}(\tanh(a+bx))^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (2*(16*b^3*x^3 - 40*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 30*b*x*ArcTanh[Tanh[a
+ b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(5*Sqrt[x])
```

Maple [A]
time = 1.33, size = 64, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{\sqrt{x}} + 12b\left(\frac{b^2x^{\frac{5}{2}}}{5} + \frac{2(\mathrm{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3} + (\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right.\right.$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{\sqrt{x}} + 12b\left(\frac{b^2x^{\frac{5}{2}}}{5} + \frac{2(\mathrm{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3} + (\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh(tanh(b*x+a))^3/x^(1/2)+12*b*(1/5*b^2*x^(5/2)+2/3*(arctanh(tanh(b
*x+a))-b*x)*b*x^(3/2)+(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.87

$$12\,b\sqrt{x}\ \mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2 - \frac{2\ \mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{\sqrt{x}} + \frac{16}{5}\left(2\,b^2x^{\frac{5}{2}} - 5\,bx^{\frac{3}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 12*b*sqrt(x)*arctanh(tanh(b*x + a))^2 - 2*arctanh(tanh(b*x + a))^3/sqrt(x)
+ 16/5*(2*b^2*x^(5/2) - 5*b*x^(3/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.37, size = 34, normalized size = 0.54

$$\frac{2\,(b^3x^3 + 5\,ab^2x^2 + 15\,a^2bx - 5\,a^3)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3\left(\tanh\left(a + bx\right)\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**(3/2), x)
```

Giac [A]
time = 0.38, size = 35, normalized size = 0.56

$$\frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Mupad [B]
time = 1.21, size = 182, normalized size = 2.89

$$\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{4\,\sqrt{x}}+\frac{2\,b^3\,x^{5/2}}{5}+\frac{3\,b\,\sqrt{x}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2}-b^2\,x^{3/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(3/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(4*x^(1/2)) + (2*b^3*x^(5/2))/5 + (3*b*x^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/2 - b^2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.189 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{32}{3}b^3x^{3/2}+16b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}$$

```
[Out] -32/3*b^3*x^(3/2)-2/3*arctanh(tanh(b*x+a))^3/x^(3/2)-4*b*arctanh(tanh(b*x+a
))^2/x^(1/2)+16*b^2*arctanh(tanh(b*x+a))*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$16b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{32}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-32*b^3*x^(3/2))/3 + 16*b^2*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (4*b*ArcTanh[
Tanh[a + b*x]]^2)/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^3)/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx$$
$$= -\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} + (8b^2)\int \frac{\tanh^{-1}(}{}$$
$$= 16b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx)) - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\tanh^{-1}(\tan}{3x}$$
$$= -\frac{32}{3}b^3x^{3/2} + 16b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx)) - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} -$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.85

$$-\frac{2\left(16b^3x^3 - 24b^2x^2\tanh^{-1}(\tanh(a+bx)) + 6bx\tanh^{-1}(\tanh(a+bx))^2 + \tanh^{-1}(\tanh(a+bx))^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-2*(16*b^3*x^3 - 24*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2))
```

Maple [A]
time = 1.22, size = 55, normalized size = 0.85

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{3x^{\frac{3}{2}}} + 4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{\sqrt{x}} + 4b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x} - \frac{2bx}{3}\right.\right.$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{3x^{\frac{3}{2}}} + 4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{\sqrt{x}} + 4b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x} - \frac{2bx}{3}\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^3/x^(3/2)+4*b*(-arctanh(tanh(b*x+a))^2/x^(1/2)+4*
b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x^(3/2)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$-\frac{4\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}} - \frac{16}{3}\left(2\,b^2x^{\frac{3}{2}} - 3\,b\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4*b*arctanh(tanh(b*x + a))^2/sqrt(x) - 16/3*(2*b^2*x^(3/2) - 3*b*sqrt(x)*a
rctanh(tanh(b*x + a)))*b - 2/3*arctanh(tanh(b*x + a))^3/x^(3/2)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.52

$$\frac{2\left(b^{3}x^{3}+9\,ab^{2}x^{2}-9\,a^{2}bx-a^{3}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A]
time = 2.05, size = 66, normalized size = 1.02

$$-\frac{32b^{3}x^{\frac{3}{2}}}{3}+16b^{2}\sqrt{x}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)-\frac{4b\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}-\frac{2\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(5/2),x)
```

```
[Out] -32*b**3*x**(3/2)/3 + 16*b**2*sqrt(x)*atanh(tanh(a + b*x)) - 4*b*atanh(tanh
(a + b*x))**2/sqrt(x) - 2*atanh(tanh(a + b*x))**3/(3*x**(3/2))
```

Giac [A]
time = 0.41, size = 34, normalized size = 0.52

$$\frac{2}{3}\,b^{3}x^{\frac{3}{2}}+6\,ab^{2}\sqrt{x}-\frac{2\left(9\,a^{2}bx+a^{3}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Mupad [B]
time = 1.22, size = 182, normalized size = 2.80

$$\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{12\,x^{3/2}}+\frac{2\,b^3\,x^{3/2}}{3}-\frac{3\,b\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2\,\sqrt{x}}-3\,b^2\,\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(5/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(12*x^(3/2)) + (2*b^3*x^(3/2))/3 - (3*b*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2)/(2*x^(1/2)) - 3*b^2*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.190 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}}\,dx$

Optimal. Leaf size=69

$$\frac{32b^3\sqrt{x}}{5}-\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}}$$

```
[Out] -4/5*b*arctanh(tanh(b*x+a))^2/x^(3/2)-2/5*arctanh(tanh(b*x+a))^3/x^(5/2)-16
/5*b^2*arctanh(tanh(b*x+a))/x^(1/2)+32/5*b^3*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 30}

$$-\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}}+\frac{32b^3\sqrt{x}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

```
[Out] (32*b^3*Sqrt[x])/5 - (16*b^2*ArcTanh[Tanh[a + b*x]])/(5*Sqrt[x]) - (4*b*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^3)/(5*x^(5/2
))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx$$

$$= -\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}(8b^2)\int \tanh^{-}$$

$$= -\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh}{5x^{5/}}$$

$$= \frac{32b^3\sqrt{x}}{5} - \frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - 2\tan$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.83

$$\frac{2\left(16b^3x^3 - 8b^2x^2\tanh^{-1}(\tanh(a+bx)) - 2bx\tanh^{-1}(\tanh(a+bx))^2 - \tanh^{-1}(\tanh(a+bx))^3\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

```
[Out] (2*(16*b^3*x^3 - 8*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 2*b*x*ArcTanh[Tanh[a +
b*x]]^2 - ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2))
```

Maple [A]
time = 1.23, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{5x^{\frac{5}{2}}} + \frac{12b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}} + \frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 2b\sqrt{x}\right)}{3}\right)}{5}$	56
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{5x^{\frac{5}{2}}} + \frac{12b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}} + \frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}} + 2b\sqrt{x}\right)}{3}\right)}{5}$	56
risch	Expression too large to display	7814

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*arctanh(tanh(b*x+a))^3/x^(5/2)+12/5*b*(-1/3*arctanh(tanh(b*x+a))^2/x^(
3/2)+4/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*b*x^(1/2)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.80

$$\frac{16}{5}\left(2\,b^{2}\sqrt{x}-\frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}\right)b-\frac{4\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{5\,x^{\frac{3}{2}}}-\frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 16/5*(2*b^2*sqrt(x) - b*arctanh(tanh(b*x + a))/sqrt(x))*b - 4/5*b*arctanh(t
anh(b*x + a))^2/x^(3/2) - 2/5*arctanh(tanh(b*x + a))^3/x^(5/2)
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.51

$$\frac{2\left(5\,b^{3}x^{3}-15\,ab^{2}x^{2}-5\,a^{2}bx-a^{3}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(5*b^3*x^3 - 15*a*b^2*x^2 - 5*a^2*b*x - a^3)/x^(5/2)
```

Sympy [A]
time = 20.16, size = 70, normalized size = 1.01

$$\frac{32b^{3}\sqrt{x}}{5}-\frac{16b^{2}\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{5\sqrt{x}}-\frac{4b\,\mathrm{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{3}{2}}}-\frac{2\,\mathrm{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(7/2),x)
```

```
[Out] 32*b**3*sqrt(x)/5 - 16*b**2*atanh(tanh(a + b*x))/(5*sqrt(x)) - 4*b*atanh(ta
nh(a + b*x))**2/(5*x**(3/2)) - 2*atanh(tanh(a + b*x))**3/(5*x**(5/2))
```

Giac [A]
time = 0.39, size = 34, normalized size = 0.49

$$2\,b^{3}\sqrt{x}-\frac{2\left(15\,ab^{2}x^{2}+5\,a^{2}bx+a^{3}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2*b^3*sqrt(x) - 2/5*(15*a*b^2*x^2 + 5*a^2*b*x + a^3)/x^(5/2)
```

Mupad [B]
time = 1.18, size = 182, normalized size = 2.64

$$\frac{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{20x^{5/2}}+2b^3\sqrt{x}+\frac{3b^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{\sqrt{x}}-\frac{b\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(7/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(20*x^(5/2)) + 2*b^3*x^(1/2) + (3*b^2*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))/x^(1/2) - (b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^(3/2))
```

3.191 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=143

$$\frac{2x^{7/2}}{7b}+\frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}+\frac{2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a\right.}{b^4}$$

```
[Out] 2/7*x^(7/2)/b+2/5*x^(5/2)*(b*x-arctanh(tanh(b*x+a)))/b^2+2/3*x^(3/2)*(b*x-a
rctanh(tanh(b*x+a)))^2/b^3-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+
a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(7/2)/b^(9/2)+2*(b*x-arctanh(tanh(b*
x+a)))^3*x^(1/2)/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2190, 2193}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}{b^{9/2}}+\frac{2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}+\frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}+\frac{2x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(7/2))/(7*b) + (2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(5*b^2) + (2
*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2)/(3*b^3) + (2*Sqrt[x]*(b*x - ArcT
anh[Tanh[a + b*x]])^3)/b^4 - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTan
h[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2))/b^(9/2)
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{7/2}}{7b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx)\right)}{b^2}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}{3b^3}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}{3b^3}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}{3b^3}$$

Mathematica [A]
time = 0.09, size = 129, normalized size = 0.90

$$\frac{2\left(176b^{7/2}x^{7/2} - 406b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx)) + 350b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 - 105\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 + 105\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}\right)}{105b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*(176*b^(7/2)*x^(7/2) - 406*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] + 350*
b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 - 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh
[a + b*x]]^3 + 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)))/(105*b^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(119) = 238$.
time = 0.12, size = 481, normalized size = 3.36

method	result
derivativedivides	$\frac{2x^{\frac{7}{2}}}{7b} - \frac{2x^{\frac{5}{2}}a}{5b^2} - \frac{2x^{\frac{5}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{5b^2} + \frac{2a^2x^{\frac{3}{2}}}{3b^3} + \frac{4a\,x^{\frac{3}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{3b^3} + \frac{2x^{\frac{3}{2}}(\text{ar}}{}$
default	$\frac{2x^{\frac{7}{2}}}{7b} - \frac{2x^{\frac{5}{2}}a}{5b^2} - \frac{2x^{\frac{5}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{5b^2} + \frac{2a^2x^{\frac{3}{2}}}{3b^3} + \frac{4a\,x^{\frac{3}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{3b^3} + \frac{2x^{\frac{3}{2}}(\text{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^(7/2)/b-2/5/b^2*x^(5/2)*a-2/5/b^2*x^(5/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/3/b^3*a^2*x^(3/2)+4/3/b^3*a*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+2/3/b^
3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-2/b^4*a^3*x^(1/2)-6/b^4*a^2*x^(1/2
)*(arctanh(tanh(b*x+a))-b*x-a)-6/b^4*a*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)
^2-2/b^4*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)^3+2/b^4/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^4+
8/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^3*(arctanh(tanh(b*x+a))-b*x-a)+12/b^4/((arctanh(ta
nh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1
/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+8/b^4/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(t
anh(b*x+a))-b*x-a)^3+2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^4
```

Maxima [A]
time = 0.47, size = 65, normalized size = 0.45

$$\frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{2\left(15\,b^3x^{\frac{7}{2}}-21\,ab^2x^{\frac{5}{2}}+35\,a^2bx^{\frac{3}{2}}-105\,a^3\sqrt{x}\right)}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^3*x^(7/2) -
 21*a*b^2*x^(5/2) + 35*a^2*b*x^(3/2) - 105*a^3*sqrt(x))/b^4
```

Fricas [A]
time = 0.36, size = 153, normalized size = 1.07

$$\left[\frac{105\,a^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(15\,b^3x^3-21\,ab^2x^2+35\,a^2bx-105\,a^3)\sqrt{x}}{105\,b^4},\ \frac{2\left(105\,a^3\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)+(15\,b^3x^3-21\,ab^2x^2+35\,a^2bx-105\,a^3)\sqrt{x}\right)}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^3*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)
) + 2*(15*b^3*x^3 - 21*a*b^2*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4, 2/10
```

```
5*(105*a^3*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*b^3*x^3 - 21*a*b^2
*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [A]
time = 0.39, size = 70, normalized size = 0.49

$$\frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{2\left(15\,b^6x^{\frac{7}{2}}-21\,ab^5x^{\frac{5}{2}}+35\,a^2b^4x^{\frac{3}{2}}-105\,a^3b^3\sqrt{x}\right)}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^6*x^(7/2) -
 21*a*b^5*x^(5/2) + 35*a^2*b^4*x^(3/2) - 105*a^3*b^3*sqrt(x))/b^7
```

Mupad [B]
time = 1.72, size = 475, normalized size = 3.32

$$\frac{2\,x^{7/2}}{7\,b}+\frac{x^{5/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5\,b^2}+\frac{x^{3/2}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{6\,b^3}+\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{4\,b^4}+\frac{\sqrt{2}\,\ln\left(\frac{64\,b^{19/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{7/2}}{16\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x)),x)
[Out] (2*x^(7/2))/(7*b) + (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(5*b^2) + (x^(3/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/(6*b^3) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(
4*b^4) + (2^(1/2)*log((64*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^
(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2))/(16*
b^(9/2))
```

3.192 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=116

$$\frac{2x^{5/2}}{5b}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{bx-\tanh^{-1}}}\right.}{}$$

[Out] 2/5*x^(5/2)/b+2/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^2-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5/2)/b^(7/2)+2*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)/b^3

Rubi [A]
time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2193}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}{b^{7/2}}+\frac{2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(5/2))/(5*b) + (2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^2) + (2
*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^3 - (2*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)
)/b^(7/2)
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{5/2}}{5b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx)\right)}{b^2}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+\right.}{b^3}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+\right.}{b^3}$$

Mathematica [A]
time = 0.08, size = 108, normalized size = 0.93

$$\frac{2\left(23b^{5/2}x^{5/2} - 35b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 15\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - 15\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}\right)}{15b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
[Out] (2*(23*b^(5/2)*x^(5/2) - 35*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 15*Sqr
t[b]*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2 - 15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(
b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)))/(
15*b^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(96) = 192.
time = 0.09, size = 330, normalized size = 2.84

method	result
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5b} - \frac{2x^{\frac{3}{2}}a}{3b^2} - \frac{2x^{\frac{3}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{3b^2} + \frac{2a^2\sqrt{x}}{b^3} + \frac{4a(\text{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^3} + 2($
default	$\frac{2x^{\frac{5}{2}}}{5b} - \frac{2x^{\frac{3}{2}}a}{3b^2} - \frac{2x^{\frac{3}{2}}(\text{arctanh}(\tanh(bx+a))-bx-a)}{3b^2} + \frac{2a^2\sqrt{x}}{b^3} + \frac{4a(\text{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^3} + 2($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)/b-2/3/b^2*x^(3/2)*a-2/3/b^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/b^3*a^2*x^(1/2)+4/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/b^3*(arc
tanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/
2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^3-6/b^3/((arcta
nh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*
b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-6/b^3/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctan
h(tanh(b*x+a))-b*x-a)^2-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b
*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
^3
```

Maxima [A]
time = 0.48, size = 54, normalized size = 0.47

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Fricas [A]
time = 0.37, size = 132, normalized size = 1.14

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\,-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.39, size = 59, normalized size = 0.51

$$-\frac{2\,a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{2\left(3\,b^{4}x^{\frac{5}{2}}-5\,ab^{3}x^{\frac{3}{2}}+15\,a^{2}b^{2}\sqrt{x}\right)}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B]
time = 1.39, size = 415, normalized size = 3.58

$$\frac{2\,x^{5/2}}{5\,b}+\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x\right)}{3\,b^{2}}+\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x\right)^{2}}{2\,b^{3}}+\frac{\sqrt{2}\,\ln\left(\frac{16\,b^{15/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x\right)-4\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x}+2\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x}}\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2\,b\,x\right)^{5/2}}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^2) + (x^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3) + (2^(1/2)*log((16*b^(15/2)*(2^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) +
2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^(5/2))/(8*b^(7/2))
```

3.193 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=89

$$\frac{2x^{3/2}}{3b}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh\right.}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(
b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))*x^(1/2
)/b^2
```

Rubi [A]

time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2193}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{5/2}}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) + (2*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 - (2*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^(3/2))/b^(5/2)
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{3/2}}{3b} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{3/2}}{3b} + \frac{2\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{b^2} + \frac{(-bx+\tanh^{-1}(\tanh(a+bx)}{b}$$

$$= \frac{2x^{3/2}}{3b} + \frac{2\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{b^2} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{bx-\tanh^{-1}(ta}}\right.}{}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.97

$$\frac{2x^{3/2}}{3b} - \frac{2\sqrt{x}\,(-bx+\tanh^{-1}(\tanh(a+bx)))}{b^2} + \frac{2\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)(-bx+\tanh^{-1}(\tanh(a+bx)))^{3/2}}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) - (2*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + (2*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^(3/2))/b^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(73) = 146.
time = 0.08, size = 207, normalized size = 2.33

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b} - \frac{2a\sqrt{x}}{b^2} - \frac{2(\text{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^2} + \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx}}\right.}{b^2\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)}}$
default	$\frac{2x^{\frac{3}{2}}}{3b} - \frac{2a\sqrt{x}}{b^2} - \frac{2(\text{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^2} + \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx}}\right.}{b^2\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b-2/b^2*a*x^(1/2)-2/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/
b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*
```

```
x+a))-b*x)*b)^(1/2))*a^2+4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(
b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x
-a)+2/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.47

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

Fricas [A]
time = 0.35, size = 103, normalized size = 1.16

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a)),x)
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.51

$$\frac{2\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\left(b^{2}x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

Mupad [B]
time = 1.85, size = 354, normalized size = 3.98

$$\frac{2\,x^{3/2}}{3\,b}+\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^{2}}+\frac{\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}}{4\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^2 + (2^(1/2)*log((
4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(4*b^(5/2))
```

3.194 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

```
[Out] 2*x^(1/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2190, 2193}

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2\sqrt{x}}{b} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2\sqrt{x}}{b} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a}{b^{3/2}}$$

Mathematica [**A**]
time = 0.03, size = 62, normalized size = 0.97

$$\frac{2\sqrt{x}}{b} - \frac{2\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(52) = 104.
time = 0.09, size = 112, normalized size = 1.75

method	result
derivativedivides	$\frac{2\sqrt{x}}{b} - \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)a}{b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}} - \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx}}\right)}{b\sqrt{(\operatorname{arctanh}(}}$
default	$\frac{2\sqrt{x}}{b} - \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)a}{b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}} - \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx}}\right)}{b\sqrt{(\operatorname{arctanh}(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)
```

Maxima [A]
time = 0.49, size = 31, normalized size = 0.48

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Fricas [A]
time = 0.34, size = 85, normalized size = 1.33

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b},-\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x)), x)
```

Giac [A]
time = 0.39, size = 31, normalized size = 0.48

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Mupad [B]
time = 2.23, size = 296, normalized size = 4.62

$$\frac{2\sqrt{x}}{b}+\frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}\right)\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(1/2))/b + (2^(1/2)*log((b^(7/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4
*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x
) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(
2*b^(3/2))
```

3.195 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(1/2)/(b*x-a
rctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2193}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b]
*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.96

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b
]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.77

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	41
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+
a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.46, size = 18, normalized size = 0.34

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.38, size = 68, normalized size = 1.28

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\,\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 18, normalized size = 0.34

$$\frac{2\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 4.01, size = 347, normalized size = 6.55

$$\frac{\sqrt{2}\,\ln\left(\frac{b^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(2\,\sqrt{2}\,a+4\,\sqrt{x}\,\sqrt{b\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\sqrt{2}\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)-2\,\sqrt{2}\,b\,x\right)}{2\,\sqrt{b\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}\,\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)}\right)}{\sqrt{b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-b\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b^2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] (2^(1/2)*log((b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(2*2^(1/2)*a + 4*x^(1/2)*(b*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))^(1/2) - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2*2^
(1/2)*b*x))/(2*(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^(1/2)*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(b*lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b^2*x)^(1/2)
```

3.196 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-a
rctanh(tanh(b*x+a)))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2194, 2193}

$$\frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]]))
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$
$$= -\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}\right.}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 0.96

$$-\frac{2\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{2}{\sqrt{x}\,\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))
```

Maple [A]
time = 0.08, size = 76, normalized size = 1.00

method	result
derivativedivides	$-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} - \frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}$
default	$-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} - \frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)
*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 31, normalized size = 0.41

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.22

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,\sqrt{x}}{ax}, \frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.40, size = 31, normalized size = 0.41

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Mupad [B]
time = 1.97, size = 464, normalized size = 6.11

$$\frac{4}{\sqrt{x}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{2\,\sqrt{2}\,\sqrt{b}\,\ln\left(\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)\,\left(\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)+4\,a^2\right)}{2\,\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right)}\right)}{{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2*2^(1/2)*b^(1/2)*log((b^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*a^2))/(2*(log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)
```

3.197 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=101

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}\right.}$$

```
[Out] -2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(5/2)+2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+2*b/(b*x-
arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2194, 2193}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{2b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + (2*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
 + b*x]])^2) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

$$= -\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}\right.}$$

Mathematica [A]
time = 0.12, size = 89, normalized size = 0.88

$$\frac{2b^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{2\left(4bx-\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])
/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(4*b*x - ArcTanh[Tanh[a + b*x
]]))/(3*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.08, size = 98, normalized size = 0.97

method	result
derivativedivides	$\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}} - \frac{2}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} +$
default	$\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}} - \frac{2}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(arctanh(tanh(b*x+a))-b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arc
tan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(arctanh(tanh(b*x+a
))-b*x)/x^(3/2)+2/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(1/2)
```

Maxima [A]
time = 0.47, size = 41, normalized size = 0.41

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.17

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\,-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 41, normalized size = 0.41

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Mupad [B]
time = 1.83, size = 642, normalized size = 6.36

$$\frac{4}{3\,x^{3/2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{8b}{\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}+\frac{4\sqrt{2}\,b^{3/2}\ln\left(\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}+2\sqrt{2}\,bx\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+24a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+16a^4-8a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{2\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)}\right)}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2) + (4*2^(1/2)*b^(3/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(
2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
- 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(5/2)
```

3.198 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=128

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}\right.}$$

```
[Out] -2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(7/2)+2/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5/x
^(5/2)/(b*x-arctanh(tanh(b*x+a)))+2*b^2/(b*x-arctanh(tanh(b*x+a)))^3/x^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2194, 2193}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) + (2*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh
[a + b*x]])^3) + (2*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 2/(5*
x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= \frac{2b^2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

$$= -\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{2b}{\sqrt{x}\,\left(bx-\tanh^{-1}\right.}$$

Mathematica [A]
time = 0.10, size = 107, normalized size = 0.84

$$-\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{2\left(23b^2x^2-11bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (2*(23*b^2*x^2 - 11*b*x*ArcTanh
[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^3)
```

Maple [A]
time = 0.09, size = 120, normalized size = 0.94

method	result
derivativedivides	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{2b^2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}+\frac{2b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}-$
default	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{2b^2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}+\frac{2b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)-2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/
x^(1/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(3/2)-2*b^3/(arctanh(tanh(b*x+
a))-b*x)^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(
tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 52, normalized size = 0.41

$$-\frac{2\,b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\,(15\,b^2x^2-5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Fricas [A]
time = 0.36, size = 144, normalized size = 1.12

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(7/2)*atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 52, normalized size = 0.41

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\,(15\,b^2x^2-5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B]
time = 1.63, size = 822, normalized size = 6.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(3*x^(3/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^2) + (16*b^2)/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (8*2^(1/2)*b^(
5/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)
*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 192*a^5*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(7/2)
```

3.199 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{7x^{5/2}}{5b^2}+\frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3}+\frac{7\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{bx-\tanh^{-1}}}\right.}{}$$

[Out] 7/5*x^(5/2)/b^2+7/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^3-7*arctanh(b^(1/2
)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5/2
)/b^(9/2)-x^(7/2)/b/arctanh(tanh(b*x+a))+7*(b*x-arctanh(tanh(b*x+a)))^2*x^(
1/2)/b^4

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2199, 2190, 2193}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}{b^{9/2}}+\frac{7\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3}-\frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{7x^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]

[Out] (7*x^(5/2))/(5*b^2) + (7*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^3) +
(7*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/
2))/b^(9/2) - x^(7/2)/(b*ArcTanh[Tanh[a + b*x]])

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{7x^{5/2}}{5b^2} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(7\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\quad}{\tan}}{2b^2}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{(}{}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+\right.}{b^4}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+\right.}{b^4}$$

Mathematica [A]
time = 0.13, size = 144, normalized size = 1.07

$$\frac{2x^{5/2}}{5b^2} - \frac{4x^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{6\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{7\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}{b^{9/2}} + \frac{\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2))/(5*b^2) - (4*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(3*b^3)
 + (6*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^(5/2))/b^(9/2) + (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)/(b^4
*ArcTanh[Tanh[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(113) = 226$.
time = 0.95, size = 452, normalized size = 3.35

method	result
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5b^2} - \frac{4a\,x^{\frac{3}{2}}}{3b^3} - \frac{4x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3b^3} + \frac{6a^2\sqrt{x}}{b^4} + \frac{12a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^4} + \frac{6(}{}$
default	$\frac{2x^{\frac{5}{2}}}{5b^2} - \frac{4a\,x^{\frac{3}{2}}}{3b^3} - \frac{4x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3b^3} + \frac{6a^2\sqrt{x}}{b^4} + \frac{12a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^4} + \frac{6(}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)/b^2-4/3/b^3*a*x^(3/2)-4/3/b^3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x
-a)+6/b^4*a^2*x^(1/2)+12/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+6/b^4*(
arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)+1/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^
3+3/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4
*x^(1/2)/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4*x^(1/2
)/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3-7/b^4/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
)*a^3-21/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctan
h(tanh(b*x+a))-b*x)*b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-21/b^4/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x-a)^2-7/b^4/((arctanh(tanh(b*x+a))-b
*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctan
h(tanh(b*x+a))-b*x-a)^3
```

Maxima [A]
time = 0.47, size = 75, normalized size = 0.56

$$\frac{6\,b^3x^{\frac{7}{2}} - 14\,ab^2x^{\frac{5}{2}} + 70\,a^2bx^{\frac{3}{2}} + 105\,a^3\sqrt{x}}{15\,(b^5x + ab^4)} - \frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/15*(6*b^3*x^(7/2) - 14*a*b^2*x^(5/2) + 70*a^2*b*x^(3/2) + 105*a^3*sqrt(x)
)/(b^5*x + a*b^4) - 7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4)
```

Fricas [A]
time = 0.34, size = 188, normalized size = 1.39

$$\left[\frac{105\,\left(a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\left(6\,b^3x^3-14\,ab^2x^2+70\,a^2bx+105\,a^3\right)\sqrt{x}}{30\,\left(b^5x+ab^4\right)},\ -\frac{105\,\left(a^2bx+a^3\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(6\,b^3x^3-14\,ab^2x^2+70\,a^2bx+105\,a^3\right)\sqrt{x}}{15\,\left(b^5x+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a^2*b*x + a^3)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a
)/(b*x + a)) + 2*(6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))
/(b^5*x + a*b^4), -1/15*(105*(a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqr
t(a/b)/a) - (6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))/(b^5
*x + a*b^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [A]
time = 0.37, size = 76, normalized size = 0.56

$$-\frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{a^3\sqrt{x}}{(bx+a)b^4}+\frac{2\,\left(3\,b^8x^{\frac{5}{2}}-10\,ab^7x^{\frac{3}{2}}+45\,a^2b^6\sqrt{x}\right)}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + a^3*sqrt(x)/((b*x + a)
*b^4) + 2/15*(3*b^8*x^(5/2) - 10*a*b^7*x^(3/2) + 45*a^2*b^6*sqrt(x))/b^10
```

Mupad [B]
time = 1.61, size = 523, normalized size = 3.87

$$\frac{2\,x^{5/2}}{5\,b^2}+\frac{2\,x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,b^3}+\frac{3\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,b^4}+\frac{7\,\sqrt{2}\,\ln\left(\frac{64\,b^{19/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{5/2}}{16\,b^{9/2}}-\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2*x^(5/2))/(5*b^2) + (2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^3) + (3*x^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (7*2^(1/2)*log((64*b^(19/2)*(2^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^(5/2))/(16*b^(9/2)) - (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - l
og(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.200 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{5x^{3/2}}{3b^2}+\frac{5\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tan\right.}{b^{7/2}}$$

```
[Out] 5/3*x^(3/2)/b^2-5*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(7/2)-x^(5/2)/b/arctanh(tanh(b*x+a))+5*
(b*x-arctanh(tanh(b*x+a)))*x^(1/2)/b^3
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2193}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}+\frac{5\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (5*x^(3/2))/(3*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - (5*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh
[Tanh[a + b*x]])^(3/2))/b^(7/2) - x^(5/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}\\ &= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))}-\frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\tanh}}{2b^2}\\ &= \frac{5x^{3/2}}{3b^2}+\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{(5}{}\\ &= \frac{5x^{3/2}}{3b^2}+\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh}}\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 119, normalized size = 1.10

$$\frac{2x^{3/2}}{3b^2}-\frac{4\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{5\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}-\frac{\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (
5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^(3/2))/b^(7/2) - (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a
 + b*x]])^2)/(b^3*ArcTanh[Tanh[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(90) = 180$.
time = 0.51, size = 294, normalized size = 2.72

method	result

derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b^2}-\frac{4a\sqrt{x}}{b^3}-\frac{4(\mathrm{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^3}-\frac{\sqrt{x}\,a^2}{b^3\,\mathrm{arctanh}(\tanh(bx+a))}-\frac{2\sqrt{x}\,a(\mathrm{arctanh}(\tanh(bx}{b^3\,\mathrm{arctanh}(\tanh(b}$
default	$\frac{2x^{\frac{3}{2}}}{3b^2}-\frac{4a\sqrt{x}}{b^3}-\frac{4(\mathrm{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^3}-\frac{\sqrt{x}\,a^2}{b^3\,\mathrm{arctanh}(\tanh(bx+a))}-\frac{2\sqrt{x}\,a(\mathrm{arctanh}(\tanh(bx}{b^3\,\mathrm{arctanh}(\tanh(b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b^2-4/b^3*a*x^(1/2)-4/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a^2-2/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a
*(arctanh(tanh(b*x+a))-b*x-a)-1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*(arctanh(t
anh(b*x+a))-b*x-a)^2+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^2+10/b^3/((arctanh(tanh(b*x+a
))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(
arctanh(tanh(b*x+a))-b*x-a)+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arct
an(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2
```

Maxima [A]
time = 0.48, size = 64, normalized size = 0.59

$$\frac{2\,b^2x^{\frac{5}{2}}-10\,abx^{\frac{3}{2}}-15\,a^2\sqrt{x}}{3\,(b^4x+ab^3)}+\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^2*x^(5/2) - 10*a*b*x^(3/2) - 15*a^2*sqrt(x))/(b^4*x + a*b^3) + 5*a
^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Fricas [A]
time = 0.38, size = 161, normalized size = 1.49

$$\left[\frac{15\,(abx+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x+ab^3)},\ \frac{15\,(abx+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{3\,(b^4x+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")

[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
x + a)) + 2(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**2,x)

[Out] Integral(x**(5/2)/atanh(tanh(a + b*x))**2, x)

Giac [A]
time = 0.38, size = 65, normalized size = 0.60

$$\frac{5\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}-\frac{a^{2}\sqrt{x}}{(bx+a)b^{3}}+\frac{2\left(b^{4}x^{\frac{3}{2}}-6\,ab^{3}\sqrt{x}\right)}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")

[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6

Mupad [B]
time = 1.53, size = 463, normalized size = 4.29

$$\frac{2\,x^{3/2}}{3\,b^2}+\frac{2\,\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}+\frac{5\,\sqrt{2}\,\ln\left(\frac{16\,b^{15/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}}{8\,b^{7/2}}-\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2\,b^3\,\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/atanh(tanh(a + b*x))^2,x)

[Out] (2*x^(3/2))/(3*b^2) + (2*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (5*2^(1/2)
*log((16*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))

```
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(8*b^(7/2)) - (x^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.201 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{3\sqrt{x}}{b^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -x^(3/2)/b/arctanh(tanh(b*x+a))+3*x^(1/2)/b^2-3*arctanh(b^(1/2)*x^(1/2)/(b*
x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2193}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a +
b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(5/2) - x^(3/2)/(b*ArcTanh[Ta
nh[a + b*x]])
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\sqrt{}}}{2b^2}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(}}{b^{5/2}}$$

Mathematica [A]
time = 0.06, size = 81, normalized size = 0.98

$$\frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]) - (3*ArcTan[(Sqrt[b]*S
qrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]])/b^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(69) = 138$.
time = 0.42, size = 160, normalized size = 1.93

method	result
derivativedivides	$\frac{2\sqrt{x}}{b^2} + \frac{\sqrt{x}\,a}{b^2\,\text{arctanh}(\tanh(bx+a))} + \frac{\sqrt{x}\,(\text{arctanh}(\tanh(bx+a))-bx-a)}{b^2\,\text{arctanh}(\tanh(bx+a))} - \frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{arctanh}(\tanh(}}\right)}{b^2\sqrt{(\text{arctanh}(\tanh(bx}}$

default	$\frac{2\sqrt{x}}{b^2}+\frac{\sqrt{x}\,a}{b^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^2\operatorname{arctanh}(\tanh(bx+a))}-\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx}\right.}{b^2\sqrt{(\operatorname{arctanh}(\tanh(bx-}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*x^(1/2)/arctanh(tanh(b*x+a))*a+1/b^2*x^(1/2)/arctanh(ta
nh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)-3/b^2/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-3/b^2/((ar
ctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*
x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.48, size = 50, normalized size = 0.60

$$\frac{2\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{b^3x+ab^2}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] (2*b*x^(3/2) + 3*a*sqrt(x))/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b
))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.40, size = 134, normalized size = 1.61

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^3x+ab^2)},\,-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.40, size = 46, normalized size = 0.55

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{a\sqrt{x}}{(bx+a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 1.77, size = 403, normalized size = 4.86

$$\frac{2\sqrt{x}}{b^2}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{b^2\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}+\frac{3\,\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}\right)\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}{4\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2*x^(1/2))/b^2 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
 + (3*2^(1/2)*log((4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)
*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(4*b^(5/2
))
```

3.202 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))-arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b
*x+a)))^(1/2))/b^(3/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2199, 2193}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sq
rt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A]
time = 0.04, size = 70, normalized size = 0.96

$$-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x
) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
)
```

Maple [A]
time = 0.39, size = 61, normalized size = 0.84

method	result
derivativedivides	$-\frac{\sqrt{x}}{b\,\mathrm{arctanh}(\tanh(bx+a))} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}$
default	$-\frac{\sqrt{x}}{b\,\mathrm{arctanh}(\tanh(bx+a))} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}$

risch	$$\frac{}{b\left(-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)+\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}-\pi \operatorname{csgn}\left(i e^{bx+a}\right)^{2}\operatorname{csg}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))+1/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 37, normalized size = 0.51

$$-\frac{\sqrt{x}}{b^2x+ab}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.39, size = 115, normalized size = 1.58

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)},\,-\frac{ab\sqrt{x}+\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**2, x)
```

Giac [A]
time = 0.38, size = 36, normalized size = 0.49

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

Mupad [B]
time = 1.77, size = 344, normalized size = 4.71

$$\frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}\right)}{2\,b^{3/2}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}-\frac{2\,\sqrt{x}}{b\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2^(1/2)*log((b^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*
b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(
2*a)*exp(2*b*x) + 1)))))/(2*b^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - (2*x^
(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1))))
```

3.203 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh
(b*x+a)))^(3/2)/b^(1/2)-1/b/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/b/arctanh(
tanh(b*x+a))/x^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(b*x
 - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{b\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{b\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh\right.}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.82

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\,\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(-
(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[x]/(ArcTanh[Tanh[a + b*x]]*(-
(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.42, size = 82, normalized size = 0.85

method	result

derivativedivides	$\frac{\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-}}$
default	$\frac{\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x
+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 35, normalized size = 0.36

$$\frac{\sqrt{x}}{abx+a^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.36, size = 116, normalized size = 1.20

$$\left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)},\ \frac{ab\sqrt{x}-\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.36

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

Mupad [B]
time = 2.18, size = 516, normalized size = 5.32

$$\frac{\sqrt{2}\,\ln\left(-\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)+4\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx}+2\sqrt{2}\,bx\right)\left(4a^2b+b\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)^2-4ab\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)\right)}{2\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}\right)}{\sqrt{b}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)^{3/2}}-\frac{4\sqrt{x}}{\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (2^(1/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)
*b*x)*(4*a^2*b + b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*b*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(3/2)) - (4*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.204 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\mathrm{t}\right.}$$

```
[Out] -1/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(3/2)/arctanh(tanh(b*x+a))+3*
arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-arct
anh(tanh(b*x+a)))^(5/2)-3/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 3/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^2) - 1/(b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(3/2)*ArcTa
nh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} - \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} +$$

$$= -\frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A]
time = 0.08, size = 104, normalized size = 0.87

$$-\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{2}{\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{b\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2) - (b*Sqrt[x])/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2)
```

Maple [A]
time = 0.46, size = 105, normalized size = 0.88

method	result
derivativedivides	$-\frac{b\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 \operatorname{arctanh}(\tanh(bx+a))}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a}}$
default	$-\frac{b\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 \operatorname{arctanh}(\tanh(bx+a))}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2*b*x^(1/2)/arctanh(tanh(b*x+a))-3/(arctanh(t
anh(b*x+a))-b*x)^2*b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/
((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2
)
```

Maxima [A]
time = 0.48, size = 51, normalized size = 0.42

$$-\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.36, size = 147, normalized size = 1.22

$$\left[\frac{3\,(bx^2+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\,(a^2bx^2+a^3x)},\ \frac{3\,(bx^2+ax)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.39, size = 49, normalized size = 0.41

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}-\frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

Mupad [B]
time = 1.83, size = 705, normalized size = 5.88

$$\frac{\sqrt{x}\left(\frac{8}{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}-\frac{24bx}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}\right)}{2bx^2-x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{6\sqrt{2}\sqrt{b}\ln\left(-\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}+2\sqrt{2}bx\right)\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4+24a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+16a^4-8a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)}{2\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)}\right)}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (x^(1/2)*(8/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
)/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (6*2^(1/2)*b^(1/2)*log(-(b^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*b^(1/
```

2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)

3.205 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\right.}$$

```
[Out] 5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-ar
ctanh(tanh(b*x+a)))^(7/2)-5/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^(5
/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(5/2)/arctanh(tanh(b*x+a))-5*b/(b*x-ar
ctanh(tanh(b*x+a)))^3/x^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{2}}-\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))}-\frac{5b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - (5*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
+ b*x]])^3) - 5/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(5/2)
*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} + \frac{}{2}$$

$$= -\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= -\frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{ta}\right.}$$

Mathematica [A]
time = 0.15, size = 120, normalized size = 0.83

$$\frac{2\left(-7bx+\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{5b^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (2*(-7*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*
x]])^3) + (5*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (b^2*Sqrt[x])/(ArcTanh
[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A]
time = 0.59, size = 128, normalized size = 0.88

method	result
derivativedivides	$\frac{b^2\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\operatorname{arctanh}(\tanh(bx+a))}+\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{(\operatorname{arctanh}(\tanh(bx+a}}$
default	$\frac{b^2\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\operatorname{arctanh}(\tanh(bx+a))}+\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{(\operatorname{arctanh}(\tanh(bx+a}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*x^(1/2)/arctanh(tanh(b*x+a))+5/(arctanh(
tanh(b*x+a))-b*x)^3*b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/
2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(arctanh(tanh(b*x+a))-b*x)^2/x
^(3/2)+4/(arctanh(tanh(b*x+a))-b*x)^3*b/x^(1/2)
```

Maxima [A]
time = 0.48, size = 64, normalized size = 0.44

$$\frac{15\,b^2x^2+10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)}+\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.39, size = 184, normalized size = 1.27

$$\left[\frac{15\,(b^2x^3+abx^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,b^2x^2+10\,abx-2\,a^2)\sqrt{x}}{6\,(a^3bx^3+a^4x^2)},-\frac{15\,(b^2x^3+abx^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2+10\,abx-2\,a^2)\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.40, size = 58, normalized size = 0.40

$$\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{b^2\sqrt{x}}{(bx+a)a^3} + \frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

Mupad [B]
time = 2.16, size = 871, normalized size = 6.01

$$\frac{\frac{32b}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}-\frac{80b^2x}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}}{\sqrt{x}\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)}-\frac{8}{3x^{3/2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}+\frac{20\sqrt{2}\,b^{3/2}\ln\left(-\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,\left(\sqrt{2}\,\ldots\right)}{\ldots}\right)}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] ((32*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (80*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))) - 8/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2
0*2^(1/2)*b^(3/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
```

```
2*b*x) + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) +
2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5
 - 192*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^(7/2)
```

3.206 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=172

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\mathrm{t}\right.}$$

```
[Out] 7*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-ar
ctanh(tanh(b*x+a)))^(9/2)-7/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3-7/5/x^
(5/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))-1
/b/x^(7/2)/arctanh(tanh(b*x+a))-7*b^2/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (7*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(9/2) - (7*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])^4) - (7*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) - 7/(5*x
^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(7/2)*(b*x - ArcTanh[Tanh
[a + b*x]])) - 1/(b*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} +$$

$$= -\frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} - \frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A]
time = 0.17, size = 139, normalized size = 0.81

$$-\frac{7b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{b^3\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4} - \frac{2\left(58b^2x^2-16bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2\right)}{15x^{5/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-7*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) - (b^3*Sqrt[x])/(ArcTanh[Tanh[a +
 b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) - (2*(58*b^2*x^2 - 16*b*x*ArcTa
```

```
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(-(b*x) + ArcT
anh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.94, size = 151, normalized size = 0.88

method	result
derivativedivides	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2} x^{\frac{5}{2}}}-\frac{6b^{2}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{4} \sqrt{x}}+\frac{4b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{3} x^{\frac{3}{2}}}-$ (arc
default	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2} x^{\frac{5}{2}}}-\frac{6b^{2}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{4} \sqrt{x}}+\frac{4b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{3} x^{\frac{3}{2}}}-$ (arc
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)-6/(arctanh(tanh(b*x+a))-b*x)^4*b^
2/x^(1/2)+4/3/(arctanh(tanh(b*x+a))-b*x)^3*b/x^(3/2)-1/(arctanh(tanh(b*x+a)
)-b*x)^4*b^3*x^(1/2)/arctanh(tanh(b*x+a))-7/(arctanh(tanh(b*x+a))-b*x)^4*b^
3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+
a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 75, normalized size = 0.44

$$-\frac{105\,b^3x^3+70\,ab^2x^2-14\,a^2bx+6\,a^3}{15\left(a^4bx^{\frac{7}{2}}+a^5x^{\frac{5}{2}}\right)}-\frac{7\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)/(a^4*b*x^(7/2) + a^
5*x^(5/2)) - 7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.35, size = 210, normalized size = 1.22

$$\left[\frac{105\left(b^3x^4+ab^2x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(105\,b^3x^3+70\,ab^2x^2-14\,a^2bx+6\,a^3\right)\sqrt{x}}{30\left(a^4bx^4+a^5x^3\right)},\ \frac{105\left(b^3x^4+ab^2x^3\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(105\,b^3x^3+70\,ab^2x^2-14\,a^2bx+6\,a^3\right)\sqrt{x}}{15\left(a^4bx^4+a^5x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/
a) - a)/(b*x + a)) - 2*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)*sq
rt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(b/a)*arc
tan(a*sqrt(b/a)/(b*sqrt(x))) - (105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6
*a^3)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(7/2)*atanh(tanh(a + b*x))**2), x)
```

Giac [A]
time = 0.40, size = 70, normalized size = 0.41

$$-\frac{7\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}-\frac{b^3\sqrt{x}}{(bx+a)a^4}-\frac{2\,(45\,b^2x^2-10\,abx+3\,a^2)}{15\,a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - b^3*sqrt(x)/((b*x + a)
*a^4) - 2/15*(45*b^2*x^2 - 10*a*b*x + 3*a^2)/(a^4*x^(5/2))
```

Mupad [B]
time = 2.14, size = 1051, normalized size = 6.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] ((96*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (224*b^3*x)/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^4)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))) - (32*b)/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b
```

```
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3) - 8/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (56*2^(1/2)*b^(5/2)*log(-(
b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^8 + 112*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 448*a
^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 1120*a^4*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^4 - 1792*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 1792*a^6*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2 + 256*a^8 - 16*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 1024*
a^7*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^(9/2)
```

3.207 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=135

$$\frac{35x^{3/2}}{12b^3}+\frac{35\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{4b^4}-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^{3/2}}{4b^{9/2}}$$

```
[Out] 35/12*x^(3/2)/b^3-35/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(
1/2))*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(9/2)-1/2*x^(7/2)/b/arctanh(tanh(b
*x+a))^2-7/4*x^(5/2)/b^2/arctanh(tanh(b*x+a))+35/4*(b*x-arctanh(tanh(b*x+a)
))*x^(1/2)/b^4
```

Rubi [A]

time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2193}

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^{3/2}}{4b^{9/2}}+\frac{35\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{4b^4}-\frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (35*x^(3/2))/(12*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b^4)
 - (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]])^(3/2))/(4*b^(9/2)) - x^(7/2)/(2*b*ArcTanh[Tanh[a +
 b*x]]^2) - (7*x^(5/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b} \\
&= -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{35\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+
\\
&= \frac{35x^{3/2}}{12b^3} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{(35(-bx
\\
&= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{4b^4} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} -
\\
&= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))}{4b^4} - \frac{35\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{bx-\tanh^{-1}}}\right.}{}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 147, normalized size = 1.09

$$
-\frac{6b^{7/2}x^{7/2}+21b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx))-140b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2+105\sqrt{b}\,\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^3-105\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{12b^{9/2}\tanh^{-1}(\tanh(a+bx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/12*(6*b^(7/2)*x^(7/2) + 21*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] - 140*
b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 + 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh
[a + b*x]]^3 - 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/(
b^(9/2)*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(107) = 214$.
time = 0.54, size = 418, normalized size = 3.10

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b^3} - \frac{6a\sqrt{x}}{b^4} - \frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^4} - \frac{13x^{\frac{3}{2}}a^2}{4b^3\operatorname{arctanh}(\tanh(bx+a))^2} - \frac{13x^{\frac{3}{2}}a(\operatorname{arctanh}(\tanh(b}{2b^3\operatorname{arctanh}(\tanh(}$
default	$\frac{2x^{\frac{3}{2}}}{3b^3} - \frac{6a\sqrt{x}}{b^4} - \frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^4} - \frac{13x^{\frac{3}{2}}a^2}{4b^3\operatorname{arctanh}(\tanh(bx+a))^2} - \frac{13x^{\frac{3}{2}}a(\operatorname{arctanh}(\tanh(b}{2b^3\operatorname{arctanh}(\tanh(}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b^3-6/b^4*a*x^(1/2)-6/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
13/4/b^3/arctanh(tanh(b*x+a))^2*x^(3/2)*a^2-13/2/b^3/arctanh(tanh(b*x+a))^2
*x^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)-13/4/b^3/arctanh(tanh(b*x+a))^2*x^(
3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)
*a^3-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a^2*(arctanh(tanh(b*x+a))-b*x-
a)-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+a))-b*x-a)^2
-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)^3+35/
4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^2+35/2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a)
)-b*x-a)+35/4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A]

time = 0.49, size = 86, normalized size = 0.64

$$\frac{8\,b^3x^{\frac{7}{2}} - 56\,ab^2x^{\frac{5}{2}} - 175\,a^2bx^{\frac{3}{2}} - 105\,a^3\sqrt{x}}{12\,(b^6x^2 + 2\,ab^5x + a^2b^4)} + \frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(8*b^3*x^(7/2) - 56*a*b^2*x^(5/2) - 175*a^2*b*x^(3/2) - 105*a^3*sqrt(x
))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(
sqrt(a*b)*b^4)
```

Fricas [A]

time = 0.35, size = 227, normalized size = 1.68

$$\left[\frac{105\,(ab^2x^2 + 2\,a^2bx + a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx + 2\,b\sqrt{x}\sqrt{-\frac{a}{b}} - a}{bx + a}\right) + 2\,(8\,b^3x^3 - 56\,ab^2x^2 - 175\,a^2bx - 105\,a^3)\sqrt{x}}{24\,(b^6x^2 + 2\,ab^5x + a^2b^4)}, \frac{105\,(ab^2x^2 + 2\,a^2bx + a^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right) + (8\,b^3x^3 - 56\,ab^2x^2 - 175\,a^2bx - 105\,a^3)\sqrt{x}}{12\,(b^6x^2 + 2\,ab^5x + a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [A]
time = 0.40, size = 77, normalized size = 0.57

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^4}-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}-9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

Mupad [B]
time = 1.98, size = 571, normalized size = 4.23

$$\frac{2\,x^{3/2}}{3\,b^3}+\frac{3\,\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^4}+\frac{35\,\sqrt{2}\,\ln\left(\frac{256\,b^{19/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^{3/2}}{32\,b^{9/2}}-\frac{13\,\sqrt{x}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{8\,b^4\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}-\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{4\,b^4\,{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) + (3*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^4 + (35*2^(1/2
)*log((256*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(
```

```
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(32*b^(9/2)) - (13*
x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(8*b^4*(log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (x^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.208 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=110

$$\frac{15\sqrt{x}}{4b^3}-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+b$$

```
[Out] -1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^2-5/4*x^(3/2)/b^2/arctanh(tanh(b*x+a))+
15/4*x^(1/2)/b^3-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1
/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(7/2)
```

Rubi [A]

time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2190, 2193}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tan
h[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/(4*b^(7/2)) - x^(5/2)/(2*
b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
```

```
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{15\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(\ldots}}{8b^2}$$

$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{(15(-b\ldots}{\ldots}$$

$$= \frac{15\sqrt{x}}{4b^3} - \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tan\ldots}}{4b^{7/2}}$$

Mathematica [A]
time = 0.07, size = 104, normalized size = 0.95

$$\frac{1}{4}\left(\frac{15\sqrt{x}}{b^3} - \frac{2x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{b^2\tanh^{-1}(\tanh(a+bx))} - \frac{15\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}{b^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
[Out] ((15*Sqrt[x])/b^3 - (2*x^(5/2))/(b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/
(b^2*ArcTanh[Tanh[a + b*x]]) - (15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + A
rcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(7/2))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(86) = 172$.
time = 0.44, size = 249, normalized size = 2.26

method	result

derivativedivides	$\frac{2\sqrt{x}}{b^3}+\frac{9x^{\frac{3}{2}}a}{4b^2\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{9x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4b^2\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7\sqrt{x}\,a^2}{4b^3\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7\sqrt{x}}{2}$
default	$\frac{2\sqrt{x}}{b^3}+\frac{9x^{\frac{3}{2}}a}{4b^2\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{9x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4b^2\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7\sqrt{x}\,a^2}{4b^3\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7\sqrt{x}}{2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2/arctanh(tanh(b*x+a))^2*x^(3/2)*a+9/4/b^2/arctanh(tanh
(b*x+a))^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a)
)^2*x^(1/2)*a^2+7/2/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+
a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x
-a)^2-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2))*a-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)
```

Maxima [A]
time = 0.49, size = 73, normalized size = 0.66

$$\frac{8\,b^2x^{\frac{5}{2}}+25\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^2*x^(5/2) + 25*a*b*x^(3/2) + 15*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x
+ a^2*b^3) - 15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Fricas [A]
time = 0.36, size = 200, normalized size = 1.82

$$\left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},-\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.40, size = 59, normalized size = 0.54

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^{3}}+\frac{2\,\sqrt{x}}{b^{3}}+\frac{9\,abx^{\frac{3}{2}}+7\,a^{2}\sqrt{x}}{4\,\left(bx+a\right)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B]
time = 1.69, size = 511, normalized size = 4.65

$$\frac{2\,\sqrt{x}}{b^3}-\frac{9\,\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}+\frac{15\,\sqrt{2}\,\ln\left(\frac{64\,b^{15/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}{16\,b^{7/2}}-\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2\,b^3\,{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*x^(1/2))/b^3 - (9*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^3*(log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1
)))) + (15*2^(1/2)*log((64*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b
^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*ex
```

```
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
+ 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(16
*b^(7/2)) - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.209 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -1/2*x^(3/2)/b/arctanh(tanh(b*x+a))^2-3/4*x^(1/2)/b^2/arctanh(tanh(b*x+a))-
3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(5/2)/(b*x-
arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2199, 2193}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/
2)*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]
]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$
$$= -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{\sqrt{x}\,\tanh^{-1}(\tan}}{8b^2}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Mathematica [A]
time = 0.05, size = 96, normalized size = 0.98

$$-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh
[a + b*x]]) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(4*b^(5/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

Maple [A]
time = 0.42, size = 85, normalized size = 0.87

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}{4b^2}}{\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{4b^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}$	85
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}{4b^2}}{\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{4b^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}$	85

risch	Expression too large to display	1075

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*(arctanh(tanh(b*x+a))-b*x)/b^2*x^(1/2))/arctanh(tanh(
b*x+a))^2+3/4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 61, normalized size = 0.62

$$-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.35, size = 185, normalized size = 1.89

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.39, size = 47, normalized size = 0.48

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2} - \frac{5\,bx^{\frac{3}{2}} + 3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

Mupad [B]
time = 1.87, size = 667, normalized size = 6.81

$$\frac{3\sqrt{2}\,\ln\left(\frac{16\,b^{11/2}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}\right)}{8\,b^{5/2}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{b^2\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^2}-\frac{\sqrt{x}\,\left(\frac{1}{b\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}+\frac{8\,\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-8\,\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+16\,b\,x}{2\,b\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}\right)\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{2\,b\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (3*2^(1/2)*log((16*b^(11/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^
(1/2)*b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2
/(exp(2*a)*exp(2*b*x) + 1)))))/(8*b^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) -
 (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - (x^(1/2)*(1/
(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)) + (8*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 8*log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 16*b*x)/(2*b*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.210 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx}$$

```
[Out] 1/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(3/2)/(b*x-
arctanh(tanh(b*x+a)))^(3/2)-1/4/b^2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/4/
b^2/arctanh(tanh(b*x+a))/x^(1/2)-1/2*x^(1/2)/b/arctanh(tanh(b*x+a))^2
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{4b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(4*b^(3/2)*(b
*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(4*b^2*Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])) - Sqrt[x]/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(4*b^2*Sqrt[x]*Arc
Tanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{4b^2\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(t}}{8b}$$

$$= -\frac{1}{4b^2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{4b^2\sqrt{}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{4b^2\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a-\right.}$$

Mathematica [A]
time = 0.16, size = 107, normalized size = 0.86

$$\frac{1}{4}\left(-\frac{2\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \frac{\sqrt{x}}{-b^2x\tanh^{-1}(\tanh(a+bx))+b\tanh^{-1}(\tanh(a+bx))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((-2*Sqrt[x])/(b*ArcTanh[Tanh[a + b*x]]^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]
)^(3/2)) + Sqrt[x]/(-(b^2*x*ArcTanh[Tanh[a + b*x]]) + b*ArcTanh[Tanh[a + b*
x]]^2))/4
```

Maple [A]
time = 0.46, size = 98, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2x^{\frac{3}{2}}}{8\,\mathrm{arctanh}(\tanh(bx+a))-8bx}-\frac{\sqrt{x}}{4b}}{\mathrm{arctanh}(\tanh(bx+a))^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{4(\mathrm{arctanh}(\tanh(bx+a))-bx)b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}$
default	$\frac{\frac{2x^{\frac{3}{2}}}{8\,\mathrm{arctanh}(\tanh(bx+a))-8bx}-\frac{\sqrt{x}}{4b}}{\mathrm{arctanh}(\tanh(bx+a))^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}\right)}{4(\mathrm{arctanh}(\tanh(bx+a))-bx)b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)\,b}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/8/(arctanh(tanh(b*x+a))-b*x)*x^(3/2)-1/8*x^(1/2)/b)/arctanh(tanh(b*x+a
))^2+1/4/(arctanh(tanh(b*x+a))-b*x)/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*
arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 64, normalized size = 0.51

$$\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(ab^3x^2+2\,a^2b^2x+a^3b)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.35, size = 186, normalized size = 1.49

$$\left[-\frac{(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\,(ab^2x-a^2b)\sqrt{x}}{8\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)},-\frac{(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(ab^2x-a^2b)\sqrt{x}}{4\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**3, x)
```

Giac [A]
time = 0.39, size = 52, normalized size = 0.42

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab}+\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\left(bx+a\right)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

Mupad [B]
time = 1.84, size = 580, normalized size = 4.64

$$\frac{\sqrt{2}\,\ln\left(-\frac{4\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)+4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx}+2\sqrt{2}\,bx\right)\left(b^3\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)^2+4a^2b^3-4ab^3\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)\right)}{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}\right)}{4\,b^{3/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)^{3/2}}-\frac{2\sqrt{x}}{b\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^2}-\frac{\sqrt{x}}{b\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2^(1/2)*log(-(4*b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/
2)*b*x)*(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 4*a^2*b^3 - 4*a*b^3*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))))/(4*b^(3/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^(3/2)) - (2*x^(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - x^(1/2)/(b*(log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.211 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=152

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{1}{4b^2x^{3/2}\left(bx-\tanh\right.}$$

```
[Out] 1/4/b^2/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+1/4/b^2/x^(3/2)/arctanh(tanh(b*x
+a))-3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arc
tanh(tanh(b*x+a)))^(5/2)/b^(1/2)+3/4/b/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
-1/2/b/arctanh(tanh(b*x+a))^2/x^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt[
b]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + 3/(4*b*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + 1/(4*b^2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*
b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2) + 1/(4*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x
]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$
$$= -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))} + \frac{3\int$$
$$= \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}$$
$$= \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\text{t}\right.}$$

Mathematica [A]
time = 0.12, size = 118, normalized size = 0.78

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{3\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt
[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)) + (3*Sqrt[x])/(4*ArcTanh[Tanh[
a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2) + Sqrt[x]/(2*ArcTanh[Tanh[a
+ b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A]
time = 0.46, size = 112, normalized size = 0.74

method	result
derivativedivides	$\frac{\sqrt{x}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\sqrt{x}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{4(\mathrm{arct}}$
default	$\frac{\sqrt{x}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\sqrt{x}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{4(\mathrm{arct}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^2+3/4/(arctanh(
tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))+3/4/(arctanh(tanh(b*x+a))-
b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh
(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 60, normalized size = 0.39

$$\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.35, size = 186, normalized size = 1.22

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\,(3\,ab^2x+5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,ab^2x+5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
```

```
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\ \operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**3), x)
```

Giac [A]
time = 0.40, size = 47, normalized size = 0.31

$$\frac{3\ \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\ a^2}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\left(bx+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

Mupad [B]
time = 1.90, size = 741, normalized size = 4.88

$$\frac{6\sqrt{x}}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}-\frac{4\sqrt{x}}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{3\sqrt{2}\ \ln\left(\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}+2\sqrt{2}\,bx\right)\left(16a^4b+b\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4-8ab\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3-32a^3b\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)+24a^2b\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2\right)}{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}\right)}{2\sqrt{b}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (6*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (4*x^(1/2))/((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (3*2^(1/2)*log((b^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
```

```
a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*(16*a^4*b + b*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^4 - 8*a*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*b
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 24*a^2*b*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a
)*exp(2*b*x) + 1)))))/(2*b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2))
```

3.212 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=176

$$-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{15}{4\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{}{4bx^{3/2}\left(bx-\tan\right.}$$

[Out] 5/4/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+3/4/b^2/x^(5/2)/(b*x-arctanh(tan
h(b*x+a)))-1/2/b/x^(3/2)/arctanh(tanh(b*x+a))^2+3/4/b^2/x^(5/2)/arctanh(tan
h(b*x+a))-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^
(1/2)/(b*x-arctanh(tanh(b*x+a)))^(7/2)+15/4/(b*x-arctanh(tanh(b*x+a)))^3/x^
(1/2)

Rubi [A]
time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2}-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{15}{4\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + 15/(4*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3) + 5/(4*b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 3/(4*
b^2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(3/2)*ArcTanh[Tanh[a
 + b*x]]^2) + 3/(4*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2}-\frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}\\
&= -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{15 \cdots}{\cdots}\\
&= \frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\cdots\\
&= \frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a-\cdots\right.}\\
&= \frac{15}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\cdots\right.}\\
&= -\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{\cdots}{4\sqrt{x}\left(bx-\tanh\cdots\right.}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 141, normalized size = 0.80

$$-\frac{15\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{2}{\sqrt{x}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{7b\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{b\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) - 2/(Sqrt[x]*(-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3) - (7*b*Sqrt[x])/(4*ArcTanh[Tanh[a + b*x]]*(-(b*x) + A
```

```
rcTanh[Tanh[a + b*x]])^3) - (b*Sqrt[x])/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*x)
 + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.62, size = 181, normalized size = 1.03

method	result
derivativedivides	$-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}-\frac{7b^2x^{\frac{3}{2}}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\operatorname{arctanh}(\tanh(bx+a))^2}-\frac{}{4(\operatorname{arctanh}(\tanh(bx+}$
default	$-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}-\frac{7b^2x^{\frac{3}{2}}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\operatorname{arctanh}(\tanh(bx+a))^2}-\frac{}{4(\operatorname{arctanh}(\tanh(bx+}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^3/x^(1/2)-7/4/(arctanh(tanh(b*x+a))-b*x)^3*b^
2/arctanh(tanh(b*x+a))^2*x^(3/2)-9/4/(arctanh(tanh(b*x+a))-b*x)^3*b/arctanh
(tanh(b*x+a))^2*x^(1/2)*a-9/4/(arctanh(tanh(b*x+a))-b*x)^3*b/arctanh(tanh(b
*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)-15/4/(arctanh(tanh(b*x+a))-b*
x)^3*b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh
(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 73, normalized size = 0.41

$$-\frac{15\,b^2x^2+25\,abx+8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}}+2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.35, size = 214, normalized size = 1.22

$$\left[\frac{15\,(b^2x^3+2\,abx^2+a^2x)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2+25\,abx+8\,a^2)\sqrt{x}}{8\,(a^3b^2x^3+2\,a^4bx^2+a^5x)},\ \frac{15\,(b^2x^3+2\,abx^2+a^2x)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2+25\,abx+8\,a^2)\sqrt{x}}{4\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \operatorname{atanh}^{3}{\left(\tanh{\left(a + bx \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**3), x)
```

Giac [**A**]
time = 0.40, size = 59, normalized size = 0.34

$$-\frac{15\, b \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\, a^{3}} - \frac{2}{a^{3}\sqrt{x}} - \frac{7\, b^{2} x^{\frac{3}{2}} + 9\, ab\sqrt{x}}{4\, (bx + a)^{2} a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

Mupad [**B**]
time = 2.17, size = 1077, normalized size = 6.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (x*((12*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (3*b*(16*log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - 16*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*b*x
))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^4) - (16*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 16*
```

```
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*b*x)/(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^3)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) + (15*2^(1/2)*b^(1/2)*log((b^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b
^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 160*a^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
+ 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 192*a^5*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (8
*b*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

3.213 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{35b}{4\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{}{12x^{3/2}\left(bx-\text{ta}\right.}$$

```
[Out] -35/4*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(9/2)+35/12/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+7/
4/b/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+5/4/b^2/x^(7/2)/(b*x-arctanh(tanh(
b*x+a)))-1/2/b/x^(5/2)/arctanh(tanh(b*x+a))^2+5/4/b^2/x^(7/2)/arctanh(tanh(
b*x+a))+35/4*b/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))}+\frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2}+\frac{35b}{4\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (35*b)/(4*Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]])^4) + 35/(12*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 7
/(4*b*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 5/(4*b^2*x^(7/2)*(b*x - A
rcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2) + 5/(4*b
^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))} + \frac{35\int$$

$$= \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} +$$

$$= \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= \frac{35b}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= -\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{3}{4\sqrt{x}\left(bx-\tanh\right.}$$

Mathematica [A]
time = 0.18, size = 156, normalized size = 0.78

$$\frac{1}{12}\left(\frac{105b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{80bx-8\tanh^{-1}(\tanh(a+bx))}{x^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{33b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{6b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((105*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) + (80*b*x - 8*ArcTanh[Tanh[a +
b*x]])/(x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (33*b^2*Sqrt[x])/(Ar
cTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (6*b^2*Sqrt[x])
/(ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/12
```

Maple [A]
time = 1.00, size = 207, normalized size = 1.03

method	result
derivativedivides	$\frac{11b^3x^{\frac{3}{2}}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^4 \operatorname{arctanh}(\tanh(bx+a))^2} + \frac{13b^2\sqrt{x}\ a}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^4 \operatorname{arctanh}(\tanh(bx+a))^2} + \frac{}{4(a}$
default	$\frac{11b^3x^{\frac{3}{2}}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^4 \operatorname{arctanh}(\tanh(bx+a))^2} + \frac{13b^2\sqrt{x}\ a}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^4 \operatorname{arctanh}(\tanh(bx+a))^2} + \frac{}{4(a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 11/4/(arctanh(tanh(b*x+a))-b*x)^4*b^3/arctanh(tanh(b*x+a))^2*x^(3/2)+13/4/(
arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*x^(1/2)*a+13/4/(arct
anh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*
x+a))-b*x-a)+35/4/(arctanh(tanh(b*x+a))-b*x)^4*b^2/((arctanh(tanh(b*x+a))-b
*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(ar
ctanh(tanh(b*x+a))-b*x)^3/x^(3/2)+6/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(1/2)
```

Maxima [A]
time = 0.48, size = 86, normalized size = 0.43

$$\frac{105\,b^3x^3 + 175\,ab^2x^2 + 56\,a^2bx - 8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}} + 2\,a^5bx^{\frac{5}{2}} + a^6x^{\frac{3}{2}}\right)} + \frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

Fricas [A]
time = 0.34, size = 250, normalized size = 1.24

$$\left[\frac{105\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3)\sqrt{x}}{24\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)},\ -\frac{105\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3)\sqrt{x}}{12\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**3), x)
```

Giac [A]
time = 0.39, size = 71, normalized size = 0.35

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

Mupad [B]
time = 2.50, size = 1362, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (x^(1/2)*((2*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^2 - (x^(1/2)*((280*b)
/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (280*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))/
(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (70*2^(1/2)*b^(3/2)*log((b^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)
*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^8 + 112*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 448*a^3*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^5 + 1120*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 17
92*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 1792*a^6*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^2 + 256*a^8 - 16*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 1024*a^7*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^(9/2)
```

3.214 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=228

$$-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+\frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5}+\frac{2}{4x^{3/2}\left(bx-\tanh\right.}$$

```
[Out] -63/4*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(11/2)+21/4*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^4+
63/20/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+9/4/b/x^(7/2)/(b*x-arctanh(tanh(
b*x+a)))^2+7/4/b^2/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))-1/2/b/x^(7/2)/arctanh
(tanh(b*x+a))^2+7/4/b^2/x^(9/2)/arctanh(tanh(b*x+a))+63/4*b^2/(b*x-arctanh(
tanh(b*x+a)))^5/x^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2199, 2194, 2193}

$$-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+\frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))}+\frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5}+\frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-63*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) + (63*b^2)/(4*Sqrt[x]*(b*x - Arc
Tanh[Tanh[a + b*x]])^5) + (21*b)/(4*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
4) + 63/(20*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 9/(4*b*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + 7/(4*b^2*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(2*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2) + 7/(4*b^2*x^(9/2)*ArcTanh
[Tanh[a + b*x]])
```

Rule 2193

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$
$$= -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))} + \frac{63}{}$$
$$= \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2}$$
$$= \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a-\right.}$$
$$= \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$
$$= \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$
$$= \frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5} + \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$
$$= -\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} + \frac{}{4\sqrt{x}\left(bx-\tanh\right.}$$

Mathematica [A]

time = 0.24, size = 174, normalized size = 0.76

$$\frac{1}{20}\left(\frac{75b^3\sqrt{x}}{(bx-\tanh^{-1}(\tanh(a+bx)))^5\tanh^{-1}(\tanh(a+bx))}-\frac{315b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\tanh^{-1}(\tanh(a+bx))}}\right)}{(-bx+\tanh^{-1}(\tanh(a+bx)))^{11/2}}-\frac{10b^3\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2(-bx+\tanh^{-1}(\tanh(a+bx)))^4}+\frac{8(36b^2x^2-7bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2)}{x^{5/2}(bx-\tanh^{-1}(\tanh(a+bx)))^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((75*b^3*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]])
 - (315*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) - (10*b^3*Sqrt[x])/(ArcTanh[
Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (8*(36*b^2*x^2 - 7*
b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/(x^(5/2)*(b*x - Arc
Tanh[Tanh[a + b*x]])^5))/20
```

Maple [A]
time = 1.82, size = 229, normalized size = 1.00

method	result
derivativedivides	$-\frac{2}{5(\text{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{5}{2}}}-\frac{12b^2}{(\text{arctanh}(\tanh(bx+a))-bx)^5\sqrt{x}}+\frac{2b}{(\text{arctanh}(\tanh(bx+a))-bx)^4 x^{\frac{3}{2}}}-\frac{}{4(\text{arc}}$
default	$-\frac{2}{5(\text{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{5}{2}}}-\frac{12b^2}{(\text{arctanh}(\tanh(bx+a))-bx)^5\sqrt{x}}+\frac{2b}{(\text{arctanh}(\tanh(bx+a))-bx)^4 x^{\frac{3}{2}}}-\frac{}{4(\text{arc}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)^3/x^(5/2)-12/(arctanh(tanh(b*x+a))-b*x)^5*b
^2/x^(1/2)+2/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(3/2)-15/4/(arctanh(tanh(b*x+
a))-b*x)^5*b^4/arctanh(tanh(b*x+a))^2*x^(3/2)-17/4/(arctanh(tanh(b*x+a))-b*
x)^5*b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*a-17/4/(arctanh(tanh(b*x+a))-b*x)^5
*b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)-63/4/(arct
anh(tanh(b*x+a))-b*x)^5*b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x
^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 97, normalized size = 0.43

$$-\frac{315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a^4}{20\left(a^5b^2x^{\frac{9}{2}}+2\,a^6bx^{\frac{7}{2}}+a^7x^{\frac{5}{2}}\right)}-\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)/
(a^5*b^2*x^(9/2) + 2*a^6*b*x^(7/2) + a^7*x^(5/2)) - 63/4*b^3*arctan(b*sqrt(
x)/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 0.35, size = 276, normalized size = 1.21

$$\left[\frac{315\,(b^4x^5+2\,ab^3x^4+a^2b^2x^3)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a^4)\sqrt{x}}{40\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)},\ \frac{315\,(b^4x^5+2\,ab^3x^4+a^2b^2x^3)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a^4)\sqrt{x}}{20\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*
sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a
^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*
x^3), 1/20*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(b/a)*arctan(a*sq
rt(b/a)/(b*sqrt(x))) - (315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*
a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 0.39, size = 80, normalized size = 0.35

$$-\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^5}-\frac{15\,b^4x^{\frac{3}{2}}+17\,ab^3\sqrt{x}}{4\,(bx+a)^2a^5}-\frac{2\,(30\,b^2x^2-5\,abx+a^2)}{5\,a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -63/4*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/4*(15*b^4*x^(3/2)
 + 17*a*b^3*sqrt(x))/((b*x + a)^2*a^5) - 2/5*(30*b^2*x^2 - 5*a*b*x + a^2)/(
a^5*x^(5/2))
```

Mupad [B]
time = 1.95, size = 2151, normalized size = 9.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] 16/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (x*(((((448*b^4)/(3*(2*a*b - b*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (16*b^3*(2*b*(
3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(3*(2*a*b - b
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))/(2*b) - (112*b^3)/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2) + (8*b^2*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1
)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14
*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)))/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^3)) - (4*b*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1))
 - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)))/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^2))/(x^(3/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - ((336*b^2)/(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^4 - (1008*b^3*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)/(x^(1/2)*(lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(
2*b*x) + 1)))) + (252*2^(1/2)*b^(5/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*
```

```
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^10 + 180*
a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8 - 960*a^3*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^7 + 3360*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 8064*a^5*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^5 + 13440*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 15360*a^7*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 11520*a^8*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2 + 1024*a^10 - 20*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^9 - 5120*a^9*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(11
/2)
```

3.215 $\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=142

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{x^{3/2}}{}$$

```
[Out] -1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(5/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-1/12*x^(3/2)*(b*x
-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b-1/8*(b*x-arctanh(tanh(b
*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}-\frac{\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{8b^2}-\frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b}+\frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] -1/8*(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTan
h[Tanh[a + b*x]])^3)/b^(5/2) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/3 - (
x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b)
 - (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/
(8*b^2)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{6}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b}$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{8b^{5/2}}$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(3b^2x^2+8bx\tanh^{-1}(\tanh(a+bx))-3\tanh^{-1}(\tanh(a+bx))^2\right)}{24b^2}+\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a + b
*x]] - 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b^2) + ((-(b*x) + ArcTanh[Tanh[a +
b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(114) = 228.
time = 0.13, size = 304, normalized size = 2.14

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}-\frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4b^2}+\frac{a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b^2}+$
default	$\frac{x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}-\frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4b^2}+\frac{a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(3/2)+1/8/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/8/b^(5/2)*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+3/8/b^(5/2)*a^2*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+1/4/b^2*a*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/8/b^(5/2)*a
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a
)^2-1/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)
+1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+
1/8/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*
x+a))-b*x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A]
time = 0.36, size = 141, normalized size = 0.99

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{3}}, -\frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 60, normalized size = 0.42

$$\frac{1}{24}\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(3/2)*atanh(tanh(a + b*x))^(1/2), x)
```

3.216 $\int \sqrt{x} \sqrt{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}}{}$$

```
[Out] -1/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(3/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)-1/4*(b*x-arctanh(
tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] -1/4*(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTan
h[Tanh[a + b*x]])^2)/b^(3/2) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/2 - (
Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{4}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int$$

$$= \frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.81

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx+\tanh^{-1}(\tanh(a+bx))\right)}{4b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(b*x + ArcTanh[Tanh[a + b*x]]))/(4*b)
 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/(4*b^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.
time = 0.12, size = 174, normalized size = 1.67

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2b} - \frac{a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b} - \frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\mathrm{t}}\right.}{4b^{\frac{3}{2}}}$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2b} - \frac{a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b} - \frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\mathrm{t}}\right.}{4b^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b*a*x^(1/2)*arctanh(tanh(b*x+a
))^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2-1/2
/b^(3/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x
+a))-b*x-a)-1/4/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))
```

```
^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(
tanh(b*x+a))-b*x-a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)*sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A]
time = 0.37, size = 114, normalized size = 1.10

$$\left[\frac{a^2\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^2},\frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 48, normalized size = 0.46

$$\frac{1}{4}\,\sqrt{bx+a}\,\left(2\,x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + 1/4*a^2*log(abs(-sqrt(b)*sqrt(x) +
sqrt(b*x + a)))/b^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(1/2), x)
```

3.217 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} \, dx$

Optimal. Leaf size=61

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}+\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+
a)))/b^(1/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] -((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{}{\sqrt{x}\ \sqrt{}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}} +$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 1.02

$$\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\log\left(b\sqrt{x}+\sqrt{b}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] + ((-(b*x) + ArcTanh[Tanh[a + b*x]])*L
og[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Maple [A]
time = 0.12, size = 75, normalized size = 1.23

method	result
derivativedivides	$\sqrt{x}\ \sqrt{\operatorname{arctanh}(\tanh(bx+a))} + \frac{\ln\left(\sqrt{b}\ \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)a}{\sqrt{b}} + \frac{\ln\left(\sqrt{b}\right.}{}$
default	$\sqrt{x}\ \sqrt{\operatorname{arctanh}(\tanh(bx+a))} + \frac{\ln\left(\sqrt{b}\ \sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)a}{\sqrt{b}} + \frac{\ln\left(\sqrt{b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/sqrt(x), x)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.52

$$\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/sqrt(x), x)
```

Giac [A]
time = 0.36, size = 36, normalized size = 0.59

$$-\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}+\sqrt{bx+a}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b) + sqrt(b*x + a)*sqrt(
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(1/2),x)
[Out] int(atanh(tanh(a + b*x))^(1/2)/x^(1/2), x)
```

3.218 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx$

Optimal. Leaf size=49

$$2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*b^(1/2)-2*arctanh(tan
h(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2199, 2196}

$$2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx = -\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= 2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 1.06

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} + 2\sqrt{b}\log\left(b\sqrt{x} + \sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] + 2*Sqrt[b]*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(37) = 74$.
time = 0.13, size = 149, normalized size = 3.04

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{2b\sqrt{x}\,\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\mathrm{arctanh}(\tanh(bx+a))-bx} + \frac{2\sqrt{b}\,\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{}\right.}{\mathrm{arctanh}(}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{2b\sqrt{x}\,\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\mathrm{arctanh}(\tanh(bx+a))-bx} + \frac{2\sqrt{b}\,\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{}\right.}{\mathrm{arctanh}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(1/2)/(arctanh(
tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a+2*b^(1/2
)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.51, size = 40, normalized size = 0.82

$$2\sqrt{b}\log\left(\frac{b\sqrt{x}}{\sqrt{ab}}+\sqrt{\frac{bx}{a}+1}\right)-\frac{2\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*log(b*sqrt(x)/sqrt(a*b) + sqrt(b*x/a + 1)) - 2*sqrt(b*x + a)/sqrt
(x)
```

Fricas [A]
time = 0.35, size = 89, normalized size = 1.82

$$\left[\frac{\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(3/2), x)
```

Giac [A]
time = 0.40, size = 57, normalized size = 1.16

$$-\sqrt{b}\log\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}\right)+\frac{4\,a\sqrt{b}}{\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*log((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2) + 4*a*sqrt(b)/((sqrt(b)*s
qrt(x) - sqrt(b*x + a))^2 - a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(3/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(1/2)/x^(3/2), x)
```

3.219 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}} \, dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2198}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}} \, dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}\left(3bx-3\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(x^(3/2)*(3*b*x - 3*ArcTanh[Tanh[a + b*x]]
))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}$	29
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A]
time = 0.36, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.
time = 0.41, size = 59, normalized size = 1.69

$$\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4}+a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + a^2*b^(3/2))/((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^3
```

Mupad [B]
time = 1.71, size = 210, normalized size = 6.00

$$\frac{2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}-2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\sqrt{x}\,\left(3\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-3\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(5/2),x)
```

```
[Out] (2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
/2)^(1/2) - 2*log(2/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/
(x^(1/2)*(3*x*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*x*log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x^2))
```

3.220 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}\,dx$

Optimal. Leaf size=72

$$\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 4/15*b*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5*
arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.67

$$\frac{2\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{5/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (2*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(
5/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.12, size = 59, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{4b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}$	59
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{4b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A]
time = 0.47, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2-abx-3\,a^2\right)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Fricas [A]
time = 0.35, size = 34, normalized size = 0.47

$$\frac{2\,(2\,b^2x^2 - abx - 3\,a^2)\sqrt{bx + a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.39, size = 112, normalized size = 1.56

$$\frac{8\left(15\,b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x} - \sqrt{bx + a}\right)^{6} + 5\,ab^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x} - \sqrt{bx + a}\right)^{4} + 5\,a^{2}b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x} - \sqrt{bx + a}\right)^{2} - a^{3}b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,\sqrt{x} - \sqrt{bx + a}\right)^{2} - a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 5*a*b^(5/2)*(sqrt(b)
*sqrt(x) - sqrt(b*x + a))^4 + 5*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a
))^2 - a^3*b^(5/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

Mupad [B]
time = 1.41, size = 174, normalized size = 2.42

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,b^2\,x^2}{15\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2} + \frac{4\,b\,x}{15\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)} - \frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(7/2),x)

[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(15*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) +
 (4*b*x)/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/5))/x^(5/2)
```

3.221 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx)}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a\right.}$$

```
[Out] 16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/35*b*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]

time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(35*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(35b^2x^2-42bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2\right)}{105x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 42*b*x*ArcTanh[Tanh[a + b*x]]
 + 15*ArcTanh[Tanh[a + b*x]]^2))/(105*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

Maple [A]
time = 0.13, size = 105, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}\right)}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}\right)}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(3/2)-8/7*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(t
anh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(
b*x+a))^(3/2))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 0.41

$$-\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Fricas [A]
time = 0.34, size = 45, normalized size = 0.41

$$-\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 138, normalized size = 1.25

$$\frac{32\left(70\,b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^8+35\,ab^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^6+21\,a^2b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-7\,a^3b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+a^4b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 32/105*(70*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 35*a*b^(7/2)*(sqrt
(b)*sqrt(x) - sqrt(b*x + a))^6 + 21*a^2*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
```

```
 + a))^4 - 7*a^3*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + a^4*b^(7/2))
/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

Mupad [B]
time = 1.59, size = 234, normalized size = 2.13

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{32\,b^2\,x^2}{105\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{128\,b^3\,x^3}{105\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}+\frac{4\,b\,x}{35\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(9/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((32*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
+ (128*b^3*x^3)/(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (4*b*x)/(35*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)) - 2/7))/x^(7/2)
```

3.222 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+b}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh\right.}$$

```
[Out] 32/315*b^3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^4+
16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+
4/21*b*arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(3/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$,
Rules used = {2202, 2198}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(315*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(21*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(9*x^(9/2)
*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A]
time = 0.05, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(105b^3x^3-189b^2x^2\tanh^{-1}(\tanh(a+bx))+135bx\tanh^{-1}(\tanh(a+bx))^2-35\tanh^{-1}(\tanh(a+bx))^3\right)}{315x^{9/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 189*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 135*b*x*ArcTanh[Tanh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))/(
315*x^(9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.13, size = 151, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,\mathrm{arc}}{15(\mathrm{arctanh}}\right.}{7(\mathrm{arctanh}(\tanh(bx+a))-b}\right.}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,\mathrm{arc}}{15(\mathrm{arctanh}}\right.}{7(\mathrm{arctanh}(\tanh(bx+a))-b}\right.}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(3/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(3/2)-4/7*b/(arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+
a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)))
```

Maxima [**A**]
time = 0.48, size = 56, normalized size = 0.38

$$\frac{2\,(16\,b^4x^4-8\,ab^3x^3+6\,a^2b^2x^2-5\,a^3bx-35\,a^4)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

Fricas [**A**]
time = 0.35, size = 56, normalized size = 0.38

$$\frac{2\,(16\,b^4x^4-8\,ab^3x^3+6\,a^2b^2x^2-5\,a^3bx-35\,a^4)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 166, normalized size = 1.12

$$\frac{64\left(315\,b^{\frac{9}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{10}+189\,ab^{\frac{9}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{8}+84\,a^2b^{\frac{9}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{6}-36\,a^3b^{\frac{9}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4}+9\,a^4b^{\frac{9}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^5b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="giac")

[Out] 64/315*(315*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^10 + 189*a*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 84*a^2*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 36*a^3*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + 9*a^4*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^5*b^(9/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^9

Mupad [B]
time = 1.52, size = 294, normalized size = 1.99

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,b^2\,x^2}{105\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{128\,b^3\,x^3}{315\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}+\frac{512\,b^4\,x^4}{315\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}+\frac{4\,b\,x}{63\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{2}{9}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(tanh(a + b*x))^(1/2)/x^(11/2),x)

[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^3*x^3)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^4*x^4)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) + (4*b*x)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/9))/x^(9/2)

3.223 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^{3/2} \, dx$

Optimal. Leaf size=177

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^4}{64b^{5/2}}-\frac{1}{8}x^{5/2}(bx-\tanh^{-1}(\tanh(a+bx)))$$

```
[Out] 3/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^4/b^(5/2)+1/4*x^(5/2)*arctanh(tanh(b*x+a))^(3/2)-1/8*x^(5/2)*(b*x-
arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/32*x^(3/2)*(b*x-arctanh(
tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)/b+3/64*(b*x-arctanh(tanh(b*x+a))
)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^4}{64b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))^3}{64b^2}+\frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))^2}{32b}-\frac{1}{8}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))+\frac{1}{4}x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^4)/(64*b^(5/2)) - (x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sq
rt[ArcTanh[Tanh[a + b*x]]])/8 + (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]])/(32*b) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b^2) + (x^(5/2)*ArcTanh[Tanh[a + b
*x]]^(3/2))/4
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{4}x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{8}\left(3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{4}x^{5/2}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}(}{}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}(}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.69

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-3b^3x^3+11b^2x^2\tanh^{-1}(\tanh(a+bx))+11bx\tanh^{-1}(\tanh(a+bx))^2-3\tanh^{-1}(\tanh(a+bx))^3\right)+3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^3*x^3 + 11*b^2*x^2*ArcT
anh[Tanh[a + b*x]] + 11*b*x*ArcTanh[Tanh[a + b*x]]^2 - 3*ArcTanh[Tanh[a + b
*x]]^3) + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*Sqr
t[ArcTanh[Tanh[a + b*x]]]])/(64*b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(143) = 286$.
time = 0.12, size = 471, normalized size = 2.66

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{4b} - \frac{a\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{8b^2} + \frac{a^2\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{32b^2} + \frac{3a^3\sqrt{x}\sqrt{\mathrm{ar}}}{}$
default	$\frac{x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{4b} - \frac{a\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{8b^2} + \frac{a^2\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{32b^2} + \frac{3a^3\sqrt{x}\sqrt{\mathrm{ar}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/8/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(5/2)+1/32/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/64/b^2*a^3*x^(
1/2)*arctanh(tanh(b*x+a))^(1/2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a^4+3/16/b^(5/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a
))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+9/64/b^2*a^2*(arctanh(tanh(b*x+a))-b
*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+9/32/b^(5/2)*a^2*ln(b^(1/2)*x^(1/2
)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+1/16/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+9/64/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/16/b^(5/2)
*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^3-1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/
2)+1/32/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/
2)+3/64/b^2*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/
2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 163, normalized size = 0.92

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.40, size = 147, normalized size = 0.83

$$\frac{1}{384}\sqrt{2}\left(8\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^{2}}{b^{2}}\right)\sqrt{x}-\frac{3a^{3}\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}\right)a+\sqrt{2}\left(\left(2\left(4\left(6x+\frac{a}{b}\right)x-\frac{5a^{2}}{b^{2}}\right)x+\frac{15a^{3}}{b^{3}}\right)\sqrt{bx+a}\sqrt{x}+\frac{15a^{4}\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(8*sqrt(2)*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(
x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a + sqrt(2)*
((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 1
5*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(3/2)*atanh(tanh(a + b*x))^(3/2), x)
```

3.224 $\int \sqrt{x}\, \tanh^{-1}(\tanh(a+bx))^{3/2}\, dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] 1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^3/b^(3/2)+1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)-1/4*x^(3/2)*(b*x-a
rctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/8*(b*x-arctanh(tanh(b*x+a
)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}+\frac{\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]])^3)/(8*b^(3/2)) - (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[
ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(8*b) + (x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \sqrt{x}\, \tanh^{-1}(\tanh(a+bx))^{3/2}\, dx = \frac{1}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{2}(bx - \tanh^{-1}(\tanh(a+bx)))\int \sqrt{x}$$

$$= -\frac{1}{4}x^{3/2}(bx - \tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{3}x^{3/2}$$

$$= -\frac{1}{4}x^{3/2}(bx - \tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{\sqrt{x}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx - \tanh^{-1}(\tanh(a+bx)))^3}{8b^{3/2}} -$$

Mathematica [A]
time = 0.05, size = 105, normalized size = 0.76

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-3b^2x^2 + 8bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2\right)}{24b} + \frac{(bx - \tanh^{-1}(\tanh(a+bx)))^3\log\left(b\sqrt{x} + \sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a +
b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b) + ((b*x - ArcTanh[Tanh[a + b*x]
])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(111) = 222$.
time = 0.12, size = 304, normalized size = 2.19

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3b} - \frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{12b} - \frac{a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b} -$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3b} - \frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{12b} - \frac{a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/12/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(3/2)-1/8/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3/2)*ln(b^(1/
```

```
2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-3/8/b^(3/2)*a^2*ln(b^(1/2)*x^(1/
2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)-1/4/b*a*(arctan
h(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-3/8/b^(3/2)*a*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-1
/12/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-1/8/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3
/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 140, normalized size = 1.01

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+14\,ab^{2}x+3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{2}},\ \frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^{3}x^{2}+14\,ab^{2}x+3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(3/2),x)
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.41, size = 122, normalized size = 0.88

$$\frac{1}{48}\,\sqrt{2}\left(6\,\sqrt{2}\left(\sqrt{bx+a}\left(2\,x+\frac{a}{b}\right)\sqrt{x}+\frac{a^{2}\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a+\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4\,x+\frac{a}{b}\right)x-\frac{3\,a^{2}}{b^{2}}\right)\sqrt{x}-\frac{3\,a^{3}\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(6*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(-s
qrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a + sqrt(2)*(sqrt(b*x + a)*(2*(4*
x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(5/2))*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\, \operatorname{atanh}(\tanh(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(3/2), x)
```

3.225 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] 3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^2/b^(1/2)+1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-3/4*(b*x-arctanh(t
anh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*Sqrt[b]) - (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*S
qrt[ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{4}(3(bx-\tanh^{-1}(\tanh(a+bx))))\int \frac{\sqrt{t}}{}$$

$$= -\frac{3}{4}\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{2}\sqrt{x}\,\tan$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^2}{4\sqrt{b}} - \frac{3}{4}$$

Mathematica [**A**]
time = 0.05, size = 83, normalized size = 0.82

$$\frac{1}{4}\left(\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(-3bx+5\tanh^{-1}(\tanh(a+bx))) + \frac{3(-bx+\tanh^{-1}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + 5*ArcTanh[Tanh[a + b*x]]) +
 (3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/Sqrt[b])/4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(79) = 158$.
time = 0.12, size = 165, normalized size = 1.63

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2} + \frac{3a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4} + \frac{3\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(t}\right.}{4\sqrt{b}}$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2} + \frac{3a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4} + \frac{3\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(t}\right.}{4\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*a*x^(1/2)*arctanh(tanh(b*x+a))^(
1/2)+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3/2*a/b
^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))
```

```
-b*x-a)+3/4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/sqrt(x), x)
```

Fricas [A]
time = 0.36, size = 119, normalized size = 1.18

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(1/2),x)
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/sqrt(x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(1/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(1/2), x)
```

3.226 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=81

$$-3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))+3b\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*
x+a)))*b^(1/2)-2*arctanh(tanh(b*x+a))^(3/2)/x^(1/2)+3*b*x^(1/2)*arctanh(tan
h(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}+3b\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}-3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```

```
[Out] -3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 3*b*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (2*ArcT
anh[Tanh[a + b*x]]^(3/2))/Sqrt[x]
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
```

```
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx$$
$$= 3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} - \frac{1}{2}(3b(bx -$$
$$= -3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.95

$$\frac{\left(3bx - 2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} + 3\sqrt{b}\left(-bx + \tanh^{-1}(\tanh(a+bx))\right)\log\left(b\sqrt{x} + \sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
[Out] ((3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] +
 3*Sqrt[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[A
rcTanh[Tanh[a + b*x]]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(65) = 130$.
time = 0.12, size = 280, normalized size = 3.46

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{2b\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{\mathrm{arctanh}(\tanh(bx+a))-bx} + \frac{3ba\sqrt{x}\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right.\right.}}{\mathrm{arctanh}(\tanh(bx+a))-bx}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{2b\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{\mathrm{arctanh}(\tanh(bx+a))-bx} + \frac{3ba\sqrt{x}\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right.\right.}}{\mathrm{arctanh}(\tanh(bx+a))-bx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3*b/(arctanh(tanh(b
*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*
x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+6*b^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)+3*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tanh(b*x
+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*x+
a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(3/2), x)
```

Fricas [A]
time = 0.35, size = 109, normalized size = 1.35

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x},\,-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(3/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(3/2), x)
```

3.227 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=70

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}$$

```
[Out] 2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*arctanh(t
anh(b*x+a))^(3/2)/x^(3/2)-2*b*arctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2199, 2196}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] 2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b*Sq
rt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x
^(3/2))
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx$$
$$= -\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} + b^2\int \frac{}{\sqrt{x}\,\sqrt{}}$$
$$= 2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Mathematica [**A**]
time = 0.05, size = 74, normalized size = 1.06

$$-\frac{2\left(3bx\sqrt{\tanh^{-1}(\tanh(a+bx))} + \tanh^{-1}(\tanh(a+bx))^{3/2} - 3b^{3/2}x^{3/2}\log\left(b\sqrt{x} + \sqrt{b}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] + ArcTanh[Tanh[a + b*x]]^(3/2) - 3*
b^(3/2)*x^(3/2)*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(3*
x^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(52) = 104.
time = 0.12, size = 315, normalized size = 4.50

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} - \frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}} + \frac{4b^{2}\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}} + \frac{2b^{2}}{}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} - \frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}} + \frac{4b^{2}\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}} + \frac{2b^{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(5/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^2/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(arctanh(
```

```
tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+4*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+2*b^2/(arctanh(tanh(b*x+a))-b*x)^2*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(ar
ctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(5/2), x)
```

Fricas [A]
time = 0.34, size = 109, normalized size = 1.56

$$\left[\frac{3\, b^{\frac{3}{2}} x^{2} \log\left(2\, bx + 2\, \sqrt{bx + a}\, \sqrt{b}\, \sqrt{x} + a\right) - 2\, (4\, bx + a)\sqrt{bx + a}\, \sqrt{x}}{3\, x^{2}}, -\frac{2\left(3\, \sqrt{-b}\, bx^{2} \arctan\left(\frac{\sqrt{bx + a}\, \sqrt{-b}}{b\sqrt{x}}\right) + (4\, bx + a)\sqrt{bx + a}\, \sqrt{x}\right)}{3\, x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a + bx\right)\right)}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(5/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(5/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(5/2), x)
```

3.228 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}} \, dx$

Optimal. Leaf size=35

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2198}

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}} \, dx = \frac{2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.07, size = 34, normalized size = 0.97

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2} \left(5bx - 5 \tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(x^(5/2)*(5*b*x - 5*ArcTanh[Tanh[a + b*x]]
))
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}$	29
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.43

$$-\frac{2 (bx+a)^{\frac{5}{2}}}{5 a x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="maxima")
[Out] -2/5*(b*x + a)^(5/2)/(a*x^(5/2))
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.89

$$-\frac{2 (b^2x^2 + 2 abx + a^2)\sqrt{bx+a}}{5 a x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="fricas")
[Out] -2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(a*x^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.39, size = 33, normalized size = 0.94

$$-\frac{2\,(bx+a)^{\frac{5}{2}}b^{6}}{5\,((bx+a)b-ab)^{\frac{5}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(b*x + a)^(5/2)*b^6/(((b*x + a)*b - a*b)^(5/2)*a*abs(b))
```

Mupad [B]
time = 1.51, size = 332, normalized size = 9.49

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{5\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+10\,b\,x}+\frac{4\,b^2\,x^2}{5\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+10\,b\,x}-\frac{4\,b\,x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+10\,b\,x}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(7/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(5*log(2/(exp(2*a)*
exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 10*b*x) + (4*b^2*x^2)/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x) - (4*b*x*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x))/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x)))/x^(5/2)
```

3.229 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}} dx$

Optimal. Leaf size=72

$$\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 4/35*b*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.67

$$\frac{2\left(7bx-5\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{7/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (2*(7*b*x - 5*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(
7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{5}{2}}}$	59
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{5}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+4/35*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2-3\,abx-5\,a^2\right)(bx+a)^{\frac{3}{2}}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2/35*(2*b^2*x^2 - 3*a*b*x - 5*a^2)*(b*x + a)^(3/2)/(a^2*x^(7/2))
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.62

$$\frac{2\left(2\,b^3x^3-ab^2x^2-8\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^2*x^(7/2)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 59, normalized size = 0.82

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\,(bx+a)b^7}{a^2}-\frac{7\sqrt{2}\,b^7}{a}\right)(bx+a)^{\frac{5}{2}}b}{35\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^7/a^2 - 7*sqrt(2)*b^7/a)*(b*x + a)^(5/2
)*b/((((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B]
time = 1.54, size = 228, normalized size = 3.17

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{6\,b\,x}{35}+\frac{4\,b^2\,x^2}{35\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{16\,b^3\,x^3}{35\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(9/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 - (6*b*x)/35 + (4*b^2*x^2)/
(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(35*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
)))/x^(7/2)
```

3.230 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 16/315*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/63*b*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(315*x^(5/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(63*x^(7/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$
$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{5/}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(63b^2x^2-90bx\tanh^{-1}(\tanh(a+bx))+35\tanh^{-1}(\tanh(a+bx))^2\right)}{315x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 90*b*x*ArcTanh[Tanh[a + b*x]]
 + 35*ArcTanh[Tanh[a + b*x]]^2))/(315*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

Maple [A]
time = 0.14, size = 105, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{5}{2}}}\right)}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{5}{2}}}\right)}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(5/2)-8/9*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(
b*x+a))^(5/2))
```

Maxima [A]
time = 0.48, size = 45, normalized size = 0.41

$$-\frac{2\,(8\,b^3x^3-12\,ab^2x^2+15\,a^2bx+35\,a^3)(bx+a)^{\frac{3}{2}}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -2/315*(8*b^3*x^3 - 12*a*b^2*x^2 + 15*a^2*b*x + 35*a^3)*(b*x + a)^(3/2)/(a^
3*x^(9/2))
```

Fricas [A]
time = 0.34, size = 56, normalized size = 0.51

$$-\frac{2\,(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4)\sqrt{bx+a}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqrt
(b*x + a)/(a^3*x^(9/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 78, normalized size = 0.71

$$-\frac{\sqrt{2}\left(\frac{63\,\sqrt{2}\,b^9}{a}+4\left(\frac{2\,\sqrt{2}\,(bx+a)b^9}{a^3}-\frac{9\,\sqrt{2}\,b^9}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*sqrt(2)*(63*sqrt(2)*b^9/a + 4*(2*sqrt(2)*(b*x + a)*b^9/a^3 - 9*sqrt(
2)*b^9/a^2)*(b*x + a))*(b*x + a)^(5/2)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B]
time = 1.64, size = 288, normalized size = 2.62

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{2\,b\,x}{21}+\frac{4\,b^2\,x^2}{105\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{32\,b^3\,x^3}{315\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{128\,b^4\,x^4}{315\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(11/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 - (2*b*x)/21 + (4*b^2*x^2)/
(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)) + (32*b^3*x^3)/(315*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2) + (128*b^4*x^4)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(9/2)
```

3.231 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}} \, dx$

Optimal. Leaf size=148

$$\frac{32b^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+b}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(\right.}$$

```
[Out] 32/1155*b^3*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^4
+16/231*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^3
+4/33*b*arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/1
1*arctanh(tanh(b*x+a))^(5/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{32b^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(1155*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(231*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(33*x^(9/2)*(b*
x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(11*x^(11
/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+b\right.}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx)}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a-\right.}$$

Mathematica [A]
time = 0.05, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(231b^3x^3-495b^2x^2\tanh^{-1}(\tanh(a+bx))+385bx\tanh^{-1}(\tanh(a+bx))^2-105\tanh^{-1}(\tanh(a+bx))^3\right)}{1155x^{11/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 495*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 385*b*x*ArcTanh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/
(1155*x^(11/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.17, size = 151, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{}{35(a}\right.}{9(\operatorname{arctanh}(\tanh(bx+}\right.}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{}{35(a}\right.}{9(\operatorname{arctanh}(\tanh(bx+}\right.}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(5/2)-12/11*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arcta
nh(tanh(b*x+a))^(5/2)-4/9*b/(arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(
b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a)
)-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)))
```

Maxima [A]
time = 0.47, size = 56, normalized size = 0.38

$$\frac{2\,(16\,b^4x^4 - 24\,ab^3x^3 + 30\,a^2b^2x^2 - 35\,a^3bx - 105\,a^4)(bx+a)^{\frac{3}{2}}}{1155\,a^4x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(16*b^4*x^4 - 24*a*b^3*x^3 + 30*a^2*b^2*x^2 - 35*a^3*b*x - 105*a^4)*
(b*x + a)^(3/2)/(a^4*x^(11/2))
```

Fricas [A]
time = 0.34, size = 67, normalized size = 0.45

$$\frac{2\,(16\,b^5x^5 - 8\,ab^4x^4 + 6\,a^2b^3x^3 - 5\,a^3b^2x^2 - 140\,a^4bx - 105\,a^5)\sqrt{bx+a}}{1155\,a^4x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4*
b*x - 105*a^5)*sqrt(b*x + a)/(a^4*x^(11/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 97, normalized size = 0.66

$$-\frac{\sqrt{2}\left(\frac{231\,\sqrt{2}\,b^{11}}{a} - 2\left(\frac{99\,\sqrt{2}\,b^{11}}{a^2} + 4\left(\frac{2\,\sqrt{2}\,(bx+a)b^{11}}{a^4} - \frac{11\,\sqrt{2}\,b^{11}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{1155\,((bx+a)b-ab)^{\frac{11}{2}}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*sqrt(2)*(231*sqrt(2)*b^11/a - 2*(99*sqrt(2)*b^11/a^2 + 4*(2*sqrt(2)
*(b*x + a)*b^11/a^4 - 11*sqrt(2)*b^11/a^3)*(b*x + a))*(b*x + a))*(b*x + a)^
(5/2)*b/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

Mupad [B]
time = 1.74, size = 348, normalized size = 2.35

$$\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{11}-\frac{2bx}{33}+\frac{4b^2x^2}{231\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{16b^3x^3}{385\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}+\frac{128b^4x^4}{1155\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}+\frac{512b^5x^5}{1155\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4}\right)}{x^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(13/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 - (2*b*x)/33 + (4*b^2*x^2
)/(231*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(385*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2) + (128*b^4*x^4)/(1155*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^5*x^5)/(11
55*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^4)))/x^(11/2)
```

3.232 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^{5/2} \, dx$

Optimal. Leaf size=174

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}(bx-\tanh^{-1}(\tanh(a+bx))$$

```
[Out] -5/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^4/b^(3/2)-5/24*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*
x+a))^(3/2)+1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)+5/32*x^(3/2)*(b*x-arctan
h(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)-5/64*(b*x-arctanh(tanh(b*x+a))
)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))^2-\frac{5}{24}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}(bx-\tanh^{-1}(\tanh(a+bx)))+\frac{1}{4}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2}-\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))^3}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^4)/(64*b^(3/2)) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])
^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/32 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b) - (5*x^(3/2)*(b*x - ArcTanh[Tan
h[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/24 + (x^(3/2)*ArcTanh[Tanh[a + b
*x]]^(5/2))/4
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^{5/2} \, dx = \frac{1}{4} x^{3/2} \tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{8} \left(5 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\right)$$

$$= -\frac{5}{24} x^{3/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{4} x$$

$$= \frac{5}{32} x^{3/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 \sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{24}$$

$$= \frac{5}{32} x^{3/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 \sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5\sqrt{}}{}$$

$$= -\frac{5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) \left(bx - \tanh^{-1}(\tanh(a+bx)\right.}{64 b^{3/2}}$$

Mathematica [A]
time = 0.06, size = 121, normalized size = 0.70

$$\frac{\sqrt{x} \sqrt{\tanh^{-1}(\tanh(a+bx))} \left(15 b^3 x^3 - 55 b^2 x^2 \tanh^{-1}(\tanh(a+bx)) + 73 b x \tanh^{-1}(\tanh(a+bx))^2 + 15 \tanh^{-1}(\tanh(a+bx))^3\right)}{192 b} - \frac{5 \left(-bx + \tanh^{-1}(\tanh(a+bx))\right)^4 \log\left(b\sqrt{x} + \sqrt{b} \sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{64 b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^3*x^3 - 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]] + 73*b*x*ArcTanh[Tanh[a + b*x]]^2 + 15*ArcTanh[Tanh[a + b*x]]^3)
)/(192*b) - (5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/(64*b^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(140) = 280.
time = 0.12, size = 471, normalized size = 2.71

method	result
derivativedivides	$\frac{\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{4b} - \frac{a\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{24b} - \frac{5a^2\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{96b} - \frac{5a^3\sqrt{x}}{}$
default	$\frac{\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{4b} - \frac{a\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{24b} - \frac{5a^2\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{96b} - \frac{5a^3\sqrt{x}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)*arctanh(tanh(b*x+a))^(7/2)/b-1/24/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(5/2)-5/96/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*a^3*x^(1/2)*
arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x
+a))^(1/2))*a^4-5/16/b^(3/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1
/2))*(arctanh(tanh(b*x+a))-b*x-a)-15/64/b*a^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/32/b^(3/2)*a^2*ln(b^(1/2)*x^(1/2)+arc
tanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-5/48/b*a*(arctanh(t
anh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-15/64/b*a*(arctanh(ta
nh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/16/b^(3/2)*a*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3-1/
24/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)-5/96/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*(
arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/
2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A]
time = 0.35, size = 162, normalized size = 0.93

$$\left[\frac{15\,a^{4}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(48\,b^{4}x^{3}+136\,ab^{3}x^{2}+118\,a^{2}b^{2}x+15\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{384\,b^{2}},\frac{15\,a^{4}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(48\,b^{4}x^{3}+136\,ab^{3}x^{2}+118\,a^{2}b^{2}x+15\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{192\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.40, size = 204, normalized size = 1.17

$$\frac{1}{384}\sqrt{2}\left(48\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a^2+16\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{3a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}\right)ab+\sqrt{2}\left(\left(2\left(4\left(6x+\frac{a}{b}\right)x-\frac{5a^2}{b^2}\right)x+\frac{15a^3}{b^3}\right)\sqrt{bx+a}\sqrt{x}+\frac{15a^4\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}\right)b^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(48*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a^2 + 16*sqrt(2)*(sqrt(b*x + a)
*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + s
qrt(b*x + a)))/b^(5/2))*a*b + sqrt(2)*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x +
 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(7/2))*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\operatorname{atanh}(\tanh(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(5/2), x)
```

3.233 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=136

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))$$

```
[Out] -5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(1/2)-5/12*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(3/
2)*x^(1/2)+1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/8*(b*x-arctanh(tanh(b*x
+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,(bx-\tanh^{-1}(\tanh(a+bx)))^2-\frac{5}{12}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}(bx-\tanh^{-1}(\tanh(a+bx)))+\frac{1}{3}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*Sqrt[b]) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^
2*Sqrt[ArcTanh[Tanh[a + b*x]]])/8 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^(3/2))/12 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)
)/3
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{6}\left(5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int$$

$$= -\frac{5}{12}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{3}\sqrt{x}$$

$$= \frac{5}{8}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{12}\sqrt{x}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.74

$$\frac{1}{24}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^2x^2-40bx\tanh^{-1}(\tanh(a+bx))+33\tanh^{-1}(\tanh(a+bx))^2\right)+\frac{5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 33*ArcTanh[Tanh[a + b*x]]^2))/24 + (5*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*Sqrt[b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(108) = 216.
time = 0.12, size = 286, normalized size = 2.10

method	result
derivativedivides	$\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3}+\frac{5a\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{12}+\frac{5a^2\sqrt{x}\,\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{8}$
default	$\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3}+\frac{5a\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{12}+\frac{5a^2\sqrt{x}\,\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/12*a*x^(1/2)*arctanh(tanh(b*x+a))^
(3/2)+5/8*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(1/2)*x^(
1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15/8*a^2/b^(1/2)*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4*a*(arctanh(tanh(
b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/8*a/b^(1/2)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+5/12*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5/8*(arctanh(
tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(
1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/sqrt(x), x)
```

Fricas [A]
time = 0.37, size = 141, normalized size = 1.04

$$\left[\frac{15\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(1/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(1/2), x)
```

3.234 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=121

$$\frac{15}{4}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^2-\frac{15}{4}b\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+$$

```
[Out] 15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2*b^(1/2)-2*arctanh(tanh(b*x+a))^(5/2)/x^(1/2)+5/2*b*arctanh(tanh(
b*x+a))^(3/2)*x^(1/2)-15/4*b*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tan
h(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{15}{4}b\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))}+\frac{15}{4}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/4 - (15*b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])
*Sqrt[ArcTanh[Tanh[a + b*x]]])/4 + (5*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2
))/2 - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/Sqrt[x]
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} + (5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} - \frac{1}{4}(15b(bx$$

$$= -\frac{15}{4}b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{2}b\sqrt{x}$$

$$= \frac{15}{4}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 0.83

$$-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^2x^2-25bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2\right)}{4\sqrt{x}} + \frac{15}{4}\sqrt{b}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

```
[Out] -1/4*(Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*
x]] + 8*ArcTanh[Tanh[a + b*x]]^2))/Sqrt[x] + (15*Sqrt[b]*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(95) = 190$.
time = 0.12, size = 460, normalized size = 3.80

method	result
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{2b\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{\operatorname{arctanh}(\tanh(bx+a))-bx} + \frac{5ba\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)} + \frac{15b}{}$

default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{2b\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{\mathrm{arctanh}(\tanh(bx+a))-bx}+\frac{5ba\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2(\mathrm{arctanh}(\tanh(bx+a))-bx)}+\frac{15b\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+2*b/(arctanh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/2*b/(arctanh(tanh(b*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*x+a))-b*x)*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+45/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+15/2*b/(arctanh(tanh(b*x+a))-b*x)*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+45/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+5/2*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="maxima")

[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(3/2), x)

Fricas [A]
time = 0.36, size = 137, normalized size = 1.13

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{bx+a}\,\sqrt{x}}{8\,x},-\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="fricas")

[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x]

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(3/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(3/2), x)
```

3.235 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=106

$$-5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))+5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh
(tanh(b*x+a)))-2/3*arctanh(tanh(b*x+a))^(5/2)/x^(3/2)-10/3*b*arctanh(tanh(b
*x+a))^(3/2)/x^(1/2)+5*b^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$-5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))+5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}}-\frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] -5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 5*b^2*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (10*b
*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(5/2
))/(3*x^(3/2))
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}} + \frac{1}{3}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx$$

$$= -\frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}} + \left(5b^2\right)\int \frac{\sqrt{}}{}$$

$$= 5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}} - \frac{2\tanh}{}$$

$$= -5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.92

$$\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^2x^2-10bx\tanh^{-1}(\tanh(a+bx))-2\tanh^{-1}(\tanh(a+bx))^2\right)}{3x^{3/2}}+5b^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]] -
 2*ArcTanh[Tanh[a + b*x]]^2))/(3*x^(3/2)) + 5*b^(3/2)*(-(b*x) + ArcTanh[Tan
h[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(82) = 164$.
time = 0.12, size = 501, normalized size = 4.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+8/3*b^2/(arct
```

```
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+10/3*b^2/(arctan
h(tanh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5*b^(3/2)/(arctan
h(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15
*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+10*b^2/(arctanh(tanh(b*x+a))-b*x
)^2*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+10/3*b^2/(arctanh(tanh(b*x+a))-b*x)
^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(a
rctanh(tanh(b*x+a))-b*x)^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(t
anh(b*x+a))^(1/2)+5*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)
+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(5/2), x)
```

Fricas [A]
time = 0.33, size = 138, normalized size = 1.30

$$\left[\frac{15\,ab^{\frac{3}{2}}x^{2}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,(3\,b^{2}x^{2}-14\,abx-2\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{6\,x^{2}},-\frac{15\,a\sqrt{-b}\,bx^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(3\,b^{2}x^{2}-14\,abx-2\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(5/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(5/2), x)
```

3.236 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=93

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

```
[Out] 2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*b*arctanh
(tanh(b*x+a))^(3/2)/x^(3/2)-2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)-2*b^2*ar
ctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2199, 2196}

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] 2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b^2*
Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*b*ArcTanh[Tanh[a + b*x]]^(3/2))/
(3*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2))
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx$$

$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}} + b^2\int \frac{\sqrt{\tan}}{}$$

$$= -\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} - \frac{2\tanh^{-1}}{}$$

$$= 2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Mathematica [A]
time = 0.04, size = 95, normalized size = 1.02

$$-\frac{2\left(15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}+5bx\tanh^{-1}(\tanh(a+bx))^{3/2}+3\tanh^{-1}(\tanh(a+bx))^{5/2}-15b^{5/2}x^{5/2}\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] (-2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 3*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*b^(5/2)*x^(5/2)*Log[b*Sqrt[x]
+ Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(15*x^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(69) = 138$.
time = 0.12, size = 532, normalized size = 5.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(7/2)-4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-16/15*b^2/(a
rctanh(tanh(b*x+a))-b*x)^3/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+16/15*b^3/(ar
ctanh(tanh(b*x+a))-b*x)^3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^3/(arcta
nh(tanh(b*x+a))-b*x)^3*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arctanh(
tanh(b*x+a))-b*x)^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(5/2)/(arcta
nh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+6
*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+4*b^3/(arctanh(tanh(b*x+a))-b*x)
^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+6*b^(5
```

```
/2)/(arctanh(tanh(b*x+a))-b*x)^3*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^
(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+4/3*b^3/(arctanh(tanh(b*x+a))-b*x)^3*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arct
anh(tanh(b*x+a))-b*x)^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh
(b*x+a))^(1/2)+2*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(7/2), x)
```

Fricas [A]
time = 0.35, size = 137, normalized size = 1.47

$$\left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)-2\,(23\,b^2x^2+11\,abx+3\,a^2)\sqrt{bx+a}\,\sqrt{x}}{15\,x^3},-\frac{2\left(15\,\sqrt{-b}\,b^2x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(23\,b^2x^2+11\,abx+3\,a^2)\sqrt{bx+a}\,\sqrt{x}\right)}{15\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^(5/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*
(23*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3, -2/15*(15*sqrt(
-b)*b^2*x^3*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (23*b^2*x^2 + 11*a
*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(7/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(7/2), x)
```

3.237 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2198\}$

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{x^{7/2}\left(7bx-7\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(x^(7/2)*(7*b*x - 7*ArcTanh[Tanh[a + b*x]]
))

Maple [A]
time = 0.13, size = 29, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}$	29
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="maxima")
[Out] -2/7*(b*x + a)^(7/2)/(a*x^(7/2))
```

Fricas [A]
time = 0.33, size = 42, normalized size = 1.20

$$-\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="fricas")
[Out] -2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/(a*x^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(9/2),x)
```

[Out] Timed out

Giac [A]
time = 0.41, size = 33, normalized size = 0.94

$$-\frac{2\,(bx+a)^{\frac{7}{2}}b^{8}}{7\,((bx+a)b-ab)^{\frac{7}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2/7*(b*x + a)^(7/2)*b^8/(((b*x + a)*b - a*b)^(7/2)*a*abs(b))
```

Mupad [B]
time = 1.65, size = 396, normalized size = 11.31

$$-\frac{\sqrt{2}\,\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\,\left(\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}+\frac{3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}-\frac{3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}\right)}{2\,x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(9/2),x)
```

```
[Out] -(2^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(14*lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 28*b*x) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))^3/(14*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 28*b*x) + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(14*log(1/(exp(2*a)*
exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 28*b*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)))/(14*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log(
(exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 28*b*x)))/(2*x^(7/2))
```

3.238 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=72

$$\frac{4b \tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 4/63*b*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{4b \tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2} \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.67

$$\frac{2\left(9bx-7\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{9/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (2*(9*b*x - 7*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(
9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.14, size = 59, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{7}{2}}}$	59
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{7}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+4/63*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

Maxima [A]
time = 0.47, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2-5\,abx-7\,a^2\right)(bx+a)^{\frac{5}{2}}}{63\,a^2x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="maxima")
```

[Out] 2/63*(2*b^2*x^2 - 5*a*b*x - 7*a^2)*(b*x + a)^(5/2)/(a^2*x^(9/2))

Fricas [A]
time = 0.35, size = 56, normalized size = 0.78

$$\frac{2\left(2\,b^4x^4-ab^3x^3-15\,a^2b^2x^2-19\,a^3bx-7\,a^4\right)\sqrt{bx+a}}{63\,a^2x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*x
 + a)/(a^2*x^(9/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 59, normalized size = 0.82

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}(bx+a)b^9}{a^2}-\frac{9\sqrt{2}\,b^9}{a}\right)(bx+a)^{\frac{7}{2}}b}{63\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^9/a^2 - 9*sqrt(2)*b^9/a)*(b*x + a)^(7/2
)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B]
time = 1.62, size = 293, normalized size = 4.07

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{19\,b\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{63}-\frac{10\,b^2\,x^2}{21}-\frac{{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{18}+\frac{4\,b^3\,x^3}{63\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{16\,b^4\,x^4}{63\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(11/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((19*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/63 - (10*b^2
*x^2)/21 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/18 + (4*b^3*x^3)/(63*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)) + (16*b^4*x^4)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(9/2)
```

3.239 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx$$

```
[Out] 16/693*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/99*b*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/11
*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(693*x^(7/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(99*x^(9/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - Ar
cTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$
$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(99b^2x^2-154bx\tanh^{-1}(\tanh(a+bx))+63\tanh^{-1}(\tanh(a+bx))^2\right)}{693x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 154*b*x*ArcTanh[Tanh[a + b*x]
] + 63*ArcTanh[Tanh[a + b*x]]^2))/(693*x^(11/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

Maple [A]
time = 0.18, size = 105, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{7}{2}}}\right)}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{7}{2}}}\right)}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(7/2)-8/11*b
/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctan
h(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(ta
nh(b*x+a))^(7/2))
```

Maxima [A]
time = 0.48, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^{3}x^{3}-20\,ab^{2}x^{2}+35\,a^{2}bx+63\,a^{3}\right)(bx+a)^{\frac{5}{2}}}{693\,a^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] -2/693*(8*b^3*x^3 - 20*a*b^2*x^2 + 35*a^2*b*x + 63*a^3)*(b*x + a)^(5/2)/(a^
3*x^(11/2))
```

Fricas [A]
time = 0.33, size = 67, normalized size = 0.61

$$-\frac{2\left(8\,b^{5}x^{5}-4\,ab^{4}x^{4}+3\,a^{2}b^{3}x^{3}+113\,a^{3}b^{2}x^{2}+161\,a^{4}bx+63\,a^{5}\right)\sqrt{bx+a}}{693\,a^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^4
*b*x + 63*a^5)*sqrt(b*x + a)/(a^3*x^(11/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 78, normalized size = 0.71

$$-\frac{\sqrt{2}\left(\frac{99\,\sqrt{2}\,b^{11}}{a}+4\left(\frac{2\,\sqrt{2}\,(bx+a)b^{11}}{a^{3}}-\frac{11\,\sqrt{2}\,b^{11}}{a^{2}}\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{693\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/693*sqrt(2)*(99*sqrt(2)*b^11/a + 4*(2*sqrt(2)*(b*x + a)*b^11/a^3 - 11*sq
rt(2)*b^11/a^2)*(b*x + a))*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(11/2)*ab
s(b))
```

Mupad [B]
time = 1.66, size = 353, normalized size = 3.21

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{23\,b\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{99}-\frac{226\,b^2\,x^2}{693}-\frac{{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{22}+\frac{4\,b^3\,x^3}{231\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{32\,b^4\,x^4}{693\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{128\,b^5\,x^5}{693\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}\right)}{x^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(13/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((23*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/99 - (226*b^
2*x^2)/693 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/22 + (4*b^3*x^3)/(231*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)) + (32*b^4*x^4)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^5*x^5
)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(11/2)
```

3.240 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}} dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{12b\tanh^{-1}(\tanh(a\ldots}{143x^{11/2}\left(bx-\tanh^{-1}(\mathrm{ta}\ldots\right.}$$

```
[Out] 32/3003*b^3*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^4
+16/429*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^3
+12/143*b*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))^2+
2/13*arctanh(tanh(b*x+a))^(7/2)/x^(13/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(3003*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(429*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (12*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(143*x^(11/2)*
(b*x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(13*x^
(13/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx}{13\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A]
time = 0.05, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(429b^3x^3-1001b^2x^2\tanh^{-1}(\tanh(a+bx))+819bx\tanh^{-1}(\tanh(a+bx))^2-231\tanh^{-1}(\tanh(a+bx))^3\right)}{3003x^{13/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 1001*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 819*b*x*ArcTanh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))
/(3003*x^(13/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.27, size = 151, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{13(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{13}{2}}}-\frac{12b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{}{63(a}\right.}{11(\mathrm{arctanh}(\tanh(bx+}\right.}{13(\mathrm{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{13(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{13}{2}}}-\frac{12b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{}{63(a}\right.}{11(\mathrm{arctanh}(\tanh(bx+}\right.}{13(\mathrm{arctanh}(\tanh(bx+a))-bx)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/13/(arctanh(tanh(b*x+a))-b*x)/x^(13/2)*arctanh(tanh(b*x+a))^(7/2)-12/13*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arc
tanh(tanh(b*x+a))^(7/2)-4/11*b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(ta
nh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x
+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)))
```

Maxima [**A**]
time = 0.47, size = 56, normalized size = 0.38

$$\frac{2\,(16\,b^4x^4-40\,ab^3x^3+70\,a^2b^2x^2-105\,a^3bx-231\,a^4)(bx+a)^{\frac{5}{2}}}{3003\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(16*b^4*x^4 - 40*a*b^3*x^3 + 70*a^2*b^2*x^2 - 105*a^3*b*x - 231*a^4)
*(b*x + a)^(5/2)/(a^4*x^(13/2))
```

Fricas [**A**]
time = 0.33, size = 78, normalized size = 0.53

$$\frac{2\,(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6)\sqrt{bx+a}}{3003\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4*
b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x + a)/(a^4*x^(13/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 97, normalized size = 0.66

$$-\frac{\sqrt{2}\left(\frac{429\sqrt{2}\,b^{13}}{a}-2\left(\frac{143\sqrt{2}\,b^{13}}{a^2}+4\left(\frac{2\sqrt{2}\,(bx+a)b^{13}}{a^4}-\frac{13\sqrt{2}\,b^{13}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{3003\,((bx+a)b-ab)^{\frac{13}{2}}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] -1/3003*sqrt(2)*(429*sqrt(2)*b^13/a - 2*(143*sqrt(2)*b^13/a^2 + 4*(2*sqrt(2
)*(b*x + a)*b^13/a^4 - 13*sqrt(2)*b^13/a^3)*(b*x + a))*(b*x + a))*(b*x + a)
^(7/2)*b/(((b*x + a)*b - a*b)^(13/2)*abs(b))
```

Mupad [B]
time = 1.74, size = 413, normalized size = 2.79

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{27\,b\,x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{143}-\frac{106\,b^2\,x^2}{429}-\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{26}+\frac{20\,b^3\,x^3}{3003\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{16\,b^4\,x^4}{1001\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{128\,b^5\,x^5}{3003\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}+\frac{512\,b^6\,x^6}{3003\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}\right)}{x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(15/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((27*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/143 - (106*b
^2*x^2)/429 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/26 + (20*b^3*x^3)/(3003*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)) + (16*b^4*x^4)/(1001*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^5
*x^5)/(3003*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^6*x^6)/(3003*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^4)))/x^(13/2)
```

3.241 $\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=145

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^3}{8b^{7/2}}+\frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b}+\frac{5x^{3/2}}{}$$

```
[Out] 5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^3/b^(7/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b+5/12*x^(3/2)*(b*
x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b^2+5/8*(b*x-arctanh(tan
h(b*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^3}{8b^{7/2}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}(bx-\tanh^{-1}(\tanh(a+bx)))^2}{8b^3}+\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}(bx-\tanh^{-1}(\tanh(a+bx)))}{12b^2}+\frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^3)/(8*b^(7/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b
) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(12*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/(8*b^3)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} - \frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\sqrt{\text{ta}}}}{6b}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\text{tan}}}{12b^2}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\text{tan}}}{12b^2}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{7/2}} + \frac{x}{}$$

Mathematica [A]
time = 0.06, size = 105, normalized size = 0.72

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(33b^2x^2-40bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2\right)}{24b^3} + \frac{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(33*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2))/(24*b^3) + (5*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(7/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(117) = 234$.
time = 0.13, size = 304, normalized size = 2.10

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3b} - \frac{5a\,x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{12b^2} + \frac{5a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\text{ta}}}{8b^3}$
default	$\frac{x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3b} - \frac{5a\,x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{12b^2} + \frac{5a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\text{ta}}}{8b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b-5/12/b^2*a*x^(3/2)*arctanh(tanh(b*
x+a))^(1/2)+5/8/b^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/8/b^(7/2)*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-15/8/b^(7/2)*a^2*ln(b^(1/2)*
x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4/b^3*a*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/8/b^(7/2
)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2-5/12/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)*arctanh(tanh(b*x+a))^(
1/2)+5/8/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1
/2)-5/8/b^(7/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A]
time = 0.37, size = 140, normalized size = 0.97

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^4},\ \frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.38, size = 64, normalized size = 0.44

$$\frac{1}{24}\sqrt{bx+a}\left(2x\left(\frac{4x}{b}-\frac{5a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}+\frac{5a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^(1/2),x)
[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.242 $\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b}+\frac{3\sqrt{\cdot}}{}$$

```
[Out] 3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^2/b^(5/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b+3/4*(b*x-arctanh
(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A]

time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^2}+\frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*b^(5/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b
) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(4*b^2)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\sqrt{\text{ta}}}}{4b}$$

$$= \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b} + \frac{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tan}}{4b^2}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}} + \frac{x}{}$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.82

$$\frac{\sqrt{b}\sqrt{x}\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b
*x]]] + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[
ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(85) = 170.
time = 0.12, size = 174, normalized size = 1.63

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{3a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b^2} + \frac{3\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\text{arc}}\right.}{}$
default	$\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{3a\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b^2} + \frac{3\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\text{arc}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b-3/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3
```

```
/2/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)-3/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arct
anh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A]
time = 0.37, size = 119, normalized size = 1.11

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^3},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^2x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
[Out] Integral(x**(3/2)/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 52, normalized size = 0.49

$$\frac{1}{4}\,\sqrt{bx+a}\,\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.243 $\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+a
)))/b^(3/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2200, 2196}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))\int \frac{}{\sqrt{x}\sqrt{}}}{2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))}{b^{3/2}} + \frac{\sqrt{x}}{}$$

Mathematica [**A**]
time = 0.04, size = 66, normalized size = 1.05

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{(-bx+\tanh^{-1}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]
])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

Maple [**A**]
time = 0.12, size = 80, normalized size = 1.27

method	result
derivativedivides	$\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b} - \frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)a}{b^{\frac{3}{2}}} - \frac{\ln\left(\sqrt{b}\right.}{}$
default	$\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b} - \frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)a}{b^{\frac{3}{2}}} - \frac{\ln\left(\sqrt{b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(t
anh(b*x+a))^(1/2))*a-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2
))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A]
time = 0.34, size = 91, normalized size = 1.44

$$\left[\frac{a\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^2},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A]
time = 0.39, size = 38, normalized size = 0.60

$$\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(1/2),x)
[Out] int(x^(1/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.244 $\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2196}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.10

$$\frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
[Out] (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Maple [A]
time = 0.12, size = 24, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{\sqrt{b}}$	24
default	$\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{\sqrt{b}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x)*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.90

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.41, size = 23, normalized size = 0.77

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)), x)
```

3.245 $\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2198\}$

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\,\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]
))
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{x}}$	29
default	$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{x}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A]
time = 0.47, size = 15, normalized size = 0.45

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.45

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.39, size = 30, normalized size = 0.91

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

Mupad [B]
time = 1.62, size = 101, normalized size = 3.06

$$\frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{\sqrt{x}\,\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(x^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.246 $\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=72

$$\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+4/3*b*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]
)^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+b}}{3\left(bx-\tanh^{-1}(\tanh(a+bx\right.}}$$

$$= \frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.64

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-3bx+\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3
/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}$	59
default	$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A]
time = 0.46, size = 33, normalized size = 0.46

$$\frac{2\left(2\,b^2x^2+abx-a^2\right)}{3\sqrt{bx+a}\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2*x^2 + a*b*x - a^2)/(sqrt(b*x + a)*a^2*x^(3/2))
```

Fricas [A]
time = 0.34, size = 23, normalized size = 0.32

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A]
time = 0.39, size = 55, normalized size = 0.76

$$\frac{8\left(3\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

Mupad [B]
time = 1.57, size = 218, normalized size = 3.03

$$\frac{\sqrt{2}\,\left(\frac{\frac{4\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}-\frac{4\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}+\frac{8\,b\,x}{3}}{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{16\,b\,x}{3\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)\,\sqrt{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}}{2\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2^(1/2)*(((4*log(2/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 + (8*b*x)/3)/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2 + (16*b*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2))*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^(1/2))/(2*x^
(3/2))
```

3.247 $\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 8/15*b*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5*
arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))+16/15*b^2*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^3/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^3) + (8*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*x^(3/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+b}}{5\left(bx-\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a\right.}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.60

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(15b^2x^2-10bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]]
 + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
3)
```

Maple [A]
time = 0.13, size = 105, normalized size = 0.95

method	result
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}} - \frac{8b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} + \frac{2b\sqrt{\operatorname{arctanh}(\tanh(bx}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}\right.}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}} - \frac{8b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} + \frac{2b\sqrt{\operatorname{arctanh}(\tanh(bx}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}\right.}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-8/5*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(t
```

```
anh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [A]
time = 0.48, size = 45, normalized size = 0.41

$$-\frac{2\,(8\,b^3x^3+4\,ab^2x^2-a^2bx+3\,a^3)}{15\,\sqrt{bx+a}\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/15*(8*b^3*x^3 + 4*a*b^2*x^2 - a^2*b*x + 3*a^3)/(sqrt(b*x + a)*a^3*x^(5/2
))
```

Fricas [A]
time = 0.33, size = 34, normalized size = 0.31

$$-\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [A]
time = 0.38, size = 77, normalized size = 0.70

$$\frac{32\left(10\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

Mupad [B]
time = 1.53, size = 227, normalized size = 2.06

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{5\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,b^2\,x^2}{15\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}+\frac{32\,b\,x}{15\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x^2)
/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (32*b*x)/(15*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
))/x^(5/2)
```

3.248 $\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 16/35*b^2*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+1
2/35*b*arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(1/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))+32/35*b^3*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (32*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^4) + (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(3/2)*(b*x - ArcTanh
[Tanh[a + b*x]])^3) + (12*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(7*x^(7/2)*
(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{7\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a-\right.}$$

$$= \frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+b}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a-\right.}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.55

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(35b^3x^3-35b^2x^2\tanh^{-1}(\tanh(a+bx))+21bx\tanh^{-1}(\tanh(a+bx))^2-5\tanh^{-1}(\tanh(a+bx))^3\right)}{35x^{7/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 35*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 21*b*x*ArcTanh[Tanh[a + b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(35*x
^(7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.13, size = 151, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}} - \frac{12b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}} - \frac{4b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh}}{3(\operatorname{arctanh}(\tanh(bx+}\right.}{}\right.}{7(\operatorname{arctanh}(\tanh(bx+a)}$

default	$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{12b\left(-\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sqrt{\operatorname{arctan}}}{3(\operatorname{arctanh}(\tanh(bx}\right.}{}\right.}{7(\operatorname{arctanh}(\tanh(bx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(1/2)-12/7*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(
tanh(b*x+a))^(1/2)-4/5*b/(arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x
+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A]
time = 0.47, size = 55, normalized size = 0.37

$$\frac{2\left(16\,b^4x^4+8\,ab^3x^3-2\,a^2b^2x^2+a^3bx-5\,a^4\right)}{35\,\sqrt{bx+a}\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(16*b^4*x^4 + 8*a*b^3*x^3 - 2*a^2*b^2*x^2 + a^3*b*x - 5*a^4)/(sqrt(b*x
 + a)*a^4*x^(7/2))
```

Fricas [A]
time = 0.34, size = 45, normalized size = 0.30

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.38, size = 103, normalized size = 0.70

$$\frac{64\left(35\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{6}-21\,a\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4}+7\,a^{2}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^{3}\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

Mupad [B]
time = 1.64, size = 287, normalized size = 1.94

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{7\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,b^{2}\,x^{2}}{35\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{3}}+\frac{512\,b^{3}\,x^{3}}{35\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{4}}+\frac{48\,b\,x}{35\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x^2)
/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^3*x^3)/(35*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^4) + (48*b*x)/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(7/2)
```

3.249 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}}-\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{7x^{5/2}}{}$$

```
[Out] 35/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(9/2)-2*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)+7/3*x^(5/2)*arcta
nh(tanh(b*x+a))^(1/2)/b^2+35/12*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arctanh(
tanh(b*x+a))^(1/2)/b^3+35/8*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)*arctanh(ta
nh(b*x+a))^(1/2)/b^4
```

Rubi [A]

time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8b^4}+\frac{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{12b^3}+\frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}-\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*b^(9/2)) - (2*x^(7/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (7*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2) + (35*x^(3/2)*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b^3) + (35*Sqrt[x
]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(8*b^4)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} - \frac{(35(-bx$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}(bx$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}(bx$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}} - $$

Mathematica [A]
time = 0.16, size = 122, normalized size = 0.73

$$\frac{\sqrt{x}\left(-48b^3x^3+231b^2x^2\tanh^{-1}(\tanh(a+bx))-280bx\tanh^{-1}(\tanh(a+bx))^2+105\tanh^{-1}(\tanh(a+bx))^3\right)}{24b^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-48*b^3*x^3 + 231*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 280*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(24*b^4*Sqrt[ArcTanh[Ta
nh[a + b*x]]]) + (35*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(134) = 268$.

time = 0.13, size = 428, normalized size = 2.58

method	result
derivativedivides	$\frac{x^{\frac{7}{2}}}{3b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{7a\,x^{\frac{5}{2}}}{12b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{35a^{2}x^{\frac{3}{2}}}{24b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(\right.\right.}}$
default	$\frac{x^{\frac{7}{2}}}{3b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{7a\,x^{\frac{5}{2}}}{12b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{35a^{2}x^{\frac{3}{2}}}{24b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)-7/12/b^2*a*x^(5/2)/arctanh(tanh(b*
x+a))^(1/2)+35/24/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4*a^3*x
^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(9/2)*a^3*ln(b^(1/2)*x^(1/2)+arcta
nh(tanh(b*x+a))^(1/2))+105/8/b^4*a^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a^2*(arctanh(tanh(b*x+a))-b*x-a)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+35/12/b^3*a*(arctanh(tanh(b*x+
a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+105/8/b^4*a*(arctanh(tanh(b*x
+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a*(arctanh(t
anh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/12/b^
2*(arctanh(tanh(b*x+a))-b*x-a)*x^(5/2)/arctanh(tanh(b*x+a))^(1/2)+35/24/b^3
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4
*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(
9/2)*(arctanh(tanh(b*x+a))-b*x-a)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))
^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.33, size = 196, normalized size = 1.18

$$\left[\frac{105\,(a^3bx+a^4)\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^4x^3-14\,ab^3x^2+35\,a^2b^2x+105\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{48\,(b^6x+ab^5)},\ \frac{105\,(a^3bx+a^4)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^4x^3-14\,ab^3x^2+35\,a^2b^2x+105\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{24\,(b^6x+ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a^3*b*x + a^4)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt
(x) + a) + 2*(8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x + 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^6*x + a*b^5), 1/24*(105*(a^3*b*x + a^4)*sqrt(-b)*arctan(s
qrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2
*x + 105*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a*b^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 75, normalized size = 0.45

$$\frac{\left(\left(2\,x\left(\frac{4\,x}{b}-\frac{7\,a}{b^2}\right)+\frac{35\,a^2}{b^3}\right)x+\frac{105\,a^3}{b^4}\right)\sqrt{x}}{24\,\sqrt{bx+a}}+\frac{35\,a^3\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*x*(4*x/b - 7*a/b^2) + 35*a^2/b^3)*x + 105*a^3/b^4)*sqrt(x)/sqrt(b*
x + a) + 35/8*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(7/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.250 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^2}{4b^{7/2}}-\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5x^{3/2}}{}$$

```
[Out] 15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(7/2)-2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)+5/2*x^(3/2)*arcta
nh(tanh(b*x+a))^(1/2)/b^2+15/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(t
anh(b*x+a))^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))^2}{4b^{7/2}}+\frac{15\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^3}+\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(7/2)) - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (5*x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b^2) + (15*Sqrt[x]*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^3)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} - \frac{(15(-bx$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} + \frac{15\sqrt{x}\,(b$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{7/2}} -$$

Mathematica [A]
time = 0.07, size = 104, normalized size = 0.81

$$-\frac{\sqrt{x}\left(8b^2x^2-25bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2\right)}{4b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{15\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] -1/4*(Sqrt[x]*(8*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[
a + b*x]]^2))/(b^3*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (15*(-(b*x) + ArcTanh[Ta
```

```
nh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b
^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(102) = 204$.
time = 0.13, size = 261, normalized size = 2.04

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{5a\,x^{\frac{3}{2}}}{4b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{15a^{2}\sqrt{x}}{4b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(b\right.\right.}}$
default	$\frac{x^{\frac{5}{2}}}{2b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{5a\,x^{\frac{3}{2}}}{4b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{15a^{2}\sqrt{x}}{4b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(b\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)-5/4/b^2*a*x^(3/2)/arctanh(tanh(b*x
+a))^(1/2)-15/4/b^3*a^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*a^2
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-15/2/b^3*a*(arctanh(tanh(b*
x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/2/b^(7/2)*a*(arctanh(tan
h(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-5/4/b^2*(ar
ctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-15/4/b^3*(arct
anh(tanh(b*x+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*(
arctanh(tanh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 175, normalized size = 1.37

$$\left[\frac{15\,(a^{2}bx+a^{3})\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{3}x^{2}-5\,ab^{2}x-15\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{8\,(b^{5}x+ab^{4})},\ -\frac{15\,(a^{2}bx+a^{3})\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{3}x^{2}-5\,ab^{2}x-15\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{4\,(b^{5}x+ab^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.40, size = 63, normalized size = 0.49

$$\frac{\left(x\left(\frac{2x}{b}-\frac{5a}{b^2}\right)-\frac{15a^2}{b^3}\right)\sqrt{x}}{4\sqrt{bx+a}}-\frac{15a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x*(2*x/b - 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/sqrt(b*x + a) - 15/4*a^2*log
(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.251 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{3\sqrt{x}\sqrt{}}{}$$

```
[Out] 3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x
+a)))/b^(5/2)-2*x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3*x^(1/2)*arctanh(tanh
(b*x+a))^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(5/2) - (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3
*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{(3(-bx+$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}} - \frac{}{b\sqrt{}}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.94

$$\frac{\sqrt{x}\left(-2bx+3\tanh^{-1}(\tanh(a+bx))\right)}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(b^2*Sqrt[ArcTanh[Tanh[a + b*
x]]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcT
anh[Tanh[a + b*x]]]])/b^(5/2)
```

Maple [A]
time = 0.12, size = 130, normalized size = 1.51

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{3a\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{3a\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctan}}\right.}{b^{\frac{5}{2}}}$
default	$\frac{x^{\frac{3}{2}}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{3a\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{3a\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctan}}\right.}{b^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3/b^2*a*x^(1/2)/arctanh(tanh(b*x+a))^(
1/2)-3/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+3/b^2*(arct
anh(tanh(b*x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-3/b^(5/2)*(arcta
nh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 145, normalized size = 1.69

$$\left[\frac{3\,(abx+a^{2})\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(b^{2}x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{2\,(b^{4}x+ab^{3})},\ \frac{3\,(abx+a^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(b^{2}x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{b^{4}x+ab^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.39, size = 48, normalized size = 0.56

$$\frac{\sqrt{x}\,\left(\frac{x}{b}+\frac{3\,a}{b^2}\right)}{\sqrt{bx+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*(x/b + 3*a/b^2)/sqrt(b*x + a) + 3*a*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.252 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=52

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(3/2)-2*x^(1/2)/b/a
rctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2199, 2196}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*Sq
rt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 1.06

$$-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{\frac{3}{2}}}$	42
default	$-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{\frac{3}{2}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+2/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 119, normalized size = 2.29

$$\left[\frac{(bx+a)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2},-\frac{2\left((bx+a)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.40, size = 39, normalized size = 0.75

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{2\,\sqrt{x}}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x
 + a)*b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(3/2),x)
[Out] int(x^(1/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.253 $\int \frac{1}{\sqrt{x}\ \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2198}

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\ \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{2\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])
)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}$	29
default	$\frac{2\sqrt{x}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 22, normalized size = 0.67

$$\frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.39, size = 15, normalized size = 0.45

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

Mupad [B]
time = 1.74, size = 163, normalized size = 4.94

$$\frac{4\,x\,\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\left(\frac{\sqrt{x}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2\,b}-\frac{\sqrt{x}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2\,b}\right)\,\left(b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-b\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (4*x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(((x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))
/(2*b) - (x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(2*
b))*(b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b^2*x))
```

3.254 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{4b\sqrt{x}}{(bx-\tanh^{-1}(\tanh(a+bx)))^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tan}}$$

```
[Out] 2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{2}{\sqrt{x}\,(bx-\tanh^{-1}(\tanh(a+bx)))\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{(bx-\tanh^{-1}(\tanh(a+bx)))^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]
]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{(2b)\int}{-bx}$$

$$= -\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{\sqrt{x}\,(bx}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.63

$$-\frac{2\left(bx+\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{x}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(b*x + ArcTanh[Tanh[a + b*x]]))/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(
-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.12, size = 59, normalized size = 0.87

method	result
derivativedivides	$-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}} - \frac{4b\sqrt{x}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\mathrm{arctan}}}$
default	$-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}} - \frac{4b\sqrt{x}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\mathrm{arctan}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b/(arcta
nh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A]
time = 0.47, size = 32, normalized size = 0.47

$$-\frac{2\left(2\,b^2x^2+3\,abx+a^2\right)}{(bx+a)^{\frac{3}{2}}a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(2*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^(3/2)*a^2*sqrt(x))
```

Fricas [A]
time = 0.32, size = 34, normalized size = 0.50

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A]
time = 0.39, size = 50, normalized size = 0.74

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^2}+\frac{4\,\sqrt{b}}{\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) + 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x
+ a))^2 - a)*a)
```

Mupad [B]
time = 1.49, size = 281, normalized size = 4.13

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,x}{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}-\frac{8\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-8\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+16\,b\,x}{2\,b\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{x^{3/2}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] -((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (8*log(2/
(exp(2*a)*exp(2*b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 16*b*x)/(2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^(3/2) - (x^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

3.255 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{8b}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+8/3*b/(b*
x-arctanh(tanh(b*x+a)))^2/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-16/3*b^2*x^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{8b}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (8*b)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[T
anh[a + b*x]]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[
Tanh[a + b*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{(4b)}{3\,(bx}$$

$$= \frac{8b}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{3x^3}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{3\sqrt{x}}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.58

$$\frac{2\left(-3b^2x^2-6bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2\right)}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*(-3*b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/
(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A]
time = 0.12, size = 105, normalized size = 0.95

method	result
derivativedivides	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{8b\left(-\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\sqrt{\operatorname{ar}}}\right.}{}$
default	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{8b\left(-\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\sqrt{\operatorname{ar}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tan
h(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a
))^(1/2))
```

Maxima [A]
time = 0.49, size = 45, normalized size = 0.41

$$\frac{2\left(8\,b^3x^3+12\,ab^2x^2+3\,a^2bx-a^3\right)}{3\,(bx+a)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(8*b^3*x^3 + 12*a*b^2*x^2 + 3*a^2*b*x - a^3)/((b*x + a)^(3/2)*a^3*x^(3/
2))
```

Fricas [A]
time = 0.33, size = 49, normalized size = 0.45

$$\frac{2\left(8\,b^2x^2+4\,abx-a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [A]
time = 0.40, size = 107, normalized size = 0.97

$$\frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}\,a^3}-\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-12\,ab^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) - 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^4 - 12*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 5*a^2*b^(3
/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^2)
```

Mupad [B]
time = 1.69, size = 286, normalized size = 2.60

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{32\,x}{3\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{4}{3\,b\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{128\,b\,x^2}{3\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}\right)}{x^{5/2}-\frac{x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((32*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(3*b
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)) - (128*b*x^2)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/(
x^(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

3.256 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(}}$$

```
[Out] 4/5*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)+2/5/x
^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+16/5*b^2/(b*x-
arctanh(tanh(b*x+a)))^3/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-32/5*b^3*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{4b}{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-32*b^3*Sqrt[x])/(5*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (16*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTan
h[Tanh[a + b*x]]]) + (4*b)/(5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]]) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqr
t[ArcTanh[Tanh[a + b*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{(6b)}{5\,(bx}$$

$$= \frac{4b}{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^{5/}}$$

$$= \frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^3}$$

$$= -\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5\sqrt{x}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.54

$$-\frac{2\left(5b^3x^3+15b^2x^2\tanh^{-1}(\tanh(a+bx))-5bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3\right)}{5x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(5*b^3*x^3 + 15*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 5*b*x*ArcTanh[Tanh[a +
 b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.12, size = 151, normalized size = 1.02

method	result
derivativedivides	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{12b\left(-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\sqrt{\operatorname{arc}}}\right.}{}$

default	$-\frac{2}{5(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}\sqrt{\text{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{12b\left(-\frac{1}{3(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\sqrt{\text{arct}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(1/2)-12/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(1/2)-4/3*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a
))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A]
time = 0.47, size = 54, normalized size = 0.36

$$-\frac{2\,\left(16\,b^{4}x^{4}+24\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-a^{3}bx+a^{4}\right)}{5\,\left(bx+a\right)^{\frac{3}{2}}a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(16*b^4*x^4 + 24*a*b^3*x^3 + 6*a^2*b^2*x^2 - a^3*b*x + a^4)/((b*x + a)
^(3/2)*a^4*x^(5/2))
```

Fricas [A]
time = 0.34, size = 58, normalized size = 0.39

$$-\frac{2\,\left(16\,b^{3}x^{3}+8\,ab^{2}x^{2}-2\,a^{2}bx+a^{3}\right)\sqrt{bx+a}\,\sqrt{x}}{5\,\left(a^{4}bx^{4}+a^{5}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 161, normalized size = 1.09

$$-\frac{2\, b^{3} \sqrt{x}}{\sqrt{bx+a}\, a^{4}} + \frac{4\left(5\, b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{8} - 30\, a b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{6} + 80\, a^{2} b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4} - 50\, a^{3} b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2} + 11\, a^{4} b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{5} a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) + 4/5*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^8 - 30*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 80*a^2*b^
(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 50*a^3*b^(5/2)*(sqrt(b)*sqrt(x)
 - sqrt(b*x + a))^2 + 11*a^4*b^(5/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2
 - a)^5*a^3)
```

Mupad [B]
time = 1.77, size = 346, normalized size = 2.34

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,x}{5\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{4}{5\,b\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,b\,x^2}{5\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}-\frac{512\,b^2\,x^3}{5\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}\right)}{x^{7/2}-\frac{x^{5/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(5*b
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)) + (128*b*x^2)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (
512*b^2*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x))/(2*b))
```

3.257 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}}-\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{}{3b^2\sqrt{}}$$

```
[Out] 35/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(9/2)-2/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-14/3*x^(5/2)/b^
2/arctanh(tanh(b*x+a))^(1/2)+35/6*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b^3+35
/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^4
```

Rubi [A]
time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^4}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{6b^3}-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(9/2)) - (2*x^(7/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3
/2)) - (14*x^(5/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (35*x^(3/2)*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(6*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x
]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^4)
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35\int \frac{}{\sqrt{\text{ta}}}}{}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35x^{3/2}\sqrt{}}{}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35x^{3/2}\sqrt{}}{}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}} -$$

Mathematica [A]
time = 0.08, size = 121, normalized size = 0.79

$$-\frac{\sqrt{x}\left(8b^3x^3+56b^2x^2\tanh^{-1}(\tanh(a+bx))-175bx\tanh^{-1}(\tanh(a+bx))^2+105\tanh^{-1}(\tanh(a+bx))^3\right)}{12b^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{35\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -1/12*(Sqrt[x]*(8*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 175*b*x*Arc
Tanh[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(b^4*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (35*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sq
rt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(119) = 238$.
time = 0.13, size = 348, normalized size = 2.27

method	result
derivativedivides	$\frac{x^{\frac{7}{2}}}{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{7a\,x^{\frac{5}{2}}}{4b^{2}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}x^{\frac{3}{2}}}{12b^{3}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}\sqrt{x}}{4b^{4}\sqrt{\mathrm{arctanh}(\tan}}$
default	$\frac{x^{\frac{7}{2}}}{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{7a\,x^{\frac{5}{2}}}{4b^{2}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}x^{\frac{3}{2}}}{12b^{3}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}\sqrt{x}}{4b^{4}\sqrt{\mathrm{arctanh}(\tan}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-7/4/b^2*a*x^(5/2)/arctanh(tanh(b*x
+a))^(3/2)-35/12/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*a^2*x^
(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*a^2*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))-35/6/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arcta
nh(tanh(b*x+a))^(3/2)-35/2/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/arcta
nh(tanh(b*x+a))^(1/2)+35/2/b^(9/2)*a*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2
)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-35/12/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*(arctanh(tanh(b*x+a))-b*x-
a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A]
time = 0.38, size = 241, normalized size = 1.58

$$\left[\frac{105\,(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4})\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(6\,b^{4}x^{3}-21\,ab^{3}x^{2}-140\,a^{2}b^{2}x-105\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{24\,(b^{7}x^{2}+2\,ab^{6}x+a^{2}b^{5})},-\frac{105\,(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4})\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(6\,b^{4}x^{3}-21\,ab^{3}x^{2}-140\,a^{2}b^{2}x-105\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{12\,(b^{7}x^{2}+2\,ab^{6}x+a^{2}b^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + 2*sqrt(b*x +
 a)*sqrt(b)*sqrt(x) + a) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 10
5*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(105
*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*
sqrt(x))) - (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 75, normalized size = 0.49

$$\frac{\left(\left(3\,x\left(\frac{2\,x}{b}-\frac{7\,a}{b^2}\right)-\frac{140\,a^2}{b^3}\right)x-\frac{105\,a^3}{b^4}\right)\sqrt{x}}{12\,(bx+a)^{\frac{3}{2}}}-\frac{35\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*x*(2*x/b - 7*a/b^2) - 140*a^2/b^3)*x - 105*a^3/b^4)*sqrt(x)/(b*x +
 a)^(3/2) - 35/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(7/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.258 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=111

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))}{b^{7/2}}-\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{}{3b^2\sqrt{\tan}}$$

```
[Out] 5*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x
+a)))/b^(7/2)-2/3*x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)-10/3*x^(3/2)/b^2/arc
tanh(tanh(b*x+a))^(1/2)+5*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2199, 2200, 2196}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)(bx-\tanh^{-1}(\tanh(a+bx)))}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(7/2) - (2*x^(5/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) -
(10*x^(3/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (5*Sqrt[x]*Sqrt[ArcTanh
[Tanh[a + b*x]]])/b^3
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2200

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Dist[n*((b*u -
a*v)/(a*(m + n + 1))), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\int \frac{}{\sqrt{\tan}}}{}$$

$$= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\sqrt{x}\,\sqrt{t}}{}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{7/2}} - \frac{}{3}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 0.91

$$-\frac{\sqrt{x}\left(2b^2x^2+10bx\tanh^{-1}(\tanh(a+bx))-15\tanh^{-1}(\tanh(a+bx))^2\right)}{3b^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(b\sqrt{x}+\sqrt{b}\,\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(2*b^2*x^2 + 10*b*x*ArcTanh[Tanh[a + b*x]] - 15*ArcTanh[Tanh[
a + b*x]]^2))/(b^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(87) = 174.
time = 0.13, size = 180, normalized size = 1.62

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{5a\,x^{\frac{3}{2}}}{3b^2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{5a\sqrt{x}}{b^3\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{5a\ln\left(\sqrt{b}\,\sqrt{\right.}}{}$
default	$\frac{x^{\frac{5}{2}}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{5a\,x^{\frac{3}{2}}}{3b^2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{5a\sqrt{x}}{b^3\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{5a\ln\left(\sqrt{b}\,\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)+5/3/b^2*a*x^(3/2)/arctanh(tanh(b*x+a))
^(3/2)+5/b^3*a*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*a*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))+5/3/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3
/2)/arctanh(tanh(b*x+a))^(3/2)+5/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A]
time = 0.36, size = 214, normalized size = 1.93

$$\left[\frac{15\,(ab^2x^2 + 2\,a^2bx + a^3)\sqrt{b}\,\log\left(2\,bx - 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\,(3\,b^3x^2 + 20\,ab^2x + 15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{6\,(b^6x^2 + 2\,ab^5x + a^2b^4)}, \frac{15\,(ab^2x^2 + 2\,a^2bx + a^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) + (3\,b^3x^2 + 20\,ab^2x + 15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{3\,(b^6x^2 + 2\,ab^5x + a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.39, size = 61, normalized size = 0.55

$$\frac{\left(x\left(\frac{3x}{b}+\frac{20a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}}+\frac{5a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*x/b + 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x + a)^(3/2) + 5*a*log(a
bs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.259 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(5/2)-2/3*x^(3/2)/b
/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2199, 2196}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(5/2) - (2*x^
(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[ArcTanh[T
anh[a + b*x]]])
```

Rule 2196

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{}{\sqrt{x}\sqrt{\mathrm{t}}}}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}} - \frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} -$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 1.04

$$-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*
x]]]])/b^(5/2)
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.79

method	result
derivativedivides	$-\frac{2x^{\frac{3}{2}}}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{2\sqrt{x}}{b^2\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}} + \frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\mathrm{arctanh}\left(\tanh\right.}\right.}{b^{\frac{5}{2}}}$
default	$-\frac{2x^{\frac{3}{2}}}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{2\sqrt{x}}{b^2\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}} + \frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\mathrm{arctanh}\left(\tanh\right.}\right.}{b^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a)
)^(1/2)+2/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A]
time = 0.39, size = 186, normalized size = 2.48

$$\left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,(4\,b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)},\,-\frac{2\left(3\,(b^2x^2+2\,abx+a^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.40, size = 49, normalized size = 0.65

$$-\frac{2\,\sqrt{x}\,\left(\frac{4\,x}{b}+\frac{3\,a}{b^2}\right)}{3\,(bx+a)^{\frac{3}{2}}}-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(4*x/b + 3*a/b^2)/(b*x + a)^(3/2) - 2*log(abs(-sqrt(b)*sqrt(x)
 + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.260 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2198\}$

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
)
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.97

$$\frac{2x^{3/2}}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]

[Out] (2*x^(3/2))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]
]))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(29) = 58$.
time = 0.12, size = 92, normalized size = 2.63

method	result
derivativedivides	$-\frac{\sqrt{x}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(\operatorname{arctanh}}\right.}{b}$
default	$-\frac{\sqrt{x}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(\operatorname{arctanh}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))^(3/2)+(arctanh(tanh(b*x+a))-b*x)/b*(1/3*x^(
1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tan
h(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(5/2), x)

Fricas [A]
time = 0.34, size = 33, normalized size = 0.94

$$\frac{2\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.40, size = 15, normalized size = 0.43

$$\frac{2\,x^{\frac{3}{2}}}{3\,\left(bx+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Mupad [B]
time = 1.70, size = 229, normalized size = 6.54

$$-\frac{4\,x^{3/2}\,\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}}{3\,b^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^2}+x^2-\frac{x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] -(4*x^(3/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*((l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2/(4*b^2) + x^2 - (x*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b))
```

3.261 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/3*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4/3*x^(1
/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{}{\sqrt{x}}}{3\left(-bx+\right.}$$

$$= -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{3\left(bx-\text{t}\right.}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.66

$$-\frac{2\sqrt{x}\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
[Out] (-2*Sqrt[x]*(b*x - 3*ArcTanh[Tanh[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/
2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A]
time = 0.12, size = 58, normalized size = 0.82

method	result
derivativedivides	$\frac{2\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right.\right.}}$
default	$\frac{2\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+4/3/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A]
time = 0.34, size = 43, normalized size = 0.61

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A]
time = 0.39, size = 25, normalized size = 0.35

$$\frac{2\,\sqrt{x}\,\left(\frac{2\,bx}{a^2}+\frac{3}{a}\right)}{3\,(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
[Out] 2/3*sqrt(x)*(2*b*x/a^2 + 3/a)/(b*x + a)^(3/2)
```

Mupad [B]
time = 1.66, size = 346, normalized size = 4.87

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,x^2}{3\,b\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}-\frac{x\,\left(48\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-48\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+96\,b\,x\right)}{12\,b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}\right)}{x^{5/2}-\frac{x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*x^2)/(3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (x
*(48*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 48*log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 96*b*x))/(12*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^
(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^2))
```

3.262 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{tan}}$$

[Out] 2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-8/3*b*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+16/3*b*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2202, 2198}

$$\frac{16b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]

[Out] (-8*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)) + (16*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + Dist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{(4b)\int \frac{}{\sqrt{}}}{-bx+}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\,(bx}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\,(bx}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.62

$$\frac{2\left(-b^2x^2+6bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2\right)}{3\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-(b^2*x^2) + 6*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A]
time = 0.12, size = 104, normalized size = 0.98

method	result
derivativedivides	$-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{8b\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(a}\right.}{\operatorname{arctanh}(t}$
default	$-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{8b\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(a}\right.}{\operatorname{arctanh}(t}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-8*b/(arcta
nh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b
*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^
(1/2))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 0.42

$$-\frac{2\,(8\,b^3x^3+20\,ab^2x^2+15\,a^2bx+3\,a^3)}{3\,(bx+a)^{\frac{5}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(8*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 3*a^3)/((b*x + a)^(5/2)*a^3*s
qrt(x))
```

Fricas [A]
time = 0.33, size = 58, normalized size = 0.55

$$-\frac{2\,(8\,b^2x^2+12\,abx+3\,a^2)\,\sqrt{bx+a}\,\sqrt{x}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [A]
time = 0.39, size = 62, normalized size = 0.58

$$-\frac{2\,\sqrt{x}\,\left(\frac{5\,b^2x}{a^3}+\frac{6\,b}{a^2}\right)}{3\,(bx+a)^{\frac{3}{2}}}+\frac{4\,\sqrt{b}}{\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(5*b^2*x/a^3 + 6*b/a^2)/(b*x + a)^(3/2) + 4*sqrt(b)/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)*a^2)
```

Mupad [B]
time = 1.69, size = 348, normalized size = 3.28

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{b^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,x^2}{3\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}-\frac{32\,x}{b\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{x^{5/2}-\frac{x^{3/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*x^2)/(
3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^3) - (32*x)/(b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^
(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^2))
```

3.263 $\int \frac{1}{x^{5/2} \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=146

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{4b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tan$$

```
[Out] 2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4*b/(b*x-
arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-16/3*b^2*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)+32/3*b^2*x^(1/2)/(
b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{32b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{4b}{\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]
]^(3/2)) + (4*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a +
 b*x]]^(3/2)) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (32*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[
ArcTanh[Tanh[a + b*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(2b)\int}{bx-}$$

$$= \frac{4b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{3x^{3/2}\,(}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\,(bx}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\,(bx}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.54

$$-\frac{2\left(b^3x^3-9b^2x^2\tanh^{-1}(\tanh(a+bx))-9bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3\right)}{3x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*(b^3*x^3 - 9*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 9*b*x*ArcTanh[Tanh[a + b*
x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)*
(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A]
time = 0.13, size = 150, normalized size = 1.03

method	result
derivativedivides	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{4b\left(-\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$
default	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{4b\left(-\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arc
tanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(
b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x
+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/
2)/arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A]
time = 0.48, size = 56, normalized size = 0.38

$$\frac{2\,(16\,b^4x^4+40\,ab^3x^3+30\,a^2b^2x^2+5\,a^3bx-a^4)}{3\,(bx+a)^{\frac{5}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(16*b^4*x^4 + 40*a*b^3*x^3 + 30*a^2*b^2*x^2 + 5*a^3*b*x - a^4)/((b*x +
a)^(5/2)*a^4*x^(3/2))
```

Fricas [A]
time = 0.33, size = 71, normalized size = 0.49

$$\frac{2\,(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [A]
time = 0.41, size = 119, normalized size = 0.82

$$\frac{2\,\sqrt{x}\,\left(\frac{8\,b^3x}{a^4}+\frac{9\,b^2}{a^3}\right)}{3\,(bx+a)^{\frac{3}{2}}}-\frac{8\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-9\,ab^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+4\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x)*(8*b^3*x/a^4 + 9*b^2/a^3)/(b*x + a)^(3/2) - 8/3*(3*b^(3/2)*(sqr
t(b)*sqrt(x) - sqrt(b*x + a))^4 - 9*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x +
 a))^2 + 4*a^2*b^(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^3)
```

Mupad [B]
time = 1.76, size = 406, normalized size = 2.78

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{3\,b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{128\,x^2}{{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}+\frac{16\,x}{b\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{512\,b\,x^3}{3\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^4}\right)}{x^{7/2}-\frac{x^{5/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{x^{3/2}\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - (128*x^2)
/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^3 + (16*x)/(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (512*
b*x^3)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x))/b + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

3.264 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{32b^2}{5\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\text{t}$$

```
[Out] 16/15*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+2/5
/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+32/5*b^2/(b*
x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-128/15*b^3*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(3/2)+256/15*b^3*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^5/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2202, 2198}

$$\frac{256b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{32b^2}{5\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{16b}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-128*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*
x]]^(3/2)) + (32*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[T
anh[a + b*x]]^(3/2)) + (16*b)/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*
ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + (256*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(8b)\int}{5\,(bx-}$$

$$= \frac{16b}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5x^{5/2}}$$

$$= \frac{32b^2}{5\sqrt{x}\,\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{15x^{3/}}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\,(}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\,(}$$

Mathematica [A]
time = 0.05, size = 100, normalized size = 0.54

$$\frac{2\left(-5b^4x^4+60b^3x^3\tanh^{-1}(\tanh(a+bx))+90b^2x^2\tanh^{-1}(\tanh(a+bx))^2-20bx\tanh^{-1}(\tanh(a+bx))^3+3\tanh^{-1}(\tanh(a+bx))^4\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-5*b^4*x^4 + 60*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 90*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^
4))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/
2))
```

Maple [A]
time = 0.13, size = 196, normalized size = 1.05

method	result

derivativedivides	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{16b\left(-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$
default	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{16b\left(-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-16/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(3/2)-2*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))
-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/
3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctan
h(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))))
```

Maxima [A]
time = 0.47, size = 67, normalized size = 0.36

$$-\frac{2\left(128\,b^{5}x^{5}+320\,ab^{4}x^{4}+240\,a^{2}b^{3}x^{3}+40\,a^{3}b^{2}x^{2}-5\,a^{4}bx+3\,a^{5}\right)}{15\,(bx+a)^{\frac{5}{2}}a^{5}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(128*b^5*x^5 + 320*a*b^4*x^4 + 240*a^2*b^3*x^3 + 40*a^3*b^2*x^2 - 5*a
^4*b*x + 3*a^5)/((b*x + a)^(5/2)*a^5*x^(5/2))
```

Fricas [A]
time = 0.45, size = 82, normalized size = 0.44

$$-\frac{2\left(128\,b^{4}x^{4}+192\,ab^{3}x^{3}+48\,a^{2}b^{2}x^{2}-8\,a^{3}bx+3\,a^{4}\right)\sqrt{bx+a}\,\sqrt{x}}{15\left(a^{5}b^{2}x^{5}+2\,a^{6}bx^{4}+a^{7}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*sq
rt(b*x + a)*sqrt(x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 173, normalized size = 0.93

$$-\frac{2\sqrt{x}\left(\frac{11\,b^4x}{a^5}+\frac{12\,b^3}{a^4}\right)}{3\,(bx+a)^{\frac{3}{2}}}+\frac{4\left(45\,b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^8-240\,ab^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^6+490\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-320\,a^3b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+73\,a^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(11*b^4*x/a^5 + 12*b^3/a^4)/(b*x + a)^(3/2) + 4/15*(45*b^(5/2)
*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 - 240*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^6 + 490*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 320*a
^3*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 73*a^4*b^(5/2))/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^5*a^4)
```

Mupad [**B**]
time = 1.86, size = 466, normalized size = 2.51

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{5\,b^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{256\,x^2}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}+\frac{64\,x}{15\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}-\frac{2048\,b\,x^3}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}+\frac{8192\,b^2\,x^4}{15\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^5}\right)}{x^{9/2}-\frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (256*x^2)
/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (64*x)/(15*b*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

```
- (2048*b*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) + (8192*b^2*x^4)/(15*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^5)))/(x^(9/2) - (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^
(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

3.265 $\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=79

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(-m,1+n;2+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(1+n)}$$

```
[Out] x^m*arctanh(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arctanh(tanh(b*x+
a))/(b*x-arctanh(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arctanh(tanh(b*x+a))))^m
)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2204\}$

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(-m,n+1;n+2;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^m*ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcTanh[Tanh[a + b*x]]))^m)
```

Rule 2204

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[u^m*(v^(n + 1)/(b*(n + 1)*(b*(u/(b*u - a*v)))^m))*Hype
rgeometric2F1[-m, n + 1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx = \frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(-m,1+n;2+\right.}{b(1+n)}$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.90

$$\frac{x^{1+m} \tanh^{-1}(\tanh(a+bx))^n \left(1+\frac{bx}{-bx+\tanh^{-1}(\tanh(a+bx))}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{-bx+\tanh^{-1}(\tanh(a+bx))}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((
b*x)/(-(b*x) + ArcTanh[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + Arc
Tanh[Tanh[a + b*x]]))^n)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{n} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] int(x^m*arctanh(tanh(b*x+a))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctanh(tanh(b*x + a))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{atanh}^{n}\left(\tanh \left(a+b x\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Integral(x**m*atanh(tanh(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\mathrm{atanh}(\tanh(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^n,x)
[Out] int(x^m*atanh(tanh(a + b*x))^n, x)
```

3.266 $\int x^4 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=165

$$\frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)} - \frac{24x \tanh^{-1}(}{b^4(1+n)(2+}$$

```
[Out] x^4*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arctanh(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+12*x^2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-24*x*a
rctanh(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)+24*arctanh(tanh(b*x+a
))^(5+n)/b^5/(n^2+7*n+12)/(n^3+8*n^2+17*n+10)
```

Rubi [A]
time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{24 \tanh^{-1}(\tanh(a+bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)} - \frac{24x \tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcTanh[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcTanh[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcTanh[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int x^4 \tanh^{-1}(\tanh(a+bx))^n\,dx &= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4\int x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)} \\
&= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12\int x^2 \tan\ldots}{b\ldots} \\
&= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh\ldots}{b^3(1+\ldots} \\
&= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh\ldots}{b^3(1+\ldots} \\
&= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh\ldots}{b^3(1+\ldots} \\
&= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh\ldots}{b^3(1+\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 146, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\left(b^4(120+154n+71n^2+14n^3+n^4)\,x^4-4b^3(60+47n+12n^2+n^3)\,x^3\tanh^{-1}(\tanh(a+bx))+12b^2(20+9n+n^2)\,x^2\tanh^{-1}(\tanh(a+bx))^2-24b(5+n)x\tanh^{-1}(\tanh(a+bx))^3+24\tanh^{-1}(\tanh(a+bx))^4\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*
x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2*
(20 + 9*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcTanh[Tanh
[a + b*x]]^3 + 24*ArcTanh[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*(5 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(165) = 330$.
time = 7.02, size = 654, normalized size = 3.96

method	result
default	$\frac{x^5 e^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{5+n}+\frac{n(\operatorname{arctanh}(\tanh(bx+a))-bx)x^4 e^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b(n^2+9n+20)}-\frac{4n\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a\ldots\right.}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(5+n)*x^5*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+9*n+20)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))-4*n*(a^2+2*a*(arctanh(tanh
(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^2/(n^3+12*n^2+47*n+60)*x^
3*exp(n*ln(arctanh(tanh(b*x+a))))+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a^5+120/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)
*exp(n*ln(arctanh(tanh(b*x+a))))*a^4*(arctanh(tanh(b*x+a))-b*x-a)+240/b^5/(
n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctan
h(tanh(b*x+a))-b*x-a)^2+240/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(a
rctanh(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^3+120/b^5/(n^3+12*n^
2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+
a))-b*x-a)^4+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(
b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^5-24*(a^2+2*a*(arctanh(tanh(b*x+a))-
b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)^2*n/b^4/(n^3+12*n^2+47*n+60)/(n^2+3*
n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))+12/b^3*(arctanh(tanh(b*x+a))-b*x)*(a
^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)*n/(2+n)
/(n^3+12*n^2+47*n+60)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))
```

Maxima [A]
time = 0.54, size = 139, normalized size = 0.84

$$\frac{\left((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+12\,(n^2+n)a^3b^2x^2-24\,a^4bnx+24\,a^5\right)(bx+a)^n}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)
*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 -
 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274
*n + 120)*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(165) = 330$.
time = 0.36, size = 374, normalized size = 2.27

$$-\frac{(24\,a^4bnx-(b^5n^4+10\,b^5n^3+35\,b^5n^2+50\,b^5n+24\,b^5)x^5-24\,a^5-(ab^4n^4+6\,ab^4n^3+11\,ab^4n^2+6\,ab^4n)x^4+4\,(a^2b^3n^3+3\,a^2b^3n^2+2\,a^2b^3n)x^3-12\,(a^3b^2n^2+a^3b^2n)x^2)\cosh\left(n\log\left(bx+a\right)\right)+(24\,a^4bnx-(b^5n^4+10\,b^5n^3+35\,b^5n^2+50\,b^5n+24\,b^5)x^5-24\,a^5-(ab^4n^4+6\,ab^4n^3+11\,ab^4n^2+6\,ab^4n)x^4+4\,(a^2b^3n^3+3\,a^2b^3n^2+2\,a^2b^3n)x^3-12\,(a^3b^2n^2+a^3b^2n)x^2)\sinh\left(n\log\left(bx+a\right)\right)}{b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x
^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*
```

(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 12*(a^3*b^2*n^2 + a^3*b^2
*n)*x^2)*cosh(n*log(b*x + a)) + (24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*
b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a
*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x
^3 - 12*(a^3*b^2*n^2 + a^3*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^5 \operatorname{atanh}^n(\tanh(a))}{5} & \text{for}\: b = 0 \\ -\frac{x^4}{4b \operatorname{atanh}^4(\tanh(a+bx))} - \frac{x^3}{3b^2 \operatorname{atanh}^3(\tanh(a+bx))} - \frac{x^2}{2b^3 \operatorname{atanh}^2(\tanh(a+bx))} - \frac{x}{b^4 \operatorname{atanh}(\tanh(a+bx))} + \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^5} & \text{for}\: n = -5 \\ \int \frac{x^4}{\operatorname{atanh}^4(\tanh(a+bx))}\, dx & \text{for}\: n = -4 \\ \int \frac{x^4}{\operatorname{atanh}^3(\tanh(a+bx))}\, dx & \text{for}\: n = -3 \\ \int \frac{x^4}{\operatorname{atanh}^2(\tanh(a+bx))}\, dx & \text{for}\: n = -2 \\ \int \frac{x^4}{\operatorname{atanh}(\tanh(a+bx))}\, dx & \text{for}\: n = -1 \\ \frac{b^4 n^4 x^4 \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} + \frac{14 b^4 n^3 x^4 \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} + \frac{71 b^4 n^2 x^4 \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} + \frac{154 b^4 n x^4 \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} + \frac{120 b^4 x^4 \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} - \text{[illegible]} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**5*atanh(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*atanh(tanh(a +
b*x))**4) - x**3/(3*b**2*atanh(tanh(a + b*x))**3) - x**2/(2*b**3*atanh(tanh
(a + b*x))**2) - x/(b**4*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/
b**5, Eq(n, -5)), (Integral(x**4/atanh(tanh(a + b*x))**4, x), Eq(n, -4)), (
Integral(x**4/atanh(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/atanh
(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/atanh(tanh(a + b*x)), x)
, Eq(n, -1)), (b**4*n**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120
*b**5) + 14*b**4*n**3*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
*5) + 71*b**4*n**2*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 154*b**4*n*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12
0*b**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*b**3*n*
*3*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 48*b**3*n*
*2*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 188*b**3*n
*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 240*b**3*x**
3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**2*n**2*x**
2*atanh(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 108*b**2*n*x**2*
atanh(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 240*b**2*x**2*atan
h(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*b*n*x*atanh(tanh(a
```

```
+ b*x))**4*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 120*b*x*atanh(tanh(a + b*x))**4
*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 24*atanh(tanh(a + b*x))**5*atanh(tanh(a +
 b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
*5*n + 120*b**5), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(165) = 330.
time = 0.39, size = 332, normalized size = 2.01

$$\frac{(bx+a)^n b^5 n^4 x^5 + (bx+a)^n a b^4 n^4 x^4 + 10\,(bx+a)^n b^5 n^3 x^5 + 6\,(bx+a)^n a b^4 n^3 x^4 + 35\,(bx+a)^n b^5 n^2 x^5 - 4\,(bx+a)^n a^2 b^3 n^3 x^3 + 11\,(bx+a)^n a b^4 n^2 x^4 + 50\,(bx+a)^n b^5 n x^5 - 12\,(bx+a)^n a^2 b^3 n^2 x^3 + 6\,(bx+a)^n a b^4 n x^4 + 24\,(bx+a)^n b^5 x^5 + 12\,(bx+a)^n a^3 b^2 n^2 x^2 - 8\,(bx+a)^n a^2 b^3 n x^3 + 12\,(bx+a)^n a^3 b^2 n x^2 - 24\,(bx+a)^n a^4 b n x + 24\,(bx+a)^n a^5}{b^5 n^5 + 15\,b^5 n^4 + 85\,b^5 n^3 + 225\,b^5 n^2 + 274\,b^5 n + 120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*n^4*x^5 + (b*x + a)^n*a*b^4*n^4*x^4 + 10*(b*x + a)^n*b^5*n
^3*x^5 + 6*(b*x + a)^n*a*b^4*n^3*x^4 + 35*(b*x + a)^n*b^5*n^2*x^5 - 4*(b*x
+ a)^n*a^2*b^3*n^3*x^3 + 11*(b*x + a)^n*a*b^4*n^2*x^4 + 50*(b*x + a)^n*b^5*
n*x^5 - 12*(b*x + a)^n*a^2*b^3*n^2*x^3 + 6*(b*x + a)^n*a*b^4*n*x^4 + 24*(b*
x + a)^n*b^5*x^5 + 12*(b*x + a)^n*a^3*b^2*n^2*x^2 - 8*(b*x + a)^n*a^2*b^3*n
*x^3 + 12*(b*x + a)^n*a^3*b^2*n*x^2 - 24*(b*x + a)^n*a^4*b*n*x + 24*(b*x +
a)^n*a^5)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 12
0*b^5)
```

Mupad [**B**]
time = 1.67, size = 546, normalized size = 3.31

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^5}{4\,b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{3\,n\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{2\,b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^4\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{2\,b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{2\,b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^3\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)/(4*b^5*(274*n + 225
*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (
n*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (n*x^3*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 +
15*n^4 + n^5 + 120)))
```

3.267 $\int x^3 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=121

$$\frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{6x \tanh^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}-\frac{6 \tanh^{-1}(\tanh}{b^4(1+n)(2+n)}$$

```
[Out] x^3*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arctanh(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+6*x*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-6*arctanh
(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)
```

Rubi [A]
time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$-\frac{6 \tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{6x \tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^3 \tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcTanh[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcTanh[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3\tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^3\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3\int x^2\tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6\int x\tan}{}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\tanh^{-}}{b^3(1+}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\tanh^{-}}{b^3(1+}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\tanh^{-}}{b^3(1+}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\left(b^3(24+26n+9n^2+n^3)\,x^3-3b^2(12+7n+n^2)\,x^2\tanh^{-1}(\tanh(a+bx))+6b(4+n)x\tanh^{-1}(\tanh(a+bx))^2-6\tanh^{-1}(\tanh(a+bx))^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*
(12 + 7*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcTanh[Tanh[a
+ b*x]]^2 - 6*ArcTanh[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(121) = 242$.
time = 2.54, size = 492, normalized size = 4.07

method	result
default	$\frac{x^4 e^{n\ln(\text{arctanh}(\tanh(bx+a)))}}{4+n} + \frac{n(\text{arctanh}(\tanh(bx+a))-bx)x^3 e^{n\ln(\text{arctanh}(\tanh(bx+a)))}}{b(n^2+7n+12)} - \frac{6\,e^{n\ln(\text{arctanh}(\tanh(bx+a)))}a^4}{b^4(n^4+10n^3+35n^2+50n+24)} - \frac{24\,e}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4+n)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+7*n+12)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))-6/b^4/(n^4+10*n^3+35*n^2+5
0*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^4-24/b^4/(n^4+10*n^3+35*n^2+50*n+
```

```
24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctanh(tanh(b*x+a))-b*x-a)-36/b^4
/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^2*(arctanh(t
anh(b*x+a))-b*x-a)^2-24/b^4/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(ta
nh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-6/b^4/(n^4+10*n^3+35*n^2+50*n
+24)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^4-3*n/b^2
*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(n^3
+9*n^2+26*n+24)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+6*n*(a^3+3*a^2*(arctanh
(tanh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a
))-b*x-a)^3)/b^3/(n^4+10*n^3+35*n^2+50*n+24)*x*exp(n*ln(arctanh(tanh(b*x+a)
)))
```

Maxima [**A**]
time = 0.53, size = 101, normalized size = 0.83

$$\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(121) = 242$.
time = 0.36, size = 255, normalized size = 2.11

$$\frac{(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)\cosh\left(n\log\left(bx+a\right)\right)+(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)\sinh\left(n\log\left(bx+a\right)\right)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*co
sh(n*log(b*x + a)) + (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4
)*x^4 - 6*a^4 + (a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2
+ a^2*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2
+ 50*b^4*n + 24*b^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4\operatorname{atanh}^n\left(\tanh\left(a\right)\right)}{4} & \text{for}\: b=0 \\ -\frac{x^3}{3b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}-\frac{x^2}{2b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}-\frac{x}{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}+\frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b^4} & \text{for}\: n=-4 \\ \int\frac{x^3}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx & \text{for}\: n=-3 \\ \int\frac{x^3}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx & \text{for}\: n=-2 \\ \int\frac{x^3}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx & \text{for}\: n=-1 \\ \frac{b^3n^3x^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{9b^3n^2x^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{26b^3nx^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{24b^3x^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{3b^2n^2x^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{21b^2nx^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{36b^2x^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{6bnx\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{24bx\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{6\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**4*atanh(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*atanh(tanh(a +
b*x))**3) - x**2/(2*b**2*atanh(tanh(a + b*x))**2) - x/(b**3*atanh(tanh(a +
b*x))) + log(atanh(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/atanh(t
anh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/atanh(tanh(a + b*x))**2, x
), Eq(n, -2)), (Integral(x**3/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b**3*n
**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*atanh(tanh(a
+ b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) + 26*b**3*n*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b
*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2
4*b**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**2*atanh(tanh
(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*atanh(tanh(a + b*x))**2*atanh(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 36*b**2*x**2*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b*n*x*atanh(t
anh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**
4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*atanh(tanh(a + b*x))**3*atanh(tanh(a
 + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 - 6*atanh(tanh(a + b*x))**4*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n
**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [A]
time = 0.39, size = 226, normalized size = 1.87

$$\frac{(bx+a)^n b^4 n^3 x^4 + (bx+a)^n a b^3 n^3 x^3 + 6\,(bx+a)^n b^4 n^2 x^4 + 3\,(bx+a)^n a b^3 n^2 x^3 + 11\,(bx+a)^n b^4 n x^4 - 3\,(bx+a)^n a^2 b^2 n^2 x^2 + 2\,(bx+a)^n a b^3 n x^3 + 6\,(bx+a)^n b^4 x^4 - 3\,(bx+a)^n a^2 b^2 n x^2 + 6\,(bx+a)^n a^3 b n x - 6\,(bx+a)^n a^4}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B]
time = 1.42, size = 418, normalized size = 3.45

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{8\,b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{3\,n\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{n\,x^3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\left(n^2+3\,n+2\right)}{2\,b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/(8*b^4*(50*n + 35*n
^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
10*n^3 + n^4 + 24) + (3*n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*b^3*(50*n + 35*n
^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(3*n + n^2 + 2))
/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.268 $\int x^2 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=82

$$\frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \tanh^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}$$

```
[Out] x^2*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arctanh(tanh(b*x+a))^(2+n)/b^2/(
1+n)/(2+n)+2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{2 \tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{2x \tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcTanh[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcTanh[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2\int x\tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\int \tanh^{-1}(}{b^2(1}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\mathrm{Subst}\left(\int x\right.}{}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\tanh^{-1}(\mathrm{ta}}{b^3(1+n)(}$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.87

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\left(b^2(6+5n+n^2)\,x^2-2b(3+n)x\tanh^{-1}(\tanh(a+bx))+2\tanh^{-1}(\tanh(a+bx))^2\right)}{b^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*Ar
cTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*(3
 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(82) = 164$.
time = 0.97, size = 315, normalized size = 3.84

method	result
default	$\frac{x^3 e^{n\ln(\mathrm{arctanh}(\tanh(bx+a)))}}{3+n} + \frac{n(\mathrm{arctanh}(\tanh(bx+a))-bx)x^2 e^{n\ln(\mathrm{arctanh}(\tanh(bx+a)))}}{b(n^2+5n+6)} + \frac{2\,e^{n\ln(\mathrm{arctanh}(\tanh(bx+a)))}a^3}{b^3(n^3+6n^2+11n+6)} + \frac{6\,e^{n\ln(}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(3+n)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+5*n+6)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+2/b^3/(n^3+6*n^2+11*n+6)*ex
p(n*ln(arctanh(tanh(b*x+a))))*a^3+6/b^3/(n^3+6*n^2+11*n+6)*exp(n*ln(arctanh
(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+6/b^3/(n^3+6*n^2+11*n+6)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+2/b^3/(n^3+
6*n^2+11*n+6)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^
```

3-2*n*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)
/b^2/(n^3+6*n^2+11*n+6)*x*exp(n*ln(arctanh(tanh(b*x+a))))

Maxima [A]
time = 0.53, size = 68, normalized size = 0.83

$$\frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(82) = 164.
time = 0.35, size = 168, normalized size = 2.05

$$-\frac{(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)\cosh\left(n\log\left(bx+a\right)\right)+(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)\sinh\left(n\log\left(bx+a\right)\right)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b
^2*n)*x^2)*cosh(n*log(b*x + a)) + (2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3
)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^3*n^3 +
 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3 \operatorname{atanh}^n\left(\tanh\left(a\right)\right)}{3} & \text{for}\: b = 0 \\ -\frac{x^2}{2b \operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)} - \frac{x}{b^2 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)} + \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b^3} & \text{for}\: n = -3 \\ \int \frac{x^2}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\, dx & \text{for}\: n = -2 \\ \int \frac{x^2}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\, dx & \text{for}\: n = -1 \\ \frac{b^2n^2x^2 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5b^2nx^2 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6b^2x^2 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2bnx \operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{6bx \operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{2 \operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*atanh(tanh(a +
b*x))**2) - x/(b**2*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**3,
 Eq(n, -3)), (Integral(x**2/atanh(tanh(a + b*x))**2, x), Eq(n, -2)), (Integ
ral(x**2/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*atanh(tanh(a
 + b*x))*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) + 5*b**2*n*x**2*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*atanh(tanh(a + b*x))*at
```

```
anh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*b*
n*x*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**
2 + 11*b**3*n + 6*b**3) - 6*b*x*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x)
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*atanh(tanh(a + b*x)
)**3*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3)
, True))
```

Giac [A]
time = 0.39, size = 140, normalized size = 1.71

$$\frac{(bx+a)^n b^3n^2x^3 + (bx+a)^n ab^2n^2x^2 + 3\,(bx+a)^n b^3nx^3 + (bx+a)^n ab^2nx^2 + 2\,(bx+a)^n b^3x^3 - 2\,(bx+a)^n a^2bnx + 2\,(bx+a)^n a^3}{b^3n^3 + 6\,b^3n^2 + 11\,b^3n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Mupad [B]
time = 1.29, size = 304, normalized size = 3.71

$$-\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}\right)^n\,\left(\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^3}{4\,b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2\,b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{n\,x^2\,\left(n+1\right)\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n^2 +
n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n + 1)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

3.269 $\int x \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=48

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

```
[Out] x*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-arctanh(tanh(b*x+a))^(2+n)/b^2/(1+n)/(
2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcTanh[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x \tanh^{-1}(\tanh(a+bx))^n \, dx = \frac{x \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\int \tanh^{-1}(\tanh(a+bx))^{1+n} \, dx}{b(1+n)}$$
$$= \frac{x \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\text{Subst}\left(\int x^{1+n} \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2(1+n)}$$
$$= \frac{x \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.85

$$\frac{\left(b(2+n)x - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{1+n}}{b^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b^
2*(1 + n)*(2 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(48) = 96.
time = 0.40, size = 175, normalized size = 3.65

method	result
default	$\frac{x^2 \mathrm{e}^{n \ln(\mathrm{arctanh}(\tanh(bx+a)))}}{2+n} + \frac{n(\mathrm{arctanh}(\tanh(bx+a)) - bx) x \, \mathrm{e}^{n \ln(\mathrm{arctanh}(\tanh(bx+a)))}}{b(n^2+3n+2)} - \frac{\mathrm{e}^{n \ln(\mathrm{arctanh}(\tanh(bx+a)))} a^2}{b^2(n^2+3n+2)} - 2\,\mathrm{e}^{n \ln(\mathrm{arc}} \cdots$
risch	$\frac{\left(\frac{1}{2}\right)^n \left(2 \ln(\mathrm{e}^{bx+a}) - \frac{i\pi \,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^2}{2} - \frac{i\pi \,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\right) \cdots}{2} \cdots\right.}{2b(1+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+n)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+3*n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))-1/b^2/(n^2+3*n+2)*exp(n*ln(ar
ctanh(tanh(b*x+a))))*a^2-2/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*
a*(arctanh(tanh(b*x+a))-b*x-a)-1/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+
a))))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.88

$$\frac{(b^2(n+1)x^2 + abnx - a^2)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.90

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\cosh\left(n\log\left(bx + a\right)\right) + (abnx + (b^2n + b^2)x^2 - a^2)\sinh\left(n\log\left(bx + a\right)\right)}{b^2n^2 + 3\,b^2n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*cosh(n*log(b*x + a)) + (a*b*n*x + (b^2
*n + b^2)*x^2 - a^2)*sinh(n*log(b*x + a)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2 \operatorname{atanh}^n\left(\tanh\left(a\right)\right)}{2} & \text{for}\ b = 0 \\ -\frac{x}{b \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)} + \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b^2} & \text{for}\ n = -2 \\ \int \frac{x}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\, dx & \text{for}\ n = -1 \\ \frac{bnx \operatorname{atanh}\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2} + \frac{2bx \operatorname{atanh}\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2} - \frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right) \operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a))**n/2, Eq(b, 0)), (-x/(b*atanh(tanh(a + b*x))
) + log(atanh(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/atanh(tanh(a +
b*x)), x), Eq(n, -1)), (b*n*x*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/
(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*atanh(tanh(a + b*x))*atanh(tanh(a +
 b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - atanh(tanh(a + b*x))**2*atanh(t
anh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Giac [A]
time = 0.40, size = 76, normalized size = 1.58

$$\frac{(bx + a)^n b^2 n x^2 + (bx + a)^n abnx + (bx + a)^n b^2 x^2 - (bx + a)^n a^2}{b^2n^2 + 3\,b^2n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Mupad [B]
time = 1.23, size = 205, normalized size = 4.27

$$-\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}\right)^n\,\left(\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^2\,\left(n^2+3\,n+2\right)}-\frac{x^2\,\left(n+1\right)}{n^2+3\,n+2}+\frac{n\,x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2 + 2))
 - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(3*n
 + n^2 + 2)))
```

3.270 $\int \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

```
[Out] arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^n \, dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n,x]
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Maple [A]
time = 0.37, size = 21, normalized size = 1.05

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^{1+n}}{b(1+n)}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^{1+n}}{b(1+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln\left(e^{bx+a}\right)-\frac{i\pi \operatorname{csgn}\left(ie^{2bx+2a}\right)\left(-\operatorname{csgn}\left(ie^{2bx+2a}\right)+\operatorname{csgn}\left(ie^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)\left(-\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\operatorname{csgn}\cdots\right.}{\cdots}\right.}{2b(1+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
[Out] arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.51, size = 21, normalized size = 1.05

$$\frac{(bx+a)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^n/(b*(n + 1))
```

Fricas [A]
time = 0.39, size = 39, normalized size = 1.95

$$\frac{(bx+a)\cosh\left(n\log\left(bx+a\right)\right)+(bx+a)\sinh\left(n\log\left(bx+a\right)\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
[Out] ((b*x + a)*cosh(n*log(b*x + a)) + (b*x + a)*sinh(n*log(b*x + a)))/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

time = 0.20, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{atanh}\left(\tanh\left(a\right)\right)} & \text{for}\: b = 0 \wedge n = -1 \\ x \operatorname{atanh}^{n}\left(\tanh\left(a\right)\right) & \text{for}\: b = 0 \\ \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)\right)}{b} & \text{for}\: n = -1 \\ \frac{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right) \operatorname{atanh}^{n}\left(\tanh\left(a + bx\right)\right)}{bn + b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x/atanh(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*atanh(tanh(a))**n, E
q(b, 0)), (log(atanh(tanh(a + b*x)))/b, Eq(n, -1)), (atanh(tanh(a + b*x))*a
tanh(tanh(a + b*x))**n/(b*n + b), True))
```

Giac [A]
time = 0.40, size = 28, normalized size = 1.40

$$\frac{(bx + a)^{n}bx + (bx + a)^{n}a}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b*x + (b*x + a)^n*a)/(b*n + b)
```

Mupad [B]
time = 1.18, size = 121, normalized size = 6.05

$$\left(\frac{1}{2}\right)^{n} \left(\frac{x}{n+1} - \frac{\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} + b\,x}{b\,\left(n+1\right)}\right) \left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n,x)
```

```
[Out] (1/2)^n*(x/(n + 1) - (log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)
))^n
```

3.271 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] arctanh(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arctanh(tanh(b*x+a))/(
b*x-arctanh(tanh(b*x+a))))/(1+n)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2195}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcTan
h[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcTanh[
Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]

time = 0.06, size = 60, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcTanh[Tanh
[a + b*x]]/(b*x)])/(n*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(tanh(b*x + a))^n/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atanh}(\tanh(a+b\,x))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x,x)
```

```
[Out] int(atanh(tanh(a + b*x))^n/x, x)
```

3.272 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}+\frac{b\tanh^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -arctanh(tanh(b*x+a))^n/x+b*arctanh(tanh(b*x+a))^n*hypergeom([1, n],[1+n],-
arctanh(tanh(b*x+a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$\frac{b\tanh^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;n+1;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^n/x) + (b*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/
(b*x - ArcTanh[Tanh[a + b*x]])
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x} + (bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x}\,dx$$
$$= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x} + \frac{b\tanh^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\tanh}{bx-\tan}\right.}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}{}_2F_1\left(1-n,-n;2-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(-1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^2,x)
[Out] int(arctanh(tanh(b*x+a))^n/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
[Out] integral(arctanh(tanh(b*x + a))^n/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**2,x)
[Out] Integral(atanh(tanh(a + b*x))**n/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="giac")
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x^2,x)
[Out] int(atanh(tanh(a + b*x))^n/x^2, x)
```

3.273 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3} dx$

Optimal. Leaf size=101

$$-\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{b^2n\tanh^{-1}(\tanh(a+bx))^{-1+n}\,{}_2F_1\left(1,-1+n;n;-\right.}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -1/2*b*n*arctanh(tanh(b*x+a))^(-1+n)/x-1/2*arctanh(tanh(b*x+a))^n/x^2+1/2*b
^2*n*arctanh(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arctanh(tanh(b*x+
a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$\frac{b^2n\tanh^{-1}(\tanh(a+bx))^{n-1}\,{}_2F_1\left(1,n-1;n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{bn\tanh^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] -1/2*(b*n*ArcTanh[Tanh[a + b*x]]^(-1 + n))/x - ArcTanh[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcTanh[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(2*(b*x - Arc
Tanh[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{1}{2}(bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x^2}\,dx$$

$$= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{1}{2}(b^2(1-n)n$$

$$= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{b^2n\tanh^{-1}(\text{ta}}{}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.66

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}{}_2F_1\left(2-n,-n;3-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(-2+n)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^3,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
[Out] integral(arctanh(tanh(b*x + a))^n/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**3,x)
[Out] Integral(atanh(tanh(a + b*x))**n/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="giac")
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{atanh}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x^3,x)
[Out] int(atanh(tanh(a + b*x))^n/x^3, x)
```

3.274 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\coth^{-1}(\tanh(a+bx))\,dx&=\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.92

$$x^m\left(\frac{bx^2}{2+m}+\frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 676, normalized size = 18.27

method	result
risch	$\frac{x\,x^m \ln\left(e^{bx+a}\right)}{1+m} - \frac{x\left(2i\pi \operatorname{csgn}\left(ie^{2bx+2a}\right)^3 + i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)m + 2i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3 m + 4bx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(2*I*Pi*csgn(I*exp(2*b*x+2*a))^3+I*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))*m+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3*m+4*b*x-2*I*Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)
)^2*m+2*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*I*Pi*m-2*I*Pi*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*m+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*I*Pi-
4*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-2*I*Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*exp(2*b*x+2*a
))^3*m-I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2*m+4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))
^2+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*m+I*Pi*csgn(I*exp(b*x+a
))^2*csgn(I*exp(2*b*x+2*a))*m)/(1+m)/(2+m)*x^m
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.89

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log{(x)} - \frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x} & \text{for}\: m=-2 \\ \int \frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x}\, dx & \text{for}\: m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2} + \frac{2xx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.39, size = 90, normalized size = 2.43

$$\frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(2
*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m + 2)
*(m + 1))
```

Mupad [B]
time = 1.60, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4}-\frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))/(6*m + 2*m^2 + 4)
```

3.275 $\int x^2 \tanh^{-1}(\coth(a+bx)) \, dx$

Optimal. Leaf size=23

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTanh[Coth[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\coth(a+bx))\,dx &= \frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Coth[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcTanh[Coth[a + b*x]]))
```

Maple [A]
time = 0.12, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^4}{12}+\frac{x^3\,\mathrm{arctanh}(\coth(bx+a))}{3}$
risch	$\frac{x^3\ln\left(\mathrm{e}^{bx+a}\right)}{3}-\frac{b\,x^4}{12}-\frac{i\pi\,x^3\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)^3}{6}-\frac{i\pi\,x^3\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3}{12}-\frac{i\pi\,x^3\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^3}{12}-\frac{i\pi\,x^3\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\mathrm{csg}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.83

$$-\frac{1}{12}\,bx^4+\frac{1}{3}\,x^3\,\mathrm{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="maxima")
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(coth(b*x + a))
```

Fricas [A]
time = 0.40, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(19) = 38$.
time = 7.95, size = 76, normalized size = 3.30

$$\begin{cases} \frac{x^3 \operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(-e^{-bx}\right) \\ \frac{x^3 \operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(e^{-bx}\right) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{atanh}\left(\frac{1}{\tanh(a+bx)}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((x**3*atanh(coth(b*x + log(-exp(-b*x))))/3, Eq(a, log(-exp(-b*x))
)), (x**3*atanh(coth(b*x + log(exp(-b*x))))/3, Eq(a, log(exp(-b*x)))), (-b*
x**4/12 + x**3*atanh(1/tanh(a + b*x))/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.39, size = 71, normalized size = 3.09

$$-\frac{1}{12}\,bx^4 + \frac{1}{6}\,x^3 \log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} + 1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 1.10, size = 19, normalized size = 0.83

$$\frac{x^3 \operatorname{atanh}(\coth(a + b\,x))}{3} - \frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(coth(a + b*x)),x)
```

```
[Out] (x^3*atanh(coth(a + b*x)))/3 - (b*x^4)/12
```

3.276 $\int x \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6376, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTanh[Coth[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\coth(a+bx))\,dx&=\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Coth[a + b*x]],x]
[Out] -1/6*(x^2*(b*x - 3*ArcTanh[Coth[a + b*x]]))
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arctanh}\left(\coth\left(bx+a\right)\right)}{2}$
risch	$\frac{x^{2}\ln\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{b\,x^{3}}{6}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)^{3}}{4}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)}{8}+\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\mathrm{e}^{2b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{1}{6}\,bx^{3}+\frac{1}{2}\,x^{2}\operatorname{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="maxima")
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(coth(b*x + a))
```

Fricas [A]
time = 0.60, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(19) = 38$.
time = 4.15, size = 160, normalized size = 6.96

$$\begin{cases} -\frac{x\log\left(-e^{-bx}\right)\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{b}-\frac{\log\left(-e^{-bx}\right)^{2}\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2b^{2}} & \text{for}\: a=\log\left(-e^{-bx}\right) \\ -\frac{x\log\left(e^{-bx}\right)\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{b}-\frac{\log\left(e^{-bx}\right)^{2}\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2b^{2}} & \text{for}\: a=\log\left(e^{-bx}\right) \\ -\frac{bx^{3}}{6}+\frac{x^{2}\operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((-x*log(-exp(-b*x))*atanh(coth(b*x + log(-exp(-b*x))))/b - log(-e
xp(-b*x))**2*atanh(coth(b*x + log(-exp(-b*x))))/(2*b**2), Eq(a, log(-exp(-b
*x)))), (-x*log(exp(-b*x))*atanh(coth(b*x + log(exp(-b*x))))/b - log(exp(-b
*x))**2*atanh(coth(b*x + log(exp(-b*x))))/(2*b**2), Eq(a, log(exp(-b*x)))),
 (-b*x**3/6 + x**2*atanh(1/tanh(a + b*x))/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.38, size = 71, normalized size = 3.09

$$-\frac{1}{6}\,bx^3 + \frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{atanh}(\coth(a+b\,x))}{2}-\frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(coth(a + b*x)),x)
```

```
[Out] (x^2*atanh(coth(a + b*x)))/2 - (b*x^3)/6
```

3.277 $\int \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

```
[Out] 1/2*arctanh(coth(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]],x]
```

```
[Out] ArcTanh[Coth[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\coth(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\coth(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2} + x\tanh^{-1}(\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTanh[Coth[a + b*x]]
```

Maple [A]
time = 0.10, size = 15, normalized size = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2b}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2b}$
risch	$x\ln\left(e^{bx+a}\right) - \frac{i\pi x\operatorname{csgn}\left(ie^{bx+a}\right)^2\operatorname{csgn}\left(ie^{2bx+2a}\right)}{4} + \frac{i\pi\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)^2 x}{2} - \frac{i\pi x\operatorname{csgn}\left(ie^{2bx+2a}\right)^3}{4} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(coth(b*x+a))^2/b
```

Maxima [A]
time = 0.30, size = 16, normalized size = 1.00

$$-\frac{1}{2}\,bx^2 + x\operatorname{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="maxima")
[Out] -1/2*b*x^2 + x*arctanh(coth(b*x + a))
```

Fricas [A]
time = 0.51, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(12) = 24$.
time = 2.23, size = 80, normalized size = 5.00

$$\begin{cases} -\frac{\log\left(-e^{-bx}\right)\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{b} & \text{for}\ a = \log\left(-e^{-bx}\right) \\ x\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right) & \text{for}\ a = \log\left(e^{-bx}\right) \\ x\operatorname{atanh}\left(\coth\left(a\right)\right) & \text{for}\ b = 0 \\ \frac{\operatorname{atanh}^2\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((-log(-exp(-b*x))*atanh(coth(b*x + log(-exp(-b*x))))/b, Eq(a, log
(-exp(-b*x)))), (x*atanh(coth(b*x + log(exp(-b*x)))), Eq(a, log(exp(-b*x)))
), (x*atanh(coth(a)), Eq(b, 0)), (atanh(1/tanh(a + b*x))**2/(2*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.
time = 0.40, size = 69, normalized size = 4.31

$$-\frac{1}{2}\,bx^2 + \frac{1}{2}\,x\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} + 1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/(
(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 0.02, size = 16, normalized size = 1.00

$$x\,\mathrm{atanh}(\coth(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x)),x)
```

```
[Out] x*atanh(coth(a + b*x)) - (b*x^2)/2
```

3.278 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$bx-\left(bx-\tanh^{-1}(\coth(a+bx))\right)\log(x)$$

```
[Out] b*x-(b*x-arctanh(coth(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$bx-\log(x)\left(bx-\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTanh[Coth[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\coth(a+bx))}{x}\,dx &= bx-\left(bx-\tanh^{-1}(\coth(a+bx))\right)\int \frac{1}{x}\,dx\\ &= bx-\left(bx-\tanh^{-1}(\coth(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$bx+\left(-bx+\tanh^{-1}(\coth(a+bx))\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x,x]
[Out] b*x + (-(b*x) + ArcTanh[Coth[a + b*x]])*Log[x]
```

Maple [A]
time = 0.08, size = 23, normalized size = 1.10

method	result
default	$\ln(x)\operatorname{arctanh}(\coth(bx+a))-b(x\ln(x)-x)$
risch	$\ln(x)\ln\left(e^{bx+a}\right)-\ln(x)xb+bx+\frac{i\ln(x)\pi\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)^2}{4}+\frac{i\pi\ln(x)\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arctanh(coth(b*x+a))-b*(x*ln(x)-x)
```

Maxima [A]
time = 0.26, size = 34, normalized size = 1.62

$$-b\left(x+\frac{a}{b}\right)\log(x)+b\left(x+\frac{a\log(x)}{b}\right)+\operatorname{artanh}(\coth(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="maxima")
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(coth(b*x + a))*log(x)
```

Fricas [A]
time = 0.35, size = 8, normalized size = 0.38

$$bx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="fricas")
[Out] b*x + a*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}(\coth(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x,x)
```

[Out] Integral(atanh(coth(a + b*x))/x, x)

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 15, normalized size = 0.71

$$bx+\frac{1}{2}\left(i\,\pi+2\,a\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="giac")

[Out] b*x + 1/2*(I*pi + 2*a)*log(x)

Mupad [B]
time = 1.20, size = 59, normalized size = 2.81

$$b\,x-\ln\left(x\right)\,\left(\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}+b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh(coth(a + b*x))/x,x)

[Out] b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)

3.279 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2}\,dx$

Optimal. Leaf size=17

$$-\frac{\tanh^{-1}(\coth(a+bx))}{x}+b\log(x)$$

[Out] -arctanh(coth(b*x+a))/x+b*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 29}

$$b\log(x)-\frac{\tanh^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[Coth[a + b*x]]/x^2,x]

[Out] -(ArcTanh[Coth[a + b*x]]/x) + b*Log[x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2}\,dx &= -\frac{\tanh^{-1}(\coth(a+bx))}{x}+b\int \frac{1}{x}\,dx\\ &= -\frac{\tanh^{-1}(\coth(a+bx))}{x}+b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.06

$$b-\frac{\tanh^{-1}(\coth(a+bx))}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^2,x]
[Out] b - ArcTanh[Coth[a + b*x]]/x + b*Log[x]
```

Maple [A]
time = 0.09, size = 18, normalized size = 1.06

method	result
default	$-\frac{\operatorname{arctanh}(\coth(bx+a))}{x}+b\ln\left(x\right)$
risch	$-\frac{\ln\left(e^{bx+a}\right)}{x}+\frac{-2i\pi\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)^{2}+i\pi\operatorname{csgn}\left(ie^{bx+a}\right)^{2}\operatorname{csgn}\left(ie^{2bx+2a}\right)+i\pi\operatorname{csgn}\left(ie^{2bx+2a}\right)^{3}+2i\pi+i\pi\operatorname{csgn}\left(ie^{2bx+2a}\right)\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^2,x,method=_RETURNVERBOSE)
[Out] -arctanh(coth(b*x+a))/x+b*ln(x)
```

Maxima [A]
time = 0.30, size = 17, normalized size = 1.00

$$b\log\left(x\right)-\frac{\operatorname{artanh}\left(\coth\left(bx+a\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="maxima")
[Out] b*log(x) - arctanh(coth(b*x + a))/x
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.76

$$\frac{bx\log\left(x\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="fricas")
[Out] (b*x*log(x) - a)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 3.96, size = 68, normalized size = 4.00

$$\begin{cases} -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{x} & \text{for}\: a=\log\left(-e^{-bx}\right) \\ -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{x} & \text{for}\: a=\log\left(e^{-bx}\right) \\ b\log\left(x\right)-\frac{\operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**2,x)
```

```
[Out] Piecewise((-atanh(coth(b*x + log(-exp(-b*x))))/x, Eq(a, log(-exp(-b*x)))),
(-atanh(coth(b*x + log(exp(-b*x))))/x, Eq(a, log(exp(-b*x)))), (b*log(x) -
atanh(1/tanh(a + b*x))/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(17) = 34$.
time = 0.40, size = 70, normalized size = 4.12

$$b\log\left(|x|\right) - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

Mupad [B]
time = 0.08, size = 17, normalized size = 1.00

$$b\,\ln\left(x\right) - \frac{\mathrm{atanh}(\coth(a+b\,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x))/x^2,x)
```

```
[Out] b*log(x) - atanh(coth(a + b*x))/x
```

3.280 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^3} \, dx$

Optimal. Leaf size=23

$$-\frac{b}{2x}-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}$$

```
[Out] -1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -1/2*b/x - ArcTanh[Coth[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\coth(a+bx))}{x^3} \, dx &= -\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2} \, dx \\ &= -\frac{b}{2x}-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$-\frac{bx+\tanh^{-1}(\coth(a+bx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -1/2*(b*x + ArcTanh[Coth[a + b*x]])/x^2
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\coth(bx+a))}{2x^2}$
risch	$-\frac{\ln(e^{bx+a})}{2x^2}-\frac{4bx+2i\pi\operatorname{csgn}(ie^{bx+a})\operatorname{csgn}(ie^{2bx+2a})^2-i\pi\operatorname{csgn}(ie^{bx+a})^2\operatorname{csgn}(ie^{2bx+2a})+2i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)^2-i\pi\operatorname{csgn}(ie^{2bx+2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{b}{2\,x}-\frac{\operatorname{artanh}\left(\coth\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arctanh(coth(b*x + a))/x^2
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(19) = 38$.
time = 7.73, size = 80, normalized size = 3.48

$$\begin{cases} -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a = \log\left(-e^{-bx}\right) \\ -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a = \log\left(e^{-bx}\right) \\ -\frac{b}{2x} - \frac{\operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**3,x)
```

```
[Out] Piecewise((-atanh(coth(b*x + log(-exp(-b*x))))/(2*x**2), Eq(a, log(-exp(-b*
x)))), (-atanh(coth(b*x + log(exp(-b*x))))/(2*x**2), Eq(a, log(exp(-b*x))))
, (-b/(2*x) - atanh(1/tanh(a + b*x))/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.40, size = 71, normalized size = 3.09

$$-\frac{b}{2\,x} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

Mupad [B]
time = 1.07, size = 16, normalized size = 0.70

$$-\frac{\operatorname{atanh}(\coth(a+b\,x))+b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x))/x^3,x)
```

```
[Out] -(atanh(coth(a + b*x)) + b*x)/(2*x^2)
```

3.281 $\int \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=27

$$-2x\tanh^{-1}(e^x)+x\tanh^{-1}(\cosh(x))-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x)$$

```
[Out] -2*x*arctanh(exp(x))+x*arctanh(cosh(x))-polylog(2,-exp(x))+polylog(2,exp(x)
)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6406, 4267, 2317, 2438}

$$-\text{Li}_2(-e^x)+\text{Li}_2(e^x)-2x\tanh^{-1}(e^x)+x\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cosh[x]],x]
```

```
[Out] -2*x*ArcTanh[E^x] + x*ArcTanh[Cosh[x]] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \tanh^{-1}(\cosh(x))\,dx &= x\tanh^{-1}(\cosh(x)) + \int x\text{csch}(x)\,dx \\
&= -2x\tanh^{-1}(e^x) + x\tanh^{-1}(\cosh(x)) - \int \log(1-e^x)\,dx + \int \log(1+e^x)\,dx \\
&= -2x\tanh^{-1}(e^x) + x\tanh^{-1}(\cosh(x)) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right) + \text{Subst}\Bigg(\int
\\
&= -2x\tanh^{-1}(e^x) + x\tanh^{-1}(\cosh(x)) - \text{Li}_2(-e^x) + \text{Li}_2(e^x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.74

$$x\tanh^{-1}(\cosh(x)) + x\left(\log\left(1-e^{-x}\right) - \log\left(1+e^{-x}\right)\right) + \text{PolyLog}\left(2, -e^{-x}\right) - \text{PolyLog}\left(2, e^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cosh[x]],x]
```

```
[Out] x*ArcTanh[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^
(-x)] - PolyLog[2, E^(-x)]
```

Maple [A]
time = 0.22, size = 21, normalized size = 0.78

method	result
default	$x\,\text{arctanh}\left(\cosh\left(x\right)\right)+2\,\text{dilog}\left(\mathrm{e}^{-x}\right)-\frac{\text{dilog}\left(\mathrm{e}^{-2x}\right)}{2}$
risch	$-\frac{ix\pi\,\text{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)^{3}}{4}-\frac{ix\pi\,\text{csgn}\left(i\left(\mathrm{e}^{x}+1\right)^{2}\right)\text{csgn}\left(i\mathrm{e}^{-x}\right)\text{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)}{4}+\frac{ix\pi\,\text{csgn}\left(i\left(\mathrm{e}^{x}+1\right)^{2}\right)\text{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)^{2}}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctanh(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.22

$$x\,\text{artanh}\left(\cosh\left(x\right)\right) - x\log\left(e^x+1\right) + x\log\left(-e^x+1\right) - \text{Li}_2(-e^x) + \text{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] x*arctanh(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog
(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.
time = 0.45, size = 58, normalized size = 2.15

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - x\log(\cosh(x)+\sinh(x)+1) + x\log(-\cosh(x)-\sinh(x)+1) + \mathrm{Li}_2(\cosh(x)+\sinh(x)) - \mathrm{Li}_2(-\cosh(x)-\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*
log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - s
inh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cosh(x)),x)
```

```
[Out] Integral(atanh(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cosh(x)),x)
```

```
[Out] int(atanh(cosh(x)), x)
```

3.282 $\int x \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=51

$$-x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e$$

```
[Out] -x^2*arctanh(exp(x))+1/2*x^2*arctanh(cosh(x))-x*polylog(2,-exp(x))+x*polylo
g(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6408, 4267, 2611, 2320, 6724}

$$-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Li}_3(-e^x)-\text{Li}_3(e^x)+x^2\left(-\tanh^{-1}\left(e^x\right)\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Cosh[x]],x]
```

```
[Out] -(x^2*ArcTanh[E^x]) + (x^2*ArcTanh[Cosh[x]])/2 - x*PolyLog[2, -E^x] + x*Pol
yLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\cosh(x))\,dx &= \frac{1}{2}x^2\tanh^{-1}(\cosh(x))+\frac{1}{2}\int x^2\text{csch}(x)\,dx\\ &= -x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))-\int x\log\left(1-e^x\right)\,dx+\int x\log\left(1+e^x\right.\\ &= -x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\int\text{Li}_2(-e^x)\\ &= -x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Subst}\Bigg(\int\frac{\text{Li}}{}\\ &= -x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Li}_3(-e^x)-\text{L}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.59

$$\frac{1}{2}\left(x^2\tanh^{-1}(\cosh(x))+x^2\log\left(1-e^{-x}\right)-x^2\log\left(1+e^{-x}\right)+2x\text{PolyLog}\left(2,-e^{-x}\right)-2x\text{PolyLog}\left(2,e^{-x}\right)+2\text{PolyLog}\left(3,-e^{-x}\right)-2\text{PolyLog}\left(3,e^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Cosh[x]],x]
```

```
[Out] (x^2*ArcTanh[Cosh[x]] + x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*Pol
yLog[2, -E^(-x)] - 2*x*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyL
og[3, E^(-x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 479, normalized size = 9.39

method	result
risch	$\frac{i\pi \operatorname{csgn}(i(\mathrm{e}^x-1))^2 \operatorname{csgn}\left(i(\mathrm{e}^x-1)^2\right) x^2}{8}-\frac{i\pi \operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^x+1)^2\right)^3 x^2}{8}-\frac{i\pi \operatorname{csgn}(i(\mathrm{e}^x-1)) \operatorname{csgn}\left(i(\mathrm{e}^x-1)^2\right)^2 x^2}{4}+\frac{i\pi \operatorname{csgn}\left(i(\mathrm{e}^x-1)^2\right) \mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)*x^2-1/8*I*Pi*csgn(I*exp(
-x)*(exp(x)+1)^2)^3*x^2-1/4*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*
x^2+1/8*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)
^2)*x^2+1/4*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^2+x*polylog(2,
exp(x))-x*polylog(2,-exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))+1/2*x^2*l
n(1-exp(x))-1/2*x^2*ln(exp(x)-1)+1/4*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^
2-1/8*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2
)*x^2-1/8*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)+1)^2)*x^2-1/8*I*Pi*csgn(
I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*csgn(I*exp(-x)*(exp(
x)-1)^2)^3*x^2-1/4*I*Pi*x^2-1/8*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^2+1/8*I*Pi*cs
gn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2-1/8*I*Pi*csgn(I*(exp(
x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2+1/8*I*Pi*csgn(I*(exp(x)-1)^2)^3
*x^2+1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2
```

Maxima [A]
time = 0.30, size = 56, normalized size = 1.10

$$\frac{1}{2}\,x^2\operatorname{artanh}\left(\cosh\left(x\right)\right)-\frac{1}{2}\,x^2\log\left(e^x+1\right)+\frac{1}{2}\,x^2\log\left(-e^x+1\right)-x\mathrm{Li}_2(-e^x)+x\mathrm{Li}_2(e^x)+\mathrm{Li}_3(-e^x)-\mathrm{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) - x
*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(42) = 84.
time = 0.42, size = 88, normalized size = 1.73

$$\frac{1}{4}\,x^2\log\left(-\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-\frac{1}{2}\,x^2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{2}\,x^2\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+x\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-x\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))-\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x*
```

```
dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cos
h(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(cosh(x)),x)
[Out] Integral(x*atanh(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="giac")
[Out] integrate(x*arctanh(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atanh}(\cosh(x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(cosh(x)),x)
[Out] int(x*atanh(cosh(x)), x)
```

3.283 $\int x^2 \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=77

$$-\frac{2}{3}x^3\tanh^{-1}(e^x)+\frac{1}{3}x^3\tanh^{-1}(\cosh(x))-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{Pol}$$

```
[Out] -2/3*x^3*arctanh(exp(x))+1/3*x^3*arctanh(cosh(x))-x^2*polylog(2,-exp(x))+x^
2*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog(
4,-exp(x))+2*polylog(4,exp(x))
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6408, 4267, 2611, 6744, 2320, 6724}

$$-x^2\text{Li}_2(-e^x)+x^2\text{Li}_2(e^x)+2x\text{Li}_3(-e^x)-2x\text{Li}_3(e^x)-2\text{Li}_4(-e^x)+2\text{Li}_4(e^x)-\frac{2}{3}x^3\tanh^{-1}(e^x)+\frac{1}{3}x^3\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Cosh[x]],x]
```

```
[Out] (-2*x^3*ArcTanh[E^x])/3 + (x^3*ArcTanh[Cosh[x]])/3 - x^2*PolyLog[2, -E^x] +
 x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyL
og[4, -E^x] + 2*PolyLog[4, E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}(\cosh(x))\,dx &= \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) + \frac{1}{3}\int x^3\operatorname{csch}(x)\,dx \\
&= -\frac{2}{3}x^3\tanh^{-1}\left(e^x\right) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - \int x^2\log\left(1-e^x\right)\,dx + \int x^2\log\left(1\right. \\
&= -\frac{2}{3}x^3\tanh^{-1}\left(e^x\right) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2\int x\text{Li}_2 \\
&= -\frac{2}{3}x^3\tanh^{-1}\left(e^x\right) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(- \\
&= -\frac{2}{3}x^3\tanh^{-1}\left(e^x\right) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(- \\
&= -\frac{2}{3}x^3\tanh^{-1}\left(e^x\right) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(-
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 109, normalized size = 1.42

$$\frac{1}{24}\left(\pi^4 - 2x^4 + 8x^3\tanh^{-1}(\cosh(x)) - 8x^3\log\left(1+e^{-x}\right) + 8x^3\log\left(1-e^x\right) + 24x^2\text{PolyLog}(2,-e^{-x}) + 24x^2\text{PolyLog}(2,e^x) + 48x\text{PolyLog}(3,-e^{-x}) - 48x\text{PolyLog}(3,e^x) + 48\text{PolyLog}(4,-e^{-x}) + 48\text{PolyLog}(4,e^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Cosh[x]],x]
```

```
[Out] (Pi^4 - 2*x^4 + 8*x^3*ArcTanh[Cosh[x]] - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[
1 - E^x] + 24*x^2*PolyLog[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyL
og[3, -E^(-x)] - 48*x*PolyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog
[4, E^x])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 501, normalized size = 6.51

method	result
risch	$-\frac{i\pi \,\mathrm{csgn}\left(i \mathrm{e}^{-x}\right) \mathrm{csgn}\left(i \mathrm{e}^{-x} \left(\mathrm{e}^{x}-1\right)^{2}\right)^{2} x^{3}}{12}-\frac{i\pi \,\mathrm{csgn}\left(i \left(\mathrm{e}^{x}+1\right)^{2}\right) \mathrm{csgn}\left(i \mathrm{e}^{-x}\right) \mathrm{csgn}\left(i \mathrm{e}^{-x} \left(\mathrm{e}^{x}+1\right)^{2}\right) x^{3}}{12}+2 x \,\mathrm{polylog}\left(3, -\mathrm{e}^{x}\right)-2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3-1/12*I*Pi*csg
n(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^3+2*x*poly
log(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog(4,-exp(x))+2*polylog(4,exp(x
))+x^2*polylog(2,exp(x))-x^2*polylog(2,-exp(x))+1/3*x^3*ln(1-exp(x))-1/3*x^
3*ln(exp(x)-1)-1/12*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)+1)^2)*x^3-1/12
*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)-1))^2*csg
n(I*(exp(x)-1)^2)*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x^
3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3-1/6*I*Pi*x^3+1/6*I*Pi*csgn(I
*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*
(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csgn(I*e
xp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3+1/6*I*Pi*csgn(I*(exp(x)+1))*csgn
(I*(exp(x)+1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(
I*exp(-x)*(exp(x)-1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(
exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3
```

Maxima [A]
time = 0.30, size = 78, normalized size = 1.01

$$\frac{1}{3}\, x^{3} \operatorname{artanh}\left(\cosh\left(x\right)\right) - \frac{1}{3}\, x^{3} \log\left(e^{x} + 1\right) + \frac{1}{3}\, x^{3} \log\left(-e^{x} + 1\right) - x^{2} \mathrm{Li}_{2}(-e^{x}) + x^{2} \mathrm{Li}_{2}(e^{x}) + 2\, x \mathrm{Li}_{3}(-e^{x}) - 2\, x \mathrm{Li}_{3}(e^{x}) - 2\, \mathrm{Li}_{4}(-e^{x}) + 2\, \mathrm{Li}_{4}(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

Fricas [A]
time = 0.42, size = 118, normalized size = 1.53

$$\frac{1}{6}\,x^{3}\log\left(-\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-\frac{1}{3}\,x^{3}\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{3}\,x^{3}\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+x^{2}\mathrm{Li}_{2}(\cosh\left(x\right)+\sinh\left(x\right))-x^{2}\mathrm{Li}_{2}(-\cosh\left(x\right)-\sinh\left(x\right))-2\,x\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+2\,x\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))+2\,\mathrm{polylog}(4,\cosh\left(x\right)+\sinh\left(x\right))-2\,\mathrm{polylog}(4,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) -
x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*pol
ylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4
, -cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\,\mathrm{atanh}\,\left(\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(cosh(x)),x)
```

```
[Out] Integral(x**2*atanh(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(cosh(x)),x)
```

```
[Out] int(x^2*atanh(cosh(x)), x)
```

3.284 $\int x^2 \tanh^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=307

$$\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x^2\mathrm{Pol}}{}$$

```
[Out] 1/3*x^3*arctanh(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d)
)/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,-(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A]
time = 0.33, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6378, 2221, 2611, 6744, 2320, 6724}

$$\frac{\mathrm{Li}_4\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\mathrm{Li}_4\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\mathrm{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\mathrm{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\mathrm{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x^2\mathrm{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{6}x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{6}x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+\frac{1}{3}x^3\tanh^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6378

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(
(1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c - d
 + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tanh^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{3}(b(1-c-d))\int\frac{e^{2a+2b}}{1-c+d+(1-}$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

Mathematica [A]
time = 8.59, size = 271, normalized size = 0.88

$$\frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx))+\frac{4b^3x^3\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-4b^3x^3\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)+6b^2x^2\text{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-6b^2x^2\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)-6bx\text{PolyLog}\left(3,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+6bx\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)+3\text{PolyLog}\left(4,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-3\text{PolyLog}\left(4,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(-1 + c - d)] - 4*b^3*x^3*Log[1 + ((1 + c + d)*E^(2*(a + b*x)
))/(1 + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1
 + c - d))] - 6*b^2*x^2*PolyLog[2, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c -
 d))] - 6*b*x*PolyLog[3, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] +
6*b*x*PolyLog[3, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))] + 3*PolyLog[
4, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - 3*PolyLog[4, -(((1 + c
 + d)*E^(2*(a + b*x)))/(1 + c - d))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.97, size = 5366, normalized size = 17.48

method	result	size
risch	Expression too large to display	5366

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.48, size = 281, normalized size = 0.92

$$\frac{1}{3}\,x^{3}\,\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)-\frac{1}{18}\,bd\left(\frac{4\,b^{3}x^{3}\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\mathrm{Li}_{3}(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})+3\,\mathrm{Li}_{4}(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^{4}d}-\frac{4\,b^{3}x^{3}\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)-6\,bx\mathrm{Li}_{3}(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1})+3\,\mathrm{Li}_{4}(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1})}{b^{4}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d +
 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4
*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog
(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^4*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(263) = 526$.
time = 0.39, size = 900, normalized size = 2.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d -
 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c +
d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c
 + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d +
1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(
2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*s
qrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
 + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6
*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + si
nh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(c
```

```
osh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(
b*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(c
osh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1
))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}(c + d \tanh(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, \mathrm{atanh}(c + d \tanh(a + b\, x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*tanh(a + b*x)), x)
```

3.285 $\int x \tanh^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=231

$$\frac{1}{2}x^2 \tanh^{-1}(c+d \tanh(a+bx))+\frac{1}{4}x^2 \log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2 \log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x\mathrm{Po}}{}$$

```
[Out] 1/2*x^2*arctanh(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*ex
p(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-
1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

Rubi [A]

time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6378, 2221, 2611, 2320, 6724}

$$-\frac{\mathrm{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\mathrm{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\mathrm{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\mathrm{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{4}x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6378

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(
(1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c - d
 + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\tanh^{-1}(c+d\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{2}(b(1-c-d))\int\frac{e^{2a+2bx}}{1-c+d+(1-\\
&= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\\
&= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\\
&= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\\
&= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-
\end{aligned}$$

Mathematica [A]
time = 6.88, size = 203, normalized size = 0.88

$$\frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx))+\frac{2b^2x^2\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-2b^2x^2\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)+2bx\text{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-2bx\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)-\text{PolyLog}\left(3,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(-1 + c - d)] - 2*b^2*x^2*Log[1 + ((1 + c + d)*E^(2*(a + b*x)
))/(1 + c - d)] + 2*b*x*PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c
 - d))] - 2*b*x*PolyLog[2, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))] -
PolyLog[3, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] + PolyLog[3, -((
(1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.33, size = 5062, normalized size = 21.91

method	result	size
risch	Expression too large to display	5062

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.48, size = 215, normalized size = 0.93

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+2\,bx\mathrm{Li}_2\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)-\mathrm{Li}_3(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1})}{b^3d}\right)+\frac{1}{2}\,x^2\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*b
*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) +
1/2*x^2*arctanh(d*tanh(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(197) = 394.
time = 0.45, size = 746, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x
+ a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d
 + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a)
 - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*co
sh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)
/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b
*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*l
og(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b
^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d
- 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)
/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d
 + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqr
t(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(c + d \tanh\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*atanh(c + d*tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atanh}(c + d \tanh(a + b\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*tanh(a + b*x)),x)
[Out] int(x*atanh(c + d*tanh(a + b*x)), x)
```

3.286 $\int \tanh^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=150

$$x\tanh^{-1}(c+d\tanh(a+bx))+\frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{2}x\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{\text{PolyLog}\left(\cdots\right.}{\cdots}$$

```
[Out] x*arctanh(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*a)
/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A]

time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6370, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6370

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[x*(E^(2*a + 2*b*x
)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(1 + c + d),
Int[x*(E^(2*a + 2*b*x)/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x))), x], x])
/; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\tanh(a+bx))\,dx = x\tanh^{-1}(c+d\tanh(a+bx)) + (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(1-c-\ldots}$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\ldots$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\ldots$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\ldots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(150) = 300.
time = 4.84, size = 366, normalized size = 2.44

$$x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{(a+bx)\log\left(1-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) + (a+bx)\log\left(1+\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) - (a+bx)\log\left(1-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) - (a+bx)\log\left(1+\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) + a\log\left(1+c-d+e^{2(a+bx)}+ce^{2(a+bx)}+de^{2(a+bx)}\right) - a\log\left(1+d+e^{2(a+bx)}-de^{2(a+bx)}-c\left(1+e^{2(a+bx)}\right)\right) + \text{PolyLog}\left(2,-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) + \text{PolyLog}\left(2,\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) - \text{PolyLog}\left(2,-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) - \text{PolyLog}\left(2,\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + ((a + b*x)*Log[1 - (Sqrt[-1 + c + d]*E^(a
+ b*x))/Sqrt[1 - c + d]] + (a + b*x)*Log[1 + (Sqrt[-1 + c + d]*E^(a + b*x))
/Sqrt[1 - c + d]] - (a + b*x)*Log[1 - (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1
 - c + d]] - (a + b*x)*Log[1 + (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1 - c +
d]] + a*Log[1 + c - d + E^(2*(a + b*x)) + c*E^(2*(a + b*x)) + d*E^(2*(a + b
*x))] - a*Log[1 + d + E^(2*(a + b*x)) - d*E^(2*(a + b*x)) - c*(1 + E^(2*(a
+ b*x)))] + PolyLog[2, -((Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[1 - c + d])] +
 PolyLog[2, (Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[1 - c + d]] - PolyLog[2, -(
(Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1 - c + d])] - PolyLog[2, (Sqrt[1 + c +
 d]*E^(a + b*x))/Sqrt[-1 - c + d]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(138) = 276.
time = 0.91, size = 361, normalized size = 2.41

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)-c-}{1-c+d}\right.}{2d}\right.}{}$
default	$-\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)-c-}{1-c+d}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arctanh(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arctanh(c+d
*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*d^2*(1/2/d*dilog((-d*tanh(b*x+a)-c
+1)/(1-c+d))+1/2/d*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c+1)/(1-c+d))-1/
2/d*dilog((-d*tanh(b*x+a)-c-1)/(-1-c+d))-1/2/d*ln(-d*tanh(b*x+a)-d)*ln((-d*
tanh(b*x+a)-c-1)/(-1-c+d))+1/2/d*dilog((-d*tanh(b*x+a)-c-1)/(-1-c-d))+1/2/d
*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c-1)/(-1-c-d))-1/2/d*dilog((-d*tan
h(b*x+a)-c+1)/(1-c-d))-1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c+1)/(
1-c-d))))
```

Maxima [A]
time = 0.48, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right)+x\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(-
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(128) = 256$.
time = 0.40, size = 552, normalized size = 3.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
 a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c
 + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(
-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2
*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sq
rt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-
sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog
(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-s
qrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt
(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c+d\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a)),x)
[Out] Integral(atanh(c + d*tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctanh(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(c+d\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tanh(a + b*x)),x)
[Out] int(atanh(c + d*tanh(a + b*x)), x)
```

3.287 $\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 12.32, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tanh(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*tanh(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(c + d\tanh(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tanh(a + b*x))/x,x)
[Out] int(atanh(c + d*tanh(a + b*x))/x, x)
```

3.288 $\int x^3 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\frac{3}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arctanh(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,-(1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6374, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left(-\left((d+1)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{8}x^4\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{4}x^4\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1+d)e^{2a+2bx}}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}(b(1+d))\int \frac{}{1+}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+\right.$$

Mathematica [**A**]
time = 3.60, size = 144, normalized size = 0.93

$$\frac{1}{16}\left(4x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - 2x^4\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Tanh[a + b*x]] - 2*x^4*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x^2*P
olyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((1 +
d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x))))])
/b^4)/16
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.28, size = 1769, normalized size = 11.41

method	result	size
risch	Expression too large to display	1769

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/16*I*x^
4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))
*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-1/8*I*x^4*P
i*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+1/20*b*x^5-1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/
16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*
(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi-1/16*I*x^4*Pi*csgn(I*d*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^3+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))+1
/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-exp
(b*x+a)*(-1-d)^(1/2))*x-1/2/b^3*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x-1/4/
b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,-(
1+d)*exp(2*b*x+2*a))*x^2-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*
a)+1)+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1+(1+d)*
exp(2*b*x+2*a))*x^4-1/8*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+1/2/b^4*a^3/
(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)
*(-1-d)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^
4/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-1
-d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))-1/4/b^4*a^3/(1+d)*p
olylog(2,-(1+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a)
)*x^3+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*poly
log(4,-(1+d)*exp(2*b*x+2*a))*x-3/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2
*a))*x-1/4/b^4*d*a^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))-3/8/b^4*d*a^4/(
1+d)*ln(1+(1+d)*exp(2*b*x+2*a))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(-1-
d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/2/b^4*d*a^
4/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^
3-1/2/b^3*d*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1
+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2
))*x-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+
1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x
+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/8*I*x^4*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/4*x^4*ln(exp
(b*x+a))-1/8*x^4*ln(d)
```

Maxima [A]
time = 0.73, size = 149, normalized size = 0.96

$$\frac{1}{4}\,x^4\,\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3\left(-(d+1)e^{(2\,bx+2\,a)}\right)+6\,bx\mathrm{Li}_4\left(-(d+1)e^{(2\,bx+2\,a)}\right)-3\,\mathrm{Li}_5\left(-(d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d + 1)
*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(135) = 270$.
time = 0.41, size = 451, normalized size = 2.91

$$\frac{2\,b^5x^5+5\,b^4x^4\log\left(-\frac{(d+2)\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}\right)-20\,b^3x^3\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-20\,b^3x^3\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\,a^4\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+\sqrt{-4\,d-4}\right)-5\,a^4\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-\sqrt{-4\,d-4}\right)+60\,b^2x^2\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+60\,b^2x^2\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\,(b^4x^4-a^4)\log\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-5\,(b^4x^4-a^4)\log\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+120\,\mathrm{polylog}\left(5,\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+120\,\mathrm{polylog}\left(5,-\frac{1}{2}\,\sqrt{-4\,d-4}\,\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{40\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*
(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**3*atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atanh}(d + d \tanh(a + b x) + 1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x^3*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.289 $\int x^2 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=128

$$\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\frac{x$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctanh(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6374, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{3}x^3\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1+d)e^{2a+2bx}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{3}(b(1+d))\int \frac{}{1+}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

Mathematica [A]
time = 4.04, size = 118, normalized size = 0.92

$$\frac{1}{24}\left(8x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - 4x^3\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Tanh[a + b*x]] - 4*x^3*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x*Pol
yLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((1 + d)*E
^(2*(a + b*x))))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 1710, normalized size = 13.36

method	result	size
risch	Expression too large to display	1710

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/3*x^3*ln(exp(b*
x+a))+1/12*b*x^4+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+
```

```
1/6*I*x^3*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/3/b^3*d/(1+d)*
ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/6*I*Pi*x^3+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(
2*b*x+2*a))*x*a^2-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^
3*a^2/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/6*ln(exp(2*b*x+2*a)*d+exp(2*
b*x+2*a)+1)*x^3-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x-1/2/b^2
*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x-1/12*I*x^3*Pi*csgn(I*d*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a)
)*x^2+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*
polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-1-d)
^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x-1/2/b^3*d*a^3/(
1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-1-
d)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^3*d*a^
2/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x
+2*a))*x*a^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*
x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d
)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/6*d/(1+d)*ln(1+
(1+d)*exp(2*b*x+2*a))*x^3+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/
b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,-(1+d)
*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b
^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a
)*(-1-d)^(1/2))-1/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/12*I*x^3
*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^3+1/12*I
*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6/(1+d)
*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*
a))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp
(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(
I*exp(2*b*x+2*a))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-
1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*
I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)
+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*x^3*ln(d)
```

Maxima [A]
time = 0.72, size = 125, normalized size = 0.98

$$\frac{1}{3}\,x^{3}\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{36}\,\left(\frac{3\,x^{4}}{d}-\frac{2\,\left(4\,b^{3}x^{3}\log\left(\left(d+1\right)e^{\left(2\,bx+2\,a\right)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_2(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)})-6\,bx\mathrm{Li}_3(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)})+3\,\mathrm{Li}_4(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)})\right)}{b^{4}d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2*b*
x + 2*a)))/(b^4*d))*b*d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(111) = 222$.
time = 0.39, size = 382, normalized size = 2.98

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a) + 2\,(d+1)\sinh(bx+a) + \sqrt{-4\,d-4}\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a) + 2\,(d+1)\sinh(bx+a) - \sqrt{-4\,d-4}\right) + 12\,bx\,\mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 12\,bx\,\mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 2\,(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 12\,\mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 12\,\mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}(d + d \tanh(a + b x) + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.290 $\int x \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=101

$$\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\frac{\text{Po}}{}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctanh(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*b*
x+2*a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6374, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\tanh^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1+d)e^{2a+2bx}}\,dx \\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}(b(1+d))\int\frac{e}{1+(1} \\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e\right. \\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e\right. \\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e\right. \\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e\right.
\end{aligned}
$$

Mathematica [A]

time = 3.61, size = 91, normalized size = 0.90

$$\frac{2b^2x^2\left(2\tanh^{-1}(1+d+d\tanh(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[3,
 -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 1627, normalized size = 16.11

method	result	size
risch	Expression too large to display	1627

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)
)^3+1/4*I*x^2*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4/b^2/(1+d
)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a
))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*ln(
1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))+
1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/4*I*Pi
*x^2+1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog
(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2)
)-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(
2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/2/b*
d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(-1-
d)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x-1/8*I*x^2*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*
x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x
^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8
/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*
x+2*a))*x*a+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*d*a/
(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2
```

```
*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a-1/4/b^2*a^2
/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^3+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*x^2*Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(e
xp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-1/4/b^2*d/(1+d)*ln(1+
(1+d)*exp(2*b*x+2*a))*a^2+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2
/b*a/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x
+a)*(-1-d)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))-1/8*I*x
^2*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*I*x^2*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2-1/4*x^2*ln(d)+1/4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)*x^2
```

Maxima [A]
time = 0.72, size = 101, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4x^3}{d}-\frac{3\left(2b^2x^2\log\left((d+1)e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2(-(d+1)e^{(2bx+2a)})-\mathrm{Li}_3(-(d+1)e^{(2bx+2a)})\right)}{b^3d}\right)bd+\frac{1}{2}x^2\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d + 1/2*x^2*arctanh(d*tanh(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(87) = 174$.
time = 0.45, size = 323, normalized size = 3.20

$$\frac{2b^3x^3+3b^2x^2\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)+\sqrt{-4d-4}\right)-3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)-\sqrt{-4d-4}\right)-3(b^2x^2-a^2)\log\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3(b^2x^2-a^2)\log\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(d \tanh\left(a+bx\right)+d+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*tanh(b*x+a)),x)
[Out] Integral(x*atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctanh(d*tanh(b*x + a) + d + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}(d+d \tanh(a+b\,x)+1)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d + d*tanh(a + b*x) + 1),x)
[Out] int(x*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.291 $\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=69

$$\frac{bx^2}{2}+x\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arctanh(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6366, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6366

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = x\tanh^{-1}(1+d+d\tanh(a+bx))+b\int \frac{x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\tanh(a+bx))-(b(1+d))\int \frac{e^{2a+2}}{1+(1+d}$$
$$= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+}\right.$$
$$= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+}\right.$$
$$= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(69) = 138.
time = 3.53, size = 201, normalized size = 2.91

$$x\tanh^{-1}(1+d+d\tanh(a+bx))+\frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-1-d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{-1-d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}+(1+d)e^{a+bx}\right)-2bx\log((2+d)\cosh(a+bx)+d\sinh(a+bx))-2\mathrm{PolyLog}\left(2,-\sqrt{-1-d}\,e^{a+bx}\right)-2\mathrm{PolyLog}\left(2,\sqrt{-1-d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*
E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
og[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)]
)/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(61) = 122.
time = 0.49, size = 265, normalized size = 3.84

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(-\frac{-d\tanh(bx+}{2d}\right.}{2d}\right.}{}$
default	$-\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(-\frac{-d\tanh(bx+}{2d}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arctanh(1+d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arctanh(1
+d+d*tanh(b*x+a))*d*ln(d*tanh(b*x+a)+d)-1/2*d^2*(1/2/d*dilog(-1/2*(-d*tanh(
b*x+a)-d)/d)+1/2/d*ln(-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2/d
*dilog((-d*tanh(b*x+a)-d-2)/(-2*d-2))-1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tan
h(b*x+a)-d-2)/(-2*d-2))+1/2/d*dilog(1/2*d*tanh(b*x+a)+1/2*d+1)+1/2/d*ln(d*t
anh(b*x+a)+d)*ln(1/2*d*tanh(b*x+a)+1/2*d+1)-1/4/d*ln(d*tanh(b*x+a)+d)^2))
```

Maxima [A]
time = 0.76, size = 72, normalized size = 1.04

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(60) = 120$.
time = 0.45, size = 239, normalized size = 3.46

$$\frac{b^2x^2+bx\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+\sqrt{-4\,d-4}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-\sqrt{-4\,d-4}\right)-(bx+a)\log\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
```

```
nh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a)),x)
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}(d+d\tanh(a+b\,x)+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*tanh(a + b*x) + 1),x)
[Out] int(atanh(d + d*tanh(a + b*x) + 1), x)
```

3.292 $\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arctanh(1+d+d*tanh(b*x+a))/x,x)

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]

[Out] Defer[Int][ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.59, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]

[Out] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(d + d\tanh(a + b\,x) + 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*tanh(a + b*x) + 1)/x,x)
[Out] int(atanh(d + d*tanh(a + b*x) + 1)/x, x)
```

3.293 $\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=168

$$\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{3}{}-$$

```
[Out] 1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d)*
exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog
(5,-(1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6374, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left(-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{8}x^4\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{4}x^4\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1-d)e^{2a+2bx}}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}(b(1-d))\int$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-\right.$$

Mathematica [**A**]
time = 3.17, size = 144, normalized size = 0.86

$$\frac{1}{16}\left(4x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - 2x^4\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{4x^3\mathrm{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x^2\mathrm{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{6x\mathrm{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3} + \frac{3\mathrm{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Tanh[a + b*x]] - 2*x^4*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Po
lyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((-1 + d)*E
^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/b^4)/1
6
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.28, size = 1773, normalized size = 10.55

method	result	size
risch	Expression too large to display	1773

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1
/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*x^4*
Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+1/16*I*x^4*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/20*b*x^5
-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/8/b^
4*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I/(exp
(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3+1/16*I*x^4*Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2
*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-3/16/b^4/(d-1)*po
lylog(5,(d-1)*exp(2*b*x+2*a))+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-1/4/
b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-1/8/b^4*d*a^4/(d-1)*ln(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/2/b^3*d*a^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))
*x+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*l
n(1+exp(b*x+a)*(d-1)^(1/2))*x+1/8*I*x^4*Pi+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2
*a))^3+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*
polylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(
1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*a^3/(d-1)*
ln(1-(d-1)*exp(2*b*x+2*a))*x-3/8/b^4*d*a^4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))
+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1+
exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))
+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/16*I*x^4*Pi*csgn(I*d
)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3+3/8/b^4*a^4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))
+1/4/b^4*a^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))+1/4/b/(d-1)*polylog(2,(d
-1)*exp(2*b*x+2*a))*x^3-3/8/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2+3
/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x+3/16/b^4*d/(d-1)*polylog(5,(
d-1)*exp(2*b*x+2*a))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/
b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/8*d/(d-1)*ln(1-(d-1)*exp(2*
b*x+2*a))*x^4-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^4*a^4/(d
-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^4*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+
2*a))*a^3-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)-1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/4*x^4*ln(exp(b
*x+a))-1/8*x^4*ln(d)
```

Maxima [A]

time = 0.72, size = 146, normalized size = 0.87

$$-\frac{1}{4}\,x^4\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)+6\,bx\mathrm{Li}_4\left((d-1)e^{(2\,bx+2\,a)}\right)-3\,\mathrm{Li}_5\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d - 1)*
e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(132) = 264$.
time = 0.37, size = 424, normalized size = 2.52

$$\frac{2\,b^5x^5 - 5\,b^4x^4\log\left(-\frac{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}{(d-2)\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 20\,b^3x^3\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 5\,a^4\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d-1}\right) - 5\,a^4\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-2\,\sqrt{d-1}\right) + 60\,b^2x^2\mathrm{polylog}\left(3,\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) + 60\,b^2x^2\mathrm{polylog}\left(3,-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 120\,bx\mathrm{polylog}\left(4,\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 120\,bx\mathrm{polylog}\left(4,-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 5\,(b^4x^4-a^4)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) - 5\,(b^4x^4-a^4)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) + 120\,\mathrm{polylog}\left(5,\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) + 120\,\mathrm{polylog}\left(5,-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{40\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^3 \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x**3*atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*tanh(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3 \operatorname{atanh}(d + d \tanh(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x^3*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.294 $\int x^2 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=139

$$\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{x}{}$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*ex
p(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6374, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{3}x^3\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1-d)e^{2a+2bx}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{3}(b(1-d))\int \frac{}{1+}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1-\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1-\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1-\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1-\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1-\right.$$

Mathematica [A]
time = 4.13, size = 119, normalized size = 0.86

$$\frac{1}{24}\left(8x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Tanh[a + b*x]] - 4*x^3*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x*Poly
Log[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((-1 + d)*E^(2*
(a + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 1716, normalized size = 12.35

method	result	size
risch	Expression too large to display	1716

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
)^2+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+e
```

```
xp(b*x+a)*(d-1)^(1/2))-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d
/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)
*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/3/b^3/(d-
1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a
))*x^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*poly
log(3,(d-1)*exp(2*b*x+2*a))*x-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2-1/3*x^3*ln(exp(b*x+a))+1/12*b*x^4-1/12*I*x^3*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I
*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a
)-1))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(e
xp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/6/b^3*a^3/(d-1)*l
n(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d
-1)^(1/2))*x-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/6/b^3*d*a
^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/6*I*x^3*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x
+2*a)*d-exp(2*b*x+2*a)-1))+1/6/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/8/b^3
/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+
a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3
*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2
*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*
d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*
x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/12*I*x^3*P
i*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn
(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))+1/2/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2+1/12*I*x^3*Pi*csgn(I
*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-
1))^3+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d*a^3/(d-1)*
ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2
))+1/12*I*x^3*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/6*I*Pi*x^3
-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^2-1/6*x^3*ln(d)+1/6*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)*x^3
```

Maxima [A]
time = 0.70, size = 123, normalized size = 0.88

$$-\frac{1}{3} x^3 \operatorname{artanh}\left(d \tanh\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3 x^4}{d}-\frac{2\left(4 b^3 x^3 \log\left(-(d-1)e^{(2bx+2a)}+1\right)+6 b^2 x^2 \mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right)-6 bx \mathrm{Li}_3\left((d-1)e^{(2bx+2a)}\right)+3 \mathrm{Li}_4\left((d-1)e^{(2bx+2a)}\right)\right)}{b^4 d}\right) bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*b*x
 + 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.
time = 0.39, size = 360, normalized size = 2.59

$$\frac{b^4x^4 - 2b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 6b^2x^2\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)+2\sqrt{d-1}\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)-2\sqrt{d-1}\right) + 12bx\,\mathrm{polylog}\left(3,\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 12bx\,\mathrm{polylog}\left(3,-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 2\left(b^3x^3+a^3\right)\log\left(\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 2\left(b^3x^3+a^3\right)\log\left(-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 12\,\mathrm{polylog}\left(4,\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 12\,\mathrm{polylog}\left(4,-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2 \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*tanh(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{atanh}(d + d \tanh(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d + d*tanh(a + b*x) - 1),x)
[Out] int(-x^2*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.295 $\int x \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=110

$$\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{\text{Po}}{}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2*b
*x+2*a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6374, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6374

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1-d)e^{2a+2bx}}\,dx$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}(b(1-d))\int\frac{e}{1+(}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e\right.$$

Mathematica [A]

time = 4.08, size = 93, normalized size = 0.85

$$\frac{2b^2x^2\left(2\tanh^{-1}(1-d-d\tanh(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 1635, normalized size = 14.86

method	result	size
risch	Expression too large to display	1635

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3+1/2/b^2*d*a/(d-1)*dilog(1-exp(b*x+a)*(d-1
)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^2*d/(d-1)*
ln(1-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a)
)*x-1/4/b^2*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1-(d-1
)*exp(2*b*x+2*a))*x*a-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/8*I*x^
2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b*d/(d-1)*
ln(1-(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))
*x+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))+1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*po
lylog(3,(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))
*a-1/4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/2/b^2*a^2/(d-1)*ln(1-exp(b*
x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8*I*x^2*
Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2/b*a/(d-1)*ln
(1+exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2
))+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1
-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/
8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*ln(1-(d-1)*exp(
2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d*a^2
/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-
1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
```

```
)+1))^3+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+
2*a)-1))^3+1/4*I*x^2*Pi-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp
(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)-1))-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)-1))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-1/4*x^2*ln(d)+1/4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)*x^2
```

Maxima [A]
time = 0.70, size = 100, normalized size = 0.91

$$\frac{1}{24}\left(\frac{4x^3}{d}-\frac{3\left(2b^2x^2\log\left(-(d-1)e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right)-\mathrm{Li}_3\left((d-1)e^{(2bx+2a)}\right)\right)}{b^3d}\right)bd-\frac{1}{2}x^2\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d - 1/2*x^2*arctanh(d*tanh(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(86) = 172.
time = 0.35, size = 306, normalized size = 2.78

$$\frac{2b^3x^3-3b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)-6bx\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6bx\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3a^2\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)+2\sqrt{d-1}\right)-3a^2\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)-2\sqrt{d-1}\right)-3\left(b^2x^2-a^2\right)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3\left(b^2x^2-a^2\right)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\,\mathrm{polylog}\left(3,\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\,\mathrm{polylog}\left(3,-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqr
t(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2
*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tanh(b*x + a) + d - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{atanh}(d + d \tanh(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.296 $\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=76

$$\frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}$$

```
[Out] 1/2*b*x^2-x*arctanh(-1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6366, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6366

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx &= x\tanh^{-1}(1-d-d\tanh(a+bx))+b\int \frac{x}{1+(1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-(b(1-d))\int \frac{e^{2a+2}}{1+(1-d}\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+}\right.\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(76) = 152.
time = 3.49, size = 200, normalized size = 2.63

$$x\tanh^{-1}(1-d-d\tanh(a+bx))+\frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-1+d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{-1+d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(-1+(-1+d)e^{2(a+bx)}\right)\right)-2bx\log((-2+d)\cosh(a+bx)+d\sinh(a+bx))-2\text{PolyLog}\left(2,-\sqrt{-1+d}\,e^{a+bx}\right)-2\text{PolyLog}\left(2,\sqrt{-1+d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1
+ d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]
] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E
^(a + b*x)])/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(66) = 132.
time = 0.51, size = 299, normalized size = 3.93

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(b}{-2}\right.}{2d}\right.}{}$
default	$-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(b}{-2}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(1/2*arctanh(-1+d+d*tanh(b*x+a))*d*ln(d*tanh(b*x+a)+d)-1/2*arctanh(-
1+d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/2/d*dilog((-d*tanh(b*x
+a)-d+2)/(-2*d+2))+1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(-2*d
+2))-1/2/d*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2/d*ln(-d*tanh(b*x+a)+d)*ln(-
1/2*(-d*tanh(b*x+a)-d)/d)+1/4/d*ln(d*tanh(b*x+a)+d)^2+1/2/d*ln(-1/2*d*tanh(
b*x+a)-1/2*d+1)*ln(1/2*d*tanh(b*x+a)+1/2*d)-1/2/d*ln(-1/2*d*tanh(b*x+a)-1/2
*d+1)*ln(d*tanh(b*x+a)+d)+1/2/d*dilog(1/2*d*tanh(b*x+a)+1/2*d)))
```

Maxima [A]
time = 0.71, size = 73, normalized size = 0.96

$$\frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left(-(d-1)e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left((d-1)e^{(2bx+2a)}\right)}{b^2d}\right)-x\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*tanh(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(61) = 122$.
time = 0.39, size = 228, normalized size = 3.00

$$\frac{b^2x^2-bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d-1}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-2\,\sqrt{d-1}\right)-(bx+a)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-(bx+a)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-\operatorname{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-\operatorname{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
```

```
h(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a)),x)
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\mathrm{atanh}(d+d\tanh(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*tanh(a + b*x) - 1),x)
[Out] int(-atanh(d + d*tanh(a + b*x) - 1), x)
```

3.297 $\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\operatorname{Int}\left(\frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.76, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{atanh}(d + d\tanh(a + b\,x) - 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*tanh(a + b*x) - 1)/x,x)
[Out] int(-atanh(d + d*tanh(a + b*x) - 1)/x, x)
```

3.298 $\int x^2 \tanh^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=303

$$\frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x^2\mathrm{Po}}{}$$

```
[Out] 1/3*x^3*arctanh(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/
b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1-c+d)
)/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A]
time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6380, 2221, 2611, 6744, 2320, 6724}

$$\frac{\mathrm{Li}_4\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\mathrm{Li}_4\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\mathrm{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\mathrm{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\mathrm{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{6}x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+\frac{1}{3}x^3\tanh^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6380

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*
((1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+}}{1-c+d+(-}$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

Mathematica [A]
time = 7.64, size = 265, normalized size = 0.87

$$\frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{1-c+d}\right) - 4b^3x^3\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{-1-c+d}\right) + 6b^2x^2\text{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) - 6bx\text{PolyLog}\left(3,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) + 6bx\text{PolyLog}\left(3,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) + 3\text{PolyLog}\left(4,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 3\text{PolyLog}\left(4,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(1 - c + d)] - 4*b^3*x^3*Log[1 + ((1 + c + d)*E^(2*(a + b*x))
)/(-1 - c + d)] + 6*b^2*x^2*PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 +
 c - d)] - 6*b^2*x^2*PolyLog[2, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]
- 6*b*x*PolyLog[3, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + 6*b*x*Pol
yLog[3, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)] + 3*PolyLog[4, ((-1 + c
+ d)*E^(2*(a + b*x)))/(-1 + c - d)] - 3*PolyLog[4, ((1 + c + d)*E^(2*(a + b
*x)))/(1 + c - d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.25, size = 5294, normalized size = 17.47

method	result	size
risch	Expression too large to display	5294

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.49, size = 277, normalized size = 0.91

$$\frac{1}{3}\,x^{3}\,\operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)-\frac{1}{18}\,bd\left(\frac{4\,b^{3}x^{3}\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)+3\,\mathrm{Li}_4\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^{4}d}-\frac{4\,b^{3}x^{3}\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)-6\,bx\mathrm{Li}_3\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)+3\,\mathrm{Li}_4\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^{4}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d +
1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b
^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(
(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)))/(b^4*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(259) = 518$.
time = 0.51, size = 880, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c -
d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d +
1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d
+ 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*si
nh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
 d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
 + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*pol
ylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*
b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
```

```
 + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt(
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) - 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) +
sinh(b*x + a))))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(c + d\coth\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*coth(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atanh}\left(c+d\,\mathrm{coth}\left(a+b\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*coth(a + b*x)), x)
```

3.299 $\int x \tanh^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=229

$$\frac{1}{2}x^2 \tanh^{-1}(c+d \coth(a+bx))+\frac{1}{4}x^2 \log \left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2 \log \left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x\text{Poly}}{}$$

```
[Out] 1/2*x^2*arctanh(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*exp
(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-1/
8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))/b^2
```

Rubi [A]
time = 0.27, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6380, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2 \log \left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{4}x^2 \log \left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+\frac{1}{2}x^2 \tanh^{-1}(d \coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6380

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*
((1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2}}{1-c+d+(-1}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)$$

Mathematica [A]
time = 6.17, size = 199, normalized size = 0.87

$$\frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{2b^2x^2\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{1-c+d}\right) - 2b^2x^2\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{-1-c+d}\right) + 2bx\text{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 2bx\text{PolyLog}\left(2,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) - \text{PolyLog}\left(3,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) + \text{PolyLog}\left(3,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(1 - c + d)] - 2*b^2*x^2*Log[1 + ((1 + c + d)*E^(2*(a + b*x))
)/(-1 - c + d)] + 2*b*x*PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c -
 d)] - 2*b*x*PolyLog[2, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)] - PolyLo
g[3, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[3, ((1 + c + d)
*E^(2*(a + b*x)))/(1 + c - d)])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.45, size = 4990, normalized size = 21.79

method	result	size
risch	Expression too large to display	4990

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a-1/4/b*c/(1+c+d)*pol
ylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x+1/2/b*a/(1+c+d)*ln((-exp(b*x+a)*c-
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x
+1/2/b*a/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(
b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/4/b*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*
b*x+2*a)/(1+c-d))*x+1/2/b^2*a^2*c/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1
+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+
c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1
+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(
(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b^2*c/(1+c+
d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*c/(1+c+d)*polylog(2,(1+
c+d)*exp(2*b*x+2*a)/(1+c-d))*a-1/4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a
)/(1+c-d))*a^2-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*
a+1/2/b^2*a^2/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))*x-1/4*I*Pi*x^2-1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*c
sgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1))*csgn(I*
((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a
)-1))-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*
b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^3+1/2/b*d*a/(1+c+d)*ln((-exp(b*x+a)*c-exp(b
*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/
b*d*a/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x
+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)
/(1+c-d))*x*a-1/2/b*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/
b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a+1/2/b*d/(c+d-1)*ln(1-(
c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a-1/2/b*c*a/(c+d-1)*ln((-exp(b*x+a)*c-exp(
```

```
b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2
/b*c*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*
x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*
x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b
*d*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+
a))/((c-d-1)*(c+d-1))^(1/2))*x+1/2/b^2*d*a/(1+c+d)*dilog((-exp(b*x+a)*c-exp
(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/
b^2*d*a/(1+c+d)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+ex
p(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*c/(1+c+d)*ln((-exp(b*x+a)*c-
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1
/2/b^2*a*c/(1+c+d)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2
)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*cs
gn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*
x+2*a)-1))+1/2/b*a*c/(1+c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d
))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/(1+c+d)*ln((exp(b
*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^
(1/2))*x+1/2/b^2*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-
1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((exp
(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1)
)^(1/2))-1/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c+
d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(2,
(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a+1/2/b^2*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp
(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/
b^2*a^2/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b
*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b
*x+2*a)/(c-d-1))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1)
)+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1
-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2-1/2/b^2*c*a/(c+d-1)*dilog((-exp(b*x+a)
*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2)
)+1/4*x^2*ln((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)-1/
4/b^2*a^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1
)-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+
2*a)-1)/(exp(2*b*x+2*a)-1))^3+1/4*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(ex
p(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2+1/2/b^2*a*c/(1+c+
d)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1
+c-d)*(1+c+d))^(1/2))+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))-1/4/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/2/b^2*c*a/(c+d-
1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c
-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d
+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1)*(...
```

Maxima [A]

time = 0.48, size = 213, normalized size = 0.93

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^3d}-\frac{2\,b^2x^2\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)-\mathrm{Li}_3\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^3d}\right)+\frac{1}{2}\,x^2\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) + 1/
2*x^2*arctanh(d*coth(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(195) = 390$.
time = 0.44, size = 730, normalized size = 3.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
 + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1)
)) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*
(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
 + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
 - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
 - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt(
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 -
a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(
c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(c + d \coth\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*coth(b*x+a)),x)
[Out] Integral(x*atanh(c + d*coth(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctanh(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atanh}(c + d \coth(a + b\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*coth(a + b*x)),x)
[Out] int(x*atanh(c + d*coth(a + b*x)), x)
```

3.300 $\int \tanh^{-1}(c + d \coth(a + bx))\, dx$

Optimal. Leaf size=150

$$x\tanh^{-1}(c+d\coth(a+bx))+\frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{2}x\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{\text{PolyLog}\big(2}{}$$

```
[Out] x*arctanh(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a)/
(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A]
time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6372, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+x\tanh^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6372

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + (-Dist[b*(1 - c - d), Int[x*(E^(2*a + 2*b*
x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*(1 + c + d),
 Int[x*(E^(2*a + 2*b*x)/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x))), x], x])
 /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\coth(a+bx))\,dx = x\tanh^{-1}(c+d\coth(a+bx)) - (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(-1+c+$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(150) = 300.
time = 3.33, size = 369, normalized size = 2.46

$$x\tanh^{-1}(c+d\coth(a+bx)) - \frac{-\left((a+bx)\log\left(1-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right)\right) - (a+bx)\log\left(1+\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) + (a+bx)\log\left(1-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + (a+bx)\log\left(1+\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + a\log\left(1+d-e^{2(a+bx)}+de^{2(a+bx)}+c\left(-1+e^{2(a+bx)}\right)\right) - a\log\left(1+c-e^{2(a+bx)}-ce^{2(a+bx)}-d\left(1+e^{2(a+bx)}\right)\right) - \mathrm{PolyLog}\left(2,-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) - \mathrm{PolyLog}\left(2,\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) + \mathrm{PolyLog}\left(2,-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + \mathrm{PolyLog}\left(2,\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] - (-((a + b*x)*Log[1 - (Sqrt[-1 + c + d]*E^(
a + b*x))/Sqrt[-1 + c - d]]) - (a + b*x)*Log[1 + (Sqrt[-1 + c + d]*E^(a + b
*x))/Sqrt[-1 + c - d]] + (a + b*x)*Log[1 - (Sqrt[1 + c + d]*E^(a + b*x))/Sq
rt[1 + c - d]] + (a + b*x)*Log[1 + (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[1 + c
 - d]] + a*Log[1 + d - E^(2*(a + b*x)) + d*E^(2*(a + b*x)) + c*(-1 + E^(2*(
a + b*x)))] - a*Log[1 + c - E^(2*(a + b*x)) - c*E^(2*(a + b*x)) - d*(1 + E^
(2*(a + b*x)))] - PolyLog[2, -((Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[-1 + c -
 d])] - PolyLog[2, (Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[-1 + c - d]] + PolyL
og[2, -((Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[1 + c - d])] + PolyLog[2, (Sqrt[
1 + c + d]*E^(a + b*x))/Sqrt[1 + c - d]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(138) = 276.
time = 1.02, size = 361, normalized size = 2.41

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^{2}\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c-}{-1-c+d}\right.}{2d}\right.}{}}{}$
default	$\frac{\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^{2}\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c-}{-1-c+d}\right.}{2d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arctanh(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*arctanh(c+d*
coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*d^2*(-1/2/d*dilog((-d*coth(b*x+a)-c
-1)/(-1-c+d))-1/2/d*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c-1)/(-1-c+d))+
1/2/d*dilog((-d*coth(b*x+a)-c+1)/(1-c+d))+1/2/d*ln(-d*coth(b*x+a)-d)*ln((-d
*coth(b*x+a)-c+1)/(1-c+d))-1/2/d*dilog((-d*coth(b*x+a)-c+1)/(1-c-d))-1/2/d*
ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c+1)/(1-c-d))+1/2/d*dilog((-d*coth(
b*x+a)-c-1)/(-1-c-d))+1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c-1)/(-
1-c-d))))
```

Maxima [A]
time = 0.49, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right)+x\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d - 1)
*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(128) = 256$.
time = 0.48, size = 540, normalized size = 3.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
(c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((
c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1
)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log(2*(c
 + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(
(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)/(c - d
+ 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt((c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt((c
 + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt((c +
d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt((c + d -
1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c + d\coth\left(a + bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(atanh(c + d*coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(c + d\coth(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*coth(a + b*x)),x)
```

```
[Out] int(atanh(c + d*coth(a + b*x)), x)
```

3.301 $\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x} \, dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(c+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x} \, dx = \int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x} \, dx$$

Mathematica [A]
time = 11.73, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c+d\coth\left(bx+a\right)\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*coth(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d\coth\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*coth(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(c + d \coth(a + b x))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*coth(a + b*x))/x,x)
[Out] int(atanh(c + d*coth(a + b*x))/x, x)
```

3.302 $\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=152

$$\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{3x^2\text{Po}}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arctanh(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,(
1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6376, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left((d+1)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{4}x^4\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1-d)e^{2a+2bx}}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}(b(1+d))\int \frac{}{1+}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d\right.$$

Mathematica [A]
time = 3.17, size = 141, normalized size = 0.93

$$\frac{1}{16}\left(4x^4\tanh^{-1}(1+d+d\coth(a+bx)) - 2x^4\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{4x^3\mathrm{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x^2\mathrm{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{6x\mathrm{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3} + \frac{3\mathrm{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Coth[a + b*x]] - 2*x^4*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Poly
Log[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((1 + d)*E^(2*
(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/b^4)/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 1741, normalized size = 11.45

method	result	size
risch	Expression too large to display	1741

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/8*I*x^4
*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-3/8/b^4*d*a^4/(1+d)*ln(1-(1
+d)*exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))+1/2
/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(
b*x+a)*(1+d)^(1/2))-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b
^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2+1/20*b*x^5-1/8/b^4*d*a^4/(1+
d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I/(ex
p(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
-1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^
2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/16*I*x^4*Pi*c
sgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2
*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/8*I*x^4*Pi+1/8*I*x^4*Pi*csgn(I*d/(exp(2*
b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(
2*b*x+2*a)-1))^3-3/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*
a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*
(1+d)^(1/2))*x+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*
a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)
-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*
(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))-1/2/b^3*d*a^3/(1+d)*ln(1-(1+d)*exp(2*b
*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*d*a^3
/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a
)*(1+d)^(1/2))-1/8*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-3/8/b^3/(1+d)*pol
ylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2
))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+3/16/b^4*d/(1+d)*polylog(
5,(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1
/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1-(1+
d)*exp(2*b*x+2*a))-1/4/b^4*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))+3/8/b^
2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-1/4/b/(1+d)*polylog(2,(1+d)*exp
(2*b*x+2*a))*x^3+3/16/b^4/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*l
n(1-(1+d)*exp(2*b*x+2*a))*x^4-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*P
i*csgn(I*exp(2*b*x+2*a))^3-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+
2*a)-1)-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/8*x^4
*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)
```

Maxima [A]

time = 0.70, size = 146, normalized size = 0.96

$$\frac{1}{4}\,x^4\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)+6\,bx\mathrm{Li}_4\left((d+1)e^{(2\,bx+2\,a)}\right)-3\,\mathrm{Li}_5\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1)*e
^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(132) = 264$.
time = 0.49, size = 424, normalized size = 2.79

$$\frac{2\,b^{5}x^{5}+5\,b^{4}x^{4}\log\left(-\frac{d\cosh\left(bx+a\right)+(d+2)\sinh\left(bx+a\right)}{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}\right)-20\,b^{3}x^{3}\mathrm{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-20\,b^{3}x^{3}\mathrm{Li}_2\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\,a^{4}\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+2\,\sqrt{d+1}\right)-5\,a^{4}\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-2\,\sqrt{d+1}\right)+60\,b^{2}x^{2}\mathrm{polylog}\left(3,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+60\,b^{2}x^{2}\mathrm{polylog}\left(3,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\,\mathrm{polylog}\left(4,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\,\mathrm{polylog}\left(4,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\,\left(b^{4}x^{4}-a^{4}\right)\log\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-5\,\left(b^{4}x^{4}-a^{4}\right)\log\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+120\,\mathrm{polylog}\left(5,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+120\,\mathrm{polylog}\left(5,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{40\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**3*atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atanh}(d + d \coth(a + b x) + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^3*atanh(d + d*coth(a + b*x) + 1), x)
```

3.303 $\int x^2 \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=126

$$\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{x\text{Poly}}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctanh(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp(2
*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6376, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{x\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{3}x^3\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1-d)e^{2a+2bx}}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}(b(1+d))\int \frac{}{1+}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d\right.$$

Mathematica [A]
time = 4.06, size = 116, normalized size = 0.92

$$\frac{1}{24}\left(8x^3\tanh^{-1}(1+d+d\coth(a+bx)) - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Coth[a + b*x]] - 4*x^3*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x*PolyLo
g[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((1 + d)*E^(2*(a +
 b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.11, size = 1684, normalized size = 13.37

method	result	size
risch	Expression too large to display	1684

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)*x^3+1/6/b^3*a^3/(1+d)*ln(exp(2*b*
x+2*a)*d+exp(2*b*x+2*a)-1)-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2+
```

```
1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,(
1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b
^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+
2*a)*d+exp(2*b*x+2*a)-1)-1/3*x^3*ln(exp(b*x+a))+1/12*b*x^4-1/12*I*x^3*Pi*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b
*x+2*a))^2-1/6*I*Pi*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*P
i*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/4/b*d/(1+d)*polylog(2,(1+
d)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+
1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1-ex
p(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/
2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+exp
(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/
2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/12*I*x^3*Pi*csgn(I/(exp
(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b
*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*
x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)
-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2+1/3/b
^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/8/b^3*d/(1+d)*polylog(4,(1+d)*e
xp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d
)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)
^(1/2))+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/12*I*x^3*Pi*csgn(I*d
)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*(exp(2
*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+
exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*
csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/12*I*x^3
*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/6*I*x^3*Pi*csgn(I*d/(exp(
2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*
x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+d
)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1
/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))+1/2/b^2/(1+d)*ln(1-(1+d)*exp(2
*b*x+2*a))*x*a^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/6*x^3*ln(d)
```

Maxima [A]

time = 0.69, size = 123, normalized size = 0.98

$$\frac{1}{3}\,x^3\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3\,x^4}{d}-\frac{2\left(4\,b^3x^3\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_4\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^4d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.
time = 0.50, size = 360, normalized size = 2.86

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a) + 2\,(d+1)\sinh(bx+a) + 2\,\sqrt{d+1}\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a) + 2\,(d+1)\sinh(bx+a) - 2\,\sqrt{d+1}\right) + 12\,bx\,\mathrm{polylog}\left(3, \sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 12\,bx\,\mathrm{polylog}\left(3, -\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,(b^3x^3+a^3)\log\left(\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 2\,(b^3x^3+a^3)\log\left(-\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 12\,\mathrm{polylog}\left(4, \sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 12\,\mathrm{polylog}\left(4, -\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}(d + d \coth(a + b x) + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d + d*coth(a + b*x) + 1), x)
```

3.304 $\int x \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=100

$$\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{\text{PolyLo}}{}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctanh(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*x+
2*a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6376, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(1+d+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1-d)e^{2a+2bx}}\,d\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}(b(1+d))\int\frac{e}{1+(-}\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e\right.\end{aligned}$$

Mathematica [A]

time = 4.06, size = 90, normalized size = 0.90

$$\frac{2b^2x^2\left(2\tanh^{-1}(1+d+d\coth(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\mathrm{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+\mathrm{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3, 1/
((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 1603, normalized size = 16.03

method	result	size
risch	Expression too large to display	1603

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/4*I*Pi*x^2
+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/8*I*x^2*Pi*csgn
(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/2/b^2*d*a^2/
(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+
2*a))*x+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(
I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)-1))^2-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+
2*a)-1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+ex
p(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
-1))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a/(1+d)*di
log(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/
4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(1-exp(b*x+
a)*(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^2*d*a^
2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2
*a))*a^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/8/b^2
/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))-1/4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))
*x^2+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1+ex
p(b*x+a)*(1+d)^(1/2))*x-1/2/b*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/8*I*
x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi
*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp
(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+
2*a))*a+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln
```

```
(1+exp(b*x+a)*(1+d)^(1/2))+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))+
1/2/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp
(b*x+a)*(1+d)^(1/2))-1/4/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+
d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))
*x^2-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)-1))^2+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/
8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a
)-1)*exp(2*b*x+2*a))-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1
)+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*x^2*ln(d)+1/
4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)*x^2
```

Maxima [A]
time = 0.71, size = 100, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4x^3}{d}-\frac{3\left(2b^2x^2\log\left(-(d+1)e^{(2bx+2a)}+1\right)+2bx\text{Li}_2\left((d+1)e^{(2bx+2a)}\right)-\text{Li}_3\left((d+1)e^{(2bx+2a)}\right)\right)}{b^3d}\right)bd+\frac{1}{2}x^2\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arctanh(d*coth(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(86) = 172.
time = 0.44, size = 306, normalized size = 3.06

$$\frac{2b^3x^3+3b^2x^2\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6bx\text{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6bx\text{Li}_2\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)+2\sqrt{d+1}\right)-3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)-2\sqrt{d+1}\right)-3\left(b^2x^2-a^2\right)\log\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3\left(b^2x^2-a^2\right)\log\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\,\text{polylog}\left(3,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\,\text{polylog}\left(3,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqr
t(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2
*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}(d+d\coth(a+b\,x)+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d + d*coth(a + b*x) + 1), x)
```

3.305 $\int \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=69

$$\frac{bx^2}{2}+x\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arctanh(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A]

time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6368, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6368

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(1+d+d\coth(a+bx))\,dx &= x\tanh^{-1}(1+d+d\coth(a+bx))+b\int \frac{x}{1+(-1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\coth(a+bx))+(b(1+d))\int \frac{e^{2a+2}}{1+(-1-}\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1+d)e^{2a+2}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1+d)e^{2a+2}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1+d)e^{2a+2}\right.\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(69) = 138.
time = 2.90, size = 197, normalized size = 2.86

$$x\tanh^{-1}(1+d+d\coth(a+bx))+\frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1+d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{1+d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(-1+(1+d)e^{2(a+bx)}\right)\right)-2bx\log(d\cosh(a+bx)+(2+d)\sinh(a+bx))-2\text{PolyLog}\left(2,-\sqrt{1+d}\,e^{a+bx}\right)-2\text{PolyLog}\left(2,\sqrt{1+d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
2*PolyLog[2, -(Sqrt[1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 + d]*E^(a +
b*x)])/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(61) = 122.
time = 0.57, size = 265, normalized size = 3.84

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(-\frac{-d\coth(bx+a}{2d}\right.}{2d}\right.}{}$
default	$-\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(-\frac{-d\coth(bx+a}{2d}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arctanh(1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arctanh(1
+d+d*coth(b*x+a))*d*ln(d*coth(b*x+a)+d)-1/2*d^2*(1/2/d*dilog(-1/2*(-d*coth(
b*x+a)-d)/d)+1/2/d*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)-1/2/d
*dilog((-d*coth(b*x+a)-d-2)/(-2*d-2))-1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*cot
h(b*x+a)-d-2)/(-2*d-2))+1/2/d*dilog(1/2*d*coth(b*x+a)+1/2*d+1)+1/2/d*ln(d*c
oth(b*x+a)+d)*ln(1/2*d*coth(b*x+a)+1/2*d+1)-1/4/d*ln(d*coth(b*x+a)+d)^2))
```

Maxima [A]
time = 0.70, size = 72, normalized size = 1.04

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\,\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(60) = 120.
time = 0.37, size = 227, normalized size = 3.29

$$\frac{b^2x^2+bx\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+2\,\sqrt{d+1}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-2\,\sqrt{d+1}\right)-(bx+a)\log\left(\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-(bx+a)\log\left(-\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-\mathrm{Li}_2\left(\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-\mathrm{Li}_2\left(-\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
```

```
h(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d \coth\left(a + bx\right) + d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(d + d\coth(a + b\,x) + 1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(atanh(d + d*coth(a + b*x) + 1), x)
```

3.306 $\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1+d+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(1+d+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*coth(a + b*x) + d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}(d + d\coth(a + b\,x) + 1)}{x} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*coth(a + b*x) + 1)/x,x)
[Out] int(atanh(d + d*coth(a + b*x) + 1)/x, x)
```

3.307 $\int x^3 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=165

$$\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{3x^2\text{Po}}{}$$

```
[Out] 1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*ex
p(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,
(1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]

time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6376, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left((1-d)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{4}x^4\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1+d)e^{2a+2bx}}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}(b(1-d))\int \frac{}{1+}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4 \tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d\right.$$

Mathematica [A]
time = 3.61, size = 147, normalized size = 0.89

$$\frac{1}{16}\left(4x^4\tanh^{-1}(1-d-d\coth(a+bx)) - 2x^4\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Coth[a + b*x]] - 2*x^4*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x^2
*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((-
1 + d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((-1 + d)*E^(2*(a + b*x)
)))])/b^4)/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.36, size = 1801, normalized size = 10.92

method	result	size
risch	Expression too large to display	1801

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/8*I*x^4
*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/20*b*x^5-1/16*I*x^4*Pi*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8/b^
4*d*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)-1/16*I*x^4*Pi*csgn(I*d)
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x
+a))^2*csgn(I*exp(2*b*x+2*a))+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/
2))*x-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*
ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*ex
p(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1
/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))
^3-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+3/8/b^3/(d-1)*polylog(
4,-(d-1)*exp(2*b*x+2*a))*x-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x
^2+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-1/2/b^4*a^4/(d-1)*ln(1+
exp(b*x+a)*(1-d)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))-1
/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+3/8/b^4*a^4/(d-1)*ln(1+(d-
1)*exp(2*b*x+2*a))-1/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4+1/4/b^4*a^3/(
d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(1-
d)^(1/2))+1/8/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylog(5
,-(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x+1/2/
b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1+ex
p(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1
/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-3/8/b^4*d/(d-1)*ln(1+(d-1)*
exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-1/4/
b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^2*d/(d-1)*polylog(3,
-(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))
*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1-
exp(b*x+a)*(1-d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/8*I*x^4*Pi+1/1
6*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)
)*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(ex
p(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(e
xp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/8/b^4*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-
exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*
x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)-1/4*x^4
*ln(exp(b*x+a))-1/8*x^4*ln(d)
```

Maxima [A]
time = 0.69, size = 149, normalized size = 0.90

$$-\frac{1}{4}\,x^4\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(\left(d-1\right)e^{\left(2\,bx+2\,a\right)}+1\right)+4\,b^3x^3\mathrm{Li}_2(-(d-1)e^{(2\,bx+2\,a)})-6\,b^2x^2\mathrm{Li}_3(-(d-1)e^{(2\,bx+2\,a)})+6\,bx\mathrm{Li}_4(-(d-1)e^{(2\,bx+2\,a)})-3\,\mathrm{Li}_5(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d - 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(135) = 270$.
time = 0.41, size = 451, normalized size = 2.73

$$\frac{2\,b^{5}x^{5}-5\,b^{4}x^{4}\log\left(-\frac{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}{d\cosh\left(bx+a\right)+\left(d-2\right)\sinh\left(bx+a\right)}\right)-20\,b^{3}x^{3}{\rm Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-20\,b^{3}x^{3}{\rm Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\,a^{4}\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)-5\,a^{4}\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)-\sqrt{-4\,d+4}\right)+60\,b^{2}x^{2}{\rm polylog}\left(3,\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+60\,b^{2}x^{2}{\rm polylog}\left(3,-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\,{\rm polylog}\left(4,\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-120\,bx\,{\rm polylog}\left(4,-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-5\left(b^{4}x^{4}-a^{4}\right)\log\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-5\left(b^{4}x^{4}-a^{4}\right)\log\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+120\,{\rm polylog}\left(5,\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+120\,{\rm polylog}\left(5,-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{40\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*
(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^{3}\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**3*atanh(d*coth(a + b*x) + d - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*coth(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3 \operatorname{atanh}(d + d \coth(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(d + d*coth(a + b*x) - 1),x)
```

```
[Out] int(-x^3*atanh(d + d*coth(a + b*x) - 1), x)
```

3.308 $\int x^2 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=137

$$\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{x\text{Poly}}{}$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*exp(
2*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6376, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{x\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{3}x^3\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1+d)e^{2a+2bx}}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}(b(1-d))\int \frac{}{1+}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d\right.$$

Mathematica [A]
time = 4.05, size = 121, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\tanh^{-1}(1-d-d\coth(a+bx)) - 4x^3\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Coth[a + b*x]] - 4*x^3*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x*P
olyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((-1 +
d)*E^(2*(a + b*x))))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 1742, normalized size = 12.72

method	result	size
risch	Expression too large to display	1742

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*ln(exp(b*x+a))+1/12*b*x^4+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a
))*a^3-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*p
```

```
olylog(2,-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*
b*x+2*a))*x-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1
)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1
/2))*x-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)
*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2
*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a
)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b^3*d*a^2
/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))
*x^3+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*d)
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I/(
exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/12*I*x^3*P
i*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(ex
p(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))-1/6/b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+
1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)
)^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*e
xp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*cs
gn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b
*x+2*a)-1)*exp(2*b*x+2*a))^3-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1
/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d
-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/
2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x
+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*
a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*
exp(2*b*x+2*a))-1/6*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/2/b^2*d/(d-1)*
ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/6*I*Pi*x^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2
*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*
x+a)*(1-d)^(1/2))*x-1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)+1))^2-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1
/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a
)*(1-d)^(1/2))+1/6*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)*x^3-1/6*x^3*ln(d)
```

Maxima [A]
time = 0.69, size = 125, normalized size = 0.91

$$-\frac{1}{3}\,x^{3}\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{36}\,\left(\frac{3\,x^{4}}{d}-\frac{2\,\left(4\,b^{3}x^{3}\log\left(\left(d-1\right)e^{\left(2\,bx+2\,a\right)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_2(-\left(d-1\right)e^{\left(2\,bx+2\,a\right)})-6\,bx\mathrm{Li}_3(-\left(d-1\right)e^{\left(2\,bx+2\,a\right)})+3\,\mathrm{Li}_4(-\left(d-1\right)e^{\left(2\,bx+2\,a\right)})\right)}{b^{4}d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(111) = 222$.
time = 0.39, size = 382, normalized size = 2.79

$$\frac{b^4x^4 - 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 2\,a^3\log\left(2\,(d-1)\cosh(bx+a)+2\,(d-1)\sinh(bx+a)+\sqrt{-4\,d+4}\right) + 2\,a^3\log\left(2\,(d-1)\cosh(bx+a)+2\,(d-1)\sinh(bx+a)-\sqrt{-4\,d+4}\right) + 12\,bx\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 12\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 2\,(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2\,\mathrm{atanh}\left(d\coth(a+bx)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*coth(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2\,\mathrm{atanh}(d+d\coth(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d + d*coth(a + b*x) - 1),x)
[Out] int(-x^2*atanh(d + d*coth(a + b*x) - 1), x)
```

3.309 $\int x \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=109

$$\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{\text{PolyLo}}{}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b*x
+2*a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6376, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6376

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1+d)e^{2a+2bx}}\,d$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}(b(1-d))\int\frac{e}{1+(-}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e\right.$$

Mathematica [A]

time = 3.51, size = 94, normalized size = 0.86

$$\frac{2b^2x^2\left(2\tanh^{-1}(1-d-d\coth(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 1659, normalized size = 15.22

method	result	size
risch	Expression too large to display	1659

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x
+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/
8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/2*x^2*ln(exp(b*x
+a))+1/6*b*x^3+1/4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)*x^2-1/4/b^2*d/(d-1
)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(ex
p(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+
2*a))*x-1/4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*exp(
2*b*x+2*a))^3-1/2/b*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*
ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))
*x+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*x^2*Pi*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*
a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-
1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/4/b^2/(d-1)*pol
ylog(2,-(d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/
2))-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+
exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/8
/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*a^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)-1))^2+1/4*I*x^2*Pi-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+
2*a))+1/4/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)-1))^3+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)+1)-1/2/b*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/4/b^2*d/(d-1)*poly
log(2,-(d-1)*exp(2*b*x+2*a))*a+1/2/b^2*d*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(
```

```
1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b*a/(d-1)*ln(1-
exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+
1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/4/b*d/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*x+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a+1/8*I*x^2
*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-
exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1
))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/8*I*x
^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b
*x+2*a)-1)*exp(2*b*x+2*a))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*
csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/4*x^2*ln(d)
```

Maxima [A]
time = 0.71, size = 101, normalized size = 0.93

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2(-(d-1)e^{(2\,bx+2\,a)})-\mathrm{Li}_3(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d - 1/2*x^2*arctanh(d*coth(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(87) = 174$.
time = 0.39, size = 323, normalized size = 2.96

$$\frac{2\,b^3x^3-3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-3\,a^2\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)-3\,a^2\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-\sqrt{-4\,d+4}\right)-3\,(b^2x^2-a^2)\log\left(\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-3\,(b^2x^2-a^2)\log\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)+6\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)+6\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x \operatorname{atanh}\left(d \coth\left(a + bx\right) + d - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+d+d*coth(b*x+a)),x)
[Out] -Integral(x*atanh(d*coth(a + b*x) + d - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(-x*arctanh(d*coth(b*x + a) + d - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{atanh}(d + d \coth(a + b\,x) - 1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d + d*coth(a + b*x) - 1),x)
[Out] int(-x*atanh(d + d*coth(a + b*x) - 1), x)
```

3.310 $\int \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=76

$$\frac{bx^2}{2}+x\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*b*x^2-x*arctanh(-1+d+d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A]

time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6368, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6368

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(1-d-d\coth(a+bx))\,dx &= x\tanh^{-1}(1-d-d\coth(a+bx))+b\int \frac{x}{1+(-1+d)e^{2a+2bx}}\,dx\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\coth(a+bx))+(b(1-d))\int \frac{e^{2a+2}}{1+(-1+}\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2}\right.\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2}\right.\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(76) = 152.
time = 3.34, size = 208, normalized size = 2.74

$$x\tanh^{-1}(1-d-d\coth(a+bx))+\frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1-d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{1-d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(1+(-1+d)e^{2(a+bx)}\right)\right)-2bx\log(d\cosh(a+bx)+(-2+d)\sinh(a+bx))-2\mathrm{PolyLog}\left(2,-\sqrt{1-d}\,e^{a+bx}\right)-2\mathrm{PolyLog}\left(2,\sqrt{1-d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
 2*PolyLog[2, -(Sqrt[1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 - d]*E^(a +
 b*x)])/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(66) = 132.
time = 0.56, size = 299, normalized size = 3.93

method	result
derivativedivides	$-\frac{\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}-\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{-d\coth}{2d}\right.}{}\right.}{}}{}$
default	$-\frac{\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}-\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{-d\coth}{2d}\right.}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(1/2*arctanh(-1+d+d*coth(b*x+a))*d*ln(d*coth(b*x+a)+d)-1/2*arctanh(-
1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*d^2*(-1/2/d*dilog(-1/2*(-d*co
th(b*x+a)-d)/d)-1/2/d*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)+1/
2/d*dilog((-d*coth(b*x+a)-d+2)/(-2*d+2))+1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*
coth(b*x+a)-d+2)/(-2*d+2))+1/4/d*ln(d*coth(b*x+a)+d)^2-1/2/d*ln(-1/2*d*coth
(b*x+a)-1/2*d+1)*ln(d*coth(b*x+a)+d)+1/2/d*ln(-1/2*d*coth(b*x+a)-1/2*d+1)*l
n(1/2*d*coth(b*x+a)+1/2*d)+1/2/d*dilog(1/2*d*coth(b*x+a)+1/2*d)))
```

Maxima [A]
time = 0.69, size = 73, normalized size = 0.96

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*coth(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(61) = 122$.
time = 0.37, size = 240, normalized size = 3.16

$$\frac{b^2x^2-bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-\sqrt{-4\,d+4}\right)-(bx+a)\log\left(\frac{1}{2}\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a
) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
```

```
h(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a)),x)
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\mathrm{atanh}(d+d\,\mathrm{coth}(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*coth(a + b*x) - 1),x)
[Out] int(-atanh(d + d*coth(a + b*x) - 1), x)
```

3.311 $\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
[Out] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] -integrate(arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a))/x,x)
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{atanh}(d + d\coth(a + b\,x) - 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*coth(a + b*x) - 1)/x,x)
[Out] int(-atanh(d + d*coth(a + b*x) - 1)/x, x)
```

3.312 $\int (e+fx)^3 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{i(e+fx)^4 \text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}+\frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f}-\frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx}{}$$

```
[Out] 1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.17, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6386, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^4 \text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{3f^3 \text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3 \text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e+fx)^2\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Ta
n[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6386

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)^3 \tanh^{-1}(\tan(a+bx))\,dx &= \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f} \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} + \frac{1}{2}\int \cdots \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e \cdots}{\cdots} \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e \cdots}{\cdots} \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e \cdots}{\cdots} \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e \cdots}{\cdots} \\
&= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e \cdots}{\cdots}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.
time = 0.74, size = 654, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Tan[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.72, size = 7429, normalized size = 24.60

method	result	size
risch	Expression too large to display	7429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*f^2
*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*f^2*x^3*e
+ 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3041 vs. 2(244) = 488.
time = 0.48, size = 3041, normalized size = 10.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(
b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I
)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b*x +
 a) + I)/(tan(b*x + a)^2 + 1)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1
) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*
x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cos
```

```
h(1)^2)*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(
tan(b*x + a)^2 + 1) + 1) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*
I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^
3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2
)*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*
x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f
*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(
1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(
1))*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cos
h(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*si
nh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*di
log(-(-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
) + 1) + 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x
 + a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a
^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^
2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(
b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*ta
n(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1
) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*
b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3
*a*b^3*cosh(1)^2)*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) +
 I - 1)/(tan(b*x + a)^2 + 1)) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^
2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*
b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(
1)^2)*sinh(1))*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan
(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3
 + 4*(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^
4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3
 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x +
a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^4*f^3*x^4 - a^4
*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f
*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*
cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 +
 a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
))*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x
 + a)^2 + 1)) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*
(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a
^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^
2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log((-(I - 1)*tan(b*x + a)^
2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^4*f^3 - 4*a^3*b*f^
2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 +
 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cos
```

```
h(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*
x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) +
6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*
f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b
^3*cosh(1)^2)*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I +
 1)/(tan(b*x + a)^2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*
f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*
b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + 3*b^4*f*x^2*cosh(1) + 3*b^4*x*c
osh(1)^2)*sinh(1))*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + 6*(-I*b*f^
3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, (I*tan(b*x + a)^2 + 2*t
an(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(-I*b*f^3*x - I*b*f^2*cosh(1) -
I*b*f^2*sinh(1))*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(b*
x + a)^2 + 1)) + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(
4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(I*b*
f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*poly...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{atanh}(\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atanh(tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arctanh(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(\tan(a+b\,x))\,{(e+f\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x)^3,x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x)^3, x)
```

3.313 $\int (e+fx)^2 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f}-\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx}{}$$

```
[Out] 1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6386, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{if^2\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Ta
n[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6386

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2\tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}$$
$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} + \frac{1}{2}\int$$
$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e}{}$$
$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e}{}$$
$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e}{}$$
$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e}{}$$

Mathematica [A]
time = 0.41, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tanh^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right) - 12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right) - 4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right) + 12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right) + 12b^3efx^2\log\left(1+ie^{2i(a+bx)}\right) + 4b^3f^2x^3\log\left(1+ie^{2i(a+bx)}\right) - 6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right) + 6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right) + 6bef\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) + 6bf^2x\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) - 6bef\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) - 6bf^2x\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) + 3if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right) - 3if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.05, size = 5543, normalized size = 23.69

method	result	size
risch	Expression too large to display	5543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1788 vs. 2(186) = 372.
time = 0.53, size = 1788, normalized size = 7.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a
) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cos
h(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh
(1))*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)
^2 + 1) + 1) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 -
I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-((I + 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*f
^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh
(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(
1))*dilog(-(-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1) + 1) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(
b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 -
```

```
 a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2
+ 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(a^3*f^2 - 3*a^2*b*f*co
sh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1
))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*
x + a)^2 + 1)) - 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b
^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x
+ a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 +
 a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b
^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*co
sh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(
b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 +
 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x
^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log((-(I - 1)*tan(b*x +
a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 + a^3
*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f
*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1
))*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x
 + a)^2 + 1)) + 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^
2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(((I - 1)*tan(b*x +
 a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(a^3*f^2 - 3*a^
2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^
2*cosh(1))*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1)) + 8*(b^3*f^2*x^3 + 3*b^3*f*x^2*cosh(1) + 3*b^3*x*cos
h(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 + 2*b^3*x*cosh(1))*sinh(1))*log(-
(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + 6*(b*f^2*x + b*f*cosh(1) + b*f*sin
h(1))*polylog(3, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 +
1)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, (I*tan(b*x + a)^2
- 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b*f^2*x + b*f*cosh(1) + b*
f*sinh(1))*polylog(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a
)^2 + 1)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, (-I*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \operatorname{atanh}(\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atanh(tan(a + b*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctanh(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(\tan(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x)^2, x)
```

3.314 $\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx}{}$$

```
[Out] 1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6386, 4266, 2611, 2320, 6724}

$$\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{f\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Ta
n[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6386

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} + \frac{1}{2}\int (e$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+$$

Mathematica [A]
time = 0.32, size = 295, normalized size = 1.82

$$ex\tanh^{-1}(\tan(a+bx)) + \frac{1}{2}fx^2\tanh^{-1}(\tan(a+bx)) - \frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)\right)}{8b} + \frac{f(4ib^2x^2\text{ArcTan}(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)))-2ibx\text{PolyLog}(2,-i\cos(2(a+bx))+\sin(2(a+bx)))-\text{PolyLog}(3,i\cos(2(a+bx))-\sin(2(a+bx)))+\text{PolyLog}(3,-i\cos(2(a+bx))+\sin(2(a+bx))))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Tan[a + b*x]] + (f*x^2*ArcTanh[Tan[a + b*x]])/2 - (e*((-4*a + P
i - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x))]
) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*
(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^2*x^
2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I*Co
s[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a + b
*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]
] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.08, size = 2543, normalized size = 15.70

method	result	size
risch	Expression too large to display	2543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a))
)/b^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(
exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x
+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f
*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))
+1))^2*x^2+1/2*I*f/b^2*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I*
f/b^2*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*f/b^2*a*(I*b*x+I*
a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*e*x-1/8*I*Pi*f*x^2-1
/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f/b^2*a*(I
*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn((1+I)*
(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/4*f/b^2*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*
x+a)))-1/4*f/b^2*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))-1/4*I*Pi*x*e*cs
gn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b*e*dilog(((-I)
^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a
)))/(-I)^(1/2))-1/4*ln(exp(2*I*(b*x+a))-I)*f*x^2+1/2*I*f/b^2*a*dilog(1+exp(
I*(b*x+a))*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2
*I*e/b*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*(I*b*x+I*a)*ln
(1-exp(I*(b*x+a))*(-1)^(3/4))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(
I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))+1))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2
*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2-1/8*I
*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*
f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*
```

```
x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-
I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+
1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/
2*ln(exp(2*I*(b*x+a))-I)*e*x-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(
2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))-1/8*
I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
+1))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csg
n((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+1/4*f/b^2*(I*b*x+I*a)^2*
ln(1-I*exp(2*I*(b*x+a)))+1/4*f/b^2*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a))
)-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b
*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))+1))^2*x^2+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))
+1))^2+1/2/b*e*a*ln(exp(2*I*(b*x+a))+I)-1/2*e/b*a*ln(-exp(2*I*(b*x+a))+I)+1
/4*f/b^2*a^2*ln(-exp(2*I*(b*x+a))+I)-1/4/b^2*f*a^2*ln(exp(2*I*(b*x+a))+I)-1
/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(I*(b*x+a)
)*(-1)^(3/4))-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(ex
p(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*
x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+
I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*f*a*dilog(((-I)^(
1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*cs
gn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/2*I*e/b*(I*b*x+I*a)
*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*(I*b*x+I*a)*ln(((-I)^
(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))+1))^3-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+
1))^3*x^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-
I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn
(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))+1))+1/2*(1/2*f*x^2+e*x)*ln(exp(2*I*(b*x+a))+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*x*e)*co
```

```
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(134) = 268$.
time = 0.38, size = 948, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(-((I + 1)*tan(b*x + a
)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(-I*b*f*x - I*b
*cosh(1) - I*b*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
 + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*d
ilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 +
1) + 1) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*f*x^2 -
 a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1))*log(((I + 1)*ta
n(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^2*f - 2
*a*b*cosh(1) - 2*a*b*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a
) + I - 1)/(tan(b*x + a)^2 + 1)) - 2*(a^2*f - 2*a*b*cosh(1) - 2*a*b*sinh(1)
)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
 1)) - 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sin
h(1))*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2
 + 1)) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*s
inh(1))*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a
)^2 + 1)) - 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b
)*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) + 2*(a^2*f - 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(((I - 1)*tan(b
*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(a^2*f - 2*
a*b*cosh(1) - 2*a*b*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a)
 + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*
sinh(1))*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - f*polylog(3, (I*tan(
b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (I*ta
n(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (-I
*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + f*polylog(3,
(-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)\operatorname{atanh}(\tan(a + bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctanh(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}(\tan(a+b\,x))\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x),x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x), x)
```

3.315 $\int \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=79

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(tan(b*x+a))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6382, 4266, 2317, 2438}

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+x\tanh^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Tan[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6382

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Tan[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \tanh^{-1}(\tan(a+bx))\,dx = x\tanh^{-1}(\tan(a+bx)) - b\int x\sec(2a+2bx)\,dx$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)\,dx - \frac{1}{2}$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2bx}\right.}{4b}$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+b}\right.}{4b}$$

Mathematica [A]
time = 0.03, size = 127, normalized size = 1.61

$$x\tanh^{-1}(\tan(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[Tan[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x
))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]
] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a
+ b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.
time = 0.60, size = 169, normalized size = 2.14

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))\,\text{arctanh}(\tan(bx+a)) + \frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2} - \frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(bx+}{1+\tan^2(bx+}\right.}{2}}{b}$
default	$\frac{\arctan(\tan(bx+a))\,\text{arctanh}(\tan(bx+a)) + \frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2} - \frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(bx+}{1+\tan^2(bx+}\right.}{2}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctan(tan(b*x+a))*arctanh(tan(b*x+a))+1/2*arctan(tan(b*x+a))*ln(1+I*(
1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2*arctan(tan(b*x+a))*ln(1-I*(1+I*tan(
b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a
)^2))+1/4*I*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(57) = 114$.
time = 0.50, size = 182, normalized size = 2.30

$$\frac{4\,(bx+a)\,\mathrm{artanh}\left(\tan\left(bx+a\right)\right)+\left(\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2},\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)-\frac{1}{2},-\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right)\right)\log\left(\tan\left(bx+a\right)^2+1\right)-(bx+a)\log\left(\frac{1}{2}\,\tan\left(bx+a\right)^2+\tan\left(bx+a\right)+\frac{1}{2}\right)+(bx+a)\log\left(\frac{1}{2}\,\tan\left(bx+a\right)^2-\tan\left(bx+a\right)+\frac{1}{2}\right)-i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan\left(bx+a\right)-\frac{1}{2}i+\frac{1}{2}\right)+i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan\left(bx+a\right)+\frac{1}{2}i+\frac{1}{2}\right)+i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan\left(bx+a\right)+\frac{1}{2}i+\frac{1}{2}\right)-i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan\left(bx+a\right)-\frac{1}{2}i+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1
/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a)
+ 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(b*
x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) - I*
dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*ta
n(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I + 1/
2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(57) = 114$.
time = 0.41, size = 499, normalized size = 6.32

$$\frac{4\,bx\log\left(-\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right)-2\,(bx+a)\log\left(\frac{(i+1)\,\tan(bx+a)^2+2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right)+2\,a\log\left(\frac{(i+1)\,\tan(bx+a)^2+2i\,\tan(bx+a)+i-1}{\tan(bx+a)^2+1}\right)-2\,a\log\left(\frac{(i+1)\,\tan(bx+a)^2-2i\,\tan(bx+a)+i-1}{\tan(bx+a)^2+1}\right)+2\,(bx+a)\log\left(\frac{(i+1)\,\tan(bx+a)^2-2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right)-2\,(bx+a)\log\left(\frac{-(i-1)\,\tan(bx+a)^2+2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right)+2\,(bx+a)\log\left(\frac{-(i-1)\,\tan(bx+a)^2-2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right)+2\,a\log\left(\frac{(i-1)\,\tan(bx+a)^2+2i\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right)-2\,a\log\left(\frac{(i-1)\,\tan(bx+a)^2-2i\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right)+i\,\mathrm{Li}_2\left(-\frac{(i+1)\,\tan(bx+a)^2+2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}+1\right)+i\,\mathrm{Li}_2\left(-\frac{(i+1)\,\tan(bx+a)^2-2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}+1\right)-i\,\mathrm{Li}_2\left(-\frac{-(i-1)\,\tan(bx+a)^2+2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}+1\right)-i\,\mathrm{Li}_2\left(-\frac{-(i-1)\,\tan(bx+a)^2-2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((
I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*a
*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x
+ a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2 + 2*
tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I - 1)*tan
(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*
log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x +
a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)
/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a)),x)
```

```
[Out] Integral(atanh(tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}(\tan(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x)),x)
```

```
[Out] int(atanh(tan(a + b*x)), x)
```

3.316 $\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(\tan(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctanh(tan(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[Tan[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 5.05, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a))/(f*x+e),x)
[Out] int(arctanh(tan(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctanh(tan(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\tan\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a))/(f*x+e),x)
[Out] Integral(atanh(tan(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(\tan(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))/(e + f*x),x)
[Out] int(atanh(tan(a + b*x))/(e + f*x), x)
```

3.317 $\int x^2 \tanh^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=395

$$\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{ix^2}{}$$

```
[Out] 1/3*x^3*arctanh(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-(1+
c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2*I*a
+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c
+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b^3-1/8
*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

Rubi [A]

time = 0.36, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6402, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix^2\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{6}x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6402

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 + c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 - c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(i(1-c)-d))\int \frac{e^{2ia+}}{1-c-id+(1}$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) -$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) -$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) -$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) -$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) -$$

Mathematica [A]
time = 0.65, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c-id)e^{2i(a+bx)}}{-1+c+id}\right) - 4b^3x^3\log\left(1+\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) - 6ib^2x^2\text{PolyLog}\left(2,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) + 6ib^2x^2\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) + 6bx\text{PolyLog}\left(3,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) - 6bx\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) + 3i\text{PolyLog}\left(4,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) - 3i\text{PolyLog}\left(4,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*
E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^
((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)
*(a + b*x)))/(1 + c + I*d))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 25.55, size = 6895, normalized size = 17.46

method	result	size
risch	Expression too large to display	6895

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*
a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^
2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c
^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
 + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
(c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4
*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^
4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (
c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*si
n(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2165 vs. 2(279) = 558.
time = 0.45, size = 2165, normalized size = 5.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 6
*I*b^2*x^2*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d
+ (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 +
 d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*di
log(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 -
```

2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^3*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 6*b*x*polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 6*b*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(-2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(-2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1

```
)) - 3*I*polylog(4, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 -
 c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*t
an(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x +
 a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c
+ I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 +
d^2 - 2*c + 1)))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(c + d\tan\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atanh}\left(c+d\,\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*tan(a + b*x)), x)
```

3.318 $\int x \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=295

$$\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{ix\mathrm{I}}{}$$

```
[Out] 1/2*x^2*arctanh(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*poly
log(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c-I*
d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2*I*b
*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b
^2
```

Rubi [A]
time = 0.28, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$,
Rules used = {6402, 2221, 2611, 2320, 6724}

$$\frac{\mathrm{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\mathrm{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\mathrm{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\mathrm{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{4}x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6402

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 + c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 - c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{2}(b(i(1-c)-d))\int\frac{e^{2ia+}}{1-c-id+(1}$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

Mathematica [A]
time = 0.44, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{2b^2x^2\log\left(1+\frac{(-1+c-id)e^{2i(a+bx)}}{-1+c+id}\right)-2b^2x^2\log\left(1+\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)-2ibx\text{PolyLog}\left(2,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right)+2ibx\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)+\text{PolyLog}\left(3,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right)-\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 2*b^2*x^2*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (2*I)*b*x*PolyLog[2, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] + (2*I)*b*x*PolyLog[2, -(((1 + c - I*d)*E^((
2*I)*(a + b*x)))/(1 + c + I*d))] + PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a +
b*x)))/(-1 + c + I*d)] - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(
1 + c + I*d))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.45, size = 6521, normalized size = 22.11

method	result	size
risch	Expression too large to display	6521

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*
x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 - d^2
 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2
+ (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d
^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2
+ 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*
```

```
d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*
x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(
c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2
*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*
c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4
*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^
2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1689 vs. $2(209) = 418$.
time = 0.44, size = 1689, normalized size = 5.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2
*I*b*x*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I
*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2
 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((
-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1
)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((I*(c - 1)*d - d
^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I
*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2
 + d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog(2*((-I*(c - 1)*d - d^2)*tan(b*x + a)
^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x
 + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c +
1) + 1) - 2*a^2*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d
 + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)
) - 2*a^2*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*
c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) + 2*
a^2*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 +
I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*lo
g(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2
- 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*x^2 - a
^2)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2
 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2
*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*log(-2*(
(-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c +
1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-2*((I*(c - 1)
```

```
*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2
 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-2*((-I*(c - 1)*d - d^2)*t
an(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c -
 I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x
+ a)^2 - c^2 - 2*I*(c + 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*
c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2
+ d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b
*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*
I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^
2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan
(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2 +
 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 +
 c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d - I*d^2
 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}(c + d \tan(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tan(b*x+a)),x)
[Out] Integral(x*atanh(c + d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctanh(d*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atanh}(c + d \tan(a + b x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*tan(a + b*x)),x)
[Out] int(x*atanh(c + d*tan(a + b*x)), x)
```

3.319 $\int \tanh^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=194

$$x\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{2}x\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{i\text{PolyI}}{}$$

```
[Out] x*arctanh(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d)
)-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-c+
I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*a+2
*I*b*x)/(1+c+I*d))/b
```

Rubi [A]

time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6394, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+x\tanh^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6394

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[x*(E^(2*I*a +
2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[
I*b*(1 - c + I*d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*
E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2
, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\tan(a+bx))\,dx = x\tanh^{-1}(c+d\tan(a+bx)) + (b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x}{1-c-id+(1-c+i$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,\mathrm{lo}$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,\mathrm{lo}$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,\mathrm{lo}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4654 vs. $2(194) = 388$.
time = 30.41, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
+ c*Cos[a + b*x] + d*Sin[a + b*x])] + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^
2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*
x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Log[(-d + Sqrt[1 - 2*
c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((-1 + c)*(I + Tan[(
a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
- 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + (a + b*x)*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(-I + T
an[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - I*Log[((-1 + c)*(I + Ta
```

n[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - (a + b*x)*Log[-((d + Sq
rt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - I*Log[((1 + c)*(-I
+ Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d +
 Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] + I*Log[((1 + c)*(
I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d
+ Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(
-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + I*Log
[((1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d
^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 +
c)] - I*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*d - I*Sqrt[1 + 2*
c + c^2 + d^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2])/(1 + c)] + I*PolyLog[2, (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan
[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d
 + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] +
 (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] + I*
PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I
 + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d + Sqrt[1 + 2*c +
 c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2
+ d^2])] + I*PolyLog[2, (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b
*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 +
 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c
)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])])*((-2*a)/(b*
(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a +
b*x)] + 2*c*d*Sin[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - Cos[2
*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a
+ b*x)]))))/(Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/
2]] + Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - Log
[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - Log[(-d +
 Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + (Log[(-d
+ Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b
x)/2]^2)/(2(1 - I*Tan[(a + b*x)/2])) - (Log[(-d + Sqrt[1 - 2*c + c^2 + d^
2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x
)/2])) + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(
1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + ((I/2)*Log[(d +
Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/
(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 +
 c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/
2)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a
 + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 +
 d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2
]) + ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/
2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2

```
]^2)/(2*((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2])) + (
(I/2)*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*
c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1
+ c) + Tan[(a + b*x)/2]) - ((I/2)*Log[-(((-1 + c)*(I + Tan[(a + b*x)/2]))/(
I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[
1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*
x)/2]^2)/(2*((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2])) +
 ((I/2)*Log[((-1 + c)*(-I + Tan[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c
+ c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c)
 + Tan[(a + b*x)/2]) - ((I/2)*Log[((-1 + c)*(I + Tan[(a + b*x)/2]))/(-I + I
*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(164) = 328$.
time = 1.28, size = 563, normalized size = 2.90

method	result
derivativedivides	$\frac{d \arctan(\tan(bx+a)) \operatorname{arctanh}(c+d\tan(bx+a)) - d^2 \left(-\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right) \ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d} + \frac{\arctan}{}\right.}{}$
default	$\frac{d \arctan(\tan(bx+a)) \operatorname{arctanh}(c+d\tan(bx+a)) - d^2 \left(-\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right) \ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d} + \frac{\arctan}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arctanh(c+d*tan(b*x+a))-d^2*(-1/2*arctan(-(c+d*
tan(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/2*arctan(-(c+d*ta
n(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)+1/4*I*ln(d*((c+d*tan(
b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))-ln((I*d
+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d+1/4*I/d*dilog((I*d-d*((c+d*tan(b
*x+a))/d-c/d))/(1+c+I*d))-1/4*I/d*dilog((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I
*d-c-1))-1/4*I*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*tan(b*x+
a))/d-c/d))/(I*d+c-1))-ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d-1/
4*I/d*dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))+1/4*I/d*dilog((I*d+
d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.

time = 0.54, size = 372, normalized size = 1.92

$$\frac{4\,(bx+a)\operatorname{artanh}\left(d\tan\left(bx+a\right)+c\right)+\left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1},\ \frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{d^2\tan(bx+a)+(c-1)d}{c^2+d^2-2\,c+1},\ \frac{(c-1)d\tan(bx+a)+c^2-2\,c+1}{c^2+d^2-2\,c+1}\right)\right)\log\left(\tan\left(bx+a\right)^2+1\right)-(bx+a)\log\left(\frac{d^2\tan(bx+a)^2+2\,(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right)+(bx+a)\log\left(\frac{d^2\tan(bx+a)^2+2\,(c-1)d\tan(bx+a)+c^2-2\,c+1}{c^2+d^2-2\,c+1}\right)-i\operatorname{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d+i}\right)+i\operatorname{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d-i}\right)-i\operatorname{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d+i}\right)+i\operatorname{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d-i}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
 (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(136) = 272.
time = 0.53, size = 1185, normalized size = 6.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
 + a)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c
^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*log(-2*((-I*
(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d
 + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*
x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-2*((I*(c - 1)*d - d^2)*
tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c +
 I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*c + 1)) + 2*(b*x + a)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^2 -
c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a)
 + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) +
 2*a*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 +
 I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log
(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 +
 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c
- 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c +
 I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d -
```

```
d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(
b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - I*dilog(2*((I*(c + 1)*d - d^2)*
tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c +
 I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 + 2*c + 1) + 1) + I*dilog(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 +
 I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*
c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) +
I*dilog(2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2
- 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*
c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - I*dilog(2*((-I*(c - 1)*
d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2
 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c + d\tan\left(a + bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a)),x)
[Out] Integral(atanh(c + d*tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arctanh(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(c + d\tan(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tan(a + b*x)),x)
[Out] int(atanh(c + d*tan(a + b*x)), x)
```

3.320 $\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.51, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tan(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d\tan\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*tan(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(c + d\tan(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tan(a + b*x))/x,x)
[Out] int(atanh(c + d*tan(a + b*x))/x, x)
```

3.321 $\int x^2 \tanh^{-1}(1 - id + d\tan(a + bx))\, dx$

Optimal. Leaf size=170

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right.\right.}{4b}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctanh(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6398, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6398

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1-id)e^{2ia+2ib}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{3}(b(i+d))\int \frac{}{1+}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

Mathematica [A]
time = 0.45, size = 155, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right) - 3i\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.80, size = 2346, normalized size = 13.80

method	result	size
risch	Expression too large to display	2346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+
a))+1))^2+1/12*I*b*x^4+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(
I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)
```

)*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*cs
gn(I/(exp(2*I*(b*x+a))+1))+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x
*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b^2*
a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/6*I*Pi*x^3+1/4*I/b*
d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/b^3*d/(I+d)*polylog(2
,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2
I(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2
I(b*x+a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*ex
p(2*I*(b*x+a)))*csgn(I*d)-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/2*I/b^2*a^2/
(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)*ln(1+I*
exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b
x+a))(-I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^
3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*d
/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/6*x^3*ln(I*exp(2*I*
(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-
I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/6*I
/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/12*I*x^3*Pi*cs
gn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I/(I+d)*ln(1-I*(I+d)*ex
p(2*I*(b*x+a)))*x^3-1/6*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*
a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilo
g(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*
I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/12*
I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))
+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+
1)*exp(2*I*(b*x+a)))^2-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a
))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/4
/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2/b^3*a^3*d/(I+d)*ln(1
-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a
))*(-I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+
I)/(exp(2*I*(b*x+a))+1))+1/3*I/b^3/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3
-1/4*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+
exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*
(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1
/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*ex
p(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*
csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-1/6*I*x^3*Pi*cs
gn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*d/(exp

```
(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x
+a)))-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/3*x^3
*ln(exp(I*(b*x+a)))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)+1/3/b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3+1/8/b^3/(I+d)*p
olylog(4,I*(I+d)*exp(2*I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(118) = 236$.
time = 0.28, size = 343, normalized size = 2.02

$$\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-id+1)}{b^2}-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)+\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2\left((id-1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3(4bx+a)\mathrm{Li}_3\left((id-1)e^{(2ibx+2ia)}\right)+6i\,\mathrm{Li}_4\left((id-1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b*
x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*a
rctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(118) = 236$.
time = 0.40, size = 345, normalized size = 2.03

$$\frac{ib^4x^4+2b^3x^3\log\left(-\frac{\left((d+i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)+6ib^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)+6ib^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-ia^4+2a^3\log\left(\frac{2(d+i)e^{(ibx+ia)}+i\sqrt{4id-4}}{2(d+i)}\right)+2a^3\log\left(\frac{2(d+i)e^{(ibx+ia)}-i\sqrt{4id-4}}{2(d+i)}\right)-12bx\,\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-12bx\,\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-2(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}+1\right)-2(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}+1\right)-12i\,\mathrm{polylog}\left(4,\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-12i\,\mathrm{polylog}\left(4,-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*
b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log
(1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + 2*a^3*log(1
/2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 12*b*x*polylo
g(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(4*I
*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^(I*b
*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)
 + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*polylog
(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}(d\tan(a+bx)-id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1-I*d+d*tan(b*x+a)),x)
[Out] Integral(x**2*atanh(d*tan(a + b*x) - I*d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*tan(a + b*x) - d*1i + 1),x)
[Out] int(x^2*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.322 $\int x \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=133

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

[Out] 1/6*I*b*x^3+1/2*x^2*arctanh(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2

Rubi [A]
time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6398, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

[In] Int[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]

[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8*b^2)

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6398

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id+d\tan(a+bx))\,dx=\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1-id)e^{2ia+2i}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}(b(i+d))\int\frac{}{1+}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-\right.$$

Mathematica [A]

time = 0.21, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.19, size = 2256, normalized size = 16.96

method	result	size
risch	Expression too large to display	2256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(
exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*d/(
exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(
I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,I*(I+d)*
exp(2*I*(b*x+a)))*a+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+
1))^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/4*I/
b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp
(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*I*Pi*x^2+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I
*(b*x+a))*(-I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2
*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(e
xp(2*I*(b*x+a))+1))-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*
a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I*(I*
exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/4*I/b^2*
d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a+1/2*I/b*a/(I+d)*ln(1-I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))*x+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*
I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x
+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csg
n(I*exp(2*I*(b*x+a)))+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2
))*x+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/4*I*x^2
*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*e
xp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(
```

```
2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))
^2-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b*a*d/(I+d)*ln(1+
I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b
*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(
I+d))^(1/2))+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(e
xp(2*I*(b*x+a))+1))^2+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x
-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*
x+a))+1))^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(
exp(2*I*(b*x+a))+1))^3-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/2
*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a-1/8*I/b^2/(I+d)*polylog(3,I*(
I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8/b
^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))+1/2/b^2*a/(I+d)*dilog(1-I*ex
p(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x
^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/8*I*x^2*P
i*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+
1)*exp(2*I*(b*x+a)))*csgn(I*d)-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*e
xp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/4/b^2*a^
2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/2/b^2*a^2*d/(I+d)*l
n(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*
x+a))*(-I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))^2*csgn(I*d)-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a)
)+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(
exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/4*
x^2*ln(d)-1/2*x^2*ln(exp(I*(b*x+a)))+1/4*x^2*ln(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(93) = 186$.
time = 0.29, size = 248, normalized size = 1.86

$$\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-id+1)}{b}-\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((id-1)e^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)+\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((id-1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I*b
*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(93) = 186$.

time = 0.39, size = 293, normalized size = 2.20

$$\frac{2i\,b^3x^3 + 3\,b^2x^2\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 3\,a^2\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}+i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right) - 3\,a^2\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}-i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right) - 3\,(b^2x^2-a^2)\log\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 3\,(b^2x^2-a^2)\log\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 6\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 6\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1/2
*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) - 3*a^2*log(1/2*(
2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 3*(b^2*x^2 - a^2)
*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*
sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d - 4)*e^(
I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*tan(a + b*x) - I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(x*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.323 $\int \tanh^{-1}(1-id+d\tan(a+bx))\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctanh(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6390, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6390

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id+d\tan(a+bx))\,dx = x\tanh^{-1}(1-id+d\tan(a+bx))+(ib)\int \frac{x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-(b(i+d))\int \frac{e^{2ia+2}}{1+(1-id}$$
$$= \frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia-}\right.$$
$$= \frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia-}\right.$$
$$= \frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia-}\right.$$

Mathematica [A]
time = 6.22, size = 84, normalized size = 0.90

$$x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{2bx\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+i\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d + d*Tan[a + b*x]] - (2*b*x*Log[1 + I/((I + d)*E^((2*I)*(a
 + b*x)))] + I*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(76) = 152.
time = 0.75, size = 321, normalized size = 3.45

method	result

derivativedivides	$\frac{\frac{i\,\mathrm{arctanh}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{i\,\mathrm{arctanh}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i(-id+d\,\mathrm{ta}}{2}\right.}{2d}\right.}{}}{}$
default	$\frac{\frac{i\,\mathrm{arctanh}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{i\,\mathrm{arctanh}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i(-id+d\,\mathrm{ta}}{2}\right.}{2d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*I*arctanh(1-I*d+d*tan(b*x+a))*d*ln(I*d+d*tan(b*x+a))-1/2*I*arcta
nh(1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))-1/2*d^2*(-1/2*I/d*dilog(1/2*
I*(-I*d+d*tan(b*x+a))/d)-1/2*I/d*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(
b*x+a))/d)+1/2*I/d*dilog(I*(I*d+d*tan(b*x+a)-I*(2*d+2*I))/(2*d+2*I))+1/2*I/
d*ln(I*d+d*tan(b*x+a))*ln(I*(I*d+d*tan(b*x+a)-I*(2*d+2*I))/(2*d+2*I))+1/4*I
/d*ln(-I*d+d*tan(b*x+a))^2-1/2*I/d*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))-1/2*I/
d*ln(-I*d+d*tan(b*x+a))*ln(1-1/2*I*d+1/2*d*tan(b*x+a))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(65) = 130$.
time = 0.48, size = 260, normalized size = 2.80

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\,\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}\right)\right)}{d}+\frac{2i\,\log(d\tan(bx+a)-i\,d+2)\log(\tan(bx+a)-i)-i\,\log(\tan(bx+a)-i)^2}{d}-\frac{2i\,\left(\log\left(\frac{1}{2}\,d\tan(bx+a)-\frac{1}{2}i\,d+1\right)\log(\tan(bx+a)-i)+\mathrm{Li}_2\left(-\frac{1}{2}\,d\tan(bx+a)+\frac{1}{2}i\,d\right)\right)}{d}+\frac{2i\,\left(\log(\tan(bx+a)-i)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)-8\,(bx+a)\,\mathrm{artanh}\,(d\tan\,(bx+a)-i\,d+1)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) - I
)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-1/2*(I*d*tan(b*x + a) +
d + 2*I)/(d + I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d + 2*I)/(d + I)))/d
+ (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b*x
+ a) - I)^2)/d - 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d + 2*I*(log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8
*(b*x + a)*arctanh(d*tan(b*x + a) - I*d + 1))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(65) = 130$.
time = 0.40, size = 218, normalized size = 2.34

$$\frac{i\,b^2x^2+bx\log\left(-\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)+a\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}+i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)+a\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}-i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + b*x*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(d
+ I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + a*log(1/2*(2*(d + I)*e
^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) + I*dilog(1/2*sqrt(4*I*d - 4)*
e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(atanh(d*tan(a + b*x) - I*d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.324 $\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(1-id+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*tan(a + b*x) - I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}(d\tan(a+b\,x)+1-d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*tan(a + b*x) - d*1i + 1)/x,x)
```

```
[Out] int(atanh(d*tan(a + b*x) - d*1i + 1)/x, x)
```

3.325 $\int x^2 \tanh^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=171

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right.\right.}{4b}$$

```
[Out] 1/12*I*b*x^4-1/3*x^3*arctanh(-1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*I*a
+2*I*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6398, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6398

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1+id)e^{2ia+2ib}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{3}(b(i-d))\int \frac{}{1+}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+\right.$$

Mathematica [A]
time = 0.30, size = 156, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right) - 3i\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a +
b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.71, size = 2456, normalized size = 14.36

method	result	size
risch	Expression too large to display	2456

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/6*I/(I-d)*l
n(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+
1/12*I*b*x^4-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^
```

```
2*csgn(I*d)-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/
(exp(2*I*(b*x+a))+1))^2-1/2*I/b^3*a^2*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*
(I-d))^(1/2))+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/
(exp(2*I*(b*x+a))+1))^3-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))
*x^2+1/6*I*Pi*x^3-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)
))^2+1/6*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/4/b/(I-d)*polylog(2,I
*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a
)))*a^2-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/6*I*
x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/2*I/b^3*a^3/(I-d
)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*
(b*x+a))*(-I*(I-d))^(1/2))+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*
I*(b*x+a))*d+I)+1/8*I/b^3*d/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))-1/2/b
^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/12*I*x^3*Pi*csgn(
I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*e
xp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/4*I/b^3*d/(I-d)*polylog(2,I*(I
-d)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*
csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1)
)+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/12*I*x^
3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))
*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/6*x
^3*ln(d)-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/8/
b^3/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))-1/2*I/b^2*a^2/(I-d)*ln(1-I*ex
p(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(I*d)
-1/2/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp
(2*I*(b*x+a))+1))-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a
))-I)/(exp(2*I*(b*x+a))+1))^3-1/3/b^3*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a))
)*a^3-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/4/b
^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x+1/2/b^3*a^3*d/(I-d)*ln(1+I
*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x
+a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1
/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*P
i*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/
12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I
*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/12*I*x^
3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))
*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/3
*I/b^3/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I-d)*polylog(3,I
*(I-d)*exp(2*I*(b*x+a)))*x+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I
-d))^(1/2))*x-1/3*x^3*ln(exp(I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x
+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a
))+1))+1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/12*I
```

```
*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(
b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+
a))+1))+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)+1/2/b^3*a^3*d/(
I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*I/b^3*a^2*d/(I-d)*dilog(1+
I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(118) = 236.
time = 0.29, size = 342, normalized size = 2.00

$$-\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-id-1)}{b^2}+\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)+\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2\left((-id-1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3\left((-id-1)e^{(2ibx+2ia)}\right)+6i\operatorname{Li}_4\left((-id-1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*
arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*
b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(118) = 236.
time = 0.43, size = 345, normalized size = 2.02

$$\frac{ib^4x^4-2b^3x^3\log\left(-\frac{de^{(2ibx+2ia)}}{(d-i)e^{(2ibx+2ia)}-i}\right)+6ib^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)+6ib^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-ia^4+2a^3\log\left(\frac{2(d-i)e^{(ibx+ia)}+i\sqrt{-4id-4}}{2(d-i)}\right)+2a^3\log\left(\frac{2(d-i)e^{(ibx+ia)}-i\sqrt{-4id-4}}{2(d-i)}\right)-12bx\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-12bx\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-2(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}+1\right)-2(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}+1\right)-12i\operatorname{polylog}\left(4,\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-12i\operatorname{polylog}\left(4,-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
 + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*lo
g(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + 2*a^3*log
(1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 12*b*x*pol
ylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt
(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d - 4)*
e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x
 + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*I
*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1-I*d+d*tan(b*x+a)),x)
[Out] Integral(x**2*atanh(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(-x^2*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*1i - d*tan(a + b*x) + 1),x)
[Out] int(x^2*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.326 $\int x \tanh^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=134

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/6*I*b*x^3-1/2*x^2*arctanh(-1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6398, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6398

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1+id)e^{2ia+2i}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}(b(i-d))\int\frac{}{1+}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+\right.$$

Mathematica [A]

time = 0.23, size = 120, normalized size = 0.90

$$\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)
))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.27, size = 2358, normalized size = 17.60

method	result	size
risch	Expression too large to display	2358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*
I*(b*x+a))+1))^2+1/8/b^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))+1/2/b^
2*a/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2/b^2*a/(I-d)*dilog(
1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+
a))-exp(2*I*(b*x+a))*d+I)-1/4*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^
2+1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2/b*d/(I-d)
*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(
-I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*
x+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)
)/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csg
n(I*d)+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I*x
^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
)^3+1/2*I/b^2*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/8*I*x^
2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2*csgn(I*d)+1/8*I*x^2*
Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))
+1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*
(b*x+a)))-1/2*I/b/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a-1/8*I*x^2*Pi*csg
n(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/2
*I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2*I/b^2*a^2/(I-d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2*I/b*a/(I-d)*ln(1+I*exp(I*(b*x+a
))*(-I*(I-d))^(1/2))*x+1/2*I/b^2*a*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-
```

```
d))^(1/2))-1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/8*
I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(
(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2
*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(I*d/(
exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x
+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1
/4*I*x^2*Pi-1/4/b/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I-d)
*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a+1/4*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*
x+a)))*x^2-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x+1/8*I*x
^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/4/b^2*d/(I-d)*ln(1-I*(I-d)*exp(
2*I*(b*x+a)))*a^2+1/4/b^2*a^2*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a)
)*d+I)-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b^
2*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a-1/8*I*x^2*Pi*csgn(I*exp(2*I
*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))-1
/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I
*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*e
xp(2*I*(b*x+a)))^2-1/4*I/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/8*I/b^2
/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a
)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))-1/4*
x^2*ln(d)+1/4*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)*x^2-1/2*x^2*ln(ex
p(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(93) = 186.
time = 0.28, size = 247, normalized size = 1.84

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-id-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((-id-1)e^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)+\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((-id-1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2*I
*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(93) = 186.

time = 0.44, size = 293, normalized size = 2.19

$$\frac{2i\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 3\,a^2\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right) - 3\,a^2\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right) - 3\,(b^2x^2-a^2)\log\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 3\,(b^2x^2-a^2)\log\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 6\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 6\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*
x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1/
2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) - 3*a^2*log(1/2
*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 3*(b^2*x^2 - a
^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-
1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*I*d -
4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/
b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(x*atanh(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.327 $\int \tanh^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}ibx^2+x\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/2*I*b*x^2-x*arctanh(-1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6390, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6390

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id-d\tan(a+bx))\,dx = x\tanh^{-1}(1+id-d\tan(a+bx)) + (ib)\int \frac{x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - (b(i-d))\int \frac{e^{2ia+2}}{1+(1+id}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia-}\right.$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia-}\right.$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia-}\right.$$

Mathematica [A]
time = 6.01, size = 85, normalized size = 0.90

$$x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{2bx\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right) + i\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (2*b*x*Log[1 - I/((-I + d)*E^((2*I)*(
a + b*x)))] + I*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(77) = 154.
time = 0.74, size = 365, normalized size = 3.88

method	result

derivativedivides	$-\frac{-\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i\operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i\operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(\right.}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(-1/2*I*arctanh(-1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*I*a
rctanh(-1-I*d+d*tan(b*x+a))*d*ln(-I*d-d*tan(b*x+a))-1/2*d^2*(-1/2*I/d*dilog
(I*(-I*d-d*tan(b*x+a)-I*(-2*d+2*I))/(-2*d+2*I))-1/2*I/d*ln(-I*d-d*tan(b*x+a
))*ln(I*(-I*d-d*tan(b*x+a)-I*(-2*d+2*I))/(-2*d+2*I))+1/2*I/d*dilog(-1/2*I*(
I*d-d*tan(b*x+a))/d)+1/2*I/d*ln(-I*d-d*tan(b*x+a))*ln(-1/2*I*(I*d-d*tan(b*x
+a))/d)-1/4*I/d*ln(-I*d+d*tan(b*x+a))^2+1/2*I/d*ln(1+1/2*I*d-1/2*d*tan(b*x+
a))*ln(-I*d+d*tan(b*x+a))-1/2*I/d*ln(1+1/2*I*d-1/2*d*tan(b*x+a))*ln(-1/2*I*
d+1/2*d*tan(b*x+a))-1/2*I/d*dilog(-1/2*I*d+1/2*d*tan(b*x+a))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(66) = 132$.
time = 0.47, size = 262, normalized size = 2.79

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}\right)\right)}{d}-\frac{2i\,\log(d\tan(bx+a)-i\,d-2)\log(\tan(bx+a)-i)-i\,\log(\tan(bx+a)-i)^2}{d}+\frac{2i\,(\log(-\frac{1}{2}\,d\tan(bx+a)+\frac{1}{2}i\,d+1)\log(\tan(bx+a)-i)+\mathrm{Li}_2(\frac{1}{2}\,d\tan(bx+a)-\frac{1}{2}i\,d))}{d}-\frac{2i\,(\log(\tan(bx+a)-i)\log(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2})+\mathrm{Li}_2(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}))}{d}\right)+8\,(bx+a)\,\mathrm{artanh}\,(d\tan\,(bx+a)-i\,d-1)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) - I
)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-1/2*(I*d*tan(b*x + a) + d
 - 2*I)/(d - I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d - 2*I)/(d - I)))/d -
 (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b*x +
 a) - I)^2)/d + 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d - 2*I*(log(tan(b*x + a) - I
)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*
(b*x + a)*arctanh(d*tan(b*x + a) - I*d - 1))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(66) = 132$.
time = 0.36, size = 219, normalized size = 2.33

$$\frac{i\,b^2x^2-bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)+a\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}+i\sqrt{-4i\,d-4}}{2\,(d-i)}\right)+a\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}-i\sqrt{-4i\,d-4}}{2\,(d-i)}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 - b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a
) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(d
 - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + a*log(1/2*(2*(d - I)
*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) + I*dilog(1/2*sqrt(-4*I*d -
 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(atanh(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.328 $\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1-id+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}{\left(- d \tan{\left(a + b x \right)} + i d + 1 \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atanh(-d*tan(a + b*x) + I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{atanh}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i - d*tan(a + b*x) + 1)/x,x)
[Out] int(atanh(d*1i - d*tan(a + b*x) + 1)/x, x)
```

3.329 $\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}+\frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f}-\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.16, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {6388, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{3f^3\text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3\text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e+fx)^2\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Co
t[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6388

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{b\int (e+fx)^4\sec(2a+2bx)\,dx}{4f}$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} + \frac{1}{2}\int (e$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.
time = 0.17, size = 654, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Cot[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.94, size = 7429, normalized size = 24.60

method	result	size
risch	Expression too large to display	7429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*f^2
*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*f^2*x^3*e
+ 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2799 vs. 2(244) = 488.
time = 0.47, size = 2799, normalized size = 9.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*poly
log(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*c
osh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*
sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*
dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*
```

```
f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3
 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cos
h(1) + b^3*cosh(1)^2)*sinh(1))*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))
 + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b
^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*
I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1
) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*
x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cos
h(1)^2)*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3
*x^4 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 +
4*b^4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*
x^3 + 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*log(-(cos(2*b*x + 2
*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 2*
(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3
- 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*
f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(cos(2*b*x + 2*a
) + I*sin(2*b*x + 2*a) + I) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*
f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^
3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)
^2)*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^
4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)^3 + 6
*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x +
a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^
2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)
*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4*
f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)
^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^
4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(
b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*
b^2*f)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2
*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*s
inh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f +
 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1)
 + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2
- a^2*b^2*f)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) +
 1) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a
*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*
b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*
cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4
*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) - sin(2*b*x +
 2*a) + 1) + 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a
*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1
)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(
-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh
```

```
(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^
2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) +
 3*a*b^3*cosh(1)^2)*sinh(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I
) + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a)) + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sin
h(1))*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 6*(-I*b*f^3*x - I
*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*
x + 2*a)) + 6*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -
I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*f^2*x*cosh(
1) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*(b^2*f^2*x + b^2*f*cosh(1))*sinh
(1))*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2
*b^2*f^2*x*cosh(1) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*(b^2*f^2*x + b^2
*f*cosh(1))*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*
(b^2*f^3*x^2 + 2*b^2*f^2*x*cosh(1) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*
(b^2*f^2*x + b^2*f*cosh(1))*sinh(1))*polylog(3,...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^3 \operatorname{atanh}(\cot(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(cot(b*x+a)),x)
[Out] Integral((e + f*x)**3*atanh(cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*arctanh(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atanh}(\cot(a+b\,x))\,{(e+f\,x)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x)^3,x)
[Out] int(atanh(cot(a + b*x))*(e + f*x)^3, x)
```

3.330 $\int (e+fx)^2 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f}-\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6388, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{if^2\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Co
t[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6388

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{b\int (e+fx)^3 \sec(2a+2bx)\,dx}{3f}$$
$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} + \frac{1}{2}\int (e$$
$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+}{}$$
$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+}{}$$

Mathematica [A]
time = 0.12, size = 409, normalized size = 1.75

$$\frac{1}{3}x(3e^2+3efx+f^2x^2)\tanh^{-1}(\cot(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right) - 12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right) - 4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right) + 12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right) + 12b^3efx^2\log\left(1+ie^{2i(a+bx)}\right) + 4b^3f^2x^3\log\left(1+ie^{2i(a+bx)}\right) - 6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right) + 6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right) + 6bef\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) + 6bf^2x\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) - 6bef\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) - 6bf^2x\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) + 3if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right) - 3if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.23, size = 5543, normalized size = 23.69

method	result	size
risch	Expression too large to display	5543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1590 vs. 2(186) = 372.
time = 0.46, size = 1590, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*
polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cos
h(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(I*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1)
 - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))
*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*
f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1
))*sinh(1))*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(I*b^2*f^2*x^
2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x
+ b^2*cosh(1))*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(
b^3*f^2*x^3 + 3*b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 +
 3*(b^3*f*x^2 + 2*b^3*x*cosh(1))*sinh(1))*log(-(cos(2*b*x + 2*a) + sin(2*b*
x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 4*(a^3*f^2 - 3*a
^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b
^2*cosh(1))*sinh(1))*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(a^
```

```
3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*
b*f - 2*a*b^2*cosh(1))*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
 I) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a
*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f
+ 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x +
2*a) + 1) + 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3
*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^
2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) - sin(2*
b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 +
3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^
2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) +
 sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(
1)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b
^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-I*cos(2*b*x +
 2*a) - sin(2*b*x + 2*a) + 1) + 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*co
sh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(-c
os(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(a^3*f^2 - 3*a^2*b*f*cosh(1)
+ 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sin
h(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 6*(b*f^2*x + b*f*co
sh(1) + b*f*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*
(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) - sin(2
*b*x + 2*a)) + 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, -I*cos(2*
b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*po
lylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \operatorname{atanh}(\cot(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(cot(b*x+a)),x)
[Out] Integral((e + f*x)**2*atanh(cot(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*arctanh(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(\cot(a+b\,x))\,{\left(e+f\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x)^2,x)
[Out] int(atanh(cot(a + b*x))*(e + f*x)^2, x)
```

3.331 $\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{i(e+fx)^2\mathrm{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f}-\frac{i(e+fx)\mathrm{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\mathrm{P}}{}$$

```
[Out] 1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6388, 4266, 2611, 2320, 6724}

$$\frac{i(e+fx)^2\mathrm{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{f\mathrm{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\mathrm{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\mathrm{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Co
t[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6388

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}\\ &= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} + \frac{1}{2}\int (\\ &= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+\\ &= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+\\ &= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+\end{aligned}$$

Mathematica [A]
time = 0.21, size = 295, normalized size = 1.82

$$ex\tanh^{-1}(\cot(a+bx)) + \frac{1}{2}fx^2\tanh^{-1}(\cot(a+bx)) - \frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)\right)}{8b} + \frac{f(4ib^2x^2\text{ArcTan}(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)))-2ibx\text{PolyLog}(2,-i\cos(2(a+bx))+\sin(2(a+bx)))-\text{PolyLog}(3,i\cos(2(a+bx))-\sin(2(a+bx)))+\text{PolyLog}(3,-i\cos(2(a+bx))+\sin(2(a+bx))))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Cot[a + b*x]] + (f*x^2*ArcTanh[Cot[a + b*x]])/2 - (e*((-4*a + P
i - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x))]
) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*
(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^2*x^
2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I*Co
s[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a + b
*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]
] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 2544, normalized size = 15.70

method	result	size
risch	Expression too large to display	2544

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a))
)/b^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((
1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/4*I*Pi*x*e*csgn(I/(exp(
2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/4*I*
Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*Pi*f*csgn(
(1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/4*I*Pi*x*e*csgn((1
-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*Pi*f*csgn(I/(exp(2*I
*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn(I*(exp
(2*I*(b*x+a))+I))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))-1/4*I*Pi*x*
e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a)
)-1))^2+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^
3-1/4*ln(exp(2*I*(b*x+a))-I)*f*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(
exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^
2*x^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(
exp(2*I*(b*x+a))-1))^2*x^2+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(
2*I*(b*x+a))-1))^3*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))^2*csgn(I*(exp(2*I*(b*x+a))+I))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*csgn(I*(exp(2*I*(b*x+a))+I))+1/8*I*Pi*f*x
^2+1/4*I*Pi*e*x+1/2*I*f/b^2*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1
/2*I*f/b^2*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/8*I*Pi*f*csgn(I*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/2*I*f/b^2*a*(I*b*x+I*a)*
ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(
I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1
/2))-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(I*(
b*x+a))*(-1)^(3/4))-1/4*f/b^2*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))+1/
4*f/b^2*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))-1/8*I*Pi*f*csgn(I/(exp(2*I*(
```

```
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2*I*e
/b*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*(I*b*x+
I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*f/b^2*a*dilog(1+exp(I
*(b*x+a))*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*
I*e/b*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*(I*b*x+I*a)*ln(
1-exp(I*(b*x+a))*(-1)^(3/4))-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I))-
1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/2*ln(exp
(2*I*(b*x+a))-I)*e*x-1/2*I*f/b^2*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)
))/(-I)^(1/2))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))
*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*x^2-1/4*I*Pi*x*e*csg
n(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*csgn(I*(exp(2*I*(b*x+a))-I
))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*csgn(I*(e
xp(2*I*(b*x+a))-I))*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/4*I
*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)-1))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-
I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*x^2+1/8*I*Pi*f*csgn((1+I)*(ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a
))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2/b*e*a*ln(exp(2*I*(b*x+a))+I)-1/2*e/b*
a*ln(-exp(2*I*(b*x+a))+I)+1/4*f/b^2*a^2*ln(-exp(2*I*(b*x+a))+I)-1/4/b^2*f*a
^2*ln(exp(2*I*(b*x+a))+I)-1/4*f/b^2*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a)))-
1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*
Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/4*f/b^2*(I
*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp
(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-
I)^(1/2))+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-
I)/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I))*x^2+1/4*I*Pi*x*e*csgn
(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*
csgn(I*(exp(2*I*(b*x+a))-I))-1/2*(-1/2*f*x^2-e*x)*ln(exp(2*I*(b*x+a))+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*x*e)*co
```

```
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(134) = 268$.
time = 0.45, size = 794, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(I*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a)) + 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(I*cos(2*
b*x + 2*a) - sin(2*b*x + 2*a)) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*di
log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 2*(I*b*f*x + I*b*cosh(1) + I*
b*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^2*f*x^2 + 2
*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
 + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 2*(a^2*f - 2*a*b*cosh(1)
 - 2*a*b*sinh(1))*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^2*f
 - 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
 + I) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*si
nh(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 - a^2*
f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1))*log(I*cos(2*b*x + 2*
a) - sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1)
 + 2*(b^2*x + a*b)*sinh(1))*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)
 - 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1)
)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*(a^2*f - 2*a*b*cosh(1
) - 2*a*b*sinh(1))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^2
*f - 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2
*a) + I) - f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + f*polylog(
3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a
) + sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)
))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{atanh}(\cot(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)*arctanh(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(\cot(a+b\,x))\,(e+f\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x),x)
[Out] int(atanh(cot(a + b*x))*(e + f*x), x)
```

3.332 $\int \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=79

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(cot(b*x+a))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6384, 4266, 2317, 2438}

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+x\tanh^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Cot[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6384

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Cot[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\cot(a+bx))\,dx &= x\tanh^{-1}(\cot(a+bx)) - b\int x\sec(2a+2bx)\,dx\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)\,dx - \cdots\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2b\cdots}\right)}{4b}\cdots\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+\cdots}\right)}{4b}\cdots\end{aligned}$$

Mathematica [A]
time = 0.05, size = 127, normalized size = 1.61

$$x\tanh^{-1}(\cot(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[Cot[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x
))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]
] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a
+ b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.
time = 0.59, size = 145, normalized size = 1.84

method	result
derivativedivides	$\frac{-\frac{i\,\mathrm{arctanh}(\cot(bx+a))\left(\ln\left(1-\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)-\ln\left(1+\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)}{4}-\frac{i\,\mathrm{dilog}\left(1-\cdots\right.}{\cdots}}{b}$
default	$\frac{-\frac{i\,\mathrm{arctanh}(\cot(bx+a))\left(\ln\left(1-\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)-\ln\left(1+\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{i(\cot(bx+a)+1)^2}{-(\cot^2(bx+a))+1}\right)}{4}-\frac{i\,\mathrm{dilog}\left(1-\cdots\right.}{\cdots}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*I*arctanh(cot(b*x+a))*(ln(1-I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1))
-ln(1+I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1)))+1/4*I*dilog(1+I*(cot(b*x+a)+1)
^2/(-cot(b*x+a)^2+1))-1/4*I*dilog(1-I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(57) = 114$.
time $= 0.51$, size $= 184$, normalized size $= 2.33$

$$\frac{4\,(bx+a)\,\mathrm{artanh}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2}\,\tan(bx+a)+\frac{1}{2},\frac{1}{2}\,\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\,\tan(bx+a)-\frac{1}{2},-\frac{1}{2}\,\tan(bx+a)+\frac{1}{2}\right)\right)\log\left(\tan(bx+a)^2+1\right) - (bx+a)\log\left(\frac{1}{2}\,\tan(bx+a)^2+\tan(bx+a)+\frac{1}{2}\right) + (bx+a)\log\left(\frac{1}{2}\,\tan(bx+a)^2-\tan(bx+a)+\frac{1}{2}\right) - i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan(bx+a)-\frac{1}{2}i+\frac{1}{2}\right) + i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan(bx+a)+\frac{1}{2}i+\frac{1}{2}\right) + i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan(bx+a)+\frac{1}{2}i+\frac{1}{2}\right) - i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan(bx+a)-\frac{1}{2}i+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(
b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) -
I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*
tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I +
1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(57) = 114$.
time $= 0.44$, size $= 389$, normalized size $= 4.92$

$$\frac{4\,bx\log\left(-\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right) + 2\,a\log\left(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)+i\right) - 2\,a\log\left(\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a)+i\right) - 2\,(bx+a)\log\left(i\,\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1\right) + 2\,(bx+a)\log\left(i\,\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1\right) - 2\,(bx+a)\log\left(-i\,\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1\right) + 2\,(bx+a)\log\left(-i\,\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1\right) + 2\,a\log\left(-\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)+i\right) - 2\,a\log\left(-\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a)+i\right) + i\,\mathrm{Li}_2(i\,\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)) + i\,\mathrm{Li}_2(i\,\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)) - i\,\mathrm{Li}_2(-i\,\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)) - i\,\mathrm{Li}_2(-i\,\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) +
 I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(
I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) +
1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b
*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*
x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a)))/b
```

Sympy [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \mathrm{atanh}\left(\cot\left(a+bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a)),x)
```

```
[Out] Integral(atanh(cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(\cot(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x)),x)
```

```
[Out] int(atanh(cot(a + b*x)), x)
```

3.333 $\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(\cot(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctanh(cot(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\cot\left(bx+a\right)\right)}{fx+e} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a))/(f*x+e),x)
[Out] int(arctanh(cot(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctanh(cot(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\cot\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a))/(f*x+e),x)
[Out] Integral(atanh(cot(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atanh}(\cot(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))/(e + f*x),x)
[Out] int(atanh(cot(a + b*x))/(e + f*x), x)
```

3.334 $\int x^2 \tanh^{-1}(c + d\cot(a + bx))\, dx$

Optimal. Leaf size=391

$$\frac{1}{3}x^3 \tanh^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3 \log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3 \log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{ix}{}$$

```
[Out] 1/3*x^3*arctanh(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1+c+
I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*a+2*
I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d
))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/8*I*po
lylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

Rubi [A]

time = 0.35, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6404, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix^2\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{6}x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6404

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 - c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 + c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{3}(b(i(1+c)-d))\int \frac{e^{2i\cdots}}{1+c-id+(-\cdots}$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \cdots$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \cdots$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \cdots$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \cdots$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \cdots$$

Mathematica [A]
time = 0.66, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{4b^3x^3\log\left(1-\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right) - 4b^3x^3\log\left(1-\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right) - 6ib^2x^2\text{PolyLog}\left(2,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right) + 6ib^2x^2\text{PolyLog}\left(2,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right) + 6bx\text{PolyLog}\left(3,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right) - 6bx\text{PolyLog}\left(3,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right) + 3i\text{PolyLog}\left(4,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right) - 3i\text{PolyLog}\left(4,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (4*b^3*x^3*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 4*b^3*x^3*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] + (6*I)*b^2*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*(a + b*x)))/(1 + c - I*d)] + 6*b*x*PolyLog[3, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] - 6*b*x*PolyLog[3, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + (3*I)*PolyLog[4, ((-1 + c + I*d)*E^((2*I)*
(a + b*x)))/(-1 + c - I*d)] - (3*I)*PolyLog[4, ((1 + c + I*d)*E^((2*I)*(a +
 b*x)))/(1 + c - I*d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 28.34, size = 6739, normalized size = 17.24

method	result	size
risch	Expression too large to display	6739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*a
)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^2
 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) + (c^
2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
+ 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4
 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (
c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
+ 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*
(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4
 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c
^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin
(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1799 vs. 2(275) = 550.
time = 0.57, size = 1799, normalized size = 4.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c^
2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2
```

```
 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*si
n(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^
2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2
 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d
^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*
sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(1/2*c^2
+ I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^3*log(1/2*c^2
 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2
*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4*a^3*log(-1/2*c
^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 4*a^3*log(-1/2
*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 6*b*x*polylo
g(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(
c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - 6*
b*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-
I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c
 + 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 +
 d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2
*a))/(c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^
2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 4*(b^3*
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c
+ 1)/(c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^
2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 4*(b^3*
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c
+ 1)/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 +
2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d
- d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c -
 I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 -
 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2
 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
+ I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(c + d\cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*cot(b*x+a)),x)
[Out] Integral(x**2*atanh(c + d*cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(c + d\cot(a + b\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*cot(a + b*x)),x)
[Out] int(x^2*atanh(c + d*cot(a + b*x)), x)
```

3.335 $\int x \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=293

$$\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{ix}{}$$

```
[Out] 1/2*x^2*arctanh(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*poly
log(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I*d)
*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*b*x)
/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^2
```

Rubi [A]
time = 0.29, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6404, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{4}x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6404

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 - c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 + c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{2}(b(i(1+c)-d))\int\frac{e^{2ia+2}}{1+c-id+(-1}$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

Mathematica [A]
time = 0.41, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx))+\frac{2b^2x^2\log\left(1-\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-2b^2x^2\log\left(1-\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)-2ibx\text{PolyLog}\left(2,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)+2ibx\text{PolyLog}\left(2,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)+\text{PolyLog}\left(3,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-\text{PolyLog}\left(3,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (2*b^2*x^2*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 2*b^2*x^2*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (2*I)*b*x*PolyLog[2, ((-1 + c + I*d)*E^((2
*I)*(a + b*x)))/(-1 + c - I*d)] + (2*I)*b*x*PolyLog[2, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + PolyLog[3, ((-1 + c + I*d)*E^((2*I)*(a + b
*x)))/(-1 + c - I*d)] - PolyLog[3, ((1 + c + I*d)*E^((2*I)*(a + b*x)))/(1 +
 c - I*d)])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.47, size = 6389, normalized size = 21.81

method	result	size
risch	Expression too large to display	6389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin(
2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x
^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*cos
(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 - d^2
- 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 +
 (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 +
 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d
```

```
)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x
 + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c
*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*
c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c
 + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*
(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2
 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(207) = 414$.
time = 0.61, size = 1463, normalized size = 4.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 +
d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
 + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x
+ 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 -
(c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1
)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
+ 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*co
s(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a)
 - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(c + 1)*d -
1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 +
 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 + I*(c - 1)*d -
 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2
- 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^2 + I*(c + 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/2*c^2 + I*(c - 1)
*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*
c + 1)) - 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
```

```
*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*
b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c +
1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I
*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(
c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2
- 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 +
2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^
2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
 + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(c + d \cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*cot(b*x+a)),x)
[Out] Integral(x*atanh(c + d*cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctanh(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atanh}\left(c+d\,\mathrm{cot}\left(a+b\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*cot(a + b*x)),x)
[Out] int(x*atanh(c + d*cot(a + b*x)), x)
```

3.336 $\int \tanh^{-1}(c+d\cot(a+bx))\,dx$

Optimal. Leaf size=194

$$x\tanh^{-1}(c+d\cot(a+bx))+\frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{2}x\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{i\text{PolyLo}}{}$$

```
[Out] x*arctanh(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d)
)-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c-I
*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+2*I
*b*x)/(1+c-I*d))/b
```

Rubi [A]

time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6396, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+x\tanh^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6396

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[x*(E^(2*I*a +
2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[
I*b*(1 + c + I*d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*
E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2
, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\cot(a+bx))\,dx = x\tanh^{-1}(c+d\cot(a+bx)) + (b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x}{1+c-id+(-1-c-$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\,\mathrm{l}$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\,\mathrm{l}$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\,\mathrm{l}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4463 vs. $2(194) = 388$.
time = 29.49, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
 + Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*
c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))/
(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c +
c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))/(-1 +
 c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 +
 d^2])/d) + Tan[(a + b*x)/2]] + (a + b*x)*Log[-((1 + c + Sqrt[1 + 2*c + c^2
 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c
- I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2
])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d +
```

```
 Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) +
 Tan[(a + b*x)/2]] - (a + b*x)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d] - I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sq
rt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(
a + b*x)/2])/d] + I*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1
- 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] + (a + b*x)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a +
 b*x)/2])/d] + I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 +
 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] - I*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*
c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)
/2])/d] - I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x
)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 + c +
Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c + I*d + Sqrt[1 - 2*
c + c^2 + d^2])] - I*PolyLog[2, (1 + c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[
(a + b*x)/2])/(1 + c + I*d - Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1
+ c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1
 + 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d
*Tan[(a + b*x)/2])/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2
, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c - I*d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2
] + d*Tan[(a + b*x)/2])/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*Poly
Log[2, (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(-1 - c +
I*d + Sqrt[1 + 2*c + c^2 + d^2])])*((2*a)/(b*(1 - c^2 - d^2 - Cos[2*(a + b*
x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])
) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x
)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/(-Log[-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + Log[-((1 + c + Sqrt[1 + 2
*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - Log[(1 - c + Sqrt[1 - 2*c + c^2 +
 d^2] + d*Tan[(a + b*x)/2])/d] + Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] +
d*Tan[(a + b*x)/2])/d] - ((I/2)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/
d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)
*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a +
b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(1 - c + Sqrt[1 - 2*c + c^2
 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]
) + ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]
*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((-1 + c + Sqrt[
1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(
a + b*x)/2]) - ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[(1 - c
+ Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I
 + Tan[(a + b*x)/2]) - ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((a + b*x)
*Sec[(a + b*x)/2]^2)/(2*(-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2])) - ((I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[
```

```
1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sqrt[1 - 2*c + c^2
+ d^2])/d) + Tan[(a + b*x)/2]) + ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(-1
+ c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/
2]^2)/(2*(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2])) + (
(I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(
a + b*x)/2]) - ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1
+ 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d
^2])/d) + Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2
 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + S...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(164) = 328$.
time = 1.36, size = 569, normalized size = 2.93

method	result
derivativedivides	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arctanh}(c+d\cot(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
default	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arctanh}(c+d\cot(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arctanh(c+d*cot(b*x+a))+d^2*(-1/2*arc
tan(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)+1/2*arcta
n(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+1/4*I*ln(d*
((c+d*cot(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1+c+I*d
))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1)))/d+1/4*I/d*dilog((I*d-d*(
(c+d*cot(b*x+a))/d-c/d))/(1+c+I*d))-1/4*I/d*dilog((I*d+d*((c+d*cot(b*x+a))/
d-c/d))/(I*d-c-1))-1/4*I*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+
d*cot(b*x+a))/d-c/d))/(I*d+c-1))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I
*d)))/d-1/4*I/d*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I*d+c-1))+1/4*I/d*d
ilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(136) = 272$.

time = 0.53, size = 392, normalized size = 2.02

$$\frac{4\left(bx+a\right)\operatorname{artanh}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+(c^2+2c+1)\tan(bx+a)}{c^2+d^2+2c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2c+1}\right)-\arctan\left(\frac{(c-1)d+(c^2-2c+1)\tan(bx+a)}{c^2+d^2-2c+1},\frac{(c-1)d\tan(bx+a)+d^2}{c^2+d^2-2c+1}\right)\right)\log\left(\tan\left(bx+a\right)^2+1\right)-\left(bx+a\right)\log\left(\frac{2(c+1)d\tan(bx+a)+(c^2+2c+1)\tan(bx+a)^2+d^2}{c^2+d^2+2c+1}\right)+\left(bx+a\right)\log\left(\frac{2(c-1)d\tan(bx+a)+(c^2-2c+1)\tan(bx+a)^2+d^2}{c^2+d^2-2c+1}\right)+i\,\mathrm{Li}_2\left(-\frac{(c+1)\tan(bx+a)-ic-i}{ic+d+i}\right)-i\,\mathrm{Li}_2\left(-\frac{(c-1)\tan(bx+a)-ic+i}{ic+d-i}\right)+i\,\mathrm{Li}_2\left(-\frac{(c-1)\tan(bx+a)+ic-i}{-ic+d+i}\right)-i\,\mathrm{Li}_2\left(-\frac{(c+1)\tan(bx+a)+ic+i}{-ic+d-i}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2 +
 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + d^
2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x +
a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x + a) +
(c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 +
d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c + d + I))
- I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dilog(-((c -
 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*tan(b*x + a
) + I*c + I)/(-I*c + d - I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1099 vs. 2(136) = 272.
time = 0.59, size = 1099, normalized size = 5.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(
2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1)
*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c - 1)*
d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c + 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(c - 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2 + d^2 -
(c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1
)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1))
 - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2
*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) +
2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2
+ 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x +
```

```
a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a)
+ (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^
2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*dilog(-(c^2 + d^2 - (c
^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d
 - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1
) - I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c +
 1)/(c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d
 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
 I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c + d \cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a)),x)
[Out] Integral(atanh(c + d*cot(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(arctanh(d*cot(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}(c + d\cot(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*cot(a + b*x)),x)
[Out] int(atanh(c + d*cot(a + b*x)), x)
```

3.337 $\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(c+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*cot(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.88, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(c+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*cot(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d\cot\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*cot(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(c + d\cot(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*cot(a + b*x))/x,x)
```

```
[Out] int(atanh(c + d*cot(a + b*x))/x, x)
```

3.338 $\int x^2 \tanh^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=168

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctanh(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*poly
log(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+2*I
*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6400, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left((id+1)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6400

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1-id)e^{2ia}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(b(i-d))\int \frac{}{1}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+\right.$$

Mathematica [A]
time = 0.31, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right) - 3i\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.95, size = 2456, normalized size = 14.62

method	result	size
risch	Expression too large to display	2456

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*
x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-
1))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*
```

x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1
))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I
*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(
I*exp(2*I*(b*x+a)))^3+1/6*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/2/b^
3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^3*a^2/(I-d)*dil
og(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2
I(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/1
2*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*
(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))
*csgn(I*exp(2*I*(b*x+a)))^2-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*
(I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1
/4*I/b^3*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csg
n(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((e
xp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/12*I*x^3*Pi
*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))
+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
+1/2*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))-1))+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2)
)*x+1/6*I*Pi*x^3-1/2/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/2/b
^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)
*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*
(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^3*a^2*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*
(I*(I-d))^(1/2))-1/4*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/2
*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/12*I*x^3*Pi*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(e
xp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))
/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*
x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*c
sgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2*csgn(I*d)+1/2/b^3*a^3*d/(I
-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^3*a^3*d/(I-d)*ln(1+I*exp(I
*(b*x+a))*(I*(I-d))^(1/2))-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*
I*(b*x+a))*d-I)-1/3/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*
d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))*x+1/8/b^3/(I-d)*polylog(4,-I*(
I-d)*exp(2*I*(b*x+a)))+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)+
1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2
I(b*x+a))-1)*exp(2*I*(b*x+a)))-1/6*x^3*ln(d)+1/12*I*x^3*Pi*csgn((exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn
(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x
+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2
I(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/6*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*

```
x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-
1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/6*I/b^3*a^3/(I
-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/8*I/b^3*d/(I-d)*polylog(4
,-I*(I-d)*exp(2*I*(b*x+a)))+1/3*I/b^3/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*
a^3-1/3*x^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(119) = 238.
time = 0.28, size = 344, normalized size = 2.05

$$\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+id+1)}{b^2}-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)-\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2((id+1)e^{(2ibx+2ia)})+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3(4bx+a)\mathrm{Li}_3((id+1)e^{(2ibx+2ia)})+6i\,\mathrm{Li}_4((id+1)e^{(2ibx+2ia)})}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b*
x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x
 + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((I*d +
 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a
^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d*s
in(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d +
 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b
^2)/b
```

Fricas [A]
time = 0.41, size = 180, normalized size = 1.07

$$\frac{2ib^4x^4+4b^3x^3\log\left(-\frac{((d-i)e^{(2ibx+2ia)}+i)e^{(-2ibx-2ia)}}{d}\right)+6ib^2x^2\mathrm{Li}_2\left(-(-id-1)e^{(2ibx+2ia)}\right)-2ia^4+4a^3\log\left(\frac{(d-i)e^{(2ibx+2ia)}+i}{d-i}\right)-6bx\mathrm{polylog}\left(3,(id+1)e^{(2ibx+2ia)}\right)-4(b^3x^3+a^3)\log\left((-id-1)e^{(2ibx+2ia)}+1\right)-3i\,\mathrm{polylog}\left(4,(id+1)e^{(2ibx+2ia)}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*
I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2
*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}(d\cot(a+bx)+id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+I*d+d*cot(b*x+a)),x)
[Out] Integral(x**2*atanh(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}(d \cot(a + b\,x) + 1 + d\,1\mathrm{i})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*1i + d*cot(a + b*x) + 1),x)
[Out] int(x^2*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.339 $\int x\tanh^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal. Leaf size=132

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctanh(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3
,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6400, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6400

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1-id)e^{2ia+2i\ldots}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(b(i-d))\int\frac{\ldots}{1+(\ldots}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id\ldots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id\ldots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id\ldots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id\ldots\right.$$

Mathematica [A]
time = 0.20, size = 119, normalized size = 0.90

$$\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 2358, normalized size = 17.86

method	result	size
risch	Expression too large to display	2358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn
(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)-1/8*I*x^2*Pi*csgn(I*d
/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I
*(b*x+a)))^2+1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/8
*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+
1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/2/b*d/(I-d
)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a))*
(I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x
+1/2/b^2*a/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/4*d/(I-d)*ln(1
+I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+
a)))*x+1/8/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*cs
gn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-
I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)
))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(e
xp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b
*x+a))-1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a
))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(d/
(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn(I*exp
(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a
))-1)*exp(2*I*(b*x+a)))^3-1/4*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/
8*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))-1/4/b^2/(I-d)*polylog(2,
-I*(I-d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I
-d))^(1/2))-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/
8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*
```

```
I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))-1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))-1/8*I*x^2*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/8*I*
x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1
/2*I/b*a/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/4*I*x^2*Pi+1/8*I*
x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^3+1/2*I/b^2*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*
a*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog
(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4*I/b^2*d/(I-d)*polylog(2,-I*(I-d)*exp(2*
I*(b*x+a)))*a-1/2*I/b/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/2*I/b*a/(I
-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x
+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a))-
exp(2*I*(b*x+a))*d-I)-1/4*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/
2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*a^2*d/(I-d
)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(
b*x+a))*(I*(I-d))^(1/2))+1/4/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2
+1/4/b^2*a^2*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/4*x^2*ln
(d)-1/2*x^2*ln(exp(I*(b*x+a)))+1/4*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))
+I)*x^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(94) = 188$.
time = 0.28, size = 249, normalized size = 1.89

$$\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+id+1)}{b}-\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((id+1)e^{(2ibx+2ia)}\right)-6\left(i(bx+a)^2-2i(bx+a)a\right)\arctan\left(d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)-\cos(2bx+2a)+1\right)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((id+1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I*b
*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x + 2*
a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*((b*x
 + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2
*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*polylog(
3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [A]
time = 0.41, size = 157, normalized size = 1.19

$$\frac{4ib^3x^3+6b^2x^2\log\left(-\frac{((d-i)e^{(2ibx+2ia)}+i)e^{(-2ibx-2ia)}}{d}\right)+4ia^3+6ibx\operatorname{Li}_2\left(-(-id-1)e^{(2ibx+2ia)}\right)-6a^2\log\left(\frac{(d-i)e^{(2ibx+2ia)}+i}{d-i}\right)-6(b^2x^2-a^2)\log\left((-id-1)e^{(2ibx+2ia)}+1\right)-3\operatorname{polylog}\left(3,(id+1)e^{(2ibx+2ia)}\right)}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)
) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a^2
)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(d \cot\left(a+bx\right)+id+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}(d \cot(a+b\,x)+1+d\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.340 $\int \tanh^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}ibx^2+x\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctanh(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6392, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6392

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id+d\cot(a+bx))\,dx = x\tanh^{-1}(1+id+d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) + (b(i-d))\int \frac{e^{2ia}}{1+(-1-}$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia}\right.$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia}\right.$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia}\right.$$

Mathematica [A]
time = 23.93, size = 83, normalized size = 0.89

$$x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{2bx\log\left(1+\frac{e^{-2i(a+bx)}}{-1-id}\right) + i\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d + d*Cot[a + b*x]] - (2*b*x*Log[1 + 1/((-1 - I*d)*E^((2*I)
*(a + b*x)))] + I*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(76) = 152.
time = 0.84, size = 321, normalized size = 3.45

method	result

derivativedivides	$\frac{-\frac{i \operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i \operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(-\frac{i(id+d\cot}{2d}\right.}{2d}\right.}{}}{}$
default	$\frac{-\frac{i \operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i \operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\,\mathrm{dilog}\left(-\frac{i(id+d\cot}{2d}\right.}{2d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*I*arctanh(1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))+1/2*I*arct
anh(1+I*d+d*cot(b*x+a))*d*ln(-I*d+d*cot(b*x+a))-1/2*d^2*(-1/2*I/d*dilog(-1/
2*I*(I*d+d*cot(b*x+a))/d)-1/2*I/d*ln(-I*d+d*cot(b*x+a))*ln(-1/2*I*(I*d+d*co
t(b*x+a))/d)+1/2*I/d*dilog(I*(-I*d+d*cot(b*x+a)-I*(-2*d+2*I))/(-2*d+2*I))+1
/2*I/d*ln(-I*d+d*cot(b*x+a))*ln(I*(-I*d+d*cot(b*x+a)-I*(-2*d+2*I))/(-2*d+2*
I))+1/4*I/d*ln(I*d+d*cot(b*x+a))^2-1/2*I/d*dilog(1+1/2*I*d+1/2*d*cot(b*x+a)
)-1/2*I/d*ln(I*d+d*cot(b*x+a))*ln(1+1/2*I*d+1/2*d*cot(b*x+a))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(66) = 132$.
time = 0.48, size = 288, normalized size = 3.10

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)-d\left(\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right)+\mathrm{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}\right)\right)}{d}+\frac{2i\left(\log\left(\frac{1}{2}\,(d-2i)\tan(bx+a)-\frac{1}{2}i\,d\right)\log(i\,\tan(bx+a)+1)+\mathrm{Li}_2\left(-\frac{1}{2}\,(d-2i)\tan(bx+a)+\frac{1}{2}i\,d+1\right)\right)}{d}-\frac{2i\,\log((i\,d+2)\tan(bx+a)+d)\log(i\,\tan(bx+a)+1)-i\,\log(i\,\tan(bx+a)+1)^2}{d}-\frac{2i\left(\log(i\,\tan(bx+a)+1)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)-8\,(bx+a)\,\mathrm{artanh}\left(i\,d+\frac{d}{\tan(bx+a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) - d*(2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x +
 a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
+ 2)))/d + 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x + a
) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d - (2*I*log((I*
d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) + 1
)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dilo
g(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
+ 1))/b
```

Fricas [A]
time = 0.38, size = 122, normalized size = 1.31

$$\frac{2i\,b^2x^2+2\,bx\log\left(-\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)+i\,\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 + 2*b*x*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1
) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d - 1
)*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(atanh(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.341 $\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(1+id+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\cot\left(a+bx\right)+id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*cot(a + b*x) + I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}(d\cot(a+b\,x)+1+d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i + d*cot(a + b*x) + 1)/x,x)
```

```
[Out] int(atanh(d*1i + d*cot(a + b*x) + 1)/x, x)
```

3.342 $\int x^2 \tanh^{-1}(1 - id - d\cot(a + bx))\, dx$

Optimal. Leaf size=169

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/12*I*b*x^4-1/3*x^3*arctanh(-1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a+2*
I*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6400, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left((1-id)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6400

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1+id)e^{2ia}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(b(i+d))\int \frac{}{1}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log(1-(1-$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log(1-(1-$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log(1-(1-$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log(1-(1-$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log(1-(1-$$

Mathematica [A]
time = 0.31, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right) + 6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right) - 3i\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.87, size = 2346, normalized size = 13.88

method	result	size
risch	Expression too large to display	2346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3*I/b^3/(
I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*
exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/
```

2))-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/12*I*x^3*P
i*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-
1)*exp(2*I*(b*x+a)))-1/4/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x
+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3
*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*b*x^4-1/6*I*Pi*x
^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2
I(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/2*I/b^3*a^2*d/(I+
d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/4*I/b*d/(I+d)*polylog(2,-I*(
I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/b^3*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*
x+a)))*a^2-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2
*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/12*I*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*ex
p(2*I*(b*x+a)))*csgn(I*d)-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d
))^(1/2))*x-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+
1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/2/b^2*d/(
I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*
(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*
x*a^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/12*
I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a)
)+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(I*d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2*csgn(I*d)+1/6/b^3*a^3*d/(I+d)*ln(I*exp
(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+
exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8/b^3/(I+d)*polylog(4,-I*(I
+d)*exp(2*I*(b*x+a)))-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a))
)-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2
I(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1
))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I/(
exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(
2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-
1))^2-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I
*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^3-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*
d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(
exp(2*I*(b*x+a))-1))-1/6*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*d/(
I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(
b*x+a))*(I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(
b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I)/(exp(2*I*(b*x+a))-1))^2-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I
*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))
)*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(
2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/3/b^3*d/(I+
d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x

```
+a))*(I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^
(1/2))-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*p
olylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/6*x^3*
ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(
b*x+a))-1)*exp(2*I*(b*x+a)))^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(119) = 238.
time = 0.30, size = 345, normalized size = 2.04

$$-\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+id-1)}{b^2}+\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)-\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2((-id+1)e^{(2ibx+2ia)})+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3((-id+1)e^{(2ibx+2ia)})+6i\operatorname{Li}_4((-id+1)e^{(2ibx+2ia)})}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b
*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*a
rctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*
b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((-I
*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [A]
time = 0.44, size = 180, normalized size = 1.07

$$\frac{2ib^4x^4-4b^3x^3\log\left(-\frac{de^{(2ibx+2ia)}}{(d+i)e^{(2ibx+2ia)}-i}\right)+6ib^2x^2\operatorname{Li}_2\left(-(id-1)e^{(2ibx+2ia)}\right)-2ia^4+4a^3\log\left(\frac{(d+i)e^{(2ibx+2ia)}-i}{d+i}\right)-6bx\operatorname{polylog}\left(3,(-id+1)e^{(2ibx+2ia)}\right)-4(b^3x^3+a^3)\log\left((id-1)e^{(2ibx+2ia)}+1\right)-3i\operatorname{polylog}\left(4,(-id+1)e^{(2ibx+2ia)}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I*
a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylog(
3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(2*I
*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2 \operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*cot(a + b*x) + I*d - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2\,\mathrm{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.343 $\int x\tanh^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal. Leaf size=133

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-$$

```
[Out] 1/6*I*b*x^3-1/2*x^2*arctanh(-1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(
3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6400, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6400

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1+id)e^{2ia+2i}}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(b(i+d))\int\frac{}{1+(}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id\right.$$

Mathematica [A]

time = 0.23, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.33, size = 2256, normalized size = 16.96

method	result	size
risch	Expression too large to display	2256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2/b*
a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/8*I*x^2*Pi*csgn(I*exp(
2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*cs
gn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)+1/8*I*x^2*Pi*csgn(I
*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3-1/2*I/b/(I+d)*ln(1+I*(I+d)*exp(2*
I*(b*x+a)))*x*a+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*
I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/4*I/b^2*a^2/(I+d)*ln
(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+
a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2
-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*
x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-
1))+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(
I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*
Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2
+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/4*I*Pi*x^2-1/2/
b*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+
a))-1))^2+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(ex
p(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*
(b*x+a))-1))^2-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))
^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I/b^2
/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2*
I*(b*x+a)))*x^2-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2
```

```
))+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2/b^2*a^2*d/(
I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*
I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/4/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+
a)))*a^2+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8/b^2
*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4*d/(I+d)*ln(1+I*(I+d)*exp(
2*I*(b*x+a)))*x^2-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^
2/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I+d)*dilog(1+I*ex
p(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+
a)))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*Pi
*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1
/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a)
)-1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I/(exp(2*
I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*
exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*c
sgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*
exp(2*I*(b*x+a)))-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2*I/
b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln
(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-
1)*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*ex
p(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))^2*csgn(I*d)-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/4*ln(I*e
xp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)*x^2-1/4*x^2*ln(d)-1/2*x^2*ln(exp(I*(b
*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(94) = 188$.
time = 0.28, size = 250, normalized size = 1.88

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+id-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((-id+1)e^{(2ibx+2ia)}\right)-6\left(i(bx+a)^2-2i(bx+a)a\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)-\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((-id+1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2*I
*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [A]

time = 0.36, size = 157, normalized size = 1.18

$$\frac{4i\,b^3x^3 - 6\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) + 4i\,a^3 + 6i\,bx\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right) - 6\,(b^2x^2-a^2)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) - 3\,\mathrm{polylog}\left(3,(-i\,d+1)e^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a))
- 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2)*
log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*b*x
 + 2*I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x\,\mathrm{atanh}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*cot(a + b*x) + I*d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x\,\mathrm{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.344 $\int \tanh^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}ibx^2+x\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2-x*arctanh(-1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6392, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6392

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id-d\cot(a+bx))\,dx = x\tanh^{-1}(1-id-d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) + (b(i+d))\int \frac{e^{2ia}}{1+(-1+}$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2i}\right.$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2i}\right.$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2i}\right.$$

Mathematica [A]
time = 20.19, size = 84, normalized size = 0.89

$$x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{2bx\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right) + i\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d - d*Cot[a + b*x]] - (2*b*x*Log[1 + 1/((-1 + I*d)*E^((2*I)
*(a + b*x)))] + I*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(77) = 154.
time = 0.82, size = 365, normalized size = 3.88

method	result

derivativedivides	$-\frac{-\frac{i\,\mathrm{arctanh}(-1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\,\mathrm{arctanh}(-1+id+d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{d^2\left(\frac{i\,\mathrm{dilog}\left(\frac{i(-id-d}{2d}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{i\,\mathrm{arctanh}(-1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\,\mathrm{arctanh}(-1+id+d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{d^2\left(\frac{i\,\mathrm{dilog}\left(\frac{i(-id-d}{2d}\right.}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(-1/2*I*arctanh(-1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))+1/2*I*ar
ctanh(-1+I*d+d*cot(b*x+a))*d*ln(I*d-d*cot(b*x+a))-1/2*d^2*(1/2*I/d*dilog(1/
2*I*(-I*d-d*cot(b*x+a))/d)+1/2*I/d*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*co
t(b*x+a))/d)-1/2*I/d*dilog(I*(I*d-d*cot(b*x+a)-I*(2*d+2*I))/(2*d+2*I))-1/2*
I/d*ln(I*d-d*cot(b*x+a))*ln(I*(I*d-d*cot(b*x+a)-I*(2*d+2*I))/(2*d+2*I))-1/4
*I/d*ln(I*d+d*cot(b*x+a))^2+1/2*I/d*ln(1-1/2*I*d-1/2*d*cot(b*x+a))*ln(I*d+d
*cot(b*x+a))-1/2*I/d*ln(1-1/2*I*d-1/2*d*cot(b*x+a))*ln(1/2*I*d+1/2*d*cot(b*
x+a))-1/2*I/d*dilog(1/2*I*d+1/2*d*cot(b*x+a))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(67) = 134$.
time = 0.49, size = 286, normalized size = 3.04

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d-2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((i\,d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}+1\right)+\mathrm{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}\right)\right)}{d}-\frac{2i\left(\log\left(-\frac{1}{2}\,(d+2i)\tan(bx+a)+\frac{1}{2}i\,d\right)\log(i\,\tan(bx+a)+1)+\mathrm{Li}_2\left(\frac{1}{2}\,(d+2i)\tan(bx+a)-\frac{1}{2}i\,d+1\right)\right)}{d}+\frac{2i\,\log((i\,d-2)\tan(bx+a)+d)\log(i\,\tan(bx+a)+1)-i\,\log(i\,\tan(bx+a)+1)^2}{d}+\frac{2i\left(\log(i\,\tan(bx+a)+1)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)+8\,(bx+a)\,\mathrm{artanh}\left(i\,d+\frac{d}{\tan(bx+a)}-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) + d*(-2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x
+ a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I*d
 - 2)))/d - 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*x +
 a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d + (2*I*log((I
*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
 - 1))/b
```

Fricas [A]
time = 0.36, size = 122, normalized size = 1.30

$$\frac{2i\,b^2x^2-2\,bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)-2i\,a^2-2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)+i\,\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 - 2*b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2
*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
+ 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)*e
^(2*I*b*x + 2*I*a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*cot(a + b*x) + I*d - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\mathrm{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.345 $\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1+id+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] -Integral(atanh(d*cot(a + b*x) + I*d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{\operatorname{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d*1i + d*cot(a + b*x) - 1)/x,x)
```

```
[Out] int(-atanh(d*1i + d*cot(a + b*x) - 1)/x, x)
```

3.346 $\int \tanh^{-1}(e^x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{2}\text{PolyLog}(2,-e^x)+\frac{1}{2}\text{PolyLog}(2,e^x)$$

```
[Out] -1/2*polylog(2,-exp(x))+1/2*polylog(2,exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 6031}

$$\frac{\text{Li}_2(e^x)}{2}-\frac{\text{Li}_2(-e^x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[E^x],x]
```

```
[Out] -1/2*PolyLog[2, -E^x] + PolyLog[2, E^x]/2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(e^x)\,dx &= \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\,dx,x,e^x\right)\\ &= -\frac{\text{Li}_2(-e^x)}{2}+\frac{\text{Li}_2(e^x)}{2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.
time = 0.18, size = 46, normalized size = 2.19

$$x\tanh^{-1}(e^x)+\frac{1}{2}(-x(-\log(1-e^x)+\log(1+e^x))-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[E^x],x]
```

```
[Out] x*ArcTanh[E^x] + (-(x*(-Log[1 - E^x] + Log[1 + E^x])) - PolyLog[2, -E^x] +
PolyLog[2, E^x])/2
```

Maple [A]
time = 0.05, size = 31, normalized size = 1.48

method	result	size
risch	$-\frac{\text{dilog}(e^x+1)}{2}+\frac{\text{dilog}(1-e^x)}{2}$	18
derivativedivides	$\ln(e^x)\operatorname{arctanh}(e^x)-\frac{\text{dilog}(e^x)}{2}-\frac{\text{dilog}(e^x+1)}{2}-\frac{\ln(e^x)\ln(e^x+1)}{2}$	31
default	$\ln(e^x)\operatorname{arctanh}(e^x)-\frac{\text{dilog}(e^x)}{2}-\frac{\text{dilog}(e^x+1)}{2}-\frac{\ln(e^x)\ln(e^x+1)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x))*arctanh(exp(x))-1/2*dilog(exp(x))-1/2*dilog(exp(x)+1)-1/2*ln(exp
(x))*ln(exp(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(13) = 26$.
time = 0.26, size = 58, normalized size = 2.76

$$-\frac{1}{2}x(\log(e^x+1)-\log(e^x-1))+x\,\text{artanh}(e^x)+\frac{1}{2}\log(-e^x)\log(e^x+1)-\frac{1}{2}x\log(e^x-1)+\frac{1}{2}\text{Li}_2(e^x+1)-\frac{1}{2}\text{Li}_2(-e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arctanh(e^x) + 1/2*log(-e^x)*log(e
^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(13) = 26$.
time = 0.45, size = 65, normalized size = 3.10

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}\text{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\text{Li}_2(-\cosh(x)-\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cos
h(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x)
 + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(e^{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(x)),x)
```

```
[Out] Integral(atanh(exp(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{atanh}(\mathrm{e}^{x})\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(exp(x)),x)
```

```
[Out] int(atanh(exp(x)), x)
```

3.347 $\int x \tanh^{-1}(e^x)\, dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x\text{PolyLog}(2,-e^x)+\frac{1}{2}x\text{PolyLog}(2,e^x)+\frac{1}{2}\text{PolyLog}(3,-e^x)-\frac{1}{2}\text{PolyLog}(3,e^x)$$

```
[Out] -1/2*x*polylog(2,-exp(x))+1/2*x*polylog(2,exp(x))+1/2*polylog(3,-exp(x))-1/
2*polylog(3,exp(x))
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6348, 2611, 2320, 6724}

$$-\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{\text{Li}_3(-e^x)}{2}-\frac{\text{Li}_3(e^x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^x],x]
```

```
[Out] -1/2*(x*PolyLog[2, -E^x]) + (x*PolyLog[2, E^x])/2 + PolyLog[3, -E^x]/2 - Po
lyLog[3, E^x]/2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(e^x)\,dx &= -\left(\frac{1}{2}\int x\log(1-e^x)\,dx\right)+\frac{1}{2}\int x\log(1+e^x)\,dx\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\int \text{Li}_2(-e^x)\,dx-\frac{1}{2}\int \text{Li}_2(e^x)\,dx\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}\,dx,x,e^x\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}\,dx,\right.\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{\text{Li}_3(-e^x)}{2}-\frac{\text{Li}_3(e^x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.65

$$\frac{1}{4}(2x^2\tanh^{-1}(e^x)+x^2\log(1-e^x)-x^2\log(1+e^x)-2x\text{PolyLog}(2,-e^x)+2x\text{PolyLog}(2,e^x)+2\text{PolyLog}(3,-e^x)-2\text{PolyLog}(3,e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^x],x]
```

```
[Out] (2*x^2*ArcTanh[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
-E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

Maple [A]
time = 0.04, size = 62, normalized size = 1.44

method	result
risch	$-\frac{x\,\text{polylog}(2,-e^x)}{2}+\frac{x\,\text{polylog}(2,e^x)}{2}+\frac{\text{polylog}(3,-e^x)}{2}-\frac{\text{polylog}(3,e^x)}{2}$
default	$\frac{x^2\,\text{arctanh}(e^x)}{2}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\,\text{polylog}(2,e^x)}{2}-\frac{\text{polylog}(3,e^x)}{2}-\frac{x^2\ln(e^x+1)}{4}-\frac{x\,\text{polylog}(2,-e^x)}{2}+\frac{\text{polylog}(3,-e^x)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(exp(x))+1/4*x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*po
lylog(3,exp(x))-1/4*x^2*ln(exp(x)+1)-1/2*x*polylog(2,-exp(x))+1/2*polylog(3
,-exp(x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.
time = 0.26, size = 59, normalized size = 1.37

$$\frac{1}{2}\,x^2\,\mathrm{artanh}\left(e^x\right)-\frac{1}{4}\,x^2\log\left(e^x+1\right)+\frac{1}{4}\,x^2\log\left(-e^x+1\right)-\frac{1}{2}\,x\mathrm{Li}_2(-e^x)+\frac{1}{2}\,x\mathrm{Li}_2(e^x)+\frac{1}{2}\,\mathrm{Li}_3(-e^x)-\frac{1}{2}\,\mathrm{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*x
*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e^x
)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.
time = 0.39, size = 95, normalized size = 2.21

$$\frac{1}{4}\,x^2\log\left(-\frac{\cosh\left(x\right)+\sinh\left(x\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)-1}\right)-\frac{1}{4}\,x^2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{4}\,x^2\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+\frac{1}{2}\,x\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\frac{1}{2}\,x\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))-\frac{1}{2}\,\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+\frac{1}{2}\,\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log
(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilog
(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, cosh
(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(e^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(x)),x)
```

```
[Out] Integral(x*atanh(exp(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\text{atanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(exp(x)),x)
```

```
[Out] int(x*atanh(exp(x)), x)
```

3.348 $\int x^2 \tanh^{-1}(e^x)\, dx$

Optimal. Leaf size=58

$$-\frac{1}{2}x^2\text{PolyLog}(2,-e^x)+\frac{1}{2}x^2\text{PolyLog}(2,e^x)+x\text{PolyLog}(3,-e^x)-x\text{PolyLog}(3,e^x)-\text{PolyLog}(4,-e^x)+\text{PolyLog}$$

```
[Out] -1/2*x^2*polylog(2,-exp(x))+1/2*x^2*polylog(2,exp(x))+x*polylog(3,-exp(x))-
x*polylog(3,exp(x))-polylog(4,-exp(x))+polylog(4,exp(x))
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6348, 2611, 6744, 2320, 6724}

$$-\frac{1}{2}x^2\text{Li}_2(-e^x)+\frac{1}{2}x^2\text{Li}_2(e^x)+x\text{Li}_3(-e^x)-x\text{Li}_3(e^x)-\text{Li}_4(-e^x)+\text{Li}_4(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^x],x]
```

```
[Out] -1/2*(x^2*PolyLog[2, -E^x]) + (x^2*PolyLog[2, E^x])/2 + x*PolyLog[3, -E^x]
- x*PolyLog[3, E^x] - PolyLog[4, -E^x] + PolyLog[4, E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tanh^{-1}(e^x)\,dx &= -\left(\frac{1}{2}\int x^2 \log(1-e^x)\,dx\right)+\frac{1}{2}\int x^2 \log(1+e^x)\,dx\\
&= -\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+\int x\mathrm{Li}_2(-e^x)\,dx-\int x\mathrm{Li}_2(e^x)\,dx\\
&= -\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\int \mathrm{Li}_3(-e^x)\,dx+\int \mathrm{Li}_3(e^x)\\
&= -\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Subst}\left(\int \frac{\mathrm{Li}_3(-x)}{x}\,dx,x,e^x\right.\\
&= -\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Li}_4(-e^x)+\mathrm{Li}_4(e^x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.60

$$\frac{1}{6}\left(2x^3\tanh^{-1}(e^x)+x^3\log(1-e^x)-x^3\log(1+e^x)-3x^2\mathrm{PolyLog}(2,-e^x)+3x^2\mathrm{PolyLog}(2,e^x)+6x\mathrm{PolyLog}(3,-e^x)-6x\mathrm{PolyLog}(3,e^x)-6\mathrm{PolyLog}(4,-e^x)+6\mathrm{PolyLog}(4,e^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^x],x]
```

```
[Out] (2*x^3*ArcTanh[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

Maple [A]
time = 0.04, size = 79, normalized size = 1.36

method	result
risch	$-\frac{x^2 \operatorname{polylog}(2,-\mathrm{e}^x)}{2}+\frac{x^2 \operatorname{polylog}(2,\mathrm{e}^x)}{2}+x \operatorname{polylog}(3,-\mathrm{e}^x)-x \operatorname{polylog}(3,\mathrm{e}^x)-\operatorname{polylog}(4,-\mathrm{e}^x)+\operatorname{polyl}$
default	$\frac{x^3 \operatorname{arctanh}(\mathrm{e}^x)}{3}+\frac{x^3 \ln(1-\mathrm{e}^x)}{6}+\frac{x^2 \operatorname{polylog}(2,\mathrm{e}^x)}{2}-x \operatorname{polylog}(3,\mathrm{e}^x)+\operatorname{polylog}(4,\mathrm{e}^x)-\frac{x^3 \ln(\mathrm{e}^x+1)}{6}-\frac{x^2 \operatorname{polylog}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arctanh(exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2,exp(x))-x*po
lylog(3,exp(x))+polylog(4,exp(x))-1/6*x^3*ln(exp(x)+1)-1/2*x^2*polylog(2,-e
xp(x))+x*polylog(3,-exp(x))-polylog(4,-exp(x))
```

Maxima [A]
time = 0.26, size = 76, normalized size = 1.31

$$\frac{1}{3}\,x^3 \operatorname{artanh}\left(e^x\right)-\frac{1}{6}\,x^3 \log\left(e^x+1\right)+\frac{1}{6}\,x^3 \log\left(-e^x+1\right)-\frac{1}{2}\,x^2 \mathrm{Li}_2(-e^x)+\frac{1}{2}\,x^2 \mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Li}_4(-e^x)+\mathrm{Li}_4(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*x
^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e^x
) - polylog(4, -e^x) + polylog(4, e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(46) = 92$.
time = 0.37, size = 120, normalized size = 2.07

$$\frac{1}{6}\,x^3 \log\left(-\frac{\cosh\left(x\right)+\sinh\left(x\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)-1}\right)-\frac{1}{6}\,x^3 \log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{6}\,x^3 \log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+\frac{1}{2}\,x^2 \mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\frac{1}{2}\,x^2 \mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))-x\operatorname{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+x\operatorname{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))+\operatorname{polylog}(4,\cosh\left(x\right)+\sinh\left(x\right))-\operatorname{polylog}(4,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log
(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dil
og(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, co
sh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) +
sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(exp(x)),x)
```

```
[Out] Integral(x**2*atanh(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(e^x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{atanh}(\mathrm{e}^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(exp(x)),x)
```

```
[Out] int(x^2*atanh(exp(x)), x)
```

3.349 $\int \tanh^{-1}\left(e^{a+bx}\right)\, dx$

Optimal. Leaf size=35

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

```
[Out] -1/2*polylog(2,-exp(b*x+a))/b+1/2*polylog(2,exp(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 6031}

$$\frac{\text{Li}_2\left(e^{a+bx}\right)}{2b}-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[E^(a + b*x)],x]
```

```
[Out] -1/2*PolyLog[2, -E^(a + b*x)]/b + PolyLog[2, E^(a + b*x)]/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \tanh^{-1}\left(e^{a+bx}\right)\, dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\, dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 69, normalized size = 1.97

$$x\tanh^{-1}\left(e^{a+bx}\right)+\frac{bx\left(\log\left(1-e^{a+bx}\right)-\log\left(1+e^{a+bx}\right)\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[E^(a + b*x)],x]
```

```
[Out] x*ArcTanh[E^(a + b*x)] + (b*x*(Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)])
 - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

Maple [A]
time = 0.06, size = 59, normalized size = 1.69

method	result	size
risch	$-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2b}+\frac{\operatorname{dilog}\left(1-\mathrm{e}^{bx+a}\right)}{2b}$	32
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$	59
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(exp(b*x+a))*arctanh(exp(b*x+a))-1/2*dilog(exp(b*x+a))-1/2*dilog(exp
(b*x+a)+1)-1/2*ln(exp(b*x+a))*ln(exp(b*x+a)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(27) = 54$.
time = 0.26, size = 107, normalized size = 3.06

$$\frac{(bx+a)\operatorname{artanh}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e^{(bx+a)}-1\right)-\mathrm{Li}_2\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctanh(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) - l
og(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)*l
og(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(27) = 54$.
time = 0.37, size = 138, normalized size = 3.94

$$\frac{bx\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh(bx+a)+\sinh(bx+a)+1\right)-a\log\left(\cosh(bx+a)+\sinh(bx+a)-1\right)+(bx+a)\log\left(-\cosh(bx+a)-\sinh(bx+a)+1\right)+\mathrm{Li}_2(\cosh(bx+a)+\sinh(bx+a))-\mathrm{Li}_2(-\cosh(bx+a)-\sinh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
 + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b*x
+ a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(e^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(b*x+a)),x)
```

```
[Out] Integral(atanh(exp(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(exp(a + b*x)),x)
```

```
[Out] int(atanh(exp(a + b*x)), x)
```

3.350 $\int x \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=71

$$-\frac{x\text{PolyLog}(2,-e^{a+bx})}{2b}+\frac{x\text{PolyLog}(2,e^{a+bx})}{2b}+\frac{\text{PolyLog}(3,-e^{a+bx})}{2b^2}-\frac{\text{PolyLog}(3,e^{a+bx})}{2b^2}$$

```
[Out] -1/2*x*polylog(2,-exp(b*x+a))/b+1/2*x*polylog(2,exp(b*x+a))/b+1/2*polylog(3
,-exp(b*x+a))/b^2-1/2*polylog(3,exp(b*x+a))/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6348, 2611, 2320, 6724}

$$\frac{\text{Li}_3(-e^{a+bx})}{2b^2}-\frac{\text{Li}_3(e^{a+bx})}{2b^2}-\frac{x\text{Li}_2(-e^{a+bx})}{2b}+\frac{x\text{Li}_2(e^{a+bx})}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -1/2*(x*PolyLog[2, -E^(a + b*x)])/b + (x*PolyLog[2, E^(a + b*x)])/(2*b) + P
olyLog[3, -E^(a + b*x)]/(2*b^2) - PolyLog[3, E^(a + b*x)]/(2*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}\left(e^{a+bx}\right)\,dx &= -\left(\frac{1}{2}\int x\log\left(1-e^{a+bx}\right)\,dx\right)+\frac{1}{2}\int x\log\left(1+e^{a+bx}\right)\,dx\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\int \text{Li}_2\left(-e^{a+bx}\right)\,dx}{2b}-\frac{\int \text{Li}_2\left(e^{a+bx}\right)\,dx}{2b}\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}\,dx,x,e^{a+bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,d\right.}{2b^2}\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Li}_3\left(-e^{a+bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{a+bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 113, normalized size = 1.59

$$\frac{2b^2x^2\tanh^{-1}\left(e^{a+bx}\right)+b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\text{PolyLog}\left(2,-e^{a+bx}\right)+2bx\text{PolyLog}\left(2,e^{a+bx}\right)+2\text{PolyLog}\left(3,-e^{a+bx}\right)-2\text{PolyLog}\left(3,e^{a+bx}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^2*x^2*ArcTanh[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*Lo
g[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a
 + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(59) = 118.
time = 0.10, size = 178, normalized size = 2.51

method	result
risch	$\frac{\ln\left(1-e^{bx+a}\right)xa}{2b}+\frac{x\,\text{polylog}\left(2,e^{bx+a}\right)}{2b}+\frac{a^2\ln\left(1-e^{bx+a}\right)}{2b^2}+\frac{\text{polylog}\left(2,e^{bx+a}\right)a}{2b^2}+\frac{a\,\text{dilog}\left(e^{bx+a}\right)}{2b^2}-\frac{\text{polylog}\left(3,e^{bx+a}\right)}{2b^2}-\frac{x\,\text{po}}{}$
default	$\frac{x^2\,\text{arctanh}\left(e^{bx+a}\right)}{2}-\frac{a^2\,\text{arctanh}\left(e^{bx+a}\right)-\frac{(bx+a)^2\ln\left(1-e^{bx+a}\right)}{2}-(bx+a)\,\text{polylog}\left(2,e^{bx+a}\right)+\text{polylog}\left(3,e^{bx+a}\right)+\frac{(bx+a)^2\ln\left(e^{bx+a}+1\right)}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctanh(exp(b*x+a))-1/2/b^2*(a^2*arctanh(exp(b*x+a))-1/2*(b*x+a)^2*
ln(1-exp(b*x+a))-(b*x+a)*polylog(2,exp(b*x+a))+polylog(3,exp(b*x+a))+1/2*(b
*x+a)^2*ln(exp(b*x+a)+1)+(b*x+a)*polylog(2,-exp(b*x+a))-polylog(3,-exp(b*x+
a))+a*(b*x+a)*ln(1-exp(b*x+a))-a*(b*x+a)*ln(exp(b*x+a)+1)-a*polylog(2,-exp(
b*x+a))+a*polylog(2,exp(b*x+a)))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 1.52

$$\frac{1}{2}\,x^2\,\operatorname{artanh}\left(e^{(bx+a)}\right)-\frac{1}{4}\,b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3}-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b
*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(57) = 114.
time = 0.36, size = 199, normalized size = 2.80

$$\frac{b^2x^2\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-2\,bx\mathrm{Li}_2(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))+a^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+\left(b^2x^2-a^2\right)\log\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)+1\right)-2\,\mathrm{polylog}(3,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+2\,\mathrm{polylog}(3,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log(-c
osh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x +
 a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\operatorname{atanh}\left(e^{a}e^{bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(b*x+a)),x)
```

```
[Out] Integral(x*atanh(exp(a)*exp(b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(exp(a + b*x)),x)
```

```
[Out] int(x*atanh(exp(a + b*x)), x)
```

3.351 $\int x^2 \tanh^{-1}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=101

$$-\frac{x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}+\frac{x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{\text{PolyLog}(4,-}{b^3}$$

```
[Out] -1/2*x^2*polylog(2,-exp(b*x+a))/b+1/2*x^2*polylog(2,exp(b*x+a))/b+x*polylog
(3,-exp(b*x+a))/b^2-x*polylog(3,exp(b*x+a))/b^2-polylog(4,-exp(b*x+a))/b^3+
polylog(4,exp(b*x+a))/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6348, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-e^{a+bx}\right)}{b^3}+\frac{\text{Li}_4\left(e^{a+bx}\right)}{b^3}+\frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -1/2*(x^2*PolyLog[2, -E^(a + b*x)])/b + (x^2*PolyLog[2, E^(a + b*x)])/(2*b)
 + (x*PolyLog[3, -E^(a + b*x)])/b^2 - (x*PolyLog[3, E^(a + b*x)])/b^2 - Pol
yLog[4, -E^(a + b*x)]/b^3 + PolyLog[4, E^(a + b*x)]/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
```

```
0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}\left(e^{a+bx}\right) dx &= -\left(\frac{1}{2}\int x^2 \log\left(1-e^{a+bx}\right) dx\right)+\frac{1}{2}\int x^2 \log\left(1+e^{a+bx}\right) dx \\
&= -\frac{x^2 \text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2 \text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\int x \text{Li}_2\left(-e^{a+bx}\right) dx}{b}-\frac{\int x \text{Li}_2\left(e^{a+bx}\right) dx}{b} \\
&= -\frac{x^2 \text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2 \text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x \text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x \text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\int \text{Li}_3\left(-e^{a+bx}\right.}{b^2} \\
&= -\frac{x^2 \text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2 \text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x \text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x \text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_3(-}{x}\right.}{} \\
&= -\frac{x^2 \text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2 \text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x \text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x \text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\text{Li}_4\left(-e^{a+bx}\right)}{b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 149, normalized size = 1.48

$$
\frac{2b^3x^3\tanh^{-1}\left(e^{a+bx}\right)+b^3x^3\log\left(1-e^{a+bx}\right)-b^3x^3\log\left(1+e^{a+bx}\right)-3b^2x^2\text{PolyLog}\left(2,-e^{a+bx}\right)+3b^2x^2\text{PolyLog}\left(2,e^{a+bx}\right)+6bx\text{PolyLog}\left(3,-e^{a+bx}\right)-6bx\text{PolyLog}\left(3,e^{a+bx}\right)-6\text{PolyLog}\left(4,-e^{a+bx}\right)+6\text{PolyLog}\left(4,e^{a+bx}\right)}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^3*x^3*ArcTanh[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Lo
g[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog
[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a +
 b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(91) = 182$.

time = 0.11, size = 326, normalized size = 3.23

method	result
risch	$-\frac{\ln(1-e^{bx+a})x\,a^2}{2b^2}+\frac{x^2\,\mathrm{polylog}(2,e^{bx+a})}{2b}-\frac{a^3\ln(1-e^{bx+a})}{2b^3}-\frac{a^2\,\mathrm{dilog}(e^{bx+a})}{2b^3}-\frac{x\,\mathrm{polylog}(3,e^{bx+a})}{b^2}-\frac{\mathrm{polylog}(2,e^{bx+a})}{2b^3}$
default	$\frac{x^3\,\mathrm{arctanh}(e^{bx+a})}{3}-\frac{-\frac{(bx+a)^3\ln\left(1-e^{bx+a}\right)}{2}-\frac{3(bx+a)^2\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{2}+3(bx+a)\,\mathrm{polylog}(3,e^{bx+a})-3\,\mathrm{polylog}(4,e^{bx+a})+\frac{(bx+a)^3}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arctanh(exp(b*x+a))-1/3/b^3*(-1/2*(b*x+a)^3*ln(1-exp(b*x+a))-3/2*(b
*x+a)^2*polylog(2,exp(b*x+a))+3*(b*x+a)*polylog(3,exp(b*x+a))-3*polylog(4,e
xp(b*x+a))+1/2*(b*x+a)^3*ln(exp(b*x+a)+1)+3/2*(b*x+a)^2*polylog(2,-exp(b*x+
a))-3*(b*x+a)*polylog(3,-exp(b*x+a))+3*polylog(4,-exp(b*x+a))-a^3*arctanh(e
xp(b*x+a))+3/2*a*(b*x+a)^2*ln(1-exp(b*x+a))+3*a*(b*x+a)*polylog(2,exp(b*x+a
))-3*a*polylog(3,exp(b*x+a))-3/2*a*(b*x+a)^2*ln(exp(b*x+a)+1)-3*a*(b*x+a)*p
olylog(2,-exp(b*x+a))+3*a*polylog(3,-exp(b*x+a))-3/2*a^2*(b*x+a)*ln(1-exp(b
*x+a))-3/2*a^2*polylog(2,exp(b*x+a))+3/2*a^2*(b*x+a)*ln(exp(b*x+a)+1)+3/2*a
^2*polylog(2,-exp(b*x+a)))
```

Maxima [A]

time = 0.27, size = 142, normalized size = 1.41

$$\frac{1}{3}\,x^3\,\mathrm{artanh}\left(e^{(bx+a)}\right)-\frac{1}{6}\,b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2(-e^{(bx+a)})-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}-\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e
^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x
+ a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(89) = 178$.

time = 0.38, size = 248, normalized size = 2.46

$$\frac{b^3x^3\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+3\,b^2x^2\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-3\,b^2x^2\mathrm{Li}_2(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))-a^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-6\,bx\mathrm{polylog}(3,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+6\,bx\mathrm{polylog}(3,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))+(b^3x^3+a^3)\log\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)+1\right)+6\,\mathrm{polylog}(4,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-6\,\mathrm{polylog}(4,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^
2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - sinh(
b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*pol
ylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a) - sinh
(b*x + a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(e^a e^{bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(exp(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(exp(a + b*x)),x)
```

```
[Out] int(x^2*atanh(exp(a + b*x)), x)
```

3.352 $\int \tanh^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=168

$$-\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\text{PolyLog}\left(2,1-\frac{\quad}{1+a+}\right.}{2d\log(f)}$$

```
[Out] -arctanh(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arctanh(a+b*f^(d*x+
c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(1+
a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c
)))/d/ln(f)
```

Rubi [A]

time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 6246, 6057, 2449, 2352, 2497}

$$\frac{\text{Li}_2\left(1-\frac{2}{bf^{c+dx}+a+1}\right)}{2d\log(f)}-\frac{\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(1-a)(bf^{c+dx}+a+1)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTanh[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcTanh[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6246

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \tanh^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{\text{Subst}\left(\int \frac{\tanh^{-1}(a+bx)}{x}\,dx,x,f^{c+dx}\right)}{d\log(f)}$$

$$=\frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx,x,a+bf^{c+dx}\right)}{bd\log(f)}$$

$$=-\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf}\right.}{d\log(f)}$$

$$=-\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf}\right.}{d\log(f)}$$

$$=-\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf}\right.}{d\log(f)}$$

Mathematica [A]
time = 0.50, size = 108, normalized size = 0.64

$$\frac{dx\log(f)\left(2\tanh^{-1}\left(a+bf^{c+dx}\right)+\log\left(\frac{-1+a+bf^{c+dx}}{-1+a}\right)-\log\left(\frac{1+a+bf^{c+dx}}{1+a}\right)\right)+\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (d*x*Log[f]*(2*ArcTanh[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(-
1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c + d*
x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

Maple [A]
time = 0.31, size = 160, normalized size = 0.95

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arctanh}\left(a+b\,f^{dx+c}\right)-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}-\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arctanh}\left(a+b\,f^{dx+c}\right)-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}-\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+}{d\ln(f)}$
risch	$\frac{x\ln\left(1+a+b\,f^{dx+c}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{1+a+b\,f^{dx}\,f^{c}}{1+a}\right)}{2\ln(f)d}-\frac{\ln\left(\frac{1+a+b\,f^{dx}\,f^{c}}{1+a}\right)x}{2}-\frac{\ln\left(\frac{1+a+b\,f^{dx}\,f^{c}}{1+a}\right)c}{2d}+\frac{c\ln\left(1+a+b\,f^{dx}\,f^{c}\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arctanh(a+b*f^(d*x+c))-1/2*dilog((-b*f^(d*x+c)-
a-1)/(-a-1))-1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a-1)/(-a-1))+1/2*dilog((
1-a-b*f^(d*x+c))/(1-a))+1/2*ln(-b*f^(d*x+c))*ln((1-a-b*f^(d*x+c))/(1-a)))
```

Maxima [A]
time = 0.26, size = 202, normalized size = 1.20

$$\frac{(dx+c)\operatorname{artanh}\left(bf^{dx+c}+a\right)}{d}-\frac{(dx+c)b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log(f)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\operatorname{Li}_2\left(\frac{bf^{dx+c}+a+1}{a+1}\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)\log\left(-\frac{bf^{dx+c}+a-1}{a-1}+1\right)+\operatorname{Li}_2\left(\frac{bf^{dx+c}+a-1}{a-1}\right)}{b}\right)}{2\,d\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arctanh(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x + c
) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x + c)
 + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x + c)
```

```
+ a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c) + a
- 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log(f))
```

Fricas [A]
time = 0.36, size = 284, normalized size = 1.69

$$\frac{dx\log\left(f\right)\log\left(-\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}\right)+c\log\left(b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1\right)\log\left(f\right)-c\log\left(b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1\right)\log\left(f\right)-\left(dx+c\right)\log\left(f\right)\log\left(\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{a+1}\right)+\left(dx+c\right)\log\left(f\right)\log\left(\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}{a-1}\right)-\mathrm{Li}_2\left(-\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(-\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}{a-1}+1\right)}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*log(f)*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) - c
*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) -
(d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh(
(d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*
sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(a+bf^{c+dx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(atanh(a + b*f**(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.58Unable to divide, perhaps due to rou
nding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,
0,0]%%%}+
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*f^(c + d*x)),x)
[Out] int(atanh(a + b*f^(c + d*x)), x)
```

3.353 $\int x\tanh^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=211

$$-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+\frac{x\text{Poly}}{}$$

```
[Out] -1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*x^2*ln(1+a+b*f^(d*x+c))+1/4*x^2*ln(1-b*f^(
d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))+1/2*x*polylog(2,b*f^(d*x+c)/(
1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*polylog(3,b*f
^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)^2
```

Rubi [A]
time = 0.11, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6348, 2612, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{4}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{4}x^2\log\left(a+bf^{c+dx}+1\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -1/4*(x^2*Log[1 - a - b*f^(c + d*x)]) + (x^2*Log[1 + a + b*f^(c + d*x)])/4
+ (x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(
1 + a)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x*Poly
Log[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b*f^(c + d*x
))/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 + a))]/(2*d^
2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}\left(a+bf^{c+dx}\right)\,dx=-\left(\frac{1}{2}\int x\log\left(1-a-bf^{c+dx}\right)\,dx\right)+\frac{1}{2}\int x\log\left(1+a+bf^{c+dx}\right)\,dx$$

$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+d}}{1-a}\right.$$

$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+d}}{1-a}\right.$$

$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+d}}{1-a}\right.$$

$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+d}}{1-a}\right.$$

Mathematica [A]
time = 0.08, size = 177, normalized size = 0.84

$$\frac{2d^2x^2\tanh^{-1}\left(a+bf^{c+dx}\right)\log^2(f)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)-2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{-1+a}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{4d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^2*x^2*ArcTanh[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*Pol
yLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-1 +
a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(195) = 390$.
time = 0.06, size = 596, normalized size = 2.82

method	result
risch	$\frac{x^2 \ln(1+a+b\,f^{dx+c})}{4} - \frac{x^2 \ln(1-a-b\,f^{dx+c})}{4} + \frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{1-a}\right)x^2}{4} + \frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{1-a}\right)xc}{2d} + \frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{1-a}\right)c^2}{4d^2} + \frac{\mathrm{polylog}\left(2, \frac{b\,f}{}\right.}{2\ln(f)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*ln(1-b*f^(d*x)*
f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(d*x)*f^c/(1-a))*x*c+1/4/d^2*ln(1-b*f^(d*x)*f
^c/(1-a))*c^2+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*po
lylog(2,b*f^(d*x)*f^c/(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a
))+1/4/d^2*c^2*ln(1-a-b*f^(d*x)*f^c)-1/2/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a
-1)/(-1+a))-1/2/d*c*ln((b*f^(d*x)*f^c+a-1)/(-1+a))*x-1/2/d^2*c^2*ln((b*f^(d
*x)*f^c+a-1)/(-1+a))-1/4*ln(1-b*f^(d*x)*f^c/(-a-1))*x^2-1/2/d*ln(1-b*f^(d*x
)*f^c/(-a-1))*x*c-1/4/d^2*ln(1-b*f^(d*x)*f^c/(-a-1))*c^2-1/2/ln(f)/d*polylo
g(2,b*f^(d*x)*f^c/(-a-1))*x-1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/(-a-1))*c
+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(-a-1))-1/4/d^2*c^2*ln(1+a+b*f^(d*
x)*f^c)+1/2/ln(f)/d^2*c*dilog((1+a+b*f^(d*x)*f^c)/(1+a))+1/2/d*c*ln((1+a+b*
f^(d*x)*f^c)/(1+a))*x+1/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))
```

Maxima [A]
time = 0.30, size = 194, normalized size = 0.92

$$-\frac{1}{4}\,bd\left(\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f\right)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f\right)-2\,\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})}{bd^3\log\left(f\right)^3}-\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log\left(f\right)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a-1}\right)\log\left(f\right)-2\,\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a-1})}{bd^3\log\left(f\right)^3}\right)\log\left(f\right)+\frac{1}{2}\,x^2\,\mathrm{artanh}\left(bf^{dx+c}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-b
*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d^3
*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*dilog
```

```
(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))/(b*
d^3*log(f)^3))*log(f) + 1/2*x^2*arctanh(b*f^(d*x + c) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(188) = 376$.
time = 0.40, size = 396, normalized size = 1.88

$$\frac{d^2x^2\log(f)^2\log\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)-c^2\log(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1)\log(f)^2+c^2\log(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1)\log(f)^2-2dx\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}+1\right)\log(f)+2dx\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}+1\right)\log(f)-(d^2x^2-c^2)\log(f)^2\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}\right)+(d^2x^2-c^2)\log(f)^2\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}\right)+2\,\mathrm{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a+1}\right)-2\,\mathrm{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a-1}\right)}{4d^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^2*x^2*log(f)^2*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*lo
g(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a -
1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 - c^2)*l
og(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)/(
a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d*x + c)*lo
g(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(a+bf^cf^{dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x*atanh(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(b*f^(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atanh}\left(a + b f^{c+dx}\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a + b*f^(c + d*x)),x)
```

```
[Out] int(x*atanh(a + b*f^(c + d*x)), x)
```

3.354 $\int x^2 \tanh^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=264

$$-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+\frac{x^2\mathrm{P}}{}$$

```
[Out] -1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*x^3*ln(1+a+b*f^(d*x+c))+1/6*x^3*ln(1-b*f^(
d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))+1/2*x^2*polylog(2,b*f^(d*x+c)
/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*polylog(3,b
*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)^2+p
olylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/(1+a))/d^3/l
n(f)^3
```

Rubi [A]
time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6348, 2612, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{Li}_4\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}-\frac{x\text{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{x^2\text{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x^2\text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{6}x^3\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{6}x^3\log\left(a+bf^{c+dx}+1\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(\frac{bf^{c+dx}}{a+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -1/6*(x^3*Log[1 - a - b*f^(c + d*x)]) + (x^3*Log[1 + a + b*f^(c + d*x)])/6
+ (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(
1 + a)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*
PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(
c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a
))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) -
PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6348

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}\left(a+bf^{c+dx}\right) dx = -\left(\frac{1}{2}\int x^2 \log\left(1-a-bf^{c+dx}\right) dx\right) + \frac{1}{2}\int x^2 \log\left(1+a+bf^{c+dx}\right) dx$$

$$= -\frac{1}{6}x^3 \log\left(1-a-bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1-\frac{bf^{c+}}{1-}\right.$$

$$= -\frac{1}{6}x^3 \log\left(1-a-bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1-\frac{bf^{c+}}{1-}\right.$$

$$= -\frac{1}{6}x^3 \log\left(1-a-bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1-\frac{bf^{c+}}{1-}\right.$$

$$= -\frac{1}{6}x^3 \log\left(1-a-bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1-\frac{bf^{c+}}{1-}\right.$$

$$= -\frac{1}{6}x^3 \log\left(1-a-bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3 \log\left(1-\frac{bf^{c+}}{1-}\right.$$

<u>Mathematica</u> [A]
time = 0.06, size = 235, normalized size = 0.89

$$\frac{2d^3x^3\tanh^{-1}\left(a+bf^{c+dx}\right)\log^3(f)+d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)-6dx\log(f)\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{-1+a}\right)+6dx\log(f)\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)+6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{-1+a}\right)-6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{1+a}\right)}{6d^3\log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^3*x^3*ArcTanh[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x^2*
Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[3, -
((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(1
+ a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((b*f^(c
 + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

<u>Maple</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(252) = 504$.
time = 0.07, size = 672, normalized size = 2.55

method	result
risch	$\frac{x^3\ln\left(1+a+b\,f^{dx+c}\right)}{6}-\frac{x^3\ln\left(1-a-b\,f^{dx+c}\right)}{6}+\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)x^3}{6}-\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)x\,c^2}{2d^2}-\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)c^3}{3d^3}+\frac{\text{polylog}\left(2\right.}{2\,\text{l}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*ln(1-b*f^(d*x)*
f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(d*
x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(f)/
d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^
c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln(1-a-
b*f^(d*x)*f^c)+1/2/ln(f)/d^3*c^2*dilog((b*f^(d*x)*f^c+a-1)/(-1+a))+1/2/d^2*
c^2*ln((b*f^(d*x)*f^c+a-1)/(-1+a))*x+1/2/d^3*c^3*ln((b*f^(d*x)*f^c+a-1)/(-1
+a))-1/6*ln(1-b*f^(d*x)*f^c/(-a-1))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-a-1))*
x*c^2+1/3/d^3*ln(1-b*f^(d*x)*f^c/(-a-1))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x
)*f^c/(-a-1))*x^2+1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-a-1))*c^2+1/ln(f)
^2/d^2*polylog(3,b*f^(d*x)*f^c/(-a-1))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*
f^c/(-a-1))+1/6/d^3*c^3*ln(1+a+b*f^(d*x)*f^c)-1/2/ln(f)/d^3*c^2*dilog((1+a+
b*f^(d*x)*f^c)/(1+a))-1/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x-1/2/d^3*c
^3*ln((1+a+b*f^(d*x)*f^c)/(1+a))
```

Maxima [A]
time = 0.29, size = 254, normalized size = 0.96

$$\frac{1}{3}\,x^3\,\mathrm{artanh}\left(bf^{dx+c}+a\right)-\frac{1}{6}\,bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f\right)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})+6\,\mathrm{Li}_4(-\frac{bf^{dx}f^c}{a+1})}{bd^4\log\left(f\right)^4}-\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log\left(f\right)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a-1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a-1})+6\,\mathrm{Li}_4(-\frac{bf^{dx}f^c}{a-1})}{bd^4\log\left(f\right)^4}\right)\log\left(f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(a
 + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 - 6*
d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*f^c
/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f
)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*polyl
og(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^
4*log(f)^4))*log(f)
```

Fricas [A]
time = 0.44, size = 480, normalized size = 1.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(d^3*x^3*log(f)^3*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
```

```
) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*cosh(
(d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*log(b*
cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 - (d^3*
x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)
) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*polylog(3, -(
b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6*d*x*log(f
)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)
) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a
+ 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))
/(a - 1)))/(d^3*log(f)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(a + bf^c f^{dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a+b*f**(d*x+c)),x)
[Out] Integral(x**2*atanh(a + b*f**c*f**(d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*arctanh(b*f^(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}\left(a + b\, f^{c+d\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a + b*f^(c + d*x)),x)
[Out] int(x^2*atanh(a + b*f^(c + d*x)), x)
```

3.355 $\int e^{c(a+bx)}\tanh^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a+}\right.}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(sinh(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {2225, 6410, 2320, 12, 1261, 646, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\sinh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\sinh(x))\,dx,x,ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\cosh(x)}{1-\sinh^2(x)}\,dx,x,ac+bcx\right.}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right.}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\right.}{2bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac}\right.}{2bc}$$

Mathematica [A]
time = 0.10, size = 153, normalized size = 1.43

$$\frac{-2e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+\log\left(1-2e^{c(a+bx)}-e^{2c(a+bx)}\right)+\log\left(1+2e^{c(a+bx)}-e^{2c(a+bx)}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (-2*E^(c*(a + b*x))*ArcTanh[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2*
Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 + E^
(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))] +
Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 868, normalized size = 8.11

method	result	size
risch	Expression too large to display	868

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/2*I/c/b*Pi
*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*
x+a))+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*ex
p(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*
I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*e
xp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*c
sgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b
*x+a))+1))*exp(c*(b*x+a))-1/2*I/c/b*exp(c*(b*x+a))*Pi+1/4*I/c/b*Pi*csgn(I*(
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+
a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a
)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*
(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x
+a))-1))^3*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*
x+a))-1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+
2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*cs
gn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a)
)-1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/c/b*ln
(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/c/b*ln(exp(2*c*(b*x+a))-(2^(1/
2)-1)^2)*2^(1/2)-2/b*a+1/2/c/b*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/c/b*l
n(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

time = 0.48, size = 184, normalized size = 1.72

$$\frac{\operatorname{artanh}\left(\sinh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}+1}{\sqrt{2}+e^{\left(bcx+ac\right)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}-1}{\sqrt{2}+e^{\left(bcx+ac\right)}+1}\right)}{2\,bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+2\,e^{\left(bcx+ac\right)}-1\right)}{2\,bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}-2\,e^{\left(bcx+ac\right)}-1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(90) = 180$.
time = 0.42, size = 234, normalized size = 2.19

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}+3\right)\sinh(bcx+ac)^2-2\sqrt{2}-3}{\cosh(bcx+ac)^2+\sinh(bcx+ac)^2-3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^2+\sinh(bcx+ac)^2-3\right)}{\cosh(bcx+ac)^2-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^
2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) +
3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
 a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh
(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)
^2)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atanh}\left(\sinh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sinh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(sinh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.53, size = 157, normalized size = 1.47

$$\frac{e^{\left(\left(bx+a\right)c\right)}\log\left(-\frac{e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}+2}{e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}-2}\right)}{2\,bc}+\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{\left(2\,bcx+2\,ac\right)}-6\right|}{\left|4\sqrt{2}+2\,e^{\left(2\,bcx+2\,ac\right)}-6\right|}\right)+\log\left(\left|e^{\left(4\,bcx+4\,ac\right)}-6\,e^{\left(2\,bcx+2\,ac\right)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2) +
2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) + lo
g(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B]

time = 1.59, size = 179, normalized size = 1.67

$$\frac{e^{ac+bcx}\,\ln\left(\frac{e^{bcx}\,e^{ac}}{2}-\frac{e^{-bcx}\,e^{-ac}}{2}+1\right)}{2\,b\,c}-\frac{e^{ac+bcx}\,\ln\left(\frac{e^{-bcx}\,e^{-ac}}{2}-\frac{e^{bcx}\,e^{ac}}{2}+1\right)}{2\,b\,c}+\frac{\ln\left(6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(sinh(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2 +
 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x
)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2)
 - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*
c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.356 $\int e^{c(a+bx)} \tanh^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 6410, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tanh^{-1}(\cosh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\cosh(x))\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{csch}(x)\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.22

$$\frac{e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.27, size = 887, normalized size = 18.10

method	result	size
risch	Expression too large to display	887

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a
))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*
```

```
I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/4*
I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/4*
I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a)
)+1)^2)^2*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*
(b*x+a))+1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c
*(b*x+a))+1)^2)^2*exp(c*(b*x+a))+1/2*I/c/b*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c
*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1))*cs
gn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x
+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/c
/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*exp
(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b
*x+a))-1)^2)*exp(c*(b*x+a))+1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*
(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*exp(-c*(b*x+a)))
*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*
exp(c*(b*x+a))-1/2*I/c/b*exp(c*(b*x+a))*Pi+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+
a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(
exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))-1/c/b*exp(
c*(b*x+a))*ln(exp(c*(b*x+a))-1)-2/b*a+1/c/b*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A]
time = 0.26, size = 64, normalized size = 1.31

$$\frac{\operatorname{artanh}\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A]
time = 0.39, size = 93, normalized size = 1.90

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(47) = 94.
time = 0.46, size = 147, normalized size = 3.00

$$\frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2\,bcx+2\,ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{2\,e^{(bcx+ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{1}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}\right)+2\,e^{(-ac)}\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x +
 a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1
) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(abs(e
^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

Mupad [B]
time = 1.52, size = 111, normalized size = 2.27

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(1-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}\right)}{2\,b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(cosh(a*c + b*c*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp(
-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*exp(
a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*c)
```

3.357 $\int e^{c(a+bx)} \tanh^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=45

$$-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(tanh(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2225, 6410}

$$\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Tanh[c*(a + b*x)]])/(b*
c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tanh^{-1}(\tanh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\ &= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(-1+\tanh^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [A]
time = 0.16, size = 68, normalized size = 1.51

method	result
default	$\frac{e^{bcx+ac}(bcx+ac)-e^{bcx+ac}+e^{bcx+ac}(\mathrm{arctanh}(\tanh(bcx+ac))-bcx-ac)}{cb}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{cb}-\frac{i\left(\pi\,\mathrm{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2c(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right)-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\mathrm{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right)^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*(exp(b*c*x+a*c)*(b*c*x+a*c)-exp(b*c*x+a*c)+exp(b*c*x+a*c)*(arctanh(ta
nh(b*c*x+a*c))-b*c*x-a*c))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.96

$$\frac{\mathrm{artanh}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A]
time = 0.33, size = 46, normalized size = 1.02

$$\frac{(bcx+ac-1)\cosh\left(bcx+ac\right)+(bcx+ac-1)\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((b*c*x + a*c - 1)*cosh(b*c*x + a*c) + (b*c*x + a*c - 1)*sinh(b*c*x + a*c))
/(b*c)
```

Sympy [A]
time = 0.78, size = 58, normalized size = 1.29

$$\begin{cases} 0 & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \\ xe^{ac} \operatorname{atanh}(\tanh(ac)) & \text{for } b = 0 \\ \frac{e^{ac}e^{bcx} \operatorname{atanh}(\tanh(ac+bcx))}{bc} - \frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(tanh(b*c*x+a*c)),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)*atanh(tanh(a*c
)), Eq(b, 0)), (exp(a*c)*exp(b*c*x)*atanh(tanh(a*c + b*c*x))/(b*c) - exp(a*
c)*exp(b*c*x)/(b*c), True))
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.78

$$\frac{(b^2c^2x + abc^2 - bc)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

Mupad [B]
time = 0.16, size = 28, normalized size = 0.62

$$\frac{e^{a\,c+b\,c\,x}\,(\operatorname{atanh}(\tanh(a\,c+b\,c\,x))-1)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(tanh(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*(atanh(tanh(a*c + b*c*x)) - 1))/(b*c)
```

3.358 $\int e^{c(a+bx)} \tanh^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=45

$$-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(coth(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2225, 6410}

$$\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Coth[c*(a + b*x)]])/(b*
c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tanh^{-1}(\coth(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\
&= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(-1+\tanh^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 349, normalized size = 7.76

method	result
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{cb}-\frac{i\left(\pi \mathrm{csgn}\left(ie^{c(bx+a)}\right)^{2}\mathrm{csgn}\left(ie^{2c(bx+a)}\right)-2\pi\,\mathrm{csgn}\left(ie^{c(bx+a)}\right)\mathrm{csgn}\left(ie^{2c(bx+a)}\right)^{2}+\pi \mathrm{csgn}\left(ie^{2c(bx+a)}\right)^{3}+\pi\,\mathrm{csgn}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a)))-1/4*I*(Pi*csgn(I*exp(c*(b*x+a)))^2*
csgn(I*exp(2*c*(b*x+a)))-2*Pi*csgn(I*exp(c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a)
))^2+Pi*csgn(I*exp(2*c*(b*x+a)))^3+Pi*csgn(I*exp(2*c*(b*x+a)))*csgn(I/(exp(
2*c*(b*x+a))-1))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))-Pi*csgn(I*ex
p(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^2+2*Pi*csgn(I
/(exp(2*c*(b*x+a))-1))^3-2*Pi*csgn(I/(exp(2*c*(b*x+a))-1))^2-Pi*csgn(I/(exp
(2*c*(b*x+a))-1))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^2+Pi*csgn(I
*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^3-4*I+2*Pi)/c/b*exp(c*(b*x+a))
```

Maxima [A]
time = 0.26, size = 43, normalized size = 0.96

$$\frac{\operatorname{artanh}\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{e^{\left(bcx+ac\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.56

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atanh}\left(\coth\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(coth(a*c + b*c*x)), x)
```

Giac [A]
time = 0.39, size = 40, normalized size = 0.89

$$\frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right)-2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.07, size = 28, normalized size = 0.62

$$\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\left(\mathrm{atanh}\left(\mathrm{coth}\left(a\,c+b\,c\,x\right)\right)-1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(coth(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*(atanh(coth(a*c + b*c*x)) - 1))/(b*c)
```

3.359 $\int e^{c(a+bx)} \tanh^{-1}(\text{sech}(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{e^{ac+bcx}\tanh^{-1}(\text{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

[Out] exp(b*c*x+a*c)*arctanh(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 6410, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]

[Out] (E^(a*c + b*c*x)*ArcTanh[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*x))]/(b*c)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tanh^{-1}(\operatorname{sech}(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^x\tanh^{-1}(\operatorname{sech}(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int e^x\operatorname{csch}(x)\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{2\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.20

$$\frac{e^{c(a+bx)}\tanh^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 872, normalized size = 17.80

method	result	size
risch	Expression too large to display	872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(exp(c*(b*x+a))-1)^2/(exp
(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2
```

```
)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a)
)+1))*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(
c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*
(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/2*I/c/b
*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a)
)-1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*c
sgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x
+a))-1)^2/(exp(2*c*(b*x+a))+1))*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(
b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I/(ex
p(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(
c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))
+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*
x+a))-1)^2)^3*exp(c*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn
(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/2*I/c/b*Pi*csgn(I*(exp(c*(b*x+a))
-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(exp
(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/c/b*exp(c*(b*x+a)
)*ln(exp(c*(b*x+a))-1)+1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-2/b*a+1/c/
b*ln(exp(2*c*(b*x+a))-1)
```

Maxima [**A**]
time = 0.28, size = 64, normalized size = 1.31

$$\frac{\operatorname{artanh}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctanh(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [**A**]
time = 0.34, size = 92, normalized size = 1.88

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\cosh\left(bcx+ac\right)+1}{\cosh\left(bcx+ac\right)-1}\right)+2\,\log\left(\frac{2\,\sinh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="fricas")
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(c
osh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))))/(b*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 0.49, size = 98, normalized size = 2.00

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a*c
) - 1))/(b*c)
```

Mupad [B]
time = 1.71, size = 119, normalized size = 2.43

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c}-\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{2\,b\,c}+\frac{\ln\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}+1\right)\,\mathrm{e}^{a\,c+b\,c\,x}}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(b*
c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c
)
```

3.360 $\int e^{c(a+bx)} \tanh^{-1}(\text{csch}(ac + bcx))\,dx$

Optimal. Leaf size=107

$$\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a}\right.}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(csch(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 6410, 2320, 12, 1261, 646, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6410

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\operatorname{csch}(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{e^x\coth(x)\operatorname{csch}(x)}{1-\operatorname{csch}^2(x)}\,dx, x, ac \ldots\right.}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x(1+x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x(1+x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1+x}{1-6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,d \ldots\right.}{2bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+}\ldots\right.}{2bc}$$

Mathematica [A]
time = 0.09, size = 150, normalized size = 1.40

$$\frac{-2\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+2e^{c(a+bx)}\tanh^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\log\left(1-2e^{c(a+bx)}-e^{2c(a+bx)}\right)+\log\left(1+2e^{c(a+bx)}-e^{2c(a+bx)}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
+ E^(c*(a + b*x)))/Sqrt[2]] + 2*E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))
/(-1 + E^(2*c*(a + b*x)))] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 842, normalized size = 7.87

method	result
risch	$\frac{e^{c(bx+a)}\ln\left(e^{2c(bx+a)}+2\,e^{c(bx+a)}-1\right)}{2cb}+\frac{i\pi\,\operatorname{csgn}\left(i\left(-e^{2c(bx+a)}+2\,e^{c(bx+a)}+1\right)\right)\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)\operatorname{csgn}\left(\frac{i\left(-e^{2c(bx+a)}+2\,e^{c(bx+a)}\right.}{e^{2c(bx+a)}-1}\right.}{4cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/c/b*Pi
*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1)
)*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c
*(b*x+a))+1/4*I/c/b*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(
I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x
+a))-1/4*I/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*
exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(
I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a)
)+1/4*I/c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/c/b*Pi*csgn(I*(-
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))
+1/4*I/c/b*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c
*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/
c/b*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3
*exp(c*(b*x+a))-1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))
-1)+1/2/c/b*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/c/b*ln(exp(2*c*(
b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2/b*a+1/2/c/b*ln(exp(2*c*(b*x+a))-(1+2^(1/2)
)^2)+1/2/c/b*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.

time = 0.47, size = 184, normalized size = 1.72

$$\frac{\operatorname{artanh}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(90) = 180$.

time = 0.38, size = 233, normalized size = 2.18

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\,\log\left(\frac{3\left(2\,\sqrt{2}+3\right)\cosh(bcx+ac)^{2}-4\left(3\,\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\,\sqrt{2}+3\right)\sinh(bcx+ac)^{2}-2\,\sqrt{2}-3}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3\right)}{\cosh(bcx+ac)^{2}-2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(s
inh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2
 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3
)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(
b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^
2)))/(b*c)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atanh}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(csch(a*c + b*c*x)), x)
```

Giac [A]

time = 0.53, size = 167, normalized size = 1.56

$$\frac{e^{\left((bx+a)c\right)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sq
rt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) -
6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B]
time = 1.62, size = 187, normalized size = 1.75

$$\frac{\ln\left(6\sqrt{2}\,e^{2c(a+bx)}-2\sqrt{2}-8\,e^{2c(a+bx)}\right)\left(\sqrt{2}+1\right)}{2bc}-\frac{e^{ac+bcx}\,\ln\left(1-\frac{1}{\frac{e^{bcx}\,e^{ac}}{2}-\frac{e^{-bcx}\,e^{-ac}}{2}}\right)}{2bc}-\frac{\ln\left(2\sqrt{2}-8\,e^{2c(a+bx)}-6\sqrt{2}\,e^{2c(a+bx)}\right)\left(\sqrt{2}-1\right)}{2bc}+\frac{\ln\left(\frac{1}{\frac{e^{bcx}\,e^{ac}}{2}-\frac{e^{-bcx}\,e^{-ac}}{2}}+1\right)e^{ac+bcx}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(1
/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2 -
(exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)
) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp(b*c
*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c)
```

3.361 $\int \frac{(a+b\tanh^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal. Leaf size=136

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{bd\text{PolyLog}(2,-cx^n)}{2n}-\frac{be\log(fx^m)\text{PolyLog}(2,-cx^n)}{2n}+\frac{bd\text{PolyLog}(2,cx^n)}{2n}+\frac{be\log$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*b*d*polylog(2,-c*x^n)/n-1/2*b*e*ln(f*x^
m)*polylog(2,-c*x^n)/n+1/2*b*d*polylog(2,c*x^n)/n+1/2*b*e*ln(f*x^m)*polylog
(2,c*x^n)/n+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)/n^2
```

Rubi [A]
time = 0.39, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 6874, 6035, 6031, 6218, 6216, 2421, 6724}

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{bd\text{Li}_2(-cx^n)}{2n}+\frac{bd\text{Li}_2(cx^n)}{2n}-\frac{be\text{Li}_2(-cx^n)\log(fx^m)}{2n}+\frac{be\text{Li}_2(cx^n)\log(fx^m)}{2n}+\frac{bem\text{Li}_3(-cx^n)}{2n^2}-\frac{bem\text{Li}_3(cx^n)}{2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - (b*d*PolyLog[2, -(c*x^n)])/(2*n) -
(b*e*Log[f*x^m]*PolyLog[2, -(c*x^n)])/(2*n) + (b*d*PolyLog[2, c*x^n])/(2*n)
 + (b*e*Log[f*x^m]*PolyLog[2, c*x^n])/(2*n) + (b*e*m*PolyLog[3, -(c*x^n)])/
(2*n^2) - (b*e*m*PolyLog[3, c*x^n])/(2*n^2)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6031

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6035

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6216

```
Int[(ArcTanh[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[Log[d*x^m]*(Log[1 + c*x^n]/x), x], x] - Dist[1/2, Int[Log[d*x^m
]*(Log[1 - c*x^n]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 6218

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTanh[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_),
x_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*Arc
Tanh[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}\left(cx^n\right)\right)}{x}+\frac{e\left(a+b\tanh^{-1}\left(cx^n\right)\right)\log\left(fx^{\cdots}\right.}{x}\right.$$

$$= d\int \frac{a+b\tanh^{-1}\left(cx^n\right)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)\log\left(f\cdots\right.}{x}$$

$$= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx + (be)\int \frac{\tanh^{-1}\left(cx^n\right)\log\left(fx^m\right)}{x}\,dx + \cdots$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\text{Li}_2\left(-cx^n\right)}{2n} + \frac{bd\text{Li}_2\left(cx^n\right)}{2n} - \frac{1\cdots}{2\cdots}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\text{Li}_2\left(-cx^n\right)}{2n} - \frac{be\log\left(fx^m\right)\text{Li}_{\cdots}}{2n}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\text{Li}_2\left(-cx^n\right)}{2n} - \frac{be\log\left(fx^m\right)\text{Li}_{\cdots}}{2n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.19, size = 114, normalized size = 0.84

$$-\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}+\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)(d+e\log(fx^m))}{n}+\frac{1}{2}a\log(x)(2d-em\log(x)+2e\log(fx^m))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*x
^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x
^(2*n)]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log[f*x^m
]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 668, normalized size = 4.91

method	result
risch	$-\frac{\ln(f)\,\mathrm{dilog}(c\,x^n+1)be}{2n}+\frac{i\pi\ln(x^n)ae\,\mathrm{csgn}(ix^m)\mathrm{csgn}(if\,x^m)^2}{2n}-\frac{i\pi\,\mathrm{dilog}(c\,x^n+1)be\,\mathrm{csgn}(if)\mathrm{csgn}(if\,x^m)^2}{4n}-\frac{i\pi\,\mathrm{dilog}(c\,x^n+1)be}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*ln(f)*dilog(c*x^n+1)*b*e+1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csg
n(I*x^m)*csgn(I*f*x^m)-1/4*I/n*Pi*dilog(1-c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*
csgn(I*f*x^m)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/
2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)/n^2-1/2*e*b/n*dilo
g(c*x^n+1)*ln(x^m)-1/2*e*b/n*dilog(c*x^n)*ln(x^m)+1/2/n*ln(f)*dilog(1-c*x^n
)*b*e+1/n*ln(f)*ln(x^n)*a*e+1/2/n*dilog(1-c*x^n)*b*d+1/n*ln(x^n)*a*d-1/2/n*
dilog(c*x^n+1)*b*d+1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/4*I/n*Pi*dilog
(c*x^n+1)*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f)*cs
gn(I*f*x^m)^2-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4
*I/n*Pi*dilog(1-c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/n*Pi*ln(x^n)*a*e
*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4*I/n*Pi*dilog(1-c*x^n)*b*e*csgn(I*x^m)*csgn
(I*f*x^m)^2+1/2*e*a/m*ln(x^m)^2+1/2*e*b*m/n*ln(x)*polylog(2,c*x^n)-1/2*e*b/
n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/2*e*b*m/n*
ln(x)*polylog(2,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)-1/2*I/n*Pi*ln(x^n)
*a*e*csgn(I*f*x^m)^3+1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f*x^m)^3-1/4*I/n*
Pi*dilog(1-c*x^n)*b*e*csgn(I*f*x^m)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*m*e*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*log(c*x^n + 1) + 1/4*(b*m*e*log(x)^2
- 2*b*e*log(x)*log(x^m) - 2*(b*e*log(f) + b*d)*log(x))*log(-c*x^n + 1) + in
tegrate(1/2*(2*b*c*n*e^(n*log(x) + 1)*log(x)*log(x^m) - (b*c*m*n*e*log(x)^2
 - 2*(b*c*n*e*log(f) + b*c*d*n)*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(123) = 246$.
time = 0.35, size = 461, normalized size = 3.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*m*n^2*cosh(1) + a*m*n^2*sinh(1))*log(x)^2 + 2*(b*d*n + (b*n*cosh(
1) + b*n*sinh(1))*log(f) + (b*m*n*cosh(1) + b*m*n*sinh(1))*log(x))*dilog(c*
cosh(n*log(x)) + c*sinh(n*log(x))) - 2*(b*d*n + (b*n*cosh(1) + b*n*sinh(1))
*log(f) + (b*m*n*cosh(1) + b*m*n*sinh(1))*log(x))*dilog(-c*cosh(n*log(x)) -
 c*sinh(n*log(x))) - ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 + 2*(b*d
*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log(c*cosh(n*log(x))
 + c*sinh(n*log(x)) + 1) + ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 +
2*(b*d*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log(-c*cosh(n*
log(x)) - c*sinh(n*log(x)) + 1) + 4*(a*d*n^2 + (a*n^2*cosh(1) + a*n^2*sinh(
1))*log(f))*log(x) + ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 + 2*(b*d
*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log(-(c*cosh(n*log(x
)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)) - 2*(
b*m*cosh(1) + b*m*sinh(1))*polylog(3, c*cosh(n*log(x)) + c*sinh(n*log(x)))
+ 2*(b*m*cosh(1) + b*m*sinh(1))*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log
(x))))/n^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}(c\,x^n))\,\left(d + e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x,x)
[Out] int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).___name___ )
        return False



def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```